A Theory of Keynesian Demand and Supply Interactions under Endogenous Firm Entry*

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Abstract

We build a macroeconomic model with endogenous firm entry where aggregate demand and supply are intertwined. Under a novel two-layers system of firms where bottom-tier firms pay random fixed costs (e.g., purchasing necessary equipment) in order to operate while a top-tier firm uses the bottom-tier produced good as a sole input and faces nominal rigidities, a bottom-tier firm relies on loans from financial markets in order to pay those fixed costs, whose rate is determined by monetary policy. In this case, a new entry by bottom-tier firms lowers the real input price for top-tier firms and shifts the aggregate supply curve down, while triggering an increase in aggregate demand through their equipment purchases, which spurs additional rounds of entries and so on. Monetary policy can be powerful as it has impacts on aggregate demand and firms' entry decisions simultaneously. Our analytical characterization expresses the equilibrium firm entry as a function of a sufficient statistic related to monetary policy.

Keywords: Monetary Policy, Satiation, Endogenous Entry

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1 Introduction

Modern macroeconomic models are usually based on a system of blocks of aggregate supply and aggregate demand, and try to disentangle shocks to those two different blocks: while positive supply shocks (e.g., positive technology or negative cost-push shocks) shifts the supply curve (e.g., New-Keynesian Phillips curve) down, positive demand shocks boosts aggregate demand and as a result, output, in the presence of nominal rigidities. However, in many cases, demand and supply shocks are convoluted (e.g., one shock leads to the other) and/or come together as in Covid-19 crisis, and econometricians usually find it hard to identify one from the other.

In this paper, we question whether supply and demand shocks in a New-Keynesian framework are separable if the firm entry is endogenous. Our motivation is as follows: for example, a negative shock to aggregate demand (e.g., monetary tightening) makes it less profitable for many potential entrants to enter the market, so the firm entry decreases, shifting aggregate supply up (or to the left). As any newly entering firm needs to build their factories and purchase necessary equipment, a drop in firm entry reduces the aggregate demand, spurring additional reduction in firm entry, and so on. During this feedback process between the two, demand shocks cause supply to shrink through firms' endogenous exits from the market, while this reduction in firm entry further lowers aggregate demand.

Figure 1, based on the classic aggregate demand (AD) and aggregate supply (AS) diagram, illustrates the opposite case: when a positive demand shock hits the economy (AD_0 to AD_1), it triggers more market entry of firms, shifting the AS curve down (AS_0 to AS_1). As new entrants purchase equipment for their operation on the market, it additionally boosts aggregate demand (AD_1 to AD_2), which spurs additional round of new firm entry (AS_1 to AS_2) and so on until the convergence happens. Note that it can explain as well cases where a positive supply shock kicks in first. Still the positive feedback between the two drives the business cycle in those cases. Monetary accommodation can particularly initiate this infinite feedback loop between AD and AS but as more and more firms have already entered the market, the strength of this positive feedback will weaken as the policy rate keeps falling. Our job in this paper is to operationalize this concept in a precise manner with the help of modern macroeconomic tools.

For that purpose, we build a New-Keynesian business cycle model with two types of firms: top- and bottom-tier. Top-tier firms produce intermediate goods that aggregate into the final consumption good and face nominal price rigidities. In contrast, bottom-tier firms have flexible prices, hire labor, and produce inputs for top-tier firms. Furthermore, bottom-tier firms, in order to operate in a given period, should pay random fixed costs in the previous period, which they draw from a given fixed cost distribution and with which they buy necessary equipment in final good terms. They also draw the next period productivity from a given productivity distribution, and enter the market only if operation can be profitable. As bottom-tier firms transfer profits to households every period, they rely on loans from financial markets to fund the fixed costs, whose rate is decided by the monetary authority.

A new market entry of bottom-tier firms raises their competition degree and thereby lowers

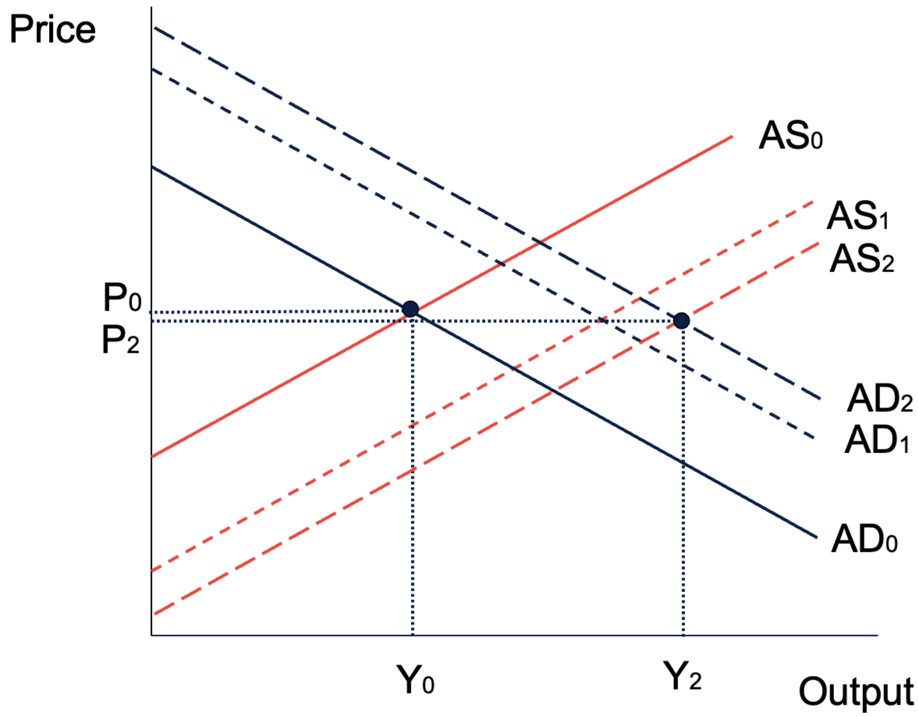

Figure 1: Convoluted aggregate demand and supply

the real (i.e., CPI-denominated) input price for top-tier firms. It lowers the real marginal cost for top-tier firms and shifts their pricing function (i.e., New-Keynesian Phillips curve or aggregate supply curve) down. In addition, it spurs more consumption demand as new entrants purchase necessary equipment in final consumption good, which invokes additional round of new entry and so on. Our steady state analysis and the calibrated impulse response exercises show that this channel is of significant power in making our results deviate from conventional macroeconomic wisdom, e.g., productivity shocks can be inflationary in our model.

Monetary accommodation can be powerful as it not only affects aggregate demand through usual channels (e.g., intertemporal substitution and wealth effects) but also it triggers firm entry by reducing the loan interest amount, adding additional upward pressures on aggregate demand. This second supply effect of monetary policy becomes weaker as the policy rate goes down, since many firms (especially those with high fixed costs or low productivity) may have already entered the market under a low interest rate.

Our novel use of extreme-value distributions (e.g., Pareto distributions for both productivity and fixed cost) allows analytic expressions for many important variables. We especially define the "satiated lower bound" (SLB) for each fixed cost type: a level of the policy rate that makes even the lowest productivity firm finds operation profitable[1] and the economy's average SLB (i.e., average across different fixed costs). It turns out that the equilibrium firm entry and

[1]In cases where the policy rate is under the satiated lower bound (SLB) for a given fixed cost type, all firms with different productivity draws that share that fixed cost will operate on the market.

other supply-related variables can be written in terms of the 'policy room': a relative ratio of the current policy rate to the economy's average SLB, implying that the average SLB acts as a benchmark interest rate in terms of monetary policy's supply-side effects. Under high levels of "policy room", a positive demand shock generates stronger supply-side (i.e., entry) responses, generating on average higher levels of the output multiplier. In addition, we decompose changes in aggregate variables into the extensive margin adjustment (i.e., new entry of firms) or intensive margin adjustment (i.e., each incumbent firm changes its behavior), which clarifies the model's interpretations.

Related literature Our business cycle setting with endogenous (bottom-tier) firm entry follows previous works in the literature, e.g., Bilbiie et al. (2007), Bergin and Corsetti (2008),[2] Stebunovs (2008), Kobayashi (2011) Bilbiie et al. (2012), Uusküla (2016), Hamano and Zanetti (2017) among others. While some papers assume equity financing for newly entering firms, e.g., Bilbiie et al. (2007), Bergin and Corsetti (2008), Bilbiie et al. (2012),[3] we assume that new firms finance their entry costs via borrowing from the financial markets, as in Stebunovs (2008), Kobayashi (2011), Uusküla (2016), so that firm entry is boosted under monetary accommodation, which aligns with the evidence presented in Colciago and Silvestrini (2022).[4] In addition, we express the equilibrium firm entry as a function of the "policy room", a sufficient statistic we devise.

Guerrieri et al. (2023) explore the circumstances under which a sectoral supply shock exhibits "Keynesian" properties. Specifically, they investigate when a supply shock prompts a shift in aggregate demand that exceeds the shock's original magnitude. Their analysis primarily revolves around two contexts: (i) the presence of multiple sectors in conjunction with incomplete markets, and (ii) scenarios where the impacted sector either complements or utilizes inputs from sectors that remain unaffected by the shock.[5] In contrast, our model's separation of the firm's sector into top-tier and bottom-tier industries, facilitates a comprehensive examination of the interplay between supply and demand. In our framework, supply shocks to bottom-tier firms influence aggregate demand via their impact on labor markets and loan demand. Conversely, demand shocks induce shifts in the bottom-tier's supply curve. These changes subsequently ripple through to

[2]Our assumption that fixed costs are paid in units of the final consumption goods aligns with the framework proposed by Bergin and Corsetti (2008). However, we deviate from their assumption of "pre-set" output procurement prices in favor of market prices.

[3]Under the assumption of equity financing for new entry, an expansionary monetary shock leads to an increase in the aggregate demand for products, raising labor demand and wages. Higher labor costs for potential entrants can lower their net present value and reduce the entry rate of new firms, which is counterfactual. For the role of "real wage rigidity" in resolving this problem, see Lewis and Poilly (2012).

[4]Colciago and Silvestrini (2022) find empirical evidence that expansionary monetary policy leads to an initial decrease and then an overshooting in the average productivity of the economy, as well as an initial increase and then undershooting in the firm's entry rate.

[5]Within the framework of Guerrieri et al. (2023), a negative supply shock to one sector engenders several countervailing effects: (i) it increases the aggregate price level, leading to a decline in overall consumption; (ii) it shifts demand towards goods from unaffected sectors. This reallocation is attenuated when the two sectors are complements, or when the unaffected sectors supply inputs to the affected sector, thereby causing the aggregate demand to decline by more than the initial supply shock itself; (iii) the decline of activity in a sector results in income losses, which, in the presence of incomplete markets and borrowing constraints, generally suppresses aggregate demand.

the top-tier industries via their influence on input prices, instigating successive shifts in demand until equilibrium is reached.

Our characterization of the satiation lower bound (SLB) hinges on the idea that (i) monetary expansion facilitates an upswing in firm entry, and (ii) upon the policy rate reaching a specified lower bound, all potential firms associated with a particular fixed entry cost have ventured into the market. Beyond this juncture, the positive supply effects stemming from further monetary accommodation and subsequent firm entry begin to wane. This phenomenon resonates with the insights of Ulate (2021) and Abadi et al. (2022), who incorporate analogous concepts in the context of banking profitability.

Layout Section 2 presents our New-Keynesian framework with endogenous firm entry. Section 3 discusses our calibration, steady-state analysis, and comparative statics. The model economy's impulse response functions to various shocks are explored in Section 4. Concluding remarks are presented in Section 5. For supplementary tables and figures, readers are directed to Appendix A. Derivations and proofs are detailed in Appendix B. A comprehensive summary of the equilibrium conditions, inclusive of the flexible-price and steady-state benchmarks, can be found in Appendix C. Lastly, Appendix D provides the derivation of the model under a simplified framework with homogeneous entry costs.

2 Model

2.1 Representative Household

The representative household maximizes lifetime utility given by

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left[\phi_{c,t} \cdot \log\left(C_t\right) - \left(\frac{\eta}{\eta+1}\right) \cdot N_t^{\left(\frac{\eta+1}{\eta}\right)} \right],$$

where C_t is consumption, N_t is labor, and $\phi_{c,t} \equiv \exp\left(u_{c,t}\right)$ is an aggregate demand shock defined as $u_{c,t} = \rho_c \cdot u_{c,t-1} + \varepsilon_{c,t},\ \ \varepsilon_{c,t} \sim N\left(0, \sigma_c^2\right)$. The household's budget constraint is

$$C_t + \frac{D_t}{P_t} + \frac{B_t}{P_t} = \frac{R_{t-1}^D D_{t-1}}{P_t} + \frac{R_{t-1}^B B_{t-1}}{P_t} + \frac{W_t N_t}{P_t} + \frac{\Upsilon_t}{P_t},$$

where D_t represents bank deposits, and B_t denotes government bonds, which are in zero net supply in equilibrium. The corresponding gross interest rates for these assets are represented by R_t^D and R_t^B, respectively. Υ_t captures lump-sum transfers to households. Such transfers may originate from various sources, including fiscal policies (such as subsidies to firms) or residual firm profits. We do not consider issues pertaining to the zero lower bound (ZLB) in this paper, so it is possible for interest rates to be negative, $R_t^D < 1$.

The first-order conditions bring the following standard intertemporal and intratemporal equations: The first-order conditions of this problem are

$$\frac{1}{R_t^D} = \frac{1}{R_t^B} = \beta E_t \left[\frac{\phi_{c,t+1}}{\phi_{c,t}} \cdot \frac{C_t}{C_{t+1}\Pi_{t+1}} \right] , \tag{1}$$

$$N_t^{\frac{1}{\eta}} = \phi_{c,t} \cdot C_t^{-1} \cdot \frac{W_t}{P_t} . \tag{2}$$

The household is indifferent between investing in bonds or deposits in equilibrium, and central bank policy via R_t^B has a one-to-one pass-through on R_t^D.

2.2 Firms

The model stratifies firms into two discrete categories: those belonging to the "top-tier" industry and those in the "bottom-tier" industry. In both layers, firms operate in an environment of monopolistic competition. Notably, only top-tier firms encounter nominal price rigidities as described by Calvo (1983). Operational dynamics are structured such that bottom-tier firms employ labor to generate intermediate input varieties, which top-tier firms subsequently incorporate into the production of consumption good varieties. Representative households own firms across both tiers.

One of the defining elements of this framework is the decision-making process for bottom-tier firms: At the beginning of each period, firms evaluate whether to continue/start operations. Should they decide to remain/enter the market, they must incur certain fixed costs that are financed through loans from the banking sector.[6]

2.2.1 Top-Tier Industry: Aggregator

A representative firm, operating under perfect competition, aggregates the differentiated products produced by a continuum of top-tier firms, denoted by u, spanning the interval [0, 1]. This can be formally expressed as:

$$Y_t = \left[\int_0^1 Y_t(u)^{\frac{\gamma-1}{\gamma}} \, \mathrm{d}u \right]^{\frac{\gamma}{\gamma-1}} .$$

The demand for each distinct variety produced by the top-tier firms, as well as the aggregate price, are given by

$$Y_t(u) = \left(\frac{P_t(u)}{P_t} \right)^{-\gamma} Y_t , \tag{3}$$

$$P_t = \left[\int_0^1 P_t(u)^{1-\gamma} \, \mathrm{d}u \right]^{\frac{1}{1-\gamma}} ,$$

where $Y_t(u)$ and $P_t(u)$ are the output and prices of top-tier varieties, respectively.

[6] This dependency on external funding effectively functions as a cash-in-advance production constraint.

Let $X_t = P_t Y_t$ represent the nominal aggregate expenditure, and $X_t(u) = P_t(u)Y_t(u)$ denote the expenditure for a specific top-tier variety u. Given these definitions, the individual demands can be reformulated as:

$$X_t(u) = \Gamma_t \cdot P_t(u)^{1-\gamma}\,, \qquad \text{where: } \Gamma_t = X_t P_t^{\gamma-1}.$$

2.2.2 Top-Tier Industry: Monopolistic Competition with Sticky Prices

Consider a firm u within the top-tier industry, belonging to the interval $[0,1]$. This firm employs $J_t(u)$ units of the aggregate product from the bottom-tier industry and produces $Y_t(u) = J_t(u)$, indicating a one-to-one transformation from input to output. Consequently, the aggregate sum of bottom-tier products, denoted as J_t, satisfies: $J_t \equiv \int_0^1 J_t(u)\,\mathrm{d}u = \int_0^1 Y_t(u)\,\mathrm{d}u$.
The profit equation for a top-tier firm u is given by

$$\Pi_t(u) = (1+\zeta^T)P_t(u)Y_t(u) - P_t^J J_t(u)\,,$$

where P_t^J represents the price of the aggregate bottom-tier product, and ζ^T stands for a production subsidy to top-tier firms. Thus, the present discounted value of profits, which the top-tier firm u seeks to maximize, can be expressed as:

$$\sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l} \left[(1+\zeta^T)P_{t+l}(u)Y_{t+l}(u) - P_{t+l}^J J_{t+l}(u) \right] \right\}\,,$$

with $Q_{t,t+l}$ being the stochastic discount factor between time t and $t+l$.
Firms in the top-tier industry face price stickiness à la Calvo (1983), characterized by a price-resetting probability of $1-\theta$. With reference to equation (3), a firm, when adjusting its price P_t^*, aims to:

$$\max_{P_t^*} \sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l}\theta^l \left[(1+\zeta^T)P_t^* - P_{t+l}^J \right] \left(\frac{P_t^*}{P_{t+l}} \right)^{-\gamma} Y_{t+l} \right\}\,,$$

where all firms that adjust their prices select P_t^* as the revised price. The resulting first-order condition can be articulated as:

$$\frac{P_t^*}{P_t} = \frac{\sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l}\theta^l \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1} \right) \left(\frac{P_{t+l}}{P_t} \right)^{\gamma+1} \left(\frac{P_{t+l}^J}{P_{t+l}} \right) Y_{t+l} \right\}}{\sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l}\theta^l \left(\frac{P_{t+l}}{P_t} \right)^{\gamma} Y_{t+l} \right\}}\,. \tag{4}$$

2.2.3 Bottom-Tier Industry: Aggregator

There exists a continuum of bottom-tier firms spanning the interval $[0,1]$, each producing a distinct variety. These firms exhibit heterogeneity in two principal dimensions: productivity, indexed by v, and operational fixed costs, indexed by m. The output of a firm, uniquely identified by the index pair mv, is defined as $J_{mv,t}$. A perfectly competitive firm aggregates these bottom-tier varieties as:

$$J_t = \left[\int_0^1 \int_{v \in \Omega_{m,t}} J_{mv,t}^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}v \, \mathrm{d}m \right]^{\frac{\sigma}{\sigma-1}} ,$$

where $\Omega_{m,t}$ denotes the subset of bottom-tier firms sharing the same operational fixed cost m that decide to produce in period t. Given significant fixed costs, only the firms with the highest productivity levels may find production viable. The demand for an individual bottom-tier variety (m, v), is:

$$J_{mv,t} = \left(\frac{P_{mv,t}^J}{P_t^J} \right)^{-\sigma} J_t \, . \tag{5}$$

Subsequently, the aggregate price index for the bottom-tier product is:

$$P_t^J = \left[\int_0^1 \underbrace{\int_{v \in \Omega_{m,t}}^1 \left(P_{mv,t}^J \right)^{1-\sigma} \mathrm{d}v}_{\equiv (P_{m,t})^{1-\sigma}} \, \mathrm{d}m \right]^{\frac{1}{1-\sigma}} = \left[\int_0^1 \left(P_{m,t}^J \right)^{1-\sigma} \, \mathrm{d}m \right]^{\frac{1}{1-\sigma}} , \tag{6}$$

where $P_{m,t}^J$ serves as the aggregate price for firms bearing the fixed costs indexed by m. We further define the nominal expenditure on a given bottom-tier variety as $X_{mv,t}^J = P_{mv,t}^J J_{mv,t}$, and the aggregate expenditure as $X_t^J = P_t^J J_t$, so

$$X_{mv,t}^J = \Gamma_t^J \cdot P_{mv,t}^{1-\sigma} , \qquad \text{where: } \Gamma_t^J = X_t^J \left(P_t^J \right)^{\sigma-1} . \tag{7}$$

Using equation (3), we can express the aggregate input demand of upper-tier firms as:

$$J_t = \int_0^1 Y_t(u) \, \mathrm{d}u = Y_t \underbrace{\int_0^1 \left(\frac{P_t(u)}{P_t} \right)^{-\gamma} \mathrm{d}u}_{\equiv \Delta_t} = Y_t \Delta_t \, , \tag{8}$$

where

$$\Delta_t = (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\gamma} + \theta \Pi_t^{\gamma} \Delta_{t-1} \, , \tag{9}$$

represents a measure of price dispersion. Utilizing equation (8), equation (7) can be expressed as $\Gamma_t^J = (P_t^J)^{\sigma} Y_t \Delta_t$.

