STATISTICAL COMPUTATIONAL METHODS

Seminar Nr. 4, Markov Chains, Applications and Simulations

1. A computer system can operate in two different modes. Every hour, it remains in the same mode or switches to a different mode according to the transition probability matrix

$$P = \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix}.$$

a) If the system is in Mode I at 5:30 pm, what is the probability that it will be in Mode I at 8:30 pm on the same day?
b) In the long run, in which mode is the system more likely to operate?

2. (Genetics) An offspring of a black dog is black with probability 0.6 and brown with probability 0.4. An offspring of a brown dog is black with probability 0.2 and brown with probability 0.8. Rex is a brown dog. What is the probability that his grandchild is black?

3. (Traffic lights) Every day, student A takes the same road from his home to the university. There are 4 street lights along his way, and he noticed the following pattern: if he sees a green light at an intersection, then 60% of the time the next light is also green (otherwise, red), and if he sees a red light, then 70% of the time the next light is also red (otherwise, green).
a) If the first light is green, what is the probability that the third light is red?
b) Student B has *many* street lights between his home and the university. If the first street light on his road is green, what is the probability that the last light is red?

4. (Shared device) A computer is shared by 2 users who send tasks to it remotely and work independently. At any minute, any connected user may disconnect with probability 0.5, and any disconnected user may connect with a new task with probability 0.2. Let $X(t)$ be the number of concurrent users at time t.
a) Find the transition probability matrix.
b) Suppose there are 2 users connected at 10:00 a.m. What is the probability that there will be 1 user connected at 10.02?
c) How many connections can be expected by noon?

5. (Weather forecast) Recall the example at the lecture, the town with sunny/rainy days (state 1 was "sunny" and state 2 was "rainy"), with transition probability matrix

$$P = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}.$$

a) Compute the probability that in that town, April 1^{st} next year is rainy.
b) If the initial forecast is 80% chance of rain, write a Matlab code to generate the forecast for the next 30 days.
c) In that town, if there are 7 or more days of sunshine, there is a water shortage, and if it rains for a week or more, there is the threat of flooding. Local authorities need to be prepared for each situation. Use the code from part b) to conduct a Monte Carlo study for estimating the probability of a water shortage and the probability of a flooding.