AERODYNAMICS AND FLIGHT MECHANICS STUDIED ON A MCDONNELL DOUGLAS DC-10

Index:

1. Introduction:

The main purpose of this report is to study the main aerodynamical characteristics of a common civil aircraft. After collecting all aircraft data and calculating the relevant parameters, we will have a first approach to the behavior of our aircraft performing in an air flow.

The aerodynamics involve four essential forces: weight, thrust, lift and drag. All these forces must be compensated to create an equilibrium that can be able to maintain the aircraft flying. The aircraft is considered as a solid heavier than air performing in a compressible and turbulent air flow, helped with its engines to create thrust and overcome the drag force created due to the friction of the air with the wings and fuselage.

By giving a determined curvature to the wing nor modifying the angle of wind impact, it is willed the appearance of pressure gradient between both wing surfaces. Consequently, as everyone connoisseur of fluid dynamics, there's some movement of air from the higher pressure zone to the lower, resulting on a rising force.This effect is known as lift and it is scrupulously strained to compensate the aircraft's weight. Although the aircraft is, for sure, more than heavy to our eyes, this lift created with the wings and an efficient pressure distribution along the boundary layer will be more than enough to make an aircraft fly.

The aircraft model we have elected is the McDonnell Douglas DC-10, a three engined aircraft introduced in the early 70s. Known by its wide fuselage, it is considered as a large range aircraft. Somehow, in spite of the elegance and sophistication of modern models such as B787 and A350, we were attracted by the number of accident reports that the DC-10 has on its registry. In fact, a remarkable accident is the famous American Airlines' flight 191, which is the one we are willing to comment additionally.

2. Aircraft characteristics:

Dimensions:	
Length overall (m)	55.29
Length of fuselage(m)	51.97
Height overall (m)	17.68
Wingspan (m)	47.34
Tailplane span (m)	21.69
Wing chord at root (m)	10.71
Wing chord at tip (m)	2.92
Wing aspect ratio	6.8
Sweep-back at quarter-chord (º)	35

Areas:	
Wings, gross (m^2)	358.7
Tailplane (m^2)	96.6
Weights:	
Maximum Take-Off Weight (kg)	195 045
Maximum Landing Weight (kg)	164 880
Operating Empty Weight (kg)	105 142
Maximum Zero Fuel Weight (kg)	151 953
Maximum Payload (kg)	46 820
Maximum Ramp Weight (kg)	196 405
Standard Fuel Capacity (litres)	82 518
Performance:	
Range With Max Payload (km)	3909
Range With Max Fuel, Zero Payload (km)	8335
Cruise Speed (km/h)	910
Maximum Speed (km/h)	932
Mach level speed	0.88
Maximum Operating Altitude (m)	10670
Take-Off Field Length (m)	2600
Landing Field Length (m)	1720
Max rate of climb at S/L (m/min)	838
Engines:	
Propulsion	3 Turbofan Engines
Engine Model	General Electric CF6-6D
Engine Power (each)	18144 kgf / 40000 lbf

All the calculations have been made for eight different heights starting at 0m (MSL) to a value below the maximum operating height (10670m).

All the performance has been calculated for three weights:

Weights:

MTOW: **195045 kg**

TOW2: **OEW + 20% PL + 70% FW = 172267 kg**

TOW3: **OEW + 50% FW = 146401 kg**

As known: density, gravity and temperature change due to an increasing height along the troposphere. The following table shows those lineal variations and gives the values to the eight different heights we had already chosen.

	h0=0m	h1=1500 m	h2=3000 m	h3=4500 m	h4=6000 m	h5=7500 m	h6=9000 m	h7=10500 m
d(kg/ m^3)	1,23	1,06	0,91	0,78	0,66	0,56	0,47	0,39
Tª(K)	288,15	278,4	268,65	258,9	249,15	239,4	229,65	219,9
g(m/s^2)	9,81	9,81	9,81	9,81	9,81	9,81	9,81	9,81

3. Aerodynamical characteristics:

For our aircraft (DC-10), the related profile we have found is the DSMA-496. For this profile, we have calculated the maximum lift coefficient $C_{lmax} = 1.6394$.

Taking care of all the aircraft components that contribute to create drag, and calculating its contribution to the whole drag coefficient, we can draw a polar incompressible curve showing the effects of lift coefficient versus the drag coefficient. We define the drag coefficient:

$$C_D = C_{D0} + \frac{{C_L}^2}{\pi A e} \qquad C_{D0} = \frac{1}{S_{REF}} \sum C_{D\pi} A_\pi$$

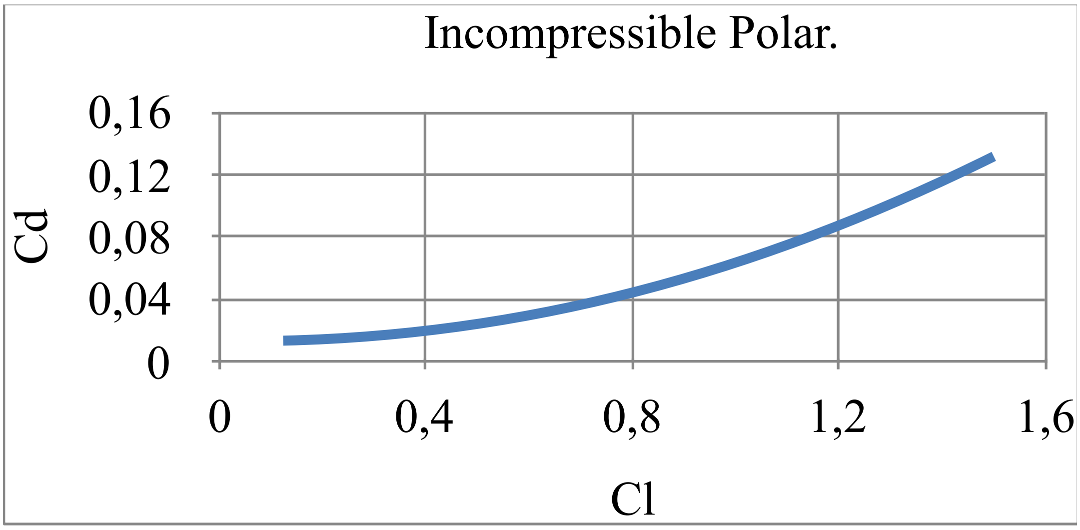
Being $C_{D\pi}$ the contribution of each component to create drag and A_π the reference areas of the components.

S_{REF} is the addition of the areas of all components that we are having in count, such as: wings, fuselage, engines, tailplane....

Finally, we obtain the incompressible polar curve for our aircraft:

Component:	$C_{D\pi i}$	$A_{\pi i}$	$C_{D\pi i} A_{\pi i}$
Wings	0,003	717,4	2,1522
Fuselage	0,0024	700,4	1,68096
Engines	0,006	65,3	0,3918
Tailplane	0,0025	193,2	0,483
Interferences	0,1	$\sum A_\pi =$	
Whole area		1676,3	
C_{D0}	0,01312		

This table harbors all the data required to draw the incompressible polar curve shown on the top of the next page.

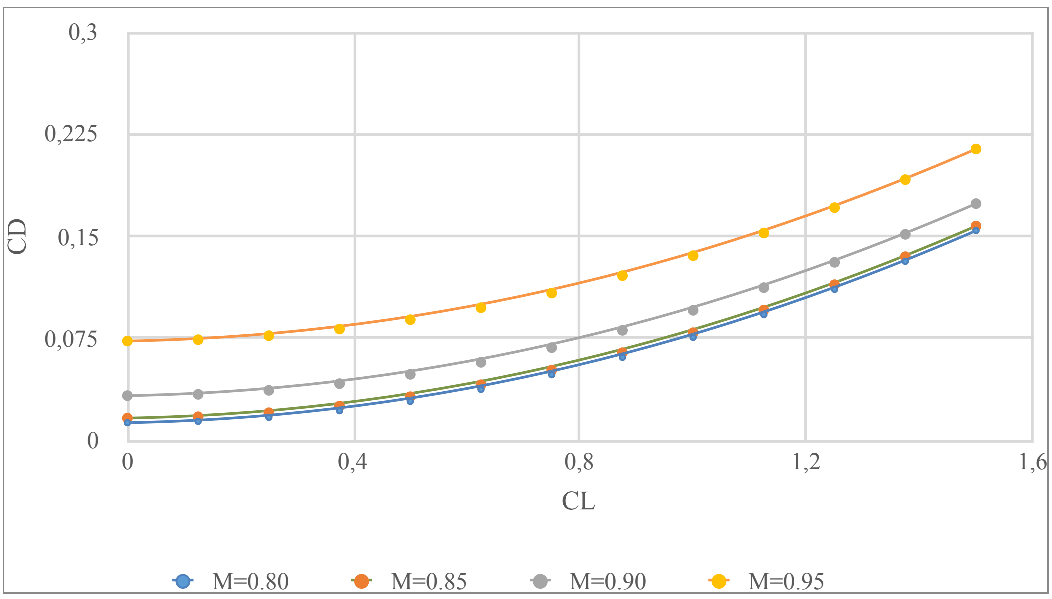


As soon as speed increases until it reaches the critical Mach number, compressibility effects become important to describe the new drag force and drag coefficient. For the DC-10, the critical Mach number is: $M_c = 0.8$.

The compressibility effects (depending on the Mach number) rise the drag coefficient, following the formula:

$$C_D(M) = C_{Dinic} + \Delta C_{Drise}(M)\ ;\ \Delta C_{Drise} = \frac{0.35}{10^3}\left[\frac{10(M - M_c)}{\left(\frac{1}{cos(\Lambda_{BA}}\right) - M_c}\right]^{\frac{3}{1+\frac{1}{1+AR}}}$$

Using the initial drag coefficient and adding the drag coefficient rise due to the mach number, we obtain new compressible polar curves for different mach numbers:



4. Polar curve slope, critical Mach number and V stall:

Once we've calculate all the polar curves for different mach numbers, we have to take into account the compressibility effects from a critical Mach number. Due to this compressibility effects, the drag coefficient begins to increase and creates additional effects on the wings.