2.2.4 Bottom Tier Industry: Monopolistic Competition, Loans, and Entry Decisions

The production function for an arbitrary firm (m, v) features diminishing returns to scale and is given by

$$J_{mv,t} = \varphi_{mv,t} \cdot N_{mv,t}^{\alpha}, \qquad \text{with } 0 < \alpha \leq 1 \, ,$$

where $N_{mv,t}$ denotes the labor employed, and $\varphi_{mv,t}$ is a firm-specific productivity assumed to be drawn from a Pareto distribution, $\varphi_{mv,t} \overset{\text{iid}}{\sim} \mathcal{P}\left(\left(\frac{\kappa-1}{\kappa}\right) A_t, \kappa\right)$, with A_t being the average aggregate productivity. A higher κ implies that the productivity distribution is more concentrated around its mean, A_t. The cumulative distribution function is given by:

$$\Psi(\varphi_{mv,t}) = 1 - \left(\frac{\left(\frac{\kappa-1}{\kappa}\right) A_t}{\varphi_{mv,t}} \right)^{\kappa} ,$$

with the probability distribution function defined as $\psi(\varphi_{mv,t}) \equiv \Psi'(\varphi_{mv,t})$.

Profit Function: Firms must pay a pre-determined in-kind fixed cost, $F_{m,t-1}$, in the preceding period (i.e., at $t-1$) to operate in period t. This cost, which might cover expenses such as equipment acquisition, is assumed to be financed through loans financed at the prevailing gross rate, R^J_{t-1}. The profit for a bottom-tier firm, if it chooses to operate in period t, is:

$$\Pi^J_{mv,t} = \underbrace{\left(1+\zeta^J\right) P^J_{mv,t} J_{mv,t}}_{\equiv r_{mv,t}} - W_t N_{mv,t} - R^J_{t-1} P_{t-1} F_{m,t-1} , \tag{10}$$

where ζ^J is a production subsidy to bottom-tier firms and $r_{mv,t}$ represents their revenue. These firms operate in a monopolistically competitive market and do not face nominal rigidities, setting prices as a constant markup over marginal costs (if they decide to produce), formally:

$$P^J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} J_{mv,t}^{\frac{1-\alpha}{\alpha}} . \tag{11}$$

By substituting the derived price equation into equation (10) and using the demand equations (5) and (7), we can rewrite the profit function as:

$$\Pi^J_{mv,t} = \Xi_t \cdot \varphi_{mv,t}^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} - R^J_{t-1} P_{t-1} F_{m,t-1} , \tag{12}$$

where

$$\Xi_t \equiv \frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{\frac{-\sigma}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} (\Gamma^J_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}} . \tag{13}$$

Entry Decision: Firms' entry decision is taken one-period ahead in $t-1$, and is based on their expected profits and associated costs in t. We assume that firms know at $t-1$ their forthcoming productivity for period t, $\varphi_{mv,t}$. However, they remain uninformed about other eventual shocks that could impact individual demand in t.[7] Should a firm decide to operate, it will subsequently hire labor in t from the spot market, realizing profits as described in equation (12). Given the

[7]This contrasts with Burnside et al. (1993), where labor decisions precede the realization of shocks. In our model, the decision to enter the market precedes the realization of other demand shocks. For simplicity, we assume that firms possess perfect foresight regarding their next period's productivity.

productivity draws, we can pinpoint the productivity threshold, $\varphi^*_{m,t}$, below which a firm would expect zero profit. Firms with the same fixed cost, $F_{m,t-1}$, and productivity draw below this threshold will opt out of market entry for period t. Using equation (12), the formal representation of $\varphi^*_{m,t}$ is:

$$E_{t-1}\left[\xi_t \cdot \Xi_t\right] \cdot \left(\varphi^*_{m,t}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} - R^J_{t-1} P_{t-1} F_{m,t-1} = 0 \,, \qquad \text{where: } \xi_t = \frac{Q_{t-1,t}}{E_{t-1}\left[Q_{t-1,t}\right]} \,. \tag{14}$$

It's important to note that this threshold, $\varphi^*_{m,t}$, is based on *ex-ante* expected profits. Once a firm (m, v) commits to market entry, unforeseen shocks could potentially push profits into negative figures. Considering the inherent lower limit on productivity, $\left(\frac{\kappa-1}{\kappa}\right) A_t$, the actual productivity threshold for entry becomes $\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right) A_t\right\}$.[8] The proportion of firms with a fixed cost $F_{m,t-1}$ that decide to operate in t is denoted as $M_{m,t}$ and is given by

$$M_{m,t} \equiv Prob\left(\varphi_{mv,t} \geq \varphi^*_{m,t}\right) = \min\left\{ \left(\frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right] \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R^J_{t-1} P_{t-1} F_{m,t-1}} \right)^{\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}}, 1 \right\} , \tag{15}$$

where we use (14) to substitute for $\varphi^*_{m,t}$ in the last expression. From this equation, we can derive the following proposition:

Proposition 1 *For bottom-tier firms with a fixed cost of $F_{m,t-1}$, $\underline{M_{m,t} = 1}$ when the policy rate R^J_{t-1} is below a threshold $R^{J,*}_{m,t-1}$ given by*

$$R^{J,*}_{m,t-1} \equiv \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right] \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{P_{t-1} F_{m,t-1}} \,. \tag{16}$$

We refer to this threshold, $R^{J,*}_{m,t-1}$, as the "satiated lower bound" (SLB) for firms of fixed cost type m.

As the policy rate, R^J_{t-1}, falls, more firms with the fixed cost $F_{m,t-1}$ opt for market entry in t due to the reduced loan repayment costs. Upon the policy rate reaching the type-specific lower bound $R^{J,*}_{m,t-1}$, all firms sharing the fixed cost $F_{m,t-1}$ (or lower) decide to become operational in t, leading to a stagnation in market entry for firms of cost type m and below. This type-specific lower bound, $R^{J,*}_{m,t-1}$, is hence termed the satiated lower bound (SLB).

In addition to the conventional intertemporal substitution effect captured by the Euler equation (1), monetary policy wields influence over the market entry decisions of bottom-tier firms. This, in turn, impacts the input market's prices and quantities, cascading onto the aggregate economy via the top-tier product markets. Upon the rate hitting the SLB for firms with the fixed cost $F_{m,t-1}$, no supplementary entries occur, rendering the supply-side effect of monetary policy ineffectual for such firms.

[8] If $\varphi^*_{m,t}$ is below $\left(\frac{\kappa-1}{\kappa}\right) A_t$, then all firms categorized by fixed cost m will operate in t.

Loan Demand: From equation (15), we derive the expression for the aggregate real loan demand of firms with a fixed cost type m:

$$\frac{L_{m,t-1}}{P_{t-1}} = M_{m,t} \cdot F_{m,t-1}. \tag{17}$$

Firms opting to operate in period t borrow an amount $L_{m,t-1}$ to acquire final goods equivalent to $F_{m,t-1}$. This acquisition connects the entry decisions of firms to the aggregate demand of the economy via the loan channel.

Fixed Cost Distribution: We assume that the fixed costs of bottom-tier firms, $F_{m,t}$, are drawn from a Pareto distribution, $F_{m,t} \overset{\text{iid}}{\sim} \mathcal{P}\left(\left(\frac{\omega-1}{\omega}\right) F_t, \omega\right)$, where F_t represents the average fixed cost associated with running a business, and $\omega > 1$ is the parameter that determines the variance of the distribution. The associated cumulative distribution function is:

$$H(F_{m,t}) = 1 - \left(\frac{\left(\frac{\omega-1}{\omega}\right) F_t}{F_{m,t}}\right)^{\omega} , \tag{18}$$

and its probability distribution function is denoted by $h(F_{m,t}) \equiv H'(F_{m,t})$. From Proposition 1, we obtain the probability measure of fixed cost types $F_{m,t-1}$ that are fully satiated, that is, the share of all firms with fixed cost $F_{m,t-1}$ that have already entered the market by time t, thus resulting in $M_{m,t} = 1$. This leads us to the following proposition:

Proposition 2 *Given the distribution in equation (18), the probability that $M_{m,t} = 1$ is:*

$$Pr\left(R^J_{t-1} \leq R^{J,*}_{m,t-1}\right) = Pr\left(F_{m,t-1} \leq \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R^J_{t-1} P_{t-1}}\right) \equiv H\left(F^*_{t-1}\right) ,$$

where F^*_{t-1} is the fixed cost threshold. All firms with a fixed cost $F_{m,t-1}$ less than or equal to F^*_{t-1}, irrespective of their productivity values $\varphi_{mv,t}$, opt to produce in period t. We term F^*_{t-1} the "full satiation fixed cost threshold".

Proposition 2 can be interpreted as follows: If a firm's fixed cost, $F_{m,t-1}$, is sufficiently low —below the threshold F^*_{t-1}— then even a firm with the lowest productivity draw, $\frac{\kappa-1}{\kappa} A_t$, would still deem operations in period t as profitable. Consequently, all firms bearing that fixed cost, regardless of their respective productivity draws, are active in period t.

Bottom-Tier Industry: Aggregation: The price aggregator for operating bottom-tier firms, denoted by P^J_t, can be expressed as:

$$\frac{P^J_t}{P_t} = \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H\left(F^*_{t-1}\right)}\right]^{\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} , \tag{19}$$

where $\Theta_3 = \frac{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}{\Theta_1\omega(\sigma-1)}$ and $\Theta_4 = \frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\omega(\sigma-1)}$ are constants. The aggregate measure of firms that operate during period t, represented by M_t, is given by

$$M_t = \int_0^1 \int_{v\in\Omega_{m,t}} 1 \, \mathrm{d}v \, \mathrm{d}m = 1 - \Theta_M \cdot \left[1 - H\left(F^*_{t-1}\right)\right] , \tag{20}$$

where $\Theta_M = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+\omega(\sigma-1)}$. Subsequently, the aggregate loan demand from operational bottom-tier firms can be derived as:

$$\frac{L_{t-1}}{P_{t-1}} = \frac{1}{P_{t-1}} \int_0^1 L_{m,t-1} \, \mathrm{d}m = F_{t-1} \cdot \left[1 - \Theta_L \cdot \left[1 - H\left(F^*_{t-1}\right)\right]^{\left(\frac{\omega-1}{\omega}\right)}\right] \tag{21}$$

where $\Theta_L = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)}$ is another model constant.

In equation (20), notice that as the satiation measure $H\left(F^*_{t-1}\right)$ rises, the number of operational firms at time t also increases. From equation (21), the aggregate real loan demand of firms is proportional to the average fixed cost, F_{t-1}, and grows with the satiation rate $H\left(F^*_{t-1}\right)$. Finally, in equation (19), the relative price of inputs from bottom-tier firms relates to the technology-adjusted real wage, $\frac{W_t}{P_t A_t}$, and the aggregate demand for inputs of top-tier firms, $\frac{Y_t \Delta_t}{A_t}$. When participation from bottom-tier firms increases, as indicated by $H(F^*_{t-1})$, this relative price decreases. This is due to more bottom-tier varieties being available to top-tier firms, leading to greater competition and a reduction in input prices. Therefore, the entry of new firms can reduce marginal costs for top-tier firms and mitigate inflationary pressures.

Average SLB: We obtain the average satiation interest rate of the economy by integrating over equation (16), and denote it by $R^{J,*}_{t-1}$. This rate serves as a measure of the satiation propensity of bottom-tier firms. When the prevailing policy rate R^J_{t-1} exceeds this average, a marginal reduction in R^J_{t-1} can induce an entry of lower-tier firms into the market. According to equation equation (19), this market entry can lower average input prices and subsequently mitigate inflation. It can also boost aggregate demand and elevate the price level, as new entrants take out loans to meet fixed costs, thus enabling the acquisition of fixed equipment for the production of final goods.

Proposition 3 *The aggregate satiation lower bound (SLB) is expressed as:*

$$R^{J,*}_{t-1} = \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{\infty} R^{J,*}_{m,t-1} \, dH\left(F_{m,t-1}\right) = \left(\frac{\omega^2}{\omega^2 - 1}\right) \cdot \frac{F^*_{t-1}}{F_{t-1}} \cdot R^J_{t-1}, \tag{22}$$

where F^*_{t-1} is the threshold fixed cost relative to the average fixed cost F_{t-1} in the economy.

If the threshold fixed cost for satiation, F^*_{t-1}, surpasses the economy's average fixed cost F_{t-1}, it signals an elevated likelihood of satiation across diverse fixed cost categories. Consequently, that results in a high value of the average SLB rate, $R^{J,*}_{t-1}$, relative to the policy rate, R^J_{t-1}. In such a situation, a minor ease in R^J_{t-1} may not substantially stimulate the entry of new bottom-tier firms.

Limit case, $\omega \to \infty$: In this calibration, the fixed cost distribution $H(F_{m,t})$ collapses to its mean value, F_t, thereby becoming degenerate. This results in a uniform fixed cost across all firms. The economy's state —whether fully satiated or not— is determined by the relative sizes of the policy rate R^J_{t-1} and the mean satiation lower bound, $R^{J,*}_{t-1}$. Specifically, should $R^J_{t-1} < R^{J,*}_{t-1}$, all bottom-tier firms enter the market and commence production in t. This simplified version of the model yields analytically tractable expressions concerning the model's equilibrium. Additional insights into the equilibrium conditions for this scenario are provided in Appendix D.

2.3 Shock Processes

The average fixed cost F_t is modeled as follows:

$$F_t = \phi_f \cdot \bar{Y}_t \cdot \exp(u_{f,t}) = \phi_f \cdot \tilde{Y} \cdot A_t \cdot \exp(u_{f,t}), \tag{23}$$

where $u_{f,t} = \rho_f \cdot u_{f,t-1} + \varepsilon_{f,t}$ and $\varepsilon_{f,t}$ is normally distributed with mean 0 and variance σ_f^2. Here, $\bar{Y}$ is the steady-state output level adjusted for technology, and $\bar{Y}_t = \bar{Y} \cdot A_t$ represents the balanced-growth path output.[9]

For technological progress, the model adopts:

$$GA_t \equiv \frac{A_{t+1}}{A_t} = (1+\mu) \cdot \exp\{u_{a,t}\} ,$$

where $u_{a,t} = \rho_a \cdot u_{a,t-1} + \varepsilon_{a,t}$, and $\varepsilon_{a,t}$ is normally distributed with mean 0 and variance σ_a^2.

Additionally, government expenditure G_t is formulated as:

$$G_t = \phi_g \cdot Y_t \cdot \exp(u_{g,t}) , \tag{24}$$

where $u_{g,t} = \rho_g \cdot u_{g,t-1} + \varepsilon_{g,t}$, and $\varepsilon_{g,t}$ is normally distributed with mean 0 and variance σ_g^2. It is assumed that the government maintains fiscal balance, levying a lump-sum tax $T_{g,t} = G_t$ on the representative household each period.[10]

[9] We assume that F_t scales with $\bar{Y}_t$, not the contemporaneous output Y_t. In practice, this assumption has minimal quantitative impact.

[10] Considering a zero net supply of government bonds, the government's dynamic budget constraint is upheld.

2.4 Central Bank

We assume that the central bank follows a Taylor rule for interest rate determination. The formal representation of this rule is given by:

$$R_t^B = R_t^J = R^J \cdot \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\tau_\pi} \left(\frac{Y_t}{\bar{Y}_t}\right)^{\tau_y} \cdot \exp\{\varepsilon_{r,t}\} \ ,$$

where $\varepsilon_{r,t}$ is a normally distributed idiosyncratic monetary policy shock with mean 0 and variance σ_r^2. The variable $\bar{Y}_t$ denotes the balanced-growth path output level, and $\bar{\Pi}$ indicates the steady-state trend inflation rate.

2.5 Aggregation

Here, we aggregate the equations presented in Section 2.2 to obtain the economy-wide conditions. Consider first the aggregate labor demand N_t, given by

$$N_t = \Theta_N \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \cdot (1 + \Theta_4 H_{t-1})^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \ , \tag{25}$$

where $H_{t-1} \equiv H(F_{t-1}^*)$ for simplicity, and

$$\begin{aligned} \Theta_N = & \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\ & \cdot \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) \Theta_3^{\left(\frac{\sigma}{\alpha(\sigma-1))}\right)} > 0 \ . \end{aligned} \tag{26}$$

From equation (25), it becomes evident that aggregate labor demand, N_t, is positively correlated with the demand for bottom-tier varieties, denoted by J_t. Conversely, the demand for labor decreases as the satiation measure, H_{t-1}, rises. An increase in H_{t-1} results in a higher aggregate measure of operating firms, M_t, as indicated in equation (20). This increase consequently stimulates employment through new entrants on the extensive margin. However, this surge in market entry also exerts downward pressure on the relative input price, $\frac{P_t^J}{P_t}$, and dampens the individual labor demand of existing firms, $N_{mv,t}$, due to intensified competition. In practice, the latter effect dominates and the reduction in labor demand at the intensive margin outweighs the increase at the extensive margin induced by new market entrants, provided that J_t is held constant.

The real wage, based on the household's intratemporal optimization condition in equation (2) and equation (25), is given by

$$\frac{W_t}{P_t A_t} = \Theta_N^{\frac{1}{\eta}} \left(\frac{C_t}{A_t}\right) \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\eta\alpha}} (1 + \Theta_4 H_{t-1})^{-\frac{\alpha+\sigma(1-\alpha)}{\eta(\sigma-1)\alpha}} \cdot \exp\{-u_{c,t}\} \ . \tag{27}$$

Substituting equation (27) into equation (19) yields:

$$\frac{P_t^J}{P_t} = \Theta_N^{\frac{1}{\eta}} \Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\frac{C_t}{A_t}\right) \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)} (1+\Theta_4 H_{t-1})^{-\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(\sigma-1)\alpha}} \cdot \exp\left\{-u_{c,t}\right\}. \tag{28}$$

Analysis of equations (25), (27), and (28) confirms that, given fixed aggregate demand measures such as C_t and J_t, an increase in H_{t-1} results in a reduction of both individual and aggregate labor demand. Consequently, this drives down the equilibrium wage. Hence, an increase in the entry of bottom-tier firms exerts a deflationary impact on the economy, signaling a positive shift in aggregate supply.

Market clearing: Market clearing in this economy is given by

$$C_t + \frac{L_t}{P_t} + G_t = Y_t \,, \tag{29}$$

which, in conjunction with equations (21), (23), and (24), can be reformulated as:

$$\frac{C_t}{Y_t} = 1 - \phi_g \cdot \exp\left\{u_{g,t}\right\} - \phi_f \cdot \left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{-1} \cdot \left[1 - \Theta_L \cdot [1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right] \cdot \exp\left\{u_{f,t}\right\}. \tag{30}$$

Notice that real loan demand is present on the left-hand side of equation (29). When bottom-tier firms opt to operate in the next period, they secure loans from financial institutions and utilize them to pay for in-kind fixed costs in terms of the final consumption good. This raises aggregate demand, exerting an inflationary influence in the economy as shown in equations (27) and (28): in those equations, stronger aggregate demand translates to inflation.[11]

Consequently, the entry of bottom-tier firms into the market has the dual effect of shifting both the aggregate supply and demand curves. Depending on the relative magnitudes of these shifts, market entry can exhibit either inflationary or deflationary tendencies. Section 4 will elaborate on the economy's short-run responses to demand and supply shocks within this framework, underscoring the inherent linkage between the two.

In the study by Guerrieri et al. (2023), a sectoral supply shock —such as the closing of high-contact sectors due to Covid-19— is more likely to become Keynesian, triggering a more substantial shift in aggregate demand than in supply, especially in multi-sector economies with incomplete markets. While their focus is primarily on an economy where the sector affected by the supply shock either complements or utilizes inputs from unaffected sectors, our dual-layered structure (comprising top-tier and bottom-tier industries) enables an exploration of the reciprocal impacts between supply and demand. Specifically, in our model, supply shocks to bottom-tier firms engender shifts in aggregate demand via the labor market and loan demand. Conversely,

[11] A Keynesian-cross structure becomes evident in equation (29) when endogenous entry of bottom-tier firms is considered. As Y_t expands, the measure of operating bottom-tier firms, M_t, along with their loan demand, $\frac{L_t}{P_t}$, rises, thus generating successive increments in demand.

demand shocks initiate shifts in the bottom-tier supply curve, affecting top-tier supply through their impact on input prices, and thereby resulting in successive rounds of demand shifts.

Average SLB and satiation: Upon substituting equation (B.22) into equation (22), we obtain an expression for the average SLB rate:

$$R_t^{J,*} = \left(\frac{\omega}{\omega+1}\right) \cdot (1-H_t)^{-\frac{1}{\omega}} \cdot R_t^J \,. \tag{31}$$

This expression allows us to interpret the "policy room", denoted as $\frac{R_t^J}{R_t^{J,*}}$, as a decreasing function of the satiation measure H_t.

Corollary 1 re-expresses the policy room $\frac{R_t^J}{R_t^{J,*}}$ as a *sufficient statistic* for the aggregate firm participation rate, M_{t+1}. Importantly, a wider policy room amplifies the impact of monetary easing on the entry of bottom-tier firms.[12] This finding rests on straightforward logic: a relatively high current policy rate R_t^J compared to the average SLB, $R_t^{J,*}$, increases the scope for additional firms to enter the market as the policy rate declines.[13] Note from equation (31) above that

$$\frac{R_t^J}{R_t^{J,*}} \leq \frac{\omega+1}{\omega} \,. \tag{32}$$

Corollary 1 *The total measure of bottom-tier firms opting to operate in period $t+1$ is:*

$$M_{t+1} = 1 - \Theta_M \cdot \left[\left(\frac{\omega}{\omega+1}\right) \cdot \frac{R_t^J}{R_t^{J,*}}\right]^{\omega} \,, \tag{33}$$

and a decrease in the policy room $\frac{R_t^J}{R_t^{J,*}}$ yields a larger increment in M_{t+1} when starting from a higher initial policy room level.

