Math 1152 Lecture Notes

June 24, 2022

Using Summation by Parts

If

- $a_n \to 0$,
- $|\sum_{n=a}^{m} \Delta b_n| \le M$, and
- $\sum_{n=a}^{\infty} b_n \Delta a_n$ converges,

then $\sum a_n \Delta b_n$ converges.

Exercise 1.

a. Show that

$$\sum_{n=0}^{m} \frac{n^2}{\sqrt{n^2+n}} - \frac{(n+2)^2}{\sqrt{(n+2)^2+n+2}}$$

is bounded (independent of m).

b. Show that

$$\sum_{n=0}^{\infty} \frac{n^2}{n\sqrt{n^2+n}} - \frac{(n+2)^2}{n\sqrt{(n+2)^2+n+2}}$$

converges.

1 Remainders - the tail of a series

Remainders, or tails, of infinite series allow us to determine how quickly the partial sums converge.

You worked out what happens in the case of geoemtric series in Written Homework 6.

The other series we have exact values for are telescoping series.

Telescoping Series

Exercise 2. Find a formula for the tail of the series

$$\sum b_n$$

if

$$b_n = a_n - a_{n-1}$$

and $a_n \to A$.

Alternating Series

Why does an alternating series which passes the Alternating Series Test converge?

Can we extract information about the tail of the series from the argument for convergence?

The Integral Test

What do we need to know in order to be able to say that the series $\sum a_n$ converges by the Integral Test?

What makes the integral test true?

Can we upgrade the qualitative information about convergence into quantitative information about the rate of convergence?

Using remainders

Exercise 3. Suppose that the series $\sum a_n$ has remainders which satisfy $|r_n| \leq \frac{1}{n}$.
How large must n be so that s_n is within 0.0006 of its limit?

Exercise 4. Suppose that $a_n, b_n \geq 0$ and the tails for the series $\sum a_n$ are bounded by 2^{-n} and for the series $\sum b_n$ are bounded by $\log(n)$.
Does $\sum a_n$ converge? Does $\sum b_n$? Does $\sum (a_n b_n)^{\frac{1}{2}}$?