



Learning objectives

After completing this topic successfully, you will be able to ...

- 1. Explain what supervised learning is
- 2. Distinguish it from unsupervised learning and reinforcement learning
- 3. Describe in detail an algorithm for decision tree induction
- 4. Apply decision tree induction to a data set
- 5. List related algorithms
- Discuss high-level concepts such as choice of hypothesis language, overfitting, underfitting and noise

Reading: Russell & Norvig 3rd Ed, Chapter 18.18.4; Kelleher et al. Chapter 4



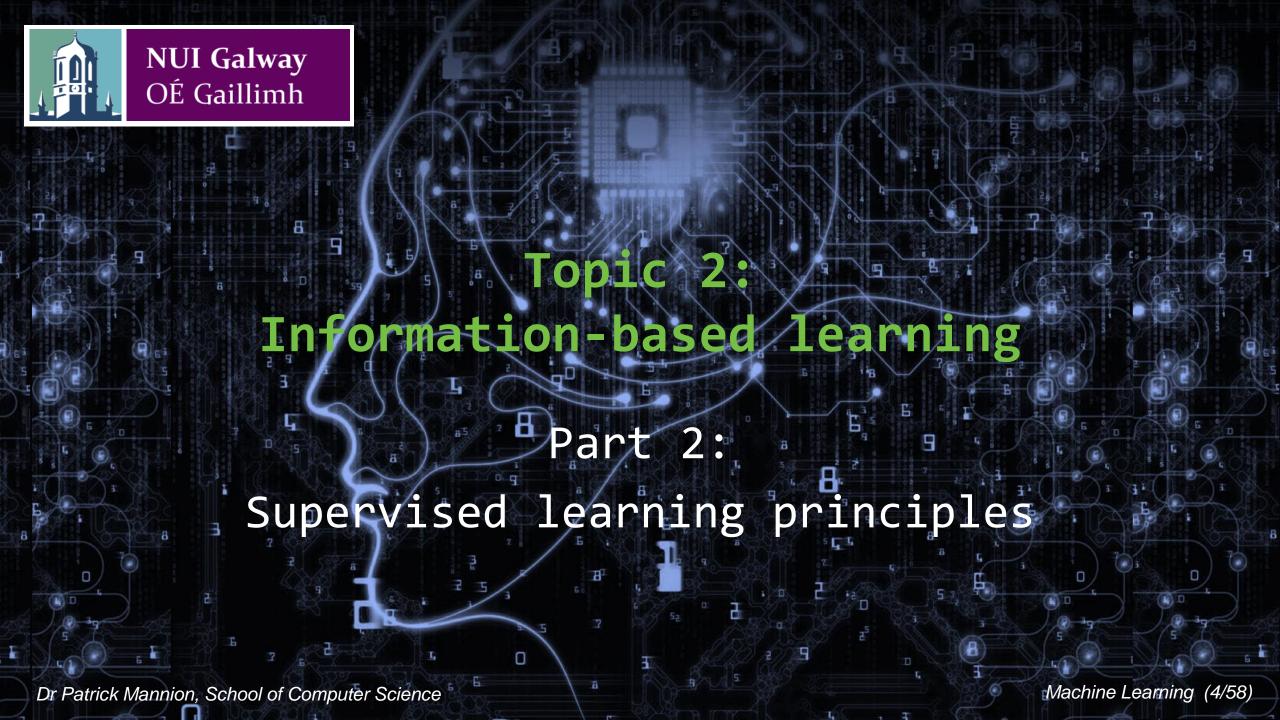
Overview of topic

This week:

- Introduction, learning objectives and overview
- 2. Supervised learning principles
- 3. Decision trees
- 4. Entropy
- 5. Information gain

Next week:

- 6. The ID3 algorithm
- 7. Issues in decision tree learning
- 8. ID3 extensions and related algorithms
- 9. Supervised learning considerations
- 10. Review of topic





Supervised learning: motivating examples

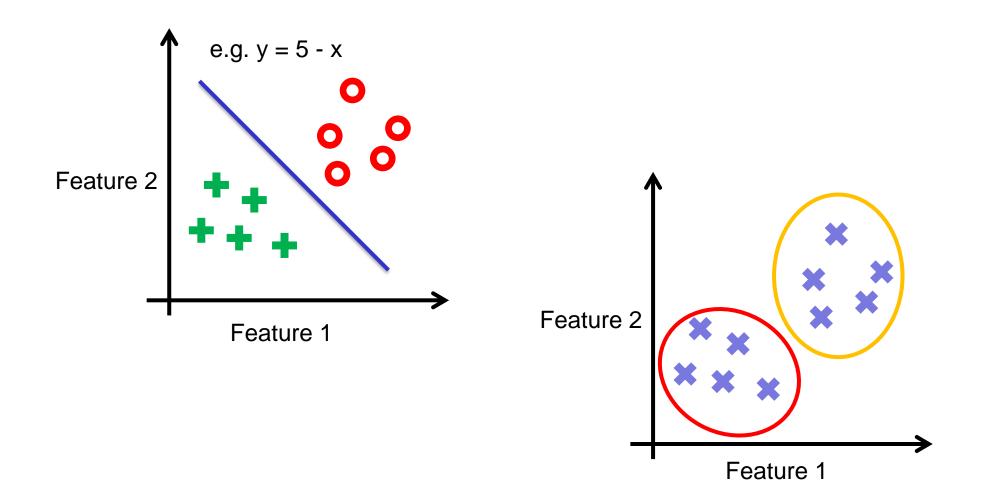
- Estimate sale price of a house, given past data of house sizes, locations and their prices
- 2. Before unlocking a tablet, determine whether a known user or somebody else is looking at the webcam
- Decide whether a chemical spectrum of a mixture has evidence of containing cocaine, based on other spectra with & without cocaine
- 4. Predict concentration of cocaine in mixture
- 5. Determine whether objects of interest are present in a scene if so, what are they? (relevant for autonomous vehicles and robotics, among other domains)

Key feature:

given "right answers" / "ground truth" as start point. Which tasks are classification, and which are regression?



Supervised vs. unsupervised learning





Supervised learning: task definition

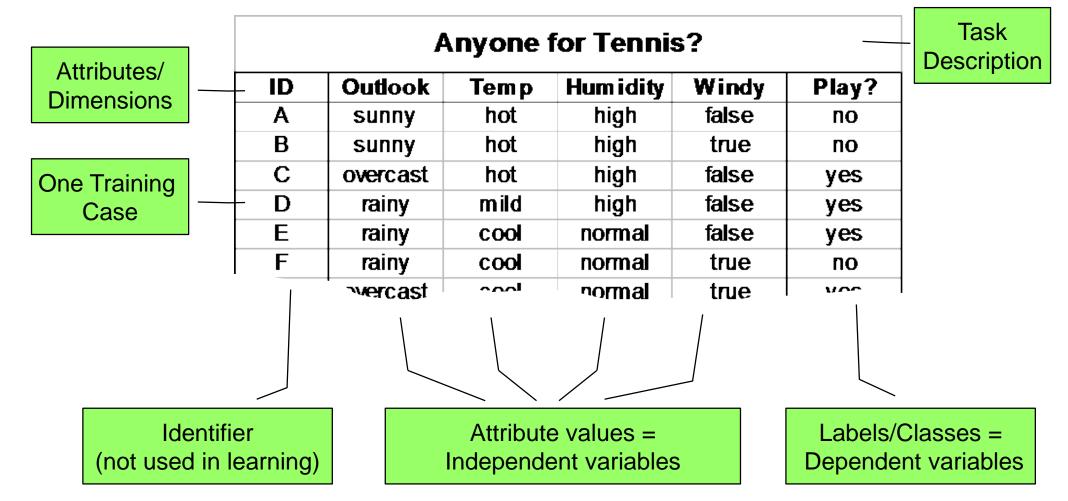
- Given examples, return function h (hypothesis) that approximates some 'true' function f that (hypothetically) generated the labels for the examples
 - Have set of examples, the training data:
 each has a label and a set of attributes that have known values
 - Consider *labels* (classes) to be *outputs* of some function *f*; the observed *attributes* are its *inputs*
 - Denote the attribute value inputs x, labels are their corresponding outputs f(x)
 - An example is a pair (x, f(x))
 - Function f is unknown; want to discover an approximation of it, h
 - Can use h to predict labels of new data: generalisation

Also known as Pure Inductive Learning – why?



