Test Flight Question 3

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Theorem. *For any integer n, the number $n^2 + n + 1$ is odd.*

Proof. All integers are either even or odd. We will show that in both cases, the expression will evaluate to an odd value. Suppose that n is even. Then there exists an integer k such that $2k = n$. We substitute this in for n:

$$(2k)^2 + 2k + 1$$

$$4k^2 + 2k + 1$$

The first two terms are even so adding 1 makes the whole expression odd and the theorem is true for all even integers.. Now let us suppose that n is odd. Then there exists an integer k such that $2k + 1 = n$. Substituting this in for n we get:

$$(2k + 1)^2 + (2k + 1) + 1$$

$$4k^2 + 4k + 1 + 2k + 1 + 1$$

$$4k^2 + 6k + 3$$

Again, the first two terms are even so adding the odd number 3 makes the whole expression odd. So the theorem is true for all odd integers as well and thus proved for all integers. □