

# A beleza matemática da criptografia

Marcel de Sena Dall'Agnol

# Como construir cripto?

(Não precisa de computador!)

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Objetivos (*threat modeling*):

- Confidencialidade
- Autenticação
- Integridade
- ...

# Como construir cripto?

(Não precisa de computador!)

Objetivos (*threat modeling*):

- Confidencialidade
- Autenticação
- Integridade
- . . .

Um componente básico: operação fácil de fazer, mas difícil de desfazer sem uma "chave".

## Cifra de César

GENERAL, ATAQUE AMANHÃ À  
MEIA NOITE.

## Cifra de César

HENERAL, ATAQUE AMANHÃ À  
MEIA NOITE.

## Cifra de César

HFNERAL, ATAQUE AMANHÃ À  
MEIA NOITE.

## Cifra de César

HFOERAL, ATAQUE AMANHÃ À  
MEIA NOITE.

## Cifra de César

HFOFRAL, ATAQUE AMANHÃ À  
MEIA NOITE.

## Cifra de César

HFOFSAL, ATAQUE AMANHÃ À  
MEIA NOITE.

## Cifra de César

HFOFSBL, ATAQUE AMANHÃ À  
MEIA NOITE.

## Cifra de César

HFOFSBM, ATAQUE AMANHÃ À  
MEIA NOITE.

# Cifra de César

HFOFSBM, BTAQUE AMANHÃ À  
MEIA NOITE.

# Cifra de César

HFOFSBM, BUQUE AMANHÃ À  
MEIA NOITE.

# Cifra de César

HFOFSBM, BUBQUE AMANHÃ À  
MEIA NOITE.

# Cifra de César

HFOFSBM, BUBRUE AMANHÃ À  
MEIA NOITE.

# Cifra de César

HFOFSBM, BUBRVÉ AMANHÃ À  
MEIA NOITE.

# Cifra de César

HFOFSBM, BUBRVF AMANHÃ À  
MEIA NOITE.

# Cifra de César

HFOFSBM, BUBRVF BNBOIB B  
NFJB OPJUF.

# Cifra de César

HFOFSBM, BUBRVF BNBOIB B  
NFJB OPJUF.

A	B	C	D	E	F	G	H	I	J	K	L	M
B	C	D	E	F	G	H	I	J	K	L	M	N
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
O	P	Q	R	S	T	U	V	W	X	Y	Z	A

## Cifra de César

GENERAL, ATAQUE AMANHÃ À  
MEIA NOITE.

# Cifra de César

KIRIVEP, EXEUYI EQERLE E QIME  
RSMXI.

A	B	C	D	E	F	G	H	I	J	K	L	M
E	F	G	H	I	J	K	L	M	N	O	P	Q
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
R	S	T	U	V	W	X	Y	Z	A	B	C	D

# Cifra de Substituição

GENERAL, ATAQUE AMANHÃ À  
MEIA NOITE.

# Cifra de Substituição

□Ø∇Ø#⊥■,  
⊥Θ⊥∀ΩØ ⊥ ΓØΞ⊥ ∇¬ΞΘØ.

A	B	C	D	E	F	G	H	I	J	K	L	M
⊥	♣	♠	§	Ø	★	□	△	Ξ	◊	♥	■	Γ
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
▽	¬	▼	∀	#	Ξ	Θ	Ω	Π	✉	✉	✉	✉

