ISSN (Print): 0974-6846 ISSN (Online): 0974-5645

Design and Implementation of Kalman Filter for GPS Receivers

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Abstract

Objective: This paper discusses the design and development of a Kalman filter for effective tracking of code and carrier in a GPS receiver. The objective is to incorporate Kalman filter in the tracking channel of a GPS receiver. **Methods/Statistical Analysis**: The tracking channel keeps synchronizing continuously, the received satellite signal and the locally generated code and carrier frequencies, using tracking loops. The first step is to develop a tracking channel and the next step is to develop a Kalman filter model suitable for the design. The reliable and standard tracking loops for tracking code and carrier are Delay Locked Loop (DLL) and Costas Loop respectively. When these loops lose the lock, i.e., when the signals are weak, the receivers can no longer retrieve the navigation data unless the code and carrier comes in lock again. This causes disturbance in tracking, i.e., the receiver can no longer determine its position until the signal becomes strong. The proposed design can solve this problem by using a Kalman filter in a phase lock loop. **Findings**: The Kalman filter, with its linear recursive filtering provides greater accuracy in estimating the phase/frequency of the received incoming signal with the initial state of the system, noise statistics of the system and the corresponding measurement errors. The replacement of Low Pass Filter (LPF) with the Kalman filter in the tracking channel provides a better estimation of tracking. The analysis reveals that the Kalman filter is the best replacement tool that can strongly influence the area of navigation in a way that minimizes the error in tracking. **Conclusion**: Thus the design ensures the continuous tracking of GPS signals by preventing the need for frequent requisition even under weak signal conditions.

Keywords: GPS Receiver, Kalman Filter, Tracking Channel, Tracking Loops, Weak Signal

1. Introduction

The GPS (Global Positioning System) based navigation system allows users to locate one's positions anywhere on earth. A constellation of 24-satellites, is used to provide an accurate position of user¹. These satellites transmit radio signals that provide their exact location, time, and other information. GPS receivers takes this information and calculates the user's exact location using a process called triangulation. The basic GPS receivers operate in two portions. One is hardware and the other is software. The hardware part of the receiver contains an antenna, an RF chain, and an ADC. Acquisition and tracking comes under software part of a receiver that can be treated as the backbone of a GPS receiver².

The first task of the software portion of a GPS receiver is to determine the satellites that are visible for every integration period. After determining the availability of a satellite signal, the GPS receiver tries to track the code and carrier components of the signal³. The GPS receiver mostly uses a Delay Lock Loop to track the C/A code sequence and a Costas loop to track the carrier of the received satellite signal. The output of the tracking loops is the decoded form of the navigation message of each satellite. It helps the user to calculate the positions of the satellites. Finally, the user calculates his position using the pseudo-range measurements from the tracking loops⁴.

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disturbance in tracking i.e., the receiver fails to locate its position until the signal becomes strong. An efficient solution for this problem is Kalman filter.

The Kalman filter forms an estimate of a process by using a form of feedback control: it estimates the state of the process at some time and then obtains feedback in the form of measurements. It integrates both the measurement data, and the system properties, to produce a pleasing estimate of the desired state variables in such a manner that minimizes the error statistically.

2. The Tracking Channel of a GPS Receiver

The received signal from a satellite is a combination of PRN code, carrier signal and navigation data. To obtain the position of a GPS receiver, the navigation data should be isolated from the above said combination. In order to achieve this, the tracking channel has to generate two replicas, one for the carrier and other for the code. The initial step is to multiply the incoming signal with the generated carrier replica. This step wipes off the carrier from the incoming signal. The next step is to multiply the signal with the generated code replica and the result of this multiplication gives the navigation data.

pseudorandom noise (PRN) codes are deterministic sequences with the properties similar to noise. These codes are generated using a clocked feedback shift register. The length (N) of the generated PRN sequences is 2^n-1 , where the GPS C/A code uses n=10. The PRN sequence repeats for every millisecond so that the length of the chip is 977.5 ns, which provides a corresponding metric length of 300m while propagating through vacuum or air.

2.1 Code Tracking

Code tracking loop is implemented to obtain a perfectly aligned replica of the code. The goal of this tracking loop is to track the phase of a specific code in the signal that is being received. The code tracking loop in the GPS receiver is a delay lock loop (DLL). It is also called as an early-late tracking loop. The signal that is obtained after the multiplication of incoming signal and a perfectly aligned local replica, is then multiplied with three code replicas, namely, early, prompt and late, separated by a spacing of ½ chip. After the two multiplications, the three outputs are integrated and dumped. The output of these integrations indicates the amount of correlation between specific code replica and the incoming signal. The prompt replica of the code has a phase shift obtained from the acquisition⁵. The early and late have a delay of $-\frac{1}{2}$ and $+\frac{1}{2}$ chip, respectively, from the prompt.

