Math 1152 Written Homework 9

Due: Thursday, July 21st in Gradescope.

- Calculators are permitted EXCEPT those calculators that have symbolic algebra or calculus capabilities.
- SHOW ALL WORK!
- A completed version of this document is due to be uploaded to Gradescope by 11:59pm on **Thursday, July 21st**.
- If you have difficulties using Gradescope, see pages under the Gradescope header in the Modules section of our Carmen page for help.
- Ideally, this can be completed on an iPad or android tablet using an app like One Note, Notability, Papyrus, etc. - if you don't have access to one of these options,then printing and scanning or using a smartphone document-scanning feature to generate a pdf to upload will also work.
- If you have difficulties uploading the assignment, email a pdf to your recitation instructor.
- This homework will be graded via random subset selection - not every part of every question will be looked at by the grader.
- Rubrics to applicable questions will be provided later.

Question 1. Sir William Timothy Gowers (generally known as Tim Gowers) is a Fields Medalist, mathematics professor at Cambridge, excellent mathematical thinker and expositor, and writes some of the most pleasant mathematics papers to read.

Here[1] is his "The Two Cultures of Mathematics". How does it relate to previous articles we have read throughout the term?

[1] https://www.dpmms.cam.ac.uk/ wtg10/2cultures.pdf

Question 2.

Using series for known functions, and the series operations of multiplication, substitution, reindexing, and subtraction, find the series for

$$\frac{x-1}{1-x^2}.$$

(This question is NOT asking you to find the Taylor Series using Taylor's Formula, but rather to work with known series directly).

Question 3. Consider the equation

$$y'' + 3xy' + xy = 0.$$

As in Friday's lecture, find a recurrence relation which must be satisfied by the sequence (a_n) if $y = \sum_{n=0}^{\infty} a_n x^n$ is a solution. **Do not solve the recurrence for a_n here.**

Bonus Question 4. We saw in lecture that a Power Series solution to the equation $y'' - xy = 0$ must satisfy the recurrence

$$2a_2 = 0,$$

$$a_{n+2} = \frac{a_{n-1}}{(n+2)(n+1)}.$$

Solve these recurrences to find two distinct power series which solve the equation $y'' - xy = 0$.

Hint: Your solution should involve a term of the form

$$\frac{\mathbf{1}}{\mathbf{(2 \cdot 3)(6 \cdot 7) \cdots ((3n)(3n+1))}}.$$