Applied Statistical Modeling (SoSe 2021) – Homework, Week 2

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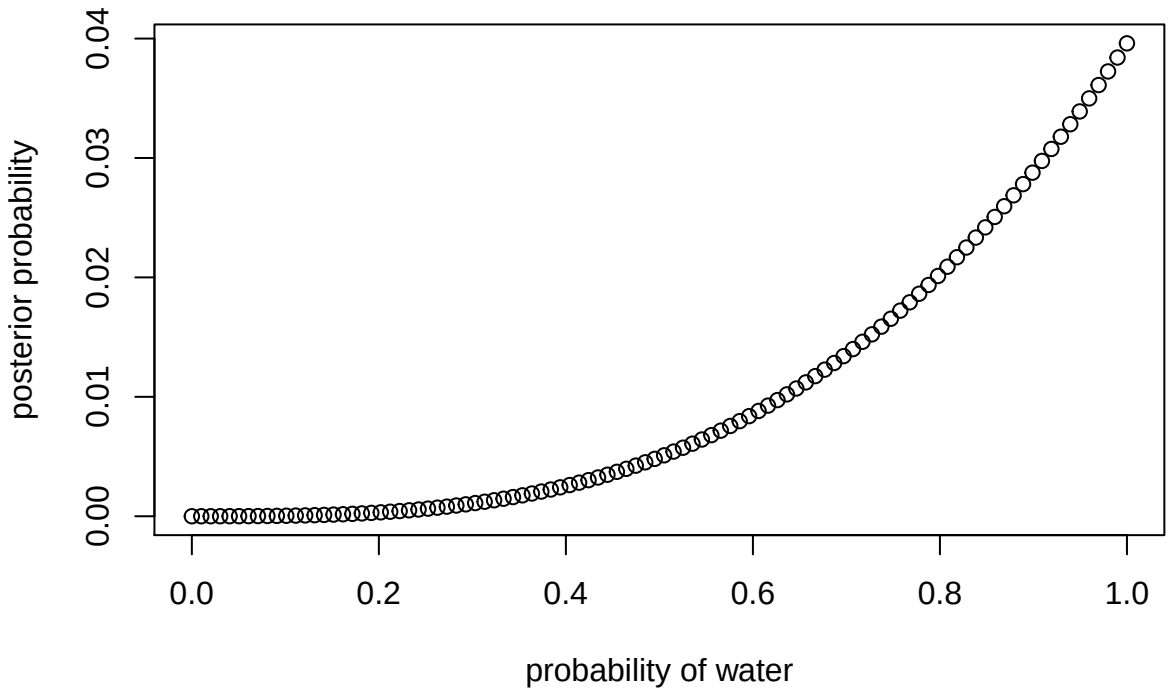
Tasks: McElreath chapter 2.6, exercises 2M(1-7), extra: 2H(1-4)

Task 2M1 – Grid Approximation with Globe Tossing Model

```
# Step 1: Define grid
p_grid <- seq(from = 0, to = 1, length.out = 100)
# Step 2: Define prior (Assumption: Uniform distribution)
prior <- rep(1 ,100)
```

