

$$\begin{aligned}
 I_n(\alpha) &= \int_0^1 \frac{x^n}{x+\alpha} dx = \int_0^1 \frac{x^{n-1}(x+\alpha-\alpha)}{x+\alpha} dx = \\
 &= \int_0^1 \frac{x^{n-1}(x+\alpha)}{x+\alpha} dx - \alpha \int_0^1 \frac{x^{n-1}}{x+\alpha} dx = \\
 &= \int_0^1 x^{n-1} dx - \alpha I_{n-1}(\alpha) = \\
 &= \frac{x^n}{n} \Big|_0^1 - \alpha I_{n-1}(\alpha) = \frac{1}{n} - \alpha I_{n-1}(\alpha)
 \end{aligned}$$

$$\begin{aligned}
 I_1(\alpha) &= \int_0^1 \frac{dx}{x+\alpha} = \int_0^1 \frac{d(x+\alpha)}{x+\alpha} = \ln(x+\alpha) \Big|_0^1 = \\
 &= \ln\left(\frac{1+\alpha}{\alpha}\right)
 \end{aligned}$$

~~$$\begin{aligned}
 I_n(\alpha) &= \frac{1}{n} - \alpha I_{n-1}(\alpha) \\
 I_{n-1}(\alpha) &= \frac{1}{n-1} - \alpha I_{n-2}(\alpha) \\
 &\vdots \\
 I_1(\alpha) &= \ln\left(\frac{1+\alpha}{\alpha}\right)
 \end{aligned}$$~~

Прямая $I_n(\alpha) = \frac{1}{n} - \alpha I_{n-1}(\alpha)$

Обратная $I_{n-1}(\alpha) = \frac{1}{\alpha n} - \frac{I_n(\alpha)}{\alpha}$

№3

$$p(n) = -p(n-1) + 6p(n-2)$$

$$p(0) = 1$$

$$p(1) = -3$$

$$p(n) = (-3)^n$$

База:

$$p(0) = 1$$

$$p(1) = -3$$

Шаг индукции

$$p(n+1) = (-3)^{n+1} = -3(-3)^n = -3p(n) =$$

~~$$-p(n) + 6p(n-1) \quad \ominus$$~~

~~$$p(n) + 3p(n-1) + 6p(n-2) \quad \ominus$$~~
$$\ominus (-3)^n + 6(-3)^{n-1} \quad \Delta$$

~~$$p(n) = -3p(n-1) \quad \Delta$$~~
$$\Delta (-3)^{n-1} (13 + 6 \cdot \text{[scribble]}) \quad \ominus$$

$$\ominus (-3)^{n-1} \cdot 9 = (-3)^{n+1}$$

$$\boxed{p(n) = (-3)^n}$$

$$p(2020) = (-3)^{2020} = \underline{\underline{3^{2020}}}$$

N4.

$$A = \begin{pmatrix} 1 & 10 \\ \delta & 1 \end{pmatrix}$$

$$\det(A - \lambda E) = 0$$

$$\begin{vmatrix} 1-\lambda & 10 \\ \delta & 1-\lambda \end{vmatrix} = 0 \Leftrightarrow$$

$$1 - 2\lambda + \lambda^2 - 10\delta = 0$$

$$\lambda^2 - 2\lambda - 10\delta + 1 = 0$$

$$b = -2$$

$$c = 1 - 10\delta$$

$$\lambda_{\max} = \frac{2 + \sqrt{4 - 4(1 - 10\delta)}}{2}$$

$$E(\delta) = \lambda_{\max} = \frac{1 + \sqrt{10\delta}}{1}$$

$$K(\delta) = \frac{dE(\delta)}{d\delta} = -\frac{\sqrt{10}}{2\sqrt{\delta}}$$

$$K(10) = -\frac{1}{2}$$

$$K(0,1) = -0,5 \cdot 10 = -5$$