Name: _____

Math 1152 Summer 2022 Midterm 1

Due: 11:59pm, May 31, 2022

Before Starting

- Complete Problems 1 and 2.
- Choose another **2** of the following problems to complete in addition to problems 1 and 2. You will not receive additional credit for completing additional problems beyond these two.
- This exam should reflect your work only and is not a group assignment.
- This exam is open-notes.
- This exam is due at **11:59pm, Tuesday May 31**.
- Full work must be shown for all problems you submit.
- **Fill in the circles below corresponding to the 2 additional problems you choose to have graded. Problems 1 and 2 have already been filled in for you.**

I am submitting for grading:

- ● **Question 1**
- ● **Question 2**
- ○ **Question 3**
- ○ **Question 4**
- ○ **Question 5**
- ○ **Question 6**
- ○ **Question 7**
- ○ **Question 8**

Question 1.

Let R be the region bounded by $y = 4 - x^2$ and $y = 0$, and suppose that a is a constant with $0 \leq a \leq 4$.

Let R_1 be the portion of R that lies *below* the line $y = a$ and R_2 be the portion of R that lies *above* the line $y = a$.

a. If the areas of R_1 and R_2 are equal, then what must be the value of a?

b. Suppose that the density, d, of a point (x, y) in R is given by $d(x, y) = \frac{\sqrt{16-y^2}}{\sqrt{4-y}}$.

i. Set up an integral representing the mass of region R_1, an integral representing the mass of region R_2, and write down an equation equating them.

Mass of R_1:

Mass of R_2:

Equation relating masses:

ii. What trigonometric substitution should you use to calculate $\int \sqrt{16 - y^2}\, dy$?

Substitution: $y =$

iii. Find $\int \sqrt{16-y^2}\,dy$ in terms of the θ from your subsitution.

$\int \sqrt{16-y^2}\,dy$ **in terms of** θ**:**

iv. Find $\int \sqrt{16-y^2}\,dy$ in terms of y.

$\int \sqrt{16-y^2}\,dy$ **in terms of** y**:**

v. Find an equation whose solution gives the value of a for which region R_1 and region R_2 have the same mass.

Equation for a:

Question 2.

Find the length of the arc of the parabola $y = x^2$ from the point $(0, 0)$ in the xy-plane to the point $(1, 1)$.

Space for Question 2 continued.

Question 3. Suppose that a solid, S, has a base in the xy-plane bounded by the curves $y^2 - 4 = x$ and $y = x/2 - 2$, and that the cross-sections perpendicular to the y-axis are semi-circles.

a. With respect to which variable should the integral be taken?

Ⓐ x Ⓑ y

b. If the integral(s) representing volume of the solid S above take the form

$$\int_a^b A(t)\,dt,$$

then the function $A(t)$ should be which of the following?

Ⓐ πt^2 Ⓑ $2t + 4 - \left(t^2 - 4\right)$ Ⓒ $\sqrt{x+4} - (x/2 - 2)$ Ⓓ None of the above

(Fill in the circle of the answer (A), (B), (C), or (D) above)

c. What should the bounds a and b be above?

$a =$ Ⓐ -12 Ⓑ -10 Ⓒ -8 Ⓓ -6 Ⓔ -4 Ⓕ -2 Ⓖ 0 Ⓗ None of the above

$b =$ Ⓐ 12 Ⓑ 10 Ⓒ 8 Ⓓ 6 Ⓔ 4 Ⓕ 2 Ⓖ 0 Ⓗ None of the above

Question 4. Suppose that a solid, S, is formed by rotating the region bounded by the curves $y^4 + 3 = 2x$ and $y^2 = x - 1$ about the line $y = -1$.

Set up a single integral which can be integrated to give the volume of the resulting solid in the space below.

Then answer the questions on the following page.

a. Should you integrate with respect to x or y?

Ⓐ x Ⓑ y

b. What is the lower bound of integration?

Ⓐ -2 Ⓑ -1 Ⓒ 0 Ⓓ 1 Ⓔ 2 Ⓕ None of the above

c. What is the upper bound of integration?

Ⓐ -2 Ⓑ -1 Ⓒ 0 Ⓓ 1 Ⓔ 2 Ⓕ None of the above

d. Which method do you use to find the volume?