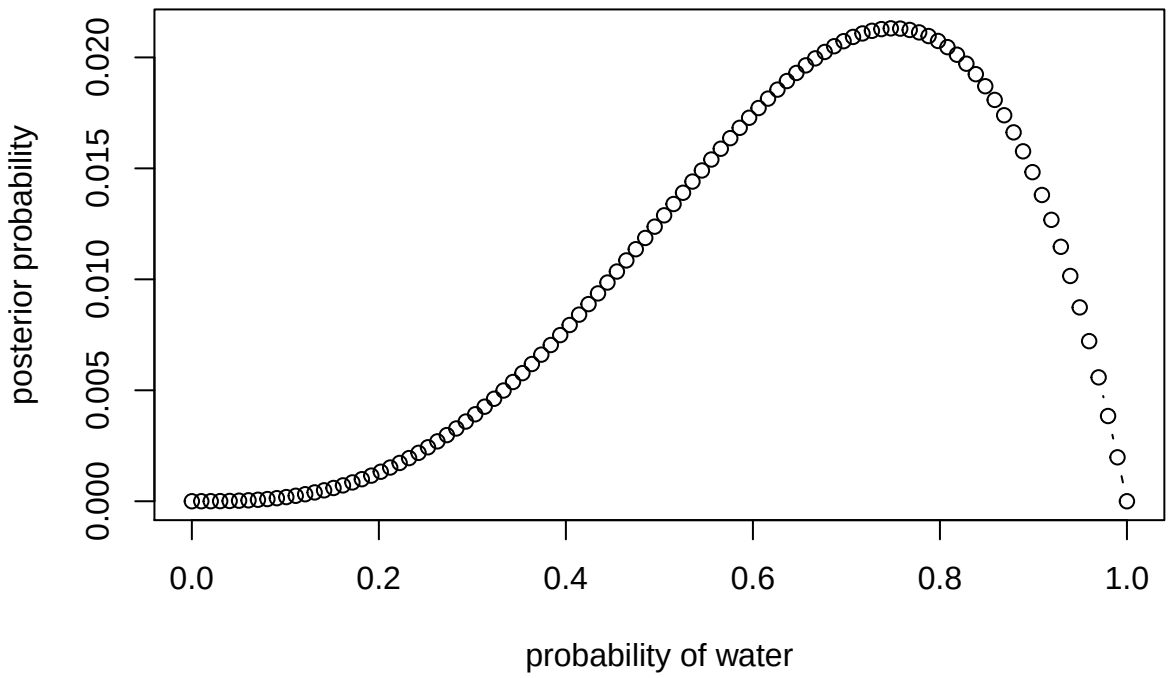
First set of observations: W W W (3/3)

```
# Step 3: Compute likelihood at each value in grid
likelihood <- dbinom(3, size=3, prob=p_grid)
# Step 4: Multiply prior with likelihood to get unstandardized posterior
unstd.posterior <- likelihood * prior
# Step 5: Standardize the posterior
posterior <- unstd.posterior / sum(unstd.posterior)

plot(p_grid, posterior, type="b" ,
      xlab="probability of water", ylab="posterior probability")
```



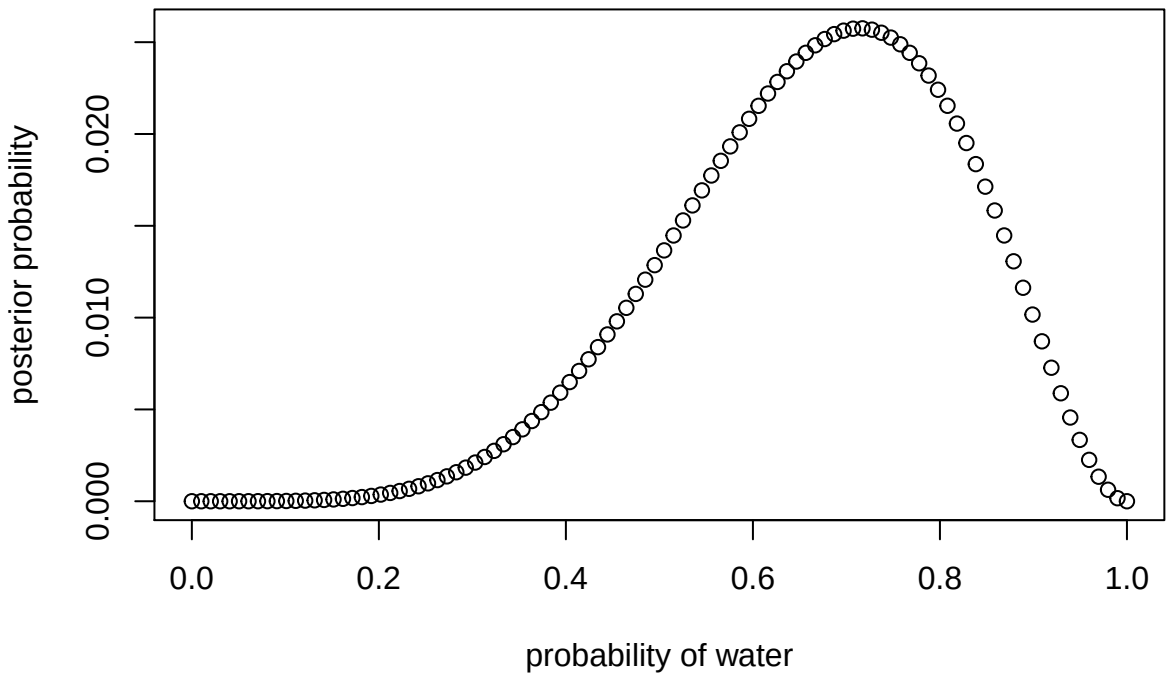
Second set of observations: W W W L (3/4)

```
# Step 3: Compute likelihood at each value in grid
likelihood <- dbinom(3, size=4, prob=p_grid)
# Step 4: Multiply prior with likelihood to get unstandardized posterior
unstd.posterior <- likelihood * prior
# Step 5: Standardize the posterior
posterior <- unstd.posterior / sum(unstd.posterior)

plot(p_grid, posterior, type="b" ,
      xlab="probability of water", ylab="posterior probability")
```



