Prerequisite Knowledge

Math 133 - Calculus I

Profs. Beineke, Carcea, Selig

Foreword

On behalf of the Department of Mathematics, welcome to Western New England University!

You are receiving this packet since you have been recommended to take MATH 109 Pre-Calculus. We understand that some students may have taken Pre-Calculus in high school and therefore may have reservations in enrolling in MATH 109. Our university level Pre-Calculus course is a rigorous, algebraically intensive course that focuses on the concepts and skills necessary to succeed in MATH 133 Calculus I.

In order to help you assess your mastery of the Pre-Calculus skills that we expect of our incoming Calculus students, we encourage you to try the problems in this packet. Please do so without the aid of a calculator. Then you may compare your solutions to those posted here (https://bit.ly/109solutions).

If you struggle with the problems, the MATH 109 course will prove to be extremely beneficial as you prepare for Calculus. While MATH 109 is offered this fall, please consider enrolling in the summer course (June 29 – August 6) at a reduced rate of $450 per credit. Successful completion of the course will enable you to enroll in MATH 133 Calculus I for the fall semester.

We look forward to working with you!

1. Use the function $f(x) = \sqrt{\dfrac{(x+1)(x-3)}{(x-3)(2x+1)}}$ to answer parts (a) - (c).

   (a) Find the domain of f.

   $$\frac{(x+1)(x-3)}{(x-3)(2x+1)} \geq 0$$

   $+ \quad 0 \; - \; \text{und} \quad + \quad \text{und} \quad +$

   $-1 \quad -\frac{1}{2} \quad 3$

   $$\underline{(-\infty, -1] \cup (-\tfrac{1}{2}, 3) \cup (3, \infty)}$$

   (b) Find all x-intercepts of f, if they exist.

   when numerator $= 0$ but denominator $\neq 0$

   $x + 1 = 0$

   $x = -1$

   $\underline{(-1, 0)}$

   (c) Find the y-intercept of f, if it exists.

   $$f(0) = \sqrt{\frac{(0+1)\cancel{(0-3)}}{\cancel{(0-3)}(2\cdot 0+1)}}$$

   $$= \sqrt{\frac{1}{1}}$$

   $$= 1$$

   $\underline{(0, 1)}$

2. Consider the piece-wise function

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 4 & \text{if } x = 2 \\ -2x + 7 & \text{if } x > 2. \end{cases}$$

(a) Evaluate $f(-1) + f(4)$.

$f(-1) = (-1)^2 - 1$
$= 1 - 1$
$= 0$

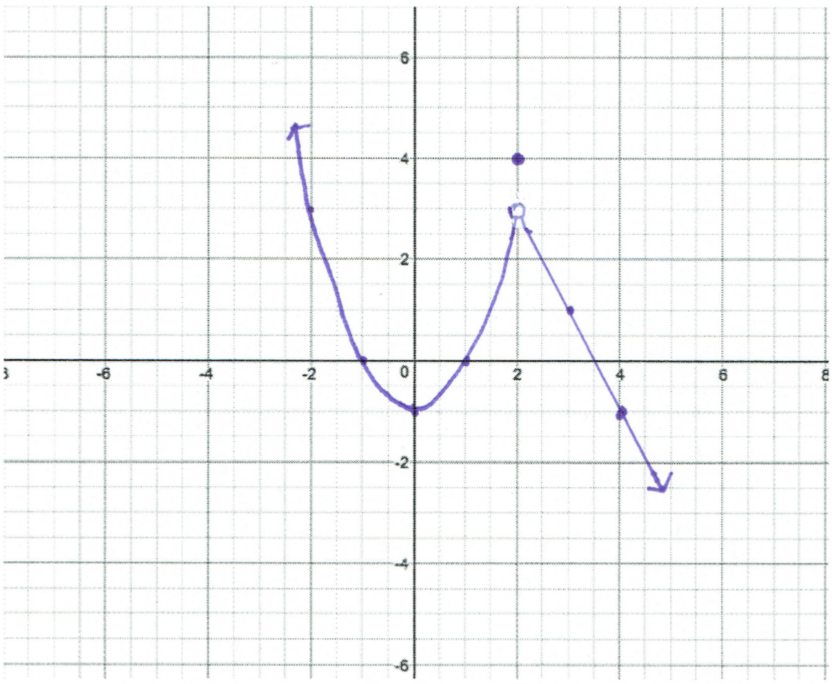
$f(4) = -2(4) + 7$
$= -8 + 7$
$= -1$

$\Rightarrow f(-1) + f(4) = 0 + -1$
$= -1$

(b) Evaluate $f(f(\sqrt{3}))$.