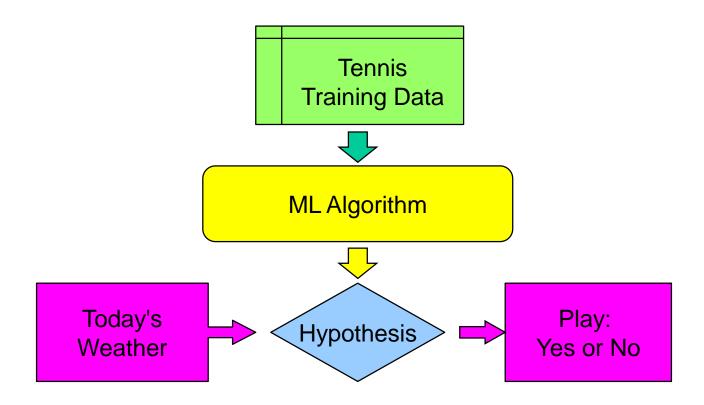


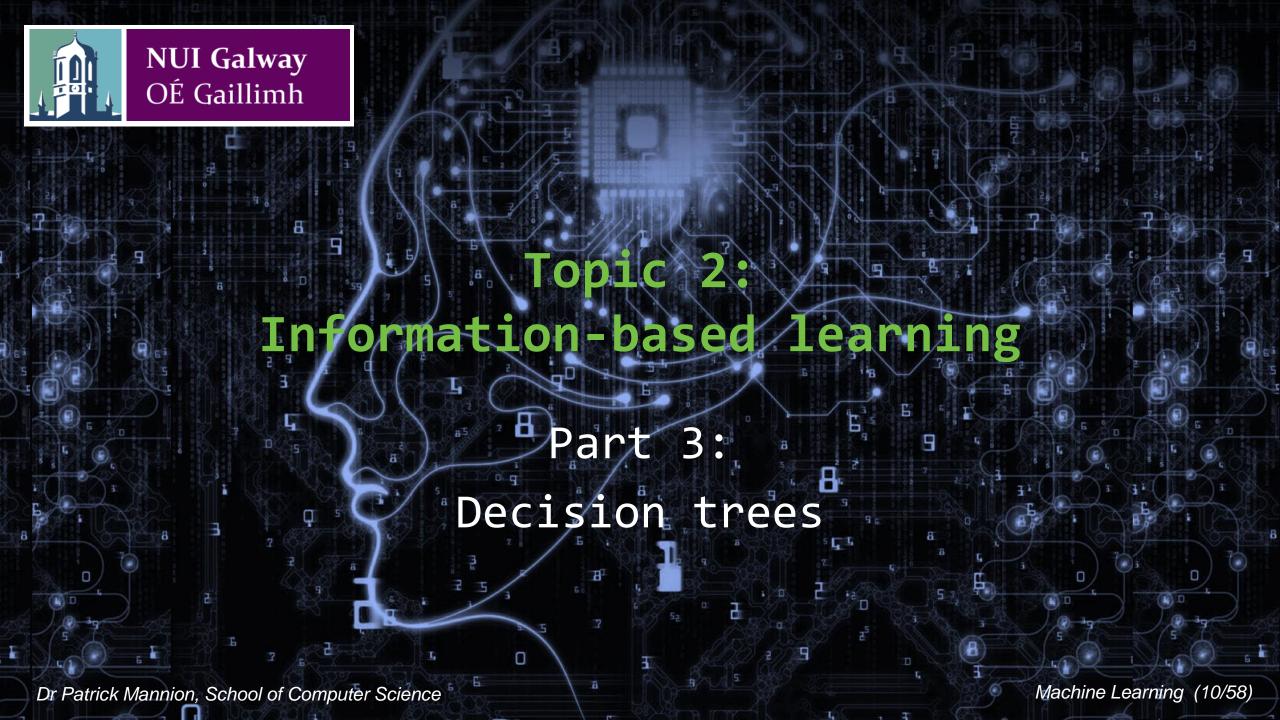
Training data example





Overview of the supervised learning process

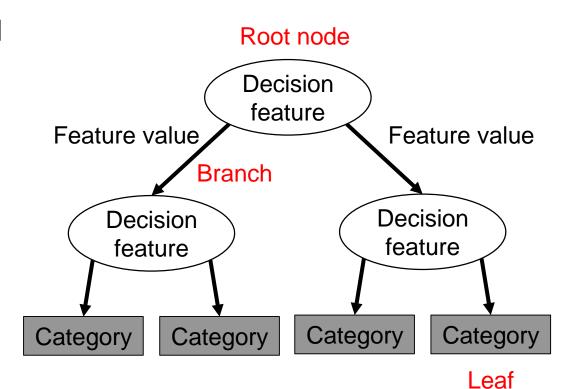






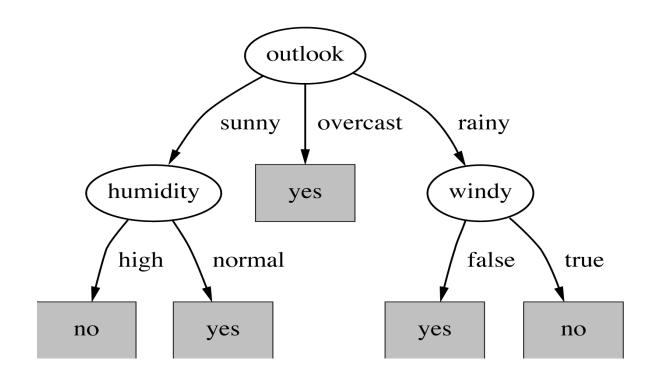
Decision trees

- Decision trees are a fundamental structure used in information-based machine learning
- Main idea: use a decision tree as a predictive model, to decide what category/label/class an item belongs to based on the values of its features
- So-called due to their tree-like structure:
 - A node (where two branches intersect) is a decision point. Nodes partition the data.
 - observations about an item (values of features) are represented using branches
 - The terminal nodes are called leaves; these specify the target label for an item





Decision tree for a sample dataset





Example dataset for induction (1)

Weather dataset

– Four attributes:

outlook: sunny / overcast / rainy

temperature: hot / mild / cool

humidity: high / normal

windy: true / false

- Used to decide whether or not to play tennis
- 14 examples in dataset
- See weather.xls (spreadsheet) or weathertxt.csv (comma separated values format)

• Objective:

- Find hypothesis that describes the cases given and can be used to make decisions in other cases
- Express the hypothesis as a decision tree.



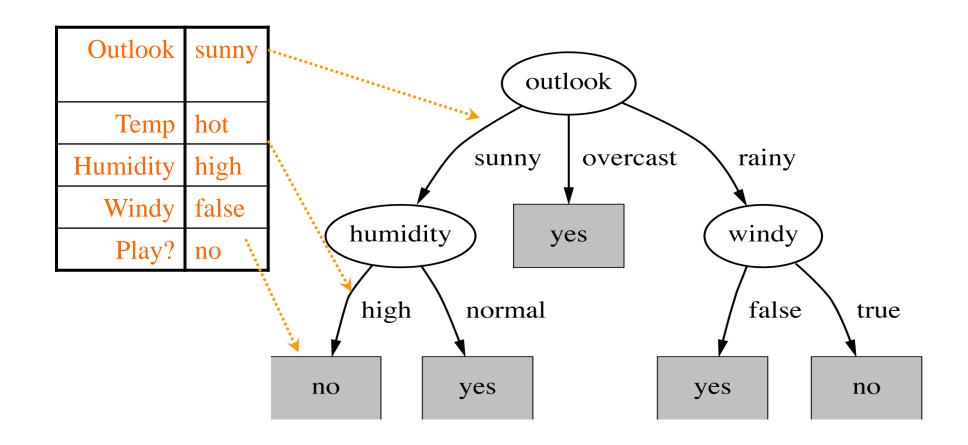
Example dataset for induction (2)

Allyone for refinition	Anyone	for	Tennis?	•
------------------------	---------------	-----	---------	---

ID	Outlook	Temp	Humidity	Windy	Play?
Α	sunny	hot	high	false	no
В	sunny	hot	high	true	no
С	overcast	hot	high	false	yes
D	rainy	mild	high	false	yes
Е	rainy	cool	normal	false	yes
F	rainy	cool	normal	true	no
G	overcast	cool	normal	true	yes
Н	sunny	mild	high	false	no
I	sunny	cool	normal	false	yes
J	rainy	mild	normal	false	yes
K	sunny	mild	normal	true	yes
L	overcast	mild	high	true	yes
M	overcast	hot	normal	false	yes
N	rainy	mild	high	true	no



Decision tree for this data (1)

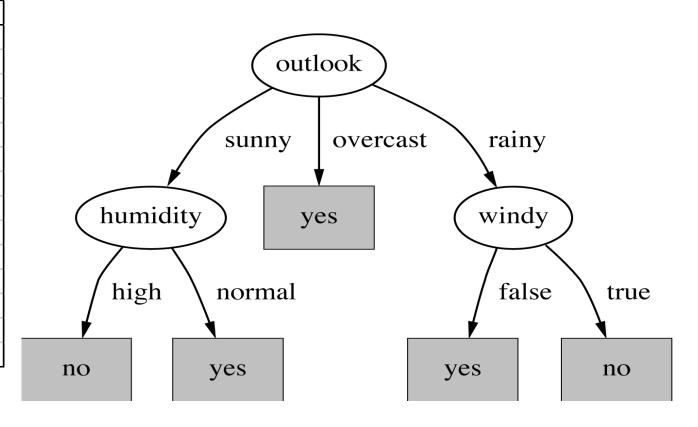




Decision tree for this data (2)

Anyone for Tennis?

