Aula 2: Methods of Analysis of Resistive Circuits (Node Voltage)

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Contents

Introduction to Electric Circuits 9th Edition by James A. Svoboda, Richard C. Dorf

Introduction

In this chapter, we consider two methods for writing a smaller set of simultaneous equations:

- The node voltage method.
- The mesh current method.

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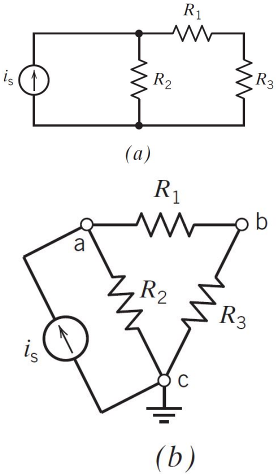
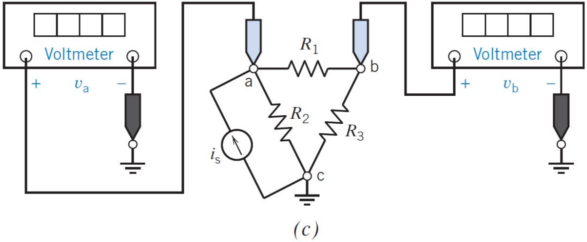
To analyze an electric circuit, we write and solve a set of equations. We apply Kirchhoff's current and voltage laws to get some of the equations. This method works well for small circuits, but the set of equations can get quite large for even moderate-sized circuits.

Node Voltage Analysis of Circuits with Current Sources



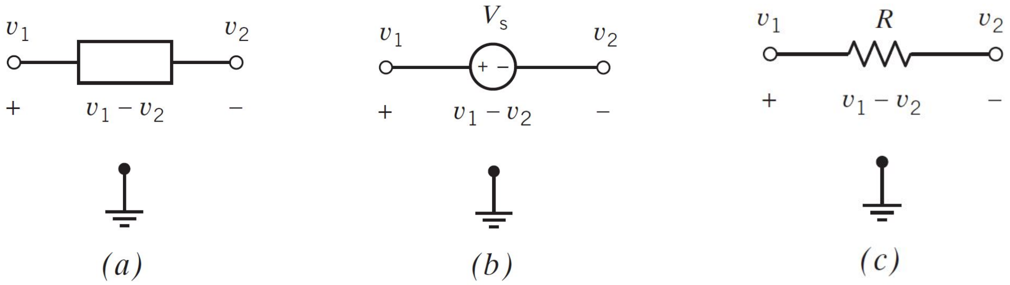


(a) A circuit with three nodes.

(b) The circuit with nodes labeled and reference node marked.

(c) Using voltmeters to measure the node voltages.

The Node Voltage

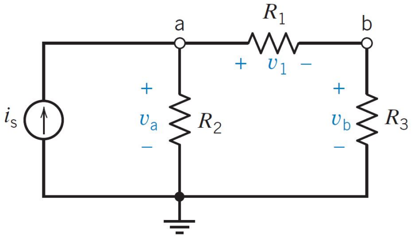


Generic element

Voltage source

Resistor

The Node Equations

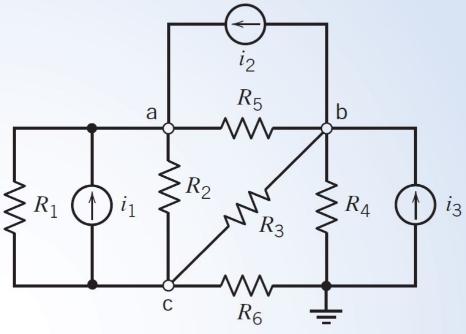


To write a set of node equations, we do two things:

- Express element currents as functions of the node voltages.
- Apply Kirchhoff's current law (KCL) at each of the nodes of the circuit except for the reference node.

Node Voltage Analysis

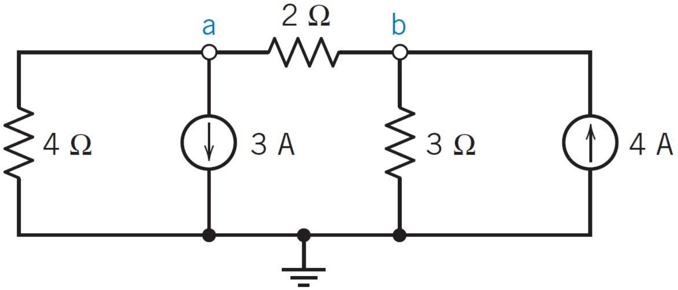
EXAMPLE 4.2-2 - Obtain the node equations for the circuit.