# O que significa "difícil" ?



# Codificação binária

A = 

B = 

C = 

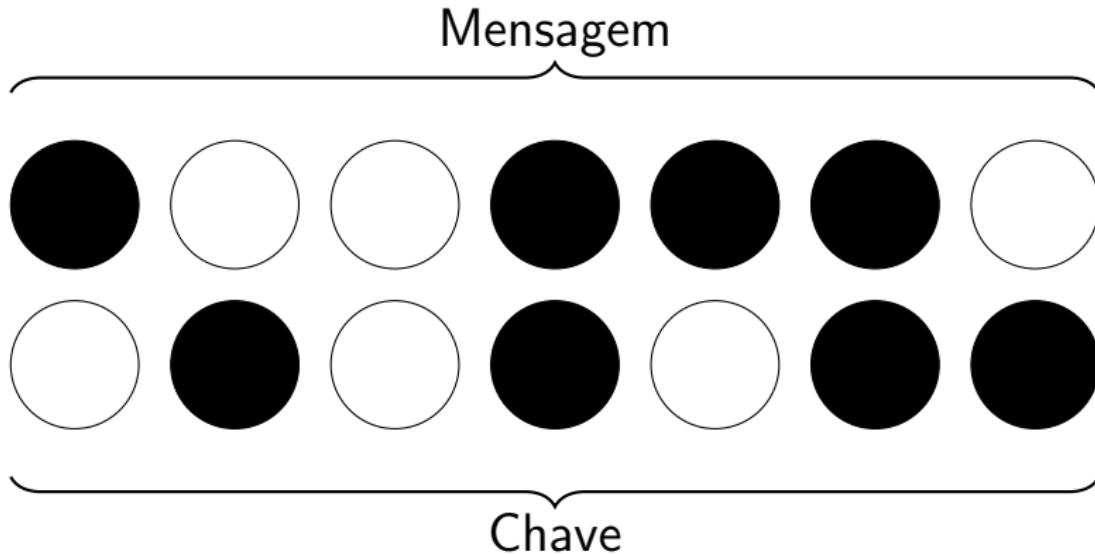
D = 

E = 

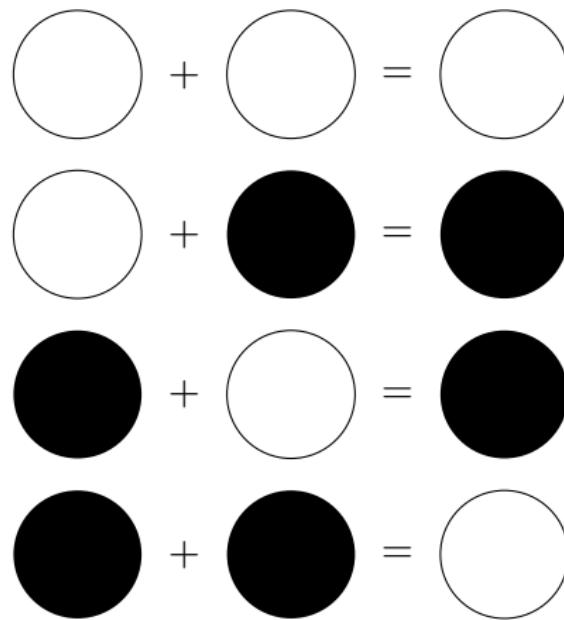
F = 

:

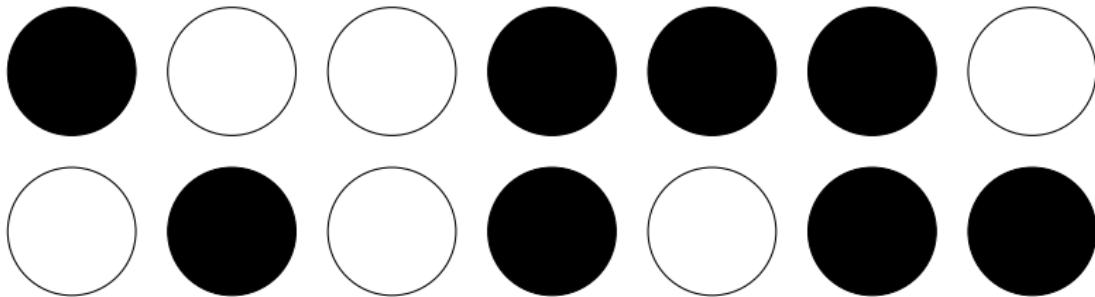
# One-Time Pad (OTP)



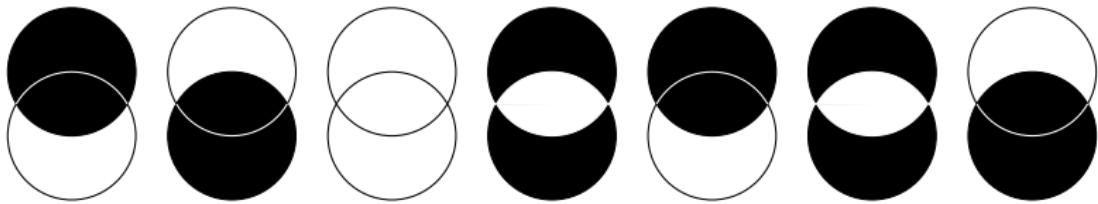
# XOR



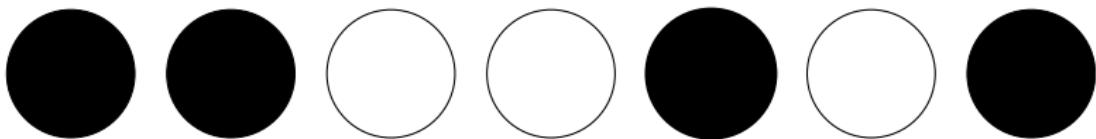
# One-Time Pad (OTP)