ID	Outlook	Temp	Humidity	Windy	Play?
Α	sunny	hot	high	false	no
В	sunny	hot	high	true	no
С	overcast	hot	high	false	yes
D	rainy	mild	high	false	yes
Е	rainy	cool	normal	false	yes
F	rainy	cool	normal	true	no
G	overcast	cool	normal	true	yes
Н	sunny	mild	high	false	no
	sunny	cool	normal	false	yes
J	rainy	mild	normal	false	yes
K	sunny	mild	normal	true	yes
L	overcast	mild	high	true	yes
М	overcast	hot	normal	false	yes
Ν	rainy	mild	high	true	no





Inductive learning of a decision tree

Step 1

• For all attributes that have not yet been used in the tree, calculate their **entropy** and **information gain** values for the training samples

Step 2

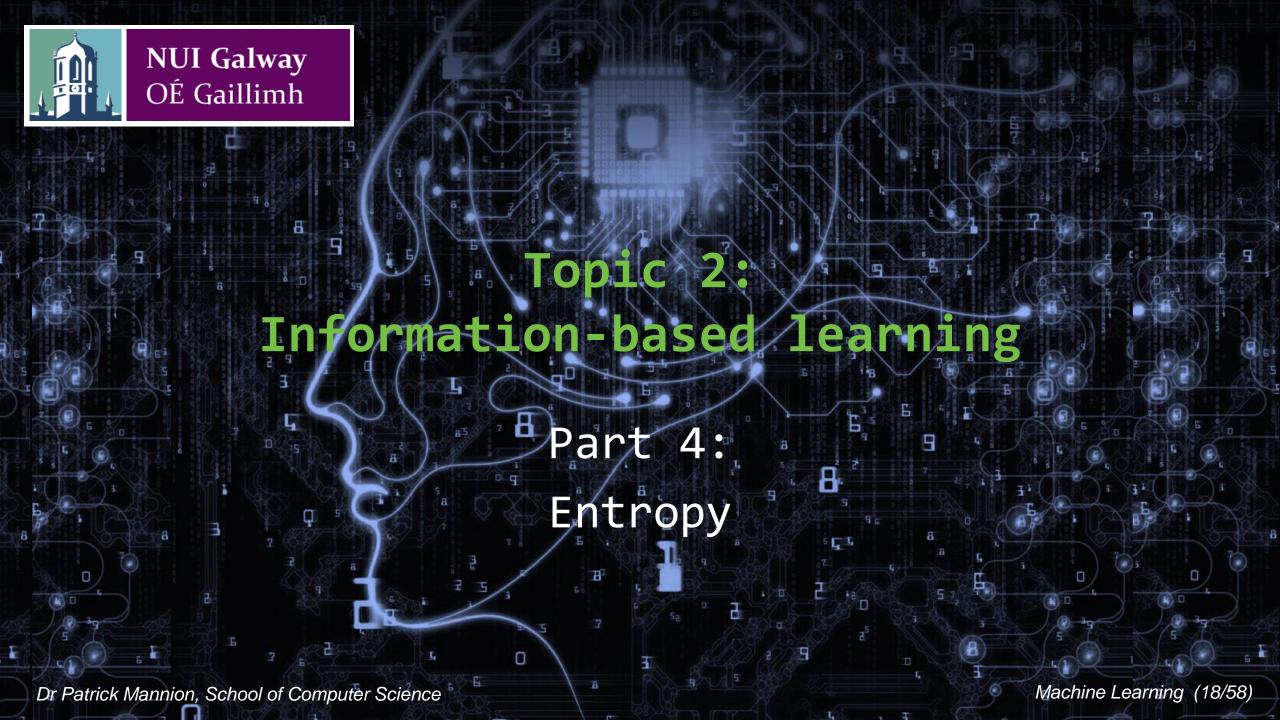
Select the attribute that has the highest information gain

Step 3

Make a tree node containing that attribute

Repeat

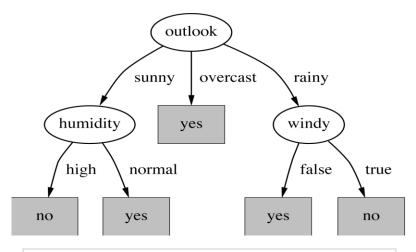
This node partitions the data:
 apply the algorithm recursively to each partition





Motivation

- We already saw how some descriptive features can more effectively discriminate between (or predict) classes which are present in the dataset
- Decision trees partition the data at each node, so it makes sense to use features which have higher discriminatory power "higher up" in a decision tree.
- Therefore we need to develop a formal measure of the discriminatory power of a given attribute
- Information gain this can be calculated using entropy

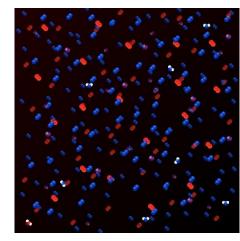


Anyone for Tennis?						
ID	Outlook	Temp	Humidity	Windy	Play?	
Α	sunny	hot	high	false	no	
В	sunny	hot	high	true	no	
С	overcast	hot	high	false	yes	
D	rainy	mild	high	false	yes	
Е	rainy	cool	normal	false	yes	
F	rainy	cool	normal	true	no	
G	overcast	cool	normal	true	yes	
Н	sunny	mild	high	false	no	
I	sunny	cool	normal	false	yes	
J	rainy	mild	normal	false	yes	
K	sunny	mild	normal	true	yes	
L	overcast	mild	high	true	yes	
M	overcast	hot	normal	false	yes	
N	rainy	mild	high	true	no	



Entropy

- Claude Shannon (often referred to as "the father of information theory") proposed a measure to of the impurity of the elements in a set, referred to as entropy
- Entropy may be used to measure of the uncertainty of a random variable
- The term entropy generally refers to disorder or uncertainty, so the use of this term in the context of information theory is analogous to the other well-known use of the term in statistical thermodynamics
- Acquisition of information (information gain) corresponds to a reduction in entropy
- "Information is the resolution of uncertainty" (Shannon)
- 1948 article "A Mathematical Theory of Communication"





Calculating entropy

• The entropy of a dataset S with n different classes may be calculated as:

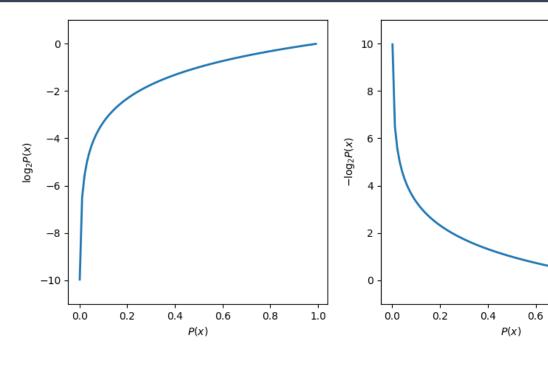
$$\operatorname{Ent}(S) = \sum_{i=1}^{n} -p_i \log_2 p_i$$

- Here p_i is the proportion of class i in the dataset.
- This is an example of a probability mass function
- Entropy is typically measured in bits (note log₂ in the equation above)
- The lowest possible entropy output from this function is 0 $(\log_2 1 = 0)$
- The highest possible entropy is $\log_2 n$ (=1 when there are only 2 classes)



Why use the binary logarithm?

