

# Raytracer technical design document

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## Creating an orthonormal coordinate frame

Let  $a$  be the viewing direction *eye-center* and let  $b$  be the *up* vector. Assume the camera is centered at the origin.

$$\begin{aligned}w &= \frac{a}{|a|} \\ u &= \frac{b \times w}{|b \times w|} \\ v &= w \times u\end{aligned}$$

## Creating a ray from the camera

Let  $width, height$  be the screen resolution.

We want to create a ray through the virtual screen, given the  $w$  vector and a pixel position given by  $i, j$ .

Let  $\alpha$  be the screen intersection in the  $u$  direction on  $[-1, 1]$  and  $\beta$  be the screen intersection in the  $v$  direction on  $[-1, 1]$ .

Therefore

$$\begin{aligned}\alpha &= \tan\left(\frac{fov_x}{2}\right) \times \left(\frac{j - (width/2)}{width/2}\right) \\ \beta &= \tan\left(\frac{fov_y}{2}\right) \times \left(\frac{(height/2) - i}{height/2}\right)\end{aligned}$$

with

$$\begin{aligned}aspect &= \frac{width}{height} \\ fov_x &= 2 * \text{atan}\left(\tan\left(\frac{fov_y}{2}\right) * aspect\right)\end{aligned}$$

and

$$ray = eye + \frac{\alpha u + \beta v - w}{|\alpha u + \beta v - w|}$$

## Ray-triangle intersection

Let  $A, B, C$  be the points of a triangle, the normal is then given by:

$$n = \frac{(C-A) \times (B-A)}{|(C-A) \times (B-A)|}$$

Using ray equation

$$\overline{P} = \overline{P}_0 + \overline{P}_1 t$$

within plane equation

$$\overline{P} \cdot \overline{n} - \overline{A} \cdot \overline{n} = 0$$

gives

$$(\overline{P}_0 + \overline{P}_1 t) \cdot \overline{n} = \overline{A} \cdot \overline{n}$$

Solving for t:

$$t = \frac{\overline{A} \cdot \overline{n} - \overline{P}_0 \cdot \overline{n}}{\overline{P}_1 \cdot \overline{n}}$$

## Ray inside triangle

Using barycentric coordinates:

$$P = \alpha A + \beta B + \gamma C$$

$$\alpha \geq 0, \beta \geq 0, \gamma \geq 0$$

$$\alpha + \beta + \gamma = 1$$

$$P - A = \beta(B - A) + \gamma(C - A)$$

$$0 \leq \beta \leq 1, 0 \leq \gamma \leq 1$$

$$\beta + \gamma \leq 1$$

## Lighting model

Let  $d$  be the distance to a light source:

$$L = \frac{L_0}{const + lin * d + quad * d^2}$$

Where *const* is the ambient term, *lin* is the linear term for direction light sources and *quad* is the quadratic term for point light sources.

## Shading model

$$I = K_a + K_e + \sum_{i=1}^n V_i L_i (K_d \max(l_i \cdot n, 0) + K_s (\max(h_i \cdot n, 0))^s)$$