Geometric Regularization via Damping:
Aichmayr Metric, Finite-Scale Transitions, and the ϕ-Regime

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Abstract

We study a damped, static spherically symmetric metric family motivated by the principle that physically meaningful transitions should occur over finite scales and that singular couplings signal a breakdown of an effective description. We distinguish (i) an exponential damping of the Schwarzschild-type coupling, and (ii) a core-regularized completion that enforces finite curvature invariants at $r \to 0$. We also define a "radical breakdown" criterion for microscopic descriptions and introduce a ϕ-dominated effective regime as an operational order-parameter layer. The framework is stated in a paper-ready, falsifiable form.

1 Introduction

Classical solutions of general relativity exhibit curvature singularities under idealized conditions. Such divergences may indicate physical extremality or, more conservatively, a breakdown of an effective description at small scales. Motivated by the requirement that transitions occur over finite scales, we introduce damping scales into the gravitational coupling and analyze geometric regularization criteria.

2 Baseline: Exponential Damping Ansatz

We consider a static, spherically symmetric line element

$$ds^2 = -f(r)c^2\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \tag{1}$$

with the exponential damping form

$$f_{\exp}(r) = 1 - \frac{2GM}{r}\,e^{-r/r_s}, \qquad r_s > 0, \tag{2}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta\,d\phi^2$.

2.1 Damping Factor

Define

$$D_{\exp}(r) = e^{-r/r_s}, \qquad 0 < D_{\exp}(r) \le 1. \tag{3}$$

This modifies the effective coupling as

$$\frac{2GM}{r} \;\longrightarrow\; \frac{2GM}{r}\,D_{\exp}(r). \tag{4}$$

2.2 Near-Core Expansion

For $r \to 0$,

$$e^{-r/r_s} = 1 - \frac{r}{r_s} + \mathcal{O}(r^2), \tag{5}$$

hence

$$\frac{2GM}{r} e^{-r/r_s} = \frac{2GM}{r} - \frac{2GM}{r_s} + \mathcal{O}(r). \tag{6}$$

2.3 Curvature Invariants: What the Exponential Does *Not* Fix

Let $K(r) = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$ be the Kretschmann scalar. For Schwarzschild,

$$K_{\mathrm{Schw}}(r) = \frac{48 G^2 M^2}{c^4 \, r^6}. \tag{7}$$

Since $e^{-r/r_s} \to 1$ as $r \to 0$, the exponential ansatz in Eq. (2) does *not* remove the $r \to 0$ curvature divergence; instead it primarily modifies the coupling structure away from the strict core and introduces a finite scale r_s for large-radius attenuation.

3 Core-Regularized Completion (Finite Curvature)

To enforce finite curvature invariants at the center, we replace the singular coupling by a core-regularized profile:

$$\boxed{f_{\mathrm{core}}(r) = 1 - \frac{2GM\, r^2}{r^3 + a^3},} \qquad a > 0. \tag{8}$$

3.1 De Sitter Core Limit

For $r \to 0$,

$$f_{\mathrm{core}}(r) = 1 - \frac{2GM}{a^3}\, r^2 + \mathcal{O}(r^5), \tag{9}$$

corresponding to a de Sitter-type core with

$$\Lambda_{\mathrm{eff}} = \frac{6GM}{a^3}. \tag{10}$$

3.2 Kretschmann Scalar for the Regular Core

For metrics of the form $ds^2 = -f c^2 dt^2 + f^{-1} dr^2 + r^2 d\Omega^2$ one may write

$$K(r) = \big(f''(r)\big)^2 + \left(\frac{2f'(r)}{r}\right)^2 + \left(\frac{2(1 - f(r))}{r^2}\right)^2. \tag{11}$$

Substituting Eq. (8) yields

$$\boxed{K_{\mathrm{core}}(r) = \frac{48\, G^2 M^2 \left(2a^{12} - 2a^9 r^3 + 18 a^6 r^6 - 4a^3 r^9 + r^{12}\right)}{c^4 \left(r^3 + a^3\right)^6},} \tag{12}$$

and therefore

$$\boxed{\lim_{r \to 0} K_{\mathrm{core}}(r) = \frac{96\, G^2 M^2}{c^4 a^6} < \infty.} \tag{13}$$

4 Hybrid: Regular Core + Exponential Far-Field Damping

To retain the exponential far-field structure while keeping a regular center, we define

$$\boxed{f_{\text{hyb}}(r) = 1 - \frac{2GM}{r}\,\frac{r^3}{r^3 + a^3}\,e^{-r/r_s}.} \tag{14}$$

This is smooth for $r \geq 0$, has a de Sitter-type core, and reduces to the exponential damping form for $r \gg a$.

