LUND UNIVERSITY
FACULTY OF ENGINEERING (LTH)

FRTN 30

NETWORK DYNAMICS

Hand In 1

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The report was handed in on: April 15, 2019

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Centrality in Input-Output Network of Goods

The first part of the hand in was a simulation of two countries economies, Sweden and Indonesia during the year 2000. To examine the different sectors three different centrality definitions were used.

a) The in-degree and out-degree centrality

As the name suggest we are only looking at the in and out-degree of each node. The centrality of the neighboring nodes are not used in this approach. The expression for in-degree and out-degree of node i are defined, respectively, as

$$w_i = \sum_{j \in V} W_{i,j} \quad \text{and} \quad w_i^- = \sum_{j \in V} W_{j,i} \tag{1}$$

See attached code for implementation. The resulting sectors are below.

Country	Most central sector (in-degree)	Most central sector (out-degree)
Sweden	1. Radio 2. Motor Vehicles 3. Other Business Activities	1. Other Business Activities 2. Real Estate Activities 3. Wholesale & retail trade; repairs
Indonesia	1. Food products 2. Construction 3. Wholesale & retail trade; repairs	3. Wholesale & retail trade; repairs 2. Agriculture 3. Mining and quarrying (energy)

The results seem to be reasonable considering the two countries, however, the in-degree centrality in Sweden "Radio" is number one, this is probably not reasonable. Furthermore, Indonesia doesn't have any tourist related sectors (except maybe Wholesale & retail trade; repairs) in its top three, neither for in or out-degree.

b) The eigenvector centrality on the largest connected component

With the eigenvector centrality the centrality of neighboring nodes factors in. The implicit expression of eigenvector centrality, z, is

$$\lambda z = W' z \tag{2}$$

See attached code for implementation. The resulting sectors are below.

Country	Most central sector (eigenvector centrality)
Sweden	1. Motor vehicles 2. Radio 3. Other Business Activities
Indonesia	1. Food products 2. Hotel & Restaurants 3. Agriculture

The most central sectors for Sweden has now been shuffled around and "Hotel & Restaurants" have been added to Indonesia, i.e. a tourist sector.

c) The Katz centrality

One problem with the eigenvector centrality is that self-loops or two nodes that have an undirected link between them will get a higher centrality. We avoid this by using the Katz centrality, the expression is defined implicitly as

$$z^{(\beta)} = \frac{(1-\beta)}{\lambda_W} W' z^{(\beta)} + \beta\mu \tag{3}$$

and explicitly as

$$z^{(\beta)} = (I - \lambda_W^{-1}(1-\beta)W')^{-1}\beta\mu \tag{4}$$

where we chose the parameter $\beta \in (0,1]$ and the nonnegative vector μ which is thought of as some intrinsic centrality.

See attached code for implementation. The resulting sectors are below.

Country	**Most central sector** $\mu_i = 1$ *for Wholesale & retail trade; repairs and zero for all else*	**Most central sector** $\mu = \mathbb{1}$
Sweden	1. Wholesale & retail trade; repairs 2. Motor Vehicles 3. Radio	1. Motor vehicles 2. Radio 3. Other Business Activities
Indonesia	1. Food products 2. Wholesale & retail trade; repairs 3. Hotels & Restaurants	1. Food products 2. Hotels & Restaurants 3. Agriculture

When $\mu_i = 1$ for "Wholesale & retail trade; repairs" and zero for the other sectors, then the WRTR sector is given a boost in relation to the other sector, look at equation 3. This results in the WRTR sector jumping up the ranking and all sectors below it jumps down one step.

Influence on Twitter

The second part of the hand in was to examine Twitter users' centrality, using the PageRank algorithm, as well as averaging dynamics with stubborn nodes, i.e. people on twitter who's opinion stays constant over time and what effect that has on the system as a whole.

a) Pagerank

The PageRank algorithm is the following

$$z^{(\beta)} = \beta \sum_{k \geq 0} (1-\beta)^k (P')^k \mu \tag{5}$$

In this case we used $\beta = 0.15$ which is custom, and $\mu = \mathbb{1}$. As k increases, i.e. the further away two nodes are from each other, $(1-\beta)^k$ decreases and since all indices in P' are between 0 and 1 $(P')^k$ also decreases with k, therefore the centrality of a node is mostly influenced by its closest neighbors.

