Raytracer technical design document

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Creating an orthonormal coordinate frame

Let a be the viewing direction $eye - center$ and let b be the up vector. Assume the camera is centered at the origin.

$$w = \frac{a}{|a|}$$
$$u = \frac{b \times w}{|b \times w|}$$
$$v = w \times u$$

Creating a ray from the camera

Let $width, height$ be the screen resolution.

We want to create a ray through the virtual screen, given the w vector and a pixel postion given by i, j.

Let α be the screen intersection in the u direction on $[-1, 1]$ and β be the screen intersection in the v direction on $[-1, 1]$.

Therefore
$$\alpha = tan\left(\frac{fovx}{2}\right) \times \left(\frac{j-(width/2)}{width/2}\right)$$
$$\beta = tan\left(\frac{fovy}{2}\right) \times \left(\frac{(height/2)-i}{height/2}\right)$$

with
$$aspect = \frac{width}{height}$$
$$fovx = 2 * atan\left(tan\left(\frac{fovy}{2}\right) * aspect\right)$$

and
$$ray = eye + \frac{\alpha u + \beta v - w}{|\alpha u + \beta v - w|}$$

Ray-sphere intersection

$$ray \equiv \overline{P} = \overline{P}_0 + \overline{P}_1 t$$
$$sphere \equiv (\overline{P} - \overline{C}) \bullet (\overline{P} - \overline{C}) - r^2 = 0$$

Substituting:
$$(\overline{P}_0 + \overline{P}_1 t - \overline{C}) \bullet (\overline{P}_0 + \overline{P}_1 t - \overline{C}) - r^2 = 0$$

Simplifying:
$$t^2(\overline{P}_1 \bullet \overline{P}_1) + 2t\overline{P}_1 \bullet (\overline{P}_0 - \overline{C}) + (\overline{P}_0 - \overline{C}) \bullet (\overline{P}_0 - \overline{C}) - r^2 = 0$$

Solving for t:
$$t = \frac{-(2(P_1 \bullet (P_0 - C)) \pm \sqrt{2(P_1 \bullet (P_0 - C))^2 - 4(P_1 \bullet P_1)((P_0 - C) \bullet (P_0 - C) - r^2)}}{2(P_1 \bullet P_1)}$$

The surface normal at the intersection is given by
$$n = \frac{P - C}{|P - C|}$$

Ray-triangle intersection

Let $A,\ B,\ C$ be the points of a triangle, the normal is then given by:
$$n = \frac{(C-A) \times (B-A)}{|(C-A) \times (B-A)|}$$

Using ray equation
$$\overline{P} = \overline{P}_0 + \overline{P}_1 t$$
within plane equation
$$\overline{P} \bullet \overline{n} - \overline{A} \bullet \overline{n} = 0$$

gives
$$(\overline{P}_0 + \overline{P}_1 t) \bullet \overline{n} = \overline{A} \bullet \overline{n}$$

Solving for t:
$$t = \frac{\overline{A} \bullet \overline{n} - \overline{P}_0 \bullet \overline{n}}{P_1 \bullet \overline{n}}$$

Ray inside triangle

Using barycentric coordinates:
$$P = \alpha A + \beta B + \gamma C$$
$$\alpha \geq 0,\ \beta \geq 0,\ \gamma \geq 0$$
$$\alpha + \beta + \gamma = 1$$

$$P - A = \beta(B - A) + \gamma(C - A)$$
$$0 \leq \beta \leq 1,\ 0 \leq \gamma \leq 1$$
$$\beta + \gamma \leq 1$$

Lighting model

Let d be the distance to a light source:
$$L = \frac{L_0}{const + lin * d + quad * d^2}$$

Where $const$ is the ambient term, lin is the linear term for direction light sources and $quad$ is the quadratic term for point light sources.

Shading model

$$I = K_a + K_e + \sum_{i=1}^{n} V_i L_i (K_d max(l_i \bullet n,\ 0) + K_s (max(h_i \bullet n,\ 0))^s)$$

Recursive shading model

$$I = K_a + K_e + \sum_{i=1}^{n} V_i L_i (K_d max(l_i \bullet n,\ 0) + K_s (max(h_i \bullet n,\ 0))^s) + K_s I_R + K_T I_T$$