Math 1152 Lecture Notes

May 11, 2022

1 Intro

Welcome to Math 1152! This is a non-coordinated course covering integral calculus, sequences and series, parametric curves, and polar coordinates.

My name is Marc Carnovale, and I look forward to working with all of you this term. A little about myself: I earned my PhD in Mathematics at OSU in 2019, and my research is in the broad branch of math known as Analysis.

Despite the popular conception, mathematics is not about numbers. I usually start each term off asking my students: "What do you think Mathematics *is*? What makes the study of something math, or what makes it fail to be math?" So take a moment, and try to answer that question for yourself. What is mathematics?

Here's my definition: Mathematics is the study of well-defined concepts.

Note that to talk about what we mean by the term 'well-defined', we need to formalize the notion of what a definition is. We won't do that in this course - you might wonder, under this definition, whether that could even be mathematics, or whether it would more properly be some *meta*mathematics.

Mathematics is the study of well-defined concepts. When we perform mathematics, we try to understand a subject area from the viewpoint of its particular rules and structures.

- So, we try to understand Euclidean Geometry using the angles and distances but end up caring more about congruences and similarity that reveal when Euclidean shapes are "the same", even if their positions or sizes have changed, and so we don't care so much about the numbers we could assign to position or size.

- We try to understand polynomials in terms of addition and multiplication, which leads ultimately to questions about divisibility and factoring - and not to particularly much focus on the specific values a polynomial takes on at a specific point. Not a focus on the numbers.

- In its graduate level formulation, we try to understand the notion of "volume" in terms of the fundamental operations we can perform on volumes, rather than caring about the exact volume of a particular object.

So why, then, do our classes care so much about numbers?

2 Course Syllabus, etc.

Lecture Notes:

- Available in Carmen under Lecture Notes module before class.

Email address:

- carnovale.2@osu.edu (not buckeyemail.osu.edu)
- 24 hour response policy (except weekends)

Office hours:

- M,W 12:30pm-1:25pm via Zoom.

Course Structure:

- Hybrid Course
- Live Zoom broadcasts, recordings uploaded after.

Grades:

- Integration Quiz (and Redux)
- 10 Written Homeworks
- Roughly Biweekly Ximera Homework, Review, and Textbook Assignments
- 2 Midterms
- Final Exam

Ximera Homework:

- Only necessary to complete 90% of it to receive full credit (there will be times when Ximera has issues, for instance not accepting the correct answer, and this will absorb those times while providing some further freedom).

Written Homework:

- Random Subselection Grading, but rubrics to full answers will be provided.

Exams:

- Exams will have large windows
- ≈ 9 hours window for midterms
- Several days window for the final.
- Midterms will be less than 1 hour of work for the average student.
- The final will be no more than 2 hours of work for the average student.

Upcoming assignments

- First Ximera assignment was due last night.
- Review 1 due tonight.
- Email me if you need extensions or Ximera reset for you.
- Monday is the Integration (Review) quiz in Carmen. There will be a second (optional) Integration quiz before the first exam which can be used to raise your Integration Quiz score.

Any questions?

3 Back to Mathematics

Definition 1. (Informally) the **limit of $f(x)$ as x approaches a**, written $\lim_{x \to a} f(x)$, is the number L (if it exists) such that the values of $f(x)$ can be made arbitrarily close to L by making x sufficiently close to a.

Definition 2. The **area under the curve** $y = f(x)$ between the vertical lines $x = a$ and $x = b$ is given by the definite integral $\int_a^b f(x)\,dx$.

Definition 3. The **definite integral** $\int_a^b f(x)\,dx$ is given as the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta_k x$$

(if the limit exists).

Some observations:

- This is very unappealing *as a formula* or a calculational tool, but conceptually it is pleasant.
- We (rarely) ever use this definition directly - just like with the limit of difference quotients definition of the derivative.

Theorem 1 (Fundamental Theorem of Calculus)**.** The value of the definite integral $\int_a^b f(x)\,dx$ is given by $F(b) - F(a)$, where F is any function where $F' = f$.

Question 1. Why is this true?

Question 2. Does the area under the curve satisfy the properties that area should satisfy? What properties *should* it satisfy?

Question 3. How should we define the area *between* two curves?

4 Generalizing the Riemann Sum

- Lets think of the area under the curve as a limit of Riemann sums, which themselves are sums of areas of rectangles approximating what the area under the curve should be, and do the same thing when the "bottom" curve isn't the x-axis.

Definition 4. The **area between the curves** $y = f(x)$ **and** $y = g(x)$ from $x = a$ to $x = b$ is given by

$$\int_a^b h(x)\,dx$$

where $h(x)$ is the (unsigned) height between the curves $y = f(x)$ and $y = g(x)$.

- If we suppose that $f(x) \geq g(x)$ for all x between a and b, and that $f(a) = g(a)$ and $f(b) = g(b)$, then $h(x) = f(x) - g(x)$ and the area between the curves is
$$\int_a^b \left(f(x) - g(x)\right)\,dx.$$
- When we are told to "find the area between $y = f(x)$ and $y = g(x)$" and nothing more is specified, we need to solve for the intersection points and determine which curve is on top. Sometimes, there may be more than one region to integrate over if the two curves cross.

Exercise 1 (Simple example)**.** Find the area between the curves $y = x^2$ and $y = x$.

Steps to finding the area between curves:

1. Solve algebraically for where the curves intersect one another.
2. Sketch a rough graph or solve the inequality in each region between intersection points to determine which curve is on top in each region.
3. Set up a (sum of) integral(s) over the region(s) which represent the area.
4. Evaluate the integral(s).

Exercise 2 (Multiple regions.)**.** Let $f(x) = \sin(\pi x)$ and $g(x) = y$. Find the area bounded by the curves $y = f(x)$, and $y = g(x)$ for x between 0 and 2.

Exercise 3 (More than two curves.)**.** Find the area bounded by the curves $y = 2x$, $y = 3 - x^2$, and $y = 1 - x$.

Observation:

- Area shouldn't change under rotations, so the area between curves shouldn't care whether y is a function of x or x is a function of y : if $x = f^{-1}(y)$ and $x = g^{-1}(y)$, then ...

- Sometimes it is easier, or might only seem possible, to compute an area integral by flipping perspective and making it about "integrating dy" instead of "dx".

Exercise 4. Find the area between the curves $y = \ln(x)$ and $y = x$.

Question 4. How does the area between curves relate to the area under the curve?

Exercise 5.

a. Find the area under the curve $y = \sin(x)$ from $x = 0$ to $x = 2\pi$.

b. Find the area between the curves $y = \sin(x)$ and $y = 0$ from $x = 0$ to $x = 2\pi$.

Exercise 6. Find the area between the curves $x = 2y$ and $x = y^2 - x$.