$f((\sqrt{3})^2 - 1)$
$f(2)$
4

(c) Graph the function f below.



3. Solve the following inequalities. Use interval notation for your solution.

(a) $\left|\frac{2}{x-7}\right| - 1 < 0$

$$\left|\frac{2}{x-7}\right| < 1$$

$$\frac{2}{x-7} < 1 \quad \text{AND} \quad \frac{2}{x-7} > -1$$

$$\frac{2}{x-7} - 1 < 0 \qquad \frac{2}{x-7} + 1 > 0$$

$$\frac{2-(x-7)}{x-7} < 0 \qquad \frac{2+x-7}{x-7} > 0$$

$$\frac{9-x}{x-7} < 0 \quad \text{AND} \quad \frac{x-5}{x-7} > 0$$

$\frac{9-x}{x-7} < 0$ — − und + 0 − (7, 9)

AND

$\frac{x-5}{x-7} > 0$ + 0 − und + (5, 7)

Looking for overlap

$\Rightarrow (-\infty, 5) \cup (9, \infty)$

(b) $3x^2 + x \geq -4x + 2$

$$3x^2 + 5x - 2 \geq 0$$

$$(3x-1)(x+2) \geq 0$$

Roots: $x = \frac{1}{3}$ $x = -2$

+ 0 − 0 +

-2 $\frac{1}{3}$

$(-\infty, -2] \cup [\frac{1}{3}, \infty)$

4. Suppose that f is an even function and g is an odd function. Use the table below to answer parts (a) - (c).

x	0	1	2	3
$f(x)$	-2	3	4	0
$g(x)$	0	2	8	2

(a) Find $f(g(0))$.

$f(g(0)) = f(0) = \underline{-2}$

(b) Find $f(g(1))$.

$f(2)$

$\underline{4}$

(c) Find $g(f(0))$.

$g(-2) = -g(2)$ since g is odd

$\underline{= -8}$

5. Suppose we have an angle θ with $\frac{3\pi}{2} < \theta < 2\pi$ such that $\cos(\theta) = \frac{5}{13}$. Use this information to answer (a) and (b).

   (a) Find the value of $\sin(\theta)$ using the Pythagorean Identity as one of the steps.

   $\sin^2\theta + \cos^2\theta = 1$

   $\sin^2\theta + \left(\frac{5}{13}\right)^2 = 1$

   $\sin^2\theta + \frac{25}{169} = 1$

   $\sin^2\theta = \frac{144}{169}$

   $\sin\theta = \pm\sqrt{\frac{144}{169}}$

   $= \pm\frac{12}{13}$

   Since θ is in Quad IV, $\sin\theta = -\frac{12}{13}$.

   (b) Find $\cot(\theta)$.

   $\cot\theta = \frac{\cos\theta}{\sin\theta}$

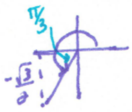
   $= \frac{5/13}{-12/13}$

   $= -5/12$

6. Find the exact indicated value. Leave answers in radical form.

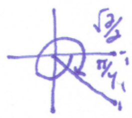
   (a) $\sin\left(\frac{4\pi}{3}\right)$

   $= -\frac{\sqrt{3}}{2}$



   (b) $\cos\left(\frac{-9\pi}{4}\right)$

   $= \frac{\sqrt{2}}{2}$



   (c) $\csc\left(\frac{\pi}{2}\right)$

   $= \frac{1}{\sin \pi/2}$

   $= \frac{1}{1}$

   $= 1$

   (d) $\cos(3\pi)$

   $= -1$

7. Use the function $f(x) = \sqrt{3x-4} - 1$ to answer (a) - (d).

   (a) Find the domain of f.

   $3x - 4 \geq 0$
   $3x \geq 4$
   $x \geq 4/3$
   $[4/3, \infty)$

   (b) Find the range of f.

   $[-1, \infty)$ (Square root function has shifted down 1 unit)

   (c) Find the inverse of $f(x)$, i.e. find $f^{-1}(x)$.