Thrid set of observations: L W W L W W W (5/7)

```
# Step 3: Compute likelihood at each value in grid
likelihood <- dbinom(5, size=7, prob=p_grid)
# Step 4: Multiply prior with likelihood to get unstandardized posterior
unstd.posterior <- likelihood * prior
# Step 5: Standardize the posterior
posterior <- unstd.posterior / sum(unstd.posterior)

plot(p_grid, posterior, type="b" ,
      xlab="probability of water", ylab="posterior probability")
```



Task 2M2 – Adjust Globe Tossing Model with binary prior shifting at the value of 0.5

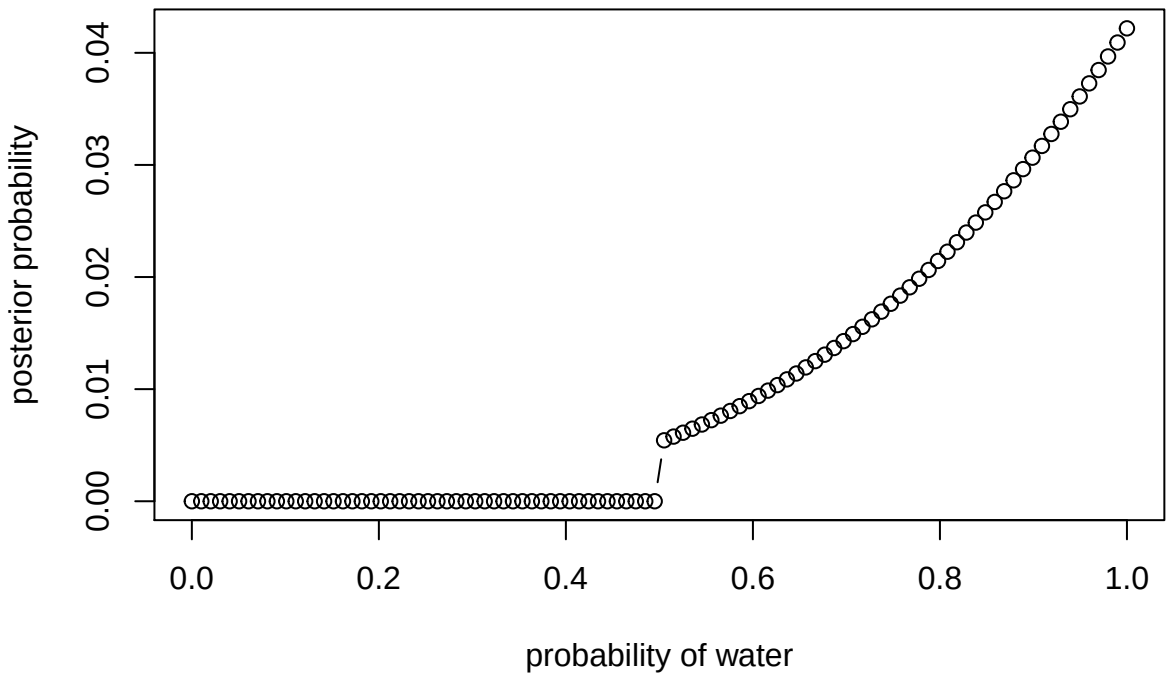
First, adjust the prior:

```
# Step 2: Define prior (Assumption: Uniform distribution)
prior <- ifelse(p_grid < 0.5, 0, 1)
```

