

ewN2HDECAY - A program for the Calculation of Electroweak One-Loop Corrections to Higgs Decays in the Next-to-Minimal Two-Higgs-Doublet Model Including State-of-the-Art QCD Corrections

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Abstract

We present in this paper our new program package `ewN2HDECAY` for the calculation of the partial decay widths and branching ratios of the Higgs bosons of the Next-to-Minimal 2-Higgs Doublet Model (N2HDM). The N2HDM is based on a general CP-conserving 2HDM which is extended by a real scalar singlet field. The program computes the complete electroweak one-loop corrections to all non-loop-induced two-body on-shell Higgs boson decays in the N2HDM and combines them with the state-of-the-art QCD corrections that are already implemented in the existing program `N2HDECAY`. Most of the independent input parameters of the electroweak sector of the N2HDM are renormalized in an on-shell scheme. The soft- Z_2 -breaking squared mass scale m_{12}^2 and the vacuum expectation value v_S of the $SU(2)_L$ singlet field, however, are renormalized with $\overline{\text{MS}}$ conditions, while for the four scalar mixing angles α_i ($i = 1, 2, 3$) and β of the N2HDM, several different renormalization schemes are applied. By giving out the leading-order and the loop-corrected partial decay widths separately from the branching ratios, the program `ewN2HDECAY` not only allows for phenomenological analyses of the N2HDM at highest precision, it can also be used for a study of the impact of the electroweak corrections and the remaining theoretical uncertainty due to missing higher-order corrections based on a change of the renormalization scheme. The input parameters are then consistently calculated with a parameter conversion routine when switching from one renormalization scheme to the other. The latest version of the program `ewN2HDECAY` can be downloaded from the URL <https://github.com/marcel-krause/ewN2HDECAY>.

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1 Introduction

The discovery of the Higgs boson by the LHC experiments ATLAS [1] and CMS [2] has been a tremendous success for particle physics. The Standard Model (SM)-like behaviour of the discovered Higgs boson [3], on the other hand, leaves many questions open. To solve these problems, extensions beyond the SM (BSM) are considered, which usually entail enlarged Higgs sectors. In view of the lack of any direct experimental sign of new physics (NP) so far, indirect searches for NP in the Higgs sector become increasingly important. Due to the very SM-like nature of the discovered Higgs boson and because of the similarity of signatures predicted by different models, such searches require sophisticated experimental techniques on the one side and high-precision predictions by theory on the other side. This renders the inclusion of higher-order corrections in the Higgs boson observables indispensable. We contribute to this effort with the program code that we present and publish here. In an earlier work [4], we have published the code `2HDECAY` for the computation of the electroweak (EW) corrections to the on-shell non-loop induced Higgs decays of the 2-Higgs-Doublet Model (2HDM). Here, we present the code `ewN2HDECAY`. It computes the EW corrections to the on-shell non-loop induced Higgs decays of the Next-to-2HDM (N2HDM). The N2HDM is based on the CP-conserving 2HDM extended by a real scalar singlet field. After electroweak symmetry breaking (EWSB) the Higgs sector consists of three neutral CP-even, one neutral CP-odd and two charged Higgs bosons. Due to its enlarged parameter space and its fewer symmetries compared with supersymmetric models it provides an interesting phenomenology with a variety of non-SM-like signatures that are still compatible with current experimental constraints [5–7]. Depending on which of its global symmetries are broken, its phenomenology can change considerably [8]. Thus it can feature a dark sector and extra sources of CP violation that only exist in the dark sector [9]. In [10], we computed the EW corrections to the Higgs boson decays in the N2HDM and provided a gauge-independent renormalization of the N2HDM. For this, we had to extend our formalism developed for the 2HDM in [11–13]¹ to the N2HDM.

We have implemented 10 different renormalization schemes in `ewN2HDECAY` for the calculation of the EW corrections to the N2HDM Higgs decays into all possible on-shell two-particle final states of the model that are not loop-induced. The program is linked with the Fortran code `N2HDECAY` [5]. Based on an extension of the Fortran code `HDECAY` [22, 23], `N2HDECAY` incorporates the state-of-the-art higher-order QCD corrections to the decays including also loop-induced and off-shell decays. We consistently combine these corrections with our newly provided N2HDM EW corrections, so that `ewN2HDECAY` provides the N2HDM Higgs boson decay widths at the presently highest possible level of precision. Moreover, we separately give out the leading order (LO) and next-to-leading order (NLO) EW-corrected decay widths so that studies can be performed on the importance of the relative EW corrections. The comparison of the results for different renormalization schemes additionally allows to estimate the remaining theoretical uncertainty due to missing higher-order corrections. For the consistent comparison we include in `ewN2HDECAY` a routine that automatically converts the input parameters from one renormalization scheme to another for all 10 implemented renormalization schemes.

The development of `ewN2HDECAY` tightly followed the development of `2HDECAY`, from which large parts of code were adapted for the calculation of the Higgs boson decays in the N2HDM. Due to the similarities of the two codes, similarities between the structure of this paper and the

¹For later works discussing the renormalization of the 2HDM, see [14–18]. An improved on-shell scheme that is essentially equivalent to the mixing angle renormalization scheme presented by our group in [11–13] is used in [19–21].

manual of 2HDECAY, *cf.* Ref. [4], are intentional. We still keep the description of `ewN2HDECAY` here as short as possible while at the same time taking care to remain self-contained. For additional details, we refer to [4] where appropriate.

The program `ewN2HDECAY` was developed and tested under Windows 10, openSUSE Leap 15.0 and macOS Sierra 10.12. In order to compile and run the program, an up-to-date version of Python 2 or Python 3 (tested with versions 2.7.14 and 3.5.0), the FORTRAN compiler `gfortran` and the GNU C compilers `gcc` (tested for compatibility with versions 6.4.0 and 7.3.1) and `g++` are required. The latest version of the package can be downloaded from

<https://github.com/marcel-krause/ewN2HDECAY> .

The paper is organized as follows. In the subsequent Sec. 2, we briefly introduce the N2HDM and its particle content, focusing solely on the differences with respect to the 2HDM due to its extended scalar sector. Moreover, we introduce the relevant input parameters and set our notation. We briefly present the counterterms of the extended scalar sector that are needed for the computation of the EW corrections as well as changes in the counterterms with respect to the 2HDM. A full list of the decays implemented in `ewN2HDECAY` is presented, the link to N2HDECAY is described and the parameter conversion is discussed. In Sec. 3, we introduce the program `ewN2HDECAY`, describe its structure in detail and provide an installation and usage guide. Moreover, we briefly describe the required format of the input and output file and the meaning of the input parameters. We complete the paper with a short summary of our work in Sec. 4. As a useful reference for the user, we print the exemplary input and output files which are included in the `ewN2HDECAY` repository, in Appendices A and B, respectively.

2 One-Loop Electroweak and QCD Corrections in the N2HDM

After a brief introduction of the N2HDM where we set up our notation and list the input parameters used for the calculation we present the renormalization of the EW sector of the N2HDM that we apply in the computation of the EW one-loop corrections to the partial decay widths of the neutral N2HDM Higgs bosons. We shortly describe the computation of these partial decay widths and explain how the EW-corrected partial decay widths are combined with the state-of-the-art QCD corrections already implemented in the code N2HDECAY.

2.1 Introduction of the N2HDM

We consider a general CP-conserving N2HDM which, in comparison to the 2HDM, is extended by adding a real $SU(2)_L$ singlet field Φ_S with hypercharge $Y = 0$. Together with the two complex $SU(2)_L$ doublets Φ_1 and Φ_2 with hypercharges $Y = +1$, it builds the scalar sector of the model. Through EWSB, the two doublet and the singlet fields develop non-negative real vacuum expectation values (VEVs) v_1 , v_2 and v_S , which in general are non-vanishing. The doublet and singlet fields can be expanded around these VEVs as

$$\Phi_1 = \begin{pmatrix} \omega_1^\pm \\ \frac{v_1 + \rho_1 + i\eta_1}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \omega_2^\pm \\ \frac{v_2 + \rho_2 + i\eta_2}{\sqrt{2}} \end{pmatrix}, \quad \Phi_S = v_S + \rho_S, \quad (2.1)$$

where ρ_i and ρ_S are three CP-even fields, η_i are two CP-odd fields and ω_i^\pm are two electromagnetically charged fields ($i = 1, 2$). The two VEVs of the doublets are connected to the SM VEV v via the relation

$$v_1^2 + v_2^2 = v^2 \approx (246.22 \text{ GeV})^2. \quad (2.2)$$

The ratio of the two VEVs defines the characteristic parameter β given by

$$\tan \beta = \frac{v_2}{v_1} . \quad (2.3)$$

The EW part of the Lagrangian relevant for our computation of the EW one-loop corrections is given by

$$\mathcal{L}_{\text{N2HDM}}^{\text{EW}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{F}} + \mathcal{L}_{\text{S}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} . \quad (2.4)$$

The Yang-Mills Lagrangian \mathcal{L}_{YM} and the fermion Lagrangian \mathcal{L}_{F} are analogous to the SM. Their explicit forms are presented *e.g.* in [24, 25]. We do not present the explicit forms of the gauge-fixing Lagrangian \mathcal{L}_{GF} and Fadeev-Popov Lagrangian \mathcal{L}_{FP} as they are not needed in the following. We remark, however, that we follow the same approach as in [26] for the 2HDM and apply the gauge-fixing procedure only after the renormalization of the N2HDM is performed so that \mathcal{L}_{GF} contains only renormalized fields and no additional counterterms for the gauge-fixing terms need to be introduced. The interaction of the Higgs bosons with the fermions is derived from the Yukawa Lagrangian \mathcal{L}_{Yuk} , with the corresponding Yukawa couplings presented *e.g.* in [5].

The scalar Lagrangian with the kinetic terms of the Higgs doublets and the scalar N2HDM potential reads

$$\mathcal{L}_{\text{S}} = \sum_{i=1}^2 (D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) + (\partial_\mu \Phi_S)(\partial^\mu \Phi_S) - V_{\text{N2HDM}} , \quad (2.5)$$

where D_μ denotes the covariant derivative

$$D_\mu = \partial_\mu + \frac{i}{2}g \sum_{a=1}^3 \sigma^a W_\mu^a + \frac{i}{2}g' B_\mu , \quad (2.6)$$

with the gauge couplings g and g' and the corresponding gauge boson fields W_μ^a and B_μ of the $SU(2)_L$ and $U(1)_Y$, respectively. The generators of the $SU(2)_L$ gauge group are given by the Pauli matrices σ^a . The scalar potential of the CP-conserving N2HDM is given by

$$V_{\text{N2HDM}} = \frac{1}{2}m_S^2 \Phi_S^2 + \frac{1}{8}\lambda_6 \Phi_S^4 + \frac{1}{2}\lambda_7 \left(\Phi_1^\dagger \Phi_1 \right) \Phi_S^2 + \frac{1}{2}\lambda_8 \left(\Phi_2^\dagger \Phi_2 \right) \Phi_S^2 + V_{\text{2HDM}} , \quad (2.7)$$

where V_{2HDM} denotes the scalar potential of the CP-conserving 2HDM, as given by [27]

$$\begin{aligned} V_{\text{2HDM}} = & m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + h.c. \right) + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 \\ & + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \frac{\lambda_5}{2} \left[\left(\Phi_1^\dagger \Phi_2 \right)^2 + h.c. \right] . \end{aligned} \quad (2.8)$$

It is obtained by imposing two \mathbb{Z}_2 symmetries on the scalar potential, under one of which, \mathbb{Z}_2 ,

$$\Phi_1 \rightarrow \Phi_1 , \quad \Phi_2 \rightarrow -\Phi_2 , \quad \Phi_S \rightarrow \Phi_S . \quad (2.9)$$

It is the trivial generalisation of the usual 2HDM \mathbb{Z}_2 symmetry (guaranteeing the absence of tree-level flavour-changing neutral currents (FCNCs) when extended to the Yukawa sector) and explicitly broken by the term proportional to m_{12}^2 . The second symmetry, \mathbb{Z}_2' , under which

$$\Phi_1 \rightarrow \Phi_1 , \quad \Phi_2 \rightarrow \Phi_2 , \quad \Phi_S \rightarrow -\Phi_S , \quad (2.10)$$

is not explicitly broken. The N2HDM potential contains twelve real-valued parameters, four mass parameters m_{11} , m_{22} , m_{12} and m_S and eight dimensionless coupling constants λ_i ($i = 1, \dots, 8$). For later convenience, we define

$$\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5 . \quad (2.11)$$

Inserting the expansion of the scalar doublet and singlet fields, Eq. (2.1), into the scalar potential of the N2HDM yields

$$V_{\text{N2HDM}} = \frac{1}{2} \begin{pmatrix} \rho_1 & \rho_2 & \rho_S \end{pmatrix} M_\rho^2 \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_S \end{pmatrix} + T_1 \rho_1 + T_2 \rho_2 + T_S \rho_S + \dots , \quad (2.12)$$

where M_ρ^2 denotes the 3×3 mass matrix in the CP-even scalar sector and T_1 , T_2 and T_S the three tadpole terms. We demand that the VEVs represent the minimum of the potential by applying the minimum condition

$$\left. \frac{\partial V_{\text{N2HDM}}}{\partial \Phi_i} \right|_{\langle \Phi_j \rangle} = 0 , \quad (2.13)$$

leading at tree level to the vanishing of the three tadpole terms,

$$T_1 = T_2 = T_S = 0 \quad (\text{at tree level}) , \quad (2.14)$$

which in terms of the Higgs potential parameters read

$$\frac{T_1}{v_1} \equiv m_{11}^2 - m_{12}^2 \frac{v_2}{v_1} + \frac{v_1^2 \lambda_1}{2} + \frac{v_2^2 \lambda_{345}}{2} + \frac{v_S^2 \lambda_7}{2} \quad (2.15)$$

$$\frac{T_2}{v_2} \equiv m_{22}^2 - m_{12}^2 \frac{v_1}{v_2} + \frac{v_2^2 \lambda_2}{2} + \frac{v_1^2 \lambda_{345}}{2} + \frac{v_S^2 \lambda_8}{2} \quad (2.16)$$

$$\frac{T_S}{v_S} \equiv m_S^2 + \frac{v_1^2 \lambda_7}{2} + \frac{v_2^2 \lambda_8}{2} + \frac{v_S^2 \lambda_6}{2} . \quad (2.17)$$

These conditions can be used to replace m_{11}^2 , m_{22}^2 and m_S^2 in favor of the three tadpole terms. The mass matrix of the CP-even scalar fields is given by

$$M_\rho^2 \equiv \begin{pmatrix} m_{12}^2 \frac{v_2}{v_1} + \lambda_1 v_1^2 & -m_{12}^2 + \lambda_{345} v_1 v_2 & \lambda_7 v_1 v_S \\ -m_{12}^2 + \lambda_{345} v_1 v_2 & m_{12}^2 \frac{v_1}{v_2} + \lambda_2 v_2^2 & \lambda_8 v_2 v_S \\ \lambda_7 v_1 v_S & \lambda_8 v_2 v_S & \lambda_6 v_S^2 \end{pmatrix} + \begin{pmatrix} \frac{T_1}{v_1} & 0 & 0 \\ 0 & \frac{T_2}{v_2} & 0 \\ 0 & 0 & \frac{T_S}{v_S} \end{pmatrix} . \quad (2.18)$$

By introducing three mixing angles α_i ($i = 1, 2, 3$) defined in the range

$$-\frac{\pi}{2} \leq \alpha_i < \frac{\pi}{2} , \quad (2.19)$$

the mass matrix can be diagonalised by means of the orthogonal matrix R parametrised as²

$$R = \begin{pmatrix} c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\ -(c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} + s_{\alpha_1} c_{\alpha_3}) & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\ -c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} + s_{\alpha_1} s_{\alpha_3} & -(c_{\alpha_1} s_{\alpha_3} + s_{\alpha_1} s_{\alpha_2} c_{\alpha_3}) & c_{\alpha_2} c_{\alpha_3} \end{pmatrix} , \quad (2.20)$$

²Throughout this paper, we use the short-hand notation $s_x \equiv \sin(x)$, $c_x \equiv \cos(x)$ and $t_x \equiv \tan(x)$.

which transforms the CP-even interaction fields into the mass eigenstates H_i ($i = 1, 2, 3$)

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix} . \quad (2.21)$$

This transformation yields the diagonalised mass matrix

$$D_\rho^2 \equiv R M_\rho^2 R^T \equiv \text{diag} (m_{H_1}^2, m_{H_2}^2, m_{H_3}^2) , \quad (2.22)$$

where we demand the three CP-even Higgs bosons H_i to be ordered by ascending mass,

$$m_{H_1} < m_{H_2} < m_{H_3} . \quad (2.23)$$

The CP-odd and charged mass matrices, not shown explicitly in Eq. (2.12) are equivalent to the ones in the 2HDM and are diagonalised by the two mixing angles β_η and β_{ω^\pm} , respectively, which, at tree level, coincide with the angle β defined in Eq. (2.3). The rotation of the corresponding fields to the mass basis yields the CP-odd and charged Higgs bosons A and H^\pm with masses m_A and m_{H^\pm} , respectively, as well as the massless CP-odd and charged Goldstone bosons G^0 and G^\pm . The quartic couplings λ_i ($i = 1, \dots, 8$) of the N2HDM potential can be written in terms of the parameters of the mass basis as [5]

$$\lambda_1 = \frac{1}{v^2 c_\beta^2} \left[\sum_{i=1}^3 m_{H_i}^2 R_{i1}^2 - \frac{s_\beta}{c_\beta} m_{12}^2 \right] \quad (2.24)$$

$$\lambda_2 = \frac{1}{v^2 s_\beta^2} \left[\sum_{i=1}^3 m_{H_i}^2 R_{i2}^2 - \frac{c_\beta}{s_\beta} m_{12}^2 \right] \quad (2.25)$$

$$\lambda_3 = \frac{1}{v^2} \left[\frac{1}{s_\beta c_\beta} \sum_{i=1}^3 m_{H_i}^2 R_{i1} R_{i2} + 2m_{H^\pm}^2 - \frac{1}{s_\beta c_\beta} m_{12}^2 \right] \quad (2.26)$$

$$\lambda_4 = \frac{1}{v^2} \left[\frac{m_{12}^2}{s_\beta c_\beta} + m_A^2 - 2m_{H^\pm}^2 \right] \quad (2.27)$$

$$\lambda_5 = \frac{1}{v^2} \left[\frac{m_{12}^2}{s_\beta c_\beta} - m_A^2 \right] \quad (2.28)$$

$$\lambda_6 = \frac{1}{v_S^2} \sum_{i=1}^3 m_{H_i}^2 R_{i3}^2 \quad (2.29)$$

$$\lambda_7 = \frac{1}{v v_{SC\beta}} \sum_{i=1}^3 m_{H_i}^2 R_{i1} R_{i3} \quad (2.30)$$

$$\lambda_8 = \frac{1}{v v_{S\beta}} \sum_{i=1}^3 m_{H_i}^2 R_{i2} R_{i3} . \quad (2.31)$$

The gauge sector of the N2HDM does not change with respect to the SM. After EWSB we have for the masses of the physical gauge bosons, the W and Z bosons and the photon,

$$m_W^2 = \frac{g^2 v^2}{4} \quad (2.32)$$

$$m_Z^2 = \frac{(g^2 + g'^2) v^2}{4} \quad (2.33)$$

$$m_\gamma^2 = 0 . \quad (2.34)$$

	u -type	d -type	leptons
I	Φ_2	Φ_2	Φ_2
II	Φ_2	Φ_1	Φ_1
lepton-specific	Φ_2	Φ_2	Φ_1
flipped	Φ_2	Φ_1	Φ_2

Table 1: The four realisations of the Yukawa couplings of the \mathbb{Z}_2 -symmetric N2HDM, depending on which Higgs doublet couples to which kind of fermions.