Searching our aircraft on the internet, we have found the maximum lift coefficient for the DSMA-496 profile. With this maximum lift coefficient we can now recalculate it for a whole wing, using the following formula:

$C_{lmax} = 1.6394$ $\qquad$ $C_{Lmax}(wing) = 0.9C_{lmax}cos(\frac{\Lambda}{4})$

We finally get:

$C_{Lmax} = 1.2086$

The polar curve slope we have obtained -> $\frac{\partial C_l}{\partial \alpha} = 0 \rightarrow C_{L\alpha} = 0.62$

Critical Mach number (taken from Jane's) -> $M_c = 0.8$

With all this data, the next step is to define the '*stall speed*'. As the speed decreases further, at some point this angle will be equal to the critical (stall) angle of attack. This speed is called the "stall speed". An aircraft flying at its stall speed cannot climb, and an aircraft flying below its stall speed cannot stop descending. Any attempt to do so by increasing angle of attack, without first increasing airspeed, will result in a stall.

$V_{stall} = \sqrt{\frac{2W}{\rho S C_{Lmax}}}$ For the different 3 weights and we have chosen and for every height, we get different stall speeds:

V_{stall}	h0	h1	h2	h3	h4	h5	h6	h7
W1	84,68	91,12	98,28	106,3	115,3	125,5	137,1	150,4
W2	79,58	85,65	92,37	99,92	108,4	118,0	128,9	141,3
W3	73,37	78,94	85,15	92,12	99,95	108,7	118,8	130,3

5. Available Thrust: $T_A = f(V, h_i)$

We define the available thrust as; the available power to fly overcoming the drag force in a mission. It only depends on the speed and height. It also depends on the type of engines used to propulse the aircraft, shown in a manufacturing thrust value at sea level. Using a throttle ratio equal to 1 (TR=1) and defining the next variables, we can easily draw the available thrust curves for different Mach numbers and heights.

Taking into account that we are using high bypass ratio engines and Mach numbers between 0 and 0.9, we get:

$$\theta_0 = \theta\left(1 + \frac{\gamma - 1}{2}M^2\right) \qquad \theta_0 \le TR : \alpha = \delta_0\left(1 - 0.49\sqrt{M}\right)$$

$$\delta_0 = \delta\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma - 1}} \qquad \theta_0 > TR : \alpha = \delta_0\left(1 - 0.49\sqrt{M} - \frac{3\left(\theta_0 - TR\right)}{1.5 + M}\right)$$

Finally $\Rightarrow \dfrac{T_h(M)}{T_{sealevel}} = \alpha\tau(\%)$

6. Required Thrust: $T_R = f\left(v, h_i, W_i\right)$

As we already defined, thrust is the force an aircraft needs to be propelled. Overcoming the drag force, engines create thrust due to burning fuel and mixing it with compressed air. The required thrust is the thrust an aircraft will need to complete a full mission.

Starting from the hypothesis of an uniform and level flight and symetric aircraft, the required thrust follows the next equation:

For incompressible flow: T=D

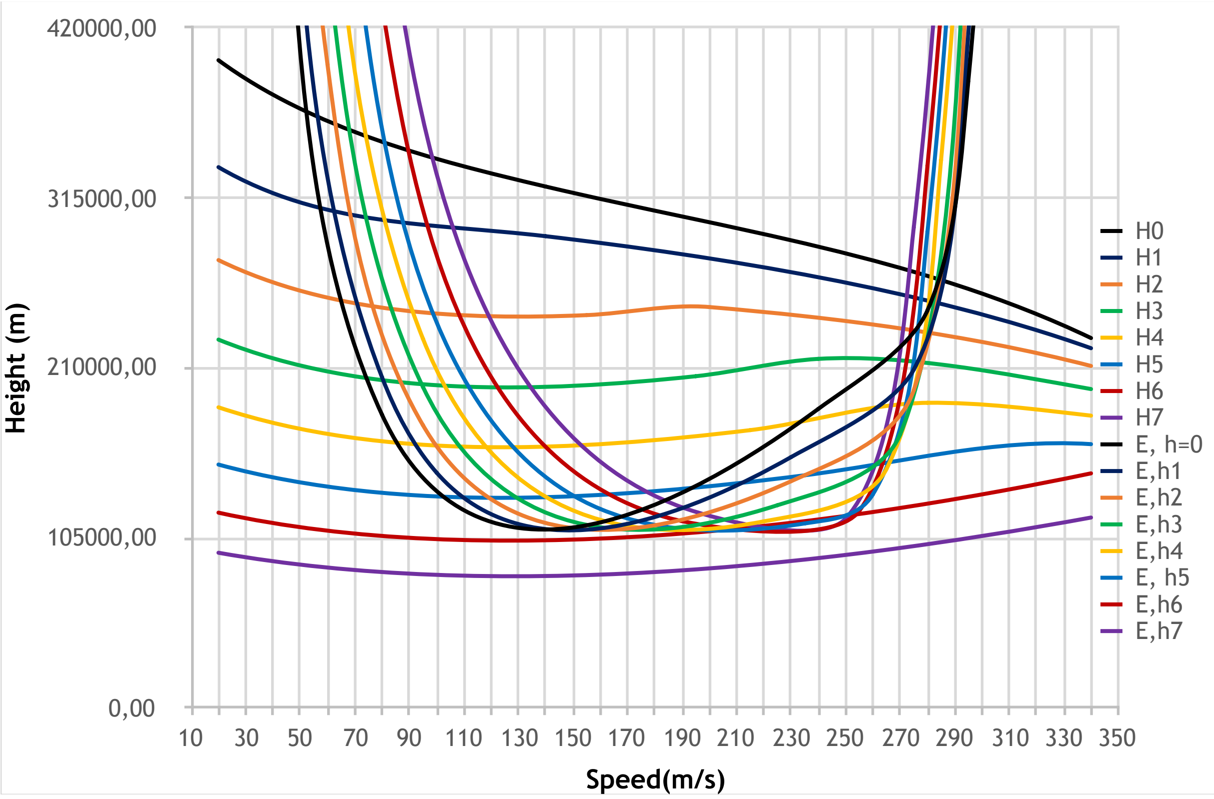
$$T_R = D = \frac{1}{2}\rho V^2 S\left(C_{D_{L=0}} + \frac{{C_L}^2}{\pi A e}\right)$$

For compressible flow: $M > M_C$

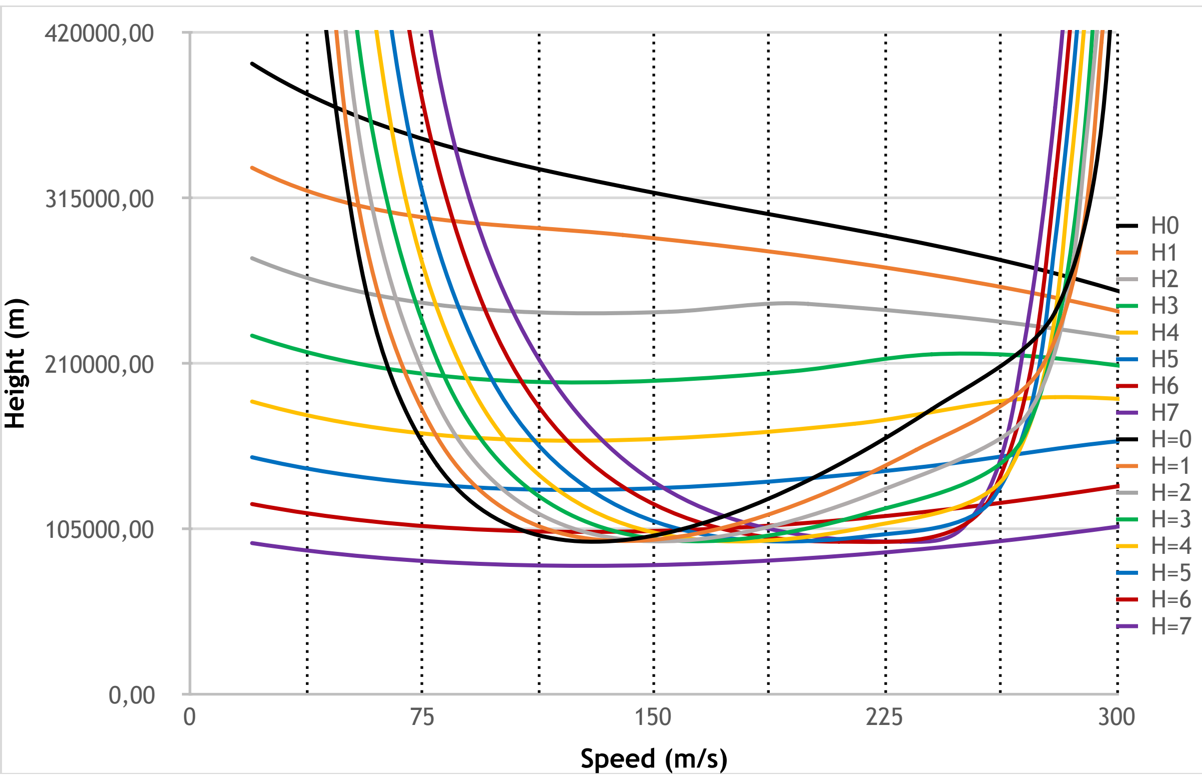
$$T_R = D = \frac{1}{2}\rho V^2 S\left(C_{D_{L=0}} + \frac{{C_L}^2}{\pi A e} + \Delta C_{Drise}\right)$$

Using those formulas, we can obtain the required thrust and the available thrust. As it follows, we can represent graphically the available thrust curves and the required thrust curves together for different heights and for each weight we have taken:

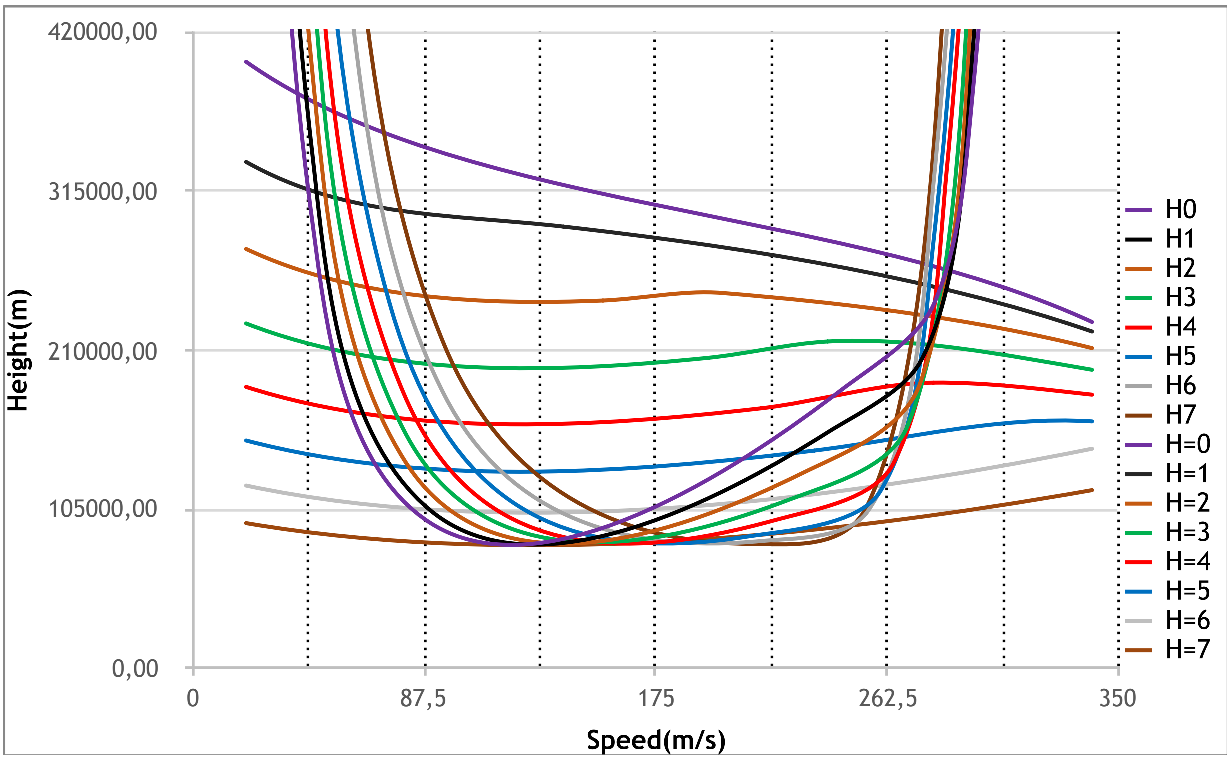
<u>FIRST WEIGHT (W1): MTOW: 195045 kg.</u>



<u>SECOND WEIGHT (W2): TOW2: OEW + 20% PL + 70% FW = 172267 kg.</u>



THIRD WEIGHT (W3): TOW3: OEW + 50% FW = 146401 kg.



It's important to remember that if we want to obtain a minimum required thrust, the only parameters that influence are: aircraft weight and aerodynamical efficiency. To get a minimum required thrust we have to decrease aircraft weight and rise the lift coefficient, making the aerodynamical efficiency maximum.

$$T_{min} = W\left(\frac{C_D}{C_L}\right)_{min}$$

7. Flight envelope:

Once we have obtained graphically the required and the available thrust representations, we can see that both curves have two intersections for low and high speeds. This shows the minimum and maximum horizontal speed, both obtained matching the required and the available thrust expressions:

Maximum horizontal speed:

$$T_R = T_A \Rightarrow V_{max} = \sqrt{\frac{\frac{T_R}{W}\frac{W}{S} + \frac{W}{S}\sqrt{\left(\frac{T_R}{W}\right)^2 - \frac{4C_{D0}}{\pi Ae}}}{\rho C_{D0}}}$$

Minimum horizontal speed:

The minimum horizontal speed is taken as the maximum speed between the minimum speed for $T_R = T_A$ and V_{stall}

$$V_{min} = max\left[V_{min} \mid T_R = T_A, V_{stall}\right]$$

Where:

- $V_{min} \mid T_R = T_A \Rightarrow V_{min} = \sqrt{\dfrac{\frac{T_A}{W}\frac{W}{S} - \frac{W}{S}\sqrt{\left(\frac{T_A}{W}\right)^2 - \frac{4C_{D0}}{\pi Ae}}}{\rho C_{D0}}}$

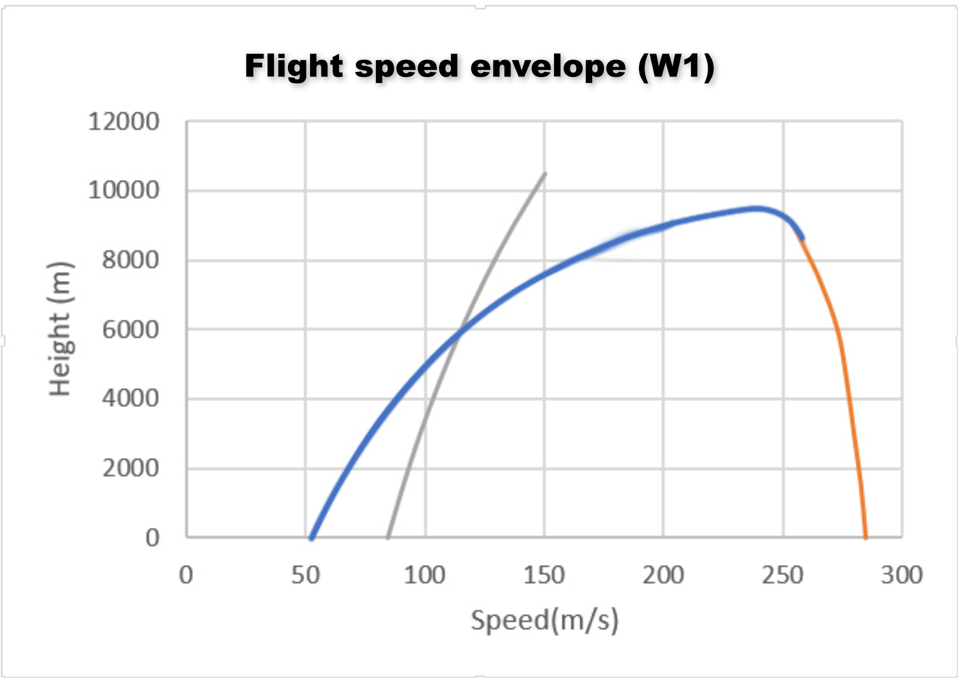
- $V_{stall} = \sqrt{\dfrac{2W}{\rho S C_{Lmax}}}$

With these speeds for different heights we can draw the flight envelope, defined as: the graphic representation of the flight performance, showing the speed and height limits to ensure a safe flight. The more centered the flight is, the more freedom to performance the aircraft has.

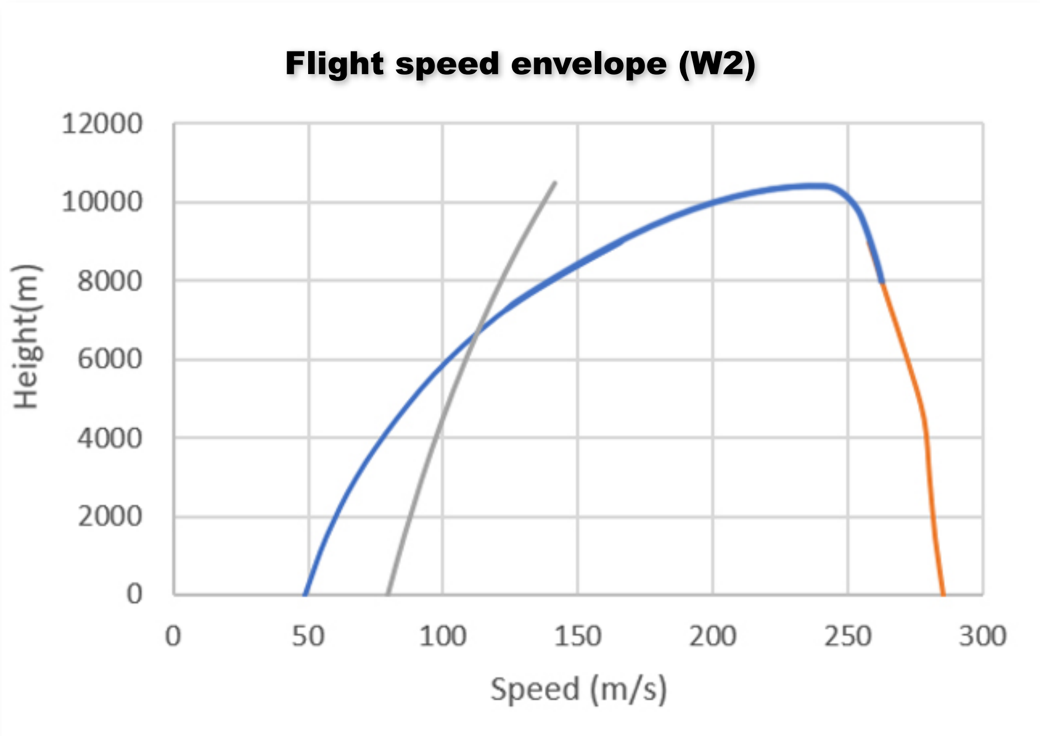
On the envelope's edges (in terms of true airspeed), the aircraft performance is limited by the maximum thrust. The lower edge shows the stall speed and the upper edge shows a critical speed related to the divergence Mach number, when compressibility effects have to be taken into account for a rising drag force.

At the top, is shown the maximum operating height of our aircraft.