- A useful measure of uncertainty should:
 - Assign high uncertainty values to outcomes with a low probability
 - Assign low uncertainty values to outcomes with a high probability
- Consider the plot to the right
 - log₂ returns large negative values when P is close to 0
 - log_2 returns small negative values when P is close to 1
- Using $-log_2$ is more convenient, as this will give positive entropy values, with 0 as the lowest entropy



0.8



Entropy worked example 1

$$\operatorname{Ent}(S) = \sum_{i=1}^{n} -p_i \log_2 p_i$$

$$Ent(S) = Ent([9+,5-])$$

$$Ent(S) = -9/14 \log_2(9/14) - 5/14 \log_2(5/14)$$

$$Ent(S) = 0.9403$$

If calculating this in a spreadsheet application such as Excel, make sure that you are using log_2 (e.g. LOG(9/14, **2**))

Anyone for Tennis?						
ID	Outlook	Temp	Humidity	Windy	Play?	
Α	sunny	hot	high	false	no	
В	sunny	hot	high	true	no	
С	overcast	hot	high	false	yes	
D	rainy	mild	high	false	yes	
Е	rainy	cool	normal	false	yes	
F	rainy	cool	normal	true	no	
G	overcast	cool	normal	true	yes	
Н	sunny	mild	high	false	no	
I	sunny	cool	normal	false	yes	
J	rainy	mild	normal	false	yes	
K	sunny	mild	normal	true	yes	
L	overcast	mild	high	true	yes	
М	overcast	hot	normal	false	yes	
N	rainy	mild	high	true	no	



Entropy worked example 2

Anyone for Tennis?

ID	Outlook	Temp	Humidity	Windy	Play?
Α	sunny	hot	high	false	no
В	sunny	hot	high	true	no
С	overcast	hot	high	false	yes
D	rainy	mild	high	false	yes
E	rainy	cool	normal	false	yes
F	rainy	cool	normal	true	no
G	overcast	cool	normal	true	yes
Н	sunny	mild	high	false	no
	sunny	cool	normal	false	yes
J	rainy	mild	normal	false	yes
K	sunny	mild	normal	true	yes
L	overcast	mild	high	true	yes
М	overcast	hot	normal	false	yes
N	rainy	mild	high	true	no



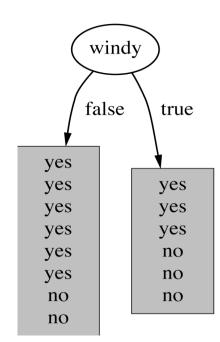
Entropy worked example 2

$$\operatorname{Ent}(S) = \sum_{i=1}^{n} -p_i \log_2 p_i$$

$$\operatorname{Ent}(S_{\text{windy=false}}) = \operatorname{Ent}([6+,2-])$$

$$= -6/8 \log_2(6/8) - 2/8 \log_2(2/8)$$

$$= 0.3112 + 0.5 = 0.8112$$

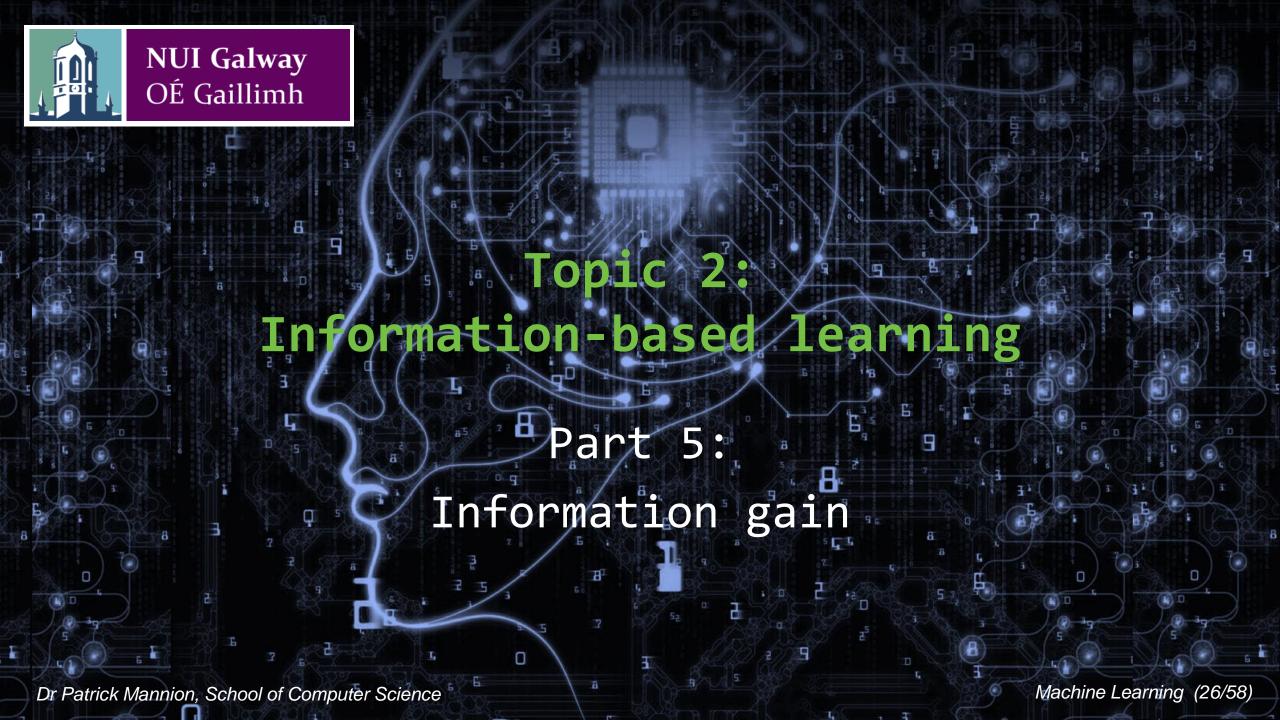


Anyone for Tennis?						
ID	Outlook	Temp	Humidity	Windy	Play?	
Α	sunny	hot	high	false	no	
В	sunny	hot	high	true	no	
С	overcast	hot	high	false	yes	
D	rainy	mild	high	false	yes	
E	rainy	cool	normal	false	yes	
F	rainy	cool	normal	true	no	
G	overcast	cool	normal	true	yes	
Н	sunny	mild	high	false	no	
l	sunny	cool	normal	false	yes	
J	rainy	mild	normal	false	yes	
K	sunny	mild	normal	true	yes	
L	overcast	mild	high	true	yes	
М	overcast	hot	normal	false	yes	
N	rainy	mild	high	true	no	

$$\operatorname{Ent}(S_{\operatorname{windy=true}}) = \operatorname{Ent}([3+,3-] =$$

$$= -3/6 \log_2(3/6) -3/6 \log_2(3/6)$$

$$= 0.5 + 0.5 = 1.0$$





Information gain

 The information gain of an attribute is the reduction in entropy from partitioning the data according to that attribute

$$Gain(S,A) = Ent(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Ent(S_v)$$

- Here S is the entire set of data being considered, and S_v refers to each partition of the data according to each possible value v for the attribute
- |S| and $|S_v|$ refer to the cardinality or size of the overall dataset, and the cardinality or size of a partition respectively
- When selecting an attribute for a node in a decision tree, use whichever attribute A
 gives the greatest information gain



Information gain worked example

Gain(S,A) = Ent(S) -
$$\sum_{v \in Values(A)} \frac{|S_v|}{|S|} Ent(S_v)$$

$$|S|=14$$
 $|S_{\text{windy=true}}|=6$

$$|S_{windy=true}| = 8$$

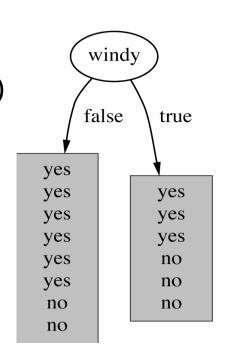
Gain(S, Windy)

= Ent(S) -
$$|S_{\text{windy=true}}|/|S|$$
 Ent($S_{\text{windy=true}}$) - $|S_{\text{windy=false}}|/|S|$ Ent($S_{\text{windy=false}}$)

$$= Ent(S) - (6/14) Ent([3+,3-]) - (8/14) Ent([6+,2-])$$

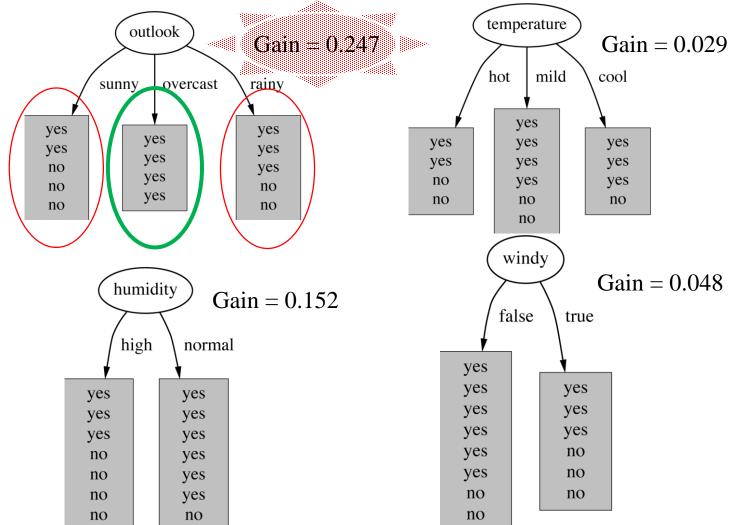
$$= 0.940 - (6/14) 1.00 - (8/14) 0.811$$

