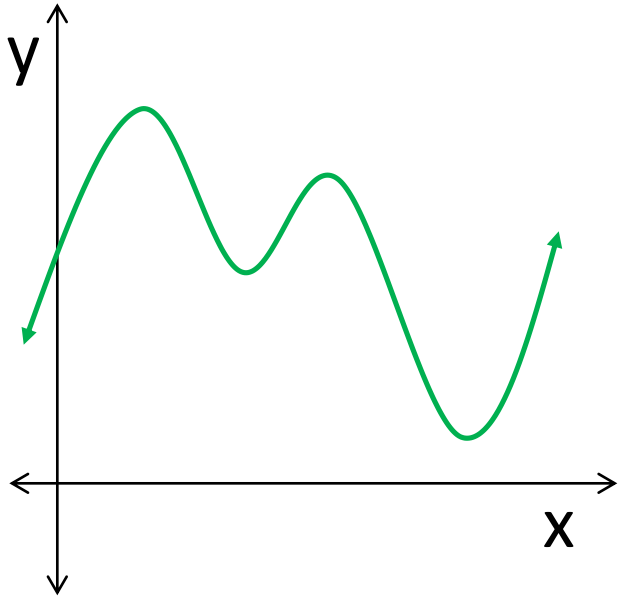
Graphing in 3D

- Purpose of 3D graphing
- Basics of 3D graphing
- Optimization in more than 1 dimension

The Purpose

So far, all the equations we've been dealing with have been in two dimensions.



Although it's common to think of x and y as both being variables, in optimization terms x is the only variable; y is the objective function.

The Purpose

But most optimization problems have more than one variable. For example, the top speed of an aircraft depends on a lot more than a single variable like the length of its wing.

In the next few lessons we'll be working with equations in two variables, but most of the techniques can also be applied to equations with more than two variables.

Graphing in 3D

When there are two variables, we call them x_1 and x_2; the objective function would be called $f(x_1, x_2)$. Here are some objective functions with two variables:

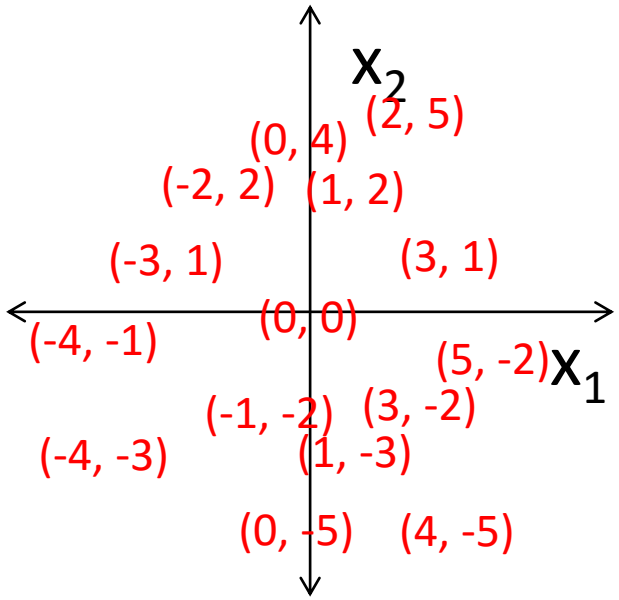
$$f(x_1, x_2) = x_1 \cdot x_2$$

$$f(x_1, x_2) = (x_1)^2 + 2(x_2)^2$$

$$f(x_1, x_2) = \frac{\sqrt{|x_1 \cdot x_2|}}{1+(x_1)^2+(x_2)^2}$$

Graphing in 3D

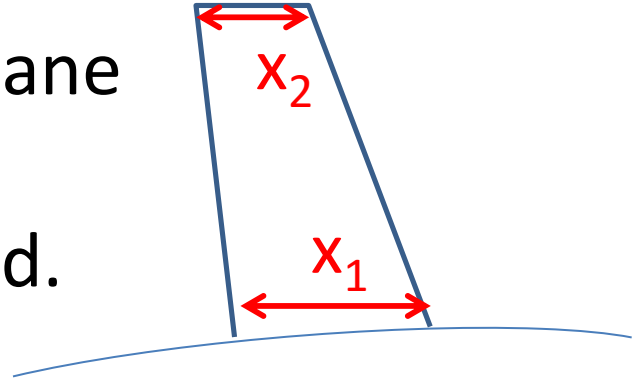
The first thing to consider is the flat plane, which is normally the axis for x and y. Now, it's the axis for x_1 and x_2.