N2HDM type	Y_i^l	Y_4^l	Y_i^d	Y_4^d	Y_i^u	Y_4^u
I	$\frac{R_{i2}}{s_\beta}$	$\frac{1}{t_\beta}$	$\frac{R_{i2}}{s_\beta}$	$\frac{1}{t_\beta}$	$\frac{R_{i2}}{s_\beta}$	$\frac{1}{t_\beta}$
II	$\frac{R_{i1}}{c_\beta}$	$-t_\beta$	$\frac{R_{i1}}{c_\beta}$	$-t_\beta$	$\frac{R_{i2}}{s_\beta}$	$\frac{1}{t_\beta}$
lepton-specific	$\frac{R_{i1}}{c_\beta}$	$-t_\beta$	$\frac{R_{i2}}{s_\beta}$	$\frac{1}{t_\beta}$	$\frac{R_{i2}}{s_\beta}$	$\frac{1}{t_\beta}$
flipped	$\frac{R_{i2}}{s_\beta}$	$\frac{1}{t_\beta}$	$\frac{R_{i1}}{c_\beta}$	$-t_\beta$	$\frac{R_{i2}}{s_\beta}$	$\frac{1}{t_\beta}$

Table 2: Definition of the Yukawa coupling parameters $Y_{i,4}^{u,d,l}$ ($i = 1, 2, 3$) for each N2HDM type.

The electromagnetic coupling e in terms of the fine-structure constant α_{em} , which we use as independent input, reads

$$e = \sqrt{4\pi\alpha_{\text{em}}} = \frac{gg'}{\sqrt{g^2 + g'^2}}. \quad (2.35)$$

Alternatively, the tree-level relation to the Fermi constant,

$$G_F \equiv \frac{\sqrt{2}g^2}{8m_W^2} = \frac{\alpha_{\text{em}}\pi}{\sqrt{2}m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right)}, \quad (2.36)$$

can be used to replace one of the parameters of the EW sector in favour of G_F .

The aforementioned softly broken \mathbb{Z}_2 symmetry, Eq. (2.9), for the two doublet fields Φ_1 and Φ_2 , extended to the Yukawa sector to avoid FCNCs at tree level, leads to four types of doublet couplings to the fermion fields as in the 2HDM. They are summarised in Tab. 1. The Yukawa couplings can be parametrised in terms of the Yukawa coupling parameters $Y_{i,4}^{u,d,l}$ ($i = 1, 2, 3$) summarised in Tab. 2, where i refers the the scalar Higgs bosons H_i and 4 to the pseudoscalar A .

Apart from the parameters introduced so far, **N2HDECAY** requires additional input parameters, namely the electromagnetic fine-structure constant α_{em} in the Thomson limit for the computation of the loop-induced decays into $\gamma\gamma$ and $Z\gamma$, the strong coupling constant α_s for the calculation of the loop-induced decays into gluons as well as for the state-of-the-art QCD corrections, and additionally the total decay widths Γ_W and Γ_Z of the W^\pm and Z bosons, respectively, for the calculation of the off-shell decays into pairs of these gauge bosons. Moreover, the CKM matrix elements V_{ij} are required as input for the computation of the decays involving charged quark currents, as well as the quark masses m_f , where $f = s, c, b, t, \mu, \tau$ – all other fermions are approximated to be massless in the computation of the partial decay widths. The fermion and gauge boson masses are defined according to the recommendations of the LHC Higgs cross section working group [28]. Note that in **N2HDECAY** the decay widths are computed in terms of the Fermi constant G_F with the exception of the decays involving external on-shell

photons which are given in terms of α_{em} in the Thomson limit. As input for our renormalization conditions of the EW corrections, we require as input parameters, however, the on-shell masses m_W and m_Z and the electromagnetic coupling at the Z boson mass scale, $\alpha_{\text{em}}(m_Z^2)$. We will come back to this in Sec. 2.4. The full set of independent input parameters in the mass basis therefore is given by ($i = 1, 2, 3$)

$$\{G_F, \alpha_s, \Gamma_W, \Gamma_Z, \alpha_{\text{em}}, m_W, m_Z, m_f, V_{ij}, t_\beta, m_{12}^2, v_S, \alpha_i, m_{H_i}, m_A, m_{H^\pm}\} . \quad (2.37)$$

For completeness, we want to mention that the three tadpole parameters T_1 , T_2 and T_S are formally independent input values, as well. However, as we explain in the upcoming Sec. 2.2.1, these parameters are either zero after renormalization or they are not present in the theory in the first place within an alternative treatment of the minimum conditions of the potential. In both cases, the parameters are effectively required to vanish and hence, we do not include them in the set of independent input parameters.

2.2 Renormalization

The ultraviolet (UV) divergences appearing in the one-loop corrections to the decay widths calculated in this work require the renormalization of the relevant parameters entering the decay widths at tree level. Apart from an extended CP-even scalar sector, the N2HDM is equivalent to the 2HDM at tree level so that the one-loop renormalization of the N2HDM can be performed analogously to the 2HDM. In the following, we only present the renormalization of the extended CP-even scalar sector of the N2HDM, *i.e.* we describe only the changes of the renormalization with respect to the 2HDM. For a thorough discussion of the renormalization of the N2HDM in general, we refer to [10] and for the definition of the 2HDM-like counterterms (CTs) as they are incorporated into `ewN2HDECAY`, we refer to [4, 11–13].³

Most of the N2HDM input parameters given in Eq. (2.37) are renormalized with on-shell (OS) conditions. For the physical masses, this leads to their counterterms being defined as the real parts of the poles of the propagators of the corresponding fields. All physical fields are equivalently renormalized in an OS approach, *i.e.* we demand that on the mass shell, no mixing of fields with the same quantum numbers takes place. The residues of the propagators associated to the corresponding fields are normalized to unity so that the fields are properly normalized as well. The soft- \mathbb{Z}_2 -breaking parameter m_{12}^2 and the singlet vacuum expectation value v_S are renormalized via $\overline{\text{MS}}$ conditions. For the scalar mixing angles α_i ($i = 1, 2, 3$) and β , several different renormalization schemes are implemented.

2.2.1 Renormalization of the Tadpoles

As discussed *e.g.* in [11, 12], the proper renormalization of the ground state of the Higgs potential is crucial for defining gauge-independent counterterms for the scalar mixing angles. We have implemented two different procedures in `ewN2HDECAY` that we briefly introduce⁴ before presenting the related renormalization conditions.

At NLO the tadpole terms are replaced by

$$T_i \longrightarrow T_i + \delta T_i \quad (i = 1, 2, S) , \quad (2.38)$$

³The renormalization of the 2HDM is also discussed in detail in [14, 15] and in [17], the renormalization of mixing angles in general and the application to the 2HDM in particular is discussed. In [18, 29], the gauge-independent renormalization of multi-Higgs models is described.

⁴For further information, see [4, 10–12] where they have been discussed in detail.

$$i\delta T_{H_i} = iT_{H_i}^{\text{loop}} = \left(\begin{array}{c} \bullet \\ | \\ H_i \end{array} \right) \quad (i = 1, 2, 3) . \quad (2.39)$$
$$\delta T_1 = \sum_{j=1}^3 R_{j1} \delta T_{H_j} \quad (2.40)$$

$$\delta T_2 = \sum_{j=1}^3 R_{j2} \delta T_{H_j} \quad (2.41)$$

$$\delta T_S = \sum_{j=1}^3 R_{j3} \delta T_{H_j} . \quad (2.42)$$

Renormalization of the tadpoles (standard scheme)

$$\delta T_{H_i H_j} = R_{i1} R_{j1} \frac{\delta T_1}{v_1} + R_{i2} R_{j2} \frac{\delta T_2}{v_2} + R_{i3} R_{j3} \frac{\delta T_S}{v_S} \quad (i, j = 1, 2, 3) \quad (2.43)$$

$$\delta T_{G^0 G^0} = c_\beta \frac{\delta T_1}{v} + s_\beta \frac{\delta T_2}{v} \quad (2.44)$$

$$\delta T_{G^0 A} = -s_\beta \frac{\delta T_1}{v} + c_\beta \frac{\delta T_2}{v} \quad (2.45)$$

$$\delta T_{AA} = \frac{s_\beta^2}{c_\beta} \frac{\delta T_1}{v} + \frac{c_\beta^2}{s_\beta} \frac{\delta T_2}{v} \quad (2.46)$$

$$\delta T_{G^\pm G^\pm} = c_\beta \frac{\delta T_1}{v} + s_\beta \frac{\delta T_2}{v} \quad (2.47)$$

$$\delta T_{G^\pm H^\pm} = -s_\beta \frac{\delta T_1}{v} + c_\beta \frac{\delta T_2}{v} \quad (2.48)$$

$$\delta T_{H^\pm H^\pm} = \frac{s_\beta^2}{c_\beta} \frac{\delta T_1}{v} + \frac{c_\beta^2}{s_\beta} \frac{\delta T_2}{v} . \quad (2.49)$$

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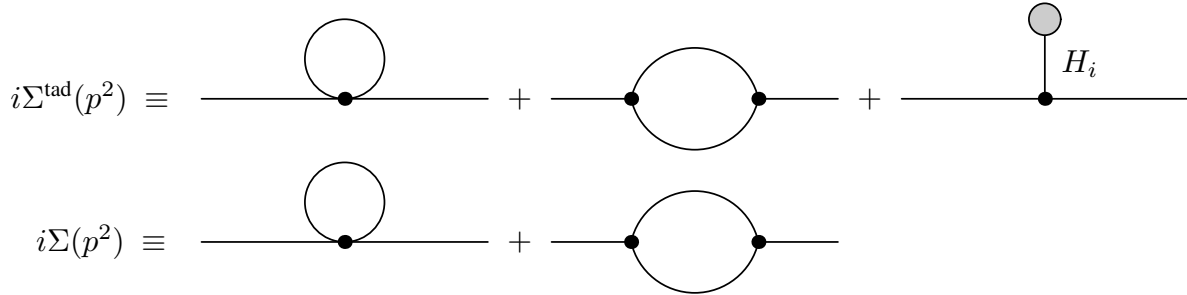


Figure 1: The topologies of the generic self-energies Σ and Σ^{tad} as functions of the squared external momentum p^2 differ by explicit one-particle-reducible tadpole contributions, mediated by the three CP-even Higgs bosons H_i ($i = 1, 2, 3$).

the scalar mass matrices, in which the tadpole terms explicitly appear and as a consequence, the tadpole CTs in the mass basis from Eqs. (2.43)-(2.49) appear in the off-diagonal scalar wave function renormalization counterterms (WFRCS) and mass CTs, *cf.* Sec. 2.2.3.

With the VEVs in the standard tadpole scheme being defined as the minimum of the loop-corrected gauge-dependent potential, they are consequently gauge-dependent as well, as are all other CTs defined through this minimum like *e.g.* the mass CTs of the Higgs and gauge bosons. While this is no problem as long as all gauge dependences in the calculation of the loop-corrected decay widths cancel, an improper renormalization condition for the mixing angle CTs in the N2HDM (and also the 2HDM) can lead to residual gauge dependences that spoil the overall gauge independence of the partial decay widths as discussed in more detail in Sec. 2.2.4.

In the framework of the *alternative (FJ) tadpole scheme*, based on the work by J. Fleischer and F. Jegerlehner in the SM, *cf.* Ref. [32], and applied to the 2HDM for the first time in Refs. [11, 12], the so-called *proper* VEVs are defined as the true ground state of the Higgs potential, *i.e.* as the renormalized all-order VEVs of the Higgs fields. In this alternative framework, the VEVs are defined through the gauge-independent tree-level potential instead of the loop-corrected gauge-dependent one so that they become manifestly gauge-independent quantities and thereby also the mass (matrix) CTs become manifestly gauge-independent. Being the fundamental quantities in the alternative tadpole schemes, the VEVs - instead of the tadpole terms - are shifted as

$$v_i \rightarrow v_i + \delta v_i, \quad (i = 1, 2, S) \quad (2.50)$$

where the CTs δv_i ($i = 1, 2, S$) are fixed by demanding that the renormalized VEVs represent the proper tree-level minima of the Higgs potential to all orders. This renormalization condition connects the CTs of the VEVs with explicit tadpole diagrams as follows:

$$\begin{pmatrix} \delta v_1 \\ \delta v_2 \\ \delta v_S \end{pmatrix} = R^T \begin{pmatrix} \frac{T_{H_1}^{\text{loop}}}{m_{H_1}^2} \\ \frac{T_{H_2}^{\text{loop}}}{m_{H_2}^2} \\ \frac{T_{H_3}^{\text{loop}}}{m_{H_3}^2} \end{pmatrix}. \quad (2.51)$$

The renormalization of the minimum of the potential in the alternative tadpole scheme implies a shift of the VEVs by additional tadpole contributions. These have to be considered

everywhere in the N2HDM where the three VEVs v_1 , v_2 and v_S appear. Consequently, all self-energies Σ^{tad} used for the definition of the CTs acquire additional one-particle-reducible tadpole contributions compared to the usual one-particle-irreducible self-energies Σ , *cf.* Fig. 1. In the calculation of the one-loop vertex corrections the tadpole diagrams have to be taken into account as well, so that the alternative tadpole scheme is characterized by the following conditions:

Renormalization of the tadpoles (alternative FJ scheme)

$$\delta T_{ij} = 0 , \quad (2.52)$$

$$\Sigma(p^2) \rightarrow \Sigma^{\text{tad}}(p^2) , \quad (2.53)$$

Tadpole diagrams have to be considered in the vertex corrections.

2.2.2 Renormalization of the Gauge Sector

The gauge sector of the N2HDM does not differ from that of the 2HDM. We implement the same renormalization conditions of the gauge sector as presented in [4] for the 2HDM. Formally, all CTs are the same as stated there, they only differ in the particle content of the two-point functions used for the definition of the CTs of the N2HDM due to the extended scalar sector. We therefore do not state them explicitly here and refer to [4] for details.

2.2.3 Renormalization of the Scalar Sector

The fields and masses of the CP-even scalar particles are shifted at one-loop level as

$$m_{H_i}^2 \rightarrow m_{H_i}^2 + \delta m_{H_i}^2 \quad (i = 1, 2, 3) , \quad (2.54)$$

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 + \frac{\delta Z_{H_1 H_1}}{2} & \frac{\delta Z_{H_1 H_2}}{2} & \frac{\delta Z_{H_1 H_3}}{2} \\ \frac{\delta Z_{H_2 H_1}}{2} & 1 + \frac{\delta Z_{H_2 H_2}}{2} & \frac{\delta Z_{H_2 H_3}}{2} \\ \frac{\delta Z_{H_3 H_1}}{2} & \frac{\delta Z_{H_3 H_2}}{2} & 1 + \frac{\delta Z_{H_3 H_3}}{2} \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} . \quad (2.55)$$

All scalar fields are renormalized through OS conditions implying the following CT and WFRC definitions,

Renormalization of the scalar sector (standard scheme)

$$\delta Z_{H_i H_j} = \frac{2}{m_{H_i}^2 - m_{H_j}^2} \text{Re} \left[\Sigma_{H_i H_j}(m_{H_j}^2) - \delta T_{H_i H_j} \right] \quad (i \neq j) , \quad (2.56)$$

$$\delta m_{H_i}^2 = \text{Re} \left[\Sigma_{H_i H_i}(m_{H_i}^2) - \delta T_{H_i H_i} \right] , \quad (2.57)$$

Renormalization of the scalar sector (alternative FJ scheme)

$$\delta Z_{H_i H_j} = \frac{2}{m_{H_i}^2 - m_{H_j}^2} \text{Re} \left[\Sigma_{H_i H_j}^{\text{tad}}(m_{H_j}^2) \right] \quad (i \neq j) , \quad (2.58)$$

$$\delta m_{H_i}^2 = \text{Re} \left[\Sigma_{H_i H_i}^{\text{tad}}(m_{H_i}^2) \right] , \quad (2.59)$$

Renormalization of the scalar sector (standard and alternative FJ scheme)
$\delta Z_{H_i H_i} = -\text{Re} \left[\frac{\partial \Sigma_{H_i H_i}(p^2)}{\partial p^2} \right]_{p^2=m_{H_i}^2}, \quad (2.60)$

where the tadpole CTs in the standard scheme are given in Eq. (2.43). The renormalization of the other scalar particles of the N2HDM, namely of the CP-odd and charged Higgs bosons A and H^\pm and the Goldstone bosons G^0 and G^\pm , is equivalent to their renormalization within the 2HDM and the CTs are implemented as presented in [4].