Now, re-run with adjusted prior:

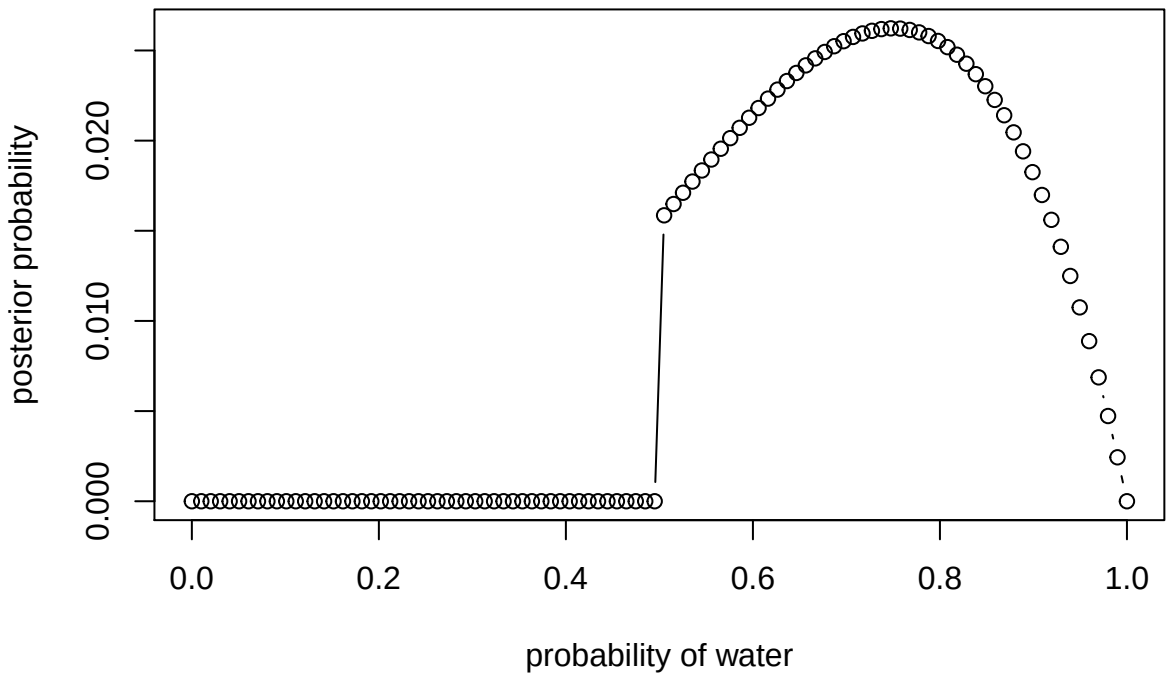
First set of observations: W W W (3/3)

```
# Step 3: Compute likelihood at each value in grid
likelihood <- dbinom(3, size=3, prob=p_grid)
# Step 4: Multiply prior with likelihood to get unstandardized posterior
unstd.posterior <- likelihood * prior
# Step 5: Standardize the posterior
posterior <- unstd.posterior / sum(unstd.posterior)

plot(p_grid, posterior, type="b" ,
      xlab="probability of water", ylab="posterior probability")
```