2.2 Carrier Tracking

An exact carrier wave replica is generated for the data demodulation using a PLL or Frequency Lock Loop (FLL). The first two multiplications, removes the Pseudo Random Noise code and the carrier component in the incoming signal. The prompt output of the E, P, L code tracking loop is used to wipe off the PRN code. The local carrier frequency is adjusted as per the feedback given by the change in the phase error.

If the GPS signal did not undergo a 50-Hz data modulation, the carrier tracking loop could use a pure PLL discriminator. But the problem with the ordinary PLL is that it is sensitive to 180 degree phase shifts^{6,7}. Due to navigation data bit transitions, a PLL has to be insensitive to 180 degree phase shifts. One such loop is the Costas loop.

2.2.1 Costas Loop

The important property of this loop is that it is insensitive to 180 degree phase shifts. This property explains that a Costas loop is insensitive for transitions in phase due to message data. This is the reason behind choosing this carrier tracking loop in GPS receivers. This loop performs two multiplications. The first multiplication is between the incoming signal and the local carrier wave and the second multiplication is between a phase-shifted carrier wave and the incoming signal. Its main aim is to keep maximum energy in the in-phase arm. To achieve this, the loop provides some kind of feedback to the oscillator^{8,9}.

The phase error of the local carrier wave can be found as:

$$\Phi = \arctan(Q_p/I_p) \tag{1}$$

From the above equation, phase error can be minimized once the correlation in the quadrature phase becomes zero and the correlation value in the in-phase arm reaches the maximum. The arctan discriminator is the most accurate of all the Costas discriminators.

3. Kalman Filter

A Kalman filter addresses the problems in which the system is considered to be a linear, white and Gaussian

model and in which the measurement noises are considered to be white and Gaussian. All the statistical parameters like mean, mode, median, etc., coincide, under these conditions, which, in fact, forms unique "best" estimate.

Navigation system becomes standard, in space applications, marine, new cars, weather science manufacturing, and many others¹⁰. Most of these navigation systems not only use the global positioning system (GPS) for finding the true way, but also an inertial navigation system (INS). These two systems allow improved navigation accuracy and reliability especially when GPS is degraded or interrupted because of buildings or tunnels. The GPS and INS complement each other to achieve the above said performance. The Kalman filter provides the basis for these applications.

As Kalman filter became an essential part of modern navigation systems, it has been labeled as "navigation's integration workhorse11,12" It is unlikely that GNSS could have been developed without the Kalman Filter.

The Kalman filter discusses the issue of estimating the state of a linear stochastic discrete process

$$x_k = A.x_{k-1} + B.u_k + w_{k-1}$$
 (2)

With its measurement that is

$$z_k = H.x_k + V_k \tag{3}$$

The random variables W₁ and V₂ represent the process and measurement noise, respectively. In general, both process noise and measurement noise are assumed to be independent, Gaussian and white. The normal probability distributions of these noises are assumed to be13.

$$P(w) \sim N(0,Q) \tag{4}$$

$$P(v) \sim N(0,R) \tag{5}$$

Where Q represents process noise covariance and R represents measurement noise covariance.

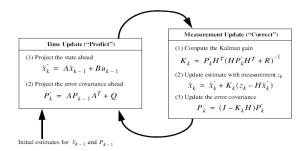


Figure 1. Basic operation of the Kalman Filter.¹³

The n*n matrix A in the state equation relates the previous state at instant k-1 to the state at the current instant, k, in the absence of either a input function or process noise. In general, the transition matrix A might change with each time step, but in the proposed design it is taken as a constant. The matrix B relates the control input to the predicted state x. The 'H' matrix in the measurement equation gives the relation between state and the corresponding measurement z_k. In general, H might change with each time step or measurement, but in the proposed design it is taken as a constant.

3.1 Kalman Filter Model

The tracking loops have certain inherent flaws. These loops use filters of fixed bandwidth, which cannot work efficiently under varying C/N levels and high user dynamics. The measurements obtained from the discriminator are all weighted equally. These measurements that are made during periods of high C/N levels are weighted equally with those made during periods of low C/N_o. The order of these tracking loop filters provides the dynamics that the loop can track with no steady state error^{14,15}.

The designer is faced with a trade off, when designing the loop filter. If the bandwidth of the filter is increased, it allows better tracking ability of the loop under high user dynamics. However, wider bandwidths make the loop more susceptible to noise and jamming.

The Kalman filter is a special type of filter whose gain varies with time. The variation of Kalman gain depends on the variation of measurement noise statistics and process noise statistics. The measurement noise statistics and its variation depend on C/N levels and jamming. The process noise statistics and its variation is dependent on user dynamics. The Kalman filter can optimally separate signal from noise, if it is provided with the relevant process and measurement noise matrices.

