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Assimilação de dados via Filtro de Kalman por Conjunto com o Sistema de Lorenz.

Data Assimilation by Ensemble Kalman Filter with the Lorenz Equations

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Resumo

Assimilação de Dados é um procedimento matemático onde se combina dados observados com informação a priori (geralmente previsão de curto prazo) considerando-se o conhecimento estatístico dos erros de observação e previsão. Neste trabalho avalia-se a performance de uma implementação do filtro de Kalman por Conjuntos na assimilação de dados sintéticos com o Modelo de Lorenz. As equações de Lorenz são amplamente utilizadas para na avaliação de esquemas de assimilação de dados por ser um sistema de baixa dimensão, mas altamente não-linear ou caótico, como a atmosfera terrestre. Com base nos resultados, conclui-se que, para esta implementação, o conjunto com 10 membros é a melhor configuração, pois com 50 e 100 membro no conjunto ocorre "overfitting". Avaliou-se a eficiência do filtro ao assimilar dados com 20% e 40% de ruído e concluiu-se que com 40% de ruído o sistema falha, principalmente no final do período de integração. Conclui-se também, que o EKF precisa de dados com boa amostragem temporal para resolver o problema da assimilação de dados em dinâmica caótica.

Palavras-chave: *Assimilação de Dados, filtro de Kalman por Conjunto, Modelo de Lorenz.*

Abstract

Data Assimilation is a procedure to get the initial condition as accurately as possible, through the statistical combination of collected observations and a background field, usually a short-range forecast. In this research a complete assimilation system for the Lorenz equations based on Ensemble Kalman Filter is presented and examined. The Lorenz model is chosen for its simplicity in structure and the dynamic similarities with primitive equations models, such as modern numerical weather forecasting. Based on results, was concluded that, in this implementation, 10 members is the best setting, because there is an overfitting for ensembles with 50 and 100 members. It was also examined if the EnKF is effective to track the control for 20% and 40% of error in the initial conditions. The results show a disagreement between the "truth" and the estimation, especially in the end of integration period, due the chaotic nature of the system. It was also concluded that EnKF have to be performed sufficiently frequently in order to produce desirable results.

Keywords: *Data Assimilation, ensemble Kalman filter, Lorenz model.*

1 Introduction

Numerical Weather Prediction (NWP) is an initial and boundary value problem, i.e. given an estimate of the initial state of the atmosphere, and appropriate surface and lateral boundary conditions, the future state can be predicted by integrating momentum equations with physical processes. Data Assimilation is a procedure to get the Initial Condition (IC) as accurately as possible, through the statistical combination of collected observations and a background field, usually a short-range forecast.

Observations are collected in limited number of observing stations, has error in measurements and error of representativeness. Numerical models are not a perfect representation of nature and the earth's atmospheric system is nonlinear, which introduces dynamically chaotic behavior. Moreover, the large number of degrees of freedom is an additional problem in operational weather forecasting. By this reason, the modern methods of data assimilation take into account the observation and background errors covariance. The state of art in data assimilation techniques are based on variational calculus and Kalman filter theory (e.g., Courtier et al., 1993; Pires et al., 1996; Gauthier, 1992; Kalman, 1960; Xue et al., 2013).

The large dimension of models can prohibit the implementation of KF in operational NWP, but its implementation with simplifications, such as ensemble Kalman filtering - EnKF (Evensen 1994; Burgers et al., 1998; Whitaker et al., 2002; Tippet et al., 2003), ensemble transform Kalman filter - ETKF (Bishop et al., 2001), ensemble adjustment Kalman filter – EAKF (Anderson, 2001), local ensemble Kalman filter (LEKF) (Ott et al., 2002, 2004), hybrid EnKF-3DVar (Gao et al., 2013) is possible due to the decrease in the computational cost.

In accordance to Miyoshi (2005), EnKF can be divided into two groups: a perturbed observation (PO) method and a square root filter (SRF) method. In this research, the author proposes to apply an EnKF based on PO method to the Lorenz equations. The PO method uses an independent data assimilation cycle for each ensemble member. In this work, likewise Burgers et al. (1998), the members of the ensemble was created adding a random Gaussian noise to the model equations. Due the random generation of the member, different implementation of EnKF must be available.

