This article reports on research I conducted in the application of deep learning in the management of defined contribution pension plans. Though this writing is my own, the research was co-authored with a colleague and our advisors. A pre-publication manuscript of our research can be found at: https://arxiv.org/abs/2306.10582

Can Deep Learning Solve the 'Nastiest, Hardest Problem in Finance'?

Marc A. Chen [a]

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Despite machine learning's ubiquitous impact, it has yet to provide effective tools to everyday investors looking to secure their retirement. It turns out that creating a machine learning solution both robust and transparent enough for retail investing has proven difficult. New research from my group at the University of Waterloo seeks to address this issue by applying deep learning in a novel way to solve one of the most important problems facing retirees: portfolio decumulation.

Decumulation, a Constant Problem?

Defined contribution (DC) plans have unquestionably become the dominant type of retirement plan for U.S. workers. In 2022, 66% of private sector workers had access to a DC plan while only 15% had access to a defined benefit plan (U.S. Bureau of Labor Statistics, 2022).

Defined contribution plans, such as 401(k)s, leave the individual investor with the problem of creating a portfolio allocation decumulation strategy. Nobel Laureate William Sharpe famously referred to portfolio decumulation as 'the nastiest, hardest problem in finance' due to its many dimensions of difficulty (Ritholz, 2017). As DC plans' ubiquity has increased, so has market uncertainty: all measures of economic uncertainty surveyed by the Federal Reserve remain significantly elevated compared to the years preceding the Covid-19 pandemic (Federal Reserve Bank of Atlanta, 2023). As such, providing DC plan investors with tools to secure their retirement has never been more important.

Currently, the prevailing rule of thumb for managing a DC plan is the '4% Rule' proposed by Bengen (1994). This rule recommends that for a 30-year period, a retiree invest in a constant 50-50 mix of stocks and bonds and withdraw 4% of initial capital each year, adjusted for inflation. In rolling historical backtests, this strategy never depletes a portfolio. The apparent safety of this strategy has made it the near-universal advice of financial planners. However, the approach has two key problems: 1) It is inefficient, since it usually leaves a significant amount of money on the table that could be funding the investor's retirement. 2) It isn't statistically sound, since backtesting on only the realized historical market cannot holistically measure risk. Even if the 4% Rule hasn't depleted a portfolio yet, that doesn't mean that it won't in the future.

[a]David R. Cheriton School of Computer Science, University of Waterloo, Waterloo ON, Canada N2L 3G1, marcandre.chen@uwaterloo.ca

A Dynamic Approach

A notable recent contribution to solving the decumulation problem is Forsyth (2022), which asks if a more efficient DC plan strategy can be formulated by allowing for dynamic asset allocation and withdrawals, instead of keeping both constant as the 4% Rule recommends. The investment problem is formulated as one in stochastic control, which is a branch of mathematics concerned with making optimal decisions under uncertainty. Forsyth (2022) assumes that each year, the investor can withdraw a variable amount within a specified range and then change the allocation of stocks and bonds. The goal is to maximize a performance criterion that measures both risk and reward. A method called dynamic programming is used to solve for the asset allocation and withdrawal that maximizes the expected value of the performance criterion. The strategy adapts depending on the total wealth in the portfolio and the time left in the investment horizon. When tested in market simulations, the strategy is shown to significantly outperform the 4% Rule both in terms of risk and reward. The dynamic programming algorithm is also provably convergent, meaning we have mathematical proof that it yields an optimal solution.

Unfortunately, dynamic programming isn't scalable or versatile enough for practical investment use. Since the algorithm is not data-driven, it requires parametric market models that are difficult to calibrate. It also suffers from the 'curse of dimensionality,' making more complex investment problems prohibitively expensive to compute. Thankfully, a machine learning approach can help us overcome both of these drawbacks.