Proof. Directly from equation (33), we find:

$$\frac{dM_{t+1}}{d\left(\frac{R_t^J}{R_t^{J,*}}\right)} = -\Theta_M \left[\left(\frac{\omega}{\omega+1}\right) \cdot \frac{R_t^J}{R_t^{J,*}}\right]^{\omega-1} \cdot \frac{\omega}{\omega+1} < 0 \,,$$

whose absolute magnitude is increasing in the level of $\frac{R_t^J}{R_t^{J,*}}$, given $\omega > 1$. ■

Flexible Price Model: Under flexible prices, the price of consumption varieties produced by top-tier firms exhibits a constant markup over the cost of bottom-tier inputs. Mathematically,

[12] This is consistent with the concave and decreasing function M_{t+1} in relation to the policy rate, R_t^J, as seen in (33).

[13] This pertains to scenarios where the fixed cost cutoff F_t^* is low, thus allowing middle-range fixed cost firms with suboptimal productivity to enter the market.

this relationship is expressed as:

$$\frac{P_t}{P_t^J} = \frac{(1+\zeta^T)^{-1}\gamma}{\gamma - 1}. \tag{34}$$

This establishes that the flexible price equilibrium is money-neutral, signifying that the policy rate R^J exerts no influence on the real allocation of resources. Additional equilibrium conditions are provided in Appendix B.

2.6 Summary Equilibrium Conditions

For analytical tractability, balanced growth path-adjusted variables are denoted with a tilde, for example, $\tilde{Y}_t \equiv \frac{Y_t}{A_t}$. In our simulation results, we assume the government implements optimal transfers to neutralize real distortions arising from monopolistic competition. Specifically, this involves setting $\zeta^T = \frac{1}{\gamma-1}$ and $\zeta^J = \frac{1}{\sigma-1}$. A comprehensive list of equilibrium conditions is provided in Appendix C.

3 Steady State Results

3.1 Calibration

The values of calibrated parameters are presented in Table 1. Our model incorporates two key factors influencing the operation of bottom-tier firms in the market: fixed costs and productivity. These variables are assumed to follow independent Pareto distributions. The model is designed such that the proportion of operating bottom-tier firms is sensitive to parameters associated with these Pareto distributions. Utilizing the calibrated parameters outlined in Table 1, our model effectively replicates the moments commonly targeted in the literature. Key steady-state values are displayed in Table 2.

Fixed cost to balanced growth path output ratio, ϕ_f: We set $\phi_f = 0.37$ based on two key considerations. First, according to the Business Dynamics Statistics (BDS), the average annual exit and entry rates from 1977 to 2016 were 10.6% and 12.3%, respectively. Our chosen value of $\phi_f = 0.37$ yields a steady-state participation rate $M = 0.9$, in which the exit rate is precisely 10%. Second, the fixed cost in our model can be interpreted as a composite of capital and non-capital costs. In the existing literature, the capital-to-output cost ratio is approximately estimated to be around 30%. According to Table 5 in Domowitz et al. (1988), the non-capital fixed cost-to-output ratio varies between 0.05 and 0.18 across industries. Our model's steady-state fixed cost-to-output ratio of 0.37 aligns well within this empirical range.

Shape parameters in Pareto distributions, κ and ω: We select $\kappa = \omega = 3.4$ based on the work of Ghironi and Melits (2005), who choose this shape parameter for the productivity distribution to align with the standard deviation of log U.S. plant sales, estimated at 1.67 by Bernard et al. (2003). In our model, the standard deviation of log sales for operating bottom-tier firms is given by equation (35),[14]

$$\sigma\left(\log r_{mv,t}\right) = \frac{\sigma - 1}{\alpha + \sigma(1-\alpha)}\sqrt{\frac{1}{\kappa^2} + \left(\frac{\alpha + \sigma(1-\alpha)}{\sigma - 1}\right)^2 \frac{1}{\omega^2}}\,. \tag{35}$$

With $\kappa = \omega = 3.4$, our model predicts the standard deviation of bottom-tier firms' revenues to be 0.51. The residual variability in Bernard et al. (2003) may stem from factors we do not account for, such as taste heterogeneity or different demand weights for product types. Additionally, their estimates are based on U.S. manufacturing plants, whereas our framework focuses on bottom-tier firms.

Regarding productivity variability, the standard deviation of log productivity for operating bottom-tier firms in our model is proportional to equation (35) and is expressed as

$$\sigma\left(\log \varphi_{mv,t}\right) = \sqrt{\frac{1}{\kappa^2} + \left(\frac{\alpha + \sigma(1-\alpha)}{\sigma - 1}\right)^2 \frac{1}{\omega^2}}, \tag{36}$$

resulting in 0.36 when $\kappa = \omega = 3.4$. According to Bernard et al. (2003), their model-generated standard deviation of log value-added per worker is 0.35, while the empirical figure stands at 0.75.[15] Given the potential for measurement errors, our calibration is closely aligned with their model-generated moment and falls within a plausible range.

Elasticity of substitution, γ and σ: We select $\gamma = \sigma = 3.79$ based on the work of Bernard et al. (2003), who calibrate the elasticity of substitution to align with U.S. plant-level and macro trade data. Specifically, the value of 3.79 is chosen to match the productivity and size advantages of U.S. exporters.[16]

The conventional calibration in existing literature suggests $\gamma = 4.3$, resulting in a 30% markup over marginal costs. In contrast, our model distinguishes between top-tier firms, which face no fixed costs and whose marginal costs equals average input costs, and bottom-tier firms which incur period-by-period fixed costs to remain operational. Consequently, for bottom-tier firms, the average total cost exceeds the marginal cost. While $\gamma = 3.79$ generates a higher markup over marginal costs, it yields a reasonable markup over average costs when both firm tiers are considered.[17]

[14] The derivation of equations (35) and (36) is provided in Appendix B.

[15] Bernard et al. (2003) note that some degree of under-prediction could result from measurement errors in Census data.

[16] Several studies, including Ghironi and Melits (2005), Bilbiie et al. (2012), and Fasani et al. (2023), also adopt this elasticity of substitution, following Bernard et al. (2003).

[17] Jones (2011) explores the substitutability and complementarity of intermediate goods by assuming two different elasticities of substitution: 3 for final goods, and 0.5 for intermediate goods. We opt for a uniform elasticity of substitution for both industry layers. The choice between γ and σ depends on the model's interpretation. If bottom-tier firms are viewed as producers of essential commodities —like electricity, transportation services, or raw materials—

	Parameter Description	Value	Source
β	Discount factor	0.99	Average annualized real interest rate of 3.5%.
η	Frisch labor supply elasticity	1	Standard.
γ	Elasticity of substitution (of top-tier market)	3.79	Calibrated by Bernard et al. (2003) to fit the US plant and macro trade data.
σ	Elasticity of substitution (of bottom-tier market)	3.79	Set to be the same as top-tier products.
α	labor share in the bottom-tier production function	0.7	Standard.
θ	Calvo (1983) price stickiness	0.75	Standard.
κ	Shape parameter: Pareto distribution of productivity	3.4	Ghironi and Melits (2005).
ω	Shape parameter: Pareto distribution of fixed cost	3.4	Keep it the same with the productivity distribution.
ϕ_f	Fixed cost - steady state output ratio	0.37	The steady state mass of firms operating in the market $M = 0.9$. The real loan to output ratio, $\frac{L}{PAY}$, equals 30%.
ϕ_g	Government spending - output ratio	18%	Smets and Wouters (2007).
τ_π	Taylor parameter (inflation)	1.5	Standard.
τ_y	Taylor parameter (output)	0.15	Standard.
μ	Long-run TFP growth rate	0.005	Match a yearly growth rate at 2%.
Π	Long-run inflation	1.02	Long-run inflation target at 2%.
ρ_a	Autoregression for TFP	0.95	Smets and Wouters (2007).
ρ_c	Autoregression for demand shock	0.6	The autocorrelation of the preference shock that affects the marginal utility of consumption estimated by Nakajima (2005).
ρ_g	Autoregression for government spending	0.97	Smets and Wouters (2007).
ρ_f	Autoregression for fixed cost	0.8	Gutiérrez et al. (2005) use data on entry, investment, and stock market valuations of the US economy to recover entry cost shocks. The estimated persistence is 0.72.
σ_a	SD for ϵ_a	0.5	Within admissible intervals in Smets and Wouters (2007).
σ_c	SD for ϵ_c	0.2	The standard deviation of the preference shock estimated by Nakajima (2005) using U.S. data on consumption, labor, and output is 0.017.
σ_g	SD for ϵ_g	0.2	In Smets and Wouters (2007), the estimated admissible interval is $[0.48, 0.58]$. For our purposes, we do not need large disturbances to generate sizable responses.
σ_f	SD for ϵ_f	0.2	Gutiérrez et al. (2005) uses data on entry, investment, and stock market valuations of the US to recover entry cost shocks. The estimated standard deviation is 0.087.
σ_r	SD for ϵ_r	0.08	In Smets and Wouters (2007), the estimated admissible interval is $[0.22, 0.27]$. For our purposes, we do not need large disturbances to generate sizable responses.

Table 1: Calibrated parameters.

Variable	Value	Description
H	0.74	Mass of productivity-irrelevant firms.
M	0.9	Mass of firms operating in the market.
R^B	1.02	Gross risk-free rate.
$R^{J,*}$	1.17	Gross satiation rate.
$\tilde{F}^*$	0.43	Cutoff fixed cost-to-output ratio.
Δ	1.0006	Price dispersion.
$\frac{W_t}{P_t A_t}$	0.67	Real wage.
$\frac{C_t}{Y_t}$	0.52	Consumption-to-output ratio.
$\frac{W_t N_t}{P_t Y_t}$	0.7	Labor cost-to-output ratio.
$\frac{L_t}{P_t Y_t}$	0.3	Loan-to-output ratio.

Table 2: Steady state values.

3.2 Comparative statics

In this section, we conduct comparative statics analyses on the steady-state equilibrium under varying parameter calibrations. This will illustrate the relationship between individual parameters and the internal mechanics of the model.

Fraction of Operating Bottom-Tier Firms: The steady-state proportion of active bottom-tier firms, denoted as M, is described by $1 - \Theta_M[1 - H]$, as derived from equation (20). Figure 2 visualizes how M responds to shifts in model parameters: κ, ω, ϕ_f, β, μ, and Π. We decompose M as follows:

$$
\begin{aligned}
M &= Prob(F < F^*) + \cancel{Prob(F > F^*)} \int_{F^*}^{\infty} \left(\frac{F_m}{F^*}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}} \frac{dH(F_m)}{\cancel{1 - H(F^*)}} \\
&= \underbrace{H(F^*)}_{\equiv M_1} + \underbrace{\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+\omega(\sigma-1)}(1 - H(F^*))}_{\equiv M_2} .
\end{aligned}
$$

Here, $M_1 = H(F^*)$ represents the mass of firms with sufficiently low fixed costs ($F_{m,t} \leq F^*$) to remain active irrespective of their productivity. M_2 comprises firms that are operational but not at the lowest fixed-cost tier; these firms do not operate if they draw a low productivity level.

The following key points can be drawn from Figure 2: (i) An increase in κ raises both M_1 and M by narrowing the productivity distribution around its mean, thereby raising the lower bound of productivity and the likelihood of satiation for any given fixed cost; (ii) An increase in ω manifests via two opposing effects on firm participation, M. On one hand, it raises the minimum fixed

their products would exhibit lower substitutability, implying $\sigma < \gamma$. Conversely, if they produce different brands of the same product, higher substitutability would suggest $\sigma > \gamma$. We remain agnostic about this interpretational aspect and choose $\gamma = \sigma = 3.79$.

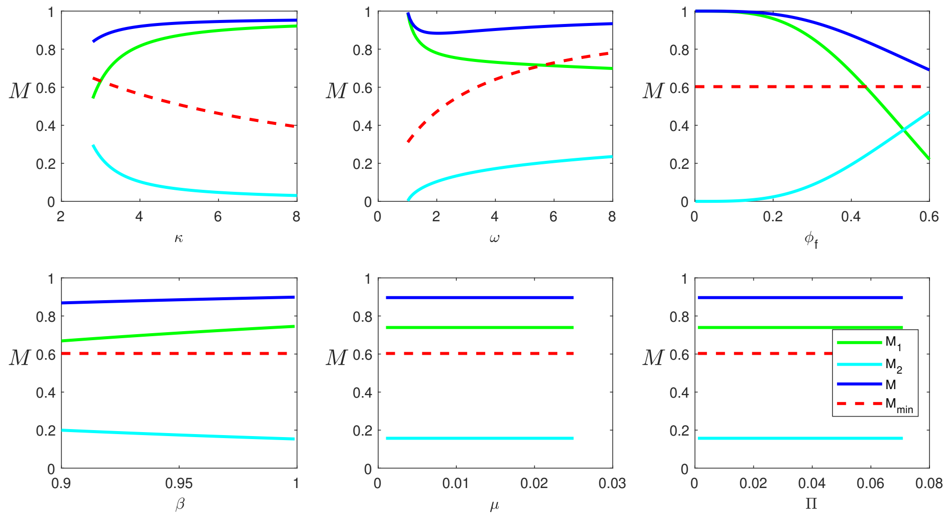

Figure 2: Comparative Statics: M

Notes: Benchmark parameters are fixed as listed in Table 1. Ranges for κ, ω, ϕ_f, β, μ, and Π are $[2.8, 8]$, $[1.01, 8]$, $[0.001, 0.6]$, $[0.9, 0.999]$, $[0.001, 0.025]$, and $[1.001, 1.0709]$, respectively.. The red dashed line marks the minimum mass of active firms, $M_{\min} = 1 - \Theta_M$, attained when no firm is satiated, $H_t = 0$. We partition M into productivity-irrelevant M_1 and jointly determined M_2 components for various parameter values.

cost $\frac{\omega-1}{\omega}F$, thereby reducing M. On the other hand, it narrows the fixed-cost distribution around its mean F, potentially reducing the mass of high fixed-cost firms and subsequently increasing M. The net effect on M depends on the relative magnitudes of these two forces. Moreover, the satiation measure M_1 typically declines as ω rises due to an increased lower bound on fixed costs, $\frac{\omega-1}{\omega}F$, affecting firms that are typically satiated. These general characteristics relating ω and M are further elaborated in Figure A.1 in Appendix A, which explores the influence of other parameters on the functional relationship between M and each parameter; (iii) An increment in ϕ_f shifts the fixed-cost distribution to the right, thereby reducing both M and M_1.
Following from equation (33), it is evident that the policy room $\frac{R^J}{R^{J,*}}$ maintains an inverse relationship with M. Variations in the parameters will produce effects on the policy room that are opposite to their impacts on M, as documented in Figure A.2 in Appendix A.

The Real Loan-to-Output Ratio: At the steady state, the following inequality is derived from equations (21) and (32):

$$\phi_f\,(1-\Theta_L) \leq \frac{L/P}{\bar{Y}} = \phi_f\left[1-\Theta_L(1-H(F^*))^{\frac{\omega-1}{\omega}}\right] = \phi_f\left[1-\Theta_L\left(\frac{\omega}{\omega+1}\frac{R^J}{R^{J,*}}\right)^{\omega-1}\right] \leq \phi_f,$$

where the real loan-to-output ratio, $\frac{L/P}{\bar{Y}}$, is a decreasing function of the policy room $\frac{R^J}{R^{J,*}}$, but increasing with respect to the satiation measure $H(F^*)$, and total firm participation, M.[18]

[18]Note that M increases with H at the steady state as per equation (20).

Figure 3 describes how $\frac{L/P}{\bar{Y}}$ varies with key model parameters: κ, ω, ϕ_f, β, μ, and Π. Our observations can be summarized as follows: (i) An increase in κ raises firm participation M, as illustrated in Figure 2, and narrows the policy room $\frac{R^J}{R^{J,*}}$, as seen in equation (33) and Figure A.2, resulting on a higher aggregate loan demand; (ii) An increase in ω gives rise to conflicting outcomes: it initially depresses firm participation M when ω is below a certain threshold, which can be attributed to an increase in the minimum fixed cost of entry, $\frac{\omega-1}{\omega}F$, as seen in Figure 2. However, this negative extensive margin effect is eventually counterbalanced by a positive intensive margin effect, where each active firm incurs a greater fixed cost, hence raising the real loan-to-output ratio; (iii) An increase in ϕ_f results in a reduction of firm participation M, evident from Figure 2, thus reducing aggregate loan demand. As before, this decrease via the extensive margin is eventually neutralized by an increase via the intensive margin, where each active firm shoulders a higher fixed cost.[19] The dynamics between the policy room $\frac{R^J}{R^{J,*}}$ and the

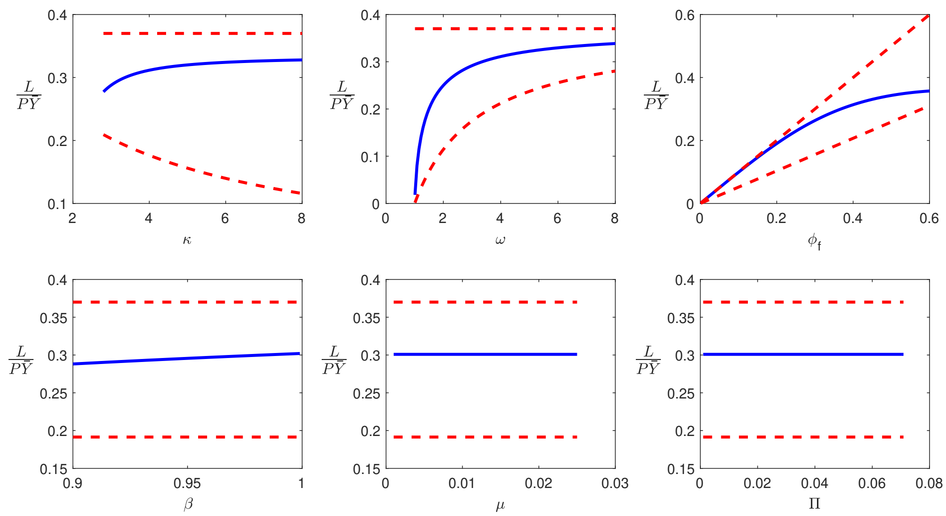

Figure 3: Comparative statistics: Output-scaled real lending

Notes: The red-dashed lines indicate the upper and lower bounds for output-scaled lending, corresponding to ϕ_f and $\phi_f(1-\Theta_L)$, respectively.

real loan-to-output ratio $\frac{L/P}{\bar{Y}}$ are captured in Figure 4. An increase in either ϕ_f or ω decreases firm participation, M, and widens the policy room, $\frac{R^J}{R^{J,*}}$, with the net effect being an increase of aggregate loan issuance. In contrast, a rise in κ raises both M and $\frac{L/P}{\bar{Y}}$, inducing a negative correlation with the policy room $\frac{R^J}{R^{J,*}}$.

[19] The functional relationship between $\frac{L/P}{\bar{Y}}$ and other parameters is further explored in Figure A.3.

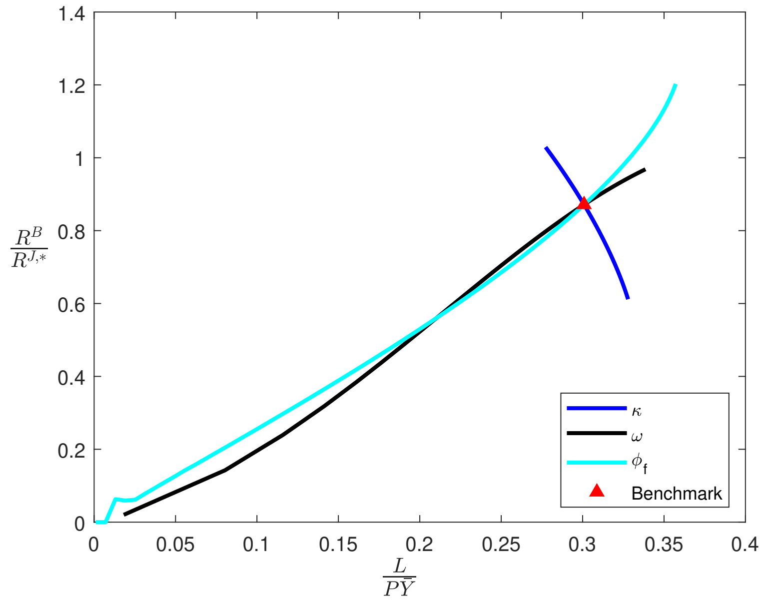

Figure 4: Policy power on output-scaled real lending

Notes: This figure illustrates the co-movements between $\frac{R^J}{R^{J,*}}$ and $\frac{\frac{L}{PA}}{\bar{Y}}$ driven by variations in κ, ω, and ϕ_f. The solid triangular marker denotes the steady-state value under benchmark calibration.

