Simplex Method: Final Steps

- Choosing the Pivot
- Putting it All Together

Choosing the Pivot

The simplex method, from start to finish, looks like this:

1. Convert a word problem into inequality constraints and an objective function.
2. Add slack variables, convert the objective function and build an initial tableau.
3. Choose a pivot. the missing link
4. Pivot.
5. Repeat steps 3 and 4 until done.

Choosing the Pivot

Recall that a typical initial standard-maximum tableau looked like this:

“real” variables zero

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 480 \\ 2 & 3 & 0 & 1 & 0 & 1200 \\ -3 & -4 & 0 & 0 & 1 & 0 \end{bmatrix}$$

this stuff positive

negatives in bottom row, called “indicators”

slack variables in basis, objective function value 0

RIGHT HERE

The starting point in finding the pivot is to find the *most negative indicator.*

Choosing the Pivot

$$\begin{bmatrix} 1 & \boxed{\begin{matrix} 1 \\ 3 \end{matrix}} & 1 & 0 & 0 & 480 \\ 2 & & 0 & 1 & 0 & 1200 \\ -3 & -4 & 0 & 0 & 1 & 0 \end{bmatrix}$$

After you find the most negative indicator, examine the column above it. The pivot will come from these numbers.

First, you must eliminate from contention any that are negative or 0. (In this tableau, both pass this test.)

Choosing the Pivot

$$\begin{array}{cc} (480) & \\ & \left[\begin{array}{cccccc} 1 & 1 & 1 & 0 & 0 & 480 \\ 2 & 3 & 0 & 1 & 0 & 1200 \\ -3 & -4 & 0 & 0 & 1 & 0 \end{array}\right] \\ (400) & \end{array}$$

With the remaining candidates, use these steps:

1. Pretend it is in the basis.
2. Find its value as if it were in the basis.
3. Choose the one with the lowest value.

That number becomes your pivot.

Practice Problem 1

Choose the pivot from the following tableaux:

1a. $$\begin{bmatrix} 3 & 1 & 5 & 1 & 0 & 0 & 0 & 30 \\ -2 & 0 & 4 & 0 & 1 & 0 & 0 & 20 \\ 5 & 2 & 1 & 0 & 0 & 1 & 0 & 15 \\ -30 & -20 & 40 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

1b. $$\begin{bmatrix} 5 & 12 & 6 & 0 & -2 & 0 & 50 \\ 0 & 2 & 1 & 3 & -3 & 0 & 8 \\ 0 & -40 & -20 & 0 & 5 & 2 & 400 \end{bmatrix}$$

What if…

- …there are no negative indicators?
  - *you're done! The tableau is maximized. Report the solution.*
- …there are negative indicators but no legal candidates for the pivot?
  - *there is no solution to the problem; the scenario is not feasible.*
- …two pivot candidates are tied for the lowest value?
  - *Either will work; protocol says to choose the one higher up in the column. After pivoting, the column value in the other row will be 0. Make sure the basic variable in that row has a positive coefficient; if not, multiply the row by -1.*
- …the solution contains decimals… but shouldn't?
  - *Test all nearby lattice (whole number) points to make sure they satisfy the constraints, then plug them into the objective function and choose the best.*

Practice Problem 2

Identify each tableau as complete, unfeasible, or neither. If complete, report the solution; if neither, choose the next pivot:

2a. $\begin{bmatrix} 0 & 1 & 1 & -2 & 0 & 500 \\ 2 & 3 & 0 & 1 & 2 & 120 \\ 0 & 4 & 0 & 4 & 1 & 1200 \end{bmatrix}$

2b. $\begin{bmatrix} 15 & 20 & 1 & 0 & 0 & 400 \\ 5 & 10 & 0 & 1 & 0 & 200 \\ 2 & -5 & 0 & 0 & 1 & 0 \end{bmatrix}$

2c. $\begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 380 \\ 2 & 0 & 0 & 1 & 0 & 500 \\ -3 & -4 & 0 & 0 & 1 & 0 \end{bmatrix}$

2d. $\begin{bmatrix} 1 & 0 & 1 & 3 & 0 & 450 \\ 0 & 3 & 0 & 1 & 0 & 100 \\ 0 & 0 & 12 & 0 & 5 & 150 \end{bmatrix}$

Practice Problem 3

Finding the pivot using code involves a lot of if/else and looping, so we won’t be writing that code; besides, there are plenty of simplex programs out there already. Instead, describe the steps for finding the pivot as if you were designing a program to do so.