Gain(S, Windy) = 0.048





Best partitioning = highest information gain



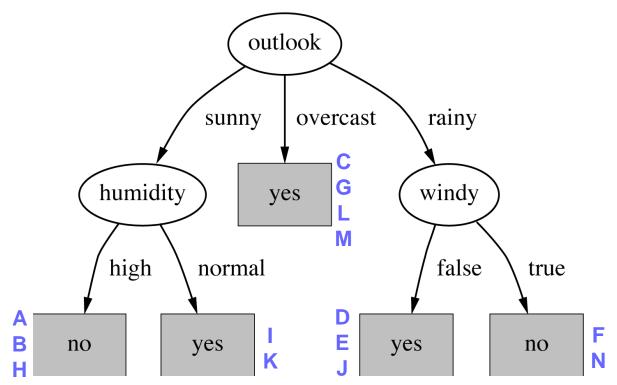
Anyone for Tennis?						
ID	Outlook	Temp	Humidity	Windy	Play?	
Α	sunny	hot	high	false	no	
В	sunny	hot	high	true	no	
С	overcast	hot	high	false	yes	
D	rainy	mild	high	false	yes	
Е	rainy	cool	normal	false	yes	
F	rainy	cool	normal	true	no	
G	overcast	cool	normal	true	yes	
Н	sunny	mild	high	false	no	
I	sunny	cool	normal	false	yes	
J	rainy	mild	normal	false	yes	
K	sunny	mild	normal	true	yes	
L	overcast	mild	high	true	yes	
M	overcast	hot	normal	false	yes	
N	rainy	mild	high	true	no	

Having found the best split for the root node, repeat the whole procedure with each subset of examples ...

S will now refer to the subset in the partition being considered, instead of the entire dataset

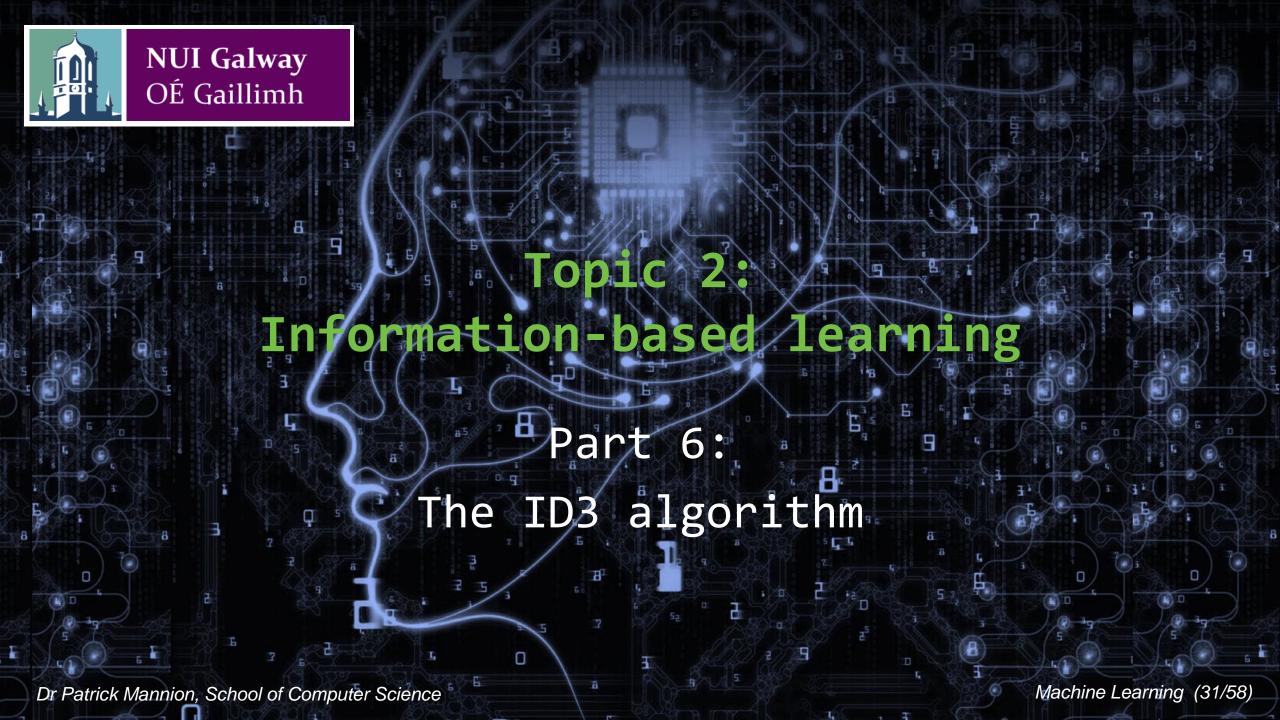


Example: complete decision tree



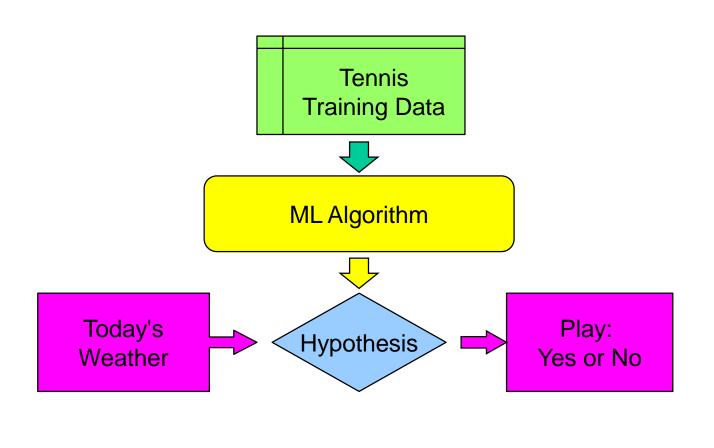
Anyone for Tennis?						
ID	Outlook	Temp	Humidity	Windy	Play?	
Α	sunny	hot	high	false	no	
В	sunny	hot	high	true	no	
С	overcast	hot	high	false	yes	
D	rainy	mild	high	false	yes	
Е	rainy	cool	normal	false	yes	
F	rainy	cool	normal	true	no	
G	overcast	cool	normal	true	yes	
Н	sunny	mild	high	false	no	
I	sunny	cool	normal	false	yes	
J	rainy	mild	normal	false	yes	
K	sunny	mild	normal	true	yes	
L	overcast	mild	high	true	yes	
M	overcast	hot	normal	false	yes	
N	rainy	mild	high	true	no	

What about Temp = {Hot, Mild, Cool}?

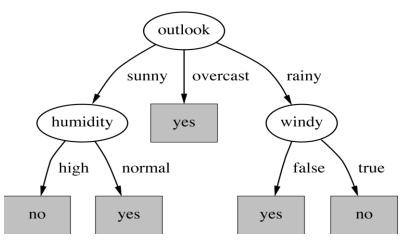




Review: the supervised learning process



Anyone for Tennis?						
ID	Outlook	Temp	Humidity	Windy	Play?	
Α	sunny	hot	high	false	no	
В	sunny	hot	high	true	no	
С	overcast	hot	high	false	yes	
D	rainy	mild	high	false	yes	
Е	rainy	cool	normal	false	yes	
F	rainy	cool	normal	true	no	
G	overcast	cool	normal	true	yes	
Н	sunny	mild	high	false	no	
ı	sunny	cool	normal	false	yes	
J	rainy	mild	normal	false	yes	
K	sunny	mild	normal	true	yes	
L	overcast	mild	high	true	yes	
М	overcast	hot	normal	false	yes	
N	rainy	mild	high	true	no	





Review: inductive learning of a decision tree

Step 1

• For all attributes that have not yet been used in the tree, calculate their **entropy** and **information gain** values for the training samples

