From Minimizing to Maximizing

- Modifying your current programs
- A nifty shortcut

Shifting the Focus

Your current saved programs from this unit are:

1. 3-point interval
2. Golden section minimum
3. Slope method minimum

All of these programs find a minimum – but what if you want to maximize?

Shifting the Focus:
3-point minimum

This program:

1. takes a starting point h and a function f, then tests x + h and x – h to determine the direction of decrease.
2. In that direction, it continues to mark off intervals until the y-values starts to increase.
3. The last three points form the three-point interval; the outer two are the endpoints.

What needs to change?

Your First Task

Change your three-point minimum program to a three-point maximum program.

Document both so you know which is which and save them separately.

Test your code!

Save this program.

Shifting the Focus:
Golden Section Minimum

This program:

1. Takes an interval [a, b] surrounding a minimum.
2. Places two interior points using the golden section
3. Tests all four points to find which three-point interval has endpoints higher than interior point.
4. Repeats on the new interval.

What needs to change?

Your Second Task

Change your Golden Section minimum program to a Golden Section maximum program.

Document both so you know which is which and save them separately.

Test your code!

Save this program.

Shifting the Focus:
Slope Method Minimum

This program:

1. Takes an interval [a, b] surrounding a minimum.
2. Finds the slope between them.
3. If slope is positive, shifts right endpoint in; if slope is negative, shifts left endpoint in; if slope is 0 shifts towards center.
4. Repeats on the new interval.

What needs to change?

Your Third Task

Change your Slope Method minimum program to a Slope Method maximum program.

Document both so you know which is which and save them separately.

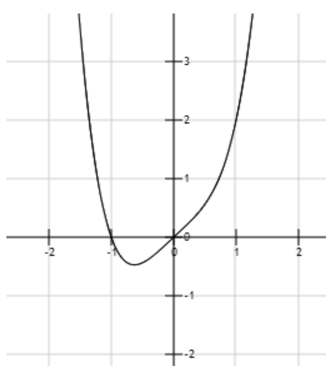
Test your code!

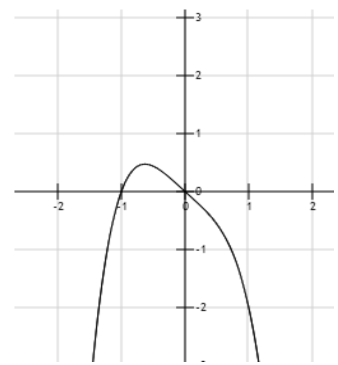
Save this program.

A Nifty Shortcut

While it's very useful to have separate programs for max and min, it's not necessary because there is a shortcut.

Compare the graph of f(x) = x^4 + x…

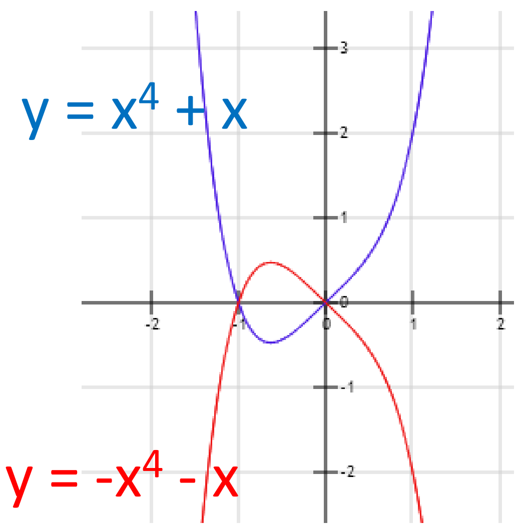




… with the graph of –f(x) = –(x^4 + x)

A Nifty Shortcut

Because the graphs of f(x) and –f(x) are *always* vertical reflections of each other, the x-value of the optimum point is the same for both.

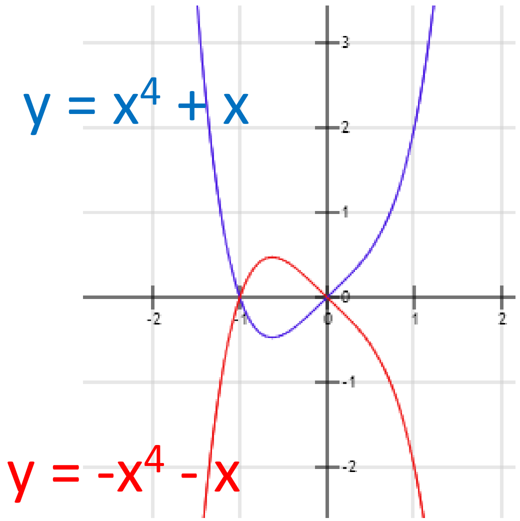


Even the y-value is the same, except one is negative.

A Nifty Shortcut

Therefore:

The location of min(f(x)) is the same as the location of max(-f(x)).



(Also, the location of max(f(x)) is the same as the location of min(-f(x)) – same thing but reversed.)

A Nifty Shortcut

So instead of modifying a program to find a maximum, you could just use an existing minimization program and enter –f(x) instead of f(x).

This trick is useful if you’re not planning on maximizing often enough to make it worth changing the program.

Practice Problem (4^{th} task)

Consider the function $f(x) = 2x^4 - 7x^3 + 2x^2 + 6x$.

It has two local minima (around 0 and 2) and one local maximum (around 1).

1. Find an interval around the local maximum using your 3-point maximum interval program, then find the actual maximum using *both* of your maximization programs.
2. Find an interval around the local maximum using your 3-point minimum interval program, then find the actual maximum using *both* of your minimization programs.
3. Verify that all four of your answers agree.
4. Repeat #1-3 for one of the local minima.