Putting It All Together

The steps, again, from start to finish:

1. Convert a word problem into inequality constraints and an objective function.
2. Add slack variables, convert the objective function and build an initial tableau.
3. Choose a pivot.
4. Pivot.
5. Repeat steps 3 and 4 until done.

Putting It All Together

Although you don't have code for the entire procedure, you do have code that completes the most difficult and error-prone step, which is pivoting. You can also run your code sequentially using the commands

```
A = pivot(A, r₁, c₁)
A = pivot(A, r₂, c₂)
println(A)
```

if you know in advance what your pivots will be.

Putting It All Together

With your pivoting-only code, what you will need to do is this:

1. Enter the initial tableau A.
2. Find the first pivot and run the code:

   B = pivot(A, r_1, c_1)
   println(B)

3. Find the next pivot and run the code from the start:

   B = pivot(A, r_1, c_1)
   B = pivot(B, r_2, c_2)
   println(B)

4. Repeat until a solution is found, then report the solution.

Practice Problem 4

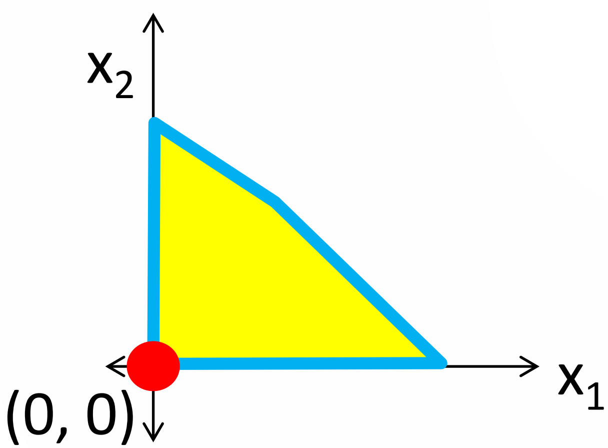
Maximize this initial tableau, from the carpenter problem in lesson 4:

$$\begin{bmatrix} 5 & 2 & 1 & 0 & 0 & 0 & 22 \\ 2 & 1 & 0 & 1 & 0 & 0 & 10 \\ 3 & -1 & 0 & 0 & 1 & 0 & 0 \\ -400 & -100 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Recalling that x_1 was the number of desks and x_2 was the number of chairs that should be built for a weekly profit f, report your answer in context.

What’s Going On

Although the simplex method is very abstract, it mimics the process of graphical linear programming almost exactly. It starts at the point (0, 0) (the initial tableau)…



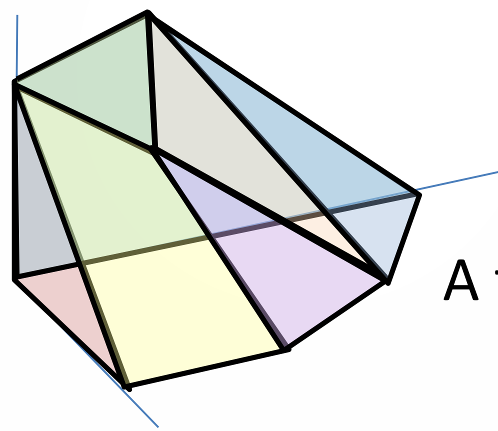
… then chooses a promising neighbor and moves there (pivoting)…

… and repeats…

… until it finds the maximum.

What's Going On

With three or more variables, the feasible region picture is no longer a two-dimensional region but a multi-dimensional polyhedron known as a *simplex* or *polytope*.

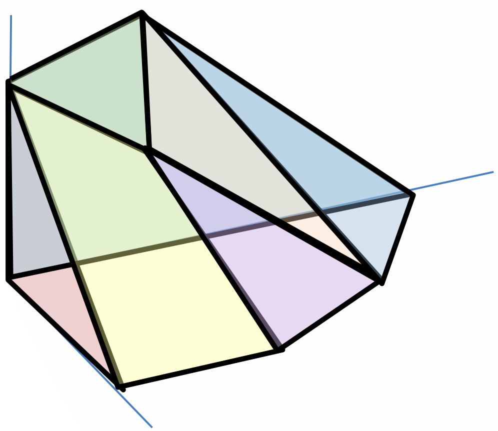


A three-variable simplex

Each constraint forms a face of the simplex and each corner point represents a potential solution.

What’s Going On

The simplex method starts at the origin and shifts from point to point, one coordinate at a time; the “basis” is simply the coordinates of the current point.



Changing the basis (pivoting) moves the point along an edge of the simplex to the most promising adjacent point.

The process repeats until a maximum is reached.

What’s Going On

Although impossible to visualize in four dimensions, the feasible region is still referred to as a “simplex” and the process is identical.