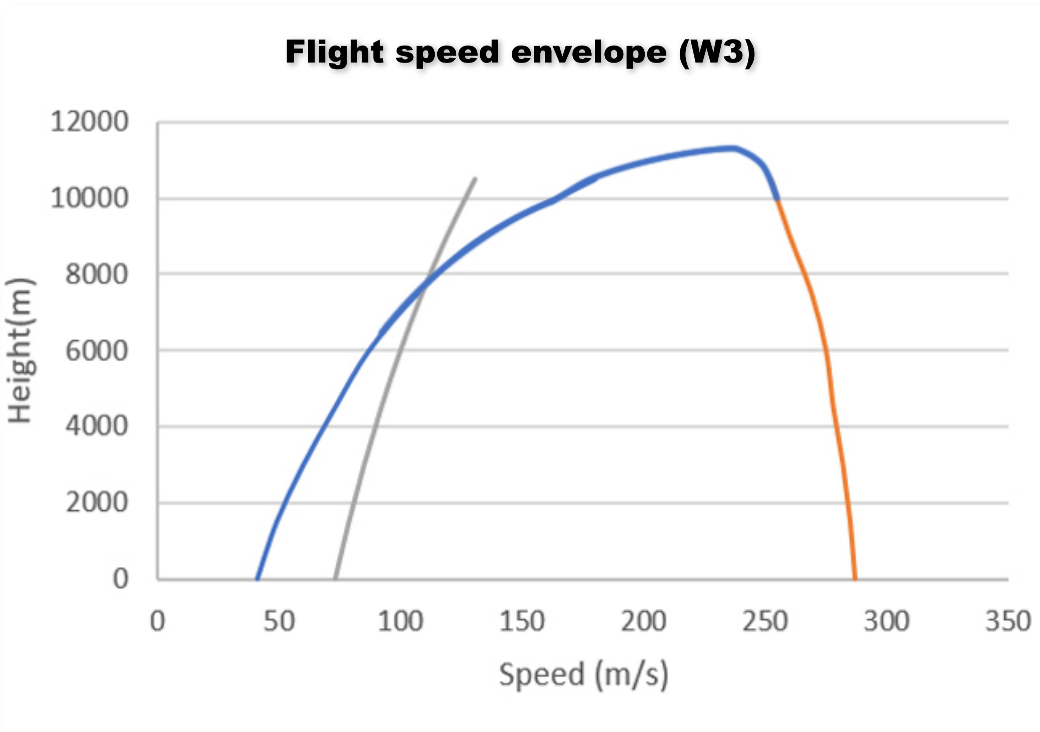
FIRST WEIGHT (W1): MTOW: 195045 kg.



SECOND WEIGHT (W2): TOW2: OEW + 20% PL + 70% FW = 172267 kg.



THIRD WEIGHT (W3): TOW3: OEW + 50% FW = 146401 kg.



8. Lift Speed:

Considering an uniform flight, symmetric aircraft and bank angle different to zero, we can define the lift speed as the speed an aircraft uses to change the flight level its flying on. The dynamical equations that describe the movement are:

$$T = Wsen(\gamma) + D \qquad Wcos(\gamma) = L$$

From these equations, we can get a first approximation to the form of lift speed: $V_S = f(T, \gamma, W, h)$. It depends on flight conditions and aircraft performance.

$$V_S = \frac{R}{C} \to \to V_S = v\left(\frac{T - D}{W}\right)$$

Lift speed appears when the thrust required to climb is bigger than zero, given by $T_A - T_R > 0$, the expression $T_A - T_R$ is called ''thrust excess''. When the horizontal speed is maximum, the thrust excess is equal to zero, it means that the aircraft doesn't have power left to climb: $T_A = T_R \to T_A - T_R = 0$.

We get the maximum lift speed when the thrust excess is maximum too: $(T_A - T_R)\,|_{max}$.

Solving the previous equation, we get an analytic expression for the lift speed:

$$V_S = v\left(\frac{T}{W} - \frac{1}{2}\rho v^2 \left(\frac{W}{S}\right)^{-1} C_{D0} - 2\frac{W}{S}\frac{\frac{1}{\pi Ae}cos^2(\gamma)}{\rho v^2}\right)$$

Due to the small errors, we can make a first approximation supposing that $\gamma << 1 \to cos(\gamma) \approx 1$, in this way, we can approximate the lift speed as:

$$V_S = \left(\frac{T_A - T_R}{W}\right)$$

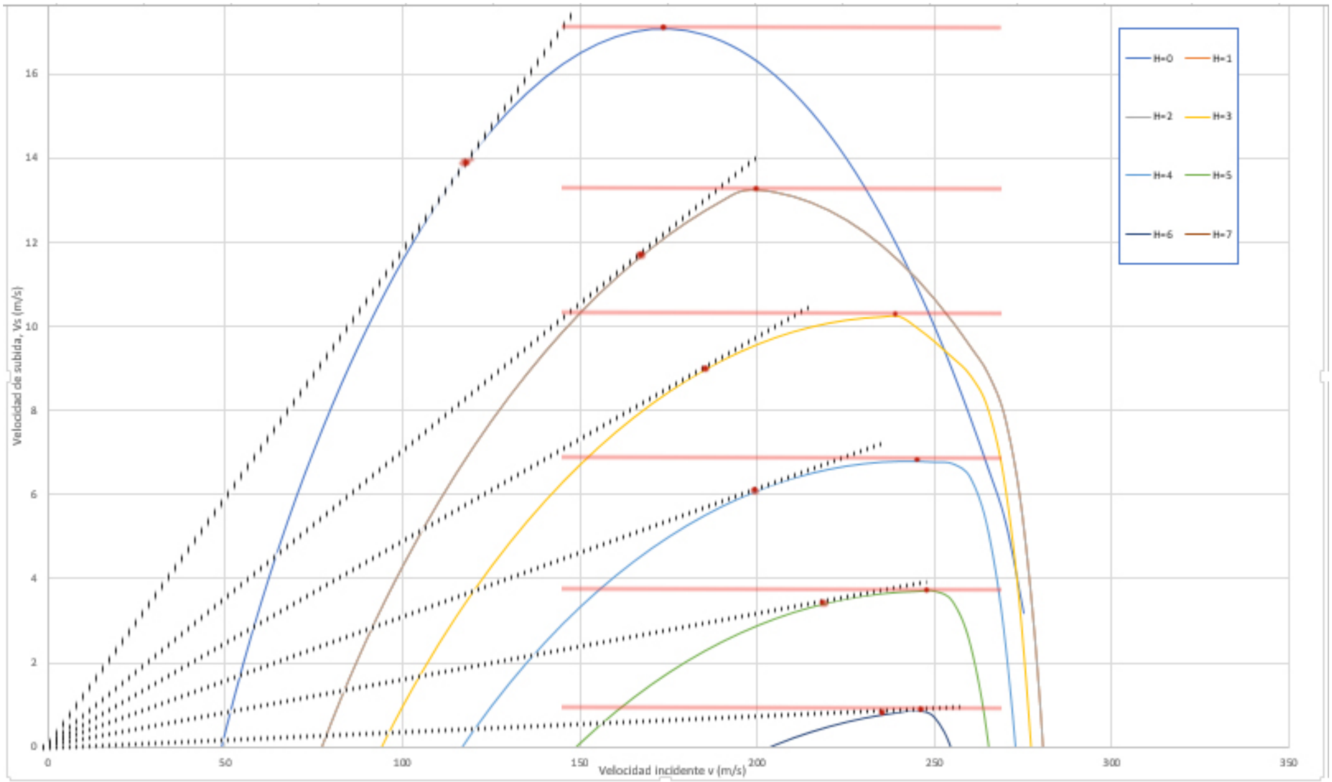
The following diagrams show the lift speed versus the incident airspeed, these diagrams decrease as height does, getting lower lift speeds for high airspeed due to the decreasing excess of power.

When the lift speed becomes zero, that means that the aircraft has reached its maximum operating height.

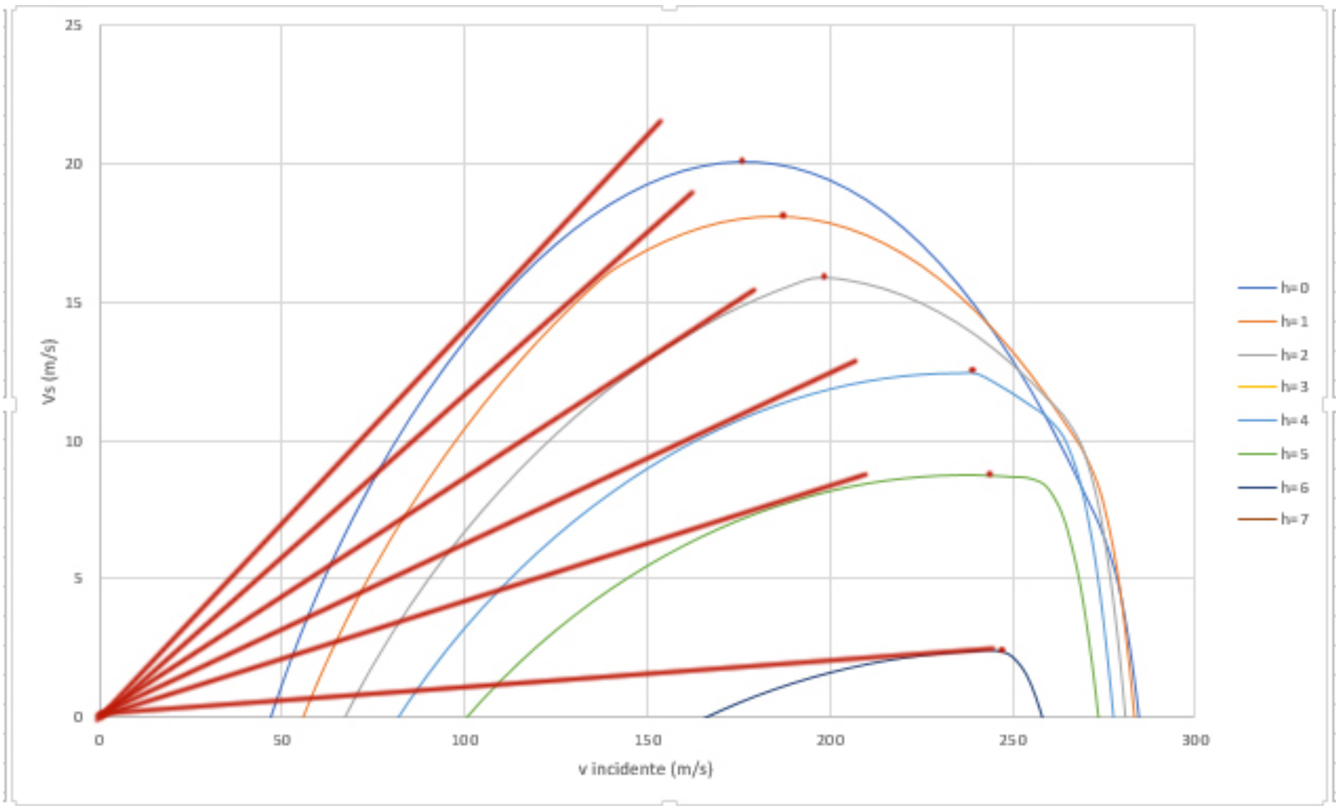
We can define two important points on the diagrams:

