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MA 181301A

2024

B.Tech. 3rd Semester End-Term Examination

MATHEMATICS III -A

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks
for the questions.

Answer question No. 1 and any *four* from the rest.

1. (I) Fill in the blanks in the following :

1 × 4

(i) The equation $\frac{\partial^2 u}{\partial x \partial y} = \left(\frac{\partial u}{\partial z}\right)^3$ is of order _____

(ii) Solution of $pq = 1$ is _____

(iii) Lagrange's auxiliary equations are _____

(iv) One dimensional wave equation is _____

(II) Choose the correct answer from the following :

1 × 6

(i) If A and B two events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.5$ then $P(A'/B')$ is equal to

(a) 1/10

(b) 3/10

(c) 3/8

(d) 6/7

(ii) For a Binomial Distribution the mean is 6 and variance is 3 then value of q is

(a) 1

(b) 2/3

(c) 1/3

(d) 1/2

(iii) T-test Z-test are used in the sample of

(a) Small size, Medium size

(b) Small size, Large size

(c) Medium size, Large size

(d) None of these

[Turn over

(iv) $L(e^{-2t} \cos t) =$

(a) $\frac{s+2}{s^2+4s+5}$

(b) $\frac{s}{s^2+4s+5}$

(c) $\frac{s-2}{s^2+4s+5}$

(d) None of these

(v) $L^{-1}\left(\frac{s}{s^2+5} + \frac{1}{s^2-4}\right) =$

(a) $\cos 5t + \frac{\sinh 2t}{2}$

(b) $\cos \sqrt{5}t + \frac{\sinh 2t}{2}$

(c) $\cos \sqrt{5}t + \frac{\sinh t}{2}$

(d) None of these

(vi) $\int_0^{\infty} t^3 e^{5t} dt =$

(a) $\frac{1}{625}$

(b) $\frac{6}{25}$

(c) $\frac{6}{625}$

(d) None of these

2. (a) Form partial differential equation whose solution is $z = f(x+it) + g(x-it)$. 3

(b) Solve any *three* of the following : 3 × 4

(i) $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x+3y)$

(ii) $(mz - ny)p + (nx - lz)q = ly - mx$

(iii) $yp = 2yx + \log q$

(iv) $x^2 p^2 + y^2 q^2 = z^2$

3. (a) Apply Charpit's method to solve $px + qy = pq$. 6

(b) Solve the Boundary value problem $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ given that $u(x, 0) = 6e^{-3x}$. 5

(c) A fair die is rolled. If 1 turns up a ball is picked up at random from bag A, If 2 or 3 turns up a ball is picked up from bag B and if 4 or 5 or 6 turns up a ball is picked up from bag C. Bag A contains 3 red and 2 white, Bag B contains 3 red and 4 white balls and Bag C contains 4 red and 5 white balls. The die is rolled and a Bag is picked up and then a ball is drawn. Find

(i) What is the probability of drawing a red ball?

(ii) If the ball drawn is red then what is the probability that the ball is picked up from bag B. 2 + 2

4. (a) (i) If $L\{f(t)\} = F(s)$ then show that $L\{e^{at}f(t)\} = F(s-a)$ 3 + 3

(ii) Find the Laplace transform of $te^{-2t} \sin t$

- (b) If $L\{f(t)\} = F(s)$ then show that $L\{f'(t)\} = sF(s) - f(0)$ where $f'(t)$ is continuous and $\lim_{t \rightarrow \infty} [e^{-st}f(t)] = 0$. 3 + 2 + 2

Hence evaluate

(i) $L\{t \sin wt\}$

(ii) $L\{wt \cos wt + \sin wt\}$

- (c) State the convolution theorem. 2

5. (a) Out of 800 families with 4 children each, how many families would you expected to have 5

(i) 2 Boys and 2 Girls

(ii) at least one Boy

(iii) at most two Girls. Assume equal probabilities for Boys and Girls.

- (b) A sample of 100 dry battery cells tested to find the length of life produced with the results mean is 12 hrs and standard deviation 3 hrs. Assuming the data to be normally distributed. What percentage of battery cells are expected to have life : 5

(i) more than 15 hrs

(ii) less than 6 hrs

(iii) between 10 and 14 hrs

- (c) Fit a straight line for the following data by least square method considering y as dependent variable. 5

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

6. (a) Evaluate $L^{-1} \left\{ \frac{s}{(s+1)^5} \right\}$ 4

- (b) A random variable X has the density function 4

$$f(x) = \begin{cases} Cx(1-x) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find :

(i) C

(ii) $P\left[\frac{1}{2} \leq X \leq \frac{3}{4}\right]$

(iii) The commutative density function.

(c) A fair coin is tossed four times. Let X denote the numbers of heads occurring and Y denote the longest string of heads occurring. Determine. 7

(i) the joint distribution of X and Y

(iii) the cov (X, Y)

(iii) the marginal distribution of X and Y .

7. (a) Solve the following differential equation by using Laplace Transforms 6

$$\frac{d^3 y}{dt^3} + y = 1, t > 0, \text{ given that}$$

$$y(0) = y'(0) = y''(0) = 0$$

(b) Define a Markov chain. A man is at an integral point of the real axis between 0 and 6. He takes a unit step to the right with probability $\frac{3}{4}$ and to the left with probability $\frac{1}{4}$ unless he is at the origin where he takes a step to the right one at the point 6 where he takes a step to the left. Let X_n denote the position after n steps.

Write the state-space and transition matrix of this Markov chain.