The main objective of this research is to examine the behavior of EnKF for different ensembles size, evaluate error in the IC and explore the assimilation over-determined by lack of observation.

Section 2 describes a brief theoretical formulation of Lorenz equations; Section 3 presents a basic introduction to EnKF; numerical experiments are shown in Section 4 and finally Sections 5 contains the concluding remarks.

2 The Lorenz Equations

Lorenz (1963) was looking for the periodic solutions of the Saltzman's model (Saltzman, 1962), considering a spectral Fourier decomposition and taking into account only low order terms. Lorenz obtained the following system of nonlinear coupled ordinary differential equations

$$dX/d\tau = -\sigma(X - Y) \tag{1}$$

$$dY/d\tau = rX - Y - XZ \tag{2}$$

$$dZ/d\tau = XY - bZ \tag{3}$$

where $\tau \equiv \pi^2 H^{-2}(1+a^2)\kappa t$ is the non-dimensional time, being H, a, κ and t respectively the layer height, thermal conductivity, wave number (diameter of the Rayleigh-Bérnard cell), and time; $\sigma \equiv \kappa^{-1}\nu$ is the Prandtl number (ν is the kinematic viscosity); $b \equiv 4(1+a^2)^{-1}$. The parameter $r = R/R_c \propto \Delta T$ is the Rayleigh number (T is the temperature), and R_c is the critical Rayleigh number.

3 Ensemble Kalman Filter

Kalman filter is the best linear unbiased estimator for a linear model under Gaussian assumption for the measurements and model random errors. The Kalman filter method applied to nonlinear models is called the Extended Kalman Filter (EKF), given by two step, as follow:

1. Forecast step

$$w^f_{n+1} = F_n w^f_n + \mu_n$$

$$P^f_{n+1} = F_n P^a_n F^T_n + Q$$

2. Analysis step

$$K_{n+1} = P^f_{n+1} H^T_{n+1} [R_{n+1} + H_{n+1} P^f_{n+1} H^T_{n+1}]^{-1}$$
$$w^a_{n+1} = w^f_{n+1} + K_{n+1} \lfloor y^0_{n+1} - H(w^f_{n+1}) \rfloor$$
$$P^a_{n+1} = [I - K_{n+1} H_{n+1}] P^f_{n+1}$$

where F^f_n is our mathematical model, μ_n is the stochastic forcing (modeling noise error), subscripts n denotes discrete time-step, and superscripts f represents the forecasting value. The observation system $\lfloor y^0_{n+1} - H(w^f_{n+1}) + y_n \rfloor$ is modeled by H matrix, and v_n is the noise associated to the observation. The typical gaussianity, zero-mean and ortogonality hypotheses for the noises are adopted. The state vector is defined as $w_{n+1} = [X_{n+1}, Y_{n+1}, Z_{n+1}]^T$, and it is estimated through the recursion $w^a_{n+1} = w^f_{n+1} + K_{n+1} \lfloor y^0_{n+1} - H(w^f_{n+1}) \rfloor$, where w^a_{n+1} is the analysis value, K_n is the Kalman gain, computed from the minimization of the estimation error variance J_{n+1}, (Jazswinski, 1970) $J_{n+1} = E\{(w^a_{n+1} - w^f_{n+1})^T (w^a_{n+1} - w^f_{n+1})\}$, being $E\{.\}$ the expected value, Q_n is the covariance of μ_n and R_n is the covariance of v_n. The assimilation is done through the sampling: $r_{n+1} \equiv z_{n+1} - z^f_{n+1} = z_{n+1} - H_n w^f_{n+1}$. "Even if a system stars with a poor initial guess of the state of the atmosphere, the EKF may go through an initial transient period of a week or so, after which it should provide the best unbiased estimate of the state of the atmosphere and its error covariance (Kalnay, 2004)." However, in according Miller et al. (1994), if the system is very unstable, and the observation are not frequent enough, it is possible for the linearization to became inaccurate, and the EKF may drift away from the true solution.