Step 2

Select the attribute that has the highest information gain

Step 3

Make a tree node containing that attribute

Repeat

 This node partitions the data: apply the algorithm recursively to each partition



The ID3 algorithm

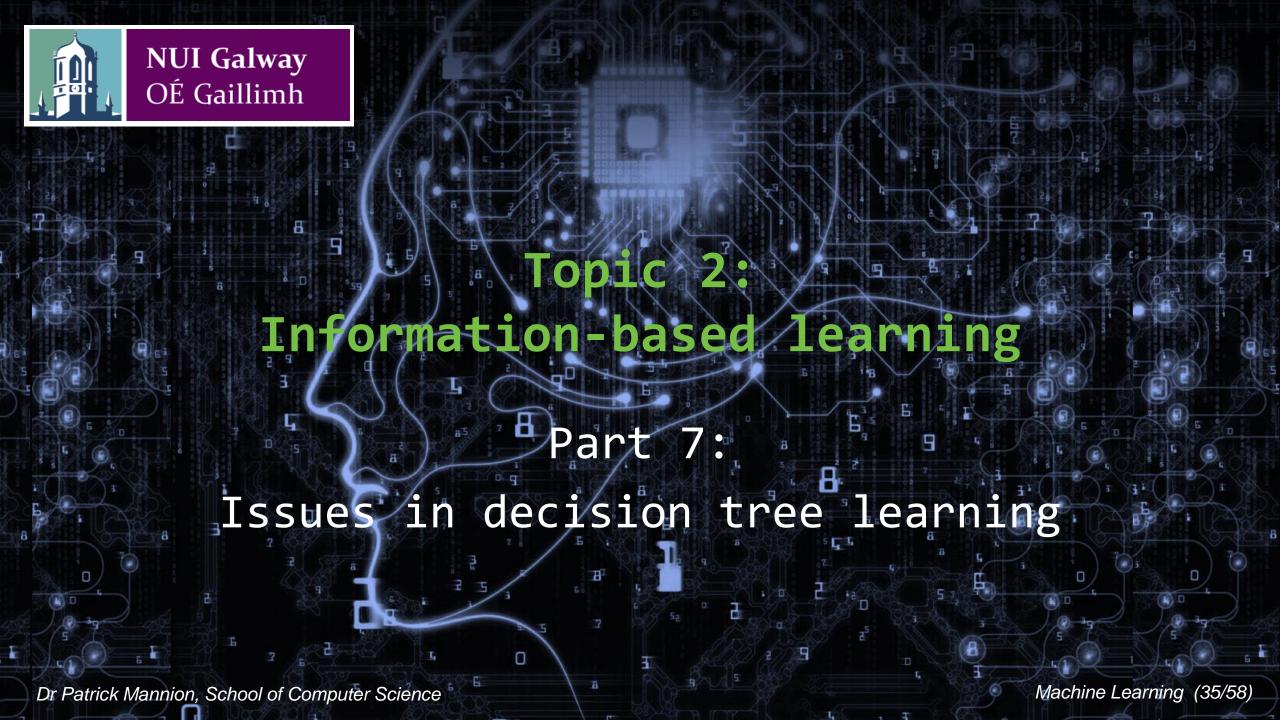
```
1. ID3(Examples, Attributes, Target):
2. Input: Examples: set of classified examples
         Attributes: set of attributes in the examples
3.
         Target: classification to be predicted
5. if Examples is empty then return a Default class
                                                                   BASE
6. else if all Examples have same class then return this class
                                                                   CASES
7. else if all Attributes are tested then return majority class
8. else:
9.
      let Best = attribute that best separates Examples relative to Target
10.
      let Tree = new decision tree with Best as root node
11.
     foreach value v<sub>i</sub> of Best:
          let Examples = subset of Examples that have Best=v;
12.
                                                                RECURSIVE
           let Subtree = ID3(Examples, Attributes-Best, Target) ← CALL
13.
14.
          add branch from Tree to Subtree with label v<sub>i</sub>
15.
     return Tree
```

Ross Quinlan, 1986



Based on
Algorithm

Decision-TreeLearning in
Russell &
Norvig textbook





Decision tree characteristics

- Popular because:
 - Relatively easy algorithm
 - Fast: greedy search without backtracking
 - Comprehensible output: important in decision-making (medical, financial, ...)
 - Practical: discrete/numeric, irrelevant attributes, noisy data, ...
- Expressiveness: what functions can a DT represent?
 - Technically, any Boolean function (propositional logic)
 - Some functions, however, require exponentially large tree (e.g. parity function)
 - Cannot consider relationships between two attributes



Dealing with noisy or missing data

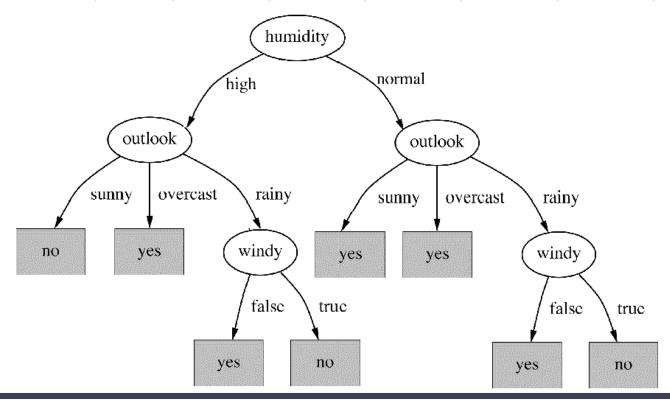
- What about inconsistent ("noisy") data?
 - Use majority class (line 7 of ID3 alg.)
 - 7. else if all Attributes are tested then return majority class
 - or interpret as probabilities
 - or return "average" target feature value
- What about missing data?
 - Given a complete decision tree, how should one classify an example that is missing one of the test attributes?
 - How should one modify the information gain formula when some training examples have unknown values for an attribute?
 - Could assign the most common value among the training examples that reach that node
 - Or could assume the attribute has all possible values, weighting each value according to its
 frequency among the training examples that reach that node



Instability of decision trees

- Hypothesis found is sensitive to training set used
 - consequence of greedy search
- Replace one example:
 - new one consistent with original tree
- Some algorithmic modifications to reduce the instability of decision tree learning were proposed by Li and Belford in their 2002 paper "Instability of decision tree classification algorithms".
- Li and Belford's main idea is to alter the attribute selection procedure, so that the tree learning algorithm is less sensitive to some % of the training dataset being replaced.

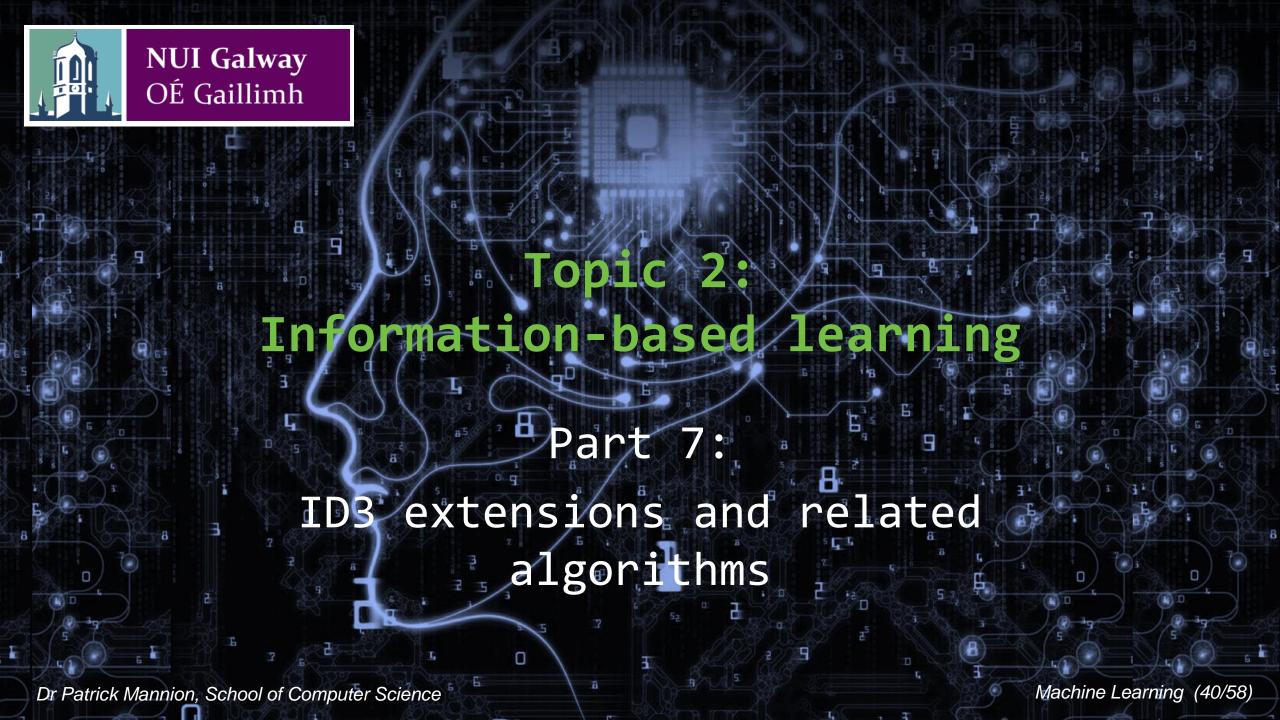
ID	Outlook	Temp	Humidity	Windy	Play?
Ð	overcast	hot	high	false	yes
0	sunny	hot	normal	true	yes





