Aula 5: Circuit Theorems

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Contents

Introduction to Electric Circuits 9th Edition by James A. Svoboda, Richard C. Dorf

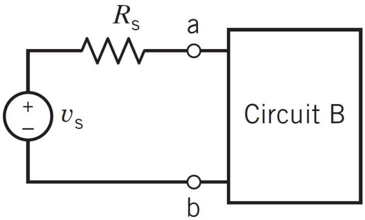
Introduction

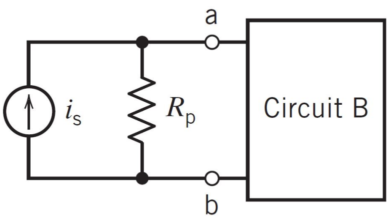
In this chapter, we consider five circuit theorems:

- Source Transformation;
- Superposition;
- Thévenin's Theorem;
- Norton's Theorem;
- The Maximum Power Transfer Theorem.

Source Transformations

The ideal voltage source is the simplest model of a voltage source, but occasionally we need a more accurate model.

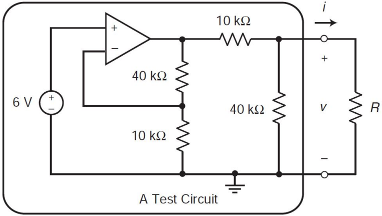