2.2.4 Renormalization of the Scalar Mixing Angles

The bare mixing angles α_i ($i = 1, 2, 3$) and β are promoted to one-loop order by introducing their CTs $\delta\alpha_i$ ($i = 1, 2, 3$) and $\delta\beta$ according to

$$\alpha_i \rightarrow \alpha_i + \delta\alpha_i \quad (i = 1, 2, 3) \quad (2.61)$$

$$\beta \rightarrow \beta + \delta\beta, \quad (2.62)$$

where the mixing angles on the right-hand side are the renormalized ones. In the following, we briefly describe the different renormalization schemes for the scalar mixing angles that are implemented in **ewN2HDECAY**. For a detailed derivation and description of these schemes, we refer to [10].

$\overline{\text{MS}}$ scheme. It has been shown in Refs. [11, 33] for the 2HDM that the one-loop corrected partial decay widths can become very large when the mixing angles are renormalized in the $\overline{\text{MS}}$ scheme and while this can also be expected in the N2HDM, we nevertheless implemented the $\overline{\text{MS}}$ scheme for the scalar mixing angles as a reference scheme. By imposing $\overline{\text{MS}}$ conditions, only UV-divergent parts are assigned to the four mixing angle CTs, but no finite parts $\delta\alpha_i|_{\text{fin}}$ ($i = 1, 2, 3$) and $\delta\beta|_{\text{fin}}$. After having checked explicitly for UV finiteness of all partial decay widths, the mixing angle CTs in the $\overline{\text{MS}}$ scheme are implemented in **ewN2HDECAY** by effectively setting them to zero:

Renormalization of $\delta\alpha_i$ and $\delta\beta$: $\overline{\text{MS}}$ scheme (both schemes)
$\delta\alpha_i _{\text{fin}} = 0 \quad (i = 1, 2, 3) \quad (2.63)$
$\delta\beta _{\text{fin}} = 0 \quad (2.64)$

As a consequence of the $\overline{\text{MS}}$ renormalization, the CTs $\delta\alpha_i$ and $\delta\beta$ become a function of the renormalization scale μ_R . This scale at which the mixing angles and their CTs are given has to be specified explicitly by the user in the input file of **ewN2HDECAY**. The one-loop corrected partial decay widths that contain these $\overline{\text{MS}}$ CTs also depend on the scale μ_R . Moreover, the one-loop partial decay widths depend on another scale μ_{out} at which the loop integrals are evaluated. The latter scale should be chosen appropriately to avoid the appearance of large logarithms in the partial decay widths. In case that the two scales μ_R and μ_{out} are chosen to be different, the parameter conversion automatically converts the scalar mixing angles from the scale μ_R to

μ_{out} , as further described in Sec. 2.5. For this conversion, the UV-divergent terms of the mixing angle CTs are required, which were extracted and implemented analytically by isolating the UV-divergent pieces of the mixing angle CTs as defined in an arbitrary other renormalization scheme.

Adapted KOSY scheme. For the renormalization of the scalar mixing angles in the 2HDM, the KOSY scheme (denoted by the authors' initials) was proposed in Ref. [30]. It can be directly adapted to the renormalization of the four mixing angles in the N2HDM as detailed in Ref. [10]. The adapted KOSY scheme connects the definition of the scalar mixing angle CTs to the off-diagonal scalar WFRCs by temporarily switching from the mass basis to the interaction basis before promoting the mixing angles to one-loop order. While the KOSY scheme not only leads to gauge-dependent mixing angle CTs but also to residual gauge dependences in the partial decay widths, we implement this scheme for comparative studies with the other schemes only. Due to the residual gauge dependences, we do not recommend it for actual use in phenomenological analyses, however. The CTs in the adapted KOSY scheme are given by

Renormalization of $\delta\alpha_i$ and $\delta\beta$: adapted KOSY scheme (standard scheme)

$$\delta\alpha_1 = \frac{c_{\alpha_3} (\text{Re} [\Sigma_{H_1 H_2}(m_{H_1}^2) + \Sigma_{H_1 H_2}(m_{H_2}^2)] - 2\delta T_{H_1 H_2})}{2c_{\alpha_2}(m_{H_1}^2 - m_{H_2}^2)} \quad (2.65)$$

$$\begin{aligned} & - \frac{s_{\alpha_3} (\text{Re} [\Sigma_{H_1 H_3}(m_{H_1}^2) + \Sigma_{H_1 H_3}(m_{H_3}^2)] - 2\delta T_{H_1 H_3})}{2c_{\alpha_2}(m_{H_1}^2 - m_{H_3}^2)} \\ \delta\alpha_2 = & \frac{s_{\alpha_3} (\text{Re} [\Sigma_{H_1 H_2}(m_{H_1}^2) + \Sigma_{H_1 H_2}(m_{H_2}^2)] - 2\delta T_{H_1 H_2})}{2(m_{H_1}^2 - m_{H_2}^2)} \quad (2.66) \\ & + \frac{c_{\alpha_3} (\text{Re} [\Sigma_{H_1 H_3}(m_{H_1}^2) + \Sigma_{H_1 H_3}(m_{H_3}^2)] - 2\delta T_{H_1 H_3})}{2(m_{H_1}^2 - m_{H_3}^2)} \end{aligned}$$

$$\begin{aligned} \delta\alpha_3 = & \frac{\text{Re} [\Sigma_{H_2 H_3}(m_{H_2}^2) + \Sigma_{H_2 H_3}(m_{H_3}^2)] - 2\delta T_{H_2 H_3}}{2(m_{H_2}^2 - m_{H_3}^2)} \quad (2.67) \\ & - \frac{s_{\alpha_2} c_{\alpha_3} (\text{Re} [\Sigma_{H_1 H_2}(m_{H_1}^2) + \Sigma_{H_1 H_2}(m_{H_2}^2)] - 2\delta T_{H_1 H_2})}{2c_{\alpha_2}(m_{H_1}^2 - m_{H_2}^2)} \\ & + \frac{s_{\alpha_2} s_{\alpha_3} (\text{Re} [\Sigma_{H_1 H_3}(m_{H_1}^2) + \Sigma_{H_1 H_3}(m_{H_3}^2)] - 2\delta T_{H_1 H_3})}{2c_{\alpha_2}(m_{H_1}^2 - m_{H_3}^2)} \end{aligned}$$

$$\delta\beta^o = - \frac{\text{Re} [\Sigma_{G^0 A}(m_A^2) + \Sigma_{G^0 A}(0)] - 2\delta T_{G^0 A}}{2m_A^2} \quad (2.68)$$

$$\delta\beta^c = - \frac{\text{Re} [\Sigma_{G^\pm H^\pm}(m_{H^\pm}^2) + \Sigma_{G^\pm H^\pm}(0)] - 2\delta T_{G^\pm H^\pm}}{2m_{H^\pm}^2} \quad (2.69)$$

Renormalization of $\delta\alpha_i$ and $\delta\beta$: adapted KOSY scheme (alternative FJ scheme)

$$\delta\alpha_1 = \frac{c_{\alpha_3} (\text{Re} [\Sigma_{H_1 H_2}^{\text{tad}}(m_{H_1}^2) + \Sigma_{H_1 H_2}^{\text{tad}}(m_{H_2}^2)])}{2c_{\alpha_2}(m_{H_1}^2 - m_{H_2}^2)} - \frac{s_{\alpha_3} (\text{Re} [\Sigma_{H_1 H_3}^{\text{tad}}(m_{H_1}^2) + \Sigma_{H_1 H_3}^{\text{tad}}(m_{H_3}^2)])}{2c_{\alpha_2}(m_{H_1}^2 - m_{H_3}^2)} \quad (2.70)$$

$$\delta\alpha_2 = \frac{s_{\alpha_3} (\text{Re} [\Sigma_{H_1 H_2}^{\text{tad}}(m_{H_1}^2) + \Sigma_{H_1 H_2}^{\text{tad}}(m_{H_2}^2)])}{2(m_{H_1}^2 - m_{H_2}^2)} + \frac{c_{\alpha_3} (\text{Re} [\Sigma_{H_1 H_3}^{\text{tad}}(m_{H_1}^2) + \Sigma_{H_1 H_3}^{\text{tad}}(m_{H_3}^2)])}{2(m_{H_1}^2 - m_{H_3}^2)} \quad (2.71)$$

$$\begin{aligned} \delta\alpha_3 = & \frac{\text{Re} [\Sigma_{H_2 H_3}^{\text{tad}}(m_{H_2}^2) + \Sigma_{H_2 H_3}^{\text{tad}}(m_{H_3}^2)]}{2(m_{H_2}^2 - m_{H_3}^2)} - \frac{s_{\alpha_2} c_{\alpha_3} (\text{Re} [\Sigma_{H_1 H_2}^{\text{tad}}(m_{H_1}^2) + \Sigma_{H_1 H_2}^{\text{tad}}(m_{H_2}^2)])}{2c_{\alpha_2}(m_{H_1}^2 - m_{H_2}^2)} \\ & + \frac{s_{\alpha_2} s_{\alpha_3} (\text{Re} [\Sigma_{H_1 H_3}^{\text{tad}}(m_{H_1}^2) + \Sigma_{H_1 H_3}^{\text{tad}}(m_{H_3}^2)])}{2c_{\alpha_2}(m_{H_1}^2 - m_{H_3}^2)} \end{aligned} \quad (2.72)$$

$$\delta\beta^o = -\frac{\text{Re} [\Sigma_{G^0 A}^{\text{tad}}(m_A^2) + \Sigma_{G^0 A}^{\text{tad}}(0)]}{2m_A^2} \quad (2.73)$$

$$\delta\beta^c = -\frac{\text{Re} [\Sigma_{G^\pm H^\pm}^{\text{tad}}(m_{H^\pm}^2) + \Sigma_{G^\pm H^\pm}^{\text{tad}}(0)]}{2m_{H^\pm}^2} \quad (2.74)$$

Like in 2HDECAY, *cf.* [4], we implemented two versions of the KOSY scheme in `ewN2HDECAY` that differ with respect to the WFRs through which the CT $\delta\beta$ is defined, namely $\delta\beta^o$ and $\delta\beta^c$ which define the CT through the WFRs of the CP-odd and the charged scalar sector, respectively.

p_* -pinched scheme. The (adapted) KOSY scheme can be modified such that a gauge-parameter-independent definition of the mixing angle CTs is achieved. This approach was suggested in Refs. [11,12] for the 2HDM and in Ref. [10] for the N2HDM. The derivation of the mixing angle CTs is analogous to the (adapted) KOSY scheme, but instead of defining them over the usual off-diagonal WFRs, the self-energies are replaced by the pinched self-energies which are derived by means of the pinch technique (PT), *cf.* Refs. [34–41]. For consistency and the cancellation of all gauge dependences, the alternative tadpole scheme is necessarily required for this renormalization scheme of the mixing angle CTs. The pinched scalar self-energies are equivalent to the self-energies $\Sigma^{\text{tad}}(p^2)$ in the alternative tadpole scheme up to additional self-energy-like contributions $\Sigma^{\text{add}}(p^2)$. In the p_* -pinched scheme, adapted from Ref. [42] in the Minimal Supersymmetric Extension of the SM (MSSM), the pinched self-energies are evaluated at the scale

$$p_{*,ij}^2 \equiv \frac{m_i^2 + m_j^2}{2}. \quad (2.75)$$

At this scale, the additional self-energy-like contributions Σ^{add} vanish. The scalar mixing angle CTs are then given as follows:

Renormalization of $\delta\alpha_i$ and $\delta\beta$: p_* -pinched scheme (alternative FJ scheme)

$$\delta\alpha_1 = \frac{c_{\alpha_3} \text{Re} [\Sigma_{H_1 H_2}^{\text{tad}}(p_{*,12}^2)]_{\xi=1}}{c_{\alpha_2}(m_{H_1}^2 - m_{H_2}^2)} - \frac{s_{\alpha_3} \text{Re} [\Sigma_{H_1 H_3}^{\text{tad}}(p_{*,13}^2)]_{\xi=1}}{c_{\alpha_2}(m_{H_1}^2 - m_{H_3}^2)} \quad (2.76)$$

$$\delta\alpha_2 = \frac{s_{\alpha_3} \text{Re} [\Sigma_{H_1 H_2}^{\text{tad}}(p_{*,12}^2)]_{\xi=1}}{m_{H_1}^2 - m_{H_2}^2} + \frac{c_{\alpha_3} \text{Re} [\Sigma_{H_1 H_3}^{\text{tad}}(p_{*,13}^2)]_{\xi=1}}{m_{H_1}^2 - m_{H_3}^2} \quad (2.77)$$

$$\delta\alpha_3 = \frac{\text{Re} [\Sigma_{H_2 H_3}^{\text{tad}}(p_{*,23}^2)]_{\xi=1}}{m_{H_2}^2 - m_{H_3}^2} + \frac{s_{\alpha_2} s_{\alpha_3} \text{Re} [\Sigma_{H_1 H_3}^{\text{tad}}(p_{*,13}^2)]_{\xi=1}}{c_{\alpha_2}(m_{H_1}^2 - m_{H_3}^2)} \quad (2.78)$$

$$- \frac{s_{\alpha_2} c_{\alpha_3} \text{Re} [\Sigma_{H_1 H_2}^{\text{tad}}(p_{*,12}^2)]_{\xi=1}}{c_{\alpha_2}(m_{H_1}^2 - m_{H_2}^2)}$$

$$\delta\beta^o = -\frac{1}{m_A^2} \text{Re} \left[\Sigma_{G^0 A}^{\text{tad}} \left(\frac{m_A^2}{2} \right) \right]_{\xi=1} \quad (2.79)$$

$$\delta\beta^c = -\frac{1}{m_{H^\pm}^2} \text{Re} \left[\Sigma_{G^\pm H^\pm}^{\text{tad}} \left(\frac{m_{H^\pm}^2}{2} \right) \right]_{\xi=1} \quad (2.80)$$

As for the adapted KOSY scheme, we implemented two different variations of the p_* -pinched scheme that differ in the definition of the CT of β . The index ' $\xi = 1$ ' means that the self-energies are evaluated in the Feynman gauge.

OS-pinched scheme. In the OS-pinched scheme, the pinched scalar self-energies are evaluated at the OS-inspired mass scale of the corresponding scalar particle. In this case, the additional UV-divergent⁵ self-energy-like contributions Σ^{add} do not vanish. They were derived for the N2HDM in Ref. [4] and read

$$\Sigma_{H_i H_j}^{\text{add}}(p^2) = -\frac{\alpha_{\text{em}} m_Z^2}{8\pi m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right)} \left(p^2 - \frac{m_{H_i}^2 + m_{H_j}^2}{2} \right) \quad (2.81)$$

$$\cdot \left\{ \mathcal{O}_{H_i H_j}^{(1)} B_0(p^2; m_Z^2, m_A^2) + \mathcal{O}_{H_i H_j}^{(2)} B_0(p^2; m_Z^2, m_Z^2) \right. \\ \left. + 2 \frac{m_W^2}{m_Z^2} \left[\mathcal{O}_{H_i H_j}^{(1)} B_0(p^2; m_W^2, m_{H^\pm}^2) + \mathcal{O}_{H_i H_j}^{(2)} B_0(p^2; m_W^2, m_W^2) \right] \right\}$$

$$\Sigma_{G^0 A}^{\text{add}}(p^2) = -\frac{\alpha_{\text{em}} m_Z^2}{8\pi m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right)} \left(p^2 - \frac{m_A^2}{2} \right) \sum_{k=1}^3 \mathcal{O}_{H_k H_k}^{(3)} B_0(p^2; m_Z^2, m_{H_k}^2) \quad (2.82)$$

$$\Sigma_{G^\pm H^\pm}^{\text{add}}(p^2) = -\frac{\alpha_{\text{em}}}{4\pi \left(1 - \frac{m_W^2}{m_Z^2}\right)} \left(p^2 - \frac{m_{H^\pm}^2}{2} \right) \sum_{k=1}^3 \mathcal{O}_{H_k H_k}^{(3)} B_0(p^2; m_Z^2, m_{H_k}^2), \quad (2.83)$$

where $i, j = 1, 2, 3$ and $\mathcal{O}_{H_i H_j}^{(x)}$ ($x = 1, \dots, 4$) is a shorthand notation for the following combinations

⁵While the additional self-energy-like contributions Σ^{add} are all separately UV-divergent, they appear only in UV-finite combinations in the definition of the mixing angle CTs $\delta\alpha_i$ ($i = 1, 2, 3$), cf. Ref. [4].