The states of the Kalman filter and the state transition matrix are shown below in the equation:

$$\begin{pmatrix}
\Theta_{K} \\
F_{K} \\
\Delta F_{K}
\end{pmatrix} = \begin{pmatrix}
1 & \Delta t & \Delta t^{2} / 2 \\
0 & 1 & \Delta t \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
\Theta_{K-1} \\
F_{K-1} \\
\Delta F_{K-1}
\end{pmatrix} + W_{K-1}$$
(6)

This is related to Kalman Filter equation as

$$X_{K} = A X_{K-1} + W_{K-1}$$
 (7)

The first state of the filter θk corresponds to the change in phase of the received carrier. The second state, F_k is the frequency of the carrier that can be obtained from the derivative of the phase of the carrier. The third state is considered as the derivative of the carrier frequency in the approach. This parameter is modeled as a linearly varying parameter with time.

The measurement provided to the filter is the output obtained from the carrier discriminator

$$Z_{v} = H X_{v}$$
 (8)

Where,

H is the observation matrix

In this design, H gives the value of the observed phase from the discriminator.

i.e.,
$$H = (1\ 0\ 0)$$
 (9)

Therefore,

$$Z = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Theta_{K} \\ F_{K} \\ \Delta F_{K} \end{pmatrix}$$
 (10)

$$\Theta_{K} = \arctan(Q_{pg} / I_{pg})$$
 (11)

The key function of the Kalman filter is to reduce the effect of measurement noise in the system. The effect of measurement noise can be reduced by properly calculating the measurement noise covariance in the system. As the designed model is assumed to be linear, the measurement noise covariance is constant over the entire system.

With the observed phase values of satellite signals from the discriminator, the measurement noise covariance in the system is found as:

$$Q = 0.05$$

4. Results and Analysis

4.1 Code tracking

To track the exact PRN code from the incoming signal, the PRN code in the incoming signal should match with the prompt sequence in the local register. It can be observed from Figure 2 that the PRN sequence from the incoming signal is aligned with the prompt sequence in the locally generated register, which are represented as "pnrc" and "prompt" respectively in the graph.

4.2 Carrier loop discriminator

The loop used in carrier tracking is the Costas loop. The discriminator used in the loop is arctan discriminator. i.e.,

Carrier Discriminator = ATAN(Q_{DS}/I_{DS})

The output from the discriminator is the phase values of the carrier signals that is calculated for every millisecond as shown in Figure 3. The difference in phase values (change in phase) is given as input to the Kalman filter.

4.3 Kalman Filter

The change in phase is given as input to the Kalman Filter. The output from the filter is the corrected change in phase of the incoming signal, which is shown in blue colour in the Figure 4. The Kalman filter minimizes the error in the measured values.

4.4 Feedback to Numerically Controlled Oscillator

The change in frequency that is calculated from Kalman Filter is given as input to the NCO, so that the NCO keeps the locally generated carrier frequency in lock with the incoming carrier frequency as shown in Figure 5

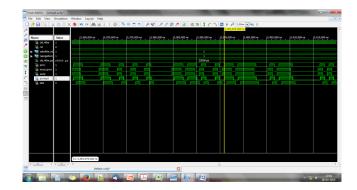


Figure 2. Code Tracking.

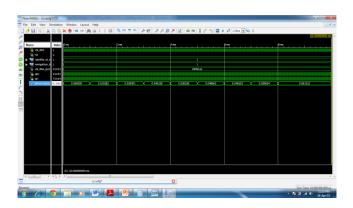


Figure 3. Carrier loop discriminator (Phase Values).

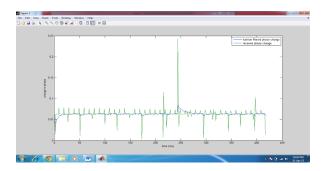


Figure 4. Kalman Filter phase error graph.

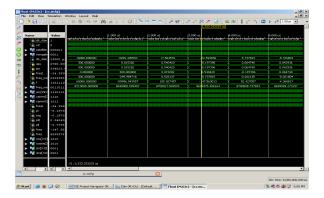


Figure 5. Feedback to Numerically Controlled Oscillator.

5. Conclusion

The major contribution of the work lies in the simulations performed for the validation of above methods for the incorporation of Kalman filter into GPS tracking loops. The major advantage lies in the capability of the presented algorithm in combating the deep fades suffered by the GPS signals due to ionospheric scintillations. The algorithm presented, and simulation results, show that the need for frequent reacquisition of the system, under weak signal conditions, is prevented. Also, there is reduction in the noise variance of the output of phase discriminator. This is possible as Kalman filter produces the optimum output using a combination of measurements and state equation. All work done was tested and results verified

using ISim and the specifications were addressed as in actual GPS receiver conditions.

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