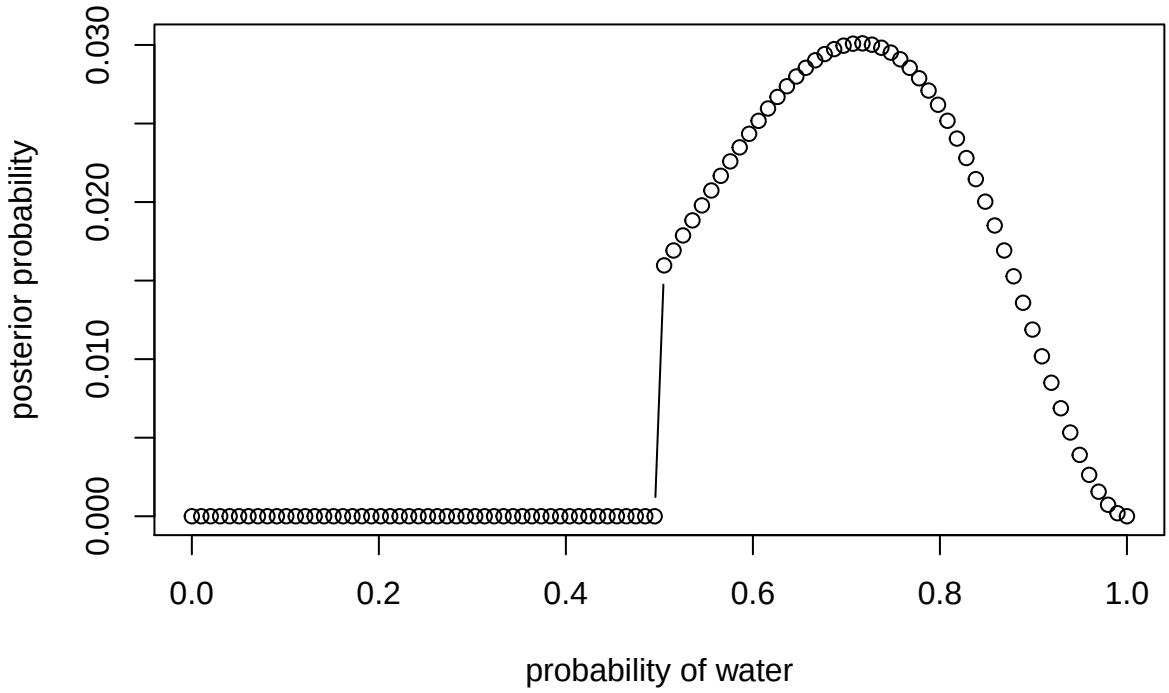
Second set of observations: W W W L (3/4)

```
# Step 3: Compute likelihood at each value in grid
likelihood <- dbinom(3, size=4, prob=p_grid)
# Step 4: Multiply prior with likelihood to get unstandardized posterior
unstd.posterior <- likelihood * prior
# Step 5: Standardize the posterior
posterior <- unstd.posterior / sum(unstd.posterior)

plot(p_grid, posterior, type="b" ,
      xlab="probability of water", ylab="posterior probability")
```



Thrid set of observations: L W W L W W W (5/7)

```
# Step 3: Compute likelihood at each value in grid
likelihood <- dbinom(5, size=7, prob=p_grid)
# Step 4: Multiply prior with likelihood to get unstandardized posterior
unstd.posterior <- likelihood * prior
# Step 5: Standardize the posterior
posterior <- unstd.posterior / sum(unstd.posterior)

plot(p_grid, posterior, type="b" ,
      xlab="probability of water", ylab="posterior probability")
```



Task 2M3 – Tossing of Earth and Mars

First, the information from the text is stored in probabilities:

```
prob_earth_water <- 0.7 # Earth covered with 70% water
prob_earth_land <- 1-prob_earth_water # Land conditional on water

prob_mars_land <- 1 # Not much water on Mars

prob_earth <- 0.5 # Both planets equally likely to be tossed
prob_mars <- 1-prob_earth
```

Now, this information can be used to compute the posterior probability that the globe was the Earth, conditional on seeing land (Pr(Earth|land)) and that it is equal to 0.23:

```
prob_land <- prob_earth_land*prob_earth + prob_mars_land*prob_mars; prob_land
```

```
## [1] 0.65
```

```
prob_land_earth <- (prob_earth_land*prob_earth)/prob_land; prob_land_earth
```

```
## [1] 0.2307692
```

Task 2M4 – Three black and white cards

A deck involves three cards with the following properties: Card 1: Black/Black Card 2: Black/White Card 3: White/White