# One-Time Pad (OTP)

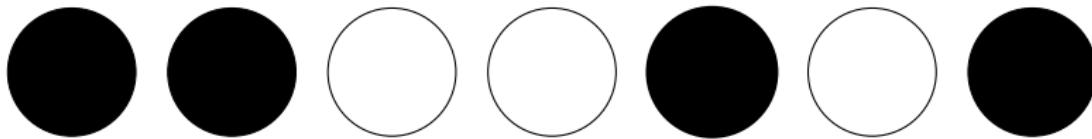


# One-Time Pad (OTP)

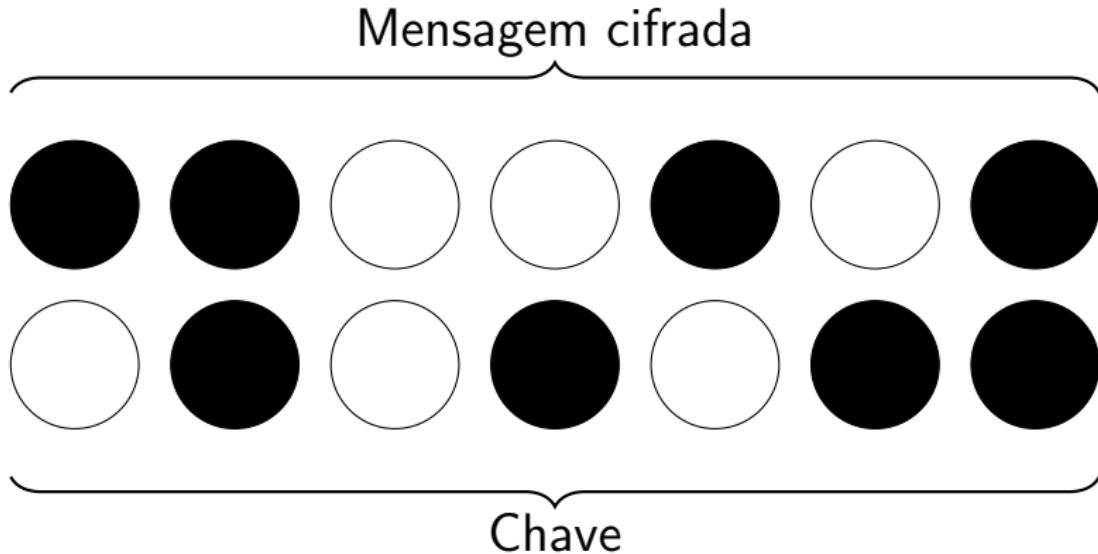


# One-Time Pad (OTP)

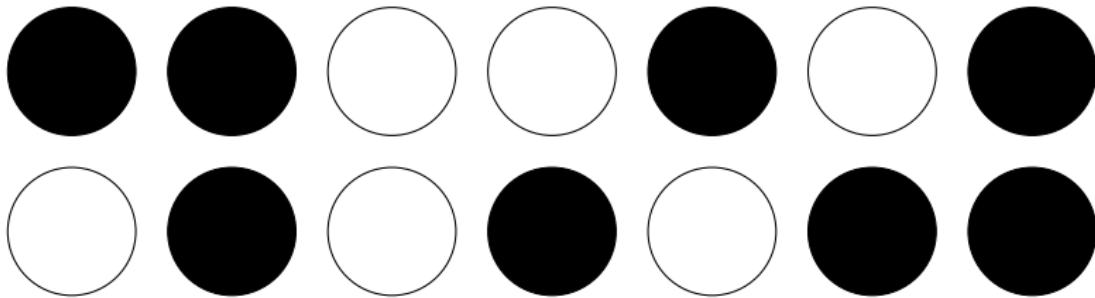
Mensagem cifrada



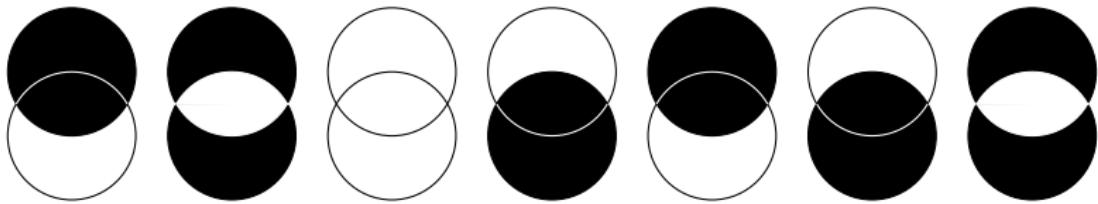
# One-Time Pad (OTP)



# One-Time Pad (OTP)



# One-Time Pad (OTP)



# One-Time Pad (OTP)

Mensagem (decifrada)



# Busca linear

Ana



Catarina

Jonas

Paula

Marco

Ana

# Busca linear

Ana



Catarina

Jonas

Paula

Marco

Ana

# Busca linear

Ana



Catarina

Jonas

Paula

Marco

Ana

# Busca linear

Ana



Catarina

Jonas

Paula

Marco

Ana

# Busca linear

Ana



Catarina

Jonas

Paula

Marco

Ana

# Busca linear

Ana



...



Ana

# Busca linear

Ana



...



# Busca linear

Ana



...



# Busca linear

Ana



# Busca linear

Ana



Ana

...

# Busca linear

Ana



...



Ana

# Busca linear

Ana



Ana

...

# Busca linear

Ana



Ana

...