- Maximum lift speed: It’s the max of the maximum lift speed.
- Optimus lift speed: When the lift angle is maximum $V_{Sopt} \to V_s(\gamma_{max})$

FIRST WEIGHT (W1): MTOW: 195045 kg.



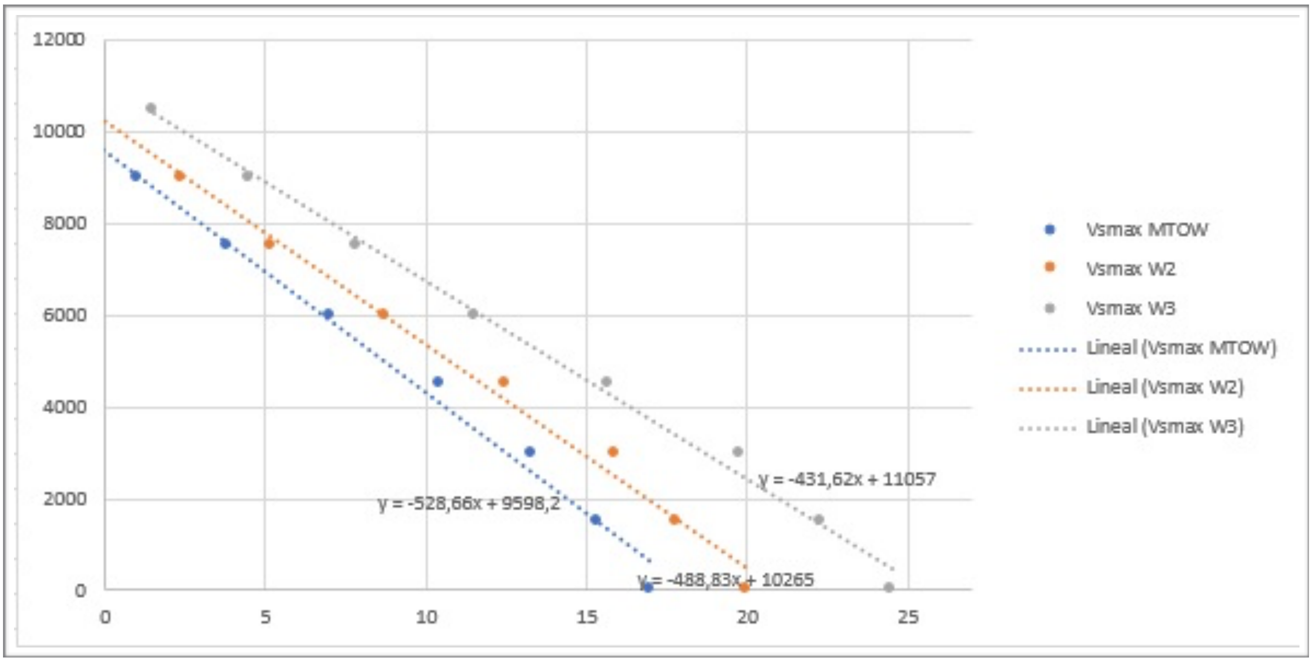
SECOND WEIGHT (W2): TOW2: OEW + 20% PL + 70% FW = 172267 kg.



9. Maximum lift speed and maximum lift angle:

As we said before, the maximum operating height of an aircraft, can be defined for a zero lift speed, showing that there's no more excess of power left to continue climbing.

The maximum lift speed increases is wing load (W/S) does it too. So, we can represent graphically the maximum lift speed for our three different weights.



The optimum lift speed is defined for a maximum bank angle (lift angle), this angle can be defined and calculated with the movement equations for an equilibrated uniform flight climbing with $V_{Sopt} = V_S(\gamma_{max})$.

$$T = D + Wsen(\gamma) \qquad sen(\gamma_{max}) = \frac{T}{W} - \sqrt{\frac{4C_{D0}}{\pi Ae}}$$

$$V_S = Vsen(\gamma) \qquad V(\gamma_{max}) = \sqrt{\frac{2}{\rho}\sqrt{\frac{1}{C_{D0}\pi Ae}}\frac{W}{S}cos(\gamma max)}$$

Finally obtaining the optimum lift speed:

$$V_S(\gamma_{max}) = V(\gamma_{max})sen(\gamma_{max}) = V_{Sopt}$$

FIRST WEIGHT (W1): MTOW: 195045 kg.

W1	H0	H1	H2	H3	H4	H5	H6
V_{Smax}	17	15	13.3	10.4	7	3.8	1
γ_{max}	7.5º	7.1º	4º	2.8º	1.7º	0.9º	0.2º

SECOND WEIGHT (W2): TOW2: OEW + 20% PL + 70% FW = 172267 kg.

W2	H0	H1	H2	H3	H4	H5	H6
V_{Smax}	20	17.8	15.9	12.5	8.7	5.2	2.4
γ_{max}	7.7º	6.6º	4.7º	3.5º	2.4º	2.2º	0.5º

THIRD WEIGHT (W3): TOW3: OEW + 50% FW = 146401 kg.

W3	H0	H1	H2	H3	H4	H5	H6
V_{Smax}	24.5	22.3	19.8	15.7	11.5	7.83	4.5
γ_{max}	10º	8.3º	6.3º	4.7º	3.3º	2.1º	1.1º

10. Minimum rise time:

With all the data shown on the previous headland, we'll be able to define a new variable among our rising movement: the minimum rise time. As its name defines totally its own meaning, it is the time our aircraft needs at least, remaining vertical velocity constant and maximum, to get to a desired height.

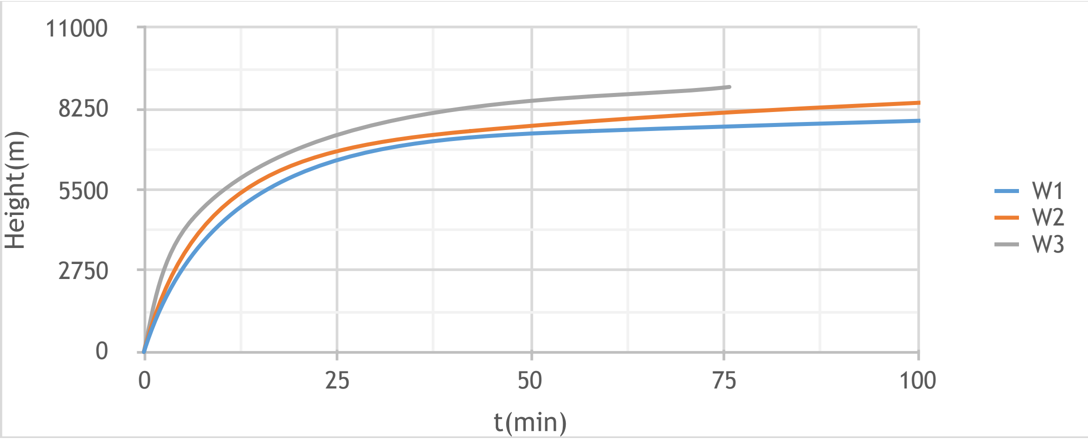
We need to remember that maximum rising velocity is not equal to the speed obtained by maintaining our greatest stable rising angle, which is consequently named as optimal speed.

Following the next mathematical expression, the minimal rise time will be revealed in units of seconds. Later on, it will be transferred into minutes so as to get more visual diagrams.

$$t = \frac{h_t}{(\frac{R}{C})_{maxSL}} ln \frac{h_t}{h_t - h}$$

As a clarification R/C is a common form to refer to our variable rising velocity. In a similar connotation, h_t is the top height our aircraft is able to reach without incurring into trouble of stability or loss of lift. In our particular case, DC-10 is restricted to FL105 as previous calculations indicate.

We'll now graph the height evolution of each of our diffetently weighted aircrafts (W1, W2 and W3 with the same values recorded all over this document) facing minimum rise time.



As it is expected, the less an airplane weights, the higher altitude it will reach.

11. Endurance and Range:

Both terms are sometimes confused and not used on its proper ambit. While range refers to the total distance (measured in ground) traveled by an aircraft with a determined value of fuel, endurance means the total time that this particular aircraft is able to be flying without refueling.

