Using Conical and Spherical Mirrors with Conventional Cameras for 360º Panorama Views in a Single Image

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***Abstract* – Conventional video cameras with projective lens have restricted fields of view. Adding mirrors of different shapes, 360º panoramic views can be achieved in a single image. For robotic football, we envisage a simple and low cost setup. The system presented uses a single home made conical mirror. The setup conditions can be easily adjusted to cover other mirror shapes, such as spherical, paraboloidal, hyperboloidal and other shapes generated by axial revolution. The single viewpoint restrictions, usually imposed for general uses, are overcome by the specific setup conditions.**

I INTRODUCTION

For middle size robotic football league (Fig. 1), restrictions in size and weight impose the use of a single camera for the vision system. A panoramic 360º view can be achieved recurring to mirrors [1] but image interpretation becomes more difficult.

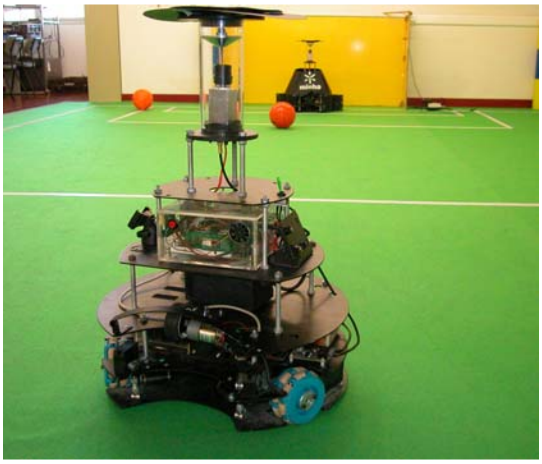

Figure 1
Example of mobile robot for middle size football league, using conical mirror

Developing further the concept of object distance measurement using a single projective video camera [2] and accounting for the mirror effect, similar angular conditions are reached. This allows distance calculations for objects at known height above the floor, where the robot moves.

II THE SETUP

The camera is mounted pointing upwards to the conical mirror. This is aligned to make lens and mirror axis coincide, in the arrangement sketched in Fig. 2.
For the projective optical system (pin-hole approximation) the object-image correspondence can be established by simple ray-optics [3]. Due to the alignment conditions, the single optical axis (vertical) and the ray tracing segments are coplanar. This means that for every object point its corresponding image point lies on the same plane as defined by the optical axis and the object point (and vice-versa). Also, if the orthogonal projections of the objects are used to define their positions (their height is assumed known) and using a radial coordinate system centered on the intersection of the horizontal plane with the optical axis, a simple distance conversion to the optical center is all that is needed to position the object once known is corresponding image. This is basically the same result that the angle invariance principle described in [2] envisages.

A Using a Conical Mirror

The vertical plane containing the optical axis and the related object and image points intersect the conical mirror surface in two straight lines as sketched in Fig. 3. Every image point can be associated to an angle α. The ray tracing method allows the localization of the point *R* where the mirror reflection occurs. Reflection laws impose the corresponding positioning of the virtual optical lens center *O’*, where the object rays should be pointed to. And this point is fixed for every object point on the plane in analysis.
Defining the conical mirror by the value of angle β (β =0 corresponds to the plane mirror) it means that for every angle a for the image point the corresponding angle for the object is

$$\theta = \alpha + 2 \cdot \beta \,. \qquad (1)$$

The angle invariance in [2] becomes “constant added to the angle α".