# Busca linear



# Ordenação (Bubble sort)



Catarina

Jonas

Paula

Marco

Ana

# Ordenação (Bubble sort)



Catarina

Jonas

Paula

Marco

Ana

# Ordenação (Bubble sort)



Catarina

Jonas

Paula

Marco

Ana

# Ordenação (Bubble sort)

Catarina

Jonas

Marco

Paula

Ana



# Ordenação (Bubble sort)



Catarina

Jonas

Marco

Ana

Paula

# Ordenação (Bubble sort)



Catarina

Jonas

Marco

Ana

Paula

# Ordenação (Bubble sort)



Catarina

Jonas

Marco

Ana

Paula

# Ordenação (Bubble sort)

Catarina

Jonas

Ana

Marco

Paula



# Ordenação (Bubble sort)



Catarina

Jonas

Ana

Marco

Paula

# Ordenação (Bubble sort)



Catarina

Jonas

Ana

Marco

Paula

# Ordenação (Bubble sort)



Catarina

Ana

Jonas

Marco

Paula

# Ordenação (Bubble sort)

Catarina

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Jonas

Marco

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# Ordenação (Bubble sort)



Catarina

Ana

Jonas

Marco

Paula

# Ordenação (Bubble sort)



Ana

Catarina

Jonas

Marco

Paula

# Ordenação (Bubble sort)



Ana

Catarina

Jonas

Marco

Paula

# Ordenação (Bubble sort)



Ana

Catarina

Jonas

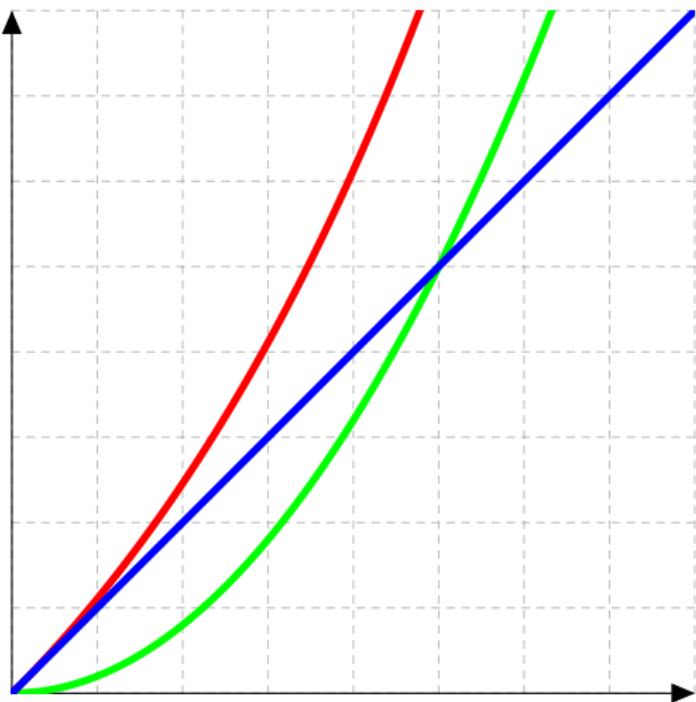
Marco

Paula

# Ordenação (Bubble sort)

$$\approx x^2$$

# Complexidade algorítmica

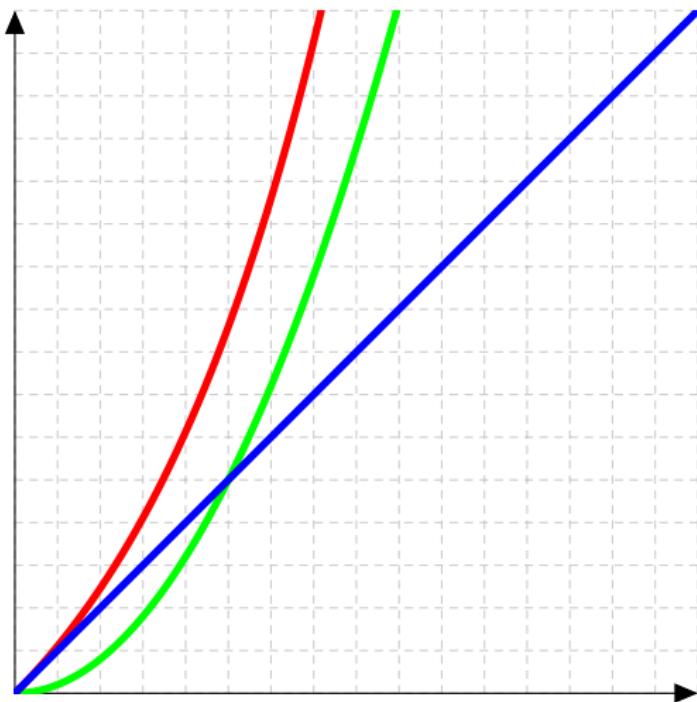


$x$

$x^2$

$e^x$

# Complexidade algorítmica



$x$

$x^2$

$e^x$

# Complexidade Algorítmica

X

# Complexidade Algorítmica

$$x^2$$

# Complexidade Algorítmica

$x^{10}$

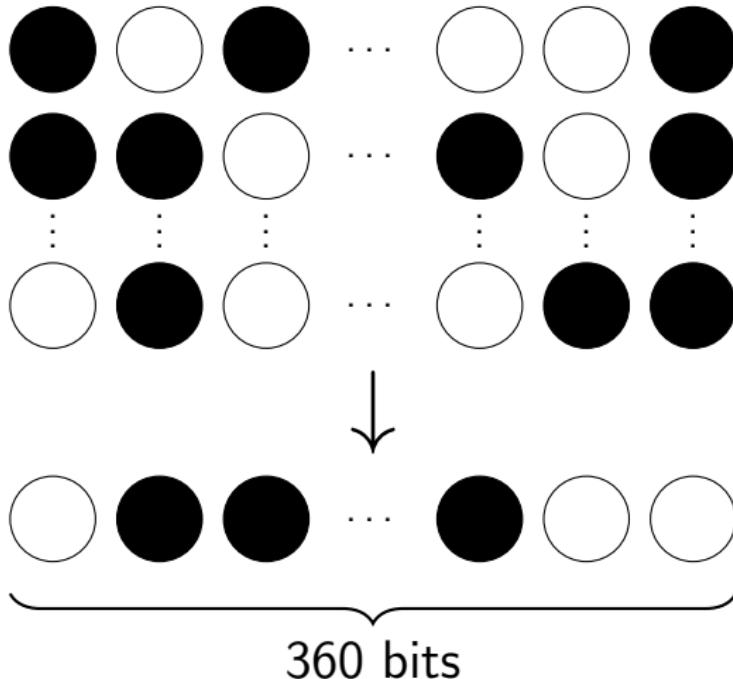
# Complexidade Algorítmica

$x^{7449279}$

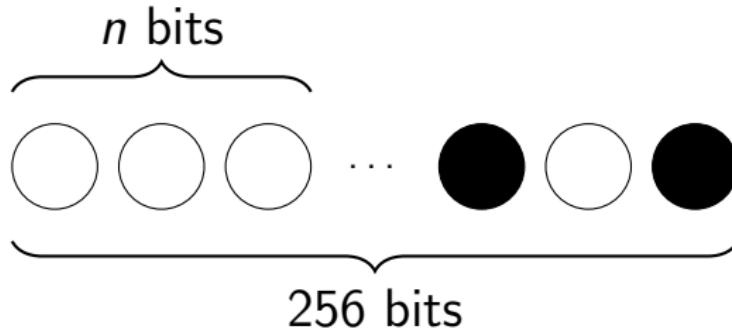
# Complexidade Algorítmica

$$e^x$$

# Pré-imagem de Hash



# Pré-imagem de Hash



# Pré-imagem de Hash

$\approx 2^{360}$

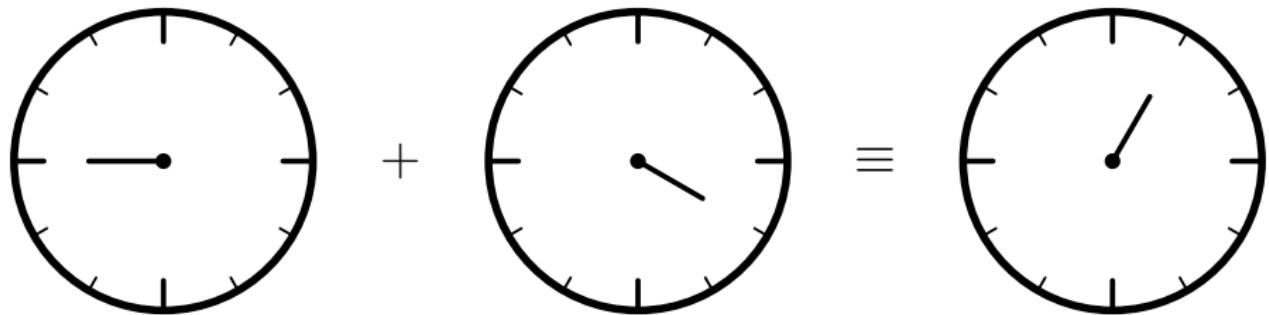
# Pré-imagem de Hash

$$\underbrace{10^{80}}_{\text{Átomos no universo}} \times \underbrace{10^{10}}_{10 \text{ GHz}} \times \underbrace{4,4 \cdot 10^{17}}_{\text{Idade do universo}}$$