$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5} & -\frac{1}{R_5} & -\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \\ -\frac{1}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} & -\frac{1}{R_3} \\ -\left(\frac{1}{R_1}+\frac{1}{R_2}\right) & -\frac{1}{R_3} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6} \end{bmatrix} \times \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} i_1+i_2 \\ i_3-i_2 \\ -i_1 \end{bmatrix}$$

Node Voltage Analysis

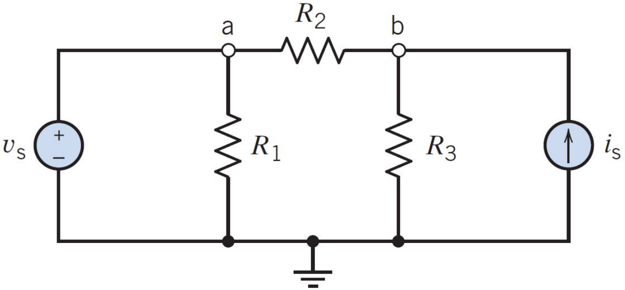
EXERCISE 4.2-2 - Determine the node voltages v_a and v_b for the circuit.



Answer: $v_a = -4/3V \; and \; v_b = 4V$

Node Voltage Analysis of Circuits with Current and Voltage Sources

First we consider the circuit with a voltage source between ground and one of the other nodes.



$$v_a = v_s \tag{1}$$

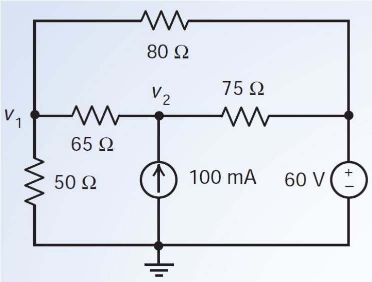
$$-i_s + \frac{v_b}{R_3} + \frac{v_b - v_a}{R_2} = 0 \tag{2}$$

$$-i_s + \frac{v_b}{R_3} + \frac{v_b - v_s}{R_2} = 0 \tag{3}$$

$$v_b = \frac{R_2 R_3 i_s + R_3 v_s}{R_2 + R_3} \tag{4}$$

Node Equations

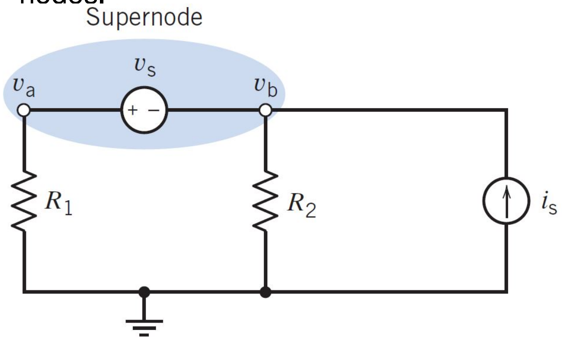
EXAMPLE 4.3-1 - Determine the values node voltages, v_1 and v_2, in the circuit.



Answer: $v_1 = 31.081V \; and \; v_2 = 47.990V$

Node Voltage Analysis of Circuits with Supernode

Next, let us consider the circuit which includes a voltage source between two nodes.



$$v_a - v_b = v_s \text{ or } v_a = v_s + v_b \quad (5)$$

$$-i_s + \frac{v_b}{R_2} + \frac{v_a}{R_1} = 0 \quad (6)$$

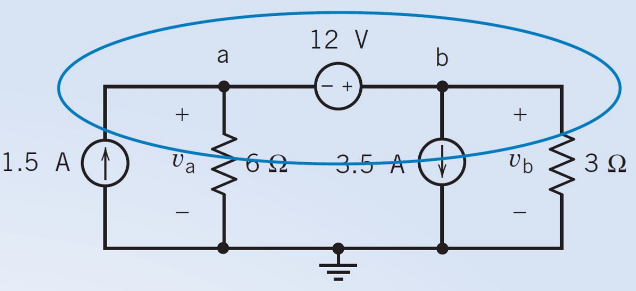
$$-i_s + \frac{v_b}{R_2} + \frac{v_s + v_b}{R_1} = 0 \quad (7)$$

$$v_b = \frac{R_1 R_2 i_s + R_2 v_s}{R_1 + R_2} \quad (8)$$

A supernode consists of two nodes connected by an independent or a dependent voltage source.

