Test Flight Question 7

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Theorem. *For any natural number n,*

$$2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2$$

Proof. We prove the theorem by induction. For the base case $n = 1$:

$$2^1 = 2^2 - 2$$
$$2 = 4 - 2$$

Now we show the induction step. If the theorem holds for n, then for $n + 1$ we have:

$$2 + 2^2 + 2^3 + \ldots + 2^n + 2^{n+1} = 2^{n+2} - 2$$
$$2(1 + 2 + 2^2 + \ldots + 2^{n-1} + 2^n) = 2(2^{n+1} - 1)$$
$$1 + 2 + 2^2 + \ldots + 2^{n-1} + 2^n = 2^{n+1} - 1$$
$$2 + 2^2 + \ldots + 2^{n-1} + 2^n = 2^{n+1} - 2$$

The identity still holds for $n + 1$ and the theorem holds for all natural numbers n. □