$$\approx 2^{358}$$

# P vs. NP

# Aritmética módulo p



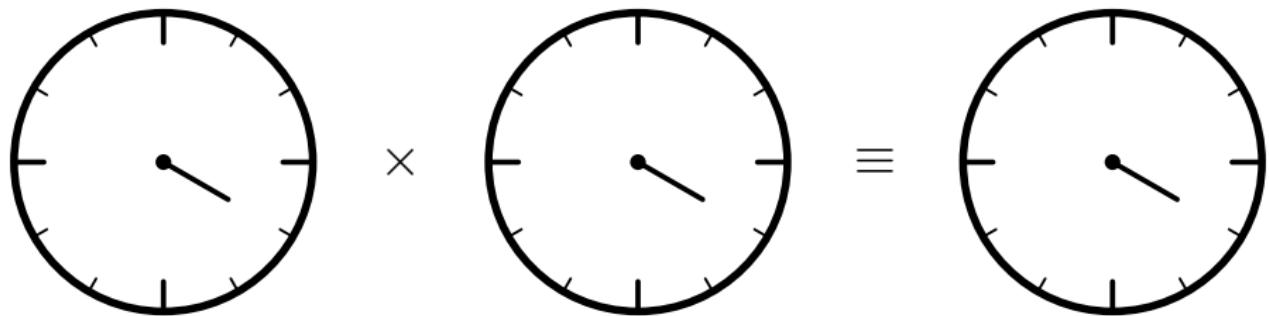
# Aritmética módulo p

$$9 + 4 \equiv 13 \pmod{12}$$

# Aritmética módulo p

$$9 + 4 \equiv 1 \pmod{12}$$

# Aritmética módulo p



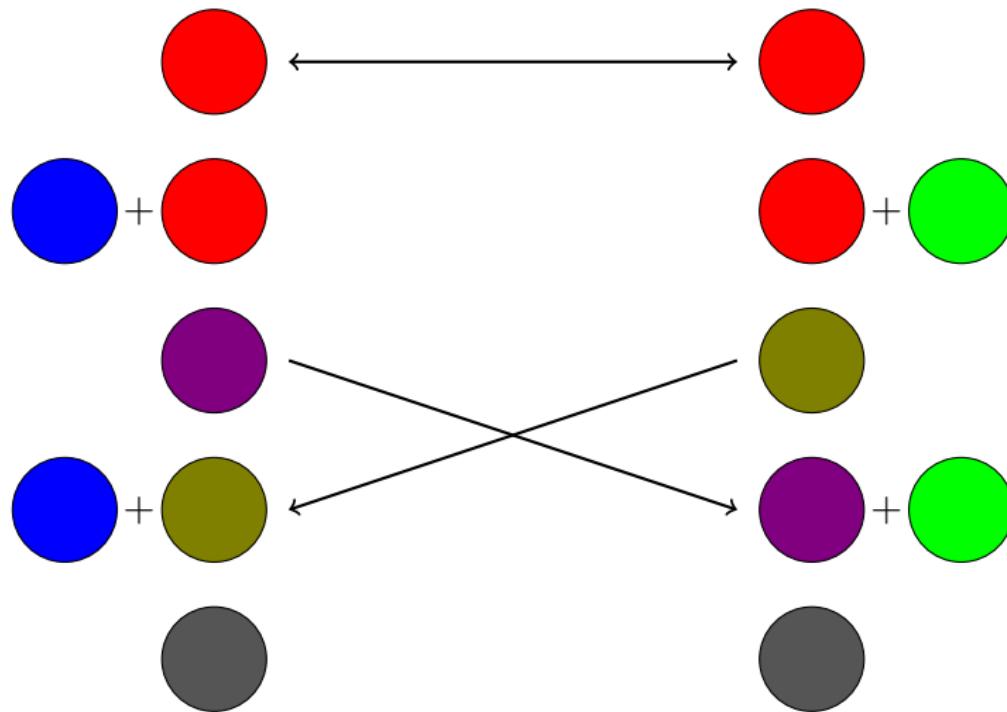
# Aritmética módulo p

$$4 \times 4 \equiv 16 \pmod{12}$$

# Aritmética módulo p

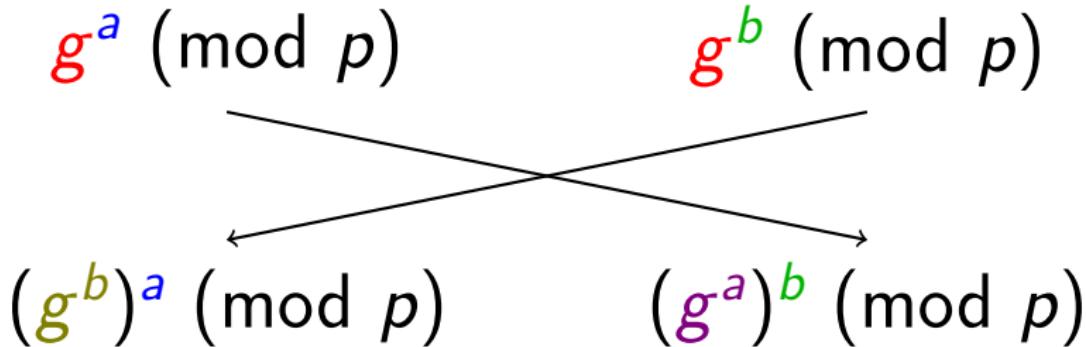
$$4 \quad \times \quad 4 \quad \equiv \quad 4 \pmod{12}$$

# Diffie-Hellman



# Diffie-Hellman

$$p, g \quad \longleftrightarrow \quad p, g$$



# RSA

$p, q$

⋮

$n = pq$

$d = \dots \longrightarrow n, e$

$e = \dots$

$$M^{de} \equiv M \pmod{n}$$

# RSA

$$(M^e)^d \pmod{n}$$

$$M^e \pmod{n}$$

# Difícil?

## Diffie-Hellman

Dados  $g$ ,  $p$  e  $H$ ,  
encontre  $a$  tal que  
 $g^a \equiv H \pmod{p}$ .

## RSA

Dado  $n$ , encontre  
 $p$  e  $q$  tais que  
 $n = pq$ .

# Difícil?

## Logaritmo discreto

Dados  $g$ ,  $p$  e  $H$ ,  
encontre  $a$  tal que  
 $g^a \equiv H \pmod{p}$ .