See attached code for implementation. The most central Twitter accounts are below, calculated using 100 steps (k goes from 0 to 100).

Rank	Username	Node	Centrality value
1.	@gustavnilsson	1	718
2.	@AVPapadopoulos	2	122
3.	@Asienfoset	112	82
4.	@Vikingafoset	9	70
5.	@bianca_grossi94	26	62

b) Stuborn nodes as the most central ones

The normalized weight matrix, P, is re-partitioned as follows

$$P = \begin{array}{cc} & \begin{array}{cc} \mathcal{R} & \mathcal{S} \end{array} \\ & \begin{pmatrix} Q & E \\ F & G \end{pmatrix} \begin{array}{c} \mathcal{R} \\ \mathcal{S} \end{array} \end{array} \tag{6}$$

where $\mathcal{S}$ is a set if stubborn nodes and $\mathcal{R} = \mathcal{V} \backslash \mathcal{S}$ is a set of regular nodes. The state vector is defined as

$$x(t) = \begin{pmatrix} \underline{x}(t) \\ u(t) \end{pmatrix} \begin{array}{c} \mathcal{R} \\ \mathcal{S} \end{array} \tag{7}$$

where $u(t) \in \mathbb{R}^S$, then the state vector $\underline{x}(t+1)$ can be calculated as

$$\underline{x}(t+1) = Q\underline{x}(t) + Eu(t) \tag{8}$$

Implementing this in MatLab resulted in the following graphs

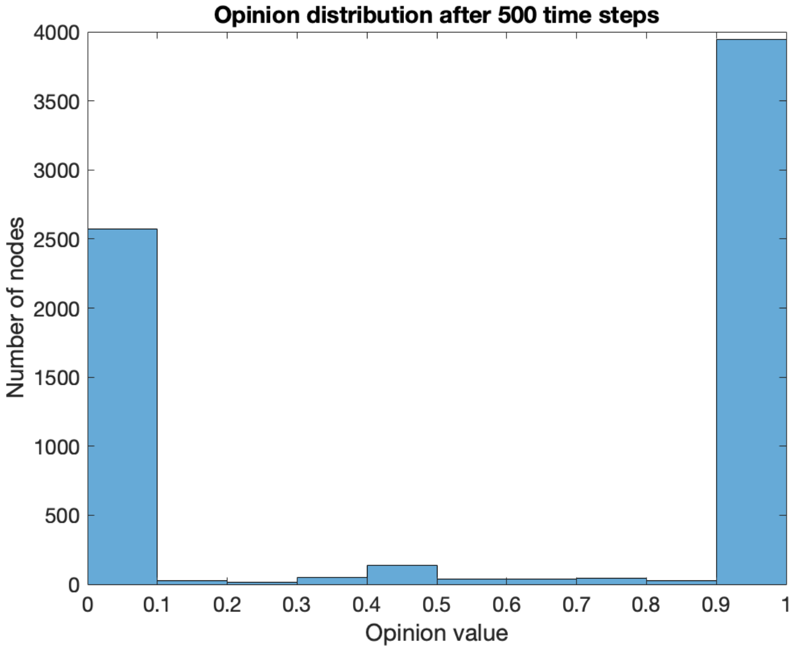

Figure 1: The opinion distribution with the two stubborn nodes 1 and 2 (the two most central ones with respect to PageRank). Node 1 has value 1 and node 2 has value 0. The regular nodes all have starting value 0.5.