Considering as main hypothesis the fact that our flight is done in standard atmosphere, both variables endurance and range can be obtained by formulas:

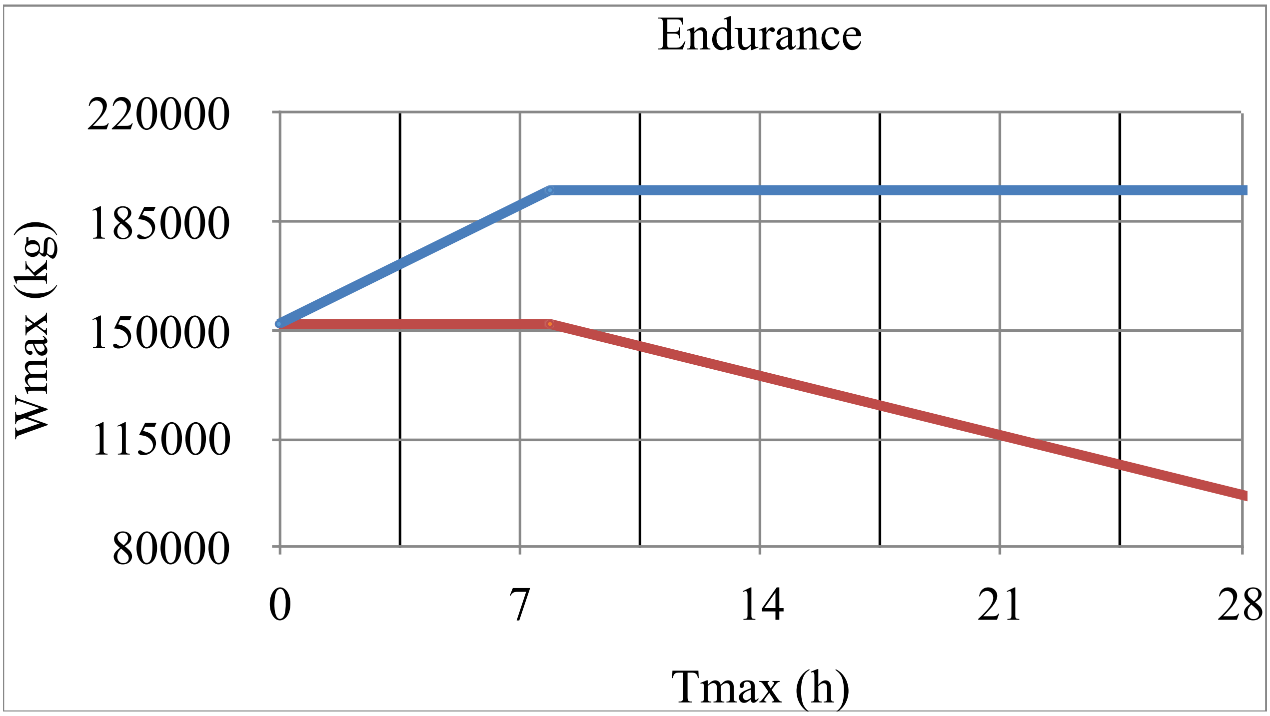
$$E_2 = \frac{1}{sfc}\frac{C_L}{C_D}Ln\left(\frac{TOW}{LW}\right)$$

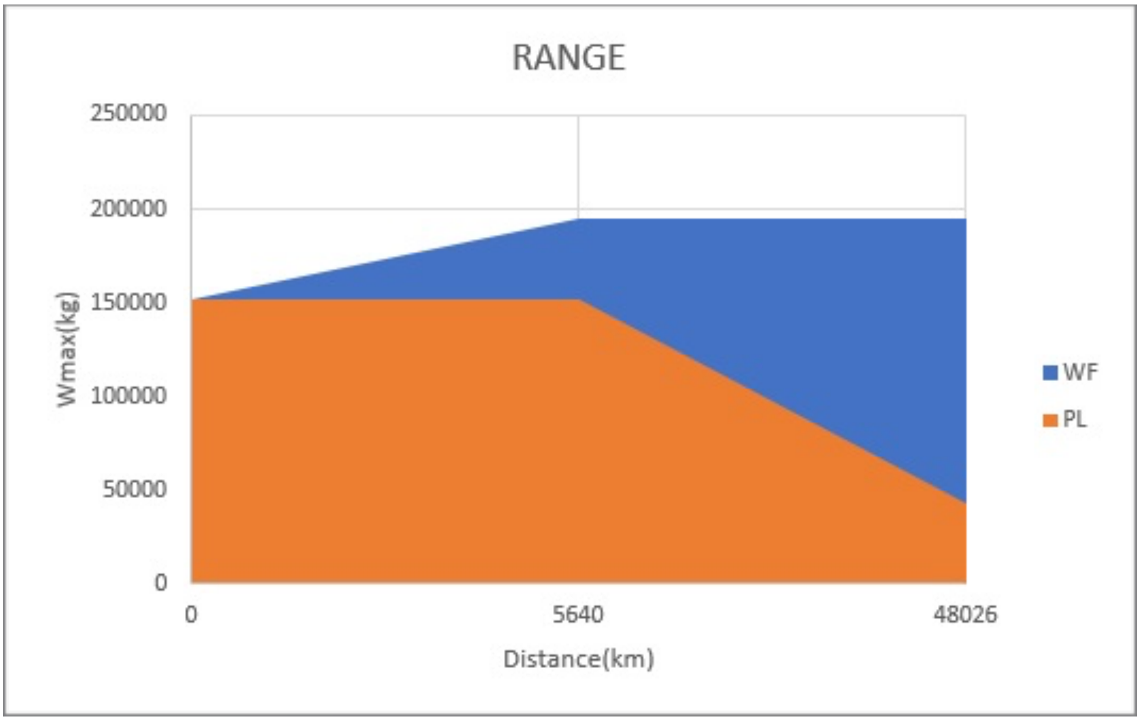
$$R_2 = \frac{1}{sfc}\sqrt{\frac{TOW}{\frac{1}{2}\rho S}}\frac{\sqrt{C_L}}{C_D}2\left(1-\sqrt{\frac{LW}{TOW}}\right)$$

Being sic a predefined relationship given as: sfc=0.88·exp(-0.05·ByPassRatio) .In our case of study, the DC-10 aircraft possesses a ByPassRatio of 4.97:1. Therefore, maximum values of endurance and range will occur when:

Maximum Range (minimum Pr/v or Tr/v)	Maximum endurance (minimum Pr or Tr)
· $(Cl^{1/2}/Cd)_{max}$ · $(C_T)_{min}$ · $(W_{TO}/(W_{TO}-W_F)_{max}$	· $(Cl/Cd)_{max}$ · $(C_T)_{min}$ · $(W_{TO}/(W_{TO}-W_F)_{max}$

Leaving these theoretical concepts behind and putting excel into work, the following diagrams can be withdrawn.





Since we represented this graph using the area diagram, we're not able (or at least we haven't found it yet) to display the range axis with more principal divisions and, thereby, more visual values of it. For instance, it would be a good manner if principal divisions could be detached by 5000 or 10000 km.

According to theoretical concepts, now payload follows a lineal decrease until getting to null. At that point, in X axis (range) will be shown the highest range our aircraft is able to reach.

12. Take Off and Landing:

Take off:

The most critical part of a flight is the take off, in which the aircraft needs to generate enough thrust and lift to complete the first climbing part. As we can see on the following picture,the take off consists of three fundamental stages:

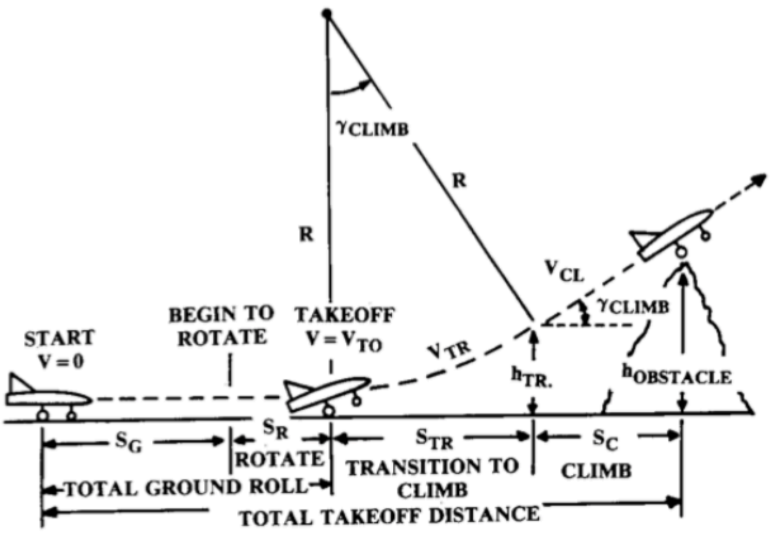


Fig. 17.17 Takeoff analysis.

-First stage: From zero speed, flaps set to take off position, brakes on and thrust set to take off configuration; until rotate speed.

The movement on this stage consist on a non sliding accelerating run. Thrust is maximum on this part of the take off due to reduce the needed runway distance.

On this first stage both angles (speed and aircraft) are zero and the thrust can be constant or not.

-Second stage: From rotate speed until lift off speed. On this stage flaps are set on take off position and thrust is maximum, set by the pilot and constant.

Here the angle formed by the speed is zero, but the angle that forms the aircraft is equal to the angle of attack.

-**Third stage:** From lift off speed until the aircraft reaches the minimum height to avoid an obstacle (35 ft). This staged is curved due to the aircraft climb. From this circular trajectory to a normal climbing route, the remarkable speed is the transition speed.

On this stage; angles, thrust and speed are all variable with time.

Next step is to calculate the required run distance to take off, for these calculations the data needed is shown below, and following the next equation we have been able to calculate take off runs for all three weights.

Data:	Value:	Data:	Value:
W_{TO}	1913391	**V stall**	69,310
μ_R	0,04	**Vref**	58,220
S	358,7	**e**	0,811
ρ	1,225	**Cl tip**	0,1
Clmáx	1,8129	**CD**	0,0292546449187716

$$S_{GR} = \frac{1.44W^2}{g\rho_{airport}SC_{Lmax}\left[T - (D + \mu_r(W - L))_{medium}\right]}$$

With all this data and the equation shown before we have obtained the following values for the take off run required for the weights selected:

-(W1): MTOW: 195045 kg._ $\rightarrow S_{GR}(W1) = 2583m = 8474ft$

-(W2): TOW2: OEW + 20% PL + 70% FW = 172267 kg. $\rightarrow S_{GR}(W2) = 2015m = 6611ft$

-(W3): TOW3: OEW + 50% FW = 146401 kg. $\rightarrow S_{GR}(W3) = 1455m = 4774ft$

<u>Landing:</u>

The other critical step of a flight is the landing. On landings the aircraft needs to get enough stability to touchdown safely. The aircraft speed is set to landing minimums and flaps are completely deployed (landing configuration 40º).