	$\kappa_{H_i VV}$	$\tilde{\kappa}_{H_i V H}$
H_1	$c_{\alpha_2} c_{\beta-\alpha_1}$	$-c_{\alpha_2} s_{\beta-\alpha_1}$
H_2	$-s_{\alpha_2} s_{\alpha_3} c_{\beta-\alpha_1} + c_{\alpha_3} s_{\beta-\alpha_1}$	$s_{\alpha_2} s_{\alpha_3} s_{\beta-\alpha_1} + c_{\alpha_3} c_{\beta-\alpha_1}$
H_3	$-s_{\alpha_2} c_{\alpha_3} c_{\beta-\alpha_1} - s_{\alpha_3} s_{\beta-\alpha_1}$	$s_{\alpha_2} c_{\alpha_3} s_{\beta-\alpha_1} - s_{\alpha_3} c_{\beta-\alpha_1}$

Table 3: Coupling constants for the Higgs-gauge sector, as defined in Ref. [10], where $\kappa_{H_i VV}$ denotes the coupling of a CP-even Higgs boson H_i ($i = 1, 2, 3$) to a pair of gauge bosons $V = W^\pm, Z$ and $\tilde{\kappa}_{H_i V H}$ denotes the coupling of a CP-even Higgs boson to a gauge boson and an additional Higgs boson $H = A, H^\pm$.

of coupling constants between the Higgs and gauge sectors as defined in Tab. 3:

$$\mathcal{O}_{H_i H_j}^{(1)} = \tilde{\kappa}_{H_i V H} \cdot \tilde{\kappa}_{H_j V H} \quad (2.84)$$

$$\mathcal{O}_{H_i H_j}^{(2)} = \kappa_{H_i V V} \cdot \kappa_{H_j V V} \quad (2.85)$$

$$\mathcal{O}_{H_i H_j}^{(3)} = \kappa_{H_i V V} \cdot \tilde{\kappa}_{H_j V H} . \quad (2.86)$$

The mixing angle CTs in the OS-pinched scheme are given by:

Renormalization of $\delta\alpha_i$ and $\delta\beta$: OS-pinched scheme (alternative FJ scheme)

$$\delta\alpha_1 = \frac{c_{\alpha_3} \left(\text{Re} [\Sigma_{H_1 H_2}^{\text{tad}}(m_{H_1}^2) + \Sigma_{H_1 H_2}^{\text{tad}}(m_{H_2}^2)]_{\xi=1} + \Sigma_{H_1 H_2}^{\text{add}}(m_{H_1}^2) + \Sigma_{H_1 H_2}^{\text{add}}(m_{H_2}^2) \right)}{2c_{\alpha_2}(m_{H_1}^2 - m_{H_2}^2)} \quad (2.87)$$

$$- \frac{s_{\alpha_3} \left(\text{Re} [\Sigma_{H_1 H_3}^{\text{tad}}(m_{H_1}^2) + \Sigma_{H_1 H_3}^{\text{tad}}(m_{H_3}^2)]_{\xi=1} + \Sigma_{H_1 H_3}^{\text{add}}(m_{H_1}^2) + \Sigma_{H_1 H_3}^{\text{add}}(m_{H_3}^2) \right)}{2c_{\alpha_2}(m_{H_1}^2 - m_{H_3}^2)}$$

$$\delta\alpha_2 = \frac{s_{\alpha_3} \left(\text{Re} [\Sigma_{H_1 H_2}^{\text{tad}}(m_{H_1}^2) + \Sigma_{H_1 H_2}^{\text{tad}}(m_{H_2}^2)]_{\xi=1} + \Sigma_{H_1 H_2}^{\text{add}}(m_{H_1}^2) + \Sigma_{H_1 H_2}^{\text{add}}(m_{H_2}^2) \right)}{2(m_{H_1}^2 - m_{H_2}^2)} \quad (2.88)$$

$$+ \frac{c_{\alpha_3} \left(\text{Re} [\Sigma_{H_1 H_3}^{\text{tad}}(m_{H_1}^2) + \Sigma_{H_1 H_3}^{\text{tad}}(m_{H_3}^2)]_{\xi=1} + \Sigma_{H_1 H_3}^{\text{add}}(m_{H_1}^2) + \Sigma_{H_1 H_3}^{\text{add}}(m_{H_3}^2) \right)}{2(m_{H_1}^2 - m_{H_3}^2)}$$

$$\delta\alpha_3 = \frac{\text{Re} [\Sigma_{H_2 H_3}^{\text{tad}}(m_{H_2}^2) + \Sigma_{H_2 H_3}^{\text{tad}}(m_{H_3}^2)]_{\xi=1} + \Sigma_{H_2 H_3}^{\text{add}}(m_{H_2}^2) + \Sigma_{H_2 H_3}^{\text{add}}(m_{H_3}^2)}{2(m_{H_2}^2 - m_{H_3}^2)} \quad (2.89)$$

$$- \frac{s_{\alpha_2} c_{\alpha_3} \left(\text{Re} [\Sigma_{H_1 H_2}^{\text{tad}}(m_{H_1}^2) + \Sigma_{H_1 H_2}^{\text{tad}}(m_{H_2}^2)]_{\xi=1} + \Sigma_{H_1 H_2}^{\text{add}}(m_{H_1}^2) + \Sigma_{H_1 H_2}^{\text{add}}(m_{H_2}^2) \right)}{2c_{\alpha_2}(m_{H_1}^2 - m_{H_2}^2)}$$

$$+ \frac{s_{\alpha_2} s_{\alpha_3} \left(\text{Re} [\Sigma_{H_1 H_3}^{\text{tad}}(m_{H_1}^2) + \Sigma_{H_1 H_3}^{\text{tad}}(m_{H_3}^2)]_{\xi=1} + \Sigma_{H_1 H_3}^{\text{add}}(m_{H_1}^2) + \Sigma_{H_1 H_3}^{\text{add}}(m_{H_3}^2) \right)}{2c_{\alpha_2}(m_{H_1}^2 - m_{H_3}^2)}$$

$$\delta\beta^o = - \frac{\text{Re} [\Sigma_{G^0 A}^{\text{tad}}(m_A^2) + \Sigma_{G^0 A}^{\text{tad}}(0)]_{\xi=1} + \Sigma_{G^0 A}^{\text{add}}(m_A^2) + \Sigma_{G^0 A}^{\text{add}}(0)}{2m_A^2} \quad (2.90)$$

$$\delta\beta^c = - \frac{\text{Re} [\Sigma_{G^\pm H^\pm}^{\text{tad}}(m_{H^\pm}^2) + \Sigma_{G^\pm H^\pm}^{\text{tad}}(0)]_{\xi=1} + \Sigma_{G^\pm H^\pm}^{\text{add}}(m_{H^\pm}^2) + \Sigma_{G^\pm H^\pm}^{\text{add}}(0)}{2m_{H^\pm}^2} \quad (2.91)$$

2.2.5 Renormalization of the Fermion Sector

The renormalization of the fermion sector in the N2HDM is performed as in the 2HDM. All CTs of the fermion masses, the CKM mixing matrix elements and the fermion WFRCs are implemented with definitions analogous to those presented in [4]. The only difference with respect to the 2HDM are the different Yukawa couplings due to the extended scalar sector of the N2HDM. The CTs of the Yukawa coupling parameters defined in Tab. 2 are given by the following relations for $f = l, u, d$ which hold independently of the chosen N2HDM type:

$$\delta Y_1^f = c_{\alpha_2} \left(c_{\alpha_3} Y_2^f - s_{\alpha_3} Y_3^f \right) \delta \alpha_1 - t_{\alpha_2} Y_1^f \delta \alpha_2 - Y_1^f Y_4^f \delta \beta \quad (2.92)$$

$$\delta Y_2^f = \left(s_{\alpha_2} Y_3^f - c_{\alpha_2} c_{\alpha_3} Y_1^f \right) \delta \alpha_1 - s_{\alpha_3} Y_1^f \delta \alpha_2 + Y_3^f \delta \alpha_3 - Y_2^f Y_4^f \delta \beta \quad (2.93)$$

$$\delta Y_3^f = \left(c_{\alpha_2} s_{\alpha_3} Y_1^f - s_{\alpha_2} Y_2^f \right) \delta \alpha_1 - c_{\alpha_3} Y_1^f \delta \alpha_2 - Y_2^f \delta \alpha_3 - Y_3^f Y_4^f \delta \beta \quad (2.94)$$

$$\delta Y_4^f = - \left(1 + \left(Y_4^f \right)^2 \right) \delta \beta . \quad (2.95)$$

2.2.6 Renormalization of the Soft- \mathbb{Z}_2 -Breaking Parameter m_{12}^2

The soft- \mathbb{Z}_2 -breaking Parameter m_{12}^2 is promoted to NLO by introducing a CT δm_{12}^2 according to

$$m_{12}^2 \rightarrow m_{12}^2 + \delta m_{12}^2 . \quad (2.96)$$

One possibility to fix the CT for m_{12}^2 is to define it via a Higgs-to-Higgs decay, since the Higgs self-couplings contain the parameter at tree level. However, such a process-dependent definition of δm_{12}^2 has the drawback that the CT is defined not only via the genuine vertex corrections of the Higgs-to-Higgs decay, but additionally as a function of several other CTs of the N2HDM due to the intricate structure of the Higgs self-couplings. As a result, such a CT can introduce very large finite contributions which leads to NLO corrections that can become very large as well. This has already been observed in Ref. [13] in the context of the 2HDM. We therefore do not implement a process-dependent scheme for δm_{12}^2 in `ewN2HDECAY` but instead fix the CT through an $\overline{\text{MS}}$ condition, *i.e.* δm_{12}^2 only contains UV-divergent and some global finite parts parts proportional to

$$\Delta \equiv \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) + \ln \left(\frac{\mu^2}{\mu_R^2} \right) , \quad (2.97)$$

where γ_E denotes the Euler-Mascheroni constant, μ denotes the mass-dimensional 't Hooft scale which cancels in the calculation of decay amplitudes and the regulator ε is introduced in the framework of dimensional regularization, *cf.* Refs. [43–47]. In `ewN2HDECAY`, we extracted the UV divergence of δm_{12}^2 by calculating the one-loop amplitude of the decay $H_1 \rightarrow AA$, including the genuine vertex corrections of the process as well as all CTs apart from δm_{12}^2 , and by considering the residual UV divergence which is then assigned to δm_{12}^2 . This yields the following analytic expression of the CT,

Renormalization of m_{12}^2 (standard and alternative FJ scheme)

$$\delta m_{12}^2 = \frac{\alpha_{\text{em}} m_{12}^2}{16\pi m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right)} \left[\frac{8m_{12}^2}{s_{2\beta}} - 2m_{H^\pm}^2 - m_A^2 + \sum_{i=1}^3 R_{i1} R_{i2} m_{H_i}^2 - 3(2m_W^2 + m_Z^2) \right. \\ \left. + \sum_u 6m_u^2 Y_4^u \left(Y_4^u - \frac{1}{t_{2\beta}}\right) + \sum_d 6m_d^2 Y_4^d \left(Y_4^d - \frac{1}{t_{2\beta}}\right) + \sum_l 2m_l^2 Y_4^l \left(Y_4^l - \frac{1}{t_{2\beta}}\right) \right] \Delta \quad (2.98)$$

where the sums are performed over all up- and down-type quarks as well as over all charged leptons, respectively. Since m_{12}^2 is a genuine parameter of the N2HDM Higgs potential before EWSB, it is gauge-independent and the CT is invariant under a change of the tadpole renormalization so that δm_{12}^2 is the same in both tadpole schemes. Due to the $\overline{\text{MS}}$ renormalization of δm_{12}^2 , the CT explicitly depends on the renormalization scale μ_R whose value has to be specified by the user⁶. In case that the renormalization scale μ_{out} at which the one-loop partial decay widths are evaluated differs from the input renormalization scale μ_R , the parameter conversion routine, *cf.* Sec. 2.5, evolves the parameter m_{12}^2 from μ_R to μ_{out} , analogous to the $\overline{\text{MS}}$ renormalized scalar mixing angles.

2.2.7 Renormalization of the singlet VEV v_S

While the singlet VEV v_S is already accounted for by the proper renormalization of the minimum of the Higgs potential, *cf.* Sec. 2.2.1, it still receives an additional CT after the minimum conditions are imposed on the one-loop potential. This is in analogy to the doublet VEVs. In the framework of *e.g.* the alternative tadpole scheme, the doublet VEVs v_1 and v_2 acquire shifts according to Eq. (2.50). These shifts ensure that the VEVs are equivalent to the gauge-independent tree-level VEVs of the potential, so that

$$v^{\text{ren}}|_{\text{FJ}} = v^{\text{tree}} = \left. \frac{2m_W}{g} \right|_{\text{FJ}}^{\text{tree}} \quad (2.99)$$

holds. After the minimum conditions are applied through the VEV shifts, the tree-level parameters m_W and g still need to be renormalized, so that v effectively acquires an additional CT Δv in form of a combination of CTs for m_W^2 and g ,

$$\left. \frac{2m_W}{g} \right|_{\text{FJ}}^{\text{tree}} \rightarrow \left. \frac{2m_W}{g} \right|_{\text{FJ}}^{\text{ren}} + \underbrace{\frac{2m_W}{g} \left(\frac{\delta m_W^2}{2m_W^2} - \frac{\delta g}{g} \right)}_{\equiv \Delta v} \Big|_{\text{FJ}}. \quad (2.100)$$

Analogously, the singlet VEV v_S is shifted according to Eq. (2.50), ensuring that v_S is equivalent to the tree-level singlet VEV in the alternative tadpole scheme, and after the minimum conditions of the potential are applied at one-loop order, v_S acquires an additional CT Δv_S through

$$v_S|_{\text{FJ}}^{\text{tree}} \rightarrow v_S|_{\text{FJ}}^{\text{ren}} + \Delta v_S. \quad (2.101)$$

For more details about the appearance of these additional CTs, we refer to Ref. [10]. Transferring the general analysis on the renormalization of spontaneously broken gauge symmetries

⁶In **ewN2HDECAY**, all $\overline{\text{MS}}$ parameters are given at the same global scale μ_R .

presented in Ref. [48] in the framework of the standard tadpole scheme to the N2HDM yields the conclusion that Δv_S cannot contain UV divergences since the corresponding singlet field obeys a rigid invariance. Therefore, Δv_S contains at most finite parts in the standard tadpole scheme which can be fixed *e.g.* in a process-dependent renormalization scheme. In **ewN2HDECAY**, we implemented $\overline{\text{MS}}$ conditions for the singlet VEV CT in order to avoid potentially large finite contributions that would be introduced in the CT when fixing it through a Higgs-to-Higgs decay. The CT is implemented by setting its finite part $\Delta v_S|_{\text{fin}}$ to zero:

Renormalization of the tree-level v_S (standard and alternative FJ scheme)	
$\Delta v_S _{\text{fin}} = 0$	(2.102)

As for $\overline{\text{MS}}$ renormalized scalar mixing angles and m_{12}^2 , the value of v_S is converted from the input renormalization scale μ_R to the scale μ_{out} at which the decays are evaluated in case that both scales differ. For this conversion the UV-divergent parts of Δv_S are needed. In the standard tadpole scheme, these UV-divergent parts are identically zero due to the rigid invariance, and as a consequence, the parameter v_S is not converted and remains the same if the two scales are different⁷. In the alternative tadpole scheme, however, Δv_S contains additional UV divergences which have been extracted by calculating the remaining UV-divergent parts of the off-shell process $H_1 \rightarrow H_1 H_1$ to one-loop order when all CTs apart from Δv_S are fixed. Due to the lengthy analytic expression of the UV-divergent parts of Δv_S , we do not state them explicitly here.

2.3 Electroweak Decay Processes at LO and NLO

For the EW corrections, we only consider OS decay processes, *i.e.* the decays of all Higgs bosons with four-momentum p_1 into two particles X_1 and X_2 with four-momenta p_2 and p_3 , respectively, for which

$$p_1^2 \geq (p_2 + p_3)^2. \quad (2.103)$$

Note that we do not include here loop-induced tree-level decays, so that the EW one-loop corrections are calculated for the following processes ($i = 1, 2, 3$),

- $H_i/A \rightarrow f\bar{f}$ ($f = c, s, t, b, \mu, \tau$)
- $H_i \rightarrow VV$ ($V = W^\pm, Z$)
- $H_i \rightarrow VS$ ($VS = ZA, W^\pm H^\mp$)
- $H_i \rightarrow SS$ ($i = 1, 2, 3, S = A, H^\pm$)
- $H_i \rightarrow H_j H_k$ ($i, j, k = 1, 2, 3$ and $i > j, k$)
- $A \rightarrow VS$ ($VS = ZH_i, W^\pm H^\mp$)
- $H^\pm \rightarrow f\bar{f}'$ ($f\bar{f}' = u/c/t + \bar{b}, u/c/t + \bar{s}, c/t + \bar{d}, \nu_\tau \bar{\tau}, \nu_\mu \bar{\mu}$)
- $H^\pm \rightarrow VS$ ($VS = W^\pm H_i, W^\pm A$)

⁷This corresponds to the vanishing of the one-loop beta function of v_S in the standard tadpole scheme.

Like in `2HDECAY`, we do not consider decays containing first-generation fermions, *i.e.* the decays $H_i/A \rightarrow f\bar{f}$ ($i = 1, 2, 3$, $f = u, d, e$) and $H^+ \rightarrow f\bar{f}$ ($f\bar{f} = u\bar{d}, \nu_e e^+$) are neglected in `ewN2HDECAY`.

From a technical point of view, the calculation of the LO and NLO decay amplitudes in `ewN2HDECAY` is identical to their calculation in the 2HDM in `2HDECAY`. In the following, we only briefly mention the tools used for the calculation. For a more detailed explanation of the calculation, we refer to Ref. [4]. The Feynman amplitudes for the LO and NLO decays as well as for the tadpole diagrams and self-energies required for the definition of the CTs were generated with `FeynArts 3.9` [49]. The N2HDM model file necessary for the generation of the amplitudes was created with `SARAH 4.14.0` [50–54]. The simplification of the Dirac algebra, the Passarino-Veltman reduction and the analytic evaluation of the decay amplitudes as well as the CTs was performed with the help of the tool `FeynCalc 8.2.0` [55, 56]. The real corrections necessary for the cancellation of all infra-red (IR) divergences were implemented analytically by applying the general results given in Ref. [57] to the N2HDM. For the evaluation of the integrals involved in the real corrections, the analytic expressions of Ref. [25] were implemented. The numerical evaluation of the loop integrals is performed by linking `LoopTools 2.14` [58].

2.4 Link to N2HDECAY

For our new tool `ewN2HDECAY` we combine the EW one-loop corrections to the N2HDM Higgs decays with the Fortran code `N2HDECAY`. This code is based on an extension of the Fortran code `HDECAY` version 6.511 [22, 23] to the N2HDM [5, 8], which includes the state-of-the-art QCD corrections in the partial decay widths.⁸ Care has to be taken when combining the two codes in order to remain consistent at higher loop level. We commented on this in detail in [4] and summarise here only very briefly the main points.

In order to consistently combine (N2)HDECAY which uses $\{G_F, m_W, m_Z\}$ as independent input parameters with the EW-corrected decays based on the set $\{\alpha_{\text{em}}(m_Z), m_W, m_Z\}$ as independent input parameters, we choose a pragmatic solution where the N2HDECAY decay widths in terms of G_F are rescaled with G_F^{calc}/G_F , with G_F^{calc} being calculated through the tree-level relation Eq. (2.36) as a function of $\alpha_{\text{em}}(m_Z^2)$.⁹ In case the user does not choose to calculate the EW corrections no such rescaling is performed. Note also that N2HDECAY includes off-shell decays in certain final states. In `ewN2HDECAY` the EW and QCD corrections are combined such that N2HDECAY computes the off-shell decay widths, but the EW corrections are only added to OS decays. We furthermore assume that the QCD and EW corrections factorize. The relative QCD corrections δ^{QCD} are defined with respect to the LO width $\Gamma^{\text{N2HD,LO}}$ that is calculated by N2HDECAY and contains *e.g.* also running quark masses in order to improve the perturbative behaviour. The relative EW corrections δ^{EW} , however, are obtained by normalizing to the LO width with OS particle masses. The QCD and EW corrected decay width into a specific final state, $\Gamma^{\text{QCD\&EW}}$, is hence calculated as

$$\Gamma^{\text{QCD\&EW}} = \frac{G_F^{\text{calc}}}{G_F} \Gamma^{\text{N2HD,LO}} [1 + \delta^{\text{QCD}}] [1 + \delta^{\text{EW}}] \equiv \frac{G_F^{\text{calc}}}{G_F} \Gamma^{\text{N2HD,QCD}} [1 + \delta^{\text{EW}}]. \quad (2.104)$$

The formula includes the aforementioned rescaling factor G_F^{calc}/G_F .

