STATISTICAL COMPUTATIONAL METHODS

Stochastic Processes

1. Markov Chains with n states $\{X_0, X_1, \ldots\}$ discrete random variables with pdf

$$X_i \begin{pmatrix} 1 & 2 & \ldots & n \\ P_i(1) & P_i(2) & \ldots & P_i(n) \end{pmatrix}, \; P_i = [P_i(1) \ldots P_i(n)], \; i = 0, 1, \ldots$$

- transition probability matrix

$$P = [p_{ij}]_{i,j=\overline{1,n}}, \text{ where } p_{ij} = P(X_{t+1} = j \mid X_t = i)$$

- h-step transition probability matrix

$$P^{(h)} = [p_{ij}^{(h)}]_{i,j=\overline{1,n}}, \text{ where } p_{ij}^{(h)} = P(X_{t+h} = j \mid X_t = i)$$

- Properties

$$P^{(h)} = P^h \text{ and } P_i = P_0 P^{(i)} = P_0 P^i$$

steady-state distribution

$$\pi_x = \lim_{h \to \infty} P_h(x), \; x = 1, \ldots,$$

found from the system

$$\pi P = \pi, \; \sum_{x=1}^{n} \pi_x = 1.$$

2. Binomial Counting Process $X(n)$, a discrete-state, discrete-time counting process; the number of successes in the first n independent Bernoulli trials, $n = 0, 1, 2, \ldots$

- frame Δ: time interval of each Bernoulli trial;

- arrival rate $\lambda = \dfrac{p}{\Delta}$: average number of successes (arrivals) per time unit;

- interarrival time $T = Y\Delta$: time between successes (arrivals); Y has Geometric(p) distribution;

- p: probability of arrival (success) during one frame (trial);

- $X(n) = X\left(\dfrac{t}{\Delta}\right)$: number of arrivals by (real) time t; X has Binomial(n, p) distribution;

- $n = \dfrac{t}{\Delta}$: number of trials (frames) during (real) time t.

3. Poisson Counting Process $X(t)$, a discrete-state, continuous-time counting process, obtained from a Binomial counting process for $\Delta \to 0$ (i.e. $n \to \infty$), while keeping λ constant

- arrival rate λ;

- $X(t)$: number of arrivals during time t; X has Poisson(λt) distribution;

- interarrival time T has Exponential(λ) distribution;

- time of the k$^{\text{th}}$ arrival T_k has Gamma($k, 1/\lambda$) distribution;

- Poisson-Gamma formula: $P(T_k \le t) = P(X(t) \ge k), \; P(T_k > t) = P(X(t) < k)$.