Node Equations

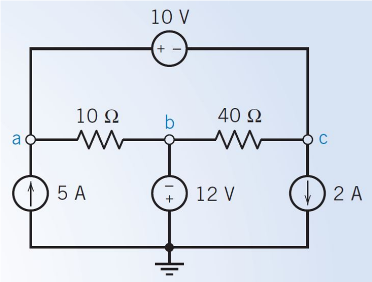
EXAMPLE 4.3-2 - Determine the values of the node voltages v_a and v_b for the circuit.



Answer: $v_a = -12V \; and \; v_b = 0V$

Node Equations

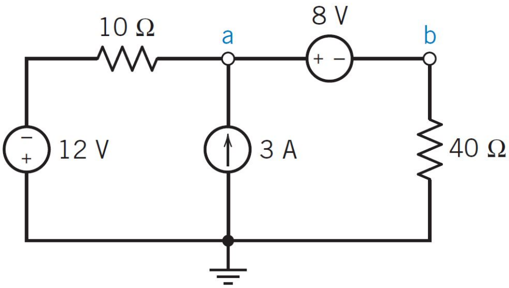
EXAMPLE 4.3-3 - Determine the node voltages for the circuit shown.



Answer: $v_a = 14V\ v_b = -12V\ and\ v_c = 4V$

Node Equations

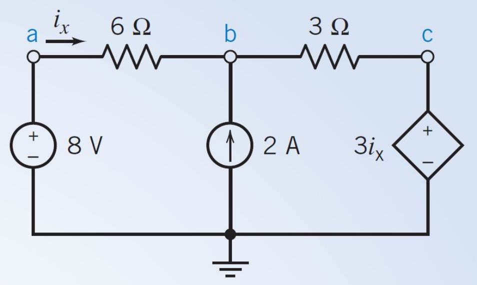
EXERCISE 4.3-2 - Find the voltages v_a and v_b for the circuit.



Answer: $v_a = 16V$ *and* $v_b = 8V$

Node Voltage Analysis with Dependent Sources

When a circuit contains a dependent source the controlling current or voltage of that dependent source must be expressed as a function of the node voltages.



$$i_x = \frac{v_a - v_b}{6} = \frac{8 - v_b}{6} \tag{9}$$

$$v_c = 3i_x = 3\left(\frac{8 - v_b}{6}\right) \tag{10}$$

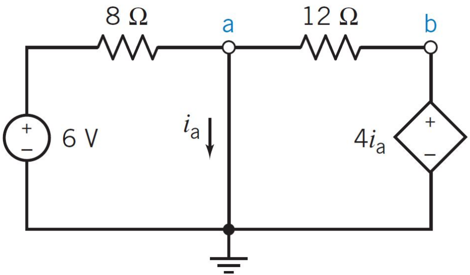
$$\frac{v_a - v_b}{6} - 2 - \frac{v_c - v_b}{3} = 0 \tag{11}$$

$$\frac{8 - v_b}{6} - 2 - \frac{3\left(\frac{8 - v_b}{6}\right) - v_b}{3} = 0 \tag{12}$$

A supernode consists of two nodes connected by an independent or a dependent voltage source. Answer: $v_a = 8V, v_b = 7V \; and \; v_c = \frac{1}{2}V$

Node Equations

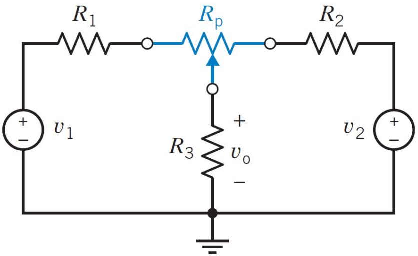
EXERCISE 4.4-1 - Find the node voltage v_b for the circuit shown.



Answer: $v_b = 4.5V$

Circuit Analysis Using MATLAB

In this section, we will use the MATLAB computer program to solve the equations.



We have seen that circuits that contain resistors and independent or dependent sources can be analyzed in the following way:

- Writing a set of node equations.
- Solving those equations simultaneously.

$R_1 = 1000\Omega,\ R_2 = 1000\Omega,\ R_3 = 5000\Omega,\ v_1 = -v_2 = 15V\ and\ R_p = 20.000\Omega$