Ⓐ Shells Ⓑ Washers Ⓒ Cross-sections

e. The function r is

Ⓐ $\big((y^4+3)/2-(y^2+1)\big)$ Ⓑ $\sqrt{4}2x-3-\sqrt{x-1}$ Ⓒ $y+1$

Ⓓ None of the above or not part of the problem.

f. The function R is

Ⓐ $\big((y^4+3)/2-(y^2+1)\big)$ Ⓑ $\sqrt{4}2x-3-\sqrt{x-1}$ Ⓒ $y+1$

Ⓓ None of the above or not part of the problem.

g. The function h is

Ⓐ $\big((y^2+1)-(y^4+3)/2\big)$ Ⓑ $\sqrt{4}2x-3-\sqrt{x-1}$ Ⓒ $y+1$

Ⓓ None of the above or not part of the problem.

Question 5.

5.I. Calculate $\displaystyle\int_{-\pi}^{0} \sqrt{1-\cos^2(\theta)}\, d\theta$

a. What was the value of the integral?

Ⓐ -2	Ⓑ -1	Ⓒ 0	Ⓓ 1	Ⓔ 2	Ⓕ None of the above

b. Which was the **1st** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	

(**5.I.** continued)

c. Which was the **2nd** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	Ⓕ I didn't use any more

d. Which was the **3rd** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	Ⓕ I didn't use any more

e. Which was the **4th** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	Ⓕ I didn't use any more

f. Which was the **5th** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	Ⓕ I didn't use any more

5.II. Calculate $\displaystyle\int \frac{1}{(x^2-9)^{3/2}}\,dx.$

a. Which was the **1st** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	

(**5.II.** continued)

b. Which was the **2nd** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	Ⓕ I didn't use any more

c. Which was the **3rd** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	Ⓕ I didn't use any more

d. Which was the **4th** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	Ⓕ I didn't use any more

e. Which was the **5th** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	Ⓕ I didn't use any more

5.III. Calculate $\int \sqrt{\cos(t)}\, dt.$

a. Which was the **1st** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	

(**5.III.** continued)

b. Which was the **2nd** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	Ⓕ I didn't use any more

c. Which was the **3rd** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	Ⓕ I didn't use any more

d. Which was the **4th** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	Ⓕ I didn't use any more

e. Which was the **5th** identity or integration technique you used?

Ⓐ an algebraic identity	Ⓑ a trigonometric identity	Ⓒ u-substitution
Ⓓ integration by parts	Ⓔ trigonometric substitution	Ⓕ I didn't use any more

Question 6. The region R is bounded above by $y = x^2 - 2x + 2$, below by $y = \frac{1}{2}\left(x^2 - 2x + 1\right)$, to the left by $y = 2 + 4x$, and to the right by $x = 1/2$.

a. Sketch these 4 curves, and the region R bounded above on four sides by them. Solve for their intersection points and list them in the boxes below.

First intersection point:
Second intersection point:
Third intersection point:
Fourth intersection point:

The sum of the three x-coordinates above which do not involve a radical is:

Ⓐ 0 Ⓑ 1 Ⓒ 2 Ⓓ 3 Ⓔ 4

Ⓕ None of the above.

b. How many integrals are needed to determine the area of R if integration is performed dx? Fill in the circle for your answer below.

Ⓐ 1 Ⓑ 2 Ⓒ 3 Ⓓ 4

c. How many integrals are needed to determine the area of R if integration is performed dy? Fill in the circle for your answer below.

Ⓐ 1	Ⓑ 2	Ⓒ 3	Ⓓ 4

d. Calculate the area of the region R using whichever method you prefer. What is the leading digit of the area?

(For example, the leading digit of 23.5 is 2 and the leading digit of 0.8 is 0). **Fill in the circle for your answer below.**

Ⓐ 0	Ⓑ 1	Ⓒ 2	Ⓓ 3	Ⓔ 4	Ⓕ 5	Ⓖ 6	Ⓗ 7	Ⓘ 8	Ⓙ 9

Question 7.

Calculate

$$\int \frac{1}{\sqrt{x^2 - 3x + 2}}\, dx.$$

Answer:

Question 8.

Show that $\lim_{n\to\infty} \int_a^b f(t)\sin(nt)\,dt = 0$ if f' is continuous on $[a, b]$.

Remember to go back to the front page and fill in the circle to indicate which problems you are submitting for grading.

Formulas:

- Pythagorean:

$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

- Double-Angle:

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1$$
$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

- Power-Reducing / Half-Angle:

$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$
$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$