The stages on a landing procedure are similar to take off stages but reverted. Starting from 1.3 V stall until the touchdown speed 1.15 V stall.

The required runway to land can be calculated using the same data we used for take off but following the next equation and giving the lift coefficient a different value corresponding to the higher flaps deployment.

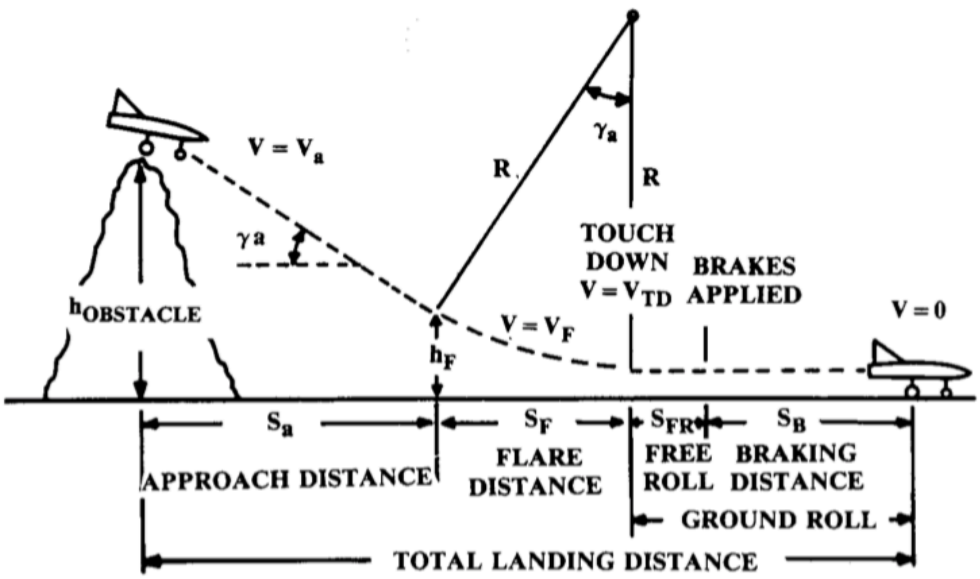


Fig. 17.18 Landing analysis.

$$S_{GR} = \frac{1.69W^2}{g\rho_{airport}SC_{Lmax}(D + \mu_r(W - L))_{medium}}$$

-(W1): MTOW: 195045 kg._ $\rightarrow S_{GR}(W1) = 846m = 2776ft$

-(W2): TOW2: OEW + 20% PL + 70% FW = 172267 kg. $\rightarrow S_{GR}(W2) = 660m = 2165ft$

-(W3): TOW3: OEW + 50% FW = 146401 kg. $\rightarrow S_{GR}(W3) = 477m = 1565ft$

13. Horizontal Turn:

Due to a non uniform (accelerated) flight, aircraft are able to make horizontal and vertical turns. The one we are going to study here is the horizontal coordinated turn with the equilibrated lift and mass forces.

On a symmetric leveled turn, the aircraft's symmetry plane is tangent to the trajectory, both V and X are tangent too. On this movement appears a rotating force that turns the aircraft.

Due to aileron deployment, the upper wing gets more lift than the downer wing, creating a lift difference that compensates the rotating force. In spite of this compensation, a yaw movement will be induced because of the turn, making the pilot use the vertical stabilizer to compensate yaw.

-Load factor, angular speed and turning radius:

For a horizontal turn, there are few remarkable variables to study, such as: angular speed for the turn, minimum turning radius, minimum load factor, stall speed for a minimum load factor…

All there variables are described on the equations below:

$$R_{min} = \frac{4k\left(\frac{W}{S}\right)}{g\rho\left(\frac{T}{W}\right)\sqrt{1 - \frac{4kC_{D_{L=0}}}{\left(\frac{T}{W}\right)^2}}} \qquad n_{max} = \left[\frac{q}{k\left(\frac{W}{S}\right)}\left(\left(\frac{T}{W}\right)_{max} - \frac{qC_{D_{L=0}}}{\left(\frac{W}{S}\right)}\right)\right]^{\frac{1}{2}}$$

$$n_{Rmin} = \sqrt{2 - \frac{4kC_{D_{L=0}}}{\left(\frac{T}{W}\right)^2}} \qquad \omega_{max} = g\sqrt{\frac{\rho}{\left(\frac{W}{S}\right)}\left[\frac{\left(\frac{T}{W}\right)}{2k} - \left(\frac{C_{D_{L=0}}}{k}\right)^{\frac{1}{2}}\right]}$$

$$V_{Rmin} = \sqrt{\frac{4k\left(\frac{W}{S}\right)}{r\left(\frac{T}{W}\right)}} \qquad V_{\omega max} = \sqrt{\frac{2\left(\frac{W}{S}\right)}{\rho}}\left(\frac{k}{C_{D_{L=0}}}\right)^{\frac{1}{4}}$$

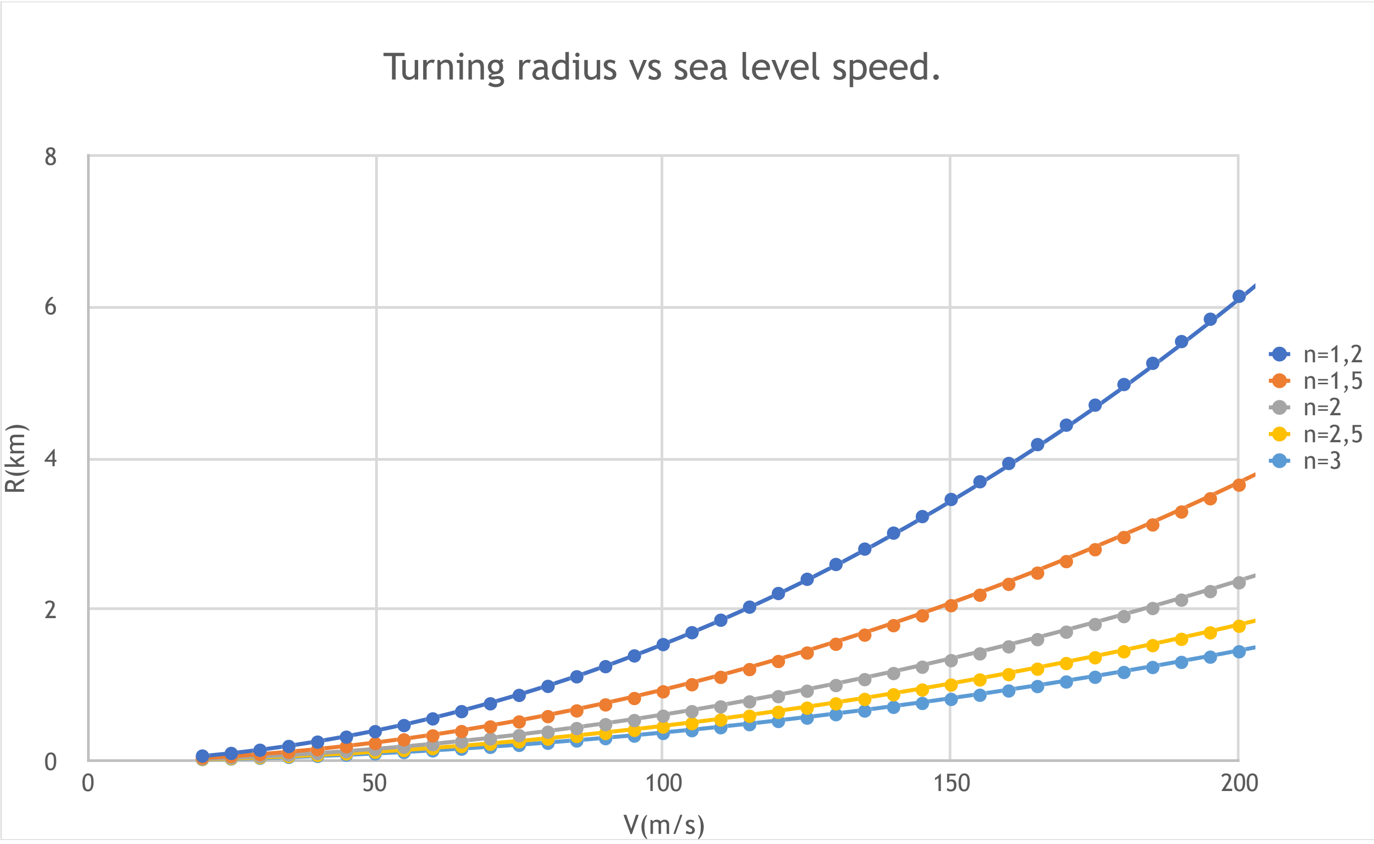
The most remarkable characteristics on a leveled turn, for the three different weights and 8 selected heights are:

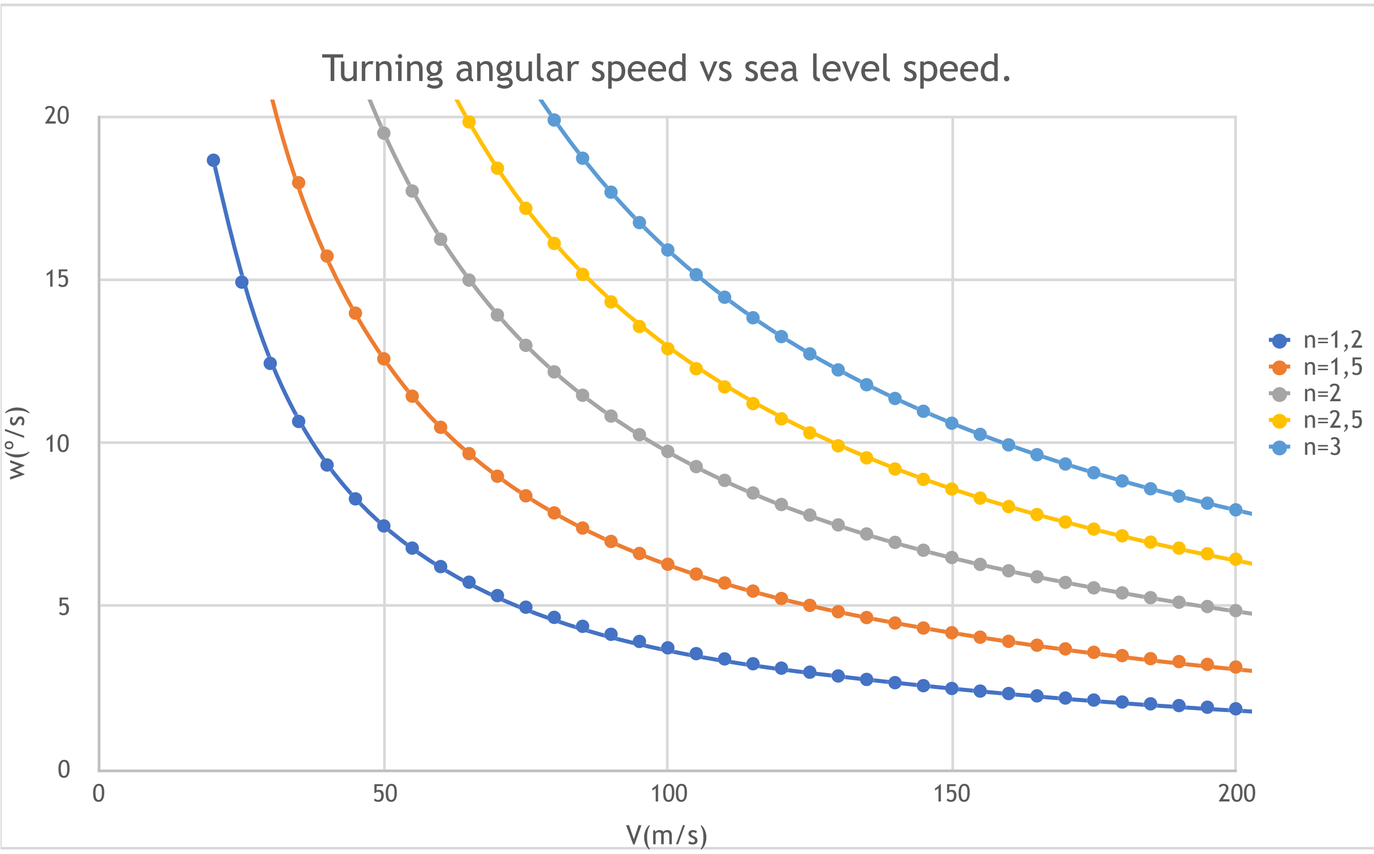
W1	H0	H1	H2	H3	H4	H5	H6	H7
V_{Rmin}	62,47	67,21	72,50	78,43	85,10	92,63	101,19	110,96
$V_{stall(n_{Rmin})}$	100,17	107,78	116,26	125,77	136,46	148,54	162,26	177,93
$V_{\omega max}$	137,70	148,15	159,81	172,88	187,57	204,18	223,05	244,59
$V_{stall}(n_{\omega max})$	145,51	156,56	168,89	182,69	198,22	215,77	235,71	258,47

W2	H0	H1	H2	H3	H4	H5	H6	H7
V_{Rmin}	55,17	59,36	64,03	69,27	75,16	81,81	89,37	98,01
$V_{stall(n_{Rmin})}$	94,25	101,41	109,39	118,33	128,39	139,76	152,67	167,42
$V_{\omega max}$	129,41	139,23	150,19	162,47	176,28	191,89	209,62	229,86
$V_{stall}(n_{\omega max})$	141,54	152,28	164,27	177,70	192,80	209,87	229,26	251,40

W3	H0	H1	H2	H3	H4	H5	H6	H7
V_{Rmin}	46,89	50,45	54,42	58,87	63,87	69,53	75,95	83,29
$V_{stall(n_{Rmin})}$	86,99	93,60	100,96	109,22	118,50	128,99	140,91	154,52
$V_{\omega max}$	119,30	128,35	138,46	149,78	162,51	176,90	193,24	211,90
$V_{stall}(n_{\omega max})$	136,40	146,76	158,31	171,25	185,81	202,26	220,95	242,28

The next graphics show the influence of the sea level speed on a horizontal turn, versus the turning angular speed and the turning radius.


Turning radius vs sea level speed.
8
6
4
2
0
R(km)
0
50
100
150
200
V(m/s)
n=1,2
n=1,5
n=2
n=2,5
n=3


Turning angular speed vs sea level speed.
20
15
10
5
0
w(°/s)
0
50
100
150
200
V(m/s)
n=1,2
n=1,5
n=2
n=2,5
n=3

14. Stability:

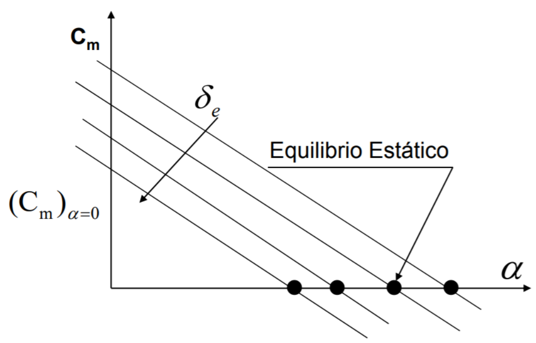
In terms of aerospace industry, stability is known as the performance or hability if an aircraft to return by itself to the equilibrium state after suffering a perturbation of its airborne variables.

But, what do we mean when we talk about stability? It's as simple as, if we reduce our aircraft's physics to a single point called Centre of Gravity (CG), torques of roll and yaw are equal to 0 by themselves an pitch momentum is 0 by the action of some controller.

It is important to differentiate stability from control, in which torques =0 all over time no matter if there are time-changing perturbations. So, this last concept is defined as the relationship between aircraft's movement and the deflections of control surfaces as a function of time. Just like stability, it can be static or dynamic.

In a similar manner,it's not the same stability and equilibrium. As a conclusion of different basic aerodynamics' expressions, while equilibrium state requires that the aerodynamical centre (CA) must be placed before the gravity centre (CG), stability appears when there's the opposite condition. So, in this contradictory scenario, what is done to acquire both?

We'll fly under stability rules (Cm,alfa<0 and CA placed behind CG) constantly looking for Cm=0 through varying the deflections of our control surfaces (spoilers,elevator and rudder). The next graphic represents a good insight into what we've just said.



with the help os this theoretical introduction to stability and the following formulas, we'll obtain the forces in roll axis (X) caused by axial speed (u), normal speed (w), and pitch angle (q)

$$\left.\begin{aligned} X_u &= -2C_D - V_0\frac{\partial C_D}{\partial V} + \frac{1}{\frac{1}{2}\rho V_0 S}\frac{\partial \tau}{\partial V} \\ X_w &= C_L - \frac{\partial C_D}{\partial \alpha} \\ X_q &= -\bar{V}_\tau \frac{\partial C_{D_\tau}}{\partial \alpha_\tau} \\ X_{\dot{w}} &= -\bar{V}_\tau \frac{\partial C_{D_\tau}}{\partial \alpha_\tau}\frac{\partial \varepsilon}{\partial \alpha} \equiv X_q \frac{\partial \varepsilon}{\partial \alpha} \end{aligned}\right\} \quad \left\langle \begin{aligned} &\frac{\partial \tau}{\partial V} = 0, \quad \frac{\partial C_{D_\tau}}{\partial \alpha_\tau} = 0 \\ &\frac{\partial C_D}{\partial V} = -k\left(\frac{2W}{\rho V^2}\right)^2 \frac{4}{V^5} \\ &\frac{\partial C_D}{\partial \alpha} = 2kC_L C_{L\alpha} \end{aligned} \right\rangle$$

15. American Airlines Flight 191:

American Airlines Flight 191 was a regularly scheduled passenger flight operated by American Airlines from Chicago's O'Hare International Airport to Los Angeles International Airport. On May 25, 1979, the McDonnell Douglas DC-10-10 operating this flight crashed just moments after takeoff from Chicago. All 258 passengers and 13 crew on board were killed, along with two people on the ground. With 273 fatalities, it is the deadliest aviation accident to have occurred in the United States.

The National Transportation Safety Board (NTSB) found that as the jet was beginning its takeoff rotation, engine number one (the left hand engine) separated from the left wing, flipping over the top of the wing and landing on the runway. As the engine separated from the aircraft, it severed hydraulic fluid lines that locked the wing's leading edge slats in place and damaged a three-foot section of the left wing's leading edge. Aerodynamic forces acting on the wing resulted in an uncommanded retraction of the outboard slats. As the jet began to climb, the damaged left wing, with no engine, produced far less lift (stalled) than the right wing, with its slats still deployed and its engine running at full takeoff speed. The disrupted and unbalanced aerodynamics of the aircraft caused it to roll abruptly to the left until it was partially inverted, reaching a bank angle of 112 degrees, before crashing in an open field by a trailer park near the end of the runway. The engine separation was attributed to damage to the pylon structure holding the engine to the wing, caused by improper maintenance procedures used at American Airlines.

After this accident, all maintenance procedures used in American Airlines were revised and changed, to avoid a new accident. Unfortunately, the DC-10 has been an aircraft with a lot of accidents related to its design and maintenance procedures, involving a lot of fatalities.

In spite of being a very problematic airplane, it was very used until the decade of the 90. Nowadays, there are few DC-10 flying regularly, and all these flights are only for cargo.

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