Test Flight Question 6

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Theorem. *The only prime triple (i.e. three primes, each 2 from the next) is 3, 5, 7.*

Proof. To find primes, we examine all triples of natural numbers (where each number is 2 from the next) greater than 1. In a given triple, either all the numbers are even or they are all odd. We rule out all even triples because the only even prime number is 2. The first odd triple, given in the theorem, is made out of prime numbers. To prove that no other prime triple exists, we will show that all odd triples contain one multiple of 3:
Our first odd triple (3, 5, 7) contains a multiple of 3 in the first slot. If we express this first number as $3k$ (where k is an integer), then the triple can be expressed as $(3k, 3k+2, 3k+4)$.
To get the next odd triple, we increase each value by 2: $(3k+2, 3k+4, 3k+6)$. It is clear that the first item is no longer divisible by 3, but now the third item is divisible by 3.
Increasing by 2 again, we get: $(3k+4, 3k+6, 3k+8)$. The third item is no longer divisible by 3 but the second item now is.
We increase by two one more time, obtaining $(3k+6, 3k+8, 3k+10)$ where the first item is divisible by 3 again and the cycle repeats.
Because the only prime multiple of 3 is 3, only the first odd triple is prime and the theorem is proved.

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