Creating an orthonormal coordinate frame

Let a be the viewing direction $eye - center$ and let b be the up vector. Assume the camera is centered at the origin.

$$w = \frac{a}{|a|}$$
$$u = \frac{b \times w}{|b \times w|}$$
$$v = w \times u$$

Creating a ray from the camera

Let $width, height$ be the screen resolution.

We want to create a ray through the virtual screen, given the w vector and a pixel postion given by i, j.

Let α be the screen intersection in the u direction on $[-1, 1]$ and β be the screen intersection in the v direction on $[-1, 1]$.

Therefore

$$\alpha = tan\left(\frac{fovx}{2}\right) \times \left(\frac{j-(width/2)}{width/2}\right)$$
$$\beta = tan\left(\frac{fovy}{2}\right) \times \left(\frac{(height/2)-i}{height/2}\right)$$

with

$$aspect = \frac{width}{height}$$
$$fovx = 2 * atan\left(tan\left(\frac{fovy}{2}\right) * aspect\right)$$

and

$$ray = eye + \frac{\alpha u + \beta v - w}{|\alpha u + \beta v - w|}$$

Ray-triangle intersection

Let A, B, C be the points of a triangle, the normal is then given by:
$n = \frac{(C-A)\times(B-A)}{|(C-A)\times(B-A)|}$

Using ray equation
$\overline{P} = \overline{P}_0 + \overline{P}_1 t$

within plane equation
$\overline{P} \bullet \overline{n} - \overline{A} \bullet \overline{n} = 0$

gives
$(\overline{P}_0 + \overline{P}_1 t) \bullet \overline{n} = \overline{A} \bullet \overline{n}$

Solving for t:
$t = \frac{\overline{A}\bullet\overline{n} - \overline{P}_0\bullet\overline{n}}{P_1\bullet\overline{n}}$

Ray inside triangle

Using bayocentric coordinates:
$P = \alpha A + \beta B + \gamma C$
$\alpha \geq 0,\ \beta \geq 0,\ \gamma \geq 0$
$\alpha + \beta + \gamma = 1$

$P - A = \beta(B - A) + \gamma(C - A)$
$0 \leq \beta \leq 1,\ 0 \leq \gamma \leq 1$
$\beta + \gamma \leq 1$