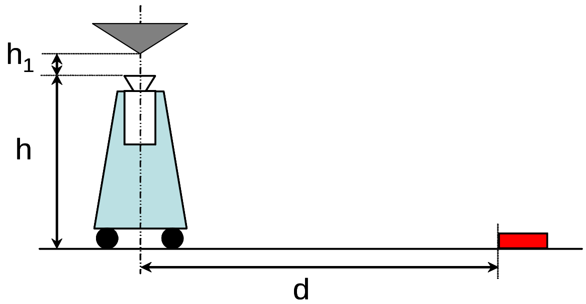

Figure 2
The setup, with the distances and heights

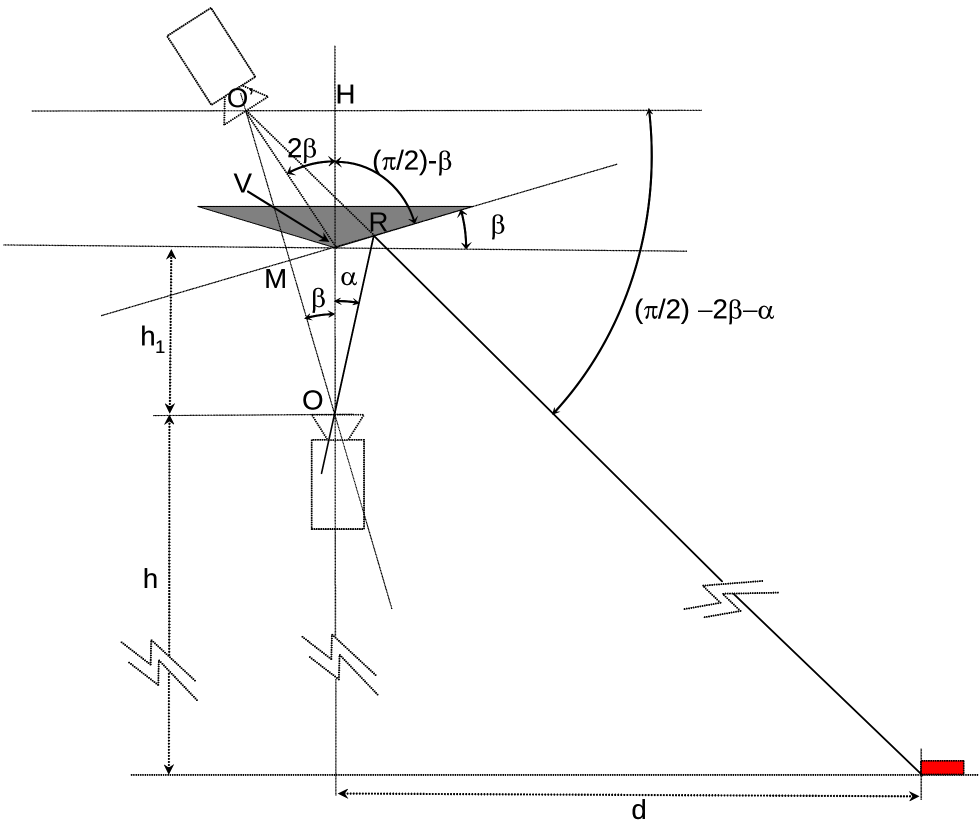

Figure 3
Detail of the geometry and variables used in the equations

B Practical Considerations for the Conical Mirror

From the image, once identified the point of interest and its coordinates (x, y) in pixel units, and considering the optical image center coordinates (x_0, y_0), and the focal distance f (also in pixel coordinates) the angle α is

$$\alpha = \tan^{-1}\left(\frac{\sqrt{(x-x_0)^2+(y-y_o)^2}}{f}\right). \qquad (2)$$

The height and position of the virtual optical center are, from Figures 2 and 3,

$$h = h_1 + 2\cdot\overline{VO}\cdot\cos^2\beta \qquad (3)$$

and

$$\overline{O'H} = \overline{VO}\cdot\sin(2\cdot\beta). \qquad (4)$$

The tilt angle to the horizontal, δ, can be calculated from a and the setup parameters as

$$\theta = \frac{\pi}{2} - (2\cdot\beta+\alpha). \qquad (5)$$

The position of the central viewpoint maintains its position in the vertical plane defined by the camera optical axis and object point under consideration.
The localization of the object points corresponding to any identified image point becomes a problem in planar geometry. Horizontal and vertical orientations are separable. The first is calculated from the angular coordinate extracted directly from the camera image; the vertical tilt positions of object points are to be calculated from the distances to the optical image center.

