Aula 3: Methods of Analysis of Resistive Circuits (Mesh Current)

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Contents

Introduction to Electric Circuits 9th Edition by James A. Svoboda, Richard C. Dorf

Introduction

In this chapter, we consider two methods for writing a smaller set of simultaneous equations:

- The node voltage method.
- The mesh current method .

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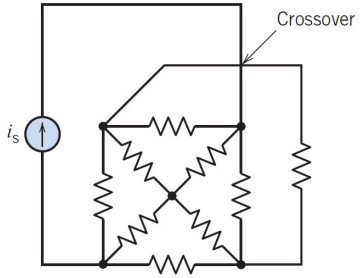
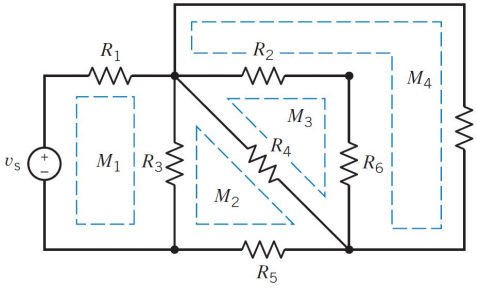
To analyze an electric circuit, we write and solve a set of equations. We apply Kirchhoff's current and voltage laws to get some of the equations. This method works well for small circuits, but the set of equations can get quite large for even moderate-sized circuits.

Mesh Current Analysis

A **mesh** is a loop that does not contain any other loops within it.



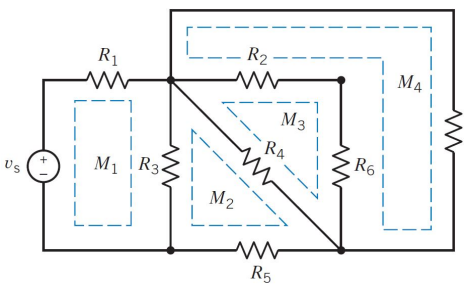
Nonplanar circuit with a crossover.



Circuit with four meshes.

Mesh Current Analysis

A **mesh** is a loop that does not contain any other loops within it.



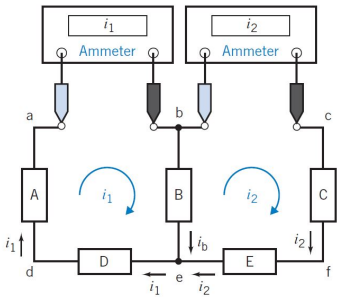
Circuit with four meshes.

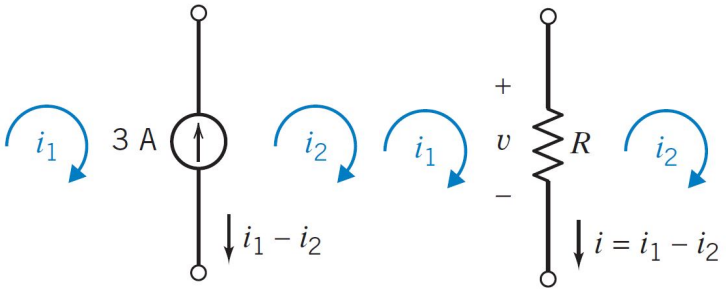
To write a set of mesh equations, we do two things:

- Express element voltages as functions of the mesh currents.
- Apply Kirchhoff's voltage law (KVL) to each of the meshes of the circuit.

Mesh Current Analysis

These equations expresses the currents i_b, -3 and i as a function of the mesh currents i_1 and i_2.





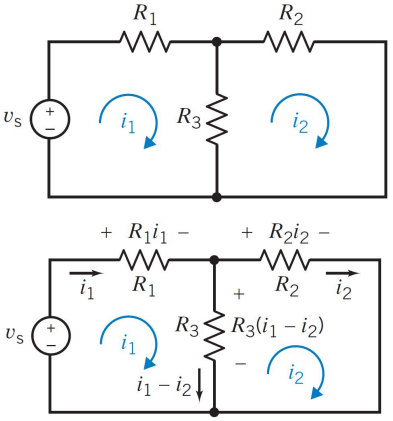
$$i_b = i_1 - i_2 \qquad (1)$$

$$-3 = i_1 - i_2 \qquad (2)$$

$$v = R(i_1 - i_2) \qquad (3)$$

Mesh Current Analysis with Independent Voltage Sources

Next, let's write mesh equations to represent the circuit shown.



