$$\frac{x^2-2x+3}{x^2+2x-3} \qquad \frac{g}{h} \qquad \frac{h\cdot g' - g\cdot h'}{h^2}$$

Analysis Aufgabe 31 / 1.

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$$\frac{(x^2+2x-3)\cdot(2x-2)-(x^2-2x+3)\cdot(2x+2)}{(x^2+2x-3)^2}$$

$$=\frac{(2x^3-2x^2+4x^2-4x-6x+6)-(2x^3+2x^2-4x^2-4x+6x+6)}{(x^2+2x-3)^2}$$

$$=\frac{2x^3+2x^2-10x+6-2x^3+2x^2-2x-6}{(x^2+2x-2)^2}=\frac{4x^2-12x}{(x^2+2x-2)^2}$$ 1. Ableitung

$$(x^2+2x-2)^2=(x^4+2x^3-2x^2+2x^3+4x^2-4x-2x^2-4x+4)$$
$$=x^4+4x^3-8x+4$$

$$\frac{4x^2-12x}{(x^2+2x-2)^2} \qquad \frac{g}{h} \qquad \frac{h\cdot g' - g\cdot h'}{h^2}$$

$\cancel{(x^2+2x-2)^2\cdot(4x^2-12x)-(4x^3+12x^2-8)}$

$$\frac{(x^4+4x^3-8x+4)\cdot(8x-12)-(4x^2-12x)\cdot(4x^3+12x^2-8)}{(x^2+2x-2)^4}$$

$$\frac{(8x^5-12x^4+32x^4-48x^3-64x^2+96x+32x-48)-(16x^5+48x^4-32x^3-48x^4-144x^3+96x)}{(x^2+2x-2)^4}$$

$$\frac{(8x^5+20x^4-48x^3-64x^2+128x-48)-(16x^5-176x^3+96x)}{(x^2+2x-2)^4}$$

$$\frac{-8x^5+20x^4+128x^3-64x^2+32x-48}{(x^2+2x-2)^4}$$ 2. Ableitung

$\frac{-5x^2+5}{x^3}=0 \quad \frac{h(x)}{g(x)}=0 \iff h(x)=0$

Analysis Aufgabe 33/2.

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Nullstellen:

$-5x^2+5=0 \quad |+5x^2$

$5=5x^2 \quad |:5$

$x^2=1 \quad |\sqrt{\ }$

$x_1=1$

$x_2=-1$

$\mathbb{L}$ Nullstellen $:=\{1;-1\}$

Ableitungen: $\frac{h\cdot g'-g\cdot h'}{h^2}$

1. Ab: $\frac{(x^3)\cdot(-10x)-(-5x^2+5)\cdot(3x^2)}{x^6}$

$=\frac{-10x^4+15x^4-15x^2}{x^6}$

$=\frac{5x^4-15x^2}{x^6}$ vereinfacht: $=\frac{5x^2-15}{x^4}$

2. Ab: $\frac{(x^6)\cdot(20x^3-30x)-(5x^4-15x^2)\cdot(6x^5)}{x^{12}}$

$=\frac{20x^9-30x^7-30x^9+90x^7}{x^{12}}=\frac{-10x^9+60x^7}{x^{12}}=\frac{-10x^2+60}{x^5}$

~~3. Ab: $(x^{12})\cdot(-10x^9+60x^7)-($~~

3. Ab: $\frac{(x^{12})\cdot(-90x^8+420x^6)-(12x^{11})\cdot(-10x^9+60x^7)}{x^{24}}$

$=\frac{-90x^{20}+420x^{18}+120x^{20}-720x^{18}}{x^{24}}$

$=\frac{30x^{20}-300x^{18}}{x^{24}}=\frac{30x^2-300}{x^6}$

Extrema:

$f'(x)=0$

$5x^2-15=0 \quad |+15$

$5x^2=15 \quad |:5$

$x^2=3 \quad |\sqrt{\ }$

$x_{1/2}=\pm\sqrt{3}$

$x_{1/2}$ in $f''(x)$

$\frac{-10\cdot(\sqrt{3})^2+60}{\sqrt{3}^5}$

$=\frac{-30+60}{3\cdot3\cdot\sqrt{3}}$

$=\frac{30}{3\cdot3\cdot\sqrt{3}}>0$ Tiefpunkt bei $x=\sqrt{3}$

$\frac{-10\cdot(-\sqrt{3})^2+60}{-\sqrt{3}^5}$

$=\frac{-30+60}{9\cdot(-\sqrt{3})}$

$=\frac{30}{9\cdot(-\sqrt{3})}$

<0

also Hochpunkt bei $x=-\sqrt{3}$

$\frac{-10\cdot(-\sqrt{3})^2+60}{-\sqrt{3}^5}=\frac{-30+60}{3\cdot3\cdot(-\sqrt{3})}=\frac{30}{9\cdot(-\sqrt{3})}$

<0

also Hochpunkt

$f''(x)=0 \quad -10x^2+60=0 \quad |+10x^2$

$60=10x^2 \quad |:10$

$6=x^2 \quad |\sqrt{\ }$

$x=\pm\sqrt{6}$

$\pm\sqrt{6}$ in $f'''(x)$

$\frac{30\cdot(\sqrt{6})^2-300}{\sqrt{6}^6}$

$=\frac{180-300}{6^3}=\frac{-120}{6^3}<0$

also links-Rechts Wendepunkt

$$\frac{30\cdot(-\sqrt{6})^2-300}{(-\sqrt{6})^6} = \frac{180-300}{6^3} = \frac{-120}{6^3} < 0$$ also links Rechts Wendepunkt.

Zusammenfassung:

Nullstellen: $\mathbb{L} := \{1; -1\}$

Extrema: $x_1 = \sqrt{3} \rightarrow$ Tiefpunkt
$x_2 = -\sqrt{3} \rightarrow$ Hochpunkt

Wendepunkte $x_{1/2} = \pm\sqrt{6} \Rightarrow$ links - Rechts Wendepunkt