The updating of Eq. (5) provides the *errors of the day*, but its computational cost makes this updating impossible in practice to carry out. Therefore the Eq. (5), has been replaced by the use of simplifying assumptions, such as ensemble mean. In this work is proposed an ensemble Kalman filter which consist of replace the forecast error covariance (Eq. 5) by:

$$P^f \approx \frac{1}{K-1} \sum_{k=1}^{K} \left(X^f_k - \bar{X}^f \right) \left(X^f_k - \bar{X}^f \right)^T$$

where the ensemble has K data assimilation cycles.

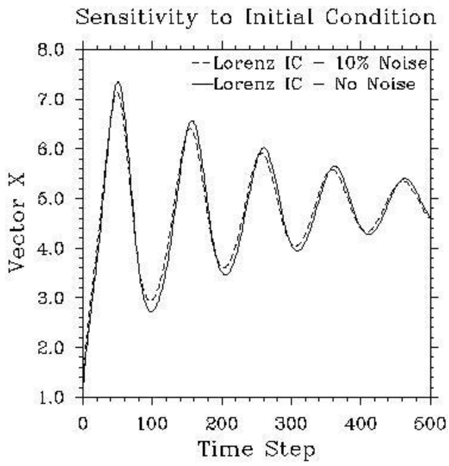
4 Numerical experiments

In the results explored forward from here, the Lorenz equations are solved by finite-differences with a non-dimensional time increment 0.01 for 500 time-steps integration length. According to Lorenz (1963) at the give σ = 10 and b = 8/3, the corresponding Rayleigh number is 24.74, which means that r larger than 24.74 will make the system a chaotic one. In this paper only the variable X is depicted, just because Y and Z take to the same conclusions.

Before to presents the results, the author show the sensitivity of the model to the initial conditions. In Figure 1a, is depicted the resulting model state trajectories for σ = 10, b = 8/3 and r = 10, assuming the IC to be X_0 = 1.00, Y_0= 3.00, Z_0 = 5.00 (Case 1) and assuming IC to be X_0 = 1.10, Y_0 = 3.30, Z_0 = 5.50 (Case 2). It means that Case 1 differ from Case 2 with an offset of 10% of noise for all model variables. In Figure 1b, this experiment is redo for chaotic regime (r = 32), Case 3 and Case 4.

This simple experiment show that for r = 10, the trajectories of Case 1 and Case 2 remain very close to each other during the integration period. By other side, for r = 32, the bifurcation in model solution occur throughout the integration period. This experiment illustrates the sensitivity of the chaotic system to IC.

The purpose of the experiments presented below is to explore the ideas delineated in the EnKF methodology.



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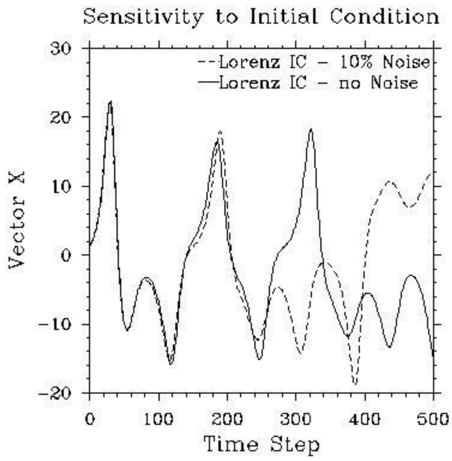
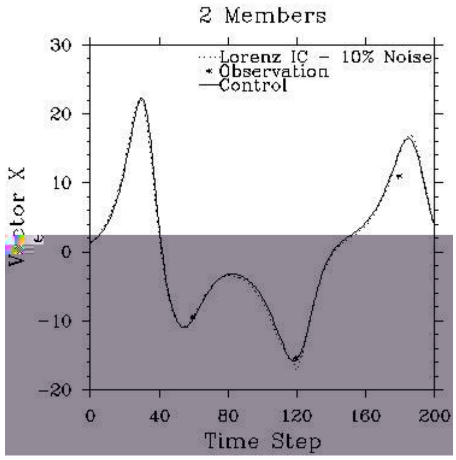


(b)

Figure 1. Resulting model state trajectories for σ = 10, b = 8/3 and r = 10, assuming the IC to be X_0 = 1.00, Y_0= 3.00, Z_0 = 5.00 (Case 1) and assuming IC to be X_0 = 1.10, Y_0 = 3.30, Z_0 = 5.50 (Case 2). It means that Case 1 differ from Case 2 with an offset of 10% of noise for all model variables. In Figure 1b, this experiment is redone for chaotic regime (r = 32), Case 3 and Case 4.