## Fatoração

Dado  $n$ , encontre  
 $p$  e  $q$  tais que  
 $n = pq$ .

# Computação e criptografia quântica

Quebra DH, RSA, ECC...

# Computação e criptografia quântica

Quebra DH, RSA, ECC...

Simétrica: BB84 + reconciliação + amplificação

Assimétrica e autenticação: Ring-LWE (NP-completo), NTRU

# Computação e criptografia quântica

Quebra DH, RSA, ECC...

Simétrica: BB84 + reconciliação + amplificação

Assimétrica e autenticação: Ring-LWE (NP-completo), NTRU

# Computação e criptografia quântica

Quebra DH, RSA, ECC...

Simétrica: BB84 + reconciliação + amplificação

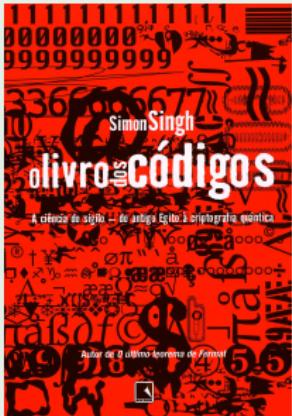
Assimétrica e autenticação: Ring-LWE (NP-completo), NTRU

Supremacia quântica?

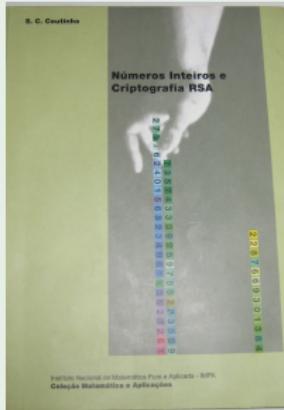
# Apelo



# Referências

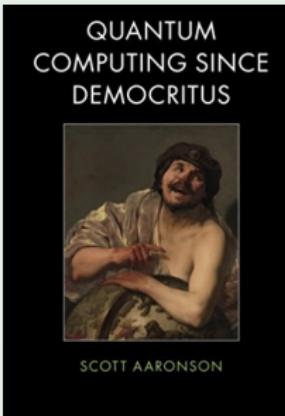


O Livro dos Códigos  
*Simon Singh*



Números Inteiros e Criptografia RSA  
*Severino Collier Coutinho*

# Referências



Quantum Computing Since  
Democritus  
*Scott Aaronson*



Summer of the Shark

Sharks are a single word or phrase is enough to expand your mental field across almost every subject. "Killing sharks" always means "killing people". "Shark attacks" always means "murders". Yesterday I heard another such phrase: "Murder of the shark".

This appears to be the summer of 2002, when factory meat processing, the media gave massive coverage to every single shark attack it could find, causing the widespread impression of an epidemic of shark attacks. In fact, there were very few shark attacks in 2002. In reality, depending on what you compare it to, the rate of shark attacks in 2002 was either about the same as in 2001, or slightly lower. As far as I can tell, the situation is that the absolute number of shark attacks has been increasing over the decades, but the rate of increase has been decreasing, so that the growth (and to aay more sharks and attacks down). The risk per person per year of being killed by a shark, for most people, appears to have been going down. This might or might not be related to the fact that shark populations are precipitately dropping, due to the fact that they're overfishing and hunting, but who's the conservation of habitat?

There's a tendency—I notice it in myself, at least—when someone says something like "there must be something going on, since otherwise you wouldn't see everyone talking about it."

The point of this little exercise of the shark is to remind you that you can be, and others, a weirdly a product of people everywhere looking for a certain story even while the actual story is something completely different, and perhaps even the opposite. Of course this has been a favorite theme of Steven Hirsch, but I don't know if ever having him here today, Omer, though he's a bit of a nut, fully grasped how much of his pervasiveness for me. If a self-reinforcing type bubble can form around a single idea, imagine how much more powerful a single number of shark attacks, imagine how common it must be with more ridiculous social phenomena.

Shtetl-Optimized  
*Scott Aaronson*

# Referências

The screenshot shows a presentation slide with the following details:

**Title:** A Few Thoughts on Cryptographic Engineering

**Author:** Matthew Green

**Abstract:** Hash-based Signatures: An illustrated Primer

**Text:** Once upon a time, I have pledged to observe from now on keep and interesting tends. The first is that we're finally starting to see the engineering of cryptosystems. For many years, the cryptosystems that we've had, they are fine every day to complete ranging from encrypted communications to cryptocurrencies.

**Text:** The second tend is that cryptographers are getting mostly for all those good results and...

**Text:** But before I get off of that... much further below... let me draw the line in not a good about the interesting tendencies, see it's about the success of cryptosystems. For many years, I've been trained to give you a good grade, much more words. This year will be the about one of the simplest card reader's I suppose.

**Text:** Hash based signature schemes were first proposed in the late 1970s by Leslie Lamport, and specifically improved by Ralph Merkle and others. For many years they largely remained as a interesting a cryptographic backbones, used in Merkle trees, digital signatures, and other interesting applications. However, in recent years these constructions have entered something of a renaissance, with the introduction of the Merkle tree and its variants together the example... But it's largely viewed as related to related questions about the theory's algorithm.