Pruning

- Overfitting occurs in a predictive model when the hypothesis learned makes predictions which are based on spurious patterns in the training data set.
 Consequence: poor generalisation to new examples.
- Overfitting may happen for a number of reasons, including sampling variance or noise present in the dataset
- Tree pruning may be used to combat overfitting in decision trees
- Tree pruning can lead to induced trees which are inconsistent with the training set
- Generally, there are two different approaches to pruning:
 - Pre-pruning (e.g. <= target # of examples in a partition, limiting tree depth, creating a new node only when information gain is above a threshold, statistical tests such as χ^2)
 - Post-pruning (e.g. target # of examples, compare error rate for model on a validation dataset with and without a given subtree; only keep a subtree if it improves the error rate, statistical tests such as χ^2 , reduced error pruning (Quinlan, 1987))





Continuous-valued attributes

- What about continuous-valued attributes?
 - Pick threshold value T for attribute A, and test whether A >T
 - Information Gain can be used to decide which T is best
 - Could select T at a midpoint where classification changes

		5		85		
Temp	40	48	60	72	80	90
Play?	No	No	Yes	Yes	Yes	No



Selecting the best attribute: alternative metrics (1)

- Earlier, we introduced the concept of information gain, which we can
 use as a metric for the discriminatory power of an attribute
- Information gain does have some drawbacks; it tends to favour attributes that can take on a large number of different values
- One alterative is to use the information gain ratio

GainRatio(
$$S, A$$
) =
$$\frac{\text{Gain}(S, A)}{\sum_{v \in \text{Values}(A)} -p_v \log_2 p_v}$$

 The divisor of this fraction measures the amount of information used to compute the gain value, and is the entropy of S with respect to A



Selecting the best attribute: alternative metrics (2)

 Another alternative is to use the Gini index instead of entropy as a measure of the impurity of a set

Gini(S) =
$$1 - \sum_{i=1}^{n} p_i^2$$

• Then the gain for a feature may be calculated based on the reduction in the Gini index (rather than a reduction in entropy):

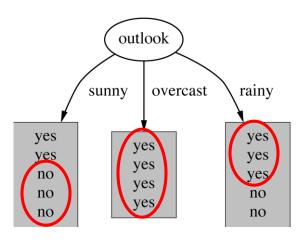
GiniGain
$$(S, A) = Gini(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Gini(S_v)$$



Related algorithms [1]

1R

- Decision tree with just one rule
- Introduced in a paper by Robert C. Holte (1993).
 "Very Simple Classification Rules Perform Well on Most Commonly Used Datasets", Computer Science Department, University of Ottawa



Decision Stump

1 rule with 1 test

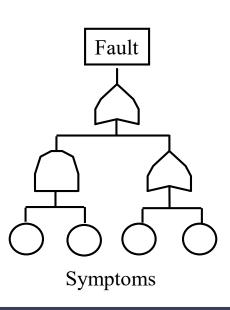
```
Outlook = Overcast => YES
Outlook != Overcast => YES
```

These are deliberately simple variants that are used within other algorithms (meta-learning; ensembles). Often referred to as "weak learners".



Related algorithms [2]

- Decision Lists
 - A set of rules (predicate logic), describing the hypothesis, that are followed in the given order
- C4.5 Rules
 - Alternative representation of C4.5 decision trees
- PART: Rules constructed with partial DTs
 - Can be more readable than standard DTs
- IFT: Induction of Fault Trees





Decision tree software

• C4.5

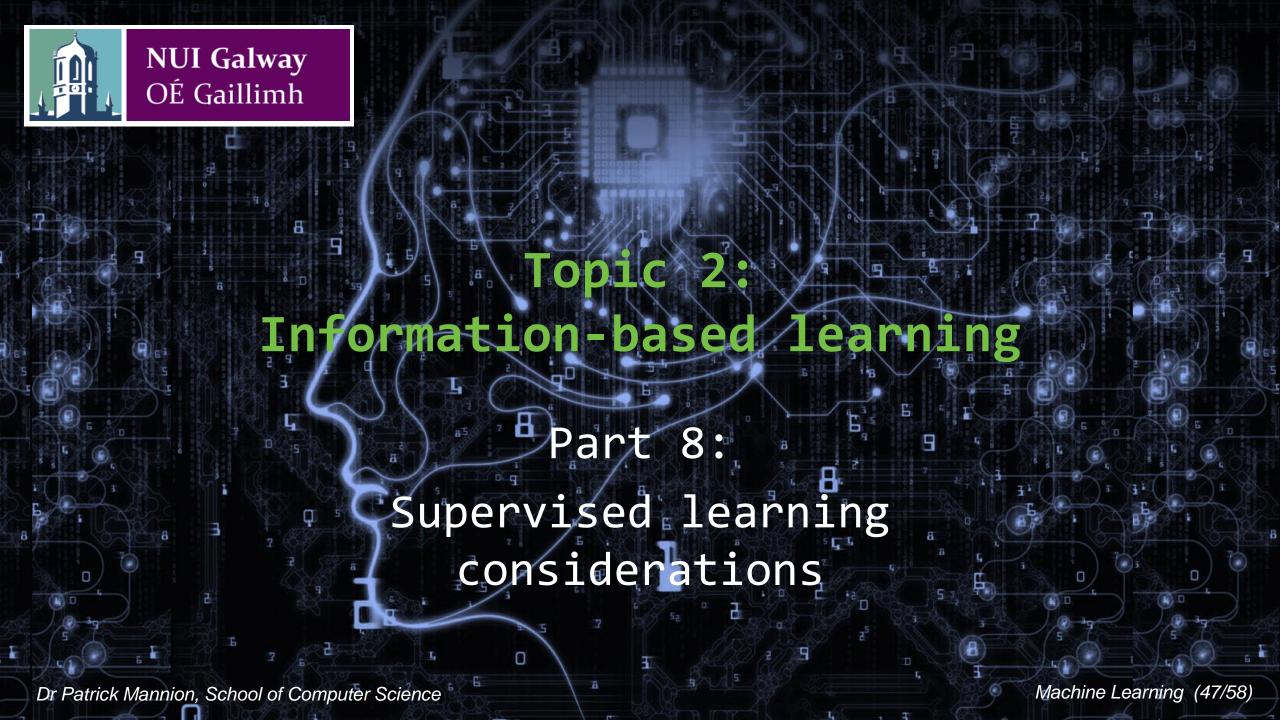
- Original implementation (C language, command line), available on Ross Quinlan's website (https://www.rulequest.com/Personal/)
- Deals with missing values in the data, high-branching attributes (e.g. ID in the weather data),
 pruning to avoid overfitting, converting a decision tree to a list of rules

• C5.0

Commercial version from RuleQuest Research, with improvements over C4.5

WEKA software

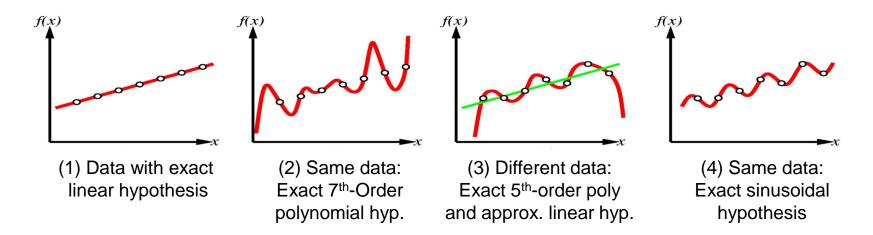
- Accompanies book by Witten & Frank, Data Mining: Practical Machine Learning Tools and Techniques
- Java implementations of many ML algorithms, including C4.5 (mysteriously called J48)
- Easy-to-use front end and utilities
- Many other implementations in Python and R...