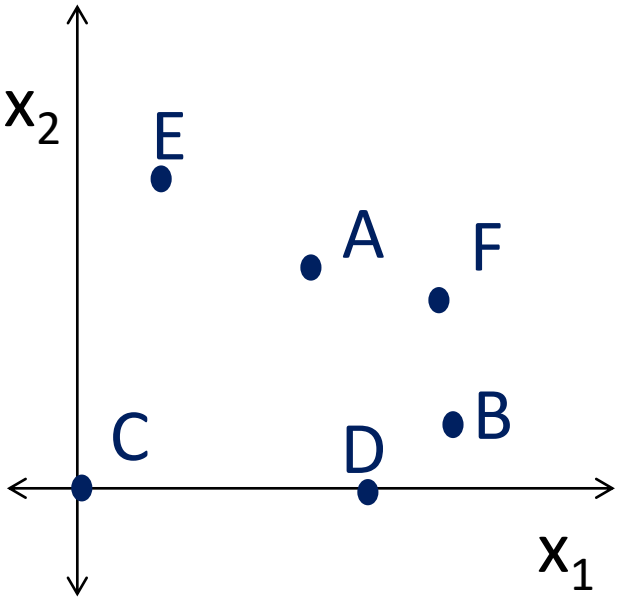


Every single point on this axis could be plugged into $f(x_1, x_2)$.

Practice Problem 1

Consider this sketch of an airplane wing, with variables x_1 and x_2 defined as the lengths indicated.





1. Sketch an airplane wing corresponding to each point on the graph at left:

a) A b) B c) C

d) D e) E f) F

Practice Problem 2

Consider the two-variable function

$$f(x_1, x_2) = (x_1)^2 + (x_2)^2.$$

a) Evaluate this function at the points (3, 1), (-2, -4), (0, 0), (5, -2), and (-4, 3).

b) Some points where this function has a value of 12 are $(3, \sqrt{3})$, $(\sqrt{12}, 0)$, $(-2, -2\sqrt{2})$, $(1, \sqrt{11})$, $(\sqrt{3}, -3)$ and $(\sqrt{10}, \sqrt{2})$. Find at least 10 more points, including positives and negatives as well as square roots, and graph them all on an x_1-x_2 coordinate plane. Sketch the resulting shape.

Graphing in 3D

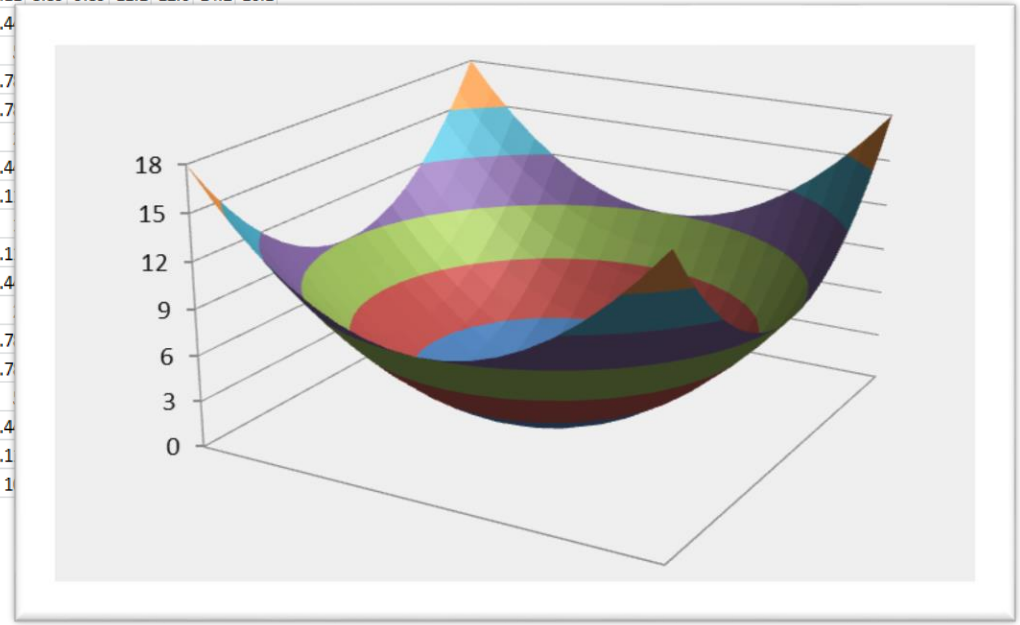
This is a picture of an Excel spreadsheet showing the results of the function on the previous page, $f(x_1, x_2)$, between -3 and 3 in both variables:

	-3	-2.7	-2.3	-2	-1.7	-1.3	-1	-0.7	-0.3	0	0.33	0.67	1	1.33	1.67	2	2.33	2.67	3
-3	18	16.1	14.4	13	11.8	10.8	10	9.44	9.11	9	9.11	9.44	10	10.8	11.8	13	14.4	16.1	18
-2.7	16.1	14.2	12.6	11.1	9.89	8.89	8.11	7.56	7.22	7.11	7.22	7.56	8.11	8.89	9.89	11.1	12.6	14.2	16.1
-2.3	14.4	12.6	10.9	9.44	8.22	7.22	6.44	5.89	5.56	5.44	5.56	5.89	6.44	7.22	8.22	9.44	10.9	12.6	14.4
-2	13	11.1	9.44	8	6.78	5.78	5	4.44	4.11	4	4.11	4.44	5	5.78	6.78	8	9.44	11.1	13
-1.7	11.8	9.89	8.22	6.78	5.56	4.56	3.78	3.22	2.89	2.78	2.89	3.22	3.78	4.56	5.56	6.78	8.22	9.89	11.8
-1.3	10.8	8.89	7.22	5.78	4.56	3.56	2.78	2.22	1.89	1.78	1.89	2.22	2.78	3.56	4.56	5.78	7.22	8.89	10.8
-1	10	8.11	6.44	5	3.78	2.78	2	1.44	1.11	1	1.11	1.44	2	2.78	3.78	5	6.44	8.11	10
-0.7	9.44	7.56	5.89	4.44	3.22	2.22	1.44	0.89	0.56	0.44	0.56	0.89	1.44	2.22	3.22	4.44	5.89	7.56	9.44
-0.3	9.11	7.22	5.56	4.11	2.89	1.89	1.11	0.56	0.22	0.11	0.22	0.56	1.11	1.89	2.89	4.11	5.56	7.22	9.11
0	9	7.11	5.44	4	2.78	1.78	1	0.44	0.11	0	0.11	0.44	1	1.78	2.78	4	5.44	7.11	9
0.33	9.11	7.22	5.56	4.11	2.89	1.89	1.11	0.56	0.22	0.11	0.22	0.56	1.11	1.89	2.89	4.11	5.56	7.22	9.11
0.67	9.44	7.56	5.89	4.44	3.22	2.22	1.44	0.89	0.56	0.44	0.56	0.89	1.44	2.22	3.22	4.44	5.89	7.56	9.44
1	10	8.11	6.44	5	3.78	2.78	2	1.44	1.11	1	1.11	1.44	2	2.78	3.78	5	6.44	8.11	10
1.33	10.8	8.89	7.22	5.78	4.56	3.56	2.78	2.22	1.89	1.78	1.89	2.22	2.78	3.56	4.56	5.78	7.22	8.89	10.8
1.67	11.8	9.89	8.22	6.78	5.56	4.56	3.78	3.22	2.89	2.78	2.89	3.22	3.78	4.56	5.56	6.78	8.22	9.89	11.8
2	13	11.1	9.44	8	6.78	5.78	5	4.44	4.11	4	4.11	4.44	5	5.78	6.78	8	9.44	11.1	13
2.33	14.4	12.6	10.9	9.44	8.22	7.22	6.44	5.89	5.56	5.44	5.56	5.89	6.44	7.22	8.22	9.44	10.9	12.6	14.4
2.67	16.1	14.2	12.6	11.1	9.89	8.89	8.11	7.56	7.22	7.11	7.22	7.56	8.11	8.89	9.89	11.1	12.6	14.2	16.1
3	18	16.1	14.4	13	11.8	10.8	10	9.44	9.11	9	9.11	9.44	10	10.8	11.8	13	14.4	16.1	18