**Text:** Hash functions and signature schemes

**Text:** In order to understand hash based signatures, it's important that you have some familiarity with cryptographic hash functions. These functions take some input at their interface as an input, a message, and produce a hash as the "output".

## A Few Thoughts on Cryptographic Engineering

### Matthew Green

## Referências

# Schneier on Security

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## LC4: Another Pen-and-Paper Cipher

Interacting systems often [LCD](#).

disnares ([@disnares](#)) is a real-life device that can run the instruction by [LCD](#). It's a microcontroller-based device that can receive and interpret such instructions. It's a great way to interact with physical objects and computers connecting to the digital realm. I thought it'd be a good time to introduce another pen-and-paper cipher that can be used with LCDs. This cipher is similar to [Diffie-Hellman](#), but it's a bit easier to implement. It's also a bit slower, but that's fine because it's a pen-and-paper cipher. This paper defines the algorithm and describes how to use it with an LCD.

For more on interacting with the physical world, see my [post](#) on how to build a [Raspberry Pi](#) robot. A pen-and-paper cipher that can do a lot of things with physical objects is the [Robot](#).

As always, I'm looking forward to hearing what you think.

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## NIST Issues Call for "Lightweight Cryptography" Algorithms

Creating three estimates to the goal of NIST's lightweight cryptography initiative, which aims to develop cryptographic algorithms capable of being implemented in low-power, low-cost, constrained environments such as sensors, actuators and microcontrollers that will function on-edge, without the need for a central server. The call for submissions is open and could be easily limited from the others found in even the simplest of forums. The NIST has issued a call for submissions for the first round of the new initiative and the Radio Frequency Identification (RFID) tag industry is responding.

All of these projects are responsive to make well-structured submissions, and the NIST is asking for them to be submitted now.

The NIST's [RFID and EPCC](#) would be very useful.

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## Bruce Schneier

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[About Bruce Schneier](#)



Bruce Schneier is a security technologist and author. He is the founder of Schneier on Security, a blog on security and privacy issues. He is a frequent speaker at security conferences and has written numerous books on security, including *Applied Cryptography*, *Secrets & Lies*, and *Liars and Outliers*. He is a fellow at the Berkman Center for Internet & Society at Harvard Law School and a member of the board of directors of the Electronic Frontier Foundation.

[View discussions](#) [Individual comments](#) [Newest first](#) [Oldest first](#) [12 comments](#)

## Lightweight Cryptography

This post has been updated.

Data is Fun. So Why Isn't Cryptography?

What Is NIST's Tesserae Initiative?

Security They Way Google Searches It

How to Implement Elliptic Curve Cryptography in Python for Security

Google's New Privacy Standard

The Data Privacy and Privacy

Smarter Data Decisions

[View discussions](#) [Individual comments](#) [Newest first](#) [Oldest first](#) [12 comments](#)

Home > Computer Science > Computer Security and Networks

# Cryptography I



**About this course:** Cryptography is an indispensable tool for protecting systems. In this course you will learn the inner workings of cryptography and how to use them in real-world applications. The course begins with a discussion of what it means for two parties to have a shared secret key and communicate securely when a

▼ More

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**Created by:** Stanford University



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**Taught by:** [Dan Boneh](#), Professor  
Computer Science

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# Cryptography I

## *Dan Boneh*

# Referências

The screenshot shows the Udacity logo at the top left. Below it, the course title "Applied Cryptography" is displayed in large green text, followed by the subtitle "Science of Secrets". A prominent blue button labeled "START FREE COURSE" is centered below the subtitle. The background is white with a light gray footer bar.

Applied Cryptography  
*Dave Evans*

The screenshot features the Caltech and TU Delft logos at the top right. The course title "Quantum Cryptography" is in large green text, with the subtitle "Learn how quantum communication provides security that is guaranteed by the laws of nature." Below this, a section titled "About this course" contains detailed course information and a rating of 4.5/5 stars. At the bottom, two faculty profiles are shown: Stephanie Wehner from Caltech and Thomas Vidick from the California Institute of Technology. The background is white with a light gray footer bar.

Quantum Cryptography  
*Stephanie Wehner e Thomas Vidick*

# Referências

Sites:

<https://www.scottaaronson.com/blog/>

<https://blog.cryptographyengineering.com/>

<https://www.schneier.com/>

<https://www.coursera.org/learn/crypto>

<https://udacity.com/course/applied-cryptography--cs387>

<https://www.edx.org/course/quantum-cryptography-caltechx-delftx-qucryptox-0>

# Referências

Imagens:

[https://www.vectorportal.com/stockvectors/Technology/  
desktop-personal-computer-vector/12976.aspx](https://www.vectorportal.com/stockvectors/Technology/desktop-personal-computer-vector/12976.aspx)

<https://bitcoin.org/img/icons/logotop.svg>

<https://images-na.ssl-images-amazon.com/images/I/819HiREnxL.jpg>

<https://www.publicdomainpictures.net/view-image.php?image=76327>