Supervised Learning Considerations [1]

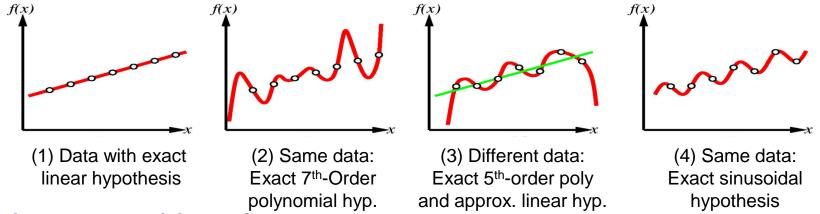
 Various hypotheses can be consistent with observations but inconsistent with each other:
 Which one should we choose?





Supervised Learning Considerations [2]

 Various hypotheses can be consistent with observations but inconsistent with each other:
 Which one should we choose?



- One solution: Ockham's Razor:
 - Prefer simplest hypothesis consistent with data
 - Definitions of simplicity (& consistency) may subject to debate
 - Depends strongly on how hypotheses are expressed



Supervised Learning Considerations [3]

- Hypothesis language is too limited?
 - Might be unable to find hypothesis that exactly matches 'true' function
 - If true function is more complex than what hypothesis can express, it will underfit the data
 - Saw this in previous slide, 3rd and 4th figures
- Hypothesis language cannot exactly match true function?
 - there will be a trade-off between complexity of hypothesis and how well it fits the data



Supervised Learning Considerations [4]

- Hypothesis language is very expressive?
 - Its search space is very large and the computational complexity of finding a good hypothesis will be high
 - Also need a large amount of data to avoid overfitting
- What can decision trees express?
 - Will learn about other algorithms that express hypotheses differently
 - In general, would like to use an algorithm for a problem that can express the true underlying function



Supervised Learning Considerations [5]

- But don't forget:
 we never know the true underlying function
- E.g. To avoid problem with poorly fitting data from a previous slide
 - Could change algorithm so that, as well as searching for coefficients of polynomials, it tries combinations of trig. functions (sin, cos, tan)
 - Learning problem will become enormously more complex, but will it solve our problems?
 - Probably not: you could easily think up some different kind of mathematical function, to generate a new dataset that the algorithm still cannot represent perfectly.
- For this reason, often use relatively simple hypothesis languages, in the absence of special knowledge about domain
 - more complex languages don't come with any real guarantees
 - more simple languages correspond to easier searching.

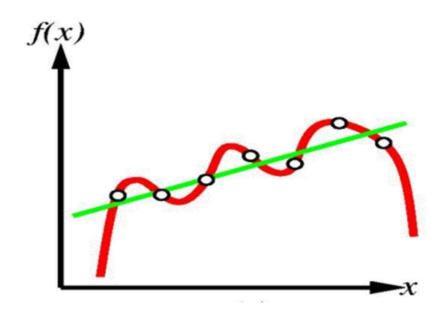


Noise, Overfitting and Underfitting [1]

• NOISE:

imprecise or incorrect attribute values or labels

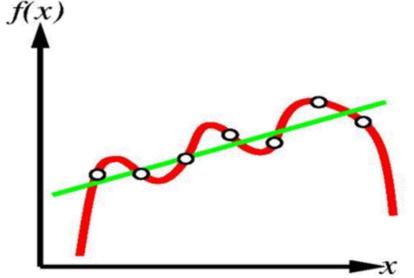
- Can't always quantify it,
 but should know from situation
 if it is present
- E.g. labels may require subjective judgement or values may come from imprecise measurements





Noise, Overfitting and Underfitting [2]

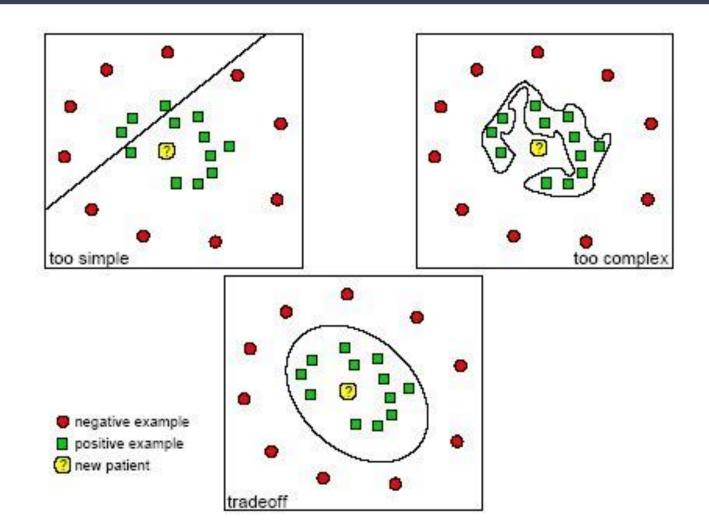
- If the data might have noise, harder to decide which hypothesis is best:
 - Linear hypothesis could not fit to it, but polynomial could
 - But which would really be the better choice?
- Complex classification methods prone to overfitting; simple ones prone to underfitting



If you increase complexity of hypothesis, you increase ability to fit to the data,
 but might also increase risk of overfitting



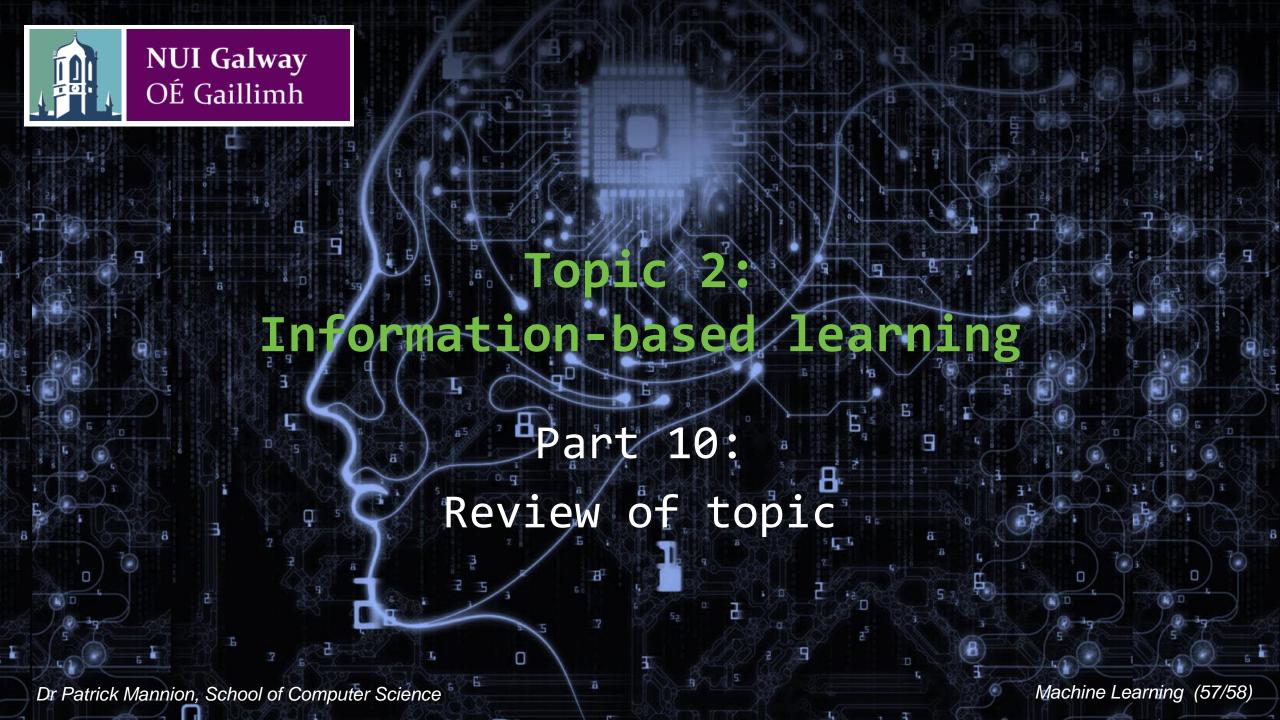
Illustration of Underfitting & Overfitting





Detecting Underfitting & Overfitting

- Previous slides have illustrated concepts only
 - In general, cannot visualise very high dimensional data: -=> can't directly observe overfitting/underfitting
- Main symptom of underfitting:
 - Poor performance even on the training data
- Main symptom of overfitting:
 - Much better performance on the training data than on independent test data
 - (Slightly better performance is to be expected)





Learning Objectives Review

After completing this topic successfully, you will be able to ...

- 1. Explain what supervised learning is
- 2. Distinguish it from unsupervised learning and reinforcement learning
- 3. Describe in detail an algorithm for decision tree induction
- 4. Demonstrate the application of decision tree induction to a data set
- 5. List related algorithms
- Discuss high-level concepts such as choice of hypothesis language, overfitting, underfitting and noise