4.1 EXPERIMENT 1 – Evaluation of Ensemble Size in the EnKF

The goal of this experiment is to show the impact of the ensemble size in the results of EnKF implemented in the Lorenz equations. Figure 2 plots the Control (X_0 = 1.00, Y_0 = 3.00, Z_0 = 5.00, r = 32 – chaotic regime), Case 4 above (X_0 = 1.10, Y_0 = 3.30, Z_0 = 5.50, r = 32 – chaotic regime) for 2, 10, 50 and 100 ensemble sizes, and three model state observations available during “observation window”. It was assumed that this observation have a Gaussian randomly distributed error of the maximum 10%. By the figure can be seen that the EnKF is able to keep the system stable, but is not possible to conclude which ensemble size is the best one. Then in Tab. 1, Root Mean Square (RMS) is presented for all ensemble sizes tested. It could be concluded that more than 10 member does make any improvement in the estimation, due the overfitting caused by large ensembles sizes.



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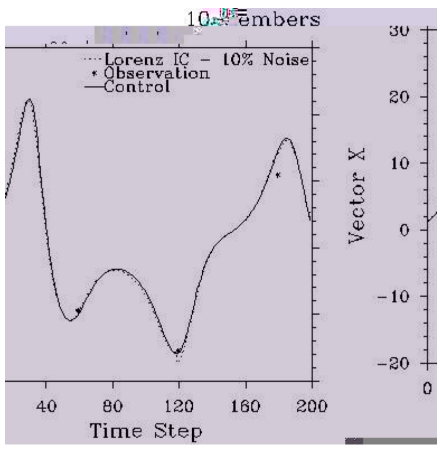

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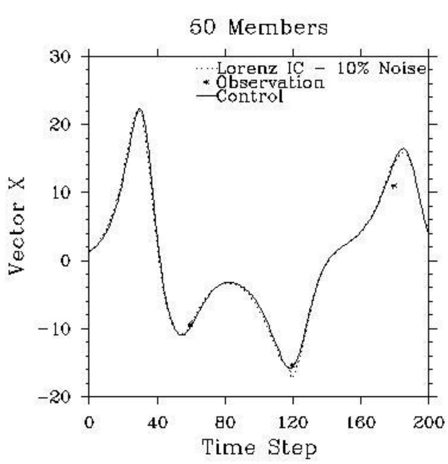

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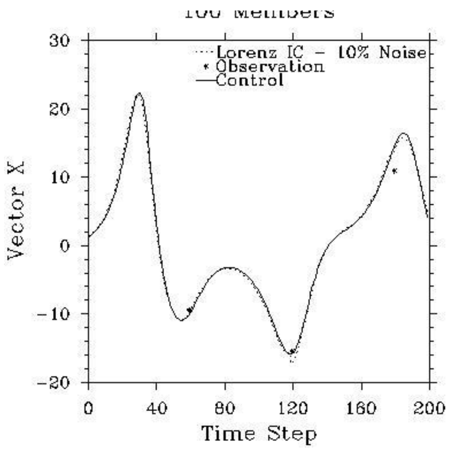


(d)

Figure 2 - Control (X_0 = 1.00, Y_0 = 3.00, Z_0 = 5.00, r = 32 – chaotic regime), Case 4 above (X_0 = 1.10, Y_0 = 3.30, Z_0 = 5.50, r = 32 – chaotic regime) for 2, 10, 50 and 100 ensemble sizes, and three model state observations available during “observation window”.

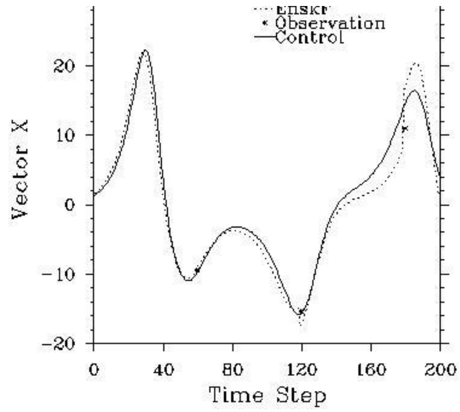
Table 1: Root Mean Square for all three Lorenz variables.

