Simplex Method: Next Steps

- Reading a Solution
- Pivoting

Reading a Solution

When reading the solution of a simplex tableau, there will always be as many active values as there are rows. An active variable is known as a *basic* variable, and is said to be *in the basis*. The objective function will always be active.

The basic variables and the objective function can be found in columns containing only one number and the rest zeroes:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 480 \\ 2 & 3 & 0 & 1 & 0 & 1200 \\ -3 & -4 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Reading a Solution

$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & f & \text{ans} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 480 \\ 2 & 3 & 0 & 1 & 0 & 1200 \\ -3 & -4 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

In this tableau, the basic variables are x_3 and x_4, and the objective function is f.

To find their values, divide the number in the last column by the number in the variable's column. All non-basic variables have value 0.

Here, $x_1 = 0$, $x_2 = 0$, $x_3 = 480$, $x_4 = 1200$, and $f = 0$. Since we are trying to maximize f, this is not a very good solution.

Reading a Solution

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & f & \text{ans} \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 280 \\ 0 & 2 & -2 & 1 & 0 & 100 \\ 0 & 0 & 30 & -3 & 4 & 1600 \end{bmatrix}$$

In this tableau, the basic variables are x_1 and x_2, and the objective function is f.

The solutions are…

$x_1 = 280$ $x_2 = 50$ $f = 400$

All non-basic variables are 0.

Practice Problem 1

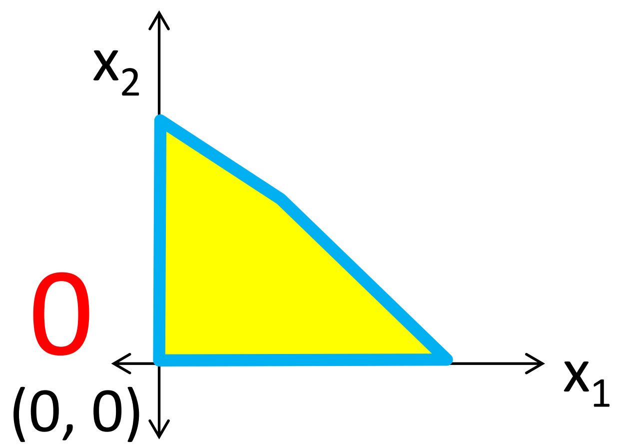
Read a solution from this simplex tableau:

$$\begin{array}{cccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & f & \text{ans} \\ \end{array}$$
$$\begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 5 & 0 & 12 \\ 0 & 0 & 3 & 2 & 0 & -1 & 0 & 13 \\ 0 & 4 & 0 & 3 & 5 & 6 & 0 & 21 \\ 0 & 5 & 0 & 4 & 0 & -2 & 14 & 200 \end{bmatrix}$$

Changing the Basis

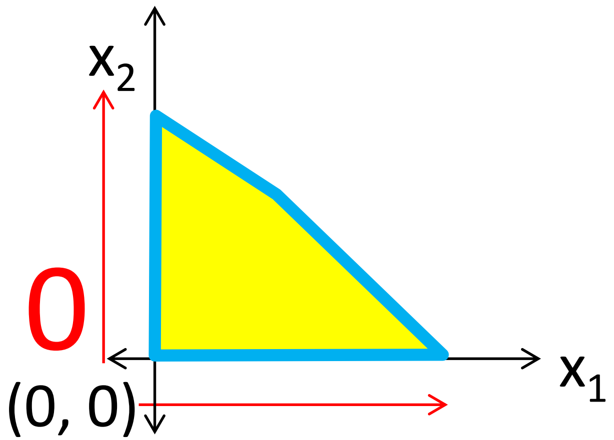
All standard-maximum simplex tableaux start with the slack variables in the basis and the objective function = 0. The actual constraint variables (x_1, x_2, etc) are also 0.

This corresponds with starting at the point (0, 0) on a graph:



Changing the Basis

In order to improve the value of f, we need to choose a better point than (0, 0). This will involve changing the variables in the basis: we need to exchange one of the slack variables for a “real” variable.



Changing the Basis: Pivoting

In order for a number to be in the basis, it must have a single number and the rest 0's in its column. This can be attained using the same rules used while *pivoting* in reduced row-echelon solving:

1. You may multiply or divide any row by any number
2. You may replace any row with the sum of itself and another row, or any multiple thereof.

The process is still called "pivoting".

Changing the Basis: Pivoting

In addition, there are a few new rules:

3. The pivot row must be used in all row-replacement operations.

4. The “answer” column must always remain positive except in the last row. In the last row, the coefficient of “f” must always remain positive. (If you get an illegal negative you can multiply the whole row by -1.)

5. It is not advisable, except when using a computer, to generate any fractions through division, including dividing the pivot row by the pivot.

Pivoting

Suppose you want to pivot this tableau around the circled value:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 480 \\ 2 & 3 & 0 & 1 & 0 & 1200 \\ -3 & -4 & 0 & 0 & 1 & 0 \end{bmatrix}$$

You could achieve that with these two operations, both of them using the pivot row as the tool:

(3 · first row) - (second row) → first row

(4 · second row) + (3 · third row) → third row

Pivoting

If you did so, the result would be:

$$\begin{bmatrix} 1 & 0 & 3 & -1 & 0 & 240 \\ 2 & 3 & 0 & 1 & 0 & 1200 \\ -1 & 0 & 0 & 4 & 3 & 4800 \end{bmatrix}$$

The solution now is…

$x_2 = 400$ $x_3 = 80$ $f = 1600$

This corresponds to the actual values $x_1 = 0$, $x_2 = 400$. On a graph this would be the point (0, 400). The value of f at that point is 1600.

Practice Problem 2

Finish the example by pivoting around the circled number. You are encouraged to use array operations in Julia to avoid mistakes in the math.

$$\begin{bmatrix} 1 & 0 & 3 & -1 & 0 & 240 \\ 2 & 3 & 0 & 1 & 0 & 1200 \\ -1 & 0 & 0 & 4 & 3 & 4800 \end{bmatrix}$$

Report the solution.

Practice Problem 3

Solve the experienced/inexperienced workers problem using this initial tableau:

$$\begin{bmatrix} 15 & 10 & 1 & 0 & 0 & 1200 \\ 1 & 2 & 0 & 1 & 0 & 120 \\ -10 & -9 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Your first pivot will be at location [1, 1].

Your second pivot will be at location [2, 2].

Use Julia to avoid mistakes in the math.

Report your answer!

Practice Problem 4

Write a program in Julia that inputs a matrix A and a pivot location [r, c], and outputs the matrix after pivoting is completed.

Be sure to include a test to fix any answer-column negatives. (The command `size(A, 2)` will output how many columns A has.)

Test your code using practice problem 3.

Practice Problem 5

Modify your program as follows:

1. Change `println(A)` near the end, to `return(A)`.

2. In addition to running the program, use it to change A. So, if your program was called pivot, you would replace

`pivot(A, 1, 1)` with `A = pivot(A, 1, 1)`

3. Following that, type in another two lines:

```
A = pivot(A, 2, 2)
println(A)
```

4. Next, recompile your code using the initial tableau from problem 3. What did that do?

Document and save your code!