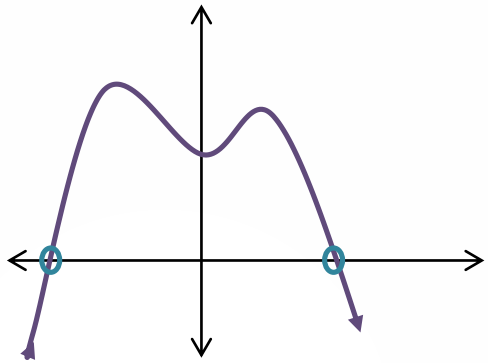
Newton's Method

- Secant Method Review
- Newton's Method Review
- Programming

Finding Roots

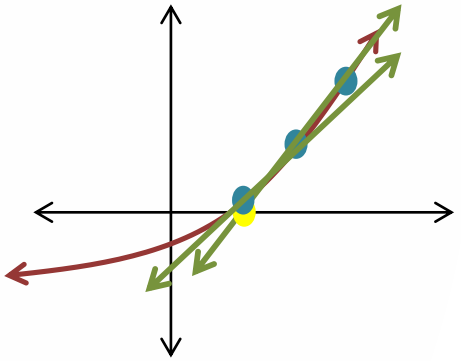


Finding the root (also known as zero, x-intercept, or solution) of a function is important in a lot of ways. Not only does it enable us to solve any graphable equation, it also has applications in calculus because there is meaning behind a derivative that is equal to zero.

Secant Method

In the very first unit of the course, we used the secant method to find zeroes. It worked like this:

1. Find two points near the root of the function.
2. Find the equation of the connecting line.
3. Find where that line crosses the axis.
4. Find the y-value of the function at that x-value.
5. Use that x-value with the better of your original points and repeat.
6. End when two successive answers are very close to each other.



Newton's Method

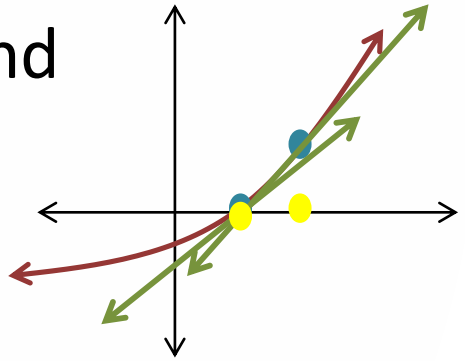
With Newton's Method, the procedure is basically the same except you do not need two points; you only need one.

The only reason we needed two points in the secant method was to get the slope of a line so we could find its equation. But with calculus you can find the slope of the line with only one point. This, of course, is known as a derivative.

Newton's Method

Newton's Method works like this:

1. Choose an x-value near the root.
2. Find the derivative at that point and use the resulting slope, plus the x and y value of the point, to write the equation of the tangent line.
3. Find where the tangent line crosses the x-axis.
4. Repeat for that x-value.



Practice Problem 1

Using the function $y = x^2 - 8$, find the positive root using three iterations of Newton's Method with a starting point of 3. Either leave the answers to each step in fraction form or round to 7 or 8 decimal places.

Compare to the correct answer of $2\sqrt{2}$.

Practice Problem 2

Using your derivative program from the first lesson in this unit, write a program that uses Newton's Method to find the root of a function f given starting point a.