MA 181202

Roll No. of candidate

2024

B.Tech. 2nd Semester End-Term Examination

MATHEMATICS

New Regulation (w.e.f. 2017-18) & New Syllabus (w.e.f. 2018-19)

Full Marks - 70

Time - Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any four from the rest.

1. Select the correct answer:

 $(10 \times 1 = 10)$

- (i) The divergence of $(3x^2i + 5xy^2j + xyz^3k)$ at the point (1, 2, 3) is
 - (a) 20

(b) 40

(c) 80

- (d) None of these
- (ii) The unit normal vector to the surface $x^2y + 2xz = 4$ at the point (2, -2, 3) is
 - (a) $\frac{1}{3}(-i+2j+2h)$
- (b) $\frac{1}{3}(i-2j+2k)$
- (c) $\frac{1}{3}(i+2j-2h)$

- (d) none of these
- (iii) The general solution of the equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$ is
 - (a) $y = Ae^{-x} + Be^{-2x}$

(b) $y = Ae^x + Be^{2x}$

(c) $y = e^{-x} + e^{-2x}$

- (d) $y = (A + Bx)i^x$
- (iv) The integrating factor of the differential equation $(1+y^2)dx = (\tan^{-1}y x)dy$ is
 - (a) etan 's

(b) etan !

(e) tan x

(d) tan-1 y

| (v) | For the differential equation | $(x^2 - 2x)\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0,$ | the point $x =$ | 0 is |
|-----|-------------------------------|---|-----------------|------|
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(a) ordinary

- (b) regular singular
- (c) irregular singular
- (d) none of these

(vi) For the Legendre polynomial $P_n(x)$, $P_n(-1)$ is

(a) 1

(b) -1

(c) $(-1)^n$

(d) none of these

(vii) The real part of $f(z) = \cos z$ is

(a) $\cosh x \cos xy$

(b) $\cos x \cosh y$

(c) $\cos x \cosh x$

(d) none of these

(viii) For the function
$$f(z) = \frac{\cos z}{z}$$
, $z = 0$ is a

- (a) pole
- (b) removable singularity
- (c) essential singularity
- (d) none of these
- (ix) The image of the circle |z|=2 under the mapping w=z+(3+2i) is a
 - (a) circle

(b) ellipse

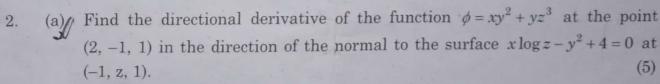
(c) pair of lines

- (d) hyperbola
- (x) Which of the following functions is analytic?
 - (a) $z + \overline{z}$

(b) $2z^2 + 2\overline{z} + c$

(c) |z|2

(d) $z^2 + 2z + c$



(b) With the help of Green's theorem evaluate the line integral $\int_C \vec{A} \cdot dc$ where $\vec{A} = (x^2 + xy^2)i + (y^2 + x^2y)j$ and C is the boundary of the region bounded by y = x and $y = x^2$.

(c) Evaluate the surface integral of $\tilde{A} = zi + xj - 3y^2zk$ over the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between z = 0 and z = 5. (5)

- (a) Solve the differential equation $(D-2)^2 y = 8(e^{2x} + \sin 2x)$. (6)
- (b) Solve any THREE from the following:

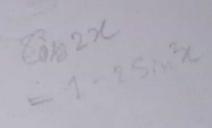
$$(3 \times 3 = 9)$$

(i)
$$(x^4 + y^4)dx - xy^3dy = 0$$

(ii)
$$P = \sin(y - px)$$

(1+x2)
$$\frac{dy}{dx} + y = \sin^{-1} x$$

(iv)
$$y = 2px + p^4x^2$$
.



- (a) Solve in series the differential equation $\frac{d^2y}{dx^2} + xy = 0$. (5)
- (b) Express the polynomial $x^4 + x^3 + x^2 + 1$ in terms of Legendre polynomials. (5)

(e) Show that
$$J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{3 - x^2}{x^2} \sin x - \frac{3}{x} \cos x \right)$$
. (5)

- (a) Prove that $u = y^3 3x^2y$ is a harmonic function. Construct its harmonic conjugate and find the corresponding analytic function f(z) in terms of z. (5)
- (b) Evaluate $\int_{c}^{z} dz$ from z = 0 to z = 4i, first along the straight line z = 0 to z = 2i and then along the line to z = 4 + 2i. (5)
- (c) State Cauchy's integral formula and hence evaluate $\oint_C \frac{e^{-2z}}{(z+1)^4}$ where C is the circle |z|=2.
- (a) Consider the transformation $w = ze^{ix/4}$ and find the region in the w-plane corresponding to the triangular region bounded by the lines x = 0, y = 0 and x + y = 1 in the z-plane. (5)
- (b) Expand the function $f(z) = \frac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region 1 < |z| < 4. (5)
- (c) State residue-theorem and apply it to evaluate $\int_{C} \frac{z^{2}dz}{(z-1)^{2}(z+2)}$ where C in the circle |z|=3.

- (a) State the generating function for Legendre polynomials and hence show that $P_n(-x) = (-1)^x P_n(x)$. (5)
- (b) Use divergence theorem to show that $\oint_S Vr^2 ds = 6V$ where S is any closed surface enclosing a volume V. (5)
- (c) Evaluate $\int_{0}^{2\pi} \frac{d\theta}{5 + 4\cos\theta}.$ (5)