Introduction to Vectors

- What is a vector?
- Vectors in optimization
- A useful physics application

What is a vector?

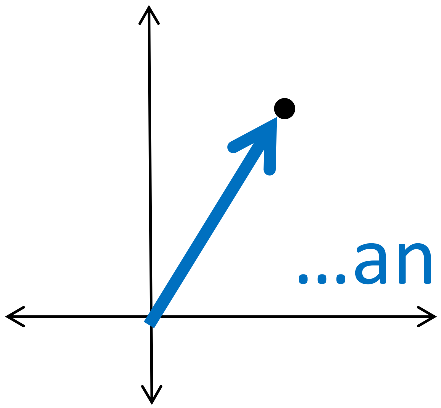
Two (of many) definitions:

1. A quantity specified by a magnitude and direction
2. A one-dimensional array (this essentially means a list of objects, usually numbers)

These two definitions are actually very closely related, as you'll see on the next page.

What is a vector?

Let’s say you have the one-dimensional array (3, 4). You could represent it graphically as the point (3, 4)…



…and connect to it from the origin…

… and that blue arrow has a magnitude (size) and a direction (that way) – so it fits the first definition of a vector.

What is a vector?

Although the previous example showed a simple two-element list, (3, 4), you could make a list three elements long like (4, 6, 8). If you graph that point in three-space you can see it also has a magnitude and direction. They are, however, harder to calculate.

When you get into four-space you can no longer represent vectors with graphs, but you can still calculate magnitude and direction.

What is a vector?

In physics, you are more likely to see vectors used as “direction + magnitude”, and typically not in four-space or five-space.

In mathematics, you are more likely to see vectors in “list of elements” form, and the number of elements is completely flexible – it could be one element or twelve or whatever.

Vectors in Optimization

In optimization, vectors are often written in matrix column form rather than point form. The list of variables x = (x_1, x_2, x_3, … x_n) would be called a vector and look like this:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdots \\ x_n \end{bmatrix}$$

It means the same thing and can still be treated as a point, it just looks different.

Sometimes, it will be written in matrix row form instead: [x_1 x_2 x_3 … x_n] which has the symbol x^T.

A useful physics application

In physics, vectors are often written with pointy brackets, like this: <4, -2>. This distinguishes them from actual points on the coordinate plane, like (4, -2).

location

vector
(magnitude and direction)

There are also quantities called *scalars*. A scalar is just a number, like 3 or -8.

A useful physics application

When changes occur to the location of an object, you can write those changes in this form:

$$\boxed{\text{New location}} = \boxed{\text{Old Location}} + \boxed{\text{scalar}} \bullet \boxed{\text{vector}}$$

The scalar is used to modify the vector so its direction is the same, but its magnitude might be larger or smaller.

A useful physics application

For example, if you had an object at (6, -1) that was being moved by a vector <2, 5> with a scalar of 0.4, you could write:

$$\boxed{?} = \boxed{(6, -1)} + \boxed{0.4} \cdot \boxed{\langle 2, 5 \rangle}$$

$$? = (6, -1) + \langle 0.8, 2.0 \rangle$$

(6.8, 1.0) is the new location.

A scalar less than 1 will reduce the magnitude of the vector; greater than 1 will increase it. A negative scalar will reverse the vector.

Practice Problem 1

1. Write each of the following vectors as a coordinate point A, a column matrix A, a row matrix A^T, and a physics-style vector $\vec{a}$.

a) 3, 4

b) 5, -2, -8

c) 6.1, 2.7, 0, 1.2, -4.9, 12.2

Practice Problems 2 and 3

2. Solve for the missing quantity:

a) Old location (5, 3), vector <-6, 2>, scalar 2.5
b) Old location (-1, -4), vector <1, 3>, scalar 0.1
c) Old location (5, 0), vector <5, 5>, scalar -2
d) New loc. (4, 1), old loc. (3, -6), scalar 1
e) New loc. (3, -2), old loc. (4, 5), scalar 0.4
f) New loc. (5, 1), old loc. (3, 6), vector <4, -10>
g) New loc. (-4, -2), vector <5, 1>, scalar 2

3. Explain why this is impossible:

New loc. (5, 1), old loc. (3, 6), vector <2, -4>

Practice Problem 4

4. Suppose that an object has moved from its old location at (6, -3) to a new location at (5, -1). You want to move it again in the same direction but only 1/3 as far as its first move.

a) Where will it end up?
b) Show your answer as a vector and scalar problem with proper notation.