4 Quantitative Analysis

4.1 Supply vs. Demand Shocks

Technology shock: Figure 5 shows how a positive technology shock, $u_{a,t}$, affects various variables in our model. Following the shock, a group of previously inactive firms enters the market, boosting aggregate firm participation M_t, the measure of productivity-insensitive entrants H_t, and aggregate loans $\frac{L_t}{P_t A_t}$.[20] As firms pay their fixed costs in units of the final consumption good, the increase in firm entry contributes to an expansion in aggregate demand, as detailed in equation (29). An uptick in market participation typically depresses the real price of inputs, $\frac{P_t^J}{P_t}$, due to heightened competition, as expressed in equation (28). Yet in this case, the rising aggregate demand dominates, increasing real input prices along with labor demand N_t and real wages. This causes inflation Π_t and interest rates R_t^J to rise, thereby narrowing the policy room $\frac{R_t^J}{R_t^{J,*}}$.[21]

We also examine the technology shock's impact under varying levels of the fixed cost parameter, ϕ_f. Higher entry costs mean a greater steady-state prevalence of inactive firms, $1 - M$. In such conditions, a positive $u_{a,t}$ shock triggers more substantial firm entry and larger increases in M_t and H_t. The increase in aggregate demand brought by stronger entry is further amplified

[20] In Figures 5 and 6, the percentage decrease in the loan-to-output ratio, $\frac{L_t/P_t}{Y_t}$, is smaller than the corresponding output increase, $\tilde{Y}_t$, resulting in a net rise in aggregate loan demand, $\frac{L_t}{P_t A_t}$. For small values of ϕ_f, changes in loan demand around the steady-state are negligible.

[21] This result is consistent with the positive correlation between the policy room, $\frac{R_t^J}{R_t^{J,*}}$, and firm participation, M_t, outlined in equation (33)

by the elevated fixed costs associated with a higher ϕ_f. Consequently, there's a sharper initial increase in loan demand, real input prices, wages, and labor demand, followed by a faster reversion to steady-state levels due to increased competition. In this setting, inflation Π_t shows a more moderate response.[22]

These dynamics align with a traditional AD-AS model as follows: (i) a positive technology shock moves the supply curve rightward; (ii) this shift leads to an outward movement of the demand curve due to increased loan and labor demands, causing more firm entry and further shifts in the supply curve; and (iii) when entry costs are high, more inactive firms enter the market after a positive supply shock. Consequently, both aggregate supply and demand curves shift more extensively rightward, resulting in moderate inflation and increased output.

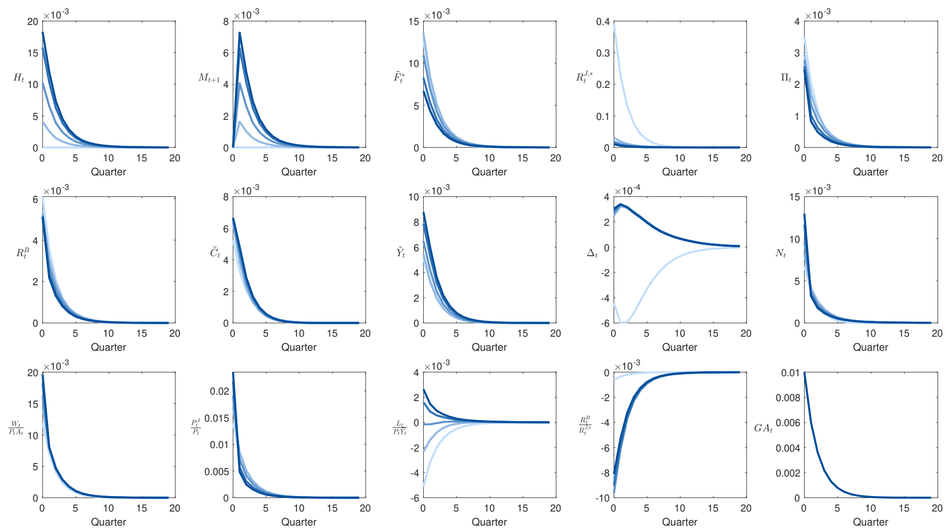

Figure 5: Impulse response functions to TFP shock

Notes: The figures display the deviation for 1 standard deviation (0.01) in $u_{a,t}$ which increases the growth rate of the average productivity for bottom-tier firms. The autoregressive coefficient is 0.6. The gradient blue lines denote the responses under calibration with varying ϕ_f. From the light blue to the dark blue, ϕ_fs are 0.02, 0.25, 0.37 (benchmark), 0.5, and 0.6, with corresponding Ms equal to 0.99, 0.96, 0.9, 0.78, and 0.69. The variables below are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{j,*}$. The rest of the variables are plotted in log deviations from their steady states. Δ is the price dispersion for the top-tier products.

Demand shock: Figure 6 illustrates the effects of a consumption demand shock, $u_{c,t}$. The figure exhibits impulse responses that are qualitatively analogous to the ones displayed in Figure 5. Specifically, a positive shock to $u_{c,t}$ prompts an increase in firm entry that results in an expansion of the aggregate supply capacity of the economy.

In summary, our framework highlights the reciprocal relationship between supply and demand that exists as a result of endogenous firm entry. Accordingly, the initial origin of the shock

[22] This observation is consistent with the findings of Cecioni (2010), who argue that greater firm entry can mitigate inflationary pressures in the U.S. economy.

—be it supply- or demand-driven— yields no qualitative distinctions in the behavior of the key variables within our model.

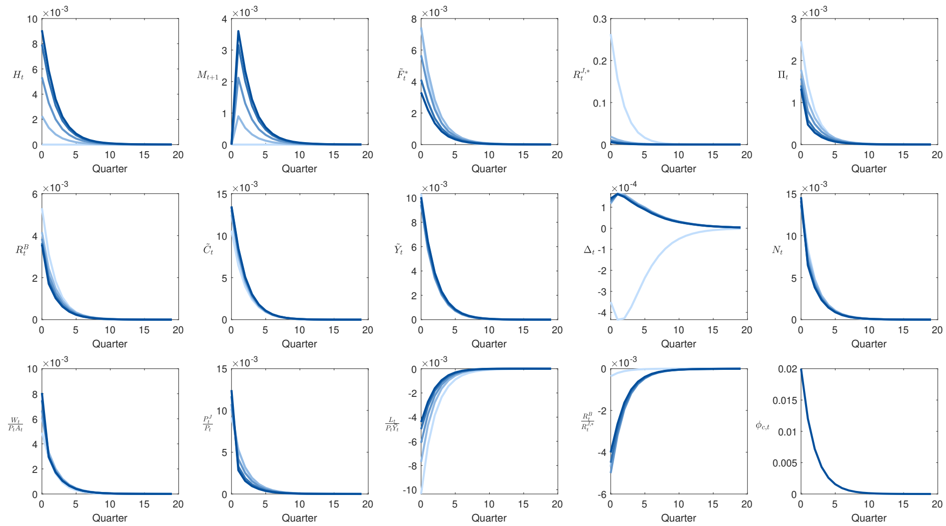

Figure 6: Impulse response functions to demand shock

Notes: The figures display the deviation for 1 standard deviation (0.08) in $u_{c,t}$, the demand shock. The autoregressive coefficient is 0.6. The gradient blue lines denote the responses under calibration with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.02, 0.25, 0.37 (benchmark), 0.5, and 0.6, with corresponding Ms equal to 0.99, 0.96, 0.9, 0.78, and 0.69. The below variables are plotted in deviations in level from their steady states: H, M, R^B, Π, and $R^{j,*}$. The rest of the variables are plotted in deviations in logs from their steady states.

Other shocks In Appendix A, impulse response functions are presented for fixed cost shocks $u_{f,t}$ (Figure A.4), monetary policy shocks $\varepsilon_{r,t}$ (Figure A.5), and government spending shocks $u_{g,t}$ (Figure A.6). A positive fixed cost shock induces a decrease in both firm entry M_t and the satiation measure H_t. This decline is attributed to the elevated productivity cutoff $\varphi^*_{m,t}$, as specified in equation (14), which rises for each firm type m due to increased entry costs. This shock has dual, opposing impacts on aggregate demand: First, reduced firm participation diminishes fixed equipment demand at the extensive margin, thereby contracting aggregate demand. Second, the increased fixed costs boost the demand from incumbent firms, thereby augmenting aggregate demand at the intensive margin. Under the model's benchmark calibration, the latter effect prevails, leading to a net expansion in aggregate demand. This subsequently results in an increase in equilibrium levels of production, labor demand, real wages, and inflation.

A negative monetary policy shock, indicative of policy loosening, yields an impulse response function akin to that produced by a consumption demand shock. A reduction in interest rates promotes a rise in aggregate participation M_t, which in turn increases loan demand, inflation, real wages, and production levels. A positive government spending shock, depicted in Figure A.6, crowds out consumption via higher real interest rates while simultaneously reducing inflation through increased participation by bottom-tier firms, as evidenced by rises in M_t and H_t. The government spending multiplier is amplified under higher values of ϕ_f, which is attributable to stronger firm entry following the shock.

4.2 Intensive vs. Extensive Margin in Labor Adjustment

Changes in aggregate labor N_t as specified in equation (25) are attributable to two primary factors: (i) variations in an operating firm's labor demand, denoted $N_{mv,t}$, over time —referred to as intensive margin adjustment; and (ii) fluctuations in the number of active bottom-tier firms M_t across business cycles —known as extensive margin adjustment. The aggregate labor N_t is formally expressed in equation (37) as:

$$N_t = \int_0^1 \int_{v \in \Omega_{m,t}} N_{mv,t} \, \mathrm{d}v \, \mathrm{d}m \, , \tag{37}$$

where the individual labor demand $N_{mv,t}$ derives from equation (B.14). We now proceed to consider a bottom-tier firm (m, v) operating across two periods t and $t + \iota$, where $\iota \geq 1$. Utilizing equation (B.14), we define:

$$g_{t,t+\iota}^{\text{Density}} \equiv \frac{N_{mv,t+i} - N_{mv,t}}{N_{mv,t}} = \left[\frac{1 + \Theta_4 \cdot H_{t-1}}{1 + \Theta_4 \cdot H_{t+\iota-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{\frac{Y_{t+\iota}\Delta_{t+\iota}}{A_{t+\iota}}}{\frac{Y_t \Delta_t}{A_t}}\right)^{\frac{1}{\alpha}} - 1 \, , \tag{38}$$

which represents the percentage change between periods t and $t + \iota$ in an individual firm (m, v)'s labor demand $N_{mv,t}$, contingent upon the firm's operation across both periods. Importantly, $g_{t,t+\iota}^{\text{Density}}$ is solely a function of aggregate variables, independent of the indices (m, v). We term $g_{t,t+\iota}^{\text{Density}}$ as the "intensive margin" adjustment in labor demand.

From equation (25), we derive an expression for the percentage change in aggregate labor, N_t, denoted as $g_{t,t+\iota}^{N}$ [23]:

$$g_{t,t+\iota}^{N} \equiv \frac{N_{t+\iota} - N_t}{N_t} = g_{t,t+\iota}^{\text{Density}} + (1 + g_{t,t+\iota}^{\text{Density}}) \cdot g_{t,t+\iota}^{\text{Entry}} \, , \tag{39}$$

where $g_{t,t+\iota}^{\text{Density}}$ is defined as in equation (38) and $g_{t,t+\iota}^{\text{Entry}}$ is given by

$$g_{t,t+\iota}^{\text{Entry}} = \frac{(H_{t+\iota-1} - H_{t-1}) + \dfrac{\omega(\sigma-1)}{\kappa[\alpha + \sigma(1-\alpha)] + (\omega-1)(\sigma-1)}(H_{t-1} - H_{t+\iota-1})}{H_{t-1} + \dfrac{\omega(\sigma-1)}{\kappa[\alpha + \sigma(1-\alpha)] + (\omega-1)(\sigma-1)}(1 - H_{t-1})} \, . \tag{40}$$

We interpret $g_{t,t+\iota}^{\text{Entry}}$ as the extensive margin adjustment in labor, triggered by changes in firm entry. According to equation (39), the total percentage change in aggregate labor comprises both intensive and extensive margin adjustments.

[23] The derivation is provided in Appendix B.

Figures 7 and 8 portray how intensive and extensive margins respond, respectively, to different

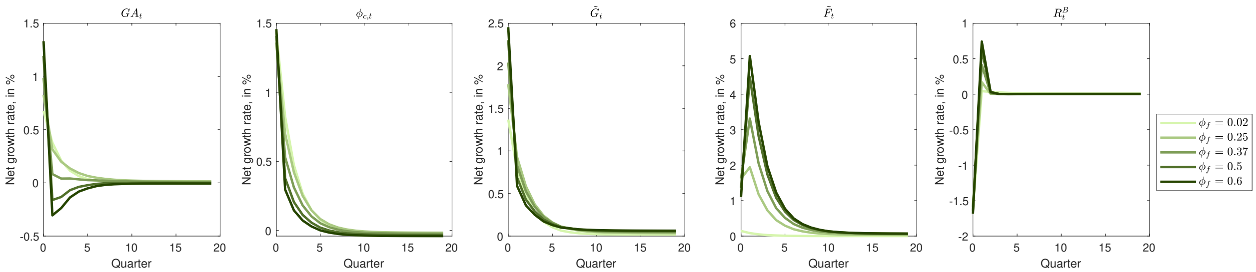

Figure 7: Decomposition of labor growth rate: isolines on intensive margin

Notes: Figures illustrate employment growth rate relative to pre-shock employment level. Gradient green lines indicate intensive margin responses with varying fixed cost parameter ϕ_f values. Growth rates are reported in net percentage terms.

shocks. For example, for a positive fixed cost shock $u_{f,t}$, we note: (i) a negative extensive margin adjustment due to the exit of less competitive firms, and (ii) an increase in per-firm labor demand corresponding to higher aggregate output, as evidenced in Figure A.4.
In contrast, a consumption demand shock, $\phi_{c,t}$, leads to positive adjustments on both labor margins due to increased market entry and output (see Figure 6). The extensive margin effect grows more salient with higher ϕ_f, while the intensive margin exhibits a non-monotonic behavior. Initially, individual firms require more workers, but as market competition intensifies, labor demand flattens, as corroborated by Figure A.4.

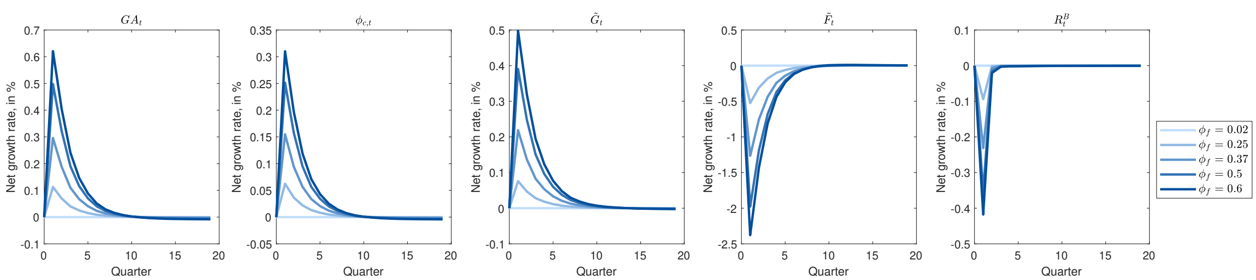

Figure 8: Decomposition of labor growth rate: isolines on extensive margin

Notes: Figures illustrate employment growth rate relative to pre-shock employment level. Gradient blue lines indicate extensive margin responses with varying fixed cost parameter ϕ_f values. Growth rates are reported in net percentage terms.

4.3 Multipliers and the Policy Room

We now examine the influence of initial policy room levels on the responses of aggregate variables to shocks, commonly termed as shock multipliers. To obtain the value of multipliers outside the steady state, we simulate the model over a span of $T = 10,000$ periods, selecting 500 unique realizations denoted as $\mathbb{Y}^{\text{original}}$. For each selected realization, we extend the model dynamics up to $h = 4$ periods ahead based on two different scenarios: (i) no additional shocks, which results in the time series $\left\{\mathbb{Y}^{\text{original}}_{t+h}\right\}_{h=0}^{h=4}$; and, (ii) an initial one standard deviation addition to the shock of interest, giving rise to the time series $\left\{\mathbb{Y}^{\text{shock}}_{t+h}\right\}_{h=0}^{h=4}$. The multiplier is subsequently computed as $\frac{|\mathbb{Y}^{\text{shock}}_{t+h} - \mathbb{Y}^{\text{original}}_{t+h}|}{\sigma(\text{shock})}$ for horizons ranging from $h = 0$ to $h = 4$.

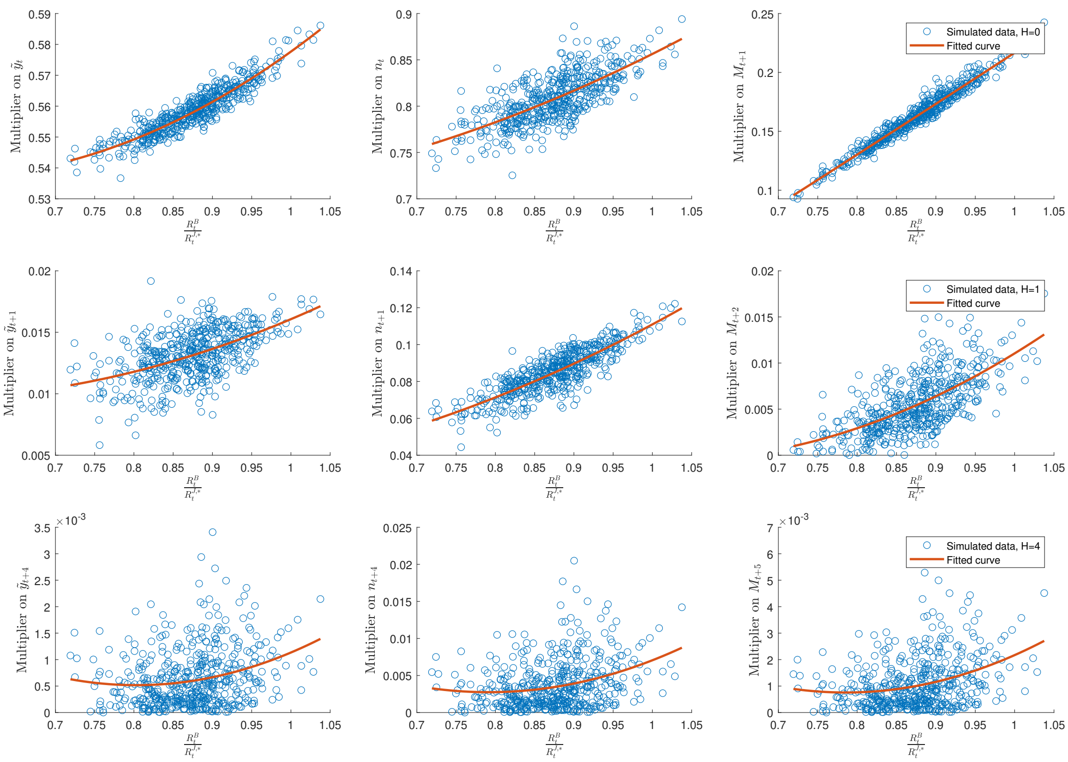

Figure 9: Scatter plot of $\frac{R_t^J}{R_t^{J,*}}$, third-order, monetary policy shock

In Figure 9, we plot the relationship between multipliers and initial policy room levels. The key findings are:

1. At $h = 0$, multipliers for output and labor positively correlate with policy room levels. This effect is due to the higher rate of firm entry in response to a monetary shock when initial policy room is larger, consistent with Corollary 1.

2. At $h = 1$, although the multipliers decline due to the shock's lack of persistence, the positive correlation with the initial policy room remains. This is explained by an increased number of firms in the market and an associated rise in supply.

3. At $h = 4$, multipliers approach zero, attributable to the lack of shock persistence.

In summary, the policy room serves as a sufficient statistic for equilibrium firm entry and is positively correlated with the multipliers for output, labor, and firm entry in response to monetary shocks. Further details can be found in Figure A.8 in Appendix A, which relates closely to the discussion here.

5 Conclusion

The primary objective of this paper is to construct a macroeconomic framework where aggregate demand and aggregate supply are mutually dependent. To this end, we introduce a two-tier system of firms. In the lower tier, firms must incur fixed costs, such as equipment purchase and factory construction, to operate in the subsequent period. The upper-tier firm utilizes the composite good produced by the lower-tier as its sole input and faces nominal rigidities. As lower-tier firms lack initial capital, they rely on financial markets for loans to meet these fixed costs, which are denominated in terms of the final consumption good. This financing requirement, upon the firm's market entry, induces a positive shock to aggregate demand.

We analytically derive the equilibrium rate of firm entry as a function of the "policy room", defined as the current interest rate relative to the average Satiation Lower Bound (SLB). The SLB represents the interest rate at which firm entry reaches saturation —i.e., maximal lower-tier market participation. Our general equilibrium model also enables the decomposition of shifts in aggregate variables, such as labor, into extensive (new firm entries) and intensive (incumbent firms) margins. This yields further insight into the model's operational mechanisms.