Graphing in 3D

One type of graph is a 3D surface.

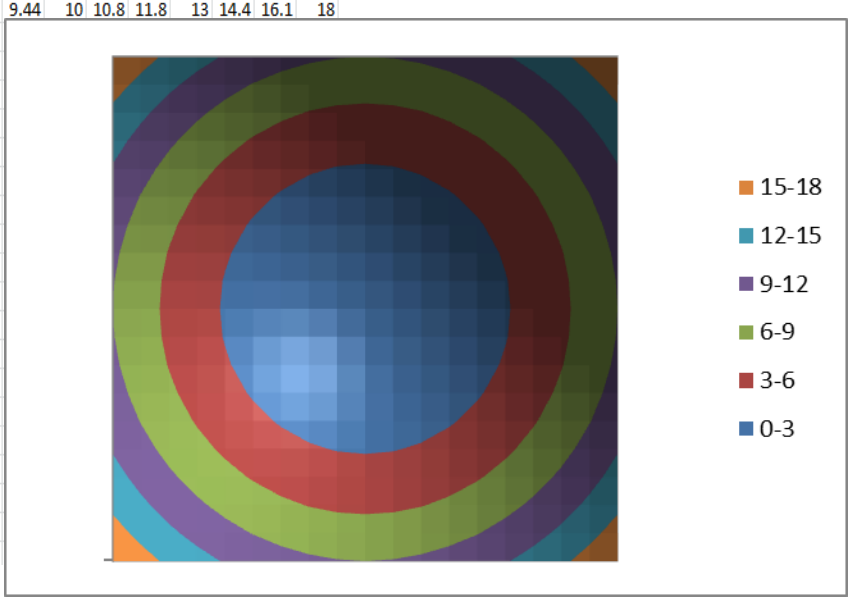
	-3	-2.7	-2.3	-2	-1.7	-1.3	-1	-0.7	-0.3	0	0.33	0.67	1	1.33	1.67	2	2.33	2.67	3
-3	18	16.1	14.4	13	11.8	10.8	10	9.44	9.11	9	9.11	9.44	10	10.8	11.8	13	14.4	16.1	18
-2.7	16.1	14.2	12.6	11.1	9.89	8.89	8.11	7.56	7.22	7.11	7.22	7.56	8.11	8.89	9.89	11.1	12.6	14.2	16.1
-2.3	14.4	12.6	10.9	9.44	8.22	7.22	6.44	5.89	5.56	5.44	5.56	5.89	6.4						
-2	13	11.1	9.44	8	6.78	5.78	5	4.44	4.11	4	4.11	4.44							
-1.7	11.8	9.89	8.22	6.78	5.56	4.56	3.78	3.22	2.89	2.78	2.89	3.22	3.7						
-1.3	10.8	8.89	7.22	5.78	4.56	3.56	2.78	2.22	1.89	1.78	1.89	2.22	2.7						
-1	10	8.11	6.44	5	3.78	2.78	2	1.44	1.11	1	1.11	1.44							
-0.7	9.44	7.56	5.89	4.44	3.22	2.22	1.44	0.89	0.56	0.44	0.56	0.89	1.4						
-0.3	9.11	7.22	5.56	4.11	2.89	1.89	1.11	0.56	0.22	0.11	0.22	0.56	1.1						
0	9	7.11	5.44	4	2.78	1.78	1	0.44	0.11	0	0.11	0.44							
0.33	9.11	7.22	5.56	4.11	2.89	1.89	1.11	0.56	0.22	0.11	0.22	0.56	1.1						
0.67	9.44	7.56	5.89	4.44	3.22	2.22	1.44	0.89	0.56	0.44	0.56	0.89	1.4						
1	10	8.11	6.44	5	3.78	2.78	2	1.44	1.11	1	1.11	1.44							
1.33	10.8	8.89	7.22	5.78	4.56	3.56	2.78	2.22	1.89	1.78	1.89	2.22	2.7						
1.67	11.8	9.89	8.22	6.78	5.56	4.56	3.78	3.22	2.89	2.78	2.89	3.22	3.7						
2	13	11.1	9.44	8	6.78	5.78	5	4.44	4.11	4	4.11	4.44							
2.33	14.4	12.6	10.9	9.44	8.22	7.22	6.44	5.89	5.56	5.44	5.56	5.89	6.4						
2.67	16.1	14.2	12.6	11.1	9.89	8.89	8.11	7.56	7.22	7.11	7.22	7.56	8.1						
3	18	16.1	14.4	13	11.8	10.8	10	9.44	9.11	9	9.11	9.44	1						