C Using a Spherical Mirror

Convex spherical mirrors can increase the viewing angles in tight places and its usage is well known in road traffic corners.
Its use in robotic vision can be analyzed in a way similar to the conical mirror setup described above.
Considering the geometry as in Figure 3 and that for every angle α there is a cone-sphere pair of surfaces that are tangent, the sphere case can be approached as an evolution

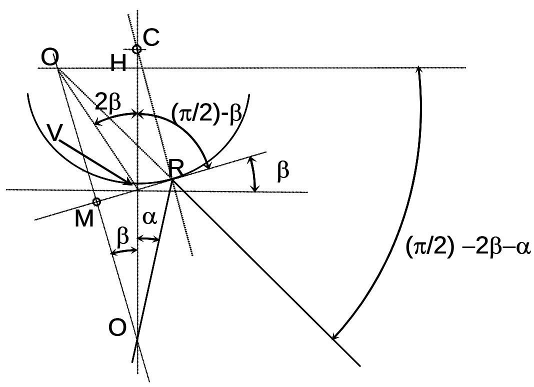

Figure 4
Detail of the geometry for the spherical mirror

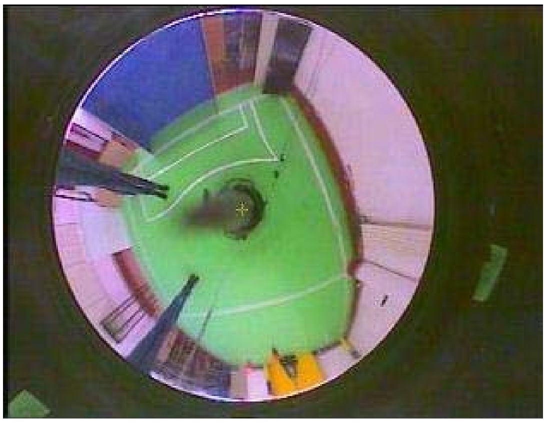

Figure 5
360º panoramic image example using conical mirror

of the conical case, where for each value of α there is a corresponding variable value of β.

D The Spherical Mirror Difference

Considering the alignment of the spherical mirror to the optical axis of the camera system, that is adjusted to obtain a reflection of the lens perfectly centered in the image, the setup is sketched in Figure 4.
Using the notations in Figure 4 and, as in Figure 3, h_1 as the distance from the lens to the mirror, r the radius of the sphere, we can obtain

$$(r + h_1 - r \cdot \cos\beta) \cdot \tan\alpha = r \cdot \sin\beta \qquad (6)$$

and, after some arrangement,

$$\tan\beta = (1 + \frac{h_1}{r}) \cdot \tan\alpha \ . \qquad (7)$$

Substituting in (5)

$$\theta = \frac{\pi}{2} - 2 \cdot \tan^{-1}\left[\left(1 + \frac{h_1}{r}\right) \cdot \tan\alpha\right] - \alpha \ . \qquad (8)$$

Also for the spherical mirror, the position of the virtual optical center *C'* varies with α for both height (4) and radial position (5).

For most cases, an approximation can be made considering just a fixed point H as the static virtual optical center for the setup.

E Other Types of Mirrors

For other types of mirror, an equivalent α to θ relationship can be established, as, for instance for the hyperboloid case, where the mirror surface is obtained by revolution of a hyperbole around its axis. Difficult and expensive to manufacture, it brings condition of the spherical case for smaller values of α combined with the conical case to the larger values.

III CONCLUSIONS

A solution to the use of mirrors for 360º panoramic views has been presented. Its main advantage is the simplicity of the calculation approach; it is also possible to put the core of the process in a Look-Up-Table, to speed-up the results. However, it relies on a careful adjustment of the setup alignment, not always granted or maintained in football robotics matches.

IV ACKNOWLEDGMENTS

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