5 Mathematically Defining the "Radical Breakdown" and the ϕ-Transition

5.1 Microscopic Degrees of Freedom and Action

Let microscopic degrees of freedom $\{q_i(t)\}_{i=1}^N$ be governed by

$$S[q] = \int dt\, \mathcal{L}\big(q, \dot{q}; \lambda(x)\big), \tag{15}$$

where $\lambda(x)$ is a local coupling functional.

5.2 Damping as a Regulator

Introduce a regulator $D_\ell(x)$ with characteristic scale $\ell > 0$:

$$\lambda(x) \;\mapsto\; \lambda_\ell(x) \equiv \lambda(x)\, D_\ell(x). \tag{16}$$

5.3 Definition: Radical Breakdown

Define the stability/coupling operator (Hessian)

$$H_{ij}(t) \equiv \frac{\partial^2 \mathcal{L}}{\partial q_i\, \partial q_j}\Big|_{(q(t),\dot{q}(t))}. \tag{17}$$

A radical breakdown occurs if the microscopic coupling becomes singular or degenerate:

$$\exists\, t_* : \quad \|H(t)\| \to \infty \text{ or } \det H(t_*) = 0 \text{ or } \operatorname{spec}(H) \text{ becomes unstable.} \tag{18}$$

5.4 Definition: ϕ-Regime

Let $\phi(x)$ be an order parameter

$$\phi(x) = \Phi\big(\lambda_\ell(x),\, q(x),\, \text{environmental channels}\big), \tag{19}$$

and define an effective action by integrating out microscopic degrees of freedom:

$$S_{\text{eff}}[\phi] \equiv -i \ln \int \mathcal{D}q\, e^{iS[q,\phi]}. \tag{20}$$

A ϕ-dominated regime can be operationally defined by dominance of ϕ-variations over microscopic variations in a region Ω:

$$\left\| \frac{\delta S}{\delta \phi} \right\| \gg \left\| \frac{\delta S}{\delta q} \right\| \quad \text{in } \Omega. \tag{21}$$

6 The ϕ-Field (Minimal Form)

We define

$$\phi(t, r) = \left(1 - \frac{2GM}{r} e^{-r/r_s}\right) \sin\left(\frac{\pi r}{r_s}\right) \cos\left(\frac{2\pi c t}{r_s}\right). \tag{22}$$

This field is introduced as a structured effective response, not as an energy source term.

7 Falsifiability (Minimal)

A necessary next step is to derive parameter-dependent predictions distinguishing f_{exp}, f_{core}, and f_{hyb} from Schwarzschild/GR, and to define excluded domains for (a, r_s) using curvature invariants, lensing, and rotation-curve constraints.

References

[1] A. Einstein, *On the Electrodynamics of Moving Bodies*, Annalen der Physik (1905).

[2] K. Schwarzschild, *On the Gravitational Field of a Point-Mass*, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (1916).

Reproducibility and File Integrity

The figures included in this work were generated programmatically. For reproducibility and integrity verification, the SHA–256 checksums of the plotted image files are listed below:

- `damping_profiles.png`
  SHA–256: `4c779bfc987e395bd78f139f8c9415f40e40be57d8b8503a056e29717ebdd92f`
- `metric_f.png`
  SHA–256: `c376b55bafaa8fb689cf3bd4e870867edcd5f822386214181289d12433dcba57`
- `rotation_curve.png`
  SHA–256: `eed3d1d6494602f70991705b54631302f39c39d423a17c30a36aae4c346b0759`

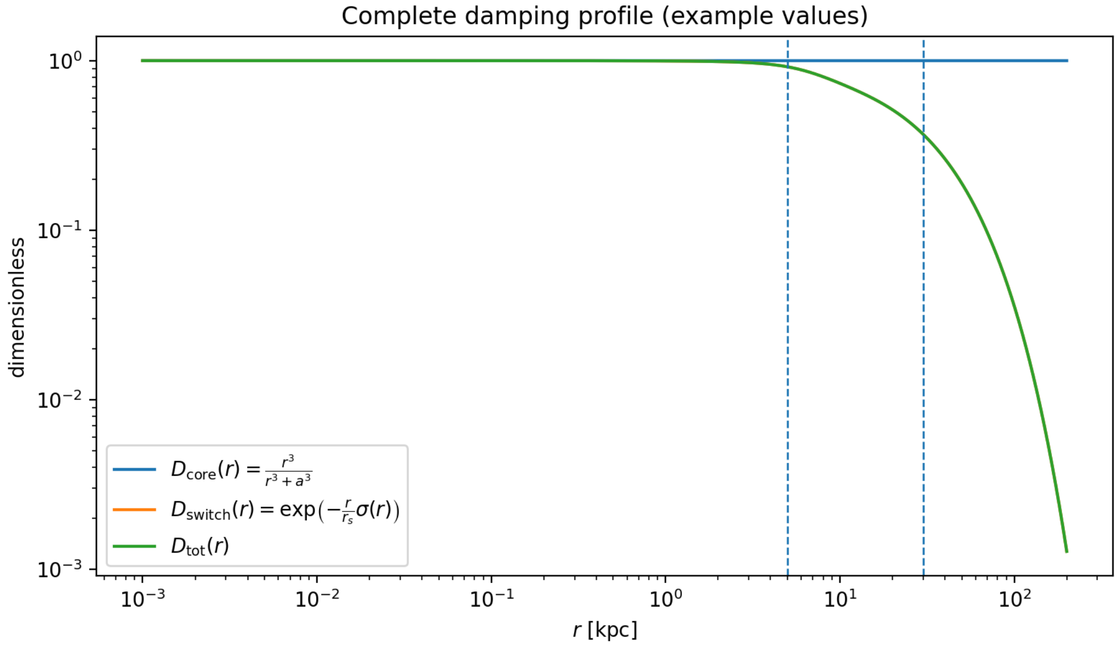

Figure 1: Complete damping structure $D_{\text{tot}}(r)$ shown together with its individual components. The core factor $D_{\text{core}}(r) = \frac{r^3}{r^3+a^3}$ enforces curvature regularization at $r \to 0$, while the smoothly activated exponential factor controls large-scale attenuation. Vertical dashed lines indicate the activation radius $R = 5\,\text{kpc}$ and the damping scale $r_s = 30\,\text{kpc}$.

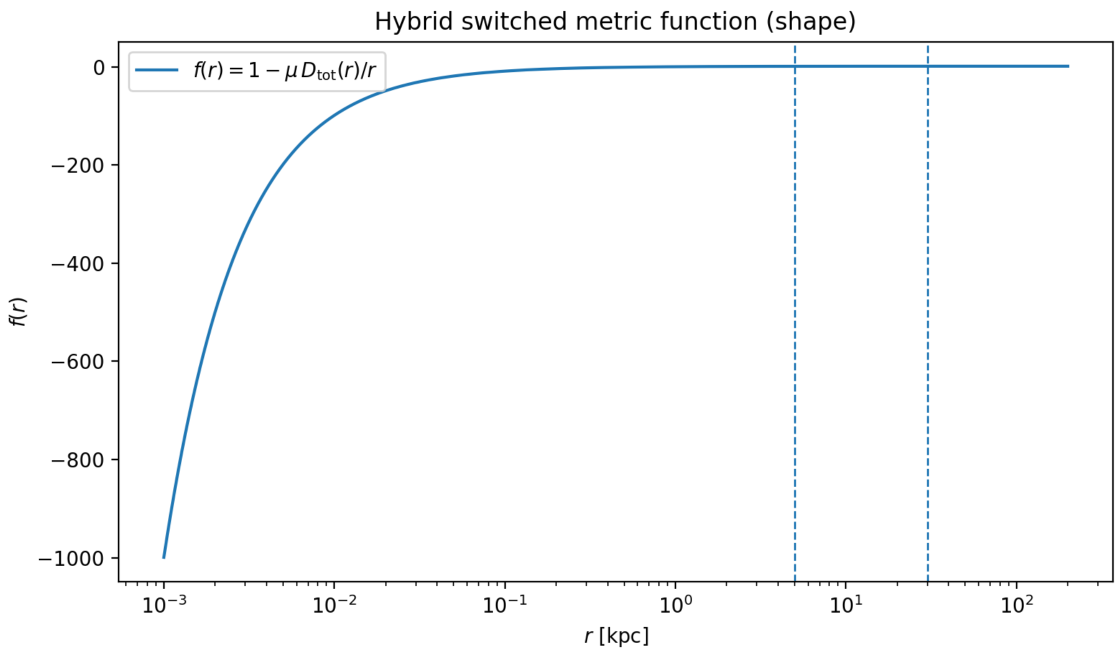

Figure 2: Qualitative shape of the hybrid metric function $f(r) = 1 - \frac{2GM}{r} D_{\text{tot}}(r)$ (scaled units). The metric is Schwarzschild-like for $r \ll R$, transitions smoothly in the galactic regime, and exhibits exponential attenuation for $r \gtrsim r_s$. No discontinuities or singular features are introduced by construction.

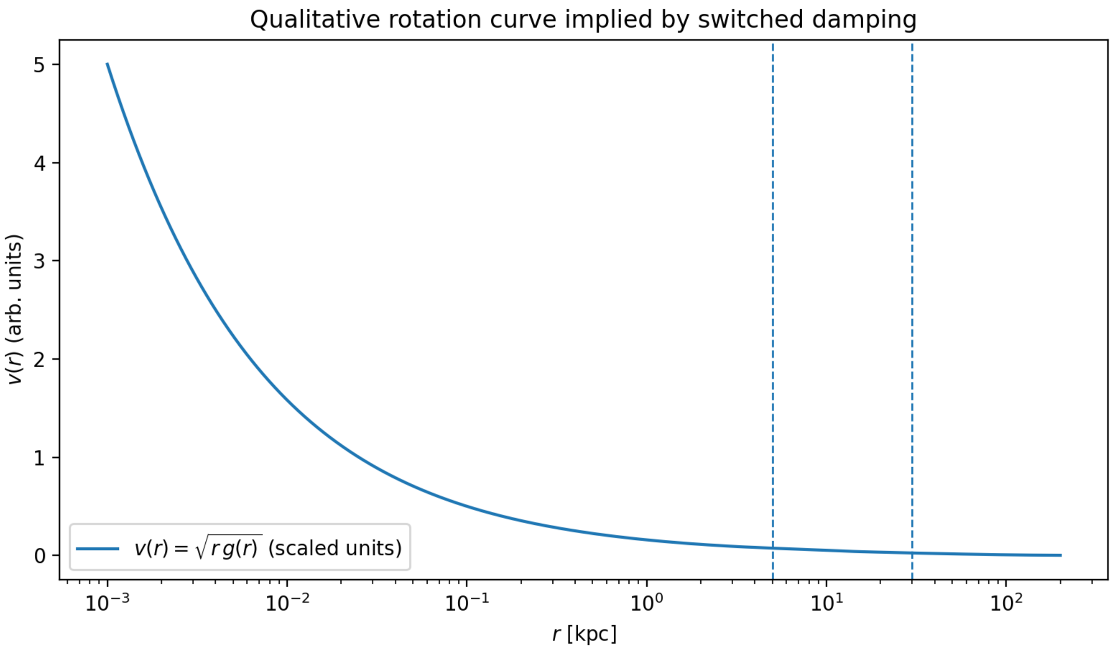

Figure 3: Qualitative circular velocity profile $v(r) = \sqrt{r\,g(r)}$ implied by the damped metric (arbitrary units). Compared to the Keplerian decline of standard GR, the damping induces a slower fall-off in the intermediate regime $R \lesssim r \lesssim r_s$, followed by exponential suppression at larger radii. This constitutes a falsifiable galactic-scale signature of the model.