Name: ______________________________

Math 1152 Summer 2022 Practice Midterm 2

Due: 11:59pm, July 5, 2022

Before Starting

- Complete Problems 1 and 2.
- Choose another **2** of the following problems to complete in addition to problems 1 and 2. You will not receive additional credit for completing additional problems beyond these two.
- This exam should reflect your work only and is not a group assignment.
- This exam is open-notes.
- This exam is due at **11:59pm, Tuesday July 5**.
- Full work must be shown for all problems you submit.
- **Fill in the circles below corresponding to the 2 additional problems you choose to have graded. Problems 1 and 2 have already been filled in for you.**

I am submitting for grading:

- ● **Question 1**
- ● **Question 2**
- ○ **Question 3**
- ○ **Question 4**
- ○ **Question 5**
- ○ **Question 6**
- ○ **Question 7**
- ○ **Question 8**

Question 1.

Let (a_n) and (b_n) be sequences and suppose that the series $\sum(-1)^n b_n$ converges and that

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 1.$$

Does $\sum a_n$ converge? Explore this question to the full extent that you can; are there variations on the conditions given that make it true or false? Give examples illustrating your reasoning.

Question 2.

Consider the series
$$\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}.$$

Find an upper bound for this series by thinking about one of our convergence tests. Do not directly use any results about tails of series in your solution.

Question 3.

Consider the series

$$\sum_{n=1}^{\infty} a_n$$

where

$$a_n = \frac{(n!)^3}{(3n)!}.$$

a. Determine whether $\sum_{n=1}^{\infty} a_n$ converges.

b. By considering just the tail

$$t_N = \sum_{n=N}^{\infty} a_n$$

for very large N, and comparing this to a more familiar series, we can get a bound on the size of the t_N and so control the rate of convergence.

For large N, what is the bound we obtain on $|t_N|$?

Question 4.

a. Find an estimate for the tail of the series

$$\sum \frac{2^n}{n!}.$$

b. Should we expect that the actual rate of convergence is faster than this? If so, see if it is possible to improve on the rate in part (a).

Question 5.

a. Find a bound for the tail of the series $\sum_{k=1}^{\infty} \frac{1}{k^{\cdot 5}}$.

What is the difference between the upper and lower bound of your approximation for $\sum_{k=1}^{n} \frac{1}{k^{\cdot 5}}$ when $n = 1000$?

Question 6.

a. Show that the sequence (a_n) given by

$$a_n = \frac{1}{n^{2/log(n)}}$$

gives a series $\sum a_n$ which satisfies the conditions of the Integral Test.

b. Explain why the Integral Test is not a good choice for determing convergence of the series $\sum a_n$.

c. Recall our derivation of the Integral Test. Instead of bounding the area of rectangles by an integral, try considering new rectangles with base lengths of 2^n. Can you use this to write down a new series which bounds the old one?