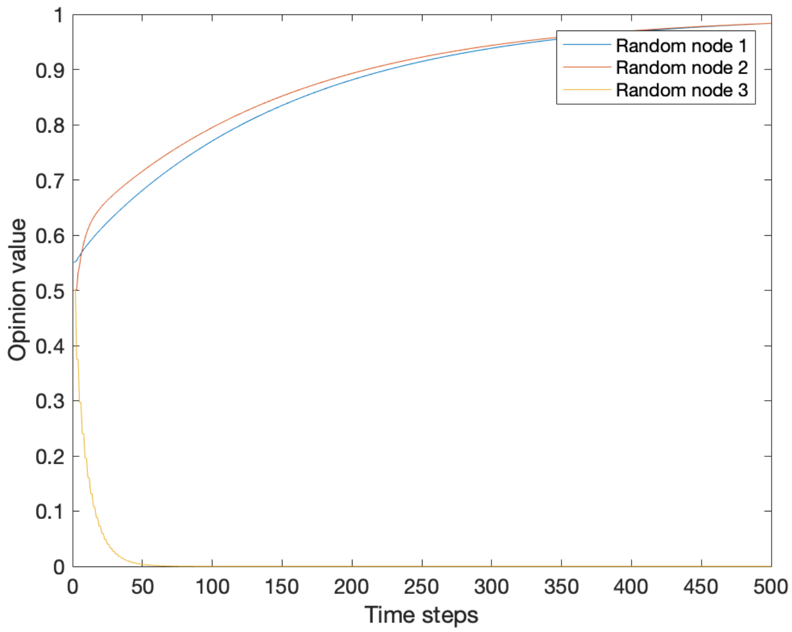

Figure 2: Plot of three, randomly chosen, nodes' opinion value over time. Random node 1 and 2 converges to 1 as time progresses and random node 3 converges to 0. The regular nodes all have starting value 0.5.

In figure 1 the nodes converge to the value 0 and 1, with 1 being the dominant value since that's the value of the more central node. The value of three random nodes over time is displayed in figure 2, either they converge to 1 or 0 depending on their proximity to the two stubborn nodes $\mathcal{S}$.

c) Stubborn nodes for different scenarios

Now the same approach as in b) but with a different set of nodes in $\mathcal{S}$. I collaborated with Ian Thorslund on this task, we chose the same nodes to be able to compare results.

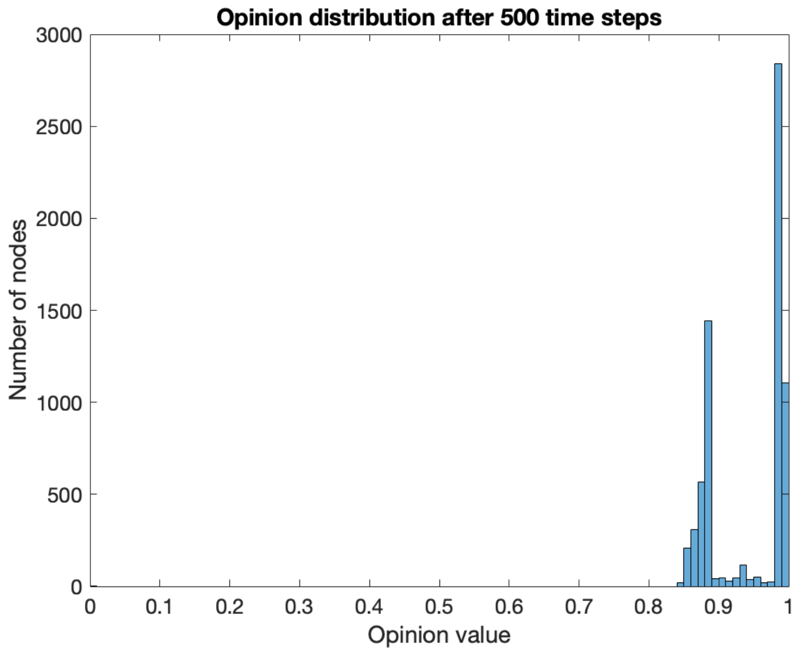

Figure 3: The most central node is stubborn and has value 1 and the least central node is stubborn and has value 0. The regular nodes all have starting value 0.5.

