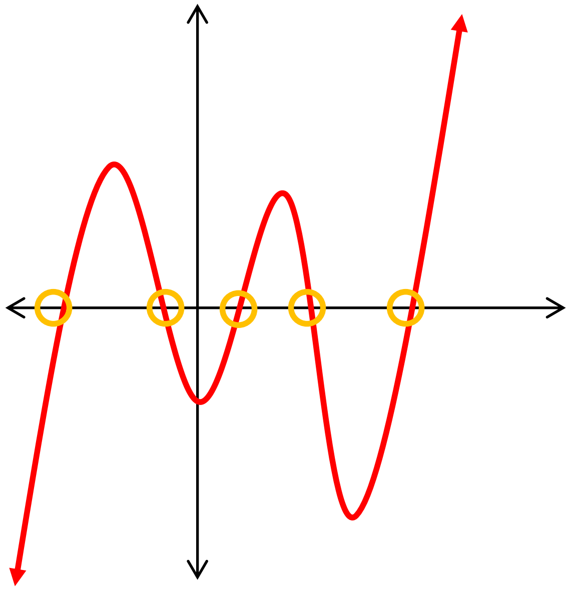
Iteration and Recursion 2

Solving using the secant method

The Problem

One of the most important things to know about an equation is its roots (also known as zeroes, x-intercepts, or solutions).



A lot of your time in math to this point has been spent learning how to find them.

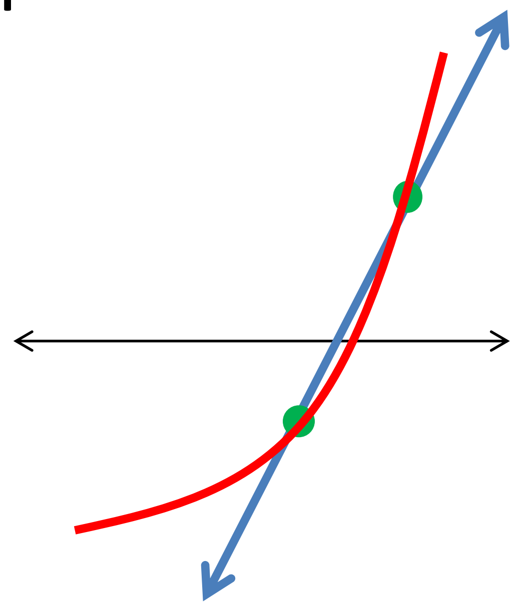
The Problem

With a quadratic like $x^2 + 6x + 8 = 0$ you can just factor it. If the quadratic isn't factorable, you can use the quadratic formula to find its roots.

But for higher-degree polynomials, as well as many non-polynomials, there is no magic formula to find the roots.

The Secant Approximation Method

However, as long as your function is smooth, you can approximate a root pretty well by using this procedure:

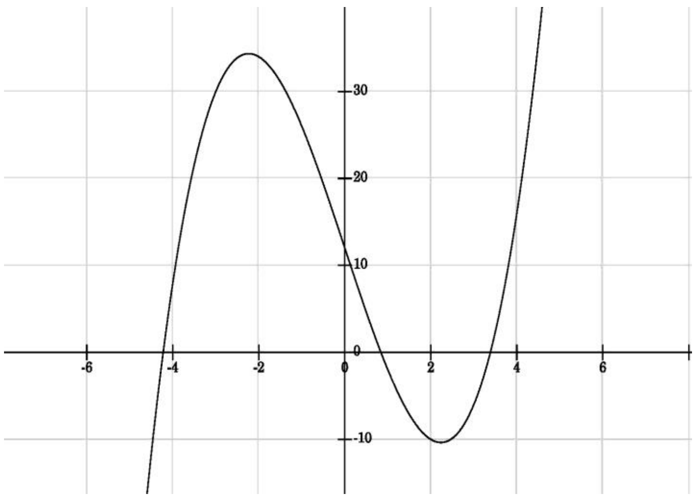


1. Find two points that are fairly close to each other and to the x-intercept of the function.
2. Find the equation of the connecting line.
3. Find where that line crosses the axis.

Notice how close this is to the actual root.

Practice Problem 1

1. Suppose $f(x) = x^3 - 15x + 12$. It appears to have roots between -5 and -4, 0 and 1, and 3 and 4.



a) Using x = 3 and x = 4, find the corresponding y-values and the equation of the connecting line. In the line equation, set y = 0 and solve to find where it crosses the x-axis.

b) repeat for x = 0, x = 1

c) repeat for x = -5, x = -4

d) Repeat for x = 4, x = 5

e) repeat for x = -1, x = 0

A better approximation

Let’s look at the answer to problem 1a), using the points (3, -6) and (4, 16). Hopefully you got an approximate root at x ≈ 3.272...

But 3.272 is not the *actual* root. How do you know? Because if it was, then f(3.272) would be 0. But when you plug 3.272… into the function, you get -2.0375…

Still, the point (3.272, -2.038) is a lot closer to the actual root than either (3, -6) or (4, 16). Hm…

A better approximation

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Ewww.

