Total No. of printed pages = 4

MA 181202

Roll No. of candidate

2024

B.Tech. 2nd Semester End-Term Examination

MATHEMATICS

New Regulation (w.e.f. 2017-18) &
New Syllabus (w.e.f. 2018-19)

Full Marks – 70 | Time – Three hours

The figures in the margin indicate full marks
for the questions.

Answer question No. 1 and any *four* from the rest.

1. Select the correct answer : (10 × 1 = 10)

(i) The divergence of $(3x^2i+5xy^2j+xyz^3k)$ at the point (1, 2, 3) is

(a) 20 (b) 40

(c) 80 (d) None of these

(ii) The unit normal vector to the surface $x^2y+2xz=4$ at the point (2, –2, 3) is

(a) $\frac{1}{3}(-i+2j+2k)$ (b) $\frac{1}{3}(i-2j+2k)$

(c) $\frac{1}{3}(i+2j-2k)$ (d) none of these

(iii) The general solution of the equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=0$ is

(a) $y=Ae^{-x}+Be^{-2x}$ (b) $y=Ae^{x}+Be^{2x}$

(c) $y=e^{-x}+e^{-2x}$ (d) $y=(A+Bx)e^{x}$

(iv) The integrating factor of the differential equation $(1+y^2)dx=(\tan^{-1}y-x)dy$ is

(a) $e^{\tan^{-1}x}$ (b) $e^{\tan^{-1}y}$

(c) $\tan^{-1}x$ (d) $\tan^{-1}y$

[Turn over

(v) For the differential equation $(x^2-2x)\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=0$, the point $x=0$ is

(a) ordinary (b) regular singular

(c) irregular singular (d) none of these

(vi) For the Legendre polynomial $P_n(x)$, $P_n(-1)$ is

(a) 1 (b) -1

(c) $(-1)^n$ (d) none of these

(vii) The real part of $f(z)=\cos z$ is

(a) $\cosh x \cos xy$ (b) $\cos x \cosh y$

(c) $\cos x \cosh x$ (d) none of these

(viii) For the function $f(z)=\frac{\cos z}{z}$, $z=0$ is a

u = cos x cos y
− sin x s

(a) pole

(b) removable singularity

(c) essential singularity

(d) none of these

(ix) The image of the circle $|z|=2$ under the mapping $w=z+(3+2i)$ is a

(a) circle (b) ellipse

(c) pair of lines (d) hyperbola

(x) Which of the following functions is analytic?

(a) $z+\bar{z}$ (b) $2z^2+2\bar{z}+c$

(c) $|z|^2$ (d) z^2+2z+c

5

2. (a) Find the directional derivative of the function $\phi=xy^2+yz^3$ at the point (2, −1, 1) in the direction of the normal to the surface $x\log z-y^2+4=0$ at (−1, z, 1). (5)

(b) With the help of Green's theorem evaluate the line integral $\int_C \vec{A}.dc$ where $\vec{A}=(x^2+xy^2)i+(y^2+x^2y)j$ and C is the boundary of the region bounded by $y=x$ and $y=x^2$. (5)

(c) Evaluate the surface integral of $\vec{A}=zi+xj-3y^2zk$ over the surface of the cylinder $x^2+y^2=16$ included in the first octant between $z=0$ and $z=5$. (5)

(a) Solve the differential equation $(D-2)^2 y = 8(e^{2x} + \sin 2x)$. (6)

(b) Solve any THREE from the following : $(3 \times 3 = 9)$

(i) $(x^4 + y^4)dx - xy^3 dy = 0$

(ii) $P = \sin(y - px)$

(iii) $(1+x^2)\frac{dy}{dx} + y = e\tan^{-1} x$

$\cos 2x = 1 - 2\sin^2 x$

=

(iv) $y = 2px + p^4 x^2$.

(a) Solve in series the differential equation $\frac{d^2y}{dx^2} + xy = 0$. (5)

(b) Express the polynomial $x^4 + x^3 + x^2 + 1$ in terms of Legendre polynomials. (5)

(c) Show that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right)$. (5)

(a) Prove that $u = y^3 - 3x^2 y$ is a harmonic function. Construct its harmonic conjugate and find the corresponding analytic function $f(z)$ in terms of z. (5)

(b) Evaluate $\int_C \bar{z}\, dz$ from $z = 0$ to $z = 4i$, first along the straight line $z = 0$ to $z = 2i$ and then along the line to $z = 4 + 2i$. (5)

(c) State Cauchy's integral formula and hence evaluate $\oint_C \frac{e^{-2z}}{(z+1)^4}$ where C is the circle $|z| = 2$. (5)

6. (a) Consider the transformation $w = ze^{i\pi/4}$ and find the region in the w-plane corresponding to the triangular region bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ in the z-plane. (5)

(b) Expand the function $f(z) = \frac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region $1 < |z| < 4$. (5)

(c) State residue-theorem and apply it to evaluate $\oint_C \frac{z^3 dz}{(z-1)^3(z+2)}$ where C in the circle $|z| = 3$. (5)

(a) State the generating function for Legendre polynomials and hence show that $P_n(-x) = (-1)^x P_n(x)$. (5)

(b) Use divergence theorem to show that $\oint_S V r^2 ds = 6V$ where S is any closed surface enclosing a volume V. (5)

(c) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$. (5)