$$-v_s + R_1 i_1 + R_3(i_1 - i_2) = 0 \qquad (4)$$

$$-R_3(i_1 - i_2) + R_2 i_2 = 0 \qquad (5)$$

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \end{bmatrix}$$

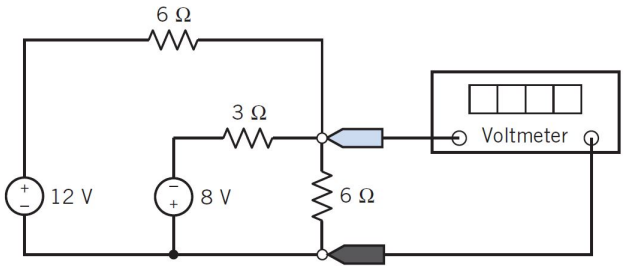
If $R_1 = R_2 = R_3 = 1\Omega$ and $v_s = 3V$, we have

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

then we have $i_1 = 2$ and $i_2 = 1$.

Mesh Current Analysis with Independent Voltage Sources

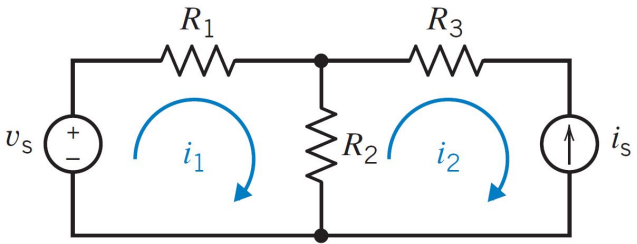
EXERCISE 4.5-1 - Determine the value of the voltage measured by the voltmeter.



Answer: -1

Mesh Current Analysis with Current and Voltage Sources

Circuit with an independent voltage source and an independent current source.



$$i_2 = -i_s \tag{6}$$

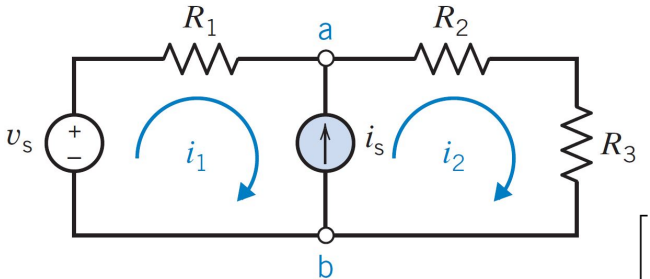
$$-v_s + R_1 i_1 + R_2(i_1 - i_2) = 0 \tag{7}$$

$$i_1 = \frac{v_s + R_2 i_2}{R_1 + R2} \tag{8}$$

$$i_1 = \frac{v_s + R_2 i_s}{R_1 + R2} \tag{9}$$

Mesh Current Analysis with Current and Voltage Sources

Circuit with an independent current source common to both meshes.



$$i_1 - i_2 = -i_s \tag{10}$$

$$-v_s + R_1 i_1 + v_{ab} = 0 \tag{11}$$

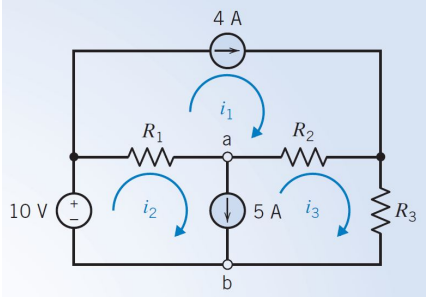
$$-v_{ab} + R_2 i_2 + R_3 i_2 = 0 \tag{12}$$

$$-v_s + R_1 i_1 = -R_2 i_2 - R_3 i_2 \tag{13}$$

$$\begin{bmatrix} R_1 & R_2 - R_3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_s \\ -i_s \end{bmatrix}$$

Mesh Current Analysis with Current and Voltage Sources

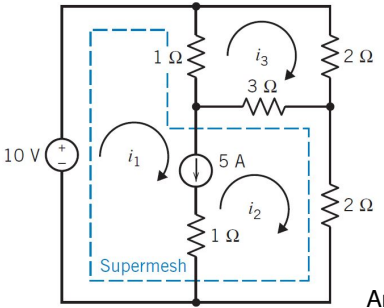
EXAMPLE 4.6-1 - Consider the circuit of Figure where $R_1 = R_2 = 1\Omega$ and $R_3 = 2\Omega$. Find the three mesh currents



Answer: $i_3 = \frac{13}{4}$ and $i_2 = \frac{33}{4}$

Supermesh Current Analysis with Current and Voltage Sources

A **supermesh** is one larger mesh created from two meshes that have an independent or dependent current source in common.



