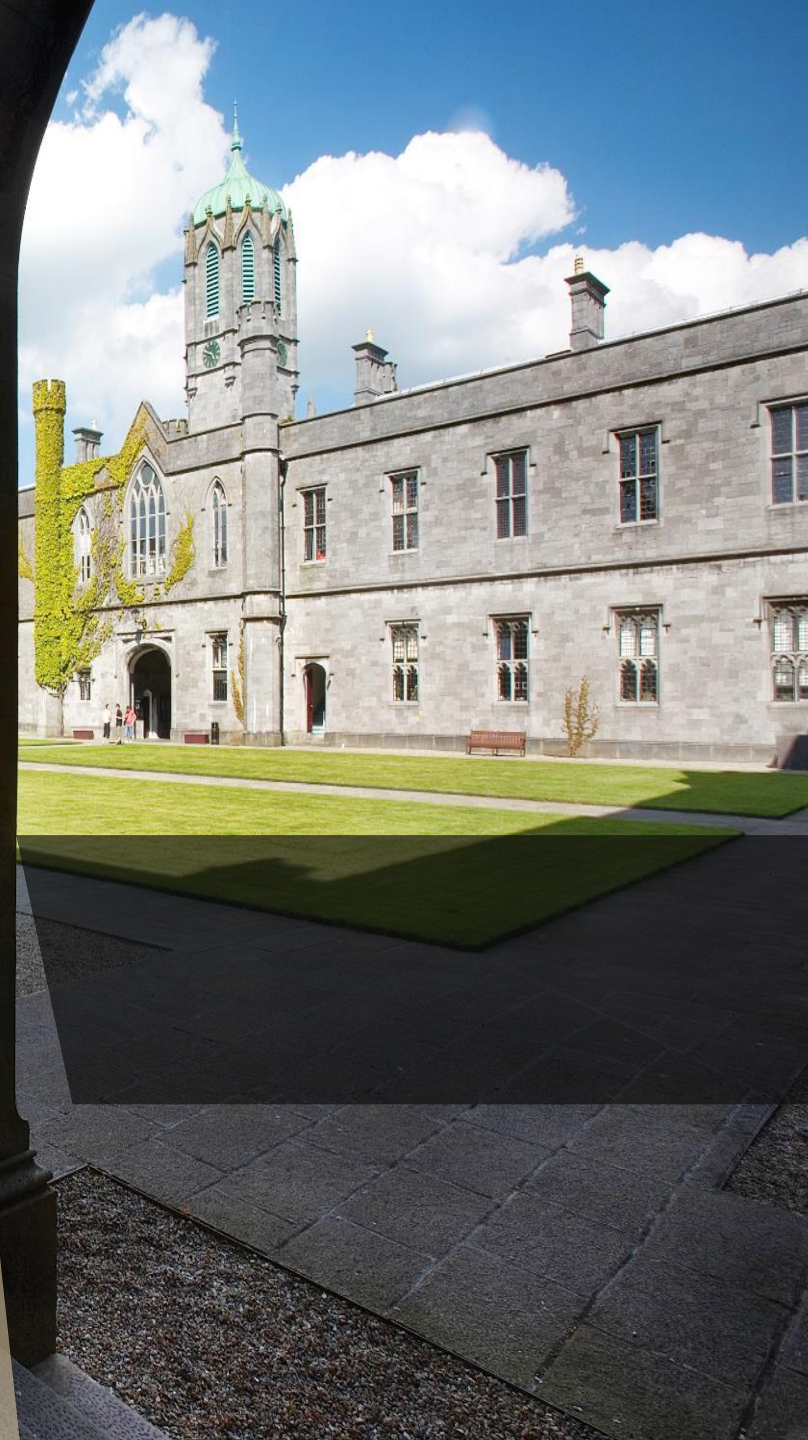
Introduction to NLP

Tagging and Hidden Markov Models

Dr. John McCrae

Data Science Institute, NUI Galway



Overview

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Summary

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Probability

We create probability over events **p(A)**

Combinations of events **p(A∩B)**

Events given other events **p(A|B∩C)**

Probability of a sentence

p("I like green eggs") is the probability of the intersection of 4 events:

$$\mathbf{p(w_1=\text{"I"} \cap w_2=\text{"like"} \cap w_3=\text{"green"} \cap w_4=\text{"eggs"})}$$