When the economy experiences a positive demand shock, it prompts lower-tier firms to enter, thereby reducing the marginal costs for upper-tier firms and lowering the aggregate supply curve. The loan-induced entry exerts additional upward pressure on aggregate demand, stimulating further firm entry. This interplay between demand and supply persists until equilibrium is achieved. Hence, in our framework, supply shocks are inherently demand shocks, and vice versa. Our model illuminates the interactive relationship between Keynesian demand factors and endogenous firm entry within a tractable macroeconomic setting.

References

Abadi, Joseph, Markus Brunnermeier, and Yann Koby, "The Reversal Interest Rate," *American Economic Review*, 2022, *113* (8), 2084–2120.

Bergin, Paul R. and Giancarlo Corsetti, "The extensive margin and monetary policy," *Journal of Monetary Economics*, 2008, *55* (7), 1222–1237.

Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum, "Plants and Productivity in International Trade," *American Economic Review*, 2003, *93* (4), 1268–1290.

Bilbiie, Florin O., Fabio Ghironi, and Marc J. Melitz, "Endogenous Entry, Product Variety, and Business Cycles," *Journal of Political Economy*, 2012, *120* (2), 304–345. Publisher: The University of Chicago Press.

_ , _ , _ , Virgiliu Midrigan, and Julio J. Rotemberg, "Monetary Policy and Business Cycles with Endogenous Entry and Product Variety [with Comments and Discussion]," *NBER Macroeconomics Annual*, 2007, *22*, 299–379. Publisher: The University of Chicago Press.

Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo, "Labor Hoarding and the Business Cycle," *Journal of Political Economy*, 1993, *101* (2), 245–273.

Calvo, Guillermo, "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 1983, *12* (3), 383–398.

Cecioni, Martina, "Firm Entry, Competitive Pressures and the U.S. Ináation Dynamics," *Temi di discussione (Economic working papers), Bank of Italy, Economic Research Department*, 2010.

Colciago, Andrea and Riccardo Silvestrini, "Monetary policy, productivity, and market concentration," *European Economic Review*, 2022, *142*, 103999.

Domowitz, Ian, Glenn Hubbard, and Bruce C. Petersen, "Market Structure and Cyclical Fluctuations in U.S. Manufacturing," *The Review of Economics and Statistics*, 1988, *70* (1), 55–66.

Fasani, Stefano, Haroon Mumtaz, and Lorenza Rossi, "Monetary policy uncertainty and firm dynamics," *Review of Economic Dynamics*, January 2023, *47*, 278–296.

Ghironi, Fabio and Marc J. Melits, "International Trade and Macroeconomic Dynamics with Heterogeneous Firms*," *The Quarterly Journal of Economics*, 2005, *120* (3), 865–915.

Guerrieri, Veronica, Guido Lorenzoni, Ludwig Straub, and Iván Werning, "Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?," *The American Economic Review*, 2023, *112* (5), 1437–1474.

Gutiérrez, Germán, Callum Jones, and Thomas Philippon, "Entry costs and aggregate dynamics," *The Review of Economics and Statistics*, 2005, *49* (5), 1331–1360.

Hamano, Masashige and Francesco Zanetti, "Endogenous product turnover and macroeconomic dynamics," *Review of Economic Dynamics*, 2017, *26*, 263–279.

Jones, Charles I, "Intermediate Goods and Weak Links in the Theory of Economic Development," *American Economic Journal: Macroeconomics*, April 2011, *3* (2), 1–28.

Kobayashi, Teruyoshi, "Firm entry, credit availability and monetary policy," *Journal of Economic Dynamics and Control*, 2011, *35* (8), 1245–1272.

Lewis, Vivien and Céline Poilly, "Firm entry, markups and the monetary transmission mechanism," *Journal of Monetary Economics*, 2012, *59* (7), 670–685.

Nakajima, Tomoyuki, "A business cycle model with variable capacity utilization and demand distur- bances," *The Review of Economics and Statistics*, 2005, *49* (5), 1331–1360.

Smets, Frank and Rafael Wouters, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 2007, *97* (3), 586–606.

Stebunovs, Viktors, "Finance as a Barrier to Entry: U.S. Bank Deregulation and Business Cycle," *working paper*, 2008.

Ulate, Mauricio, "Going Negative at the Zero Lower Bound: The Effects of Negative Nominal Interest Rates," *American Economic Review*, 2021, *111* (1), 1–40.

Uusküla, Lenno, "Monetary transmission mechanism with firm turnover," *Journal of Macroeconomics*, 2016, *50*, 1–18.

Appendix A Additional Tables and Figures

A.1 Section 3.2

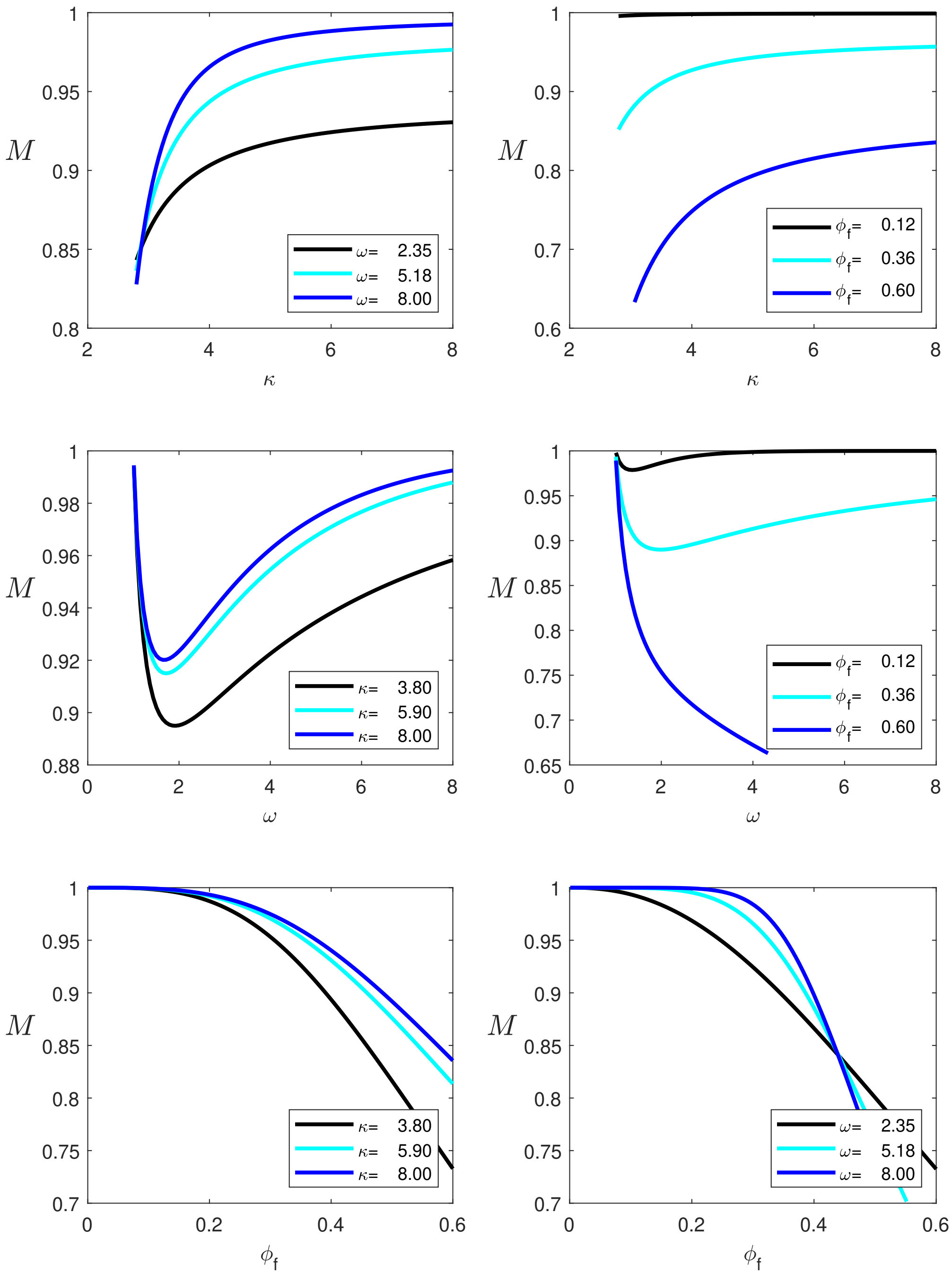

Figure A.1: Comparative Statics: M

Notes: This figure displays how variations in other structural parameters affect the relation between M and the structural parameters

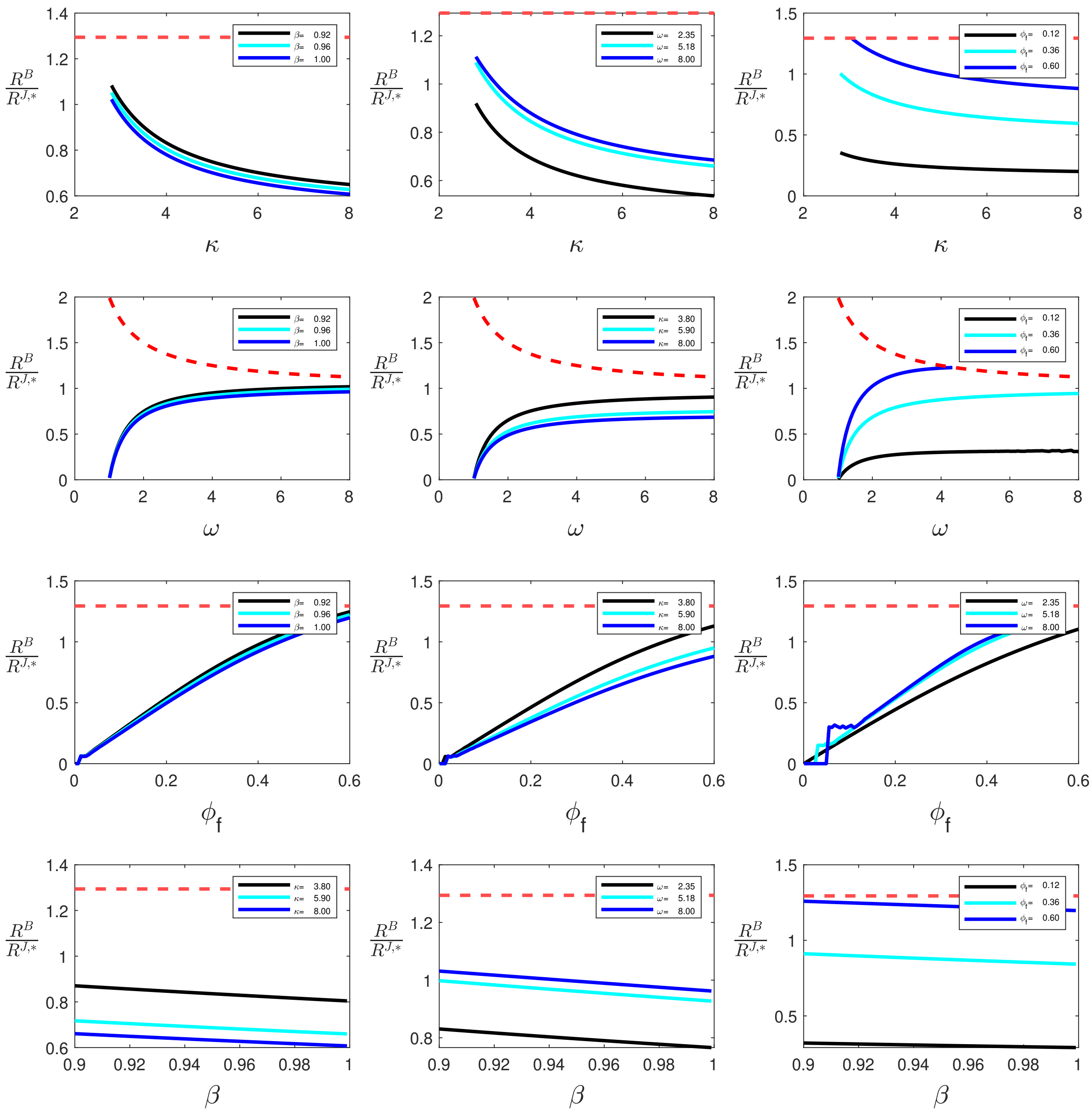

Figure A.2: Comparative Statics: Policy Room

Notes: This figure display how κ, ω, and ϕ_f affect the relationship between the policy room and the parameters

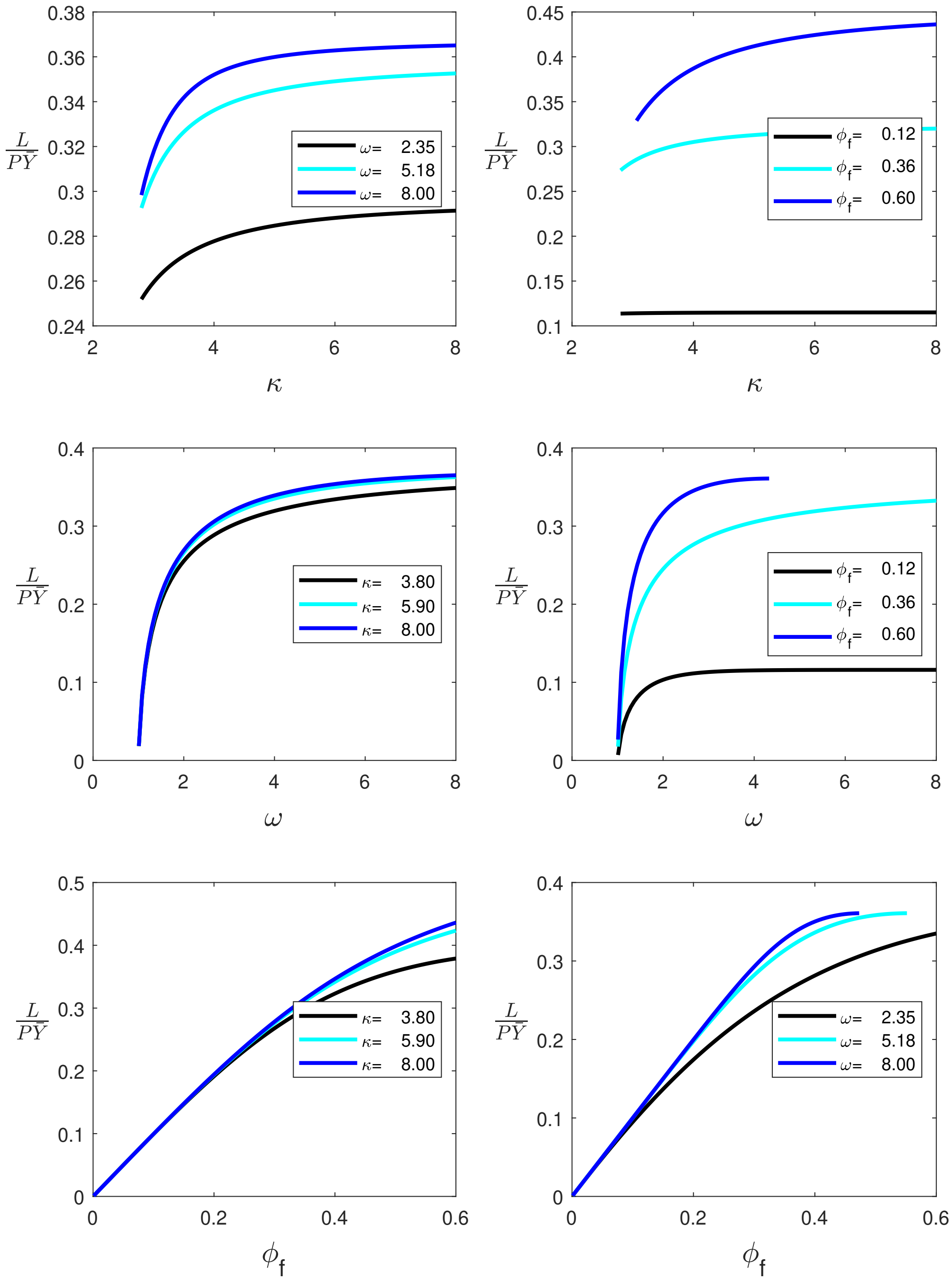

Figure A.3: Comparative Statics: Output-Scaled Loan

Notes: This figure display how κ, ω, and ϕ_f affect the relationship between $\frac{\frac{L}{PA}}{Y}$ and the parameters

A.2 Section 4.1

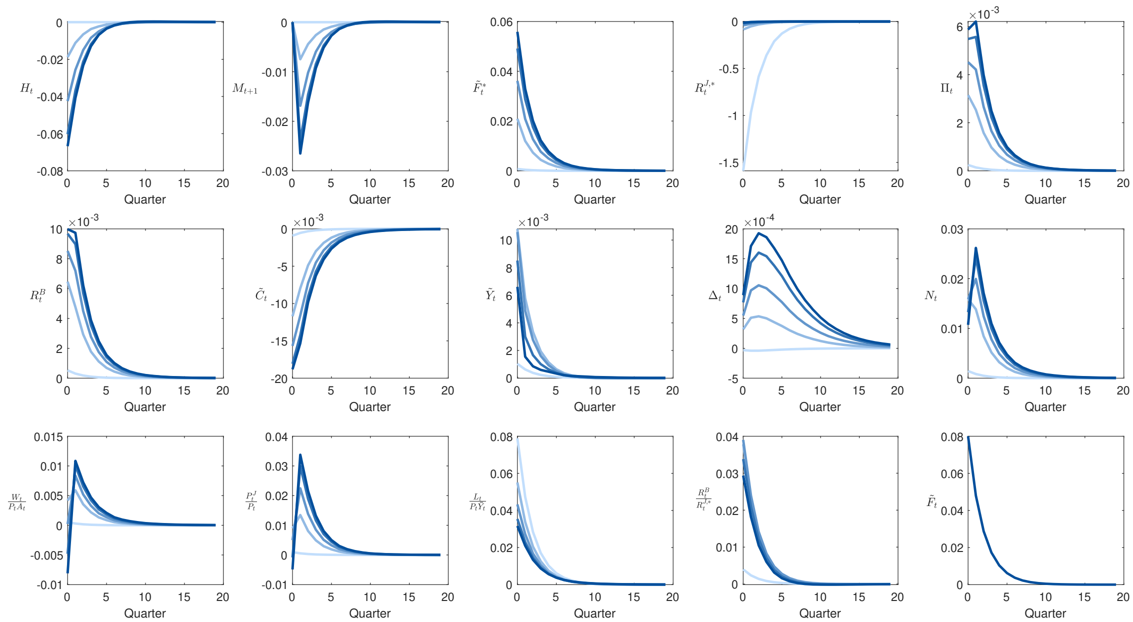

Figure A.4: Impulse response functions to fixed cost shock

Notes: The figures display the deviation for 1 positive standard deviation (0.08) in $u_{f,t}$, the fixed cost shock. The autoregressive coefficient is 0.6. The gradient blue lines denote the responses under calibrations with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.02, 0.25, 0.37 (benchmark), 0.5, and 0.6, with corresponding Ms equal to 0.99, 0.96, 0.9, 0.78, and 0.69. The variables below are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{j,*}$ (net interest rate). The rest of the variables are plotted in log deviations from their steady states. Δ is the price dispersion for the top-tier products. $W_t/(P_t A_t)$ is the real wage. P_t^J/P_t measures the aggregate price for the bottom-tier products or the input price for the top-tier firms.

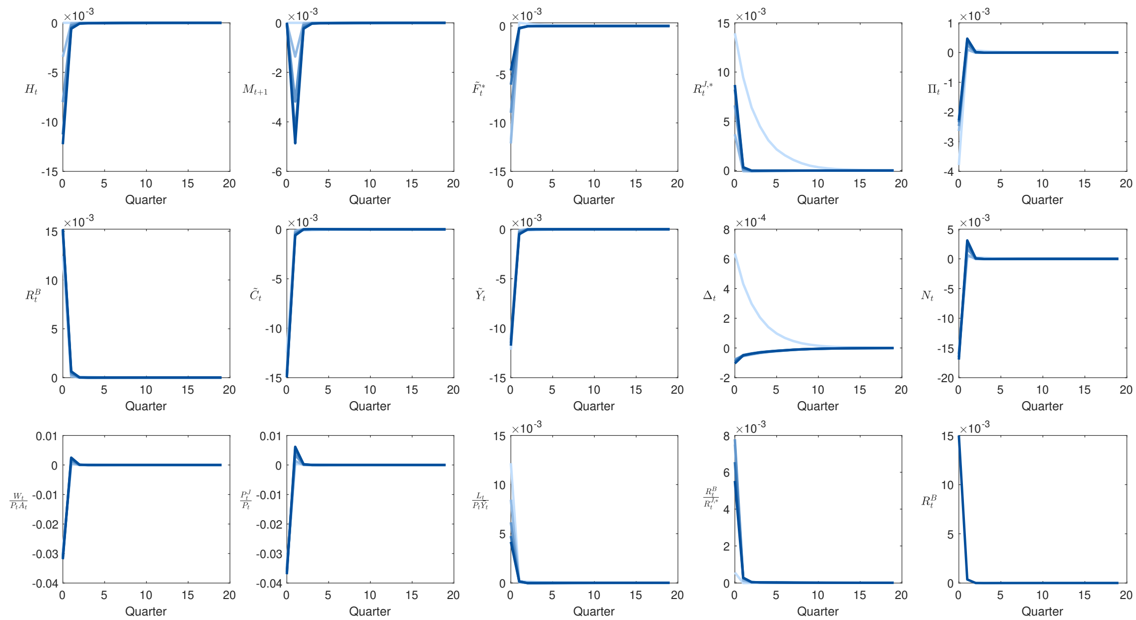

Figure A.5: Impulse response functions to monetary policy shock

Notes: The figures display the deviation for 1 positive standard deviation (0.02) in $\epsilon_{r,t}$, the monetary policy shock. The gradient blue lines denote the responses under calibrations with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.02, 0.25, 0.37 (benchmark), 0.5, and 0.6, with corresponding Ms equal to 0.99, 0.96, 0.9, 0.78, and 0.69. The variables below are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{j,*}$ (net interest rate). The remaining variables are plotted in log deviations from their steady states. Δ is the price dispersion for the top-tier products.$W_t/(P_tA_t)$ is the real wage. P_t^J/P_t measures the aggregate price for the bottom-tier products or the input price for the top-tier firms.

