Test Flight Question 8

Marcel Goh

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Theorem. *Prove that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.*

Proof. We say that the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L if for any real number $\epsilon > 0$, there exists a natural number N such that for all $n \geq N$, $|a_n - L| < \epsilon$. Now we want to prove that for a fixed number $M > 0$, $\{Ma_n\}_{n=1}^{\infty}$ tends to limit ML, so for any real number $\epsilon > 0$, there must exist a natural number N such that for all $n \geq N$, $|Ma_n - ML| < \epsilon$.
Let $\epsilon > 0$ be given. Now pick any N such that $|a_N - L| < \frac{\epsilon}{M}$. Then for any $n \geq N$, $|a_n - L| < \frac{\epsilon}{M}$ Manipulating the inequality as follows:

$$\begin{aligned} |a_n - L| &< \frac{\epsilon}{M} \\ M\,|a_n - L| &< \epsilon \\ |Ma_n - ML| &< \epsilon \end{aligned}$$

So for any arbitrary value of ϵ greater than 0, the existence of an N to satisfy the inequality is shown and the theorem is proved. □