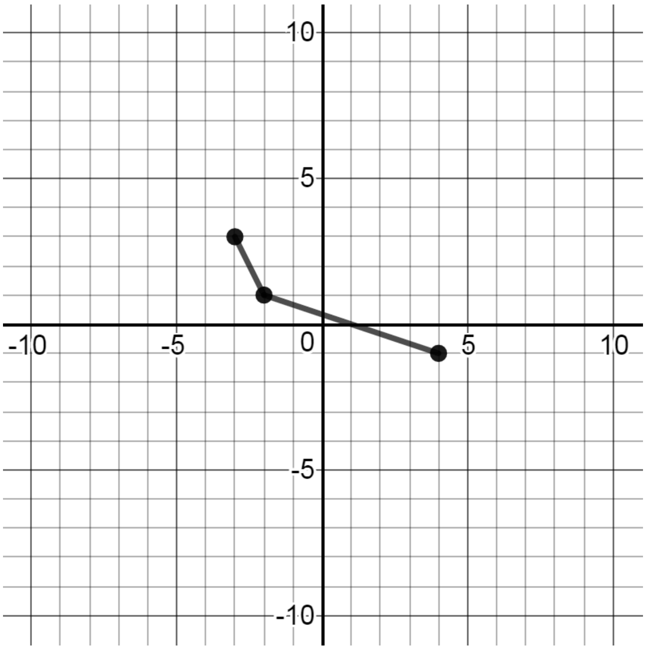
   $y = \sqrt{3x-4} - 1$
   $x = \sqrt{3y-4} - 1$
   $x + 1 = \sqrt{3y-4}$
   $(x+1)^2 = 3y - 4$
   $(x+1)^2 + 4 = 3y$

   $y = f^{-1}(x) = \dfrac{(x+1)^2 + 4}{3}$ or $\frac{1}{3}(x+1)^2 + \frac{4}{3}$

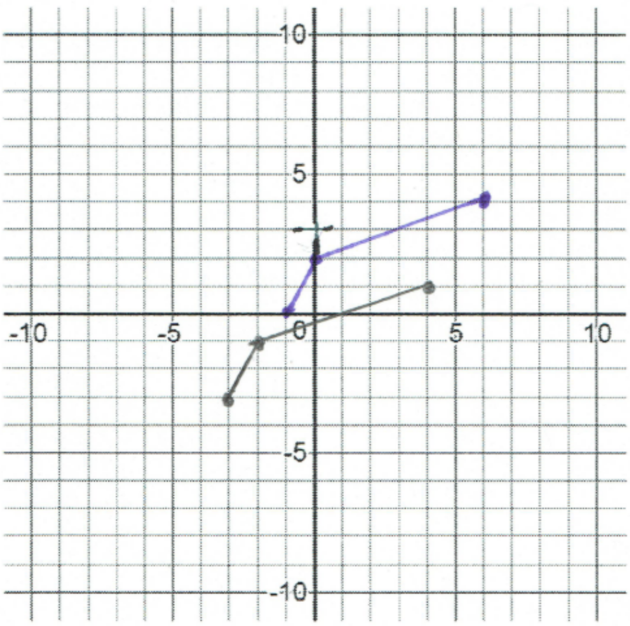
   (d) Find the domain and range of $f^{-1}(x)$.

   Domain of $f^{-1}(x)$ = Range of $f(x) = [-1, \infty)$
   Range of $f^{-1}(x)$ = Domain of $f(x) = [\frac{4}{3}, \infty)$

8. The graph of $f(x)$ is shown below. Use the graph to answer (a) - (c).



(a) Graph the following transformation of f: $y = -f(x-2)+3$



In pencil:
$y = -f(x)$
Reflect over x-axis

In purple
$y = -f(x-2)+3$
Shift 2 right and 3 up

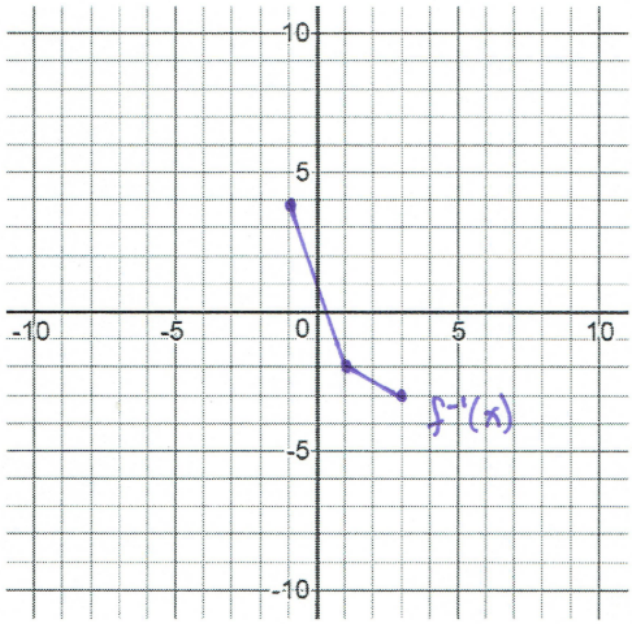
(b) Evaluate $y = -3f(x-5)+2$ at $x = 3$.