Learning, not Assuming

To avoid needing a parametric market model, we are led to learn an optimal strategy directly from market data. Turning to machine learning is therefore a natural next step. Neural networks in particular have exceptional expressive power to model complex relationships that can be learned from data. Neural networks are compositions of linear functions and non-linear activation functions. The linear functions each have a weight and bias, which are modified by training the model to optimize a specified performance criterion. Our research in Chen et al. (2023) asks whether a neural network approach can effectively learn the same optimal strategy from data with respect to the same performance criterion as in Forsyth (2022). If so, we'll have a far more versatile method that doesn't need to make assumptions about the underlying market.

Breaking the 'Curse of Dimensionality'

The other main advantage of a machine learning approach is that it is far more scalable. Dynamic programming relies on a brute-force search step across a grid to find the optimal decision at each time step. This grid represents the probability space of the system, and its dimensionality depends on the number of random processes being modeled. The investment problem we've been discussing has two random processes: the stock and bond returns. For an accurate solution, 2048 possible values might be considered for each process, creating a 2048×2048 grid to represent all combinations. At each point on the grid, the algorithm exhaustively searches for an optimal strategy that maximizes a specified performance criterion. While this is an expensive operation, it's still feasible. But what if an investor wanted to consider investing in a third asset, such as an international stock index? The grid would balloon to $2048 \times 2048 \times 2048$. The necessary computation has increased by an order of magnitude! This makes dynamic programming with more than even two stochastic variables infeasible in most cases.

What if we could bypass this high-dimensional step? It would allow us to solve for optimal strategies for more complex problems, such as those with a higher number of assets or more complex performance criteria. Our research in Chen et al. (2023) shows that neural networks allow us to do just that. A neural network is a universal map from the domain of its inputs to the range of its output activation functions—meaning that it can be used to directly model optimal strategies. The catch is that a neural network needs to be successfully trained before it can generate the mapping between its input domain and an optimal investment strategy.

Convergence!

In machine learning research, it is a rare opportunity to have a convergent solution to use as a ground-truth. By using the results from Forsyth (2022) as a benchmark to guide our experimentation, we developed a framework that reliably trains and closely approximates the optimal solution. This is remarkable, given that the two methods operate completely differently. Dynamic programming algorithm is not a learning model; it is a numerical algorithm that solves partial differential equations. That our NN model is able to learn the same patterns from data is a significant step forward in efforts to create robust ML methods for retail investing.

With an NN framework that can robustly model an optimal strategy, computing high-dimensional conditional probabilities is no longer necessary. This opens the door for us to solve optimal strategies in more complex market models.

Optimizing, not Predicting

It's important to note that our NN model does not attempt to predict asset returns. Only the most sophisticated quantitative researchers might consistently find success in using machine learning to predict stock movements, and even then the advantage gained is usually niche and temporary. Trying to predict future returns is clearly not the right approach for the average investor. This is a primary principle in our approach; the NN model is explicitly prohibited from attempting to make predictions. Asset prices are never shown to the NN model as input features so that it can never try to 'learn' an asset's behavior. The NN is instead given the total portfolio wealth over time. This way, the NN learns how the wealth is *expected* to evolve over time when following the prescribed asset allocation and withdrawal schedule. Through training, the NN can adjust its strategy and learn how to *optimize* expected investment performance. Return data for individual assets is only used in the investment simulation, not given to the NN model directly. Once the NN is trained, the optimal training performance is validated on testing data from another distribution to ensure the model isn't overfitting and performs well on market data it hasn't seen before.

A Solid Foundation

We believe these developments represent an exciting step forward in demonstrating the robustness of machine learning methods for helping everyday investors secure their retirements. Our research not only makes use of the convergent solution in Forsyth (2022) to ensure its optimality, but also tests the model's performance on both synthetic and historic market data to ensure far lower measures of risk and increased wealth withdrawals for the investor. But this is just the beginning. With this solid foundation, the approach can be extended to problems considering more assets or more complex objective functions to seek even further improved performance. We look forward to the continued development of this research.

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