Take a moment in pairs/groups or alone to perform the calculation (find the slope, the linear equation, and the x-intercept of the line).

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Take a moment in pairs/groups or alone to perform the calculation (find the slope, the linear equation, and the x-intercept of the line).

Hopefully, you got x = 3.413 ish.

An even better approximation

Hopefully you didn't see this coming…

Now we have two good points: (3.272, -2.038) and (3.413ish, 0.561ish).

Yes, we're going to do it again.

The answer? $x \approx 3.384$.

How close are we getting?

No, we're not going to do it again. But we could.

For now, let's look at all the x-coordinates we examined, ordered by distance from the actual root:

*actual root: **3.3844…**

	distance:
(**4**, 16)	0.6156
(**3**, -6)	0.3844
(**3.273**, -2.038)	0.1117
(**3.413**, 0.561)	0.0286
(**3.383**, -0.034)	0.0017

In Formal Language

The iterative procedure for the secant method of finding zeroes could be written like this:

- Start with:
- End when:
- Do this:
- Loop: using ______, return to ______.

How is this process iterative? How is it recursive?

Practice Problem 2

This isn't actually practice, it's a step to make your life easier on the next problem.

2. Given two points (x_1, y_1) and (x_2, y_2),

a) write an equation for the slope

b) write an equation for the line between them

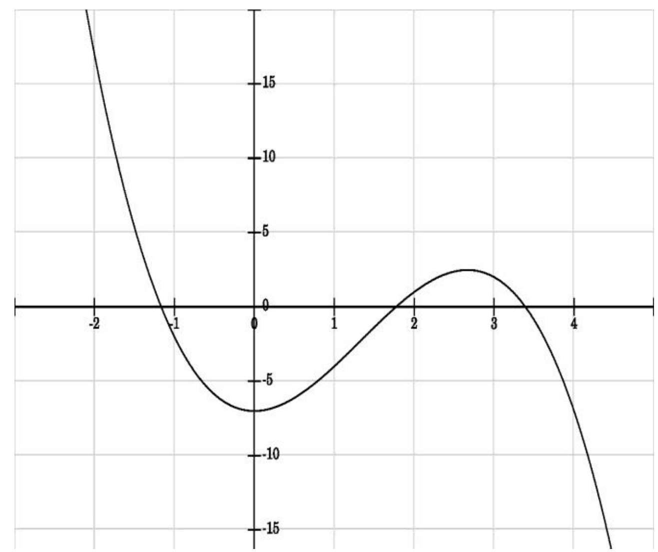
c) set $y = 0$ in that equation and solve for x

d) simplify to get an equation $x =$ __________, where the right hand side contains only x_1, y_1, x_2 and y_2. This equation can be used with any two points to directly find the x-intercept. Ugly, but useful.

e) use your equation to find the x-intercept of the line between (3, -6) and (4, 14).

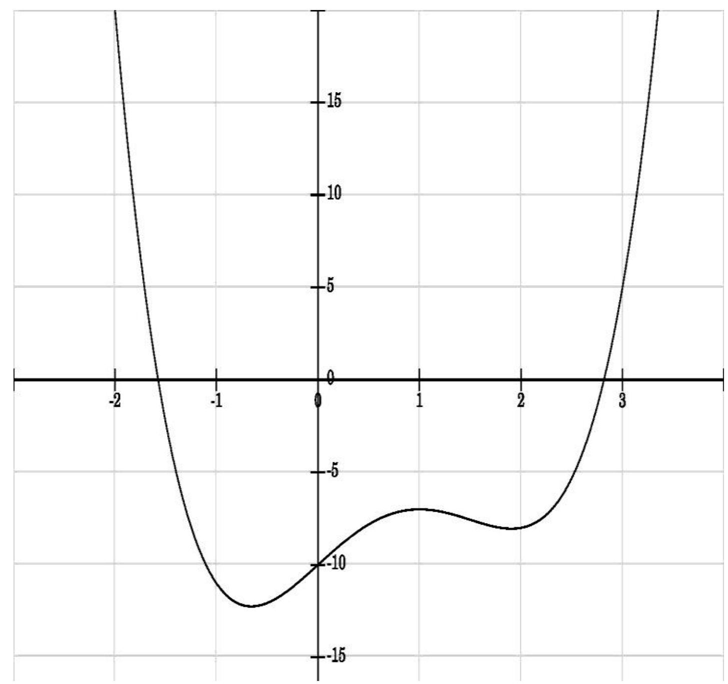
Practice Problem 3

3. Run three iterations of the secant procedure to approximate any one of the three roots of $f(x) = -x^3 + 4x^2 - 7$. Show your two original points as well as the three points obtained during iteration.



Practice Problem 4

4. Run three iterations of the secant procedure to approximate either of the two roots of $f(x) = x^4 - 3x^3 + 5x - 10$. Show your two original points as well as the three points obtained during iteration.



Practice Problem 5

Find at least one solution to the equation

$$e^{2x} - 3 = 4\sin x$$

to within 0.001 using successive iterations of the secant approximation procedure.