$y = -3f(3-5)+2$
$= -3f(-2)+2$
$= -3(1)+2$
$= -3+2$
$= -1$

find $f(-2)$ from graph of $f(x)$
$f(-2)=1$

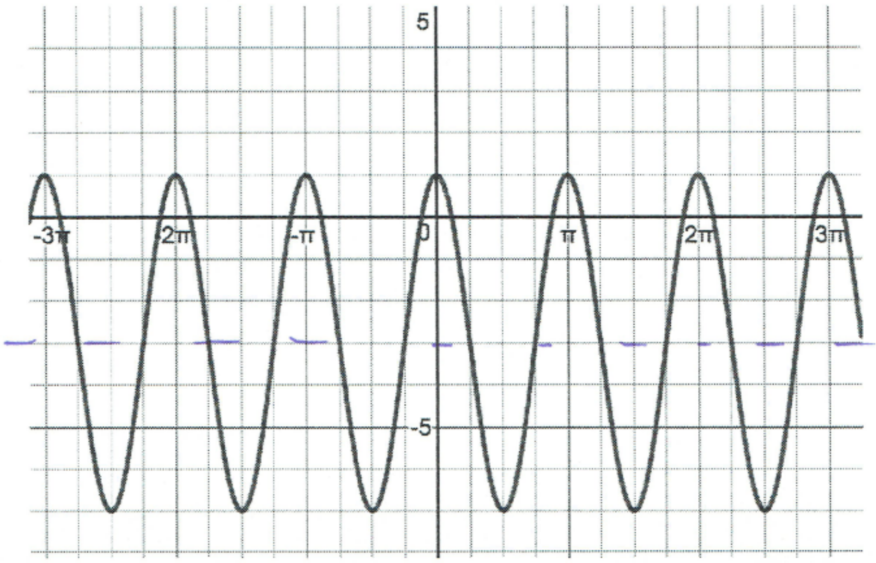
(or: transform $f(x)$ graph and then find y coordinate when $x=3$)

(c) Is $f(x)$ invertible? If yes, graph $f^{-1}(x)$ below. If not, state why it is not invertible.



$f(x)$ is invertible since it is one-to-one

9. The graph of $f(x)$ is shown below. Use the graph to answer (a) - (c).



Axis of wave $y = -3$

(a) What is the amplitude of f?

4

(b) What is the period of f?

π

(c) What is an equation for $f(x)$?

$f(x) =$ $4\cos(2x) - 3$ OR $y = -4\cos\left[2\left(x \pm \frac{\pi}{2}\right)\right] - 3$

Could also do sine equation

Period $= \frac{2\pi}{B}$

$\pi = \frac{2\pi}{B}$

$B\pi = 2\pi$

$B = 2$

10. (a) Write the expression as a sum/or difference of logarithms. Express powers as factors.

$$\ln\left[\frac{5x^2\sqrt[3]{1-x}}{4(x+1)^2}\right]$$

$= \ln\left(5x^2\sqrt[3]{1-x}\right) - \ln\left(4(x+1)^2\right)$

$= \ln 5x^2 + \ln\sqrt[3]{1-x} - \left(\ln 4 + \ln(x+1)^2\right)$

$= \ln 5 + \ln x^2 + \ln\sqrt[3]{1-x} - \ln 4 - \ln(x+1)^2$

$= \underline{\ln 5 + 2\ln x + \frac{1}{3}\ln(1-x) - \ln 4 - 2\ln(x+1)}$

(b) Solve the following equation.

$$\log_4(x+3) + \log_4(2-x) = 1$$

$\log_4\left((x+3)(2-x)\right) = 1$

$\log_4\left(-x^2 - x + 6\right) = 1$

$-x^2 - x + 6 = 4^1$

$-x^2 - x + 2 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$\underline{x = -2 \quad , \quad x = 1}$

Both values are in the domain. ✓