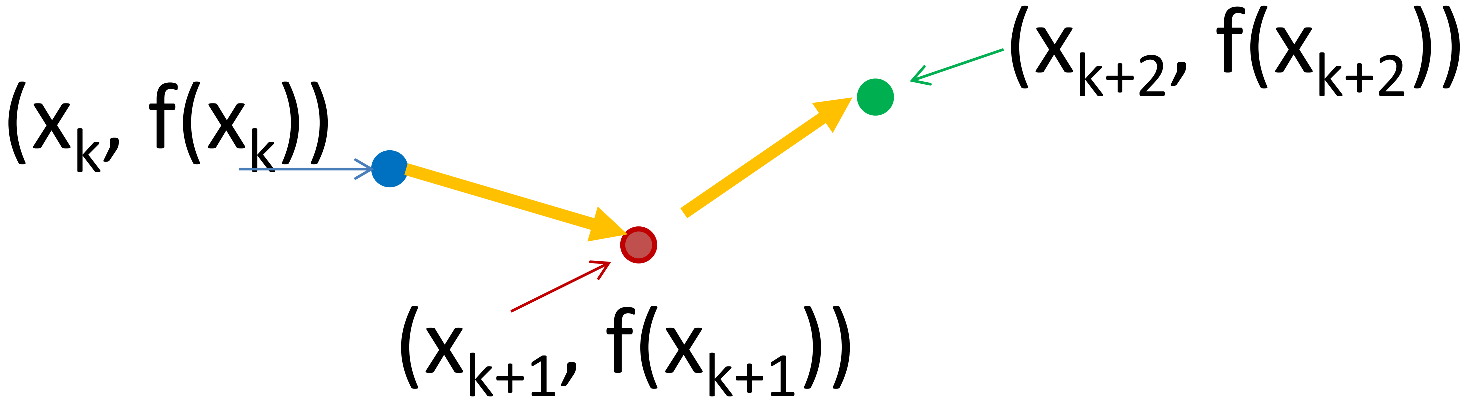
Finding a 3-Point Interval

- Review of the premise
- Writing a program
- Refining the program

The Premise



A 3-point interval around an extreme gives you an idea of where that extreme might be, even if you don't know the graph. It gives you an important first step for calculations of the actual minimum or maximum.

The Premise

To find a 3-point interval around a *minimum*, these are the most basic steps:

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1. Pick a starting point x and interval h. Test x - h and x + h to determine the direction of decrease.
2. Continue to take steps in that direction until the y-values start going the other way.
3. The last 3 points form your 3-point interval.

The Program – Problem 1

Here is the instruction loop we used in unit 1. This loop will *minimize* f(x).

1. Start with (x, f(x)).
2. Find (x+h, f(x+h)) and check if f(x+h) is lower than f(x). If not, find (x-h, f(x-h)).
3. Take one more step h or –h.
4. Check if you have reached the goal (middle point lower than both endpoints).
   – If yes, stop.
   – If no, return to step 3.

1. Write a program to accomplish this task, given a preloaded function f, a starting point and an interval.

Be sure to test your code.

Analyzing the Program

Although the execution of the loop was really fast, it's still useful to know how many iterations it had to run. Add a counter to your program to report how many iterations it takes, then find the number of iterations for…

- $f(x) = x^2 - 4x$, $x = 5$, $h = 0.1$
- $f(x) = x^2 - 4x$, $x = 5$, $h = 0.5$
- $f(x) = x^2 - 3x + 5$, $x = -12$, $h = 0.01$
- $f(x) = x^2 - 3x + 5$, $x = -12$, $h = 1$

Refining the Program

In the past unit we considered two potential problems with h:

- If h is too big, your margin of error is large and you risk skipping over the maximum or minimum completely.
- If h is too small, the calculations become too numerous.

Refining the Program

The solution to the problem with h is to start with a small h and make it larger as the loops count up. Our modified loop was this:

1. Start with (x, f(x)).
2. Find (x+h, f(x+h)) and check if it's going the right direction. If not, find (x-h, f(x-h)).
3. Take one more step h or –h.
4. Check if you have reached the goal (middle point higher/lower than both endpoints).
   – If yes, stop.
   – If no, increase h and return to step 3.

Refining the Program – Problem 2

Modify your program so that the value of h increases each time you loop through.

First attempt – use a multiplier of 1.5 for h.

Second attempt – increment using Fibonacci number multiples of h.

Test your code with low initial values of h; the iteration counter should be much lower.

Save this program! Document your code!