Test Flight Question 9

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Theorem. *There exists a family of intervals* $A_n, n = 1, 2, \ldots,$ *such that* $A_{n+1} \subset A_n$ *for all* n *and*

$$\bigcap_{n=1}^{\infty} A_n = \emptyset$$

Proof. Let A_n be the family of open intervals $(0, \frac{1}{n})$. For all natural numbers n, $\frac{1}{n+1} < \frac{1}{n}$, so the interval $(0, \frac{1}{n+1}) \subset (0, \frac{1}{n})$. Because each interval in the sequence is subset of the previous interval,

$$\bigcap_{n=1}^{\infty} A_n = (0, L)$$

where L is the limit as n approaches infinity of the sequence $\{\frac{1}{n}\}_{n=1}^{\infty}$. This limit approaches 0, so we have:

$$\bigcap_{n=1}^{\infty} A_n = (0, 0) = \emptyset$$

The existence of such a family of intervals is shown and the theorem is proved. □