$$current\ source: i_1 - i_2 = 5 \tag{14}$$

$$mesh\ 3: 2i_3 - 3(i_2 - i_3) - 1(i_1 - i_3) = 0 \tag{15}$$

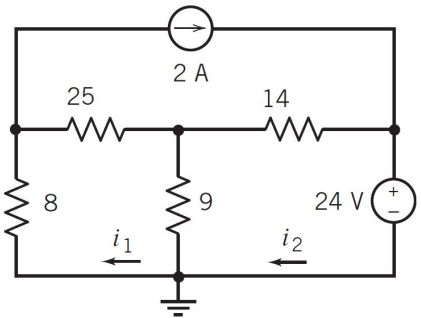
$$supermesh: -10 + 1(i_1 - i_3) + 3(i_2 - i_3) + 2i_2 = 0 \tag{16}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & -3 & 6 \\ 1 & 5 & -4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix}$$

Answer: $i_1 = 7.5V,\ i_2 = 2.5V\ and\ i_3 = 2.5V$

Mesh Current Analysis with Current and Voltage Sources

PROBLEM 4.9-5 - Determine the mesh currents i_1 and i_2 for the circuit.



Mesh Current Analysis with Dependent Sources

When a circuit contains a dependent source, the controlling current or voltage of that dependent source must be expressed as a function of the mesh currents.

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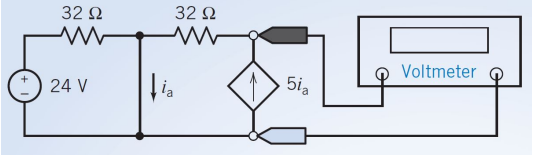
EXAMPLE 4.7-1 - Consider the circuit. Find the value of the voltage measured by the voltmeter.



Answer: $v_m = 30V$

$$i_a = i_1 - i_2$$

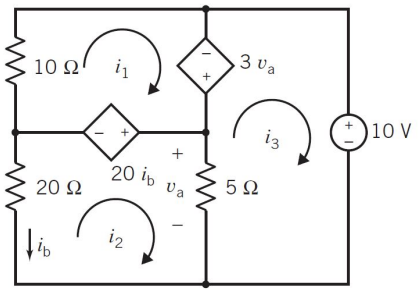
$$-i_2 = 5i_a = 5(i_1 - i_2)$$

$$-24 + 32i_1 = 0$$

$$\begin{bmatrix} 5 & -4 \\ 32 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 24 \end{bmatrix}$$

Mesh Current Analysis with Current and Voltage Sources

PROBLEM 4.7-7 - The currents i_1, i_2, and i_3 are the mesh currents of the circuit shown. Determine the values of i_1, i_2, and i_3.



The Node Voltage Method and Mesh Current Method Compared

In some cases, one method is clearly preferred over another. For example:

- When the circuit contains only voltage sources, it is probably easier to use the mesh current method.
- When the circuit contains only current sources, it will usually be easier to use the node voltage method.

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The Node Voltage Method and Mesh Current Method Compared

If a circuit has both current sources and voltage sources, it can be analyzed by either method. One approach is to compare the number of equations required for each method.

- If the circuit has fewer nodes than meshes, it may be wise to select the node voltage method.
- If the circuit has fewer meshes than nodes, it may be easier to use the mesh current method.

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The Node Voltage Method and Mesh Current Method Compared

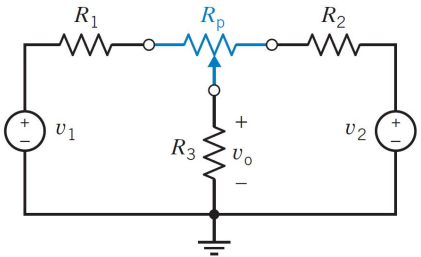
Another point to consider when choosing between the two methods is what information is required.

- If you need to know several currents, it may be wise to proceed directly with mesh current analysis.
- If you need to know several voltages, it may be wise to proceed directly with node voltage analysis.

It is often helpful to determine which method is more appropriate for the problem requirements and to consider both methods.

Circuit Analysis Using MATLAB

In this section, we will use the MATLAB computer program to solve the equations.



We have seen that circuits that contain resistors and independent or dependent sources can be analyzed in the following way:

- Writing a set of node equations.
- Solving those equations simultaneously.

$R_1 = 1000\Omega,\ R_2 = 1000\Omega,\ R_3 = 5000\Omega,\ v_1 = -v_2 = 15V\ and\ R_p = 20.000\Omega$