⁸Details can be found in [22, 23].

⁹The proper conversion between the $\{G_F, m_W, m_Z\}$ and $\{\alpha_{\text{em}}(m_Z), m_W, m_Z\}$ scheme would require the inclusion of N2HDM higher-order corrections in the conversion formulae. We expect the differences with respect to our pragmatic approach to be small.

For each input file generically called `inputfilename.in` in the following¹⁰, the program `ewN2HDECAY` provides two separate output files which adopt the filename of the input file together with additional suffices as explained in the following. The file `inputfilename.BR.out` contains both the total widths and branching ratios as calculated by `N2HDECAY` without the EW corrections¹¹ and those including the EW corrections based on the formula Eq. (2.104). In the file `inputfilename.EW.out` the LO and EW-corrected NLO decay widths are given out. Note that the LO widths do not contain any running quark masses here in decays into quark pair final states. Furthermore, the widths are calculated in the $\{\alpha_{\text{em}}, m_W, m_Z\}$ scheme. The widths given out here are not useful for phenomenological analyses as they do not include any QCD corrections nor loop-induced or off-shell decays. They can be used, however, for the study on the importance of the EW corrections and an estimate of the remaining theoretical uncertainty due to missing higher-order corrections by changing the renormalization schemes.

We finally remark that for certain parameter choices the EW-corrected decay widths can become negative. This can be due to a small LO width, due to an artificial enhancement of the EW corrections because of a badly chosen renormalization scheme or due to parametrically enhanced EW corrections because of large involved couplings. In this case, the NLO corrections cannot be trusted of course and should be discarded. For further details and discussions, we refer to [4, 10, 12, 13].

2.5 Parameter Conversion

The one-loop corrected partial decay widths explicitly depend on the renormalization scale μ_{out} at which the integrals of the higher-order corrections are evaluated. In `ewN2HDECAY`, this scale can be chosen by the user to be either at a fixed global value or to be equal to the mass of the decaying particle. Additionally, all $\overline{\text{MS}}$ renormalized parameters introduce a dependence on the input renormalization scale μ_R at which these parameters are defined. This scale has to be given by the user in the input file. In case the two scales μ_R and μ_{out} differ, the $\overline{\text{MS}}$ parameters need to be converted from the input scale μ_R to the output scale μ_{out} , which is done by means of the formula

$$\varphi(\{\mu_{\text{out}}\}) \approx \varphi(\{\mu_R\}) + \ln\left(\frac{\mu_{\text{out}}^2}{\mu_R^2}\right) \delta\varphi^{\text{div}}(\{\varphi\}) \quad (2.105)$$

where φ and $\delta\varphi$ denote all $\overline{\text{MS}}$ parameters (m_{12}^2 , v_S and α_i ($i = 1, 2, 3$) and β , if the latter are defined in an $\overline{\text{MS}}$ scheme) and their respective CTs. The superscript 'div' indicates that only the UV-divergent part of the CT is taken into account.

Apart from the conversion of the $\overline{\text{MS}}$ parameters from one scale to another, an additional parameter conversion of the scalar mixing angles has to be performed if the renormalization scheme at which their input is given is different from the renormalization scheme with which the one-loop partial decay widths are evaluated. Since the 10 different renormalization schemes implemented in `ewN2HDECAY` differ only in their definition of the mixing angle CTs, the scalar mixing angles are the only parameters affected by the parameter conversion. The values of the mixing angles given in the reference scheme, *i.e.* φ_{ref} , and their values φ_i in another different renormalization scheme are connected to each other via the bare values of the mixing angles, which are independent of the renormalization scheme. Together with the corresponding CTs

¹⁰The input filename can be chosen arbitrarily by the user.

¹¹We remind the reader that they include loop-induced and off-shell decays as well as QCD corrections where applicable. They are furthermore rescaled by G_F^{calc}/G_F in case the flag for the EW corrections is turned on in the input file.

$\delta\varphi_{\text{ref}}$ and $\delta\varphi_i$ in the two different schemes, the conversion of the values is given by

$$\varphi_i(\{\mu_{\text{out}}\}) \approx \varphi_{\text{ref}}(\{\mu_R\}) + \delta\varphi_{\text{ref}}(\{\varphi_{\text{ref}}, \mu_R\}) - \delta\varphi_i(\{\varphi_{\text{ref}}, \mu_{\text{out}}\}) . \quad (2.106)$$

This linearized relation holds approximately up to higher-order terms, since the CT $\delta\varphi_i$ on the right-hand side is evaluated with the parameters φ given in the reference scheme instead of the other renormalization scheme in which φ_i is defined. We want to emphasize that Eq. (2.106) also contains a dependence on the scales μ_R and μ_{out} which is of importance if the scalar mixing angles are defined in an $\overline{\text{MS}}$ scheme.

3 Program Description

In this section, we describe the system requirements for `ewN2HDECAY` and provide a guide for installing and using the program. Moreover, we describe the format of the input and output files in detail.

3.1 System Requirements

The Python/FORTRAN program code `ewN2HDECAY` was developed under Windows 10 and openSUSE Leap 15.0. The following operating systems are supported:

- Windows 7 and Windows 10 (tested with Cygwin 2.10.0)
- Linux (tested with openSUSE Leap 15.0)
- macOS (tested with macOS Sierra 10.12)

The compilation of `ewN2HDECAY` under Windows requires an installed up-to-date version of Cygwin (together with the packages `cURL`, `find`, `gcc`, `g++` and `gfortran`). For the compilation of `ewN2HDECAY` the GNU C compilers `gcc` (tested with versions 6.4.0 and 7.3.1), `g++` and the FORTRAN compiler `gfortran` are needed. Additionally, an up-to-date version of either Python 2 or Python 3 are required (tested with versions 2.7.14 and 3.5.0).

3.2 License

`ewN2HDECAY` is released under the GNU General Public License (GPL) (`GNU GPL-3.0-or-later`). `ewN2HDECAY` is free software, which means that anyone can redistribute it and/or modify it under the terms of the GNU GPL as published by the Free Software Foundation, either version 3 of the License, or any later version. `ewN2HDECAY` is distributed without any warranty. A copy of the GNU GPL is included in the `LICENSE.md` file in the root directory of `ewN2HDECAY`.

3.3 Download

The latest version of the program package `ewN2HDECAY` can always be obtained from the url <https://github.com/marcel-krause/ewN2HDECAY>. The user can either clone the repository or download the whole program as a zip archive. The directory for the installation that is chosen by the user will in the following be referred to as `$ewN2HDECAY`. The main folder of `ewN2HDECAY` contains several subfolders:

BuildingBlocks Here the analytic EW one-loop corrections for all considered decays can be found, as well as the CTs and real corrections needed for the UV and IR finiteness of the decay widths.

Documentation Contains this documentation.

N2HDECAY This subfolder contains a modified version of N2HDECAY [5,22,23]. It computes the LO and (where applicable) QCD corrected decay widths as well as off-shell decay widths and the loop-induced decay widths into gluon and photon pair final states and into $Z\gamma$. N2HDECAY also computes the branching ratios.

Input Here, at least one or more input files which shall be used for the computation are stored. In Sec.3.5 the format of the input file is explained. In the Github repository, we provide the exemplary input file depicted in App. A in the folder **Input**.

Results In this subfolder the results of a successful run of **ewN2HDECAY** are stored as output files. They have the same name as the corresponding input files in the **Input** folder, but with the file extension **.in** replaced by **.out** and a suffix “_BR” and “_EW” for the branching ratios and EW partial decay widths, respectively. In the Github repository, we provide the exemplary output files which are printed in App. B in the folder **Results**.

The main folder **\$ewN2HDECAY** also contains several files:

ewN2HDECAY.py This is the main program file of **ewN2HDECAY**. It serves as a wrapper file that calls N2HDECAY for the conversion of the charm and bottom quark masses from the $\overline{\text{MS}}$ input values to the corresponding OS values and for the computation of the LO widths, QCD corrections, off-shell and loop-induced decays and the branching ratios. It also calls **electroweakCorrections** for the calculation of the EW one-loop corrections.

Changelog.md Documents all changes made in the **ewN2HDECAY** since version 1.0.0.

CommonFunctions.py A library of functions frequently used in the different files of the program **ewN2HDECAY**.

Config.py Main configuration file. In case **LoopTools** is not installed automatically by the installer of **ewN2HDECAY**, the paths to the **LoopTools** executables and libraries have to be set manually in this file.

constants.F90 Library for all constants used in **ewN2HDECAY**.

counterterms.F90 Here all fundamental CTs necessary for the EW one-loop renormalization of the Higgs boson decays are defined. These CTs require the analytic results saved in the **BuildingBlocks** subfolder.

electroweakCorrections.F90 Main file for the calculation of the EW one-loop corrections to the Higgs boson decays. The EW one-loop corrections to the decay widths are combined with the necessary CTs and real corrections for the EW contributions to the tree-level decay widths that are then combined with the QCD corrections in N2HDECAY.

getParameters.F90 Routine to read in the input values given by the user in the input files that are needed by **ewN2HDECAY**.

LICENSE.md Contains the full GNU General Public License (GNU GPL-3.0-or-later) agreement.

README.md Provides an overview over basic information about the program as well as a quick-start guide.

setup.py Main setup and installation file of **ewN2HDECAY**. For a guided installation, this file should be called after downloading the program.

3.4 Installation

For the installation of **ewN2HDECAY**, we recommend to use the automatic installation script **setup.py** which is part of the **ewN2HDECAY** repository and which guides the user through the installation. If **ewN2HDECAY** is installed under **Windows**, the user should first install **Cygwin** and check whether the path to the **Cygwin** executable in line 36 of the configuration file **\$ewN2HDECAY/Config.py** is correct and modify it, if necessary. In order to use the automatic installation script, the user opens a terminal, navigates to the main folder **\$ewN2HDECAY** and executes the following command:

```
python setup.py
```

First, the script asks whether the **LoopTools** version specified in line 37 of **\$2HDECAY/Config.py** should be downloaded and installed. By typing **y**, the specified version of **LoopTools** is automatically installed in a subdirectory of **ewN2HDECAY**. For a more detailed description of the installation of the program, we refer to [58]. The installation of **LoopTools** is optional if the user already has a current working version of the program on the system. The installation of **LoopTools** can then be skipped. However, in this case the lines 33-35 of the configuration file **\$2HDECAY/Config.py** need to be modified in order to specify the path to the **LoopTools** executables and libraries. Moreover, line 32 has to be set to

```
useRelativeLoopToolsPath = False
```

This step is crucial if **LoopTools** is not installed automatically with the install script, since otherwise the paths to the **LoopTools** libraries are incorrectly set in the makefile of **ewN2HDECAY** and the make process of the program will fail.

In the next step of the installation, the script asks the user whether the makefile and the file **electroweakCorrections.F90** should be created and whether the program should be compiled, which should be responded to by typing **y**. The script then executes the makefile and the program is compiled. The make process may take several minutes to finish. Finally, the script provides a convenient way to 'make clean' the installation, which is optional.

After the installation, the user can type

```
python ewN2HDECAY.py
```

into the terminal for a quick check whether the installation was successful. The exemplary input file provided in the **2HDECAY** repository is then used for the calculation of the partial decay widths and branching ratios and the exemplary output files provided in the **\$2HDECAY/Results** subdirectory should be reproduced. If the program terminates without printing an error to the terminal, the installation was successful.

Line	Input name	Allowed values and meaning
6	OMIT ELW2	0: electroweak corrections (N2HDM) are calculated 1: electroweak corrections (N2HDM) are neglected
10	N2HDM	0: considered model is not the N2HDM 1: considered model is the N2HDM
58	TYPE	1: N2HDM type I 2: N2HDM type II 3: N2HDM lepton-specific 4: N2HDM flipped
76	RENSCHEM	0: all renormalization schemes are calculated 1-10: only the chosen scheme (<i>cf.</i> Tab. 6) is calculated
77	REFSCHEM	1-10: the input values of α_i , β , m_{12}^2 and v_S (<i>cf.</i> Tab. 5) are given in the chosen reference scheme and at the scale μ_R given by INSCALE in case of $\overline{\text{MS}}$ parameters; the values of α_i , β , m_{12}^2 and v_S in all other schemes and at the scale μ_{out} at which the decays are calculated, are evaluated using Eqs. (2.105) and (2.106)

Table 4: Set of input parameters for the basic control of **ewN2HDECAY**. The first column depicts the line number at which the input value is specified in the input file. For the calculation of the one-loop EW corrections in the N2HDM, the parameter **OMIT ELW2** has to be set to 0. In this case, the value of **N2HDM** is set to 1 automatically internally and the provided input value is ignored. All input values presented in this table have to be entered as integer values.

3.5 Input File Format

The format of the input file of **ewN2HDECAY** is directly adopted from the input file format of **N2HDECAY** [5], which again is adopted from the format of **HDECAY** [22, 23]. In comparison to the **N2HDECAY** input file, minor modifications were implemented to account for the EW corrections. In the input file, two classes of input parameters are provided. The first class are the integer-valued input parameters needed for the basic control of **ewN2HDECAY**, as given in Tab. 4. The second class of input parameters are the values of the independent physical input parameters, provided in **FORTRAN** double-precision format, as shown in Tab. 5. Since the input file format of **ewN2HDECAY** is mostly equivalent to the one of **2HDECAY** (apart from the extended scalar sector), we refer to [4] for an in-depth explanation of the meaning of the input parameters. However, we want to emphasize here again that the input parameters **M_12^2** and **V_SING**, corresponding to the values of the $\overline{\text{MS}}$ parameters m_{12}^2 and v_S , respectively, as well as **alpha1**, **alpha2**, **alpha3** and **TGBET2HDM**, corresponding to the values of α_i ($i = 1, 2, 3$) and $\tan \beta$ (in case that they are $\overline{\text{MS}}$ renormalized) all explicitly depend on the input renormalization scale μ_R , given by **INSCALE**. This scale can be set to an arbitrary double-precision value by the user. The EW one-loop corrected decay widths are evaluated at the output scale μ_{out} , provided by **OUTSCALE**, which can be set to either a global fixed value or to the mass of the decaying particle by setting **OUTSCALE=MIN**. In case that both scales differ, all $\overline{\text{MS}}$ parameters are converted from μ_R to μ_{out} by means of Eq. (2.105). Moreover, if the reference renormalization scheme given by **REFSCHEM** is different from the renormalization scheme **RENSCHEM** within which the partial decay widths are calculated, then the values of the scalar mixing angles are automatically converted from one scheme to the other by means of Eq. (2.106). The user has the possibility to store as many input files in the input file subdirectory **\$2HDECAY/Input** as wanted. The input files are allowed to have arbitrary non-empty filenames and filename extensions. After a successful run, all output

Line	Input name	Name in Sec. 2	Allowed values and meaning
19	ALS(MZ)	$\alpha_s(m_Z)$	strong coupling constant (at m_Z)
20	MSBAR(2)	$m_s(2 \text{ GeV})$	s -quark $\overline{\text{MS}}$ mass at 2 GeV in GeV
21	MCBAR(3)	$m_c(3 \text{ GeV})$	c -quark $\overline{\text{MS}}$ mass at 3 GeV in GeV
22	MBBAR(MB)	$m_b(m_b)$	b -quark $\overline{\text{MS}}$ mass at m_b in GeV
23	MT	m_t	t -quark pole mass in GeV
24	MTAU	m_τ	τ -lepton pole mass in GeV
25	MMUON	m_μ	μ -lepton pole mass in GeV
26	1/ALPHA	$\alpha_{\text{em}}^{-1}(0)$	inverse fine-structure constant (Thomson limit)
27	ALPHAMZ	$\alpha_{\text{em}}(m_Z)$	fine-structure constant (at m_Z)
30	GAMW	Γ_W	partial decay width of the W boson
31	GAMZ	Γ_Z	partial decay width of the Z boson
32	MZ	m_Z	Z boson on-shell mass in GeV
33	MW	m_W	W boson on-shell mass in GeV
34-42	Vij	V_{ij}	CKM matrix elements ($i \in \{u, c, t\}$, $j \in \{d, s, b\}$)
60	TGBET2HDM	t_β	ratio of the VEVs in the 2HDM
61	M_12^2	m_{12}^2	squared soft- \mathbb{Z}_2 -breaking scale in GeV^2
66	MHA	m_A	CP-odd Higgs boson mass in GeV
67	MH+-	m_{H^\pm}	charged Higgs boson mass in GeV
78	INSCALE	μ_R	renormalization scale for $\overline{\text{MS}}$ inputs in GeV
79	OUTSCALE	μ_{out}	renormalization scale for the evaluation of the partial decay widths in GeV or in terms of MIN
80	MH1	m_{H_1}	mass of the CP-even Higgs boson H_1 in GeV
81	MH2	m_{H_2}	mass of the CP-even Higgs boson H_2 in GeV
82	MH3	m_{H_3}	mass of the CP-even Higgs boson H_3 in GeV
83	alpha1	α_1	CP-even Higgs mixing angle α_1 in radians
84	alpha2	α_2	CP-even Higgs mixing angle α_2 in radians
85	alpha3	α_3	CP-even Higgs mixing angle α_3 in radians

Table 5: All input parameters necessary for running `ewN2HDECAY`. The number in the first column denotes the line in the input file at which the input value is specified. Almost all input parameters are entered as integers. Exceptions are the renormalization scales `INSCALE`, entered as a double-precision number, and `OUTSCALE`, entered as either a double-precision number or in terms of the mass MIN of the decaying particle. Note that the 2HDM values t_β , m_{12}^2 , m_A and m_{H^\pm} are used as N2HDM input parameters if N2HDM is set to 1 in the input file.

files are stored in the output file subdirectory `$2HDECAY/Results` under the same name as the corresponding input file, but with the filename extension replaced by `.out`. For each input file that is used for the calculation, two corresponding output files are created. One output file, indicated by the filename suffix `'_BR'`, contains the calculated branching ratios, while the other, indicated by the filename suffix `'_EW'`, contains the electroweak partial decay widths, *cf.* Subsection 2.4.

