UNIVERSITAT POLITÈCNICA DE CATALUNYA

DELIVERABLE 1: CONTEXT AND SCOPE OF THE PROJECT

Design of an environment for solving pseudo-boolean optimization problems

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Chapter 1

Introduction

In this chapter a first introduction . . .

1.1 Context

Boolean satisfiability problems *(SAT from now on)* is the problem of finding a model[1] for a *Boolean Formula*. In other words, it is the result of evaluating the *Boolean Formula* after replacing its variables for *true* or *false*.
SAT is widely used in Computer Science because it was the first problem proved to be NP-Complete[1][2] which allowed a lot of NP[3] to be reduced to it.

1.1.1 What is a Pseudo-Boolean Formula?

In propositional logic, a boolean formula is defined as following[2]:
Let P be a set of predicate symbols like $p, q, r, ...$

- All predicate symbol of P is a formula.

- If F and G are formulae, then $(F \wedge G)$ and $(F \vee G)$ are formulae to.

- If F is a formula, then $(\neg F)$ is a formula.

- Nothing else is a formula.

This representation has some limitations because it can only express properties which are *true* or *false*.

Cuidado que crec que estic barrejant Formulas amb Functions

Pseudo-Boolean Formulas are functions of the form $f : B^n \rightarrow \mathbb{R}$. For example, the following formula is a *Pseudo-Boolean Formula*: $3x + 5y$. Therefore, *Boolean Formulas* are a special case of *Pseudo-Boolean Formula* where the domain $d = \{0, 1\}$.

1.1.2 Pseudo-Boolean formulae minimization

Explicar que es un problema NP-Hard i en que consisteix

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[1] An interpretation which satisfies the formula.
[2] NP and NP-hard.
[3] Nondeterministic polynomial time.

1.2 Background

During the past semester (Q1 2017/2018) I had been developing a C++ library called _____.

This tool allows the users to represent *Boolean Formulas* in a C++ program in a intuitive way, do operations between them and convert them into *BDDs*. However, the main functionality of this library is the conversion from a *Boolean Formula* to a *CNF*. As previously explained, *CNF* is a particular type of a *Boolean Formula*, a conjunction of disjunctions. *CNF* is an important format because it is the standard input for *SAT Solvers*A.1.

As shown in this paper, *Mitchell, Selman, and Levesque*[3], there is a correlation between the number of variables, the number of clauses and the hardness of solving the *CNF*.
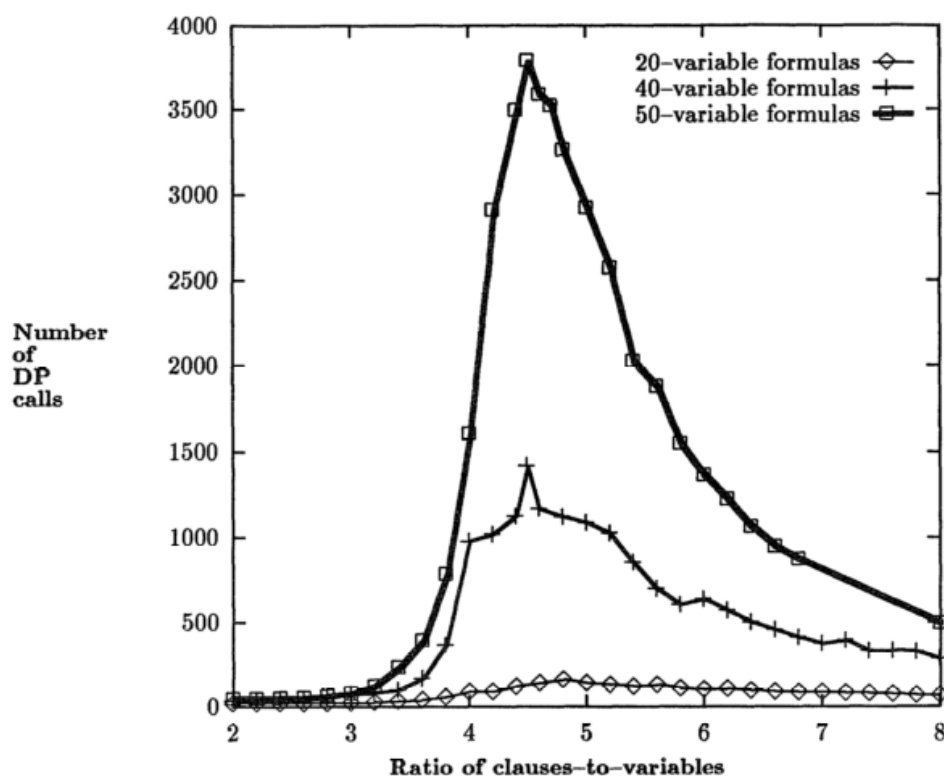
FIGURE 1.1: Median number of recursive DP calls for Random 3-SAT formulas, as a function of the ratio of clauses-to-variables. Extracted from Mitchell, Selman, and Levesque[3]