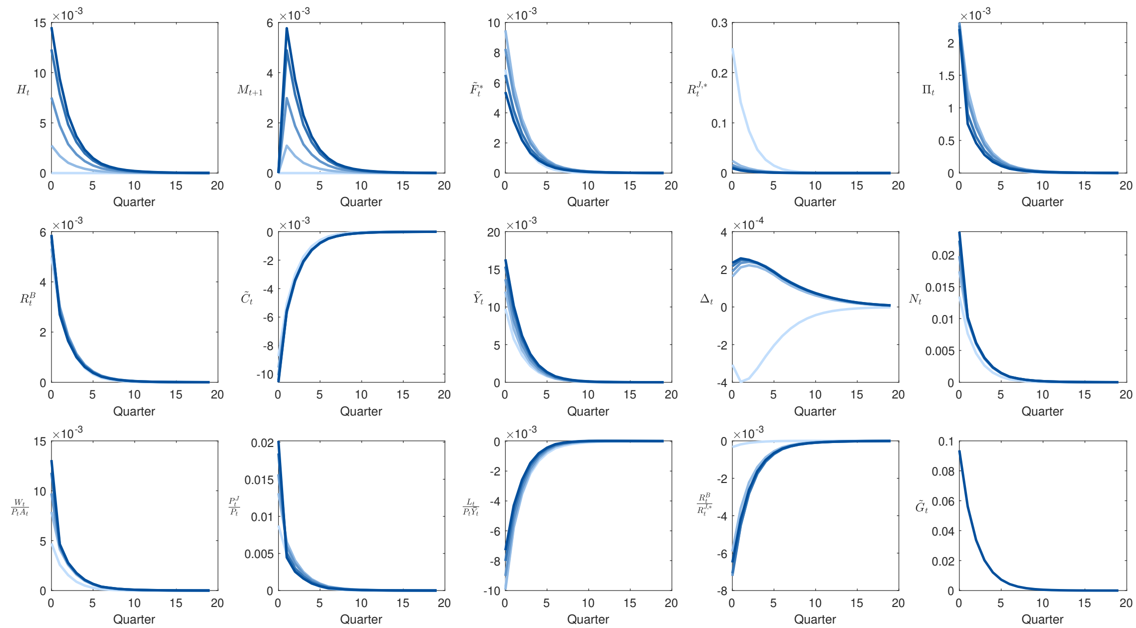

Figure A.6: Impulse response functions to government spending shock

Notes: The figures display the deviation for 1 positive standard deviation (0.08) in $u_{g,t}$ which denotes the government spending shock. The autoregressive coefficient is 0.97. The gradient blue lines denote the responses under calibration with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.02, 0.25, 0.37 (benchmark), 0.5, and 0.6, with corresponding Ms equal to 0.99, 0.96, 0.9, 0.78, and 0.69. The variables below are plotted in level deviations from their steady states: H, M, R^B, Π, and $R^{j,*}$ (net interest rate). The remaining variables are plotted in log deviations from their steady states. Δ is the price dispersion for the top-tier products. $W_t/(P_t A_t)$ is the real wage. P_t^J/P_t measures the aggregate price for the bottom-tier products or the input price for the top-tier firms.

A.3 Section 4.3

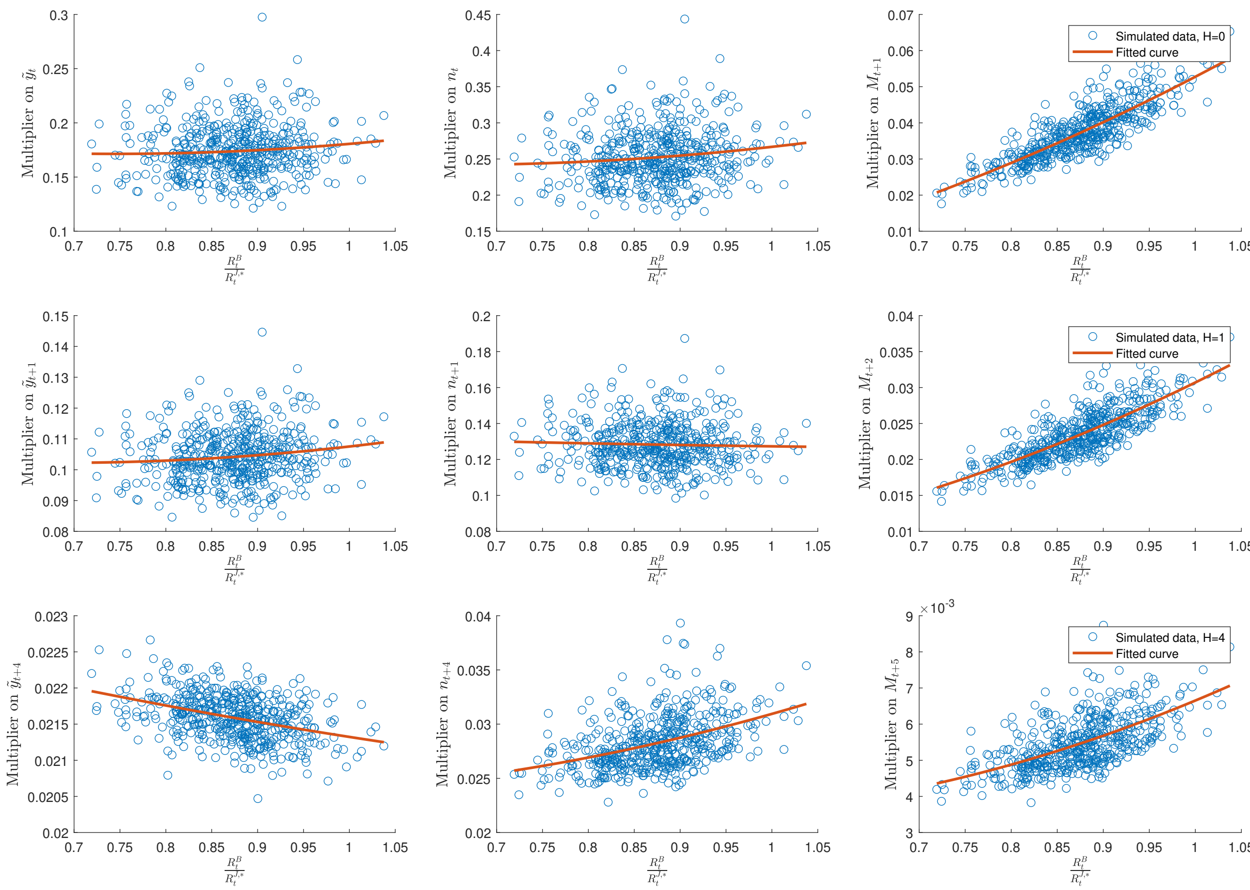

Figure A.7: Scatter plot of $\frac{R_t^B}{R_t^{J,*}}$ and multipliers, third-order, government spending shock

Notes: The multiplier is calculated as follows: we call the previously constructed data set as $\mathbb{Y}^{\text{original}}$. Then we propagate the model with a different size of initial shock by adding one extra standard deviation to the shock at t. We only modify the size of one shock and keep the other shocks the same. The propagated data set is denoted with $\mathbb{Y}^{\text{shock}}$. The multiplier is calculated as $\frac{|\mathbb{Y}^{\text{shock}} - \mathbb{Y}^{\text{original}}|}{\sigma(\text{shock})}$. The figure displays the case with $h = 0, 4, 16$, from top to bottom respectively. The output and employment are in logs, so the multipliers are $\frac{|\tilde{y}_{t+h}^{\text{shock}} - \tilde{y}_{t+h}^{\text{original}}|}{\sigma(\text{shock})} = \frac{|\log(\tilde{Y}_{t+h}^{\text{shock}}) - \log(\tilde{Y}_{t+h}^{\text{original}})|}{\sigma(\text{shock})}$ and $\frac{|n_{t+h}^{\text{shock}} - n_{t+h}^{\text{original}}|}{\sigma(\text{shock})} = \frac{|\log(N_{t+h}^{\text{shock}}) - \log(N_{t+h}^{\text{original}})|}{\sigma(\text{shock})}$ for $t = 0, 4, 16$, correspondingly. The mass of firms in the one-period ahead is in level, with the multipliers equal to $\frac{|M_{t+1+h}^{\text{shock}} - M_{t+1+h}^{\text{original}}|}{\sigma(\text{shock})}$ for $t = 0, 4, 16$, correspondingly. The multipliers are displayed in level and there is no percentage adjustment.

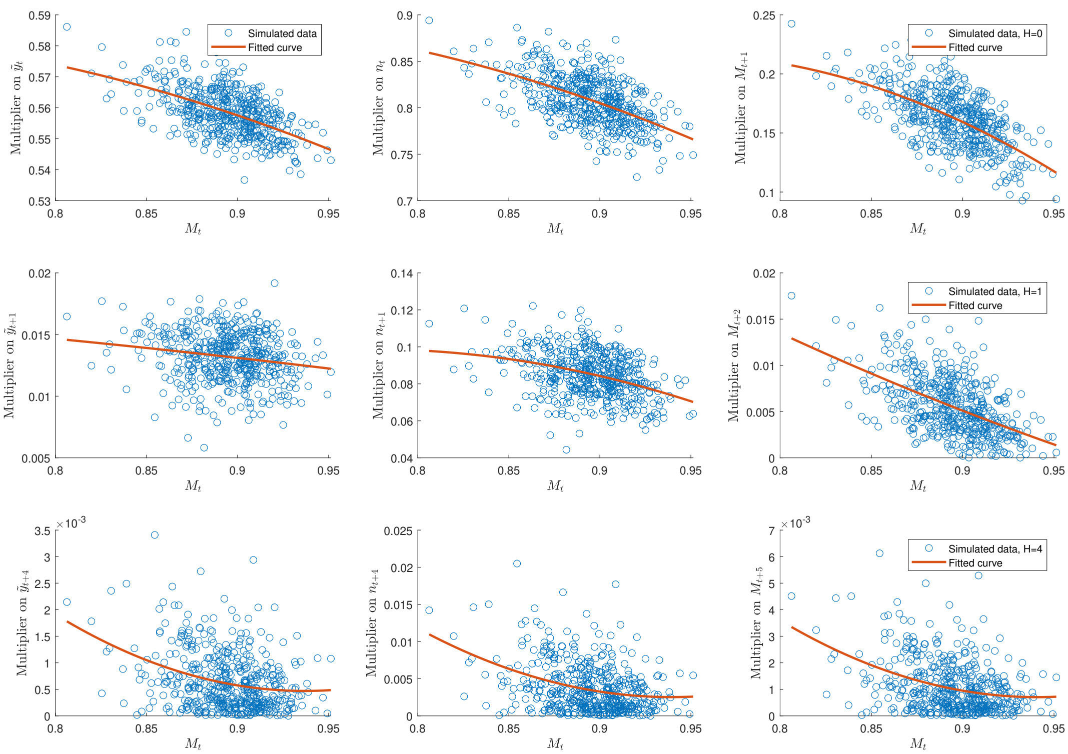

Figure A.8: Scatter plot of M_t and multipliers, third-order, monetary policy shock

Notes: The multiplier is calculated as follows: we call the previously constructed data set as $\mathbb{Y}^{\text{original}}$. Then we propagate the model with a different size of initial shock by adding one extra standard deviation to the shock at t. We only modify the size of one shock and keep the other shocks the same. The propagated data set is denoted with $\mathbb{Y}^{\text{shock}}$. The multiplier is calculated as $\frac{|\mathbb{Y}^{\text{shock}} - \mathbb{Y}^{\text{original}}|}{\sigma(\text{shock})}$. The figure displays the case with $h = 0, 4, 16$, from top to bottom respectively. The output and employment are in logs, so the multipliers are $\frac{|\tilde{y}^{\text{shock}}_{t+h} - \tilde{y}^{\text{original}}_{t+h}|}{\sigma(\text{shock})} = \frac{|\log(\tilde{Y}^{\text{shock}}_{t+h}) - \log(\tilde{Y}^{\text{original}}_{t+h})|}{\sigma(\text{shock})}$ and $\frac{|n^{\text{shock}}_{t+h} - n^{\text{original}}_{t+h}|}{\sigma(\text{shock})} = \frac{|\log(N^{\text{shock}}_{t+h}) - \log(N^{\text{original}}_{t+h})|}{\sigma(\text{shock})}$ for $t = 0, 4, 16$, correspondingly. The mass of firms in the one-period ahead is in level, with the multipliers equal to $\frac{|M^{\text{shock}}_{t+1+h} - M^{\text{original}}_{t+1+h}|}{\sigma(\text{shock})}$ for $t = 0, 4, 16$, correspondingly. The multipliers are displayed in level and there is no percentage adjustment.

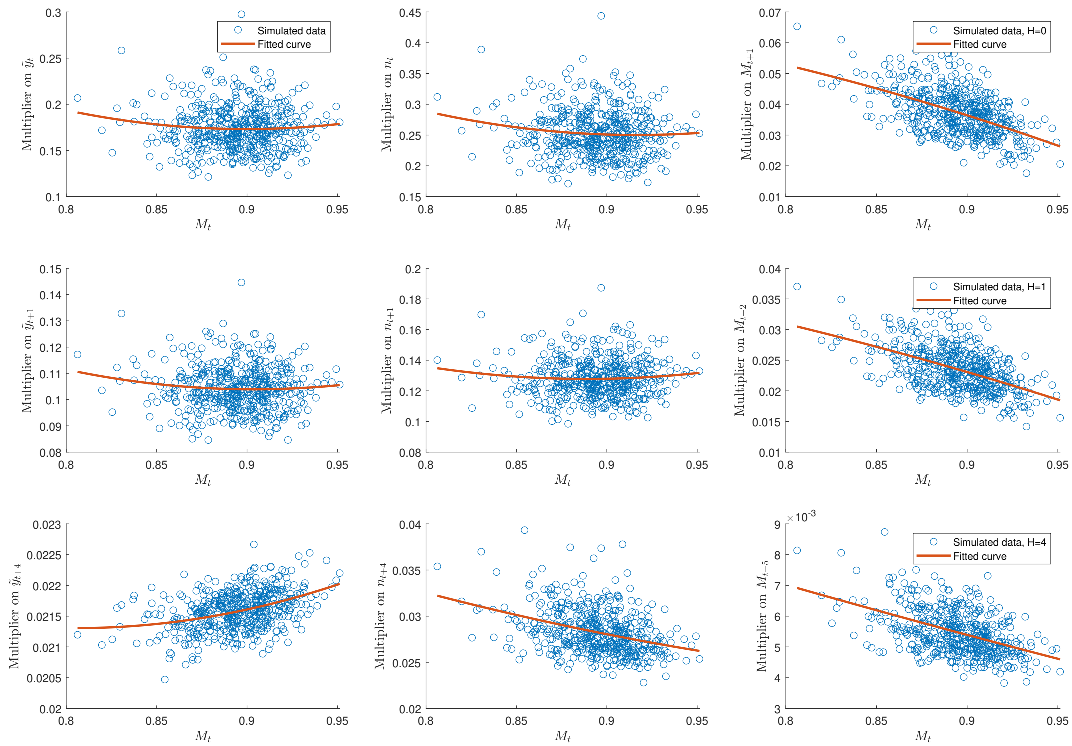

Figure A.9: Scatter plot of M_t and multipliers, third-order, government spending shock

Notes: The multiplier is calculated as follows: we call the previously constructed data set as $\mathbb{Y}^{\text{original}}$. Then we propagate the model with a different size of initial shock by adding one extra standard deviation to the shock at t. We only modify the size of one shock and keep the other shocks the same. The propagated data set is denoted with $\mathbb{Y}^{\text{shock}}$. The multiplier is calculated as $\frac{|\mathbb{Y}^{\text{shock}}-\mathbb{Y}^{\text{original}}|}{\sigma(\text{shock})}$. The figure displays the case with $h = 0, 4, 16$, from top to bottom respectively. The output and employment are in logs, so the multipliers are $\frac{|\tilde{y}^{\text{shock}}_{t+h}-\tilde{y}^{\text{original}}_{t+h}|}{\sigma(\text{shock})} = \frac{|\log(\tilde{Y}^{\text{shock}}_{t+h})-\log(\tilde{Y}^{\text{original}}_{t+h})|}{\sigma(\text{shock})}$ and $\frac{|n^{\text{shock}}_{t+h}-n^{\text{original}}_{t+h}|}{\sigma(\text{shock})} = \frac{|\log(N^{\text{shock}}_{t+h})-\log(N^{\text{original}}_{t+h})|}{\sigma(\text{shock})}$ for $t = 0, 4, 16$, correspondingly. The mass of firms in the one-period ahead is in level, with the multipliers equal to $\frac{|M^{\text{shock}}_{t+1+h}-M^{\text{original}}_{t+1+h}|}{\sigma(\text{shock})}$ for $t = 0, 4, 16$, correspondingly. The multipliers are displayed in level and there is no percentage adjustment.

Appendix B Derivation and Proofs

B.1 Detailed Derivation in Section 2.2

Derivation of equations (12) and (13) We start from the price setting of a firm (m, v), given by

$$P^J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} J_{mv,t}^{\frac{1-\alpha}{\alpha}} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} \left[(P^J_{mv,t})^{-\sigma}\Gamma^J_t\right]^{\frac{1-\alpha}{\alpha}},$$

in which we can solve for $P^J_{mv,t}$ as

$$(P^J_{mv,t})^{\frac{\alpha+\sigma(1-\alpha)}{\alpha}} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} (\Gamma^J_t)^{\frac{1-\alpha}{\alpha}}$$

from which we obtain

$$P^J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \varphi_{mv,t}^{-\frac{1}{\alpha+\sigma(1-\alpha)}} (\Gamma^J_t)^{\frac{(1-\alpha)}{\alpha+\sigma(1-\alpha)}}. \tag{B.1}$$

To get the revenue function $r_{mv,t}$, we start from

$$P^J_{mv,t} J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} J_{mv,t}^{\frac{1}{\alpha}},$$

which leads to

$$\begin{aligned} r_{mv,t} &= (1+\zeta^J) P^J_{mv,t} J_{mv,t} = \left(\frac{\sigma}{(\sigma-1)\alpha}\right) W_t N_{mv,t} = (1+\zeta^J) P^J_{mv,t} \left(\frac{P^J_{mv,t}}{P^J_t}\right)^{-\sigma} J_t \qquad \text{(B.2)} \\ &= (1+\zeta^J)(P^J_{mv,t})^{1-\sigma}\Gamma^J_t = (1+\zeta^J) \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \varphi_{mv,t}^{-\frac{(1-\sigma)}{\alpha+\sigma(1-\alpha)}} (\Gamma^J_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}. \end{aligned}$$

Finally, we obtain the formula for the profit $\Pi^J_{mv,t}$, which is given by

$$\Pi^J_{mv,t} = r_{mv,t} - W_t N_{mv,t} - R^J_{t-1} P_{t-1} F_{m,t-1} = \frac{\alpha+\sigma(1-\alpha)}{\sigma} r_{mv,t} - R^J_{t-1} P_{t-1} F_{m,t-1}.$$