3.6 Structure of the Program

The program `ewN2HDECAY` combines the EW one-loop corrections to the partial decay widths with the state-of-the-art QCD corrections already available in the tool `N2HDECAY`. In Fig. 2, the work flow of the main wrapper file of `ewN2HDECAY`, *i.e.* `ewN2HDECAY.py`, is depicted. The wrapper file iterates over all given input files and first calls `N2HDECAY` in a so-called minimal run in which

Input ID	Tadpole scheme	$\delta\alpha_i$	$\delta\beta$	Gauge-par.-indep. Γ
1	standard	KOSY	KOSY (odd)	\times
2	standard	KOSY	KOSY (charged)	\times
3	alternative (FJ)	KOSY	KOSY (odd)	\times
4	alternative (FJ)	KOSY	KOSY (charged)	\times
5	alternative (FJ)	p_* -pinched	p_* -pinched (odd)	\checkmark
6	alternative (FJ)	p_* -pinched	p_* -pinched (charged)	\checkmark
7	alternative (FJ)	OS-pinched	OS-pinched (odd)	\checkmark
8	alternative (FJ)	OS-pinched	OS-pinched (charged)	\checkmark
9	standard	$\overline{\text{MS}}$	$\overline{\text{MS}}$	\times
10	alternative (FJ)	$\overline{\text{MS}}$	$\overline{\text{MS}}$	\checkmark

Table 6: Overview over all renormalization schemes for the mixing angles α_i ($i = 1, 2, 3$) and β that are implemented in **ewN2HDECAY**. By setting **RENSCHEM** in the input file, *cf.* Tab. 4, equal to the Input ID the corresponding renormalization scheme is chosen. In case of 0, however, the results for all renormalization schemes are given out. The definition of the CTs $\delta\alpha_i$ and $\delta\beta$ in each scheme is explained in Sec. 2.2.4. The crosses and check marks in the column for gauge independence indicate whether the chosen scheme in general yields explicitly gauge-independent partial decay widths or not.

the charm and bottom masses are converted from their $\overline{\text{MS}}$ values to the corresponding pole masses. These are then used together with all other given input values to calculate the EW corrections to the partial decay widths. In a second run of **N2HDECAY**, the LO widths and QCD corrections to the decays are computed. Additionally, the off-shell decay widths and the loop-induced decays to final-state pairs of gluons or photons and $Z\gamma$ are calculated and the branching ratios are evaluated. The results of these computations are then consistently combined with the EW corrections as described in Subsection 2.4. This procedure is repeated for every individual input file in the input file list. The work flow of **ewN2HDECAY** is analogous to that of **2HDECAY**. For a more detailed description, we refer to [4].

3.7 Usage

Before running **ewN2HDECAY**, all input files need to be stored in the input file subdirectory **\$ewN2HDECAY/Input**. The input files have to be formatted exactly as described in Sec. 3.5, otherwise, the program might either crash with a segmentation error or input values are read in incorrectly. For convenience, the user may use the exemplary input file printed in App. A which is part of the **ewN2HDECAY** repository as a template to generate own input files.

In each run of **ewN2HDECAY**, the output file subfolder **\$ewN2HDECAY/Results** is emptied to make space for the output files generated in a new run. The user is therefore advised to check the output file folder for any output files of a previous run before starting a new run and to create backups of them, if necessary.

In order to start a run of **ewN2HDECAY**, the user opens a terminal, navigates to the main **\$ewN2HDECAY** folder and executes the following command:

```
python ewN2HDECAY.py
```

Provided that **ewN2HDECAY** was installed successfully and that all input files stored in the subdirectory **\$ewN2HDECAY/Input** have the correct format, **ewN2HDECAY** now iterates over all input files and computes the EW and/or QCD corrections as indicated by the flowchart in Fig. 2. Intermediate results and some additional information are printed on the terminal. After a suc-

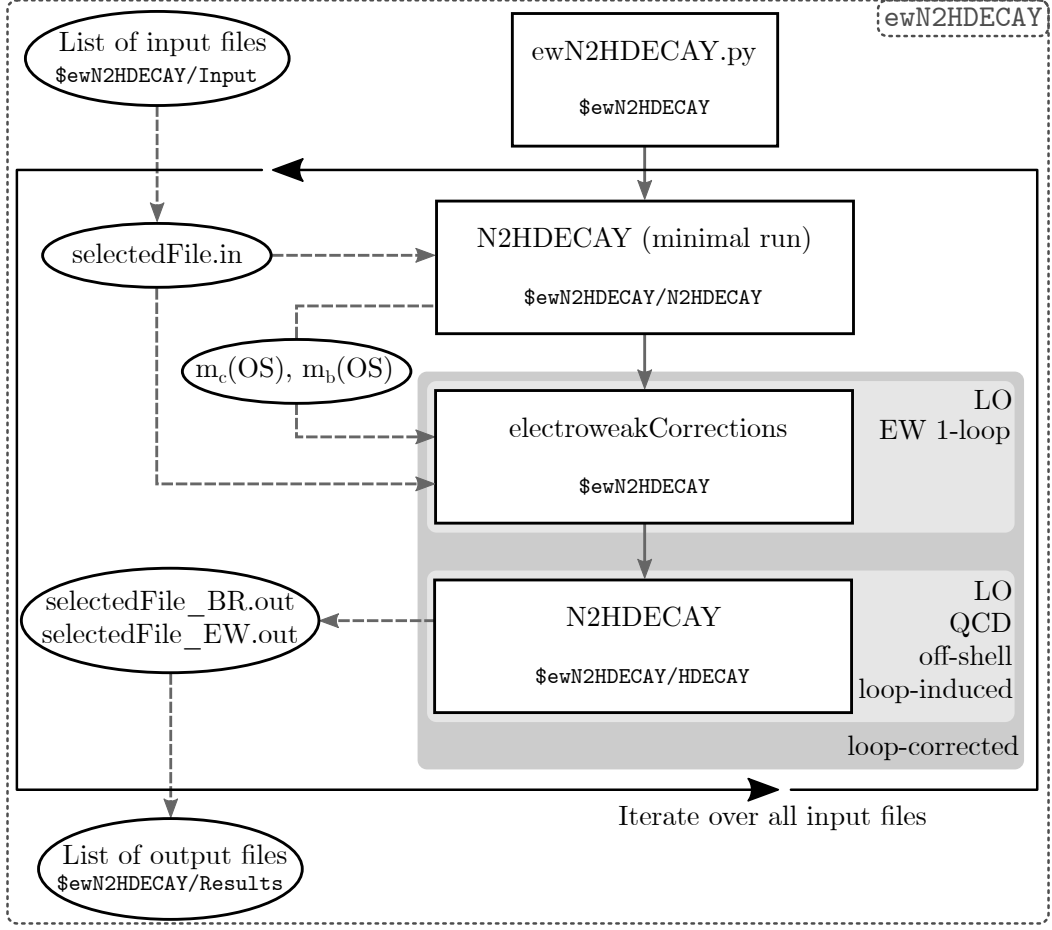


Figure 2: Flowchart of **ewN2HDECAY**. The main wrapper file **ewN2HDECAY.py** generates a list of input files, provided by the user in the subfolder **\$ewN2HDECAY/Input**, and iterates over the list. For each selected input file in the list, the wrapper calls **N2HDECAY** and the subprogram **electroweakCorrections**. The computed branching ratios including the EW and QCD corrections as described in the text are written to the output file with suffix '**_BR**', the calculated LO and NLO EW-corrected partial decay widths are given out in the output file with suffix '**_EW**'.

cessful run, **ewN2HDECAY** terminates with no errors printed on the terminal and the output files are stored in the subdirectory **\$ewN2HDECAY/Results**.

3.8 Output File Format

After a successful run, **ewN2HDECAY** creates two output files for each individual input file, indicated in their filenames by the suffixes '**_QCD**' and '**_EW**' for the branching ratios and electroweak partial decay widths, respectively. Exemplary output files are printed in App. B. The output file format is a modified SLHA format¹². The format of the output files is completely analogous to the format of the output files in **2HDECAY**. For a detailed description, we refer to [4].

¹²The original SLHA output format [59–61] has only been designed for supersymmetric models. For **ewN2HDECAY**, we modified the SLHA format to account for the EW corrections calculated in the N2HDM.

4 Summary

We have presented the program package `ewN2HDECAY` for the calculation of the EW corrections to the Higgs boson decays in the N2HDM. The program allows for the computation of NLO EW corrections to the two-body on-shell decays of all N2HDM Higgs bosons that are not loop-induced. For the scalar mixing angles α_i ($i = 1, 2, 3$) and β of the N2HDM, 10 different renormalization schemes are implemented. The other parameters of the N2HDM necessary for the computation of the EW corrections are renormalized in an OS scheme, except for the soft- \mathbb{Z}_2 -breaking scale m_{12}^2 and the singlet VEV v_S which are $\overline{\text{MS}}$ renormalized. The EW corrections are consistently combined with the state-of-the-art QCD corrections that are obtained from `N2HDECAY` and the EW&QCD-corrected total decay widths and branching ratios are given out in an SLHA-inspired output file format. Additionally, `ewN2HDECAY` provides a separate SLHA-inspired output of the LO and NLO EW partial decay widths to all OS non-loop-induced N2HDM Higgs decays. The implementation of 10 different renormalization schemes for the scalar mixing angles and a routine to automatically convert the scalar mixing angles from one scheme to another, allows to compare NLO partial decay widths calculated within these different schemes. The $\overline{\text{MS}}$ input parameters are moreover evolved from a user-given input renormalization scale μ_R to a user-given output scale μ_{out} at which the partial decay widths are evaluated. Thus the remaining theoretical uncertainty due to missing higher-order corrections can be estimated from a scale change or the change of the renormalization scheme. Because of its fast numerical computation, our new program `ewN2HDECAY` allows for efficient phenomenological studies of the N2HDM Higgs sector at high precision required for the analysis of indirect new physics effects in the Higgs sector and for the identification of the underlying model in case additional Higgs bosons are discovered.

Acknowledgments

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A Exemplary Input File

We present an exemplary input file `ewn2hdecay.in` which is also included in the program subfolder `$ewN2HDECAY/Input` in the `ewN2HDECAY` repository. The first integer in each line represents the line number and is not part of the actual input file, but given here for convenience. The role of the various input parameters has been specified in Sec. 3.5. With respect to the input file format of the unmodified `N2HDECAY` program [5], the lines 6, 27, 29, and 76-79 are new, but the remainder of the input file format is unchanged. Note that the value `GFCALC` in the input file is overwritten by the program and thus not an input value that is provided by the user, but it is calculated by `ewN2HDECAY` internally. The sample N2HDM parameter point has been checked to be in accordance with all relevant theoretical and experimental constraints. Thus, it features an SM-like Higgs boson with a mass of 125.09 GeV [62] which is given by the lightest CP-even neutral Higgs boson H_1 . For details on the applied constraints, we refer to Ref. [5]. Note that we understood all running $\overline{\text{MS}}$ input parameters to be given by the mass of the SM-like Higgs boson by specifying the value `INSCALE` accordingly. The EW-corrected decays on the other hand

are calculated at the scale of the decaying particle, *i.e.* OUTSCALE=MIN.

```

1  SLHAIN   = 1
2  SLHAOUT  = 1
3  COUPVAR  = 1
4  HIGGS    = 5
5  OMIT ELW = 1
6  OMIT ELW2= 0
7  SM4      = 0
8  FERMPHOB = 0
9  2HDM     = 1
10 N2HDM    = 1
11 MODEL    = 1
12 TGBET    = 5.07403e+01
13 MABEG     = 4.67967e+02
14 MAEND     = 4.67967e+02
15 NMA       = 1
16 ***** hMSSM (MODEL = 10) *****
17 MHL       = 125.D0
18 *****
19 ALS(MZ)   = 1.18000e-01
20 MSBAR(2)  = 9.50000e-02
21 MCBAR(3)  = 0.98600e+00
22 MBBAR(MB)= 4.18000e+00
23 MT        = 1.73200e+02
24 MTAU      = 1.77682e+00
25 MMUON     = 1.056583715e-01
26 1/ALPHA   = 1.37036e+02
27 ALPHAMZ   = 7.754222173973729e-03
28 GF        = 1.1663787e-05
29 GFCALC    = 0.000000000
30 GAMW      = 2.08500e+00
31 GAMZ      = 2.49520e+00
32 MZ        = 9.11876e+01
33 MW        = 8.0385e+01
34 VTB       = 9.9910e-01
35 VTS       = 4.040e-02
36 VTD       = 8.67e-03
37 VCB       = 4.12e-02
38 VCS       = 9.7344e-01
39 VCD       = 2.252e-01
40 VUB       = 3.51e-03
41 VUS       = 2.2534e-01
42 VUD       = 9.7427e-01
43 ***** 4TH GENERATION *****
44 SCENARIO FOR ELW. CORRECTIONS TO H -> GG (EVERYTHING IN GEV):
45 GG.ELW = 1: MTP = 500      MBP = 450      MNUP = 375      MEP = 450
46 GG.ELW = 2: MBP = MNUP = MEP = 600      MTP = MBP+50*(1+LOG(M.H/115)/5)
47
48 GG.ELW   = 1
49 MTP      = 500.D0
50 MBP      = 450.D0
51 MNUP     = 375.D0
52 MEP      = 450.D0
53 ***** 2 Higgs Doublet Model *****
54 TYPE: 1 (I), 2 (II), 3 (lepton-specific), 4 (flipped)
55 PARAM: 1 (masses), 2 (lambda_i)
56

```

```

57 PARAM      = 1
58 TYPE       = 2
59 *****
60 TGBET2HDM= 1.32331D0
61 M_12^2      = 182778.0D0
62 ***** PARAM=1:
63 ALPHA_H     = 10.D0
64 MHL         = 10.D0
65 MHH         = 10.D0
66 MHA         = 722.000D0
67 MH+         = 758.918D0
68 ***** PARAM=2:
69 LAMBDA1     = 0D0
70 LAMBDA2     = 0D0
71 LAMBDA3     = 0D0
72 LAMBDA4     = 0D0
73 LAMBDA5     = 0D0
74 ***** N2HDM *****
75 *** needs TYPE, TGBET2HDM, M12^2, MHA and MH+ from the 2HDM block ***
76 RENSCHEM   = 7
77 REFSCHEM   = 5
78 INSCALE    = 125.09D0
79 OUTSCALE   = MIN
80 MH1        = 125.09D0
81 MH2        = 286.094D0
82 MH3        = 648.564D0
83 alpha1     = 0.909079D0
84 alpha2     = -0.155397D0
85 alpha3     = -1.54459D0
86 V_SING     = 2440.84D0
87 *****
88 SUSYSCALE= 1000.D0
89 MU         = 1000.D0
90 M2         = 1000.D0
91 MGLUINO    = 1000.D0
92 MSL1       = 1000.D0
93 MER1       = 1000.D0
94 MQL1       = 1000.D0
95 MUR1       = 1000.D0
96 MDR1       = 1000.D0
97 MSL        = 1000.D0
98 MER        = 1000.D0
99 MSQ        = 1000.D0
100 MUR        = 1000.D0
101 MDR        = 1000.D0
102 AL         = 1000.D0
103 AU         = 1000.D0
104 AD         = 1000.D0
105 ON-SHELL   = 0
106 ON-SH-WZ   = 0
107 IPOLE      = 0
108 OFF-SUSY   = 0
109 INDIDEC    = 0
110 NF-GG      = 5
111 IGOLD      = 0
112 MPLANCK    = 2.4D18
113 MGOLD      = 1.D-13
114 ***** VARIATION OF HIGGS COUPLINGS *****

```

```

115 ELWK      = 0
116 CW        = 1.D0
117 CZ        = 1.D0
118 Ctau      = 1.D0
119 Cmu       = 1.D0
120 Ct        = 1.D0
121 Cb        = 1.D0
122 Cc        = 1.D0
123 Cs        = 1.D0
124 Cgaga     = 0.D0
125 Cgg       = 0.D0
126 CZga      = 0.D0
127 ***** 4TH GENERATION *****
128 Ctp       = 1.D0
129 Cbp       = 1.D0
130 Cnup      = 1.D0
131 Cep       = 1.D0

```

B Exemplary Output Files

We give here the exemplary output files `ewn2hdecay_BR.out` and `ewn2hdecay_EW.out` that have been generated from the sample input file `ewn2hdecay.in` and are included in the subfolder `$ewn2HDECAY/Results` in the `ewn2HDECAY` repository. The suffixes “_BR” and “_EW” stand for the branching ratios and EW partial decay widths, given out in the respective files. Again, the first integer in each line represents the line number and is not part of the actual output file, but printed here for convenience. The output file format is explained in detail in Sec. 3.8. The exemplary output file was generated for a specific choice of the renormalization scheme, namely we have set `RENSCHEM = 7` in line 76 of the input file, *cf.* App. A. For `RENSCHEM = 0`, the output file becomes considerably longer, since the EW corrections are then calculated for all 10 implemented renormalization schemes. As reference scheme we chose the renormalization scheme number 5, *cf.* Tab. 6 for its definition.