Therefore, an improvement of the input *CNF* of the *SAT Solver* can reduce a lot the hardness of the problem.

This is the main goal of the library, try to reduce the size of the final *CNF* resulting from applying different converting methods on the original *Boolean Formula*.

1.3 Sate-of-art

1.4 Motivation

Informatics Logic is taught in this[4] faculty. In that course I realized how important

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[4]Facultat Informàtica de Barcelona

is *logic* through its lecturer, Dr.Robert Nieuwenhuis, and its activities.

In the first coursework we had to code a *SAT Solver* which used *Unit Propagation*A.2. With this activity I comprehended how hard and substantial is the study of *logic* and all its context. For example, how *logic* is used in Artificial Intelligence and Planners.

When the time of deciding the *TFG* arrived , I contacted my actual supervisor, Dr. Jordi Cortadella, and he proposed me some topics and ideas for projects. Finally, we agreed on doing this project.

1.5   Stakeholders

In this section the Stakeholders of the project are defined. Stakeholders are entities which are effected, directly or indirectly, by the solution developed in this project.

1.5.1   Target audience

This tools tools targets all the entities (researchers, companies, . . .) which works with Pseudo-Boolean Minimization.

1.5.2   Users

The users will be C++ programmers due this tool is developed in this language/

1.5.3   Beneficiaries

All those entities which works with Pseudo-Boolean Minimization. For example AI, SAT Solvers, Planners, . . .

Chapter 2

Project Formulation

As mentioned before, this project is an extension of a previous C++ library. The main goal of this project is improve the time required to solve a *minimization* problems. To achieve this goal, the following objectives have been established.

2.1 General objectives

2.1.1 Pseudo-Boolean minimization

For the problems of the form $min(c_1 x_1 + c_2 x_2 + \ldots + c_n x_n \leq k)$, the goal is to find an assignment for $\{x_1, x_2, \ldots, x_n\}$ so that k is minimum.

Previously[1], it has been explained that this types of problems are *NP-Hard*. This project will try to reduce the time to solve this problems by trying two approaches:

- Binary search:
  Implement the well known *Binary Search*[1] algorithm to find the minimum value for k.

- Linear search:
  Some *SAT Solvers* can learn and derive new restrictions from previous problems. To take advantage of this ability it is necessary to implement a *Linear Search* algorithm.

2.1.2 Timeout

For some problems it is more important to find a solution before a deadline than finding the best possible solution. For instance, a delivery company must have all the route planned for all trucks before the journey starts, therefore, they care more about having a solution than finding the best one.
For this, a *Timeout strategy* will be implemented in case that a good enough solution has been found or the problem does not seem to have a solution.

2.1.3 Multi-threading

This tool will take advantage of multi-core processors trying to split the problem and solving each part separately.

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[1]Binary search is a search algorithm that finds the position of a target value within a sorted array. (more)

Chapter 3

Scope

3.1 What and how?

To achieve all the general objectives2 of the project, the following stages have been established:

- Analyze, refactor[1] and test the existing code to have a solid base.

- Add the functionality of representing *Pseudo-Boolean Formulae*.

- Study PBLib library to see which functionalities has to work with *minimization*.

- Implement *minimization* strategies.

- Study timeout strategies and implement them.

- Study and implement multithreading.

3.2 Possible obstacles

In this section the possible obstacles and its solutions are exposed.

