Math 1152 Lecture Notes

May 18, 2022

1 Solids of Revolution - Washers, Cont'd.

Volume of a Solid of Revolution - Washer Method (integrating dy)

Suppose that the region R lies between $y = a$ and $y = b$ and does not intersect the line $x = c$. Then if R is rotated about $x = c$, the volume of the resulting solid is given by

$$\int_a^b \pi \left(R^2(y) - r^2(y)\right)\, dy$$

where $R(t)$ and $r(t)$ are the inside and outside distances from the line $x = c$ to the region R with a y-coordinate of t.

Volume of a Solid of Revolution - Washer Method (integrating dx)

Suppose that the region R lies between $x = a$ and $x = b$ and does not intersect the line $y = c$. Then if R is rotated about $y = c$, the volume of the resulting solid is given by

$$\int_a^b \pi \left(R^2(x) - r^2(x)\right)\, dx$$

where $R(t)$ and $r(t)$ are the inside and outside distances from the line $y = c$ to the region R with a x-coordinate of t.

Exercise 1. Suppose that the circle $(x-2)^2 + y^2 = 1$ is rotated about the x-axis to form a shape. If you can, think about what that shape is. Set up an integral that will represent the volume of this shape.

Observations:

- We do not need to be able to visualize the end-result in order to answer one of these questions.
- We first reduce to the two-dimensional region being rotated around a given line
- and we next reduce to the *line-segment* being rotated around that forms a given washer.
- **Washers occur when the line-segment is perpendicular to the axis of rotation.**

2 Solids of Revolution - Shells

- What happens if we take a *vertical* slice when the axis of rotation is vertical?
- The shape that we get when rotating a line segment about a parllalel axis is called a **shell**.

Area of a Shell:

$$A = 2\pi r h$$

where

$$r = |t - c|,$$

and

$$h = f(t) - g(t) = \text{length of the line-segment.}$$

The Method of Shells

Volume of a Solid of Revolution - Shell Method (integrating dx)

Suppose that the region R lies between $x = a$ and $x = b$ and does not intersect the line $x = c$. Then if R is rotated about $x = c$, the volume of the resulting solid is given by

$$\int_a^b 2\pi r(x)h(x)\,dx$$

where $r(x) = |c - x|$ is the distance of the parameter x to the axis of rotation, and $h(x)$ is the height of the vertical line segment with coordinate x whose endpoints lie on the top and bottom of the region.

Volume of a Solid of Revolution - Shell Method (integrating dy)

Suppose that the region R lies between $y = a$ and $y = b$ and does not intersect the line $y = c$. Then if R is rotated about $y = c$, the volume of the resulting solid is given by

$$\int_a^b 2\pi r(y)h(y)\,dy$$

where $r(y) = |c - y|$ is the distance of the parameter y to the axis of rotation, and $h(y)$ is the width of the horizontal line segment with coordinate y whose endpoints lie on the left and right side of the region.

Exercise 2. Consider the region, R, between the lines $y = x^2$ and $y = x^3$ from $x = 0$ to $x = 1$. Suppose R is rotated in space around the line $x = 0$, creating a solid, S. Find the volume of S using the Method of Shells.

- Geometrically, what do the Riemann sums corresponding to volume via shells correspond to?

3 Putting it all together

Exercise 3. The region R is bounded by the curves $x = y^2 - 4$ and $y = 2 - x$.

a. Find the **area** of the region R.

b. If the **cross-sections** of a solid with base R taken perpendicular to the x-axis are semi-circles with base on the region R, set up an integral or sum of integrals which represent the **volume** of that solid.

c. If the **cross-sections** of a solid with base R taken parallel to the x-axis are right-isosceles triangles with hypotenuse on the region R, set up an integral or sum of integrals which represent the **volume** of that solid.

d. If the region R is rotated about the line $y = 5$ to form a solid, set up an integral or sum of integrals which would yield it's volume via the Washer Method.

e. If the region R is rotated about the line $y = 5$ to form a solid, set up an integral or sum of integrals which would yield it's volume via the **Shell Method**.

For next time: Can we find the *perimeter* of the region R in the problem above?