B.1 Exemplary Output File for the Branching Ratios

The exemplary output file `ewn2hdecay_BR.out` contains the branching ratios without and with the electroweak corrections. The content of the file is presented in the following.

```

1 #
2 BLOCK DCINFO # Decay Program information
3     1 ewN2HDECAY # decay calculator
4     2 1.0.0      # version number
5 #
6 BLOCK SMINPUTS # Standard Model inputs
7     2 1.19596488E-05 # G_F [GeV^-2]
8     3 1.18000000E-01 # alpha_S(M_Z)^MSbar
9     4 9.11876000E+01 # M_Z on-shell mass
10    5 4.18000000E+00 # mb(mb)^MSbar
11    6 1.73200000E+02 # mt pole mass
12    7 1.77682000E+00 # mtau pole mass
13    8 4.84141297E+00 # mb pole mass
14    9 1.43141297E+00 # mc pole mass
15   10 1.05658372E-01 # muon mass
16   11 8.03850000E+01 # MW on-shell mass

```



```

17      12      2.08500000E+00  # W boson total width
18      13      2.49520000E+00  # Z boson total width
19 #
20 BLOCK N2HDMINPUTS  # N2HDM inputs of reference scheme
21      1      2      # N2HDM type
22      2      1.32331000E+00  # tan(beta)
23      3      1.82778000E+05  # M_12^2
24      4      1.25090000E+02  # M_H1
25      5      2.86094000E+02  # M_H2
26      6      6.48564000E+02  # M_H3
27      7      7.22000000E+02  # M_A
28      8      7.58918000E+02  # M_CH
29      9      9.09079000E-01  # alpha1
30     10     -1.55397000E-01  # alpha2
31     11     -1.54459000E+00  # alpha3
32     12      2.44084000E+03  # v_S
33     13      7      # renormalization scheme EW corrs
34     14      5      # reference ren. scheme EW corrs
35     15      125.09D0      # ren. scale of MSbar parameters
36     16      MIN      # ren. scale at which decay is evaluated
37 #
38 BLOCK VCKMIN  # CKM mixing
39      1      9.99100000E-01  # V_tb
40      2      4.04000000E-02  # V_ts
41      3      8.67000000E-03  # V_td
42      4      4.12000000E-02  # V_cb
43      5      9.73440000E-01  # V_cs
44      6      2.25200000E-01  # V_cd
45      7      3.51000000E-03  # V_ub
46      8      2.25340000E-01  # V_us
47      9      9.74270000E-01  # V_ud
48 #
49 #      PDG      Width QCD Only
50 DECAY QCD      25      4.14310207E-03  # H1 decays with QCD corrections only
51      7      # Renormalization Scheme Number
52      0.90516083E+00  # Corresponding mixing angle alpha1
53     -0.15530063E+00  # Corresponding mixing angle alpha2
54     -0.15445673E+01  # Corresponding mixing angle alpha3
55      0.13029492E+01  # Corresponding tan(beta)
56      0.18277800E+06  # Corresponding m_12^2
57      0.24408400E+04  # Corresponding v_S
58 #      BR      NDA      ID1      ID2
59      5.98141907E-01  2      5      -5  # BR(H1 -> b      bb      )
60      6.43400735E-02  2     -15     15  # BR(H1 -> tau+    tau-    )
61      2.27785781E-04  2     -13     13  # BR(H1 -> mu+    mu-    )
62      2.25766183E-04  2      3     -3  # BR(H1 -> s      sb      )
63      2.79834641E-02  2      4     -4  # BR(H1 -> c      cb      )
64      7.45745241E-02  2     21     21  # BR(H1 -> g      g      )
65      2.21308229E-03  2     22     22  # BR(H1 -> gam     gam     )
66      1.54082518E-03  2     22     23  # BR(H1 -> Z      gam     )
67      2.05127755E-01  2     24    -24  # BR(H1 -> W+     W-      )
68      2.56248172E-02  2     23     23  # BR(H1 -> Z      Z      )
69 #
70 #      PDG      Width QCD and EW
71 DECAY QCD&EW      25      4.04594148E-03  # H1 decays with QCD and EW corrections
72      7      # Renormalization Scheme Number
73      0.90516083E+00  # Corresponding mixing angle alpha1
74     -0.15530063E+00  # Corresponding mixing angle alpha2

```

75	-0.15445673E+01	#	Corresponding	mixing angle	alpha3			
76	0.13029492E+01	#	Corresponding	tan(beta)				
77	0.18277800E+06	#	Corresponding	m_12^2				
78	0.24408400E+04	#	Corresponding	v_S				
79	#	BR	NDA	ID1	ID2			
80	5.92535099E-01	2		5	-5	#	BR(H1 -> b	bb)
81	6.29924711E-02	2		-15	15	#	BR(H1 -> tau+	tau -)
82	2.18132813E-04	2		-13	13	#	BR(H1 -> mu+	mu-)
83	2.25522035E-04	2		3	-3	#	BR(H1 -> s	sb)
84	2.75253978E-02	2		4	-4	#	BR(H1 -> c	cb)
85	7.63653816E-02	2		21	21	#	BR(H1 -> g	g)
86	2.26622799E-03	2		22	22	#	BR(H1 -> gam	gam)
87	1.57782707E-03	2		22	23	#	BR(H1 -> Z	gam)
88	2.10053761E-01	2		24	-24	#	BR(H1 -> W+	W-)
89	2.62401801E-02	2		23	23	#	BR(H1 -> Z	Z)
90	#							
91	#	PDG	Width QCD Only					
92	DECAY QCD	35	2.51908281E-01	#	H2 decays with QCD corrections only			
93		7		#	Renormalization Scheme Number			
94	0.90516083E+00	#	Corresponding	mixing angle	alpha1			
95	-0.15530063E+00	#	Corresponding	mixing angle	alpha2			
96	-0.15445673E+01	#	Corresponding	mixing angle	alpha3			
97	0.13029492E+01	#	Corresponding	tan(beta)				
98	0.16715360E+06	#	Corresponding	m_12^2				
99	0.24408400E+04	#	Corresponding	v_S				
100	#	BR	NDA	ID1	ID2			
101	6.94510276E-04	2		5	-5	#	BR(H2 -> b	bb)
102	8.77457425E-05	2		-15	15	#	BR(H2 -> tau+	tau-)
103	3.10346729E-07	2		-13	13	#	BR(H2 -> mu+	mu-)
104	2.60430448E-07	2		3	-3	#	BR(H2 -> s	sb)
105	1.65447031E-05	2		4	-4	#	BR(H2 -> c	cb)
106	5.64810018E-06	2		6	-6	#	BR(H2 -> t	tb)
107	3.17272086E-04	2		21	21	#	BR(H2 -> g	g)
108	1.04971784E-05	2		22	22	#	BR(H2 -> gam	gam)
109	4.57638077E-05	2		22	23	#	BR(H2 -> Z	gam)
110	4.67693293E-01	2		24	-24	#	BR(H2 -> W+	W-)
111	2.05903660E-01	2		23	23	#	BR(H2 -> Z	Z)
112	3.25224495E-01	2		25	25	#	BR(H2 -> H1	H1)
113	#							
114	#	PDG	Width QCD and EW					
115	DECAY QCD&EW	35	2.87330365E-01	#	H2 decays with QCD and EW correctis			
116		7		#	Renormalization Scheme Number			
117	0.90516083E+00	#	Corresponding	mixing angle	alpha1			
118	-0.15530063E+00	#	Corresponding	mixing angle	alpha2			
119	-0.15445673E+01	#	Corresponding	mixing angle	alpha3			
120	0.13029492E+01	#	Corresponding	tan(beta)				
121	0.16715360E+06	#	Corresponding	m_12^2				
122	0.24408400E+04	#	Corresponding	v_S				
123	#	BR	NDA	ID1	ID2			
124	5.72230258E-04	2		5	-5	#	BR(H2 -> b	bb)
125	7.18420289E-05	2		-15	15	#	BR(H2 -> tau+	tau -)
126	2.48410041E-07	2		-13	13	#	BR(H2 -> mu+	mu-)
127	2.18867293E-07	2		3	-3	#	BR(H2 -> s	sb)
128	1.38893457E-05	2		4	-4	#	BR(H2 -> c	cb)
129	4.95180244E-06	2		6	-6	#	BR(H2 -> t	tb)
130	2.78158787E-04	2		21	21	#	BR(H2 -> g	g)
131	9.20308634E-06	2		22	22	#	BR(H2 -> gam	gam)
132	4.01220460E-05	2		22	23	#	BR(H2 -> Z	gam)

133	4.13024816E-01	2	24	-24	# BR(H2 -> W+	W-)
134	1.81037672E-01	2	23	23	# BR(H2 -> Z	Z)
135	4.04946647E-01	2	25	25	# BR(H2 -> H1	H1)
136	# PDG	Width QCD Only					
137	DECAY QCD	38	1.46796633E+01	# H3 decays with QCD corrections only			
138		7		# Renormalization Scheme Number			
139	0.90516083E+00	# Corresponding	mixing angle	alpha1			
140	-0.15530063E+00	# Corresponding	mixing angle	alpha2			
141	-0.15445673E+01	# Corresponding	mixing angle	alpha3			
142	0.13029492E+01	# Corresponding	tan(beta)				
143	0.15169628E+06	# Corresponding	m_12^2				
144	0.24408400E+04	# Corresponding	v_S				
145	# BR	NDA	ID1	ID2			
146	1.05566345E-03	2	5	-5	# BR(H3 -> b	bb)
147	1.55539709E-04	2	-15	15	# BR(H3 -> tau+	tau-)
148	5.50024113E-07	2	-13	13	# BR(H3 -> mu+	mu-)
149	3.86057017E-07	2	3	-3	# BR(H3 -> s	sb)
150	1.93693674E-05	2	4	-4	# BR(H3 -> c	cb)
151	9.92185598E-01	2	6	-6	# BR(H3 -> t	tb)
152	2.31958632E-03	2	21	21	# BR(H3 -> g	g)
153	6.71149392E-06	2	22	22	# BR(H3 -> gam	gam)
154	1.86917571E-06	2	22	23	# BR(H3 -> Z	gam)
155	1.29809388E-03	2	24	-24	# BR(H3 -> W+	W-)
156	6.32155300E-04	2	23	23	# BR(H3 -> Z	Z)
157	1.79993258E-03	2	25	25	# BR(H3 -> H1	H1)
158	4.83996982E-04	2	25	35	# BR(H3 -> H1	H2)
159	4.05475613E-05	2	35	35	# BR(H3 -> H2	H2)
160	#						
161	# PDG	Width QCD and EW					
162	DECAY QCD&EW	38	1.42137962E+01	# H3 decays with QCD and EW correctis			
163		7		# Renormalization Scheme Number			
164	0.90516083E+00	# Corresponding	mixing angle	alpha1			
165	-0.15530063E+00	# Corresponding	mixing angle	alpha2			
166	-0.15445673E+01	# Corresponding	mixing angle	alpha3			
167	0.13029492E+01	# Corresponding	tan(beta)				
168	0.15169628E+06	# Corresponding	m_12^2				
169	0.24408400E+04	# Corresponding	v_S				
170	# BR	NDA	ID1	ID2			
171	9.71729023E-04	2	5	-5	# BR(H3 -> b	bb)
172	1.43688626E-04	2	-15	15	# BR(H3 -> tau+	tau -)
173	4.96243881E-07	2	-13	13	# BR(H3 -> mu+	mu-)
174	3.68118807E-07	2	3	-3	# BR(H3 -> s	sb)
175	1.82112984E-05	2	4	-4	# BR(H3 -> c	cb)
176	9.81534840E-01	2	6	-6	# BR(H3 -> t	tb)
177	2.39561238E-03	2	21	21	# BR(H3 -> g	g)
178	6.93146781E-06	2	22	22	# BR(H3 -> gam	gam)
179	1.93043925E-06	2	22	23	# BR(H3 -> Z	gam)
180	2.36691905E-03	2	24	-24	# BR(H3 -> W+	W-)
181	6.73441154E-04	2	23	23	# BR(H3 -> Z	Z)
182	6.46052130E-03	2	25	25	# BR(H3 -> H1	H1)
183	5.11304746E-03	2	25	35	# BR(H3 -> H1	H2)
184	3.12263142E-04	2	35	35	# BR(H3 -> H2	H2)
185	# PDG	Width QCD Only					
186	DECAY QCD	36	2.06377878E+01	# A decays with QCD corrections only			
187		7		# Renormalization Scheme Number			
188	0.90516083E+00	# Corresponding	mixing angle	alpha1			
189	-0.15530063E+00	# Corresponding	mixing angle	alpha2			
190	-0.15445673E+01	# Corresponding	mixing angle	alpha3			

191	0.13029492E+01	#	Corresponding	$\tan(\beta)$			
192	0.14967045E+06	#	Corresponding	m_{12}^2			
193	0.24408400E+04	#	Corresponding	v_S			
194	#	BR	NDA	ID1	ID2		
195	8.54499665E-04	2	5	-5	#	BR(A \rightarrow b	bb)
196	1.26183976E-04	2	-15	15	#	BR(A \rightarrow tau+	tau -)
197	4.46200865E-07	2	-13	13	#	BR(A \rightarrow mu+	mu-)
198	3.21263372E-07	2	3	-3	#	BR(A \rightarrow s	sb)
199	1.46395489E-05	2	4	-4	#	BR(A \rightarrow c	cb)
200	9.93049035E-01	2	6	-6	#	BR(A \rightarrow t	tb)
201	2.49656795E-03	2	21	21	#	BR(A \rightarrow g	g)
202	7.27483596E-06	2	22	22	#	BR(A \rightarrow gam	gam)
203	2.39953028E-06	2	22	23	#	BR(A \rightarrow Z	gam)
204	6.31266102E-04	2	23	25	#	BR(A \rightarrow Z	H1)
205	2.64866795E-03	2	23	35	#	BR(A \rightarrow Z	H2)
206	2.37168383E-04	2	23	38	#	BR(A \rightarrow Z	H3)
207	#						
208	#	PDG	Width QCD and EW				
209	DECAY QCD&EW	36	1.97238031E+01	#	A decays with QCD and EW corrections		
210			7	#	Renormalization Scheme Number		
211	0.90516083E+00	#	Corresponding	mixing angle	alpha1		
212	-0.15530063E+00	#	Corresponding	mixing angle	alpha2		
213	-0.15445673E+01	#	Corresponding	mixing angle	alpha3		
214	0.13029492E+01	#	Corresponding	$\tan(\beta)$			
215	0.14967045E+06	#	Corresponding	m_{12}^2			
216	0.24408400E+04	#	Corresponding	v_S			
217	#	BR	NDA	ID1	ID2		
218	7.80198466E-04	2	5	-5	#	BR(A \rightarrow b	bb)
219	1.15123849E-04	2	-15	15	#	BR(A \rightarrow tau+	tau -)
220	3.97333090E-07	2	-13	13	#	BR(A \rightarrow mu+	mu-)
221	3.02897142E-07	2	3	-3	#	BR(A \rightarrow s	sb)
222	1.36486632E-05	2	4	-4	#	BR(A \rightarrow c	cb)
223	9.92644607E-01	2	6	-6	#	BR(A \rightarrow t	tb)
224	2.61225684E-03	2	21	21	#	BR(A \rightarrow g	g)
225	7.61194583E-06	2	22	22	#	BR(A \rightarrow gam	gam)
226	2.51072252E-06	2	22	23	#	BR(A \rightarrow Z	gam)
227	9.68311638E-04	2	23	25	#	BR(A \rightarrow Z	H1)
228	2.67851520E-03	2	23	35	#	BR(A \rightarrow Z	H2)
229	1.76515248E-04	2	23	38	#	BR(A \rightarrow Z	H3)
230	#	PDG	Width QCD Only				
231	DECAY QCD	37	2.17468065E+01	#	H+ decays with QCD corrections only		
232			7	#	Renormalization Scheme Number		
233	0.90516083E+00	#	Corresponding	mixing angle	alpha1		
234	-0.15530063E+00	#	Corresponding	mixing angle	alpha2		
235	-0.15445673E+01	#	Corresponding	mixing angle	alpha3		
236	0.13029492E+01	#	Corresponding	$\tan(\beta)$			
237	0.14872861E+06	#	Corresponding	m_{12}^2			
238	0.24408400E+04	#	Corresponding	v_S			
239	#	BR	NDA	ID1	ID2		
240	1.41706224E-06	2	4	-5	#	BR(H+ \rightarrow c	bb)
241	1.25872256E-04	2	-15	16	#	BR(H+ \rightarrow tau+	nu_tau)
242	4.45098078E-07	2	-13	14	#	BR(H+ \rightarrow mu+	nu_mu)
243	1.01136265E-08	2	2	-5	#	BR(H+ \rightarrow u	bb)
244	1.54202891E-08	2	2	-3	#	BR(H+ \rightarrow u	sb)
245	7.02521008E-07	2	4	-1	#	BR(H+ \rightarrow c	db)
246	1.34140274E-05	2	4	-3	#	BR(H+ \rightarrow c	sb)
247	9.51944776E-01	2	6	-5	#	BR(H+ \rightarrow t	bb)
248	1.55493512E-03	2	6	-3	#	BR(H+ \rightarrow t	sb)

249	7.16118317E-05	2	6	-1	# BR(H+ -> t	db)
250	7.13310769E-04	2	24	25	# BR(H+ -> W+	H1)
251	3.15960499E-03	2	24	35	# BR(H+ -> W+	H2)
252	4.24092631E-02	2	24	38	# BR(H+ -> W+	H3)
253	4.62195001E-06	2	24	36	# BR(H+ -> W+	A)
254	#						
255	#	PDG	Width QCD and EW				
256	DECAY QCD&EW	37	1.99772998E+01		# H+ decays with QCD and EW corrections		
257			7		# Renormalization Scheme Number		
258	0.90516083E+00				# Corresponding mixing angle	alpha1	
259	-0.15530063E+00				# Corresponding mixing angle	alpha2	
260	-0.15445673E+01				# Corresponding mixing angle	alpha3	
261	0.13029492E+01				# Corresponding tan(beta)		
262	0.14872861E+06				# Corresponding m_12^2		
263	0.24408400E+04				# Corresponding v_S		
264	#	BR	NDA	ID1	ID2		
265	1.27362693E-06	2	4	-5	# BR(H+ -> c	bb)
266	1.14241461E-04	2	-15	16	# BR(H+ -> tau+	nu_tau)
267	3.93843731E-07	2	-13	14	# BR(H+ -> mu+	nu_mu)
268	8.01271883E-09	2	2	-5	# BR(H+ -> u	bb)
269	1.45819891E-08	2	2	-3	# BR(H+ -> u	sb)
270	6.65248800E-07	2	4	-1	# BR(H+ -> c	db)
271	1.27376814E-05	2	4	-3	# BR(H+ -> c	sb)
272	9.50944592E-01	2	6	-5	# BR(H+ -> t	bb)
273	1.55565603E-03	2	6	-3	# BR(H+ -> t	sb)
274	7.13525053E-05	2	6	-1	# BR(H+ -> t	db)
275	7.79767060E-04	2	24	25	# BR(H+ -> W+	H1)
276	3.14065971E-03	2	24	35	# BR(H+ -> W+	H2)
277	4.33736066E-02	2	24	38	# BR(H+ -> W+	H3)
278	5.03134325E-06	2	24	36	# BR(H+ -> W+	A)

As can be inferred from the file, the branching ratios of the lightest CP-even Higgs boson H_1 are SM-like. When these are compared with those generated by the code N2HDECAY [5], there are differences due to the rescaling factor $G_F^{\text{calc}}/G_F = 1.025366$, which is applied in **ewN2HDECAY** to consistently combine the EW-corrected decay widths with the decay widths generated by N2HDECAY. Additional differences arise in the decay widths into massive vector bosons $V = W, Z$, of around 2%. This is because N2HDECAY throughout computes these decay widths using the double off-shell formula while **ewN2HDECAY** uses the on-shell formula for Higgs boson masses above the threshold.

B.2 Exemplary Output File for the Electroweak Partial Decay Widths

The exemplary output file **ewn2hdecay_EW.out** contains the LO and electroweak NLO partial decay widths. The content of the file is presented in the following.

```

1 #
2 BLOCK DCINFO # Decay Program information
3   1   ewN2HDECAY # decay calculator
4   2   1.0.0      # version number
5 #
6 BLOCK SMINPUTS # Standard Model inputs
7   2   1.19596488E-05 # G_F [GeV^-2]
8   3   1.18000000E-01 # alpha_S(M_Z)^MSbar
9   4   9.11876000E+01 # M_Z on-shell mass
10  5   4.18000000E+00 # mb(mb)^MSbar
11  6   1.73200000E+02 # mt pole mass

```

```

12      7      1.77682000E+00  # mtau pole mass
13      8      4.84141297E+00  # mb pole mass
14      9      1.43141297E+00  # mc pole mass
15     10      1.05658372E-01  # muon mass
16     11      8.03850000E+01  # MW on-shell mass
17     12      2.08500000E+00  # W boson total width
18     13      2.49520000E+00  # Z boson total width
19 #
20 BLOCK N2HDMINPUTS  # N2HDM inputs of reference scheme
21      1      2      # N2HDM type
22      2      1.32331000E+00  # tan(beta)
23      3      1.82778000E+05  # M_12^2
24      4      1.25090000E+02  # M_H1
25      5      2.86094000E+02  # M_H2
26      6      6.48564000E+02  # M_H3
27      7      7.22000000E+02  # M_A
28      8      7.58918000E+02  # M_CH
29      9      9.09079000E-01  # alpha1
30     10     -1.55397000E-01  # alpha2
31     11     -1.54459000E+00  # alpha3
32     12      2.44084000E+03  # v_S
33     13      7      # renormalization scheme EW corrs
34     14      5      # reference ren. scheme EW corrs
35     15      125.09D0      # ren. scale of MSbar parameters
36     16      MIN      # ren. scale at which decay is evaluated
37 #
38 BLOCK VCKMIN  # CKM mixing
39      1      9.99100000E-01  # V_tb
40      2      4.04000000E-02  # V_ts
41      3      8.67000000E-03  # V_td
42      4      4.12000000E-02  # V_cb
43      5      9.73440000E-01  # V_cs
44      6      2.25200000E-01  # V_cd
45      7      3.51000000E-03  # V_ub
46      8      2.25340000E-01  # V_us
47      9      9.74270000E-01  # V_ud
48 #
49 #      PDG
50 LO DECAY WIDTH 25  # H1 non-zero LO EW decay widths of on-shell and non-loop
    induced decays
51      7      # Renormalization Scheme Number
52      0.90516083E+00  # Corresponding mixing angle alpha1
53     -0.15530063E+00  # Corresponding mixing angle alpha2
54     -0.15445673E+01  # Corresponding mixing angle alpha3
55      0.13029492E+01  # Corresponding tan(beta)
56      0.18277800E+06  # Corresponding m_12^2
57      0.24408400E+04  # Corresponding v_S
58 #      WIDTH      NDA      ID1      ID2
59      5.89110455E-03  2      5      -5  # GAM(H1 -> b      bb )
60      2.66567492E-04  2     -15     15  # GAM(H1 -> tau+   tau)
61      9.43739741E-07  2     -13     13  # GAM(H1 -> mu+   mu-)
62      2.28882843E-06  2      3      -3  # GAM(H1 -> s      sb )
63      4.96099310E-04  2      4      -4  # GAM(H1 -> c      cb )
64 #
65 #      PDG
66 NLO DECAY WIDTH 25  # H1 non-zero NLO EW decay widths of on-shell and non-loop
    induced decays
67      7      # Renormalization Scheme Number

```

68	0.90516083E+00	#	Corresponding mixing angle	alpha1	
69	-0.15530063E+00	#	Corresponding mixing angle	alpha2	
70	-0.15445673E+01	#	Corresponding mixing angle	alpha3	
71	0.13029492E+01	#	Corresponding tan(beta)		
72	0.18277800E+06	#	Corresponding m_12^2		
73	0.24408400E+04	#	Corresponding v_S		
74	#	WIDTH	NDA	ID1	ID2
75	5.69902477E-03	2	5	-5	# GAM(H1 -> b bb)
76	2.54863852E-04	2	-15	15	# GAM(H1 -> tau+ tau-)
77	8.82552598E-07	2	-13	13	# GAM(H1 -> mu+ mu-)
78	2.23273559E-06	2	3	-3	# GAM(H1 -> s sb)
79	4.76534904E-04	2	4	-4	# GAM(H1 -> c cb)
80	#				
81	#	PDG			
82	LO DECAY WIDTH	35	#	H2 non-zero LO EW decay widths of on-shell and non-loop induced decays	
83		7	#	Renormalization Scheme Number	
84	0.90516083E+00	#	Corresponding mixing angle	alpha1	
85	-0.15530063E+00	#	Corresponding mixing angle	alpha2	
86	-0.15445673E+01	#	Corresponding mixing angle	alpha3	
87	0.13029492E+01	#	Corresponding tan(beta)		
88	0.16715360E+06	#	Corresponding m_12^2		
89	0.24408400E+04	#	Corresponding v_S		
90	#	WIDTH	NDA	ID1	ID2
91	4.91587719E-04	2	5	-5	# GAM(H2 -> b bb)
92	2.21038791E-05	2	-15	15	# GAM(H2 -> tau+ tau-)
93	7.81789109E-08	2	-13	13	# GAM(H2 -> mu+ mu-)
94	1.89605235E-07	2	3	-3	# GAM(H2 -> s sb)
95	2.08960570E-05	2	4	-4	# GAM(H2 -> c cb)
96	1.17815813E-01	2	24	-24	# GAM(H2 -> W+ W-)
97	5.18688369E-02	2	23	23	# GAM(H2 -> Z Z)
98	8.19267433E-02	2	25	25	# GAM(H2 -> H1 H1)
99	#				
100	#	PDG			
101	NLO DECAY WIDTH	35	#	H2 non-zero NLO EW decay widths of on-shell and non-loop induced decays	
102		7	#	Renormalization Scheme Number	
103	0.90516083E+00	#	Corresponding mixing angle	alpha1	
104	-0.15530063E+00	#	Corresponding mixing angle	alpha2	
105	-0.15445673E+01	#	Corresponding mixing angle	alpha3	
106	0.13029492E+01	#	Corresponding tan(beta)		
107	0.16715360E+06	#	Corresponding m_12^2		
108	0.24408400E+04	#	Corresponding v_S		
109	#	WIDTH	NDA	ID1	ID2
110	4.61989652E-04	2	5	-5	# GAM(H2 -> b bb)
111	2.06423964E-05	2	-15	15	# GAM(H2 -> tau+ tau-)
112	7.13757478E-08	2	-13	13	# GAM(H2 -> mu+ mu-)
113	1.81751713E-07	2	3	-3	# GAM(H2 -> s sb)
114	2.00090392E-05	2	4	-4	# GAM(H2 -> c cb)
115	1.18674571E-01	2	24	-24	# GAM(H2 -> W+ W-)
116	5.20176206E-02	2	23	23	# GAM(H2 -> Z Z)
117	1.16353468E-01	2	25	25	# GAM(H2 -> H1 H1)
118	#				
119	#	PDG			
120	LO DECAY WIDTH	38	#	H3 non-zero LO EW decay widths of on-shell and non-loop induced decays	
121		7	#	Renormalization Scheme Number	
122	0.90516083E+00	#	Corresponding mixing angle	alpha1	

123	-0.15530063E+00	#	Corresponding mixing angle	alpha2	
124	-0.15445673E+01	#	Corresponding mixing angle	alpha3	
125	0.13029492E+01	#	Corresponding tan(beta)		
126	0.15169628E+06	#	Corresponding m_12^2		
127	0.24408400E+04	#	Corresponding v_S		
128	#	WIDTH	NDA	ID1	ID2
129	5.08405647E-02	2	5	-5	# GAM(H3 -> b bb)
130	2.28327056E-03	2	-15	15	# GAM(H3 -> tau+ tau-)
131	8.07416876E-06	2	-13	13	# GAM(H3 -> mu+ mu-)
132	1.95820644E-05	2	3	-3	# GAM(H3 -> s sb)
133	1.64155530E-03	2	4	-4	# GAM(H3 -> c cb)
134	1.45228111E+01	2	6	-6	# GAM(H3 -> t tb)
135	1.90555811E-02	2	24	-24	# GAM(H3 -> W+ W-)
136	9.27982692E-03	2	23	23	# GAM(H3 -> Z Z)
137	2.64224042E-02	2	25	25	# GAM(H3 -> H1 H1)
138	7.10491271E-03	2	25	35	# GAM(H3 -> H1 H2)
139	5.95224546E-04	2	35	35	# GAM(H3 -> H2 H2)
140	#				
141	#	PDG			
142	NLO DECAY WIDTH	35	# H3 non-zero NLO EW decay widths of on-shell and non-loop induced decays		
143		7	# Renormalization Scheme Number		
144	0.90516083E+00	#	Corresponding mixing angle	alpha1	
145	-0.15530063E+00	#	Corresponding mixing angle	alpha2	
146	-0.15445673E+01	#	Corresponding mixing angle	alpha3	
147	0.13029492E+01	#	Corresponding tan(beta)		
148	0.15169628E+06	#	Corresponding m_12^2		
149	0.24408400E+04	#	Corresponding v_S		
150	#	WIDTH	NDA	ID1	ID2
151	4.53131283E-02	2	5	-5	# GAM(H3 -> b bb)
152	2.04236084E-03	2	-15	15	# GAM(H3 -> tau+ tau-)
153	7.05350937E-06	2	-13	13	# GAM(H3 -> mu+ mu-)
154	1.80796084E-05	2	3	-3	# GAM(H3 -> s sb)
155	1.49442795E-03	2	4	-4	# GAM(H3 -> c cb)
156	1.39109721E+01	2	6	-6	# GAM(H3 -> t tb)
157	3.36429049E-02	2	24	-24	# GAM(H3 -> W+ W-)
158	9.57215530E-03	2	23	23	# GAM(H3 -> Z Z)
159	9.18285329E-02	2	25	25	# GAM(H3 -> H1 H1)
160	7.26758144E-02	2	25	35	# GAM(H3 -> H1 H2)
161	4.43844464E-03	2	35	35	# GAM(H3 -> H2 H2)
162	#				
163	#	PDG			
164	LO DECAY WIDTH	36	# A non-zero LO EW decay widths of on-shell and non-loop induced decays		
165		7	# Renormalization Scheme Number		
166	0.90516083E+00	#	Corresponding mixing angle	alpha1	
167	-0.15530063E+00	#	Corresponding mixing angle	alpha2	
168	-0.15445673E+01	#	Corresponding mixing angle	alpha3	
169	0.13029492E+01	#	Corresponding tan(beta)		
170	0.14967045E+06	#	Corresponding m_12^2		
171	0.24408400E+04	#	Corresponding v_S		
172	#	WIDTH	NDA	ID1	ID2
173	5.79978912E-02	2	5	-5	# GAM(A -> b bb)
174	2.60415812E-03	2	-15	15	# GAM(A -> tau+ tau-)
175	9.20859876E-06	2	-13	13	# GAM(A -> mu+ mu-)
176	2.23333665E-05	2	3	-3	# GAM(A -> s sb)
177	1.75923370E-03	2	4	-4	# GAM(A -> c cb)
178	2.25987982E+01	2	6	-6	# GAM(A -> t tb)

179	1.30279359E-02	2	23	25	# GAM(A -> Z	H1)
180	5.46626471E-02	2	23	35	# GAM(A -> Z	H2)
181	#						
182	#	PDG					
183	NLO DECAY WIDTH	36	# A non-zero NLO EW decay widths of on-shell and non-loop induced decays				
184		7	# Renormalization Scheme Number				
185	0.90516083E+00		# Corresponding mixing angle alpha1				
186	-0.15530063E+00		# Corresponding mixing angle alpha2				
187	-0.15445673E+01		# Corresponding mixing angle alpha3				
188	0.13029492E+01		# Corresponding tan(beta)				
189	0.14967045E+06		# Corresponding m_12^2				
190	0.24408400E+04		# Corresponding v_S				
191	#	WIDTH	NDA	ID1	ID2		
192	5.06096011E-02	2	5	-5	# GAM(A -> b	bb)
193	2.27068012E-03	2	-15	15	# GAM(A -> tau+	tau-)
194	7.83691964E-06	2	-13	13	# GAM(A -> mu+	mu-)
195	2.01240631E-05	2	3	-3	# GAM(A -> s	sb)
196	1.56752139E-03	2	4	-4	# GAM(A -> c	cb)
197	2.15891704E+01	2	6	-6	# GAM(A -> t	tb)
198	1.90987881E-02	2	23	25	# GAM(A -> Z	H1)
199	5.28305065E-02	2	23	35	# GAM(A -> Z	H2)
200	#						
201	#	PDG					
202	LO DECAY WIDTH	37	# H+ non-zero LO EW decay widths of on-shell and non-loop induced decays				
203		7	# Renormalization Scheme Number				
204	0.90516083E+00		# Corresponding mixing angle alpha1				
205	-0.15530063E+00		# Corresponding mixing angle alpha2				
206	-0.15445673E+01		# Corresponding mixing angle alpha3				
207	0.13029492E+01		# Corresponding tan(beta)				
208	0.14872861E+06		# Corresponding m_12^2				
209	0.24408400E+04		# Corresponding v_S				
210	#	WIDTH	NDA	ID1	ID2		
211	1.06619789E-04	2	4	-5	# GAM(H+ -> c	bb)
212	2.73731960E-03	2	-15	16	# GAM(H+ -> tau+	nu_tau)	
213	9.67946175E-06	2	-13	14	# GAM(H+ -> mu+	nu_mu)	
214	7.51082775E-07	2	2	-5	# GAM(H+ -> u	bb)
215	1.19203342E-06	2	2	-3	# GAM(H+ -> u	sb)
216	9.37817403E-05	2	4	-1	# GAM(H+ -> c	db)
217	1.77450991E-03	2	4	-3	# GAM(H+ -> c	sb)
218	2.43287119E+01	2	6	-5	# GAM(H+ -> t	bb)
219	3.97057154E-02	2	6	-3	# GAM(H+ -> t	sb)
220	1.82863984E-03	2	6	-1	# GAM(H+ -> t	db)
221	1.55122312E-02	2	24	25	# GAM(H+ -> W+	H1)
222	6.87113183E-02	2	24	35	# GAM(H+ -> W+	H2)
223	9.22266037E-01	2	24	38	# GAM(H+ -> W+	H3)
224	#						
225	#	PDG					
226	NLO DECAY WIDTH	37	# H+ non-zero NLO EW decay widths of on-shell and non-loop induced decays				
227		7	# Renormalization Scheme Number				
228	0.90516083E+00		# Corresponding mixing angle alpha1				
229	-0.15530063E+00		# Corresponding mixing angle alpha2				
230	-0.15445673E+01		# Corresponding mixing angle alpha3				
231	0.13029492E+01		# Corresponding tan(beta)				
232	0.14872861E+06		# Corresponding m_12^2				
233	0.24408400E+04		# Corresponding v_S				

234	#	WIDTH	NDA	ID1	ID2		
235		8.80303489E-05	2	4	-5	# GAM($H^+ \rightarrow c$	bb)
236		2.28223592E-03	2	-15	16	# GAM($H^+ \rightarrow \tau u +$	nu_τ)
237		7.86793427E-06	2	-13	14	# GAM($H^+ \rightarrow \mu u +$	nu_μ)
238		5.46640859E-07	2	2	-5	# GAM($H^+ \rightarrow u$	bb)
239		1.03550925E-06	2	2	-3	# GAM($H^+ \rightarrow u$	sb)
240		8.15801254E-05	2	4	-1	# GAM($H^+ \rightarrow c$	db)
241		1.54792859E-03	2	4	-3	# GAM($H^+ \rightarrow c$	sb)
242		2.23256376E+01	2	6	-5	# GAM($H^+ \rightarrow t$	bb)
243		3.64918286E-02	2	6	-3	# GAM($H^+ \rightarrow t$	sb)
244		1.67376283E-03	2	6	-1	# GAM($H^+ \rightarrow t$	db)
245		1.55776403E-02	2	24	25	# GAM($H^+ \rightarrow W^+$	$H1$)
246		6.27419005E-02	2	24	35	# GAM($H^+ \rightarrow W^+$	$H2$)
247		8.66487541E-01	2	24	38	# GAM($H^+ \rightarrow W^+$	$H3$)

As can be read off from the output file, the EW corrections reduce the decay widths of the SM-like Higgs boson H_1 . The relative NLO EW corrections, $\Delta^{\text{EW}} = (\Gamma^{\text{EW}} - \Gamma^{\text{LO}})/\Gamma^{\text{LO}}$, range between -6.5 and -2.5% for the decays $\Gamma(H_1 \rightarrow \mu^+\mu^-)$ and $\Gamma(H_1 \rightarrow s\bar{s})$, respectively. For the heavier Higgs bosons the NLO EW corrections can be considerably larger. We remind the reader also that only LO and NLO EW-corrected decay widths are given out for on-shell and non-loop induced decays.

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