3.2.1 Base project

This project will be build on top of an existing one, as explained in *Background section*1. The existing project could be a source of bugs and other problems caused by not testing the code. For this reason and to solve possible issues, the first stage of the project will be focused on solving this problem.

3.2.2 Schedule

Due to the circumstances in which this project will be developed (Erasmus) possible delays could appear. To fix this circumstances, a realistic schedule with weekly communication will be planned. This will support a continuous development and detect as soon as possible delays.

3.2.3 PBLib

One of the main requirements of this project, *Pseudo-Boolean minimization*, is planned to be done with *PBLib* library. It could happen that this library does not fit as expected with the project forcing to find a substitute.

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[1]Code refactoring is the process of restructuring existing computer code—changing the factoring—without changing its external behavior. (more)

3.2.4 Correctness

As explained in *Rigor and Validation*4, correctness in this project is very important because of the context it is in.
Guarantee correctness could be hard and take more time than expected. If this happens, formal correctness could be delayed or reduce the level of formalization.

3.2.5 Unforeseen circumstances

3.2.6 Computational power

Chapter 4

Methodology and Rigor

Research is a vast process with no clear path between *a* and *b*. For this, it is important to follow some directions. Methodology will provide some guidelines to avoid possible problems, be more efficient and do the project more manageable.

4.1   Methodology

The methodology adopted for this project will be Agile[1]. It is important to clarify that this methodology will not be followed strictly but adapted to this project where there is only one developer and all the objectives are well defined. The main characteristics followed from Agile in this project will be:

- Short cycles

- TDD (Test-Driven Development)

- Weekly scrums with the supervisor

4.2   Tools

In this chapter the development tools for this project will be introduced.

4.2.1   Git

Git is a well known version control system developed by Linus Torvalds[2].
Git will be used in this project because it allows to maintain a tracking of all the changes made (commits), and what is more important, return to them at any time. In addition to this, it enforces a short cycle development (because commits are small units of work) and the developer has to document them which matches perfect with Agile methodology.

4.2.2   Trello

Trello is a web board which helps to organize tasks and its state. It will be used for this project to define which tasks need to be done, its deadline and what its their current state.

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[1]Methodology based on the on the adaptability in front of any change to improve exit possibilities.
[2]Linux creator. (more)

4.3 Communication

Due to my conditions, I'm currently studying abroad in an Erasmus program, all the communication will be made through electronic means. The majority of it will be made using e-mail but if it is necessary a video conference could be done.
The minimum communication with the supervisor will be a weekly e-mail report where all the tasks done during the week will be explained. Problems or questions will be also exposed, if any.

4.4 Rigor and Validation

Rigor and Validation for this project is relevant.
The surrounding of it, such as *Artificial Intelligence, Planners, Cryptographic Protocols verification, ...*, are widely used nowadays and have been becoming more popular lately. This means that this project could have a big repercussion and be used by some professionals. For this, it is important to guarantee the validation and correctness of the project.
During the development, TDD will be used to avoid unnecessary code (possible origin of bugs) and assure the correctness of the implementation. It is also possible to formalize and prove all the operations done by the software.
Finally, my supervisor could give me orientation and validate, if necessary, the operations done.

Appendix A

More information

A.1 Why SAT Solvers use CNF as input format?

There are two mains reasons for this: Equisatisfiability and Computational Complexity. Let us start with the first one:

Two *Boolean Formulas* are **equisatisfiable** if and only if both have the same *models*. This may seem the same as equality but it is not because in an equality relationship both *Boolean Formulas* have to have the same variables.
This is important because between a *Boolean Formula* and its result from a *CNF* transformation the equisatisfiability is preserved which means that if the *SAT Solver* finds a *model* for the *CNF*, then this *interpretation* will be also a *model* for the original *Boolean Formula*.

The second reason is computational complexity. Let us have a look at the following table:

	DNF	CNF
TAUT	NP	P
SAT	P	NP

TABLE A.1: Complexity of deciding if a *Boolean Formula* is SAT or TAUT depending of its format.

So as a *Boolean Formula* can be converted into a *CNF* in linear time while preserving equisatisfiability, *SAT Solvers* will use them to target satisfiability.

A.2 What is Unit Propagation?

Unit propagation

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