Distributing the Heat Equation

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1 Heat equation

$$\frac{\partial x}{\partial t} = \nabla^2 x$$

2 Cellular automata

Question 1. There is N^2 cells in $[\![0, N-1]\!]^2$. Therefore, tN^2 applications of the function δ are necessary to compute X^t on $[\![0, N-1]\!]^2$.

Question 2.

Question 3.

3 Average automata

Let $A = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}$ and $B = \begin{pmatrix} b_{1,1} & b_{1,2} & b_{1,3} \\ b_{2,1} & b_{2,2} & b_{2,3} \\ b_{3,1} & b_{3,2} & b_{3,3} \end{pmatrix}$.

$$\begin{aligned}
\delta(A) + \lambda\delta(B) &= (1-p)a_{2,2} + p\frac{a_{2,1} + a_{1,2} + a_{3,2} + a_{2,3}}{4} + \lambda\left((1-p)b_{2,2} + p\frac{b_{2,1} + b_{1,2} + b_{3,2} + b_{2,3}}{4}\right) \\
&= (1-p)(a_{2,2} + \lambda b_{2,2}) + p\frac{a_{2,1} + a_{1,2} + a_{3,2} + a_{2,3} + \lambda(b_{2,1} + b_{1,2} + b_{3,2} + b_{2,3})}{4} \\
&= \delta(A + \lambda B)
\end{aligned}$$