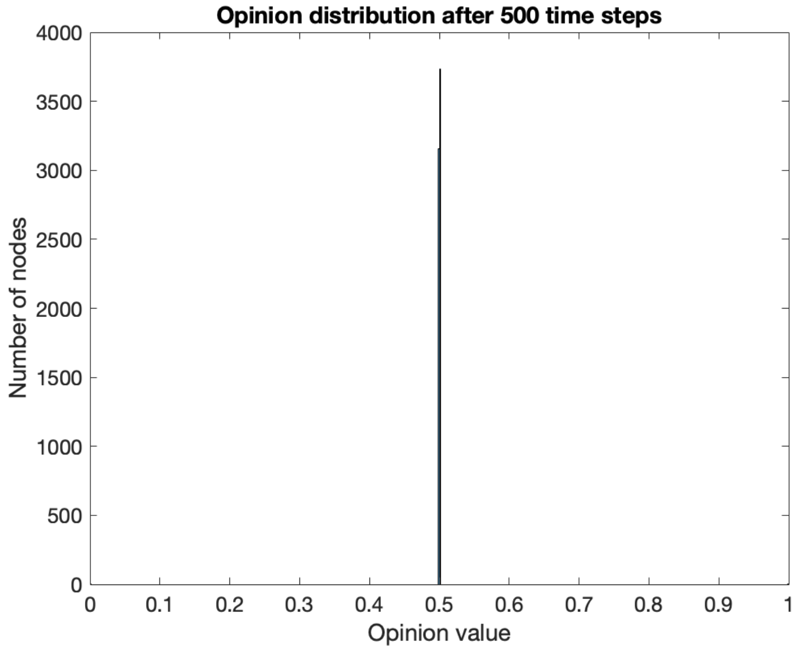

Figure 4: The two least central nodes are stubborn and has respective value 0 and 1. The regular nodes all have starting value 0.5.

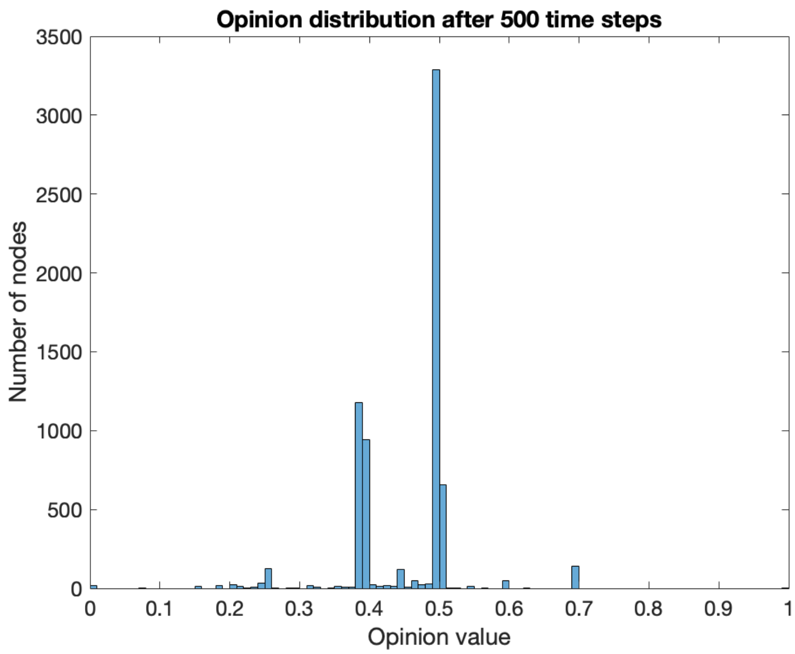

Figure 5: The 99^{th} and 100^{th} most central nodes are stubborn and has respective value 1 and 0. The regular nodes all have starting value 0.5.

All the graphs make sense intuitively.