Calculating $P^J_{m,t}$ in (6): the price aggregator for firms of fixed $F_{m,t-1}$ From our notation in (6), we know that among firms with fixed cost $F_{m,t-1}$, a set of operating ones at t would be given by $\Omega_{m,t} = \left\{\varphi_{mv,t} \in \left[\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right) A_t\right\}, \infty\right]\right\}$. The cumulative distribution function of productivities of bottom-tier firms that decide to produce is $\frac{\Psi(\varphi_{m,t})}{1-\Psi(\varphi^*_{m,t})}$, a truncated Pareto distribution which is itself a Pareto distribution. With the individual firm (m, v)'s pricing equation (B.1), we now can compute the aggregate price of bottom-tier firms with fixed cost $F_{m,t-1}$ as:

$$
\begin{aligned}
\left(\frac{P^J_{m,t}}{P_t}\right)^{1-\sigma} &= \cancel{M_{m,t}} \cdot \int_{\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right)A_t\right\}}^{\infty} \left(\frac{P^J_{mv,t}}{P_t}\right)^{1-\sigma} \frac{\mathrm{d}\,\Psi(\varphi_{mv,t})}{\cancel{1-\Psi(\varphi^*_{m,t})}} \qquad \text{(B.3)}\\
&= \int_{\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right)A_t\right\}}^{\infty} \left(\frac{P^J_{mv,t}}{P_t}\right)^{1-\sigma} \mathrm{d}\,\Psi(\varphi_{mv,t})\\
&= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{(\sigma-1)}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{P_t A_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\\
&\quad \cdot \left(\frac{\Gamma^J_t}{(P^J_t)^\sigma A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \int_{\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right)A_t\right\}}^{\infty} \varphi_{mv,t}^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} \mathrm{d}\,\Psi(\varphi_{mv,t})\\
&= \Theta_1 \left(\frac{W_t}{P_t A_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \max\left\{\frac{\varphi^*_{m,t}}{\left(\frac{\kappa-1}{\kappa}\right)A_t}, 1\right\}^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\alpha+\sigma(1-\alpha)}}\\
&= \Theta_1 \left(\frac{W_t}{P_t A_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \min\left\{\left(\frac{R^J_{t-1}P_{t-1}F_{m,t-1}}{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}}, 1\right\}
\end{aligned}
$$

where we define

$$
\Theta_1 = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right).
$$

Reexpressing Ξ_t in equation (13) Combining equation (13) with $\Gamma^J_t = (P^J_t)^\sigma Y_t\Delta_t$, we obtain

$$
\begin{aligned}
\Xi_t &= \frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{-\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}}\\
&\quad \cdot \left(\frac{W_t}{A_t P_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} P_t(Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \qquad \text{(B.4)}\\
&= \Theta_2 \cdot \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{A_t P_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} P_t(Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}
\end{aligned}
$$

where we define

$$
\Theta_2 = \frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{-\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}}.
$$

Derivation of P_t^J in (19) We start from the full satiation threshold of the fixed cost F_{t-1}^* defined in Proposition 2:

$$
\begin{aligned}
F_{t-1}^* &= \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R_{t-1}^J P_{t-1}} \\
&= \Theta_2 E_{t-1}\left[\xi_t \left(\frac{P_t^J}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{A_t P_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\Pi_t (Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}}{R_{t-1}^J}\right)\right]
\end{aligned}
\tag{B.5}
$$

where the second equality is from equation (B.4). From (14) and (B.5), we obtain

$$
\varphi_{m,t}^* = \left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}} \left(\frac{\kappa-1}{\kappa}\right) A_t, \quad R_{m,t-1}^{J,*} = \left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{-1} R_{t-1}^J. \tag{B.6}
$$

From (15), we obtain

$$
M_{m,t} = \min\left\{\left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}\right)}, 1\right\}. \tag{B.7}
$$

Using equation (B.3) and (B.5), we obtain

$$
\left(\frac{P_{m,t}^J}{P_t}\right)^{1-\sigma} = \Theta_1 \cdot \left(\frac{W_t}{P_t A_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P_t^J}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \min\left\{\left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)}, 1\right\} \tag{B.8}
$$

We rearrange equation (6) as:

$$
\begin{aligned}
\left(\frac{P_t^J}{P_t}\right)^{1-\sigma} &= \int_0^1 \left(\frac{P_{m,t}^J}{P_t}\right)^{1-\sigma} \mathrm{d}m \\
&= Prob\left(F_{m,t-1} \le F_{t-1}^*\right) E_t\left[\left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} | F_{m,t-1} \le F_{t-1}^*\right] \\
&\quad + Prob\left(F_{m,t-1} > F_{t-1}^*\right) E_t\left[\left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} | F_{m,t-1} > F_{t-1}^*\right] \\
&= \cancel{H(F_{t-1}^*)} \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F_{t-1}^*} \left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} \frac{\mathrm{d}H(F_{m,t-1})}{\cancel{H(F_{t-1}^*)}} + \cancel{[1-H(F_{t-1}^*)]} \int_{F_{t-1}^*}^{\infty} \left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} \frac{\mathrm{d}H(F_{m,t-1})}{\cancel{1-H(F_{t-1}^*)}} \\
&= \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F_{t-1}^*} \left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} \mathrm{d}H(F_{m,t-1}) + \int_{F_{t-1}^*}^{\infty} \left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} \mathrm{d}H(F_{m,t-1})
\end{aligned}
\tag{B.9}
$$

where $\frac{P^J_{m,t}}{P^J_t}$ is given by (B.8). Plugging (B.8) into (B.9), we obtain

$$\left(\frac{P^J_t}{P_t}\right)^{1-\sigma} = \Theta_1 \cdot \left(\frac{W_t}{P_t A_t}\right)^{\left(\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{P^J_t}{P_t}\right)^{\left(\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)}$$
$$\cdot \left[\int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F^*_{t-1}} 1 \, \mathrm{d}H\left(F_{m,t-1}\right) + \int_{F^*_{t-1}}^{\infty} \left(\frac{F_{m,t-1}}{F^*_{t-1}}\right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)} \mathrm{d}H\left(F_{m,t-1}\right)\right], \tag{B.10}$$

which leads to

$$\left(\frac{P^J_t}{P_t}\right)^{\left(\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} = \Theta_1 \cdot \left(\frac{W_t}{P_t A_t}\right)^{\left(\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)}$$
$$\cdot \left[H\left(F^*_{t-1}\right) + \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) \cdot \left[1 - H\left(F^*_{t-1}\right)\right]\right] \tag{B.11}$$

Rearranging equation (B.11), we finally obtain:

$$\frac{P^J_t}{P_t} = \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H\left(F^*_{t-1}\right)}\right]^{\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} \tag{B.12}$$

where we define

$$\Theta_3 = \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}{\Theta_1 \omega(\sigma-1)}\right), \quad \Theta_4 = \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\omega(\sigma-1)}\right).$$

Derivation of M_t and L_{t-1} in (20) and (21)

$$M_t = \int_0^1 \int_{v \in \Omega_{m,t}} 1 \, \mathrm{d}v \, \mathrm{d}m = \int_0^1 M_{m,t} \, \mathrm{d}m = \int_0^1 M_{m,t} \cdot dH(F_{m,t-1}) \tag{B.13}$$
$$= \underbrace{Prob\left(F_{t-1} \leq F^*_{t-1}\right)}_{=H(F^*_{t-1})} \cdot 1 + \cancel{Prob\left(F_{t-1} > F^*_{t-1}\right)} \cdot \int_{F^*_{t-1}}^{\infty} \left(\frac{F_{m,t-1}}{F^*_{t-1}}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}} \frac{\mathrm{d}H(F_{m,t-1})}{\cancel{1 - H\left(F^*_{t-1}\right)}}$$
$$= 1 - \Theta_M \cdot \left[1 - H\left(F^*_{t-1}\right)\right]$$

where

$$\Theta_M = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+\omega(\sigma-1)}.$$

To derive equation (17), we start from

$$
\begin{aligned}
\frac{L_{t-1}}{P_{t-1}} &= \frac{\int_0^1 L_{m,t-1}\,\mathrm{d}m}{P_{t-1}} \\
&= Prob\left(F_{m,t-1} \le F^*_{t-1}\right) \cdot \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F^*_{t-1}} F_{m,t-1}\,\frac{\mathrm{d}H\left(F_{m,t-1}\right)}{H\left(F^*_{t-1}\right)} \\
&\quad + Prob\left(F_{m,t-1} > F^*_{t-1}\right) \cdot \int_{F^*_{t-1}}^{\infty} \left(F^*_{t-1}\right)^{\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}\right)} \cdot F_{m,t-1}^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)}\,\frac{\mathrm{d}H\left(F_{m,t-1}\right)}{1-H\left(F^*_{t-1}\right)} \\
&= \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F^*_{t-1}} F_{m,t-1}\,\mathrm{d}H\left(F_{m,t-1}\right) + \int_{F^*_{t-1}}^{\infty} \left(F^*_{t-1}\right)^{\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}\right)} \cdot F_{m,t-1}^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)}\,\mathrm{d}H\left(F_{m,t-1}\right),
\end{aligned}
$$

which leads to

$$
\begin{aligned}
\frac{L_{t-1}}{P_{t-1}} &= F_{t-1} - \left(\frac{\omega}{\omega-1}\right)\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)}\right) \cdot F^*_{t-1} \cdot \left[1-H\left(F^*_{t-1}\right)\right] \\
&= F_{t-1} \cdot \left[1 - \Theta_L \cdot \left[1-H\left(F^*_{t-1}\right)\right]^{\left(\frac{\omega-1}{\omega}\right)}\right]
\end{aligned}
$$

where

$$
\Theta_L = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)}.
$$

Derivation of N_t in equation (25) Labor $N_{mv,t}$ employed by a producing bottom-tier firm (m,v) is given by

$$
\begin{aligned}
N_{mv,t} &= J_{mv,t}^{\frac{1}{\alpha}} \varphi_{mv,t}^{-\frac{1}{\alpha}} = \varphi_{mv,t}^{-\frac{1}{\alpha}} \cdot \left[\left(\frac{P^J_{mv,t}}{P^J_t}\right)^{-\sigma} \cdot J_t\right]^{\frac{1}{\alpha}} \qquad \text{(B.14)} \\
&= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{\varphi_{mv,t}}{\left(\frac{\kappa-1}{\kappa}\right)A_t}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \\
&\quad \cdot \left(\frac{W_t}{P_t A_t}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{P^J_t}{P_t}\right)^{\left(\frac{\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{Y_t\Delta_t}{A_t}\right)^{\left(\frac{1}{\alpha+\sigma(1-\alpha)}\right)} \\
&= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\varphi_{mv,t}}{\left(\frac{\kappa-1}{\kappa}\right)A_t}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left[\frac{\Theta_3}{1+\Theta_4 H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}},
\end{aligned}
$$

where we use equation (5) in the second equality, equations (8) and (11) for the third equality, and equation (19) to obtain the fourth one. For convenience we define $H_{t-1} \equiv H\left(F^*_{t-1}\right)$. Now we integrate labor in (B.14) across all producing firms to obtain the aggregate labor N_t. First,

$$
\begin{aligned}
N_t &= \int_0^1 \int_{v\in\Omega_{m,t}} N_{mv,t} \, \mathrm{d}v \, \mathrm{d}m \\
&= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{1-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \\
&\quad \cdot \left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \int_0^1 \int_{v\in\Omega_{m,t}} \varphi_{mv,t}^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \, \mathrm{d}v \, \mathrm{d}m \\
&= \square \int_0^1 \int_{v\in\Omega_{m,t}} \varphi_{mv,t}^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \, \mathrm{d}v \, \mathrm{d}m,
\end{aligned}
\tag{B.15}
$$

where

$$
\square = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{1-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}}.
\tag{B.16}
$$

Now, (37) leads to

$$
\begin{aligned}
N_t &= \square \int_0^1 \int_{\max\left(\varphi^*_{m,t},\frac{\kappa-1}{\kappa}A_t\right)} \varphi_{mv,t}^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \kappa \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\kappa} \varphi_{mv,t}^{-(\kappa+1)} \, \mathrm{d}\varphi_{mv,t} \, \mathrm{d}m \\
&= \square \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\kappa} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\alpha+\sigma(1-\alpha)}\right)} \int_0^1 {\color{red}\max} \left(\frac{\varphi^*_{m,t}}{\frac{\kappa-1}{\kappa}A_t}, 1\right)^{\left(-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\alpha+\sigma(1-\alpha)}\right)} \mathrm{d}m \\
&= \square \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \int_0^1 \min\left(\left(\frac{F_{m,t-1}}{F^*_{t-1}}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}}, 1\right) \mathrm{d}m \\
&= \square \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left[H_{t-1} + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})\right] \\
&= \square \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) [1+\Theta_4 H_{t-1}] \\
&= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) [1+\Theta_4 H_{t-1}] \left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \\
&= \Theta_N \cdot \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \cdot (1+\Theta_4 H_{t-1})^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}},
\end{aligned}
\tag{B.17}
$$

where Θ_N is defined in (26).

Equilibrium conditions for top-tier firms Plugging equation (28) and the expression for $Q_{t,t+l}$ into (4), we can express the resetting price in (4) in a recursive fashion as: with

$$O_t = \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\left(\frac{Y_t}{A_t}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)}\Delta_t^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)}(1+\Theta_4 H_{t-1})^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}}\exp\{-u_{c,t}\}$$
$$+\beta\theta E_t\left[\exp\{u_{c,t+1}-u_{c,t}\}\cdot\Pi_{t+1}^{\gamma}\cdot O_{t+1}\right] \tag{B.18}$$

and

$$V_t = \left(\frac{C_t}{Y_t}\right)^{-1} + \beta\theta\cdot E_t\left[\exp\{u_{c,t+1}-u_{c,t}\}\cdot\Pi_{t+1}^{\gamma-1}\cdot V_{t+1}\right], \tag{B.19}$$

we obtain

$$\frac{P_t^*}{P_t} = \frac{O_t}{V_t}. \tag{B.20}$$

Due to price stickiness à la Calvo (1983), the aggregate price level can be recursively expressed as:

$$P_t^{1-\gamma} = (1-\theta)\left(P_t^*\right)^{1-\gamma} + \theta\left(P_{t-1}\right)^{1-\gamma}$$

which can be re-expressed as:

$$\frac{P_t^*}{P_t} = \left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}} \tag{B.21}$$

Plugging equation (B.20) into equation (9) and equation (B.21), we obtain

$$\frac{O_t}{V_t} = \left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}}, \quad \Delta_t = (1-\theta)\left(\frac{O_t}{V_t}\right)^{-\gamma} + \theta\Pi_t^{\gamma}\Delta_{t-1}$$

Equilibrium conditions for households We can write F_t^* as a function of H_t by using the cumulative distribution function of fixed costs in (18) and (23):

$$F_t^* = [1-H_t]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\tilde{Y}A_t\cdot\exp\{u_{f,t}\}. \tag{B.22}$$

Using the above (B.22), we can rearrange equation (B.5) (i.e., equation about F_t^* as:

$$R_t^J = E_t\left[\xi_{t+1}\cdot\left(\frac{P_{t+1}^J}{P_{t+1}}\right)^{\left(\frac{\sigma}{\alpha+\sigma(1-\alpha)}\right)}\left(\frac{w_{t+1}}{P_{t+1}A_{t+1}}\right)^{\left(\frac{(1-\sigma)\alpha}{\alpha+\sigma(1-\alpha)}\right)}\frac{1}{\tilde{Y}}\Pi_{t+1}GA_{t+1}\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\left(\frac{1}{\alpha+\sigma(1-\alpha)}\right)}\right]$$
$$\cdot\left(\frac{\Theta_2}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}[1-H_t]^{\frac{1}{\omega}}\cdot\exp\{-u_{f,t}\} \tag{B.23}$$

Plugging (27) and (28) into the above (B.23), we obtain:

$$R_t^J = \left(\frac{\Theta_2 \Theta_N^{\frac{1}{\eta}} \Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right) \cdot \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)} (1+\Theta_4 H_t)^{\left(\frac{\alpha+\sigma(1-\alpha)+\sigma\eta}{\eta(1-\sigma)\alpha}\right)} \cdot (1-H_t)^{\frac{1}{\omega}}$$
$$\cdot E_t \left[\xi_{t+1}\Pi_{t+1} \left(\frac{\frac{C_{t+1}}{A_{t+1}}}{\frac{Y_{t+1}}{A_{t+1}}}\right)\left(\frac{\frac{Y_{t+1}}{A_{t+1}}}{\bar{Y}}\right)\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)} \cdot GA_{t+1} \cdot \exp\left\{-(u_{f,t}+u_{c,t+1})\right\}\right]. \quad \text{(B.24)}$$

Finally, we can rearrange the Euler equation in (1), using (30) as follows:

$$\frac{1}{R_t^J} = \beta E_t \left[\frac{\left(\frac{C_t}{Y_t}\right)}{\left(\frac{C_{t+1}}{Y_{t+1}}\right)\widetilde{GY}_{t+1} GA_{t+1}\Pi_{t+1}} \cdot \exp\left\{u_{c,t+1} - u_{c,t}\right\}\right] \quad \text{(B.25)}$$

where $\widetilde{GY}_{t+1} = \frac{Y_{t+1}}{Y_{t+1}}\frac{A_t}{A_{t+1}}$ and $GA_{t+1} = \frac{A_{t+1}}{A_t}$. Combining equation (B.24) and equation (B.25), we obtain

$$\exp\left\{u_{f,t}+u_{c,t}\right\} = \beta\left(\frac{\Theta_2 \cdot \Theta_N^{\frac{1}{\eta}} \cdot \Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)} \cdot (1+\Theta_4 H_t)^{\left(\frac{[\alpha+\sigma(1-\alpha)]+\sigma\eta}{\eta(1-\sigma)\alpha}\right)}$$
$$\cdot (1-H_t)^{\frac{1}{\omega}} \cdot \left(\frac{\frac{C_t}{A_t}}{\bar{Y}}\right) \cdot E_t\left[\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)}\right] \quad \text{(B.26)}$$

Flexible price equilibrium Plugging (34) into (19), we obtain

$$\frac{W_t}{P_t A_t} = \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-1} \cdot \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{\alpha-1}{\alpha}} \cdot \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_{t-1}}\right]^{\left(\frac{\alpha+\sigma(1-\alpha)}{1-\sigma}\right)} \quad \text{(B.27)}$$

Plugging (19) and (B.27) into (B.5) (i.e., equation about the cutoff fixed cost F_t^*), and based on the fact that there is no price dispersion under flexible prices, i.e., $\Delta_t = 1$, we obtain:

$$F_t^* = \Theta_2 \cdot \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-1} \cdot \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_t}\right] E_t\left[\xi_{t+1}\left(\frac{\Pi_{t+1}Y_{t+1}}{R_t^J}\right)\right]. \quad \text{(B.28)}$$

By the definition of the distribution function of the fixed costs (see eq. equation (18)), we can express:

$$[1-H_t]^{-\frac{1}{\omega}} = \frac{F_t^*}{\left(\frac{\omega-1}{\omega}\right)F_t} = \frac{F_t^*}{\left(\frac{\omega-1}{\omega}\right)\cdot\phi_f\cdot\bar{Y}A_t\cdot\exp\left\{u_{f,t}\right\}}. \quad \text{(B.29)}$$

Plugging equation (B.29) into equation (B.28), we obtain:

$$1 = \left(\frac{\beta\Theta_2}{\left(\frac{\omega-1}{\omega}\right)\cdot\phi_f}\right)\cdot\left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-1}\cdot\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\cdot\left[\frac{\Theta_3}{1+\Theta_4\cdot H_t}\right]$$
$$\cdot\left[1-H_t\right]^{\frac{1}{\omega}}\cdot E_t\left[\left(\frac{\tilde{Y}_t}{\bar{Y}}\right)\left(\frac{\frac{C_t}{Y_t}}{\frac{C_{t+1}}{Y_{t+1}}}\right)\cdot\exp\left\{u_{c,t+1}-\left(u_{f,t}+u_{c,t}\right)\right\}\right] \tag{B.30}$$

Finally, plugging (34) into (28) and based on no price dispersion under flexible prices, i.e., $\Delta_t = 1$, we obtain

$$\frac{Y_t}{A_t} = \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)}\Theta_N^{-\left(\frac{\alpha}{(1-\alpha)\eta+1}\right)}\Theta_3^{-\frac{\eta[\alpha+\sigma(1-\alpha)]}{[(1-\alpha)\eta+1](\sigma-1)}}\cdot\left(\frac{C_t}{A_t}\right)^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)}$$
$$\cdot\left(1+\Theta_4 H_{t-1}\right)^{-\frac{(1+\eta)[\alpha+\sigma(1-\alpha)])}{(1-\sigma)[(1-\alpha)\eta+1]}}\cdot\exp\left\{\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)\cdot u_{c,t}\right\} \tag{B.31}$$

From (B.30) and (B.31), we can see that the flexible price equilibrium is money-neutral.