This can be hard to estimate for long sentences

Conditional Probabilities

We can express this in terms of conditional probabilities

p("I like green eggs") =
p("eggs"|"I like green") ×
p("green"|"I like") ×
p("like"|"I") ×
p("I")

N-gram approximations

We can express this in terms of conditional probabilities

p("I like green eggs") ≈
p("eggs"|"green") ×
p("green"|"like") ×
p("like"|"I") ×
p("I")

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Syntax

Goal of a language model is to predict whether a sentence is in English (or another language)

Syntax is important

Correct:	*Happy children play*	(**Adj Noun Verb)**
Incorrect:	*Children happy play*	**(Noun Adj Verb)**

Trees next week

Tags as probabilities

We now have two kinds of events:

$$p(w_1=\text{"happy"} \cap w_2=\text{"children"} \cap w_3=\text{"play"} \cap t_1=\text{Adj} \cap t_2=\text{Noun} \cap t_3=\text{Verb})$$

This is not possible to calculate for long sentences => Approximation

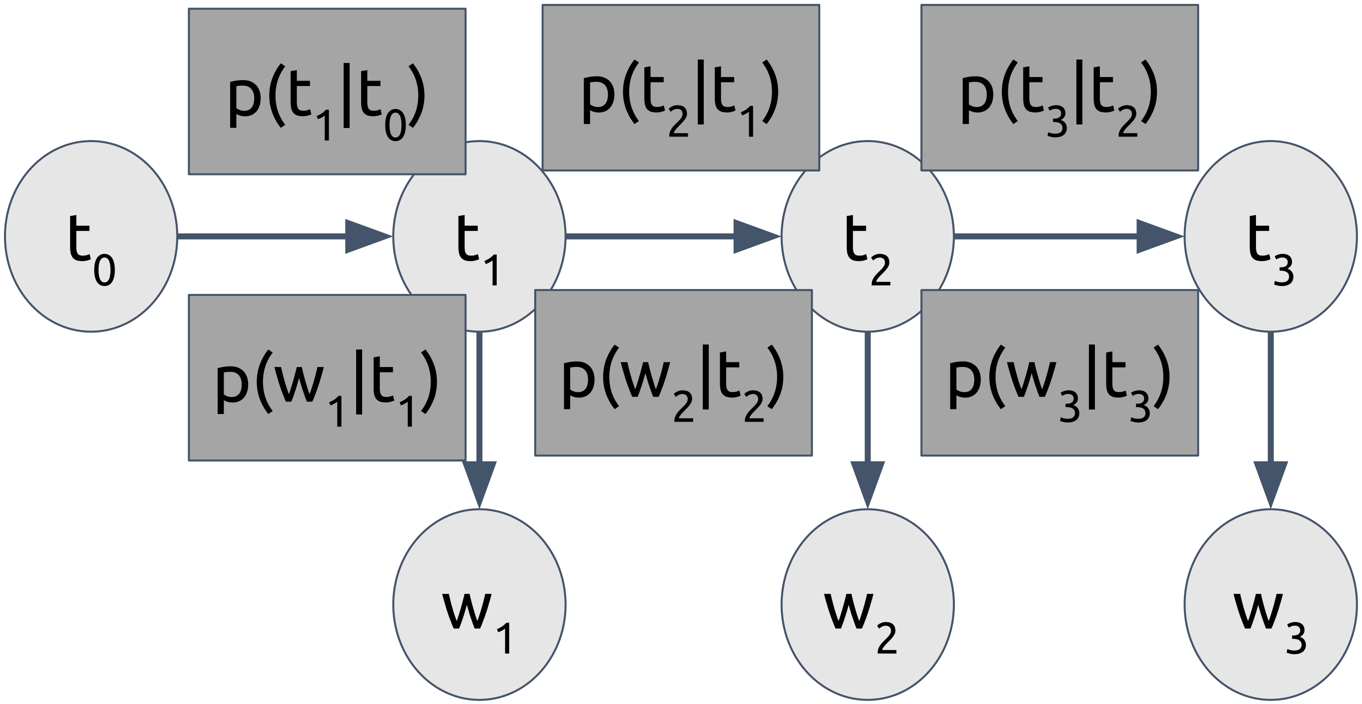
Hidden Markov Model (without approximation)

$$\begin{gathered}
p(w_1,w_2,w_3,t_1,t_2,t_3) = \\
p(w_3|t_3,t_2,t_1,w_1,w_2) \times \\
p(t_3|t_2,t_1,w_1,w_2) \times \\
p(w_2|t_2,t_1,w_1) \times \\
p(t_2|t_1,w_1) \times \\
p(w_1|t_1) \times \\
p(t_1)
\end{gathered}$$