Ensemble Size (members)	2	10	50	100
RMS	2.014	1.678	1.751	1.818

4.2 EXPERIMENT 2 – Evolution of the EnKF for poor IC

In this experiment, shown in Fig. 4, EnKF assimilates the same evolution trajectories as the EXPERIMENT 1, but the initial guess is rather poor, 20% (X_0 = 1.20, Y_0 = 3.60, Z_0 = 6.00) and 40% (X_0 = 1.40, Y_0 = 4.20, Z_0 = 7.00).

One may observe, from plots in Fig. 3, that with 20% of noise, the differences between the simulated trajectories and the observations as well as “true trajectories” are rather small. However, the differences are increasingly significant at the later part of the integration period, due the chaotic behaviour of the system. However, for the case with 40% error in the IC, the EnKF is not able to track the Control with only 3 observation ingested. In fact, given 40% error in the IC, the chaotic nature of model solution, demands for a relatively large number of observations.



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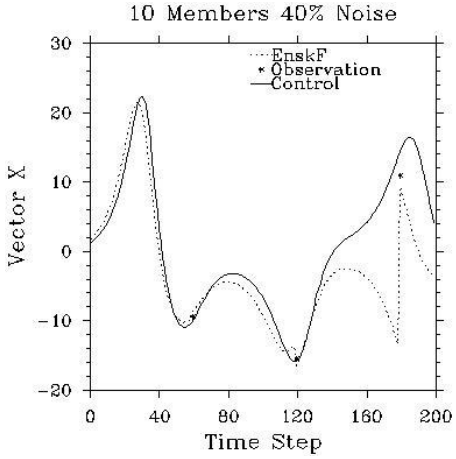

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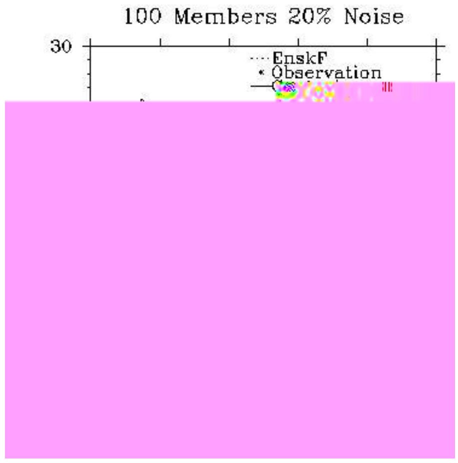

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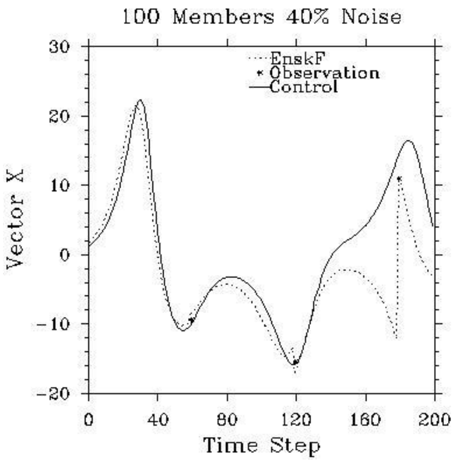

(d)

Figure 3 – EnKF assimilates the same evolution trajectories as the EXPERIMENT 1, but the initial guess is rather poor, 20% (X = 1.20, Y = 3.60, Y = 6.00) and 40% (X = 1.40, Y = 4.20, Y = 7.00).

4.3 EXPERIMENT 3 – Assimilation over-determined by lack of observation

Numerical Weather Forecasting is of the order of 16^6-10^7 degrees of freedom, whereas the total number of conventional observation of the variables used in the models is of the order of 10^4 (Kalnay, 2004). Considering this fact, the amount of variables observation applied hitherto is somewhat overestimated. By this reason, the Experiment 3 represents a more realist [illegible]

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