Finding a Local Minimum

Method 2: Golden Ratio Intervals

The Premise

Clearly, brute force is not the best way to find the minimum of a function on an interval.

A better way would be to divide the given interval into smaller intervals, then test those to see where the minimum occurs.

The Problem

Recall that we need three points to verify that a minimum lies within an interval. If you just take the midpoint of the interval…

then you still don’t know if the actual minimum lies left or right of the endpoint.

The Solution

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Although it would seem obvious to divide the segment into equal thirds, with points at .33 and .67 across the segment, there is a better way.

The Golden Ratio

The Golden Ratio, symbolized by ϕ (the Greek letter *phi*) is a famous number related to the Fibonacci numbers. It is irrational and has two possible values: 1.61803... and 0.61803.... We will be using the second value.

The non-decimal representation is $(-1+\sqrt{5})/2$. It has the interesting property that $\phi^2 = 1-\phi$, among others.

The Solution

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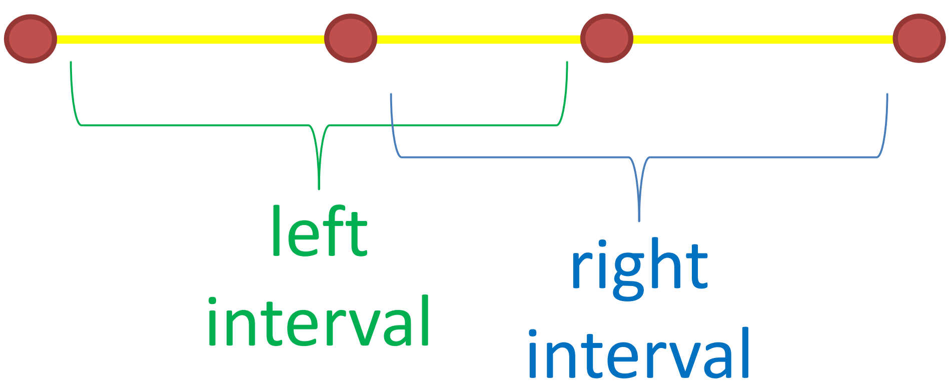
Dividing a segment with these proportions is known as creating a Golden Section.

The reason for its usefulness will be explained soon.

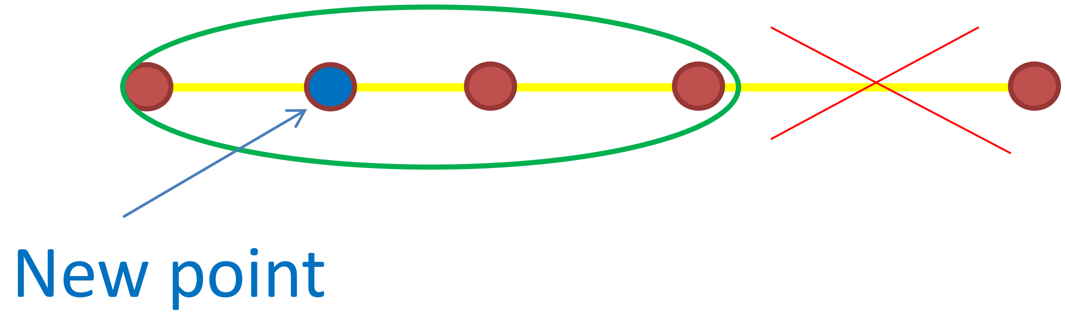
The Solution

After dividing the segment, there are two testable intervals, left and right. Whichever one has the middle point lower than the two endpoints, becomes our new interval for the minimum.



The Next Step

Then we will repeat the procedure again.



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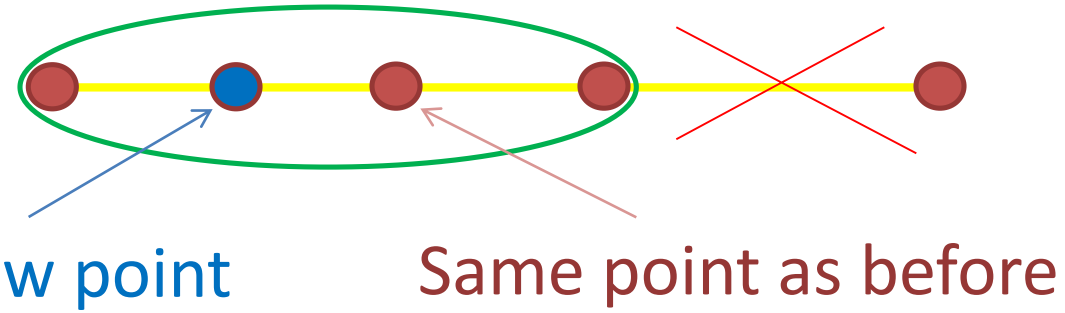

The cool property of the Golden Ratio is that if we use the exact value of ϕ, one of the two interior points can be used again – it's already there.

Practice Problem 1

1. Write the steps of a process for approximating a minimum using Golden Section intervals:

1. Start with:
2. End when:
3. Do this:
4. Using ________, loop back to step ___.

Practice Problem 2

Write the program.

Test your code.

Save this program! Document your code!