STATISTICAL COMPUTATIONAL METHODS

Seminar Nr. 5, Counting Processes

1. On the average, 2 airplanes per minute land at a certain international airport. Assume the number of landings is modeled by a Binomial counting process.
a) what frame length should be used to guarantee that the probability of a landing does not exceed 0.1?
b) using the chosen frames, compute the probability of no landings during the next 3 minutes;
c) using the chosen frames, compute the probability of more than 100 landed airplanes during the next hour.

2. Messages arrive at a communications center according to a Binomial counting process with 30 frames per minute. The average arrival rate is 40 messages per hour. How many messages can be expected to arrive between 10 a.m. and 10:30 a.m.? What is the standard deviation of that number of messages?

3. An internet service provider offers special discounts to every third connecting customer. Its customers connect to the internet according to a Poisson process with the rate of 5 customers per minute. Compute
a) the probability that no offer is made during the first 2 minutes;
b) the probability that no customers connect for 20 seconds;
c) expectation and standard deviation of the time of first offer.

4. On the average, Mr. X drinks and drives once in 4 years. He knows that

- every time he drinks and drives, he is caught by the police;
- according to the law of his state, the third time he is caught drinking and driving, he loses his driver's license;
- a Poisson counting process models such "rare events" as drinking and driving

What is the probability that Mr. X will keep his driver's license for at least 10 years?

5. Simulation and illustration of Binomial and Poisson counting processes.
a) Given sample path size N and probability of arrival p, simulate a Binomial counting process $X(t)$.
Application: For a frame size of 1 second, simulate the number of airplane landings from Problem 1., for 1 minute.
b) Given frequency λ and a time frame $[0, T_{max}]$, simulate a Poisson counting process $X(t)$.
Application: Simulate the number of internet connections from Problem 3., for a period of half an hour.