B.2 Detailed Derivations in Section 3.1

Derivations on the cross-sectional standard deviations of sales and productivities in (35) and (36) We start from the formula for the revenue $r_{mv,t}$ generated by a firm (m, v) in (B.2):

$$r_{mv,t} = (1+\zeta^J)\left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}(\Gamma_t^J)^{\frac{1}{\alpha+\sigma(1-\alpha)}}\varphi_{mv,t}^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}, \tag{B.32}$$

where

$$\varphi_{m,t}^* = \left(\frac{R_{t-1}^J P_{t-1} F_{m,t-1}}{E_{t-1}\left[\xi_t\cdot\Xi_t\right]}\right)^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}}. \tag{B.33}$$

We can calculate the cross-sectional standard deviation of an individual firm's revenue and productivity by calculating the variance:

$$\sigma^2\left(\log r_{mv,t}\right) = \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2\sigma^2\left(\log\varphi_{mv,t}\right) = \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2\sigma^2\left(\log\frac{\varphi_{mv,t}}{\varphi_{m,t}^*}+\log\varphi_{m,t}^*\right)$$
$$= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2\left[\sigma^2\left(\log\frac{\varphi_{mv,t}}{\varphi_{m,t}^*}\right)+\sigma^2\left(\log\varphi_{m,t}^*\right)\right], \tag{B.34}$$

where for the second line we use the property that (i) $\phi_{mv,t}|\phi_{mv,t} \geq \phi^*_{m,t}$ follows a Pareto distribution; (ii) distributions of productivities and fixed costs are independent of each other. Therefore,

$$\sigma^2\left(\log r_{mv,t}\right) = \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2\left[\sigma^2\left(\log\frac{\varphi_{mv,t}}{\varphi^*_{m,t}}\right) + \left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2\sigma^2\left(\log F_{m,t-1}\right)\right]$$
$$= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2\left[\frac{1}{\kappa^2} + \left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2\frac{1}{\omega^2}\right],$$

which implies

$$\sigma\left(\log r_{mv,t}\right) = \frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\sqrt{\frac{1}{\kappa^2} + \left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2\frac{1}{\omega^2}},$$

and

$$\sigma\left(\log\varphi_{mv,t}\right) = \sqrt{\frac{1}{\kappa^2} + \left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2\frac{1}{\omega^2}}.$$

B.3 Detailed Derivation in Section 4.2

Intensive vs. extensive margin labor adjustments: derivation of (39) From (37), (B.16), and (B.17), we know that the aggregate labor N_t can be written as

$$N_t = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)}\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)}\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \tag{B.35}$$
$$\cdot\left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}}\left[H_{t-1} + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})\right]$$
$$= \Theta_{DN}\left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}}\underbrace{\left[H_{t-1} + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})\right]}_{\equiv SN^I_t}$$
$$= \Theta_{DN}\left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}}\cdot SN^I_t \tag{B.36}$$

Where

$$\Theta_{DN} \equiv \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)}\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)}\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right). \tag{B.37}$$

From (B.35), we obtain for $\forall\iota$

$$\frac{N_{t+\iota}-N_t}{N_t}=\underbrace{\left[\frac{1+\Theta_4\cdot H_{t-1}}{1+\Theta_4\cdot H_{t+\iota-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{Y_{t+\iota}\Delta_{t+\iota}/A_{t+\iota}}{Y_t\Delta_t/A_t}\right)^{\frac{1}{\alpha}}-1}_{=g^{\text{Density}}_{t,t+\iota}} \tag{B.38}$$

$$+\left\{1+\underbrace{\left[\left[\frac{1+\Theta_4\cdot H_{t-1}}{1+\Theta_4\cdot H_{t+\iota-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{Y_{t+\iota}\Delta_{t+\iota}/A_{t+\iota}}{Y_t\Delta_t/A_t}\right)^{\frac{1}{\alpha}}-1\right]}_{=g^{\text{Density}}_{t,t+\iota}}\right\}\cdot\underbrace{\frac{SN^E_{t,t+\iota}}{SN^I_t}}_{\equiv g^{\text{Entry}}_{t,t+\iota}} \tag{B.39}$$

Therefore, by (38) and the definition of the decomposition in (39), we obtain

$$g^{\text{Entry}}_{t,t+\iota}\equiv\frac{SN^E_{t,t+\iota}}{SN^I_t}=\frac{(H_{t+\iota-1}-H_{t-1})+\dfrac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(H_{t-1}-H_{t+\iota-1})}{H_{t-1}+\dfrac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})}, \tag{B.40}$$

which proves (40).

Appendix C Summary of Equilibrium Conditions

C.1 Sticky Price Equilibrium (i.e., Original Model)

$$\exp\left\{u_{f,t}+u_{c,t}\right\}=\beta\left(\frac{\Theta_2\cdot\Theta_N^{\frac{1}{\eta}}\cdot\Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\cdot\left(1+\Theta_4H_t\right)^{\left(\frac{\alpha+\sigma(1-\alpha)+\sigma\eta}{\eta(1-\sigma)\alpha}\right)}$$
$$\cdot\left(1-H_t\right)^{\frac{1}{\omega}}\cdot\left(\frac{\tilde{C}_t}{\bar{Y}}\right)\cdot E_t\left[\left(\tilde{Y}_{t+1}\Delta_{t+1}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)}\right]$$

$$\frac{1}{R_t^J}=\beta E_t\left[\frac{\tilde{C}_t}{\tilde{C}_{t+1}GA_{t+1}\Pi_{t+1}}\cdot\exp\left\{u_{c,t+1}-u_{c,t}\right\}\right]$$

$$\frac{\tilde{C}_t}{\tilde{Y}_t}=1-\phi_g\cdot\exp\left\{u_{g,t}\right\}-\phi_f\cdot\left(\frac{\tilde{Y}_t}{\bar{Y}}\right)^{-1}\cdot\left[1-\Theta_L\cdot\left[1-H_t\right]^{\left(\frac{\omega-1}{\omega}\right)}\right]\cdot\exp\left\{u_{f,t}\right\}$$

$$O_t=\left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\tilde{Y}_t^{\left(\frac{\eta+1}{\eta\alpha}\right)}\Delta_t^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)}\left(1+\Theta_4H_{t-1}\right)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}}\exp\left\{-u_{c,t}\right\}$$
$$+\beta\theta E_t\left[\exp\left\{u_{c,t+1}-u_{c,t}\right\}\cdot\Pi_{t+1}^{\gamma}\cdot O_{t+1}\right]$$

$$V_t=\left(\frac{\tilde{C}_t}{\tilde{Y}_t}\right)^{-1}+\beta\theta\cdot E_t\left[\exp\left\{u_{c,t+1}-u_{c,t}\right\}\cdot\Pi_{t+1}^{\gamma-1}\cdot V_{t+1}\right]$$

$$\frac{O_t}{V_t}=\left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}}$$

$$\Delta_t=(1-\theta)\left(\frac{O_t}{V_t}\right)^{-\gamma}+\theta\Pi_t^{\gamma}\Delta_{t-1}$$

$$R_t^J=R^J\cdot\left(\frac{\Pi_t}{\Pi}\right)^{\tau_\pi}\left(\frac{\tilde{Y}_t}{\bar{Y}}\right)^{\tau_y}\cdot\exp\left\{\varepsilon_{r,t}\right\},\ \varepsilon_{r,t}\sim N\left(0,\sigma_r^2\right)$$

$$\tilde{F}_t^*=\left[1-H_t\right]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\bar{Y}\cdot\exp\left\{u_{f,t}\right\}$$

$$R_t^{J,*}=\left(\frac{\omega}{\omega+1}\right)\cdot\left(1-H_t\right)^{-\frac{1}{\omega}}\cdot R_t^J$$

$$N_t=\Theta_N\cdot\left(\tilde{Y}_t\Delta_t\right)^{\frac{1}{\alpha}}\cdot\left(1+\Theta_4H_{t-1}\right)^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}}$$

$$g_{t,t+1}^{Density}=\left[\frac{1+\Theta_4\cdot H_{t-1}}{1+\Theta_4\cdot H_t}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{\tilde{Y}_{t+1}\Delta_{t+1}}{\tilde{Y}_t\Delta_t}\right)^{\frac{1}{\alpha}}-1$$

$$g_{t,t+1}^{Entry}=\frac{(H_t-H_{t-1})+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(H_{t-1}-H_t)}{H_{t-1}+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})}$$

$$\frac{W_t}{P_tA_t}=\Theta_N^{\frac{1}{\eta}}\left(\tilde{C}_t\right)\left(\tilde{Y}_t\Delta_t\right)^{\frac{1}{\eta\alpha}}\left(1+\Theta_4H_{t-1}\right)^{\frac{\alpha+\sigma(1-\alpha)}{\eta(1-\sigma)\alpha}}\cdot\exp\left\{-u_{c,t}\right\}$$

$$
\begin{aligned}
\frac{P_t^J}{P_t} &= \Theta_N^{\frac{1}{\eta}} \Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\tilde{C}_t\right) \left(\tilde{Y}_t \Delta_t\right)^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)} \left(1+\Theta_4 H_{t-1}\right)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}} \cdot \exp\left\{-u_{c,t}\right\} \\
M_{t+1} &= 1-\Theta_M \cdot [1-H_t] \\
\frac{\frac{L_t}{A_t P_t}}{\bar{Y}} &= \phi_f \cdot \left[1-\Theta_L \cdot [1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right] \\
GA_t &= (1+\mu) \cdot \exp\left\{u_{a,t}\right\}
\end{aligned}
$$

Shock processes:

$$
\begin{aligned}
u_{a,t} &= \rho_a \cdot u_{a,t-1} + \varepsilon_{a,t},\ \varepsilon_{a,t} \sim N\left(0,\ \sigma_a^2\right) \\
u_{c,t} &= \rho_c \cdot u_{c,t-1} + \varepsilon_{c,t},\ \varepsilon_{c,t} \sim N\left(0, \sigma_c^2\right) \\
u_{g,t} &= \rho_g \cdot u_{g,t-1} + \varepsilon_{g,t},\ \varepsilon_{g,t} \sim N\left(0,\ \sigma_g^2\right) \\
u_{f,t} &= \rho_f \cdot u_{f,t-1} + \varepsilon_{f,t},\ \varepsilon_{f,t} \sim N\left(0,\ \sigma_f^2\right)
\end{aligned}
$$

Parameters:

$$
\begin{aligned}
\Theta_1 &= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
\Theta_2 &= \frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{-\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}} \\
\Theta_3 &= \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}{\Theta_1 \omega(\sigma-1)}\right) \\
\Theta_4 &= \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\omega(\sigma-1)}\right) \\
\Theta_N &= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) \Theta_3^{\left(\frac{\sigma}{\alpha(\sigma-1))}\right)} > 0 \\
\Theta_M &= \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+\omega(\sigma-1)} \\
\Theta_L &= \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)}
\end{aligned}
$$

C.2 Flexible Price Equilibrium

$$
\begin{aligned}
1 &= \left(\frac{\beta\Theta_2}{\left(\frac{\omega-1}{\omega}\right)\cdot\phi_f}\right)\cdot\left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-1}\cdot\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\cdot\left[\frac{\Theta_3}{1+\Theta_4\cdot H_t}\right][1-H_t]^{\frac{1}{\omega}} \\
&\quad \cdot E_t\left[\left(\frac{\tilde{Y}_t}{\bar{Y}}\right)\left(\frac{\tilde{C}_t/\tilde{Y}_t}{\tilde{C}_{t+1}/\tilde{Y}_{t+1}}\right)\cdot\exp\left\{u_{c,t+1}-\left(u_{f,t}+u_{c,t}\right)\right\}\right] \\
\tilde{Y}_t &= \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)}\Theta_N^{-\left(\frac{\alpha}{(1-\alpha)\eta+1}\right)}\Theta_3^{-\frac{\eta[\alpha+\sigma(1-\alpha)]}{[(1-\alpha)\eta+1](\sigma-1)}}\cdot\tilde{C}_t^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)} \\
&\quad \cdot(1+\Theta_4 H_{t-1})^{-\frac{(1+\eta)[\alpha+\sigma(1-\alpha)])}{(1-\sigma)[(1-\alpha)\eta+1]}}\cdot\exp\left\{\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)\cdot u_{c,t}\right\} \\
\frac{\tilde{C}_t}{\tilde{Y}_t} &= 1-\phi_g\cdot\exp\left\{u_{g,t}\right\}-\phi_f\cdot\left(\frac{\tilde{Y}_t}{\bar{Y}}\right)^{-1}\cdot\left[1-\Theta_L\cdot[1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right]\cdot\exp\left\{u_{f,t}\right\} \\
\tilde{F}_t^{*} &= [1-H_t]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\bar{Y}\cdot\exp\left\{u_{f,t}\right\} \\
R_t^J &= R^J\cdot\left(\frac{\Pi_t}{\Pi}\right)^{\tau_\pi}\left(\frac{\tilde{Y}_t}{\bar{Y}}\right)^{\tau_y}\cdot\exp\left\{\varepsilon_{r,t}\right\} \\
R_t^{J,*} &= \left(\frac{\omega}{\omega+1}\right)\cdot(1-H_t)^{-\frac{1}{\omega}}\cdot R_t^J
\end{aligned}
$$

Shock processes:

$$
\begin{aligned}
GA_t &= (1+\mu)\cdot exp\left\{u_{a,t}\right\} \\
u_{a,t} &= \rho_a\cdot u_{a,t-1}+\varepsilon_{a,t} \\
u_{c,t} &= \rho_c\cdot u_{c,t-1}+\varepsilon_{c,t} \\
u_{g,t} &= \rho_g\cdot u_{g,t-1}+\varepsilon_{g,t} \\
u_{f,t} &= \rho_f\cdot u_{f,t-1}+\varepsilon_{f,t} \\
\varepsilon_{c,t} &\sim N\left(0,\sigma_c^2\right) \\
\varepsilon_{a,t} &\sim N\left(0,\ \sigma_a^2\right) \\
\varepsilon_{g,t} &\sim N\left(0,\ \sigma_g^2\right) \\
\varepsilon_{f,t} &\sim N\left(0,\ \sigma_f^2\right) \\
\varepsilon_{r,t} &\sim N\left(0,\sigma_r^2\right)
\end{aligned}
$$

C.3 Steady State Conditions

$$
\begin{aligned}
R^B &= \beta^{-1}(1+\mu)\Pi \\
\Delta &= \left(\frac{1-\theta}{1-\theta\Pi^{\gamma}}\right)\left(\frac{1-\theta\Pi^{\gamma-1}}{1-\theta}\right)^{\left(\frac{\gamma}{\gamma-1}\right)} \\
\frac{\Theta_3\cdot[1-H]^{\frac{1}{\omega}}}{1+\Theta_4\cdot H} &= \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\left[\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right]\left[\frac{1-\theta\Pi^{\gamma}}{1-\theta\Pi^{\gamma-1}}\right]\left[\frac{1-\beta\theta\Pi^{\gamma-1}}{1-\beta\theta\Pi^{\gamma}}\right]\left(\frac{\left(\frac{\omega-1}{\omega}\right)\phi_f}{\beta\cdot\Theta_2}\right) \\
\bar{Y} &= \frac{\left(\frac{\beta\Theta_2\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)^{-\left(\frac{\eta\alpha}{\eta+1}\right)}\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{-\eta\alpha(\sigma-1)(1-\alpha)}{[\alpha+\sigma(1-\alpha)](\eta+1)}\right)}(1+\Theta_4H)^{-\frac{[\alpha+\sigma(1-\alpha)]+\sigma\eta}{(\eta+1)(1-\sigma)}}(1-H)^{-\frac{\eta\alpha}{\omega(\eta+1)}}}{\Delta\cdot\left[1-\phi_g-\phi_f\cdot\left[1-\Theta_L\cdot[1-H]^{\left(\frac{\omega-1}{\omega}\right)}\right]\right]^{\left(\frac{\eta\alpha}{\eta+1}\right)}} \\
\tilde{C} &= \left[1-\phi_g-\phi_f\cdot\left[1-\Theta_L\cdot[1-H]^{\left(\frac{\omega-1}{\omega}\right)}\right]\right]\cdot\bar{Y} \\
M &= 1-\Theta_M\cdot[1-H] \\
\tilde{F}^* &= [1-H]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\bar{Y} \\
R^{J,*} &= \left(\frac{\omega}{\omega+1}\right)\cdot(1-H)^{-\frac{1}{\omega}}\cdot\beta^{-1}(1+\mu)\Pi \\
\frac{R^{J,*}}{R^B} &= \left(\frac{\omega}{\omega+1}\right)\cdot(1-H)^{-\frac{1}{\omega}} \\
N &= \Theta_N\cdot\bar{Y}^{\frac{1}{\alpha}}\cdot\Delta^{\frac{1}{\alpha}}\cdot(1+\Theta_4H)^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}} \\
\frac{W}{PA} &= \Theta_N^{\frac{1}{\eta}}\tilde{C}\bar{Y}^{\frac{1}{\eta\alpha}}\Delta^{\frac{1}{\eta\alpha}}(1+\Theta_4H)^{\frac{\alpha+\sigma(1-\alpha)}{\eta(1-\sigma)\alpha}} \\
\frac{P_t^J}{P_t} &= \Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\tilde{C}(\bar{Y}\Delta)^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)}(1+\Theta_4H)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}} \\
O &= \frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\cdot\frac{\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\bar{Y}^{\left(\frac{\eta+1}{\eta\alpha}\right)}\Delta^{\frac{(1-\alpha)\eta+1}{\eta\alpha}}(1+\Theta_4H)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}}}{1-\beta\theta\Pi^{\gamma}} \\
V &= \frac{\left(\frac{\tilde{C}}{\bar{Y}}\right)^{-1}}{1-\beta\theta\Pi^{\gamma-1}} \\
\frac{\frac{L}{PA}}{\bar{Y}} &= \phi_f\left[1-\Theta_L(1-H)^{\frac{\omega-1}{\omega}}\right]
\end{aligned}
$$

Appendix D Limiting Case with $\omega \to \infty$

When $\omega \to +\infty$,the Pareto distribution $H(F_{m,t})$ of the fixed costs collapse to its mean, F_t. In this scenario, it is trivial to see that $P^J_{m,t} = P^J_t$. For P^J_t, we plug equation (B.4) into equation (B.3), and obtain

$$\frac{P^J_t}{P_t} = \begin{cases} \Theta_1^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \\ \quad \cdot \left(\frac{R^J_{t-1} F_{t-1}}{\Theta_2 E_{t-1} \left[\xi_t \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{P_t A_t}\right)^{-\frac{(\sigma-1)\alpha}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa} A_t\right) \Pi_t \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \right]} \right)^{\frac{[\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)](\alpha+\sigma(1-\alpha))}{(\sigma-1)^2\alpha}} \\ \underline{\text{if } R^J_t > R^{J,*}_t,} \\ \\ \Theta_1^{-\left(\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}\right)} \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \quad \underline{\text{if } R^J_t \leq R^{J,*}_t}. \end{cases} \tag{D.1}$$

Plugging (D.1) into (B.4), we can obtain

$$\Xi_t = \begin{cases} \Theta_5 \cdot \left(\frac{W_t}{P_t A_t}\right) \left[\left(\frac{\kappa-1}{\kappa}\right) A_t \right]^{\frac{1}{\alpha+\sigma(1-\alpha)}} \cdot P_t \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \\ \quad \cdot \left(\frac{R^J_{t-1} F_{t-1}}{\Theta_2 E_{t-1} \left[\xi_t \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{P_t A_t}\right)^{-\frac{(\sigma-1)\alpha}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa} A_t\right)^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \Pi_t (Y_t \Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \right]} \right)^{\frac{\sigma}{\sigma-1}\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{(\sigma-1)\alpha}\right)} \\ \underline{\text{if } R^J_t > R^{J,*}_t,} \\ \\ \Theta_5 \cdot \left(\frac{W_t}{P_t A_t}\right) \left[\left(\frac{\kappa-1}{\kappa}\right) A_t \right]^{\frac{1}{\alpha+\sigma(1-\alpha)}} \cdot P_t \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \quad \underline{\text{if } R^J_t \leq R^{J,*}_t}, \end{cases} \tag{D.2}$$

where we define

$$\Theta_5 = \Theta_1^{-\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \Theta_2 \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(1-\sigma)-1}{\alpha+\sigma(1-\alpha)}}.$$

Now that $M_t = M_{m,t}$, $L_t = L_{m,t}$, $R^{J,*}_t = R^{J,*}_{m,t}$ and $\varphi^*_t = \varphi^*_{m,t}$, we can substitute (D.2) into (14), (15), (16), and (17) to obtain following analytical expressions:

$$R_t^{J,*} = \Theta_5 \cdot E_t \left[\xi_{t+1} \left(\frac{\kappa - 1}{\kappa} A_{t+1} \right)^{\frac{\sigma}{\alpha + \sigma(1-\alpha)}} \left(\frac{w_{t+1}}{P_{t+1} A_{t+1}} \right) \frac{\Pi_{t+1}}{F_t} \left(\frac{Y_{t+1} \Delta_{t+1}}{A_{t+1}} \right)^{\frac{1}{\alpha}} \right], \tag{D.3}$$

$$\varphi_t^* = \left(\frac{R_t^J}{R_t^{J,*}} \right)^{\left(\frac{\alpha + \sigma(1-\alpha)}{\sigma - 1} \right)} \left[\left(\frac{\kappa - 1}{\kappa} \right) A_{t+1} \right], \tag{D.4}$$

$$M_{t+1} = \begin{cases} \left(\frac{R_t^J}{R_t^{J,*}} \right)^{-\left(\frac{\kappa[\alpha + \sigma(1-\alpha)]}{\sigma - 1} \right)} & \underline{\text{if } R_t^J > R_t^{J,*}}, \\ 1 & \underline{\text{if } R_t^J \leq R_t^{J,*}}, \end{cases} \tag{D.5}$$

$$L_t = \begin{cases} \left(\frac{R_t^J}{R_t^{J,*}} \right)^{-\left(\frac{\kappa[\alpha + \sigma(1-\alpha)]}{\sigma - 1} \right)} \cdot F_t & \underline{\text{if } R_t^J > R_t^{J,*}}, \\ F_t & \underline{\text{if } R_t^J \leq R_t^{J,*}}. \end{cases} \tag{D.6}$$

We observe: if $R_t^J \leq R_t^{J,*}$, where $R_t^{J,*}$ is defined in (22), all firms are satiated and the loan amount made to firms is equal to F_t, the fixed cost that operating firms need to pay one period in advance.