Graphing in 3D

A second type of graph is a contour map.

	-3	-2.7	-2.3	-2	-1.7	-1.3	-1	-0.7	-0.3	0	0.33	0.67	1	1.33	1.67	2	2.33	2.67	3
-3	18	16.1	14.4	13	11.8	10.8	10	9.44	9.11	9	9.11	9.44	10	10.8	11.8	13	14.4	16.1	18
-2.7	16.1	14.2	12.6	11.1	9.89	8.89	8.11	7.56	7.22	7.11	7.22								
-2.3	14.4	12.6	10.9	9.44	8.22	7.22	6.44	5.89	5.56	5.44	5.56								
-2	13	11.1	9.44	8	6.78	5.78	5	4.44	4.11	4	4.11								
-1.7	11.8	9.89	8.22	6.78	5.56	4.56	3.78	3.22	2.89	2.78	2.89								
-1.3	10.8	8.89	7.22	5.78	4.56	3.56	2.78	2.22	1.89	1.78	1.89								
-1	10	8.11	6.44	5	3.78	2.78	2	1.44	1.11	1	1.11								
-0.7	9.44	7.56	5.89	4.44	3.22	2.22	1.44	0.89	0.56	0.44	0.56								
-0.3	9.11	7.22	5.56	4.11	2.89	1.89	1.11	0.56	0.22	0.11	0.22								
0	9	7.11	5.44	4	2.78	1.78	1	0.44	0.11	0	0.11								
0.33	9.11	7.22	5.56	4.11	2.89	1.89	1.11	0.56	0.22	0.11	0.22								
0.67	9.44	7.56	5.89	4.44	3.22	2.22	1.44	0.89	0.56	0.44	0.56								
1	10	8.11	6.44	5	3.78	2.78	2	1.44	1.11	1	1.11								
1.33	10.8	8.89	7.22	5.78	4.56	3.56	2.78	2.22	1.89	1.78	1.89								
1.67	11.8	9.89	8.22	6.78	5.56	4.56	3.78	3.22	2.89	2.78	2.89								
2	13	11.1	9.44	8	6.78	5.78	5	4.44	4.11	4	4.11								
2.33	14.4	12.6	10.9	9.44	8.22	7.22	6.44	5.89	5.56	5.44	5.56								
2.67	16.1	14.2	12.6	11.1	9.89	8.89	8.11	7.56	7.22	7.11	7.22								
3	18	16.1	14.4	13	11.8	10.8	10	9.44	9.11	9	9.11								



Here, the colors indicate different constant values of the objective function.

Practice Problem 3

Consider the function $f(x_1, x_2) = (x_1)^2 + x_2$.

a) Sketch a contour map with dividing lines at f = 10, 5, 1, and 17. (To do this, find enough points for which $(x_1)^2 + x_2 = 10$ to see a pattern, then connect those points; repeat for 5, 1, 17.)

b) Sketch a 3D surface.