One card is randomly chosen and put on the table. The upper side is black, now show that the probability

that the other side is also black is 2/3:

```
d <- tibble(front = c(0, 0, 1),
            back = c(0, 1, 1))

d %>% mutate(first_black= front+back) %>%
      mutate(prob = first_black/(sum(first_black)))
```

```
## # A tibble: 3 x 4
##   front  back first_black  prob
##   <dbl> <dbl>       <dbl> <dbl>
## 1     0     0           0 0
## 2     0     1           1 0.333
## 3     1     1           2 0.667
```

Task 2M5 – Now four cards

The deck from the previous task is supplemented by another Black/Black card. Therefore the overall deck is:

Card 1: Black/Black Card 2: Black/White Card 3: White/White Card 4: Black/Black

```
d <- tibble(front = c(0, 0, 1, 1),
            back = c(0, 1, 1, 1))

d %>% mutate(first_black= front+back) %>%
      mutate(prob = first_black/(sum(first_black)))
```

```
## # A tibble: 4 x 4
##   front  back first_black  prob
##   <dbl> <dbl>       <dbl> <dbl>
## 1     0     0           0   0
## 2     0     1           1   0.2
## 3     1     1           2   0.4
## 4     1     1           2   0.4
```

When adding up the probabilities where the first side is black (lines 3 and 4) we arrive at a probability of 0.8 (0.4 + 0.4) that the other side is black as well.

Task 2M6 – Unequal probabilities to draw black

Now, the deck from task 2M4 is used again but this time the probabilities to draw a card depend on the color since the ink makes the cards with black sides heavier:

Card 1: Black/Black Card 2: Black/White Card 3: White/White

For every way to pull the Black/Black card from the bag, there are 2 ways to pull the Black/White card and 3 ways to pull the White/White card.

```
d <- tibble(front = c(0, 0, 1),
            back = c(0, 1, 1),
            weight = c(3, 2, 1))

d %>% mutate(first_black= front+back) %>%
      mutate(prob = first_black*weight/(sum(first_black*weight)))
```

```
## # A tibble: 3 x 5
##   front  back weight first_black  prob
##   <dbl> <dbl>  <dbl>       <dbl> <dbl>
```

```
## 1     0     0      3           0   0
## 2     0     1      2           1   0.5
## 3     1     1      1           2   0.5
```

From the table it can be seen that if the front is black, then the probability with the adjusted weight of also having black on the back drops to 0.5.

Task 2M7 – Three black and white cards, again, but different setting

Deck: Card 1: Black/Black Card 2: Black/White Card 3: White/White

Setting: A first card is drawn from the deck and is black in the front. Now a second card is drawn and the front is white. In this setting, what is the probability that the backside of the first card is also black?

```
card <- c("Black/Black", "Black/White", "White/White")
prior <- 1
ways <- c(6, 2, 0)
likelihood <- ways * prior
prob <- likelihood / sum(likelihood); prob
```

```
## [1] 0.75 0.25 0.00
```

Note: The code above doesn't automatically produces the results but builds on the logically derived, hard-coded "ways" vector.

First card: Black/Black (2) –> Second card: Black/White or White/White (3) –> there are 6 (2 * 3) possible ways to produce the observed outcome First card: Black/White (1) –> Second card: Black/Black or White/White (2) –> there are 2 (1 * 2) possible ways to produce the observed outcome First card: White/White (0) –> No way to produce the observed outcome

Test citations:

(Zahran et al. 2008) Rosenzweig (2010)

Rosenzweig, Cynthia, William Solecki, Stephen A. Hammer, and Shagun Mehrotra. 2010. "Cities Lead the Way in Climate–Change Action." *Nature* 467: 909–11. https://doi.org/10.1038/467909a.

Zahran, Sammy, Samuel D Brody, Arnold Vedlitz, Himanshu Grover, and Caitlyn Miller. 2008. "Vulnerability and Capacity: Explaining Local Commitment to Climate-Change Policy." *Environment and Planning C: Government and Policy* 26 (3): 544–62. https://doi.org/10.1068/c2g.