Test Flight Question 10

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Theorem. *There exists a family of intervals $A_n, n = 1, 2, \ldots,$ such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n$ consists of a single real number.*

Proof. Let A_n be the family of closed intervals $[0, \frac{1}{n}]$. For all natural numbers n, $\frac{1}{n+1} < \frac{1}{n}$, so the interval $[0, \frac{1}{n+1}] \subset [0, \frac{1}{n}]$. Because each interval in the sequence is subset of the previous interval,

$$\bigcap_{n=1}^{\infty} A_n = [0, L]$$

where L is the limit as n approaches infinity of the sequence $\{\frac{1}{n}\}_{n=1}^{\infty}$. This limit approaches 0, so we have:

$$\bigcap_{n=1}^{\infty} A_n = [0, 0] = 0$$

The intersection is the single real number 0 so the existence of such a family of intervals is shown and the theorem is proved. □