Hidden Markov Model (with approximation)

$$p(w_1,w_2,w_3,t_1,t_2,t_3) \approx$$
$$p(w_3|t_3) \times$$
$$p(t_3|t_2) \times$$
$$p(w_2|t_2) \times$$
$$p(t_2|t_1) \times$$
$$p(w_1|t_1) \times$$
$$p(t_1)$$

Hidden Markov Model



Hidden Markov Model

The general form for this is:

$$p(w_1, \ldots, w_n, t_1, \ldots, t_n) \approx \prod_{i=1,\ldots,n} p(t_i|t_{i-1})p(w_i|t_i)$$

We introduce a variable t_0 which always has the same value ('START')

HMM Example

For "John likes Mary", the probability of tagging it as "NOUN VERB NOUN" is

$$\mathbf{p(w_1{=}John, w_2{=}likes, w_3{=}Mary, t_1{=}N, t_2{=}V, t_3{=}N) = p(N|V) \times p(V|N) \times p(N|Start) \times p(John|N) \times p(likes|V) \times p(Mary|N)}$$

Stochastic Transition Matrices

	V	**N**
Start	0.2	0.8
V	0.2	0.6
N	0.6	0.4

	John	***likes***	***Mary***
V	0.2	0.6	0.2
N	0.4	0.2	0.4

HMM Example

$$p(w_1{=}\text{John}, w_2{=}\text{likes}, w_3{=}\text{Mary}, t_1{=}N, t_2{=}V, t_3{=}N) =$$
$$p(N|V) \times p(V|N) \times p(N|\text{Start}) \times$$
$$p(\text{John}|N) \times p(\text{likes}|V) \times p(\text{Mary}|N) =$$
$$0.6 \times 0.6 \times 0.8 \times 0.4 \times 0.6 \times 0.4 = 0.028$$

Formal Definition

An HMM is a 4-tuple $\mu=(S,K,A,B)$

Set of states (tags)	$S=\{s_1(=\sigma),\ldots,s_N\}$
Output alphabet	$K=\{k_1,\ldots,k_M\}$
State transition probabilities	$A=\{a_{ij}\}\ i,j \in S$
Symbol Emission probabilities	$B=\{b_{ij}\}\ i \in S, j \in K$
State sequences (hidden)	$T=\{t_1, \ldots, t_T\}\ y_T \in S$
Output sequence (observed)	$W=\{w_1,\ldots,w_T\}\ x_T \in K$

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Three fundamental problems for HMMs

1. What is the probability of a sequence given an observation and a model?
   a. What is **P(DET N VBZ DET N|the cat chases the mouse, μ)**?

2. What is the probability of an observation given the model?
   a. What is **P(the cat chases the mouse|μ)**?

3. What is the model that maximizes the likelihood of the observed data and known sequences?
   a. What μ maximizes **P(the cat chases the mouse, DET N VBZ DET N|μ)**?
   b. What μ maximizes **P(the cat chases the mouse|μ)**?

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Tagging

Tagging is the process of assigning (exactly) one class to (each) word

Part-of-Speech Tagging

This	*is*	*an*	*example*
DT	VBZ	DT	NN

Named Entity Recognition

in	*New*	*York*	*City*	*yesterday*
O	B	I	I	O

HMMs for Tagging

How do we use HMMs for tagging?

$$\arg\max_{t_1,\ldots,t_n} p(t_1,\ldots,t_n|w_1,\ldots,w_n)$$

Example

If *S={V,N}*, what is the most likely tagging of "cats drink"?

p(N,N,cats,drink) = p(N|Start) × p(N|N) × p(cats|N) × p(drink|N)

p(N,N,cats,drink)	0.023
p(N,V,cats,drink)	0.034
p(V,N,cats,drink)	0.017
p(V,V,cats,drink)	0.021

Complexity is $\mathcal{O}(TN^T)$

Example

What about three words, e.g., "*cats drink milk*"?

p(N,N,N,cats,drink,milk) = p(N|Start) × p(N|N) × p(N|N)
p(cats|N) × p(drink|N) × p(milk|N)

p(V,N,N,cats,drink,milk) = p(V|Start) × p(N|V) × p(N|N)
p(cats|V) × p(drink|N) × p(milk|N)

Dynamic Programming

a.k.a., caching
a.k.a., DRY
a.k.a., remembering the results of previous calculations

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Recursive algorithm for HMMs

$$\pi_{s,i} = \max_{s' \in S} p(t_i|s')p(w_i|t_i)\pi_{s',i-1}$$

$$\max p(w_1, \ldots, w_n, t_1, \ldots, t_n) = \max \pi_{t_n,n}$$

Still exponential complexity

Example (Python)

```
def pi(s, i):
    if i == 0 && s == “start”:
    return 1.0
  else if i == 0:
    return 0.0
  return max(a(s,t) * b(w[i],s) * pi(t, i -1)
                  for t in states)
```

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Dynamic programming (Python)

```
def pi(s, i):
  if i == 0 && s == “start”:
    return 1.0
  else if i == 0:
    return 0.0
  else if mem[s,i]:
    return mem[s,i]
  else:
    p = max(a(s,t) * b(w[i],s) * pi(t, i -1)
            for t in states)
    mem[s,i] = p
    return p
```

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Viterbi algorithm

Set $\pi_{s,0}$=0 except for $\pi_{Start,0}$=1

Set y_s = []

For i from 1 to T

  For s ∈ S

    Set $\pi_{s,i} = \max_{t \in S} \pi_{t,i-1} p(s|t) p(w_i|s)$

   Append *t* to y_s

Return y_s + s where s

  maximizes $\pi_{s,T}$

Can be more efficiently implemented using only two vectors for $\pi_{s,i}$ and $\pi_{s,i-1}$

Viterbi Algorithm Example - Table

	John	*cat*	*has*	*a*
N	0.3	0.3	0.1	0.3
V	0.1	0.1	0.7	0.1

	p(N|.)	**p(V|.)**
N	0.7	0.3
V	0.5	0.5
Start	0.9	0.1

"John has"

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Viterbi Algorithm Example

Previous state (i=0)

Start	N	V
1.0	0.0	0.0
[]	[]	[]

Next state (i=1)

Start	N	V
0.0	max(0.9 × 0.3, 0.0, 0.0) = 0.27	max(0.1 × 0.1, 0.0, 0.0) = 0.01
	[Start]	[Start]

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Viterbi Algorithm Example

Previous state (i=1)

	N	V
	0.27	0.01
	[Start]	[Start]

Next state (i=2)

N

max(0.27 × 0.7 × 0.1,
0.01 × 0.5 × 0.1) = 0.019

[Start, N]

V

max(0.27 × 0.3 × 0.7,
0.01 × 0.5 × 0.7) = 0.057

[Start,N]

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Viterbi Algorithm Example

Final state:

As $\pi_{V,2} > \pi_{N,2}$
return [Start, N, V]

Complexity is $\mathcal{O}(TN^2)$

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HMM as a language model

Recall the law of total probability

$$p(A = a) = \sum_{b} p(A = a \cap B = b)$$

Hidden Markov Models can also work as language models

$$p(w_1, \ldots, w_n) = \sum_{t_1 \in S, \ldots, t_n \in S} p(w_1, \ldots, w_n, t_1, \ldots, t_n)$$

Calculating HMM LM probability

We can follow same principle of calculating all values

$$p(w_1, \ldots, w_n) = \sum_{t_1 \in S} \cdots \sum_{t_n \in S} \prod_{i=1,\ldots,n} p(t_i|t_{i-1})p(w_i|t_i)$$

Direct calculation of this is exponential.

Instead we modify the Viterbi algorithm.

Forward algorithm

Set $\alpha_{s,0}$=0 except for $\alpha_{Start,0}$=1

For i from 1 to T

For s ∈ S

Set $\alpha_{s,i} = \sum_{t \in S} \alpha_{t,i-1} p(s|t) p(w_i|s)$

Return $\Sigma\alpha_{s,T}$

$\alpha_{s,i}$ is the sum of probabilities up to state *s* at word *i*

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Forward algorithm Example

Previous state (i=0)

Start	N	V
1.0	0.0	0.0

Next state (i=1)

Start	N	V
0.0	$0.9 \times 0.3 + 0.0 + 0.0 = 0.27$	$0.1 \times 0.1 + 0.0 + 0.0 = 0.01$

Forward Algorithm Example

Previous state (i=1)

N	**V**
0.27	0.01

Next state (i=2)

N

$0.27 \times 0.7 \times 0.1 +$
$0.01 \times 0.5 \times 0.1 = 0.019$

V

$0.27 \times 0.3 \times 0.7 +$
$0.01 \times 0.5 \times 0.7 = 0.060$

Final **p = 0.019 + 0.060 = 0.0796**

Backward Algorithm

Alternatively we can calculate probabilities backwards:

$$\beta_{s,i} = P(t_i = s, w_{t+1} \dots w_T | \mu)$$

$$\beta_{s,T} = 1$$

$$\beta_{s,i} = \sum_{t \in S} \beta_{t,i+1} p(t|s) p(w_{i+1}|t)$$

These backward probabilities can be calculated using a dynamic programming approach much as for the forward variables.

Forward-Backward Procedure

If we combine these forward and backward variables, we can find the probability of any single tag

$$P(T_i = s, W|\mu) = \alpha_{s,i}\beta_{s,i}$$

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Supervised Learning

If we now the tags for some set of words we can directly obtain the probabilities by counting

$$p(s_i|s_j) = \frac{c(t_{i-1} = s_j, t_i = s_i)}{\sum_{s'} c(t_{i-1} = s_j, t_i = s')}$$

$$p(w|s) = \frac{c(w_i = w, t_i = s)}{c(t_i = s)}$$

Supervised Learning Example

V	**N**	**V**	**O**	**N**	**O**	**N**
Do	you	like	green	eggs	and	ham

N	**N**	**V**	**N**	**O**
Mr	green	eggs	you	on

N	**V**	**O**	**N**
you	are	like	ham

Counts

	N	V	O
Do	0	1	0
You	3	0	0
Like	0	1	1
Green	1	0	1
Eggs	1	1	0
And	0	0	1
Ham	2	0	0
Mr	1	0	0
on	0	0	1
are	0	1	0

	N	V	O
N	1	3	2
V	2	0	2
O	3	0	0
Start	2	1	0

Probabilities

	N	V	O
Do	0/8	1/4	0/4
You	3/8	0/4	0/4
Like	0/8	1/4	1/4
Green	1/8	0/4	1/4
Eggs	1/8	1/4	0/4
And	0/8	0/4	1/4
Ham	2/8	0/4	0/4
Mr	1/8	0/4	0/4
on	0/8	0/4	1/4
are	0/8	1/4	0/4

	N	V	O
N	1/6	3/6	2/6
V	2/4	0/4	2/4
O	3/3	0/3	0/3
Start	2/3	1/3	0/3

Unsupervised Learning

The **unsupervised** case is where we don't have tags

In the supervised case we set probabilities using observed counts

$$p(t|s) = \frac{c(t, s)}{c(s)}$$

In the unsupervised count we can instead use **expected counts**

$$p^*(t|s) = \frac{E[c(t, s)]}{E[c(s)]}$$

Expected Counts

If we throw a coin 4 times, we **expect** to see 2 heads on average

$$E[c(heads)] = \sum_{i=1}^{4} p(heads) = 2$$

We can say the same about our tags, e.g.,

$$E[c(s)] = \sum_{i} P(t_i = s)$$

Unsupervised probabilities

This means we can calculate probabilities in an unsupervised setting by simply using expectancies

$$p^*(s_i|s_j) = \frac{E[c(t_{i-1} = s_j, t_i = s_i)]}{\sum_{s'} E[c(t_{i-1} = s_j, t_i = s'))])}$$

$$p^*(w|s) = \frac{E[c(w_i = w, t_i = s_i)]}{E[c(t_i = s))])}$$

Updating probabilities

Problem: we need the transmission and emissions probabilities, *A, B*, in order to calculate $p^*(s_i|s_j)$ and $p^*(w|s)$, which we then use to update *A, B*! This is a chicken and egg problem.

Solution: Start with random A and B and iterate until convergence

NB. It can be a bit mind-blowing that this actually works, it can be shown formally why, but this is beyond the scope here

Baum-Welch Algorithm

1. Initialize A and B at random
2. Repeat until convergence
   a. Calculate $p^*(s_i|s_j)$ and $p^*(w|s)$ using A,B
   b. Update $A \leftarrow p^*(s_i|s_j)$, $B \leftarrow p^*(w|s)$

This algorithm is a special case of the Expectation-Maximization (E-M) algorithm

A detailed (non-examinable) walkthrough of the Baum-Welch algorithm is here: https://colab.research.google.com/drive/1iDWy-5T-84eD61LhLdL7vwvmfhaYNg7J?usp=sharing

Unsupervised learning

Finding the model that maximizes the likelihood of the data involves estimating the parameters of the HMM. This problem can be formulated as follows. Given a set of states *S*, an output alphabet T and observation *W*, compute the parameters *a* and b of the HMM so that the probability of the observed data is maximized:

$$\mu = argmax_{a,b} P(W|\mu)$$

Learning the model's parameters

There is no known analytic method to select the μ that maximizes the above formula.

We can use an iterative hill-climbing algorithm known as the **Baum-Welch algorithm** that is a special case of the so called **Expectation-Maximization** algorithm.

Expectation-Maximization Algorithm

Chicken and egg problem: To estimate the model we need counts of random variables that we can not directly observe.

The E-M Algorithm applies the following procedure:

1. Initialize the model
2. Estimate the counts
3. Re-compute the model given the estimated counts
4. Repeat 2 and 3 until convergence

Baum-Welch Algorithm

Recall our previous definition of forward and backward variables, and let

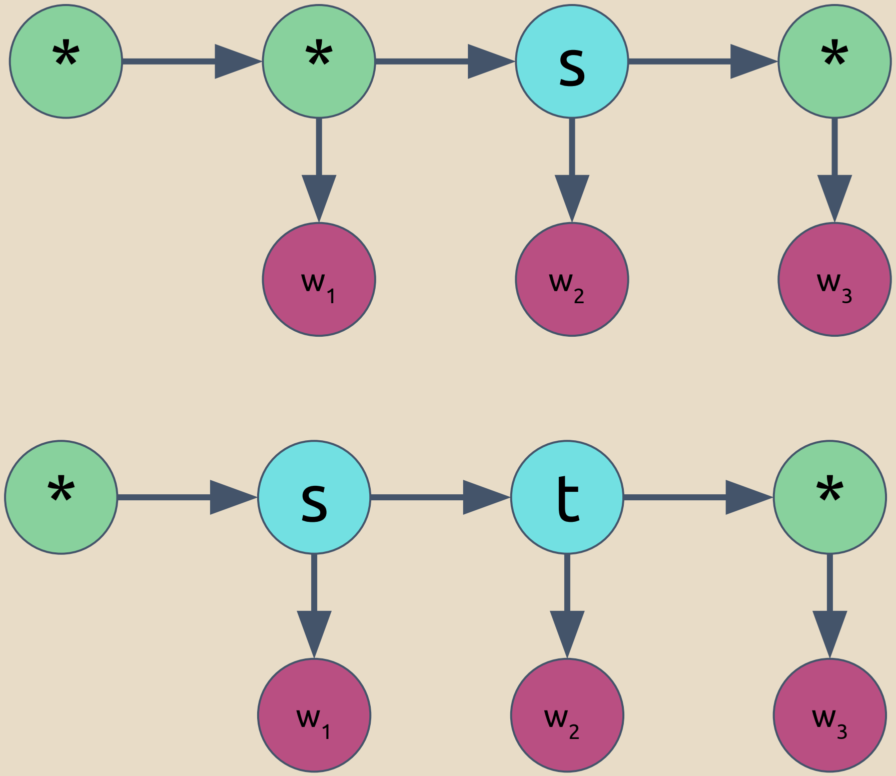
$$\gamma_i(t) = P(t_i = t | W, \mu) = \frac{\alpha_{t,i}\beta_{t,i}}{\sum_{s \in S} \alpha_{s,i}\beta_{s,j}}$$

(Probability of each state)

$$\xi_i(s,t) = P(t_i = s, t_{i+1} = t | W, \mu) = \frac{\alpha_{s,i} a_{s,t} \beta_{t,i+1} b_{t,w_{i+1}}}{\sum_{s' \in S} \sum_{t' \in T} \alpha_{s',i} a_{s',t'} \beta_{t',i+1} b_{t',w_{i+1}}}$$

(Transition probability)

Gamma and Xi



$\gamma_i(s)$ is the probability of all models where the state is $t_i=s$

$\xi_i(s)$ is the probability of all models where the state is $t_{i-1}=s$ and $t_i=t$

Baum-Welch Algorithm

The most likely estimators of our probabilities are thus:

$$a^*_{s,t} = \frac{\sum_{i=1}^{T-1} P(t_i = s, t_{i+1} = t | W, \mu)}{\sum_{i=1}^{T-1} P(t_i = s | W, \mu)} = \frac{\sum_{i=1}^{T-1} \xi_i(s,t)}{\sum_{i=1}^{T-1} \gamma_i(s)}$$

$$b^*_{s,w} = \frac{\sum_{i=1}^{T} \mathbf{1}(w_i = w)\gamma_i(s)}{\sum_{i=1}^{T} \gamma_i(s)}$$

$$\mathbf{1}(w_i = w) = \begin{cases} 1 & \text{if } w_i = w \\ 0 & \text{otherwise} \end{cases}$$

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Baum-Welch for Speech Recognition

In practice, Baum-Welch is most useful as a semi-supervised method

Some tags are observed, some not

Most frequently in speech recognition:

- Train the system on known data
- Adapt to an individual speaker

Semi-supervised Baum-Welch

Semi-supervised Baum-Welch, combines the supervised and unsupervised modes.

We use the expected counts for the unlabelled data and the observed counts for the labelled data, e.g.,

$$p^*(w|s) = \frac{E[c(w_i = w, t_i = s_i)] + c(w_i = w, t_i = s_i)}{E[c(t_i = s)]) + c(t_i = s)}$$

Similarly for emission probabilities

Example of semi-supervised Baum-Welch

We may have some corpus with WAV files and IPA Transcription

> iz ðɪs ɹaɪt

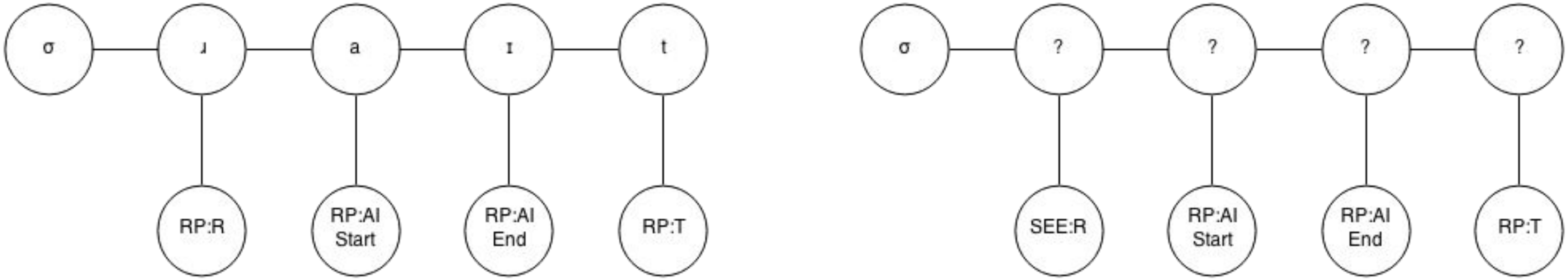
And wave transcription (not shown here)

We may then come across a speaker who pronounces it like this (R-labialization)

> iz ðɪs ʊaɪt

Example of semi-supervised Baum-Welch

By Baum-Welch training we can recognise `SEE:R` as coming from the class 'ɹ'



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Syntax is important for understanding sentences
Sequence model is a good model for:

- Part-of-speech
- Named Entity Recognition
- Speech Recognition

Sequence model needs to be approximated, HMM is most widely used approximation

$$p(t_1, \ldots, t_n, w_1, \ldots, w_n) \approx \prod p(t_i|t_{i-1})p(w_i|t_i)$$

There are an exponential number of *hidden states,* so naive application of this is exponential

Summary

Dynamic programming approaches enable polynomial time calculation of

- Most likely sequence (and its probability)
- Total probability of a sequence (language model)

HMMs are frequently learnt in a supervised setting (with known tags)

In some applications (speech recognition) unsupervised learning is applied

- Baum-Welch algorithm is a special case of expectation-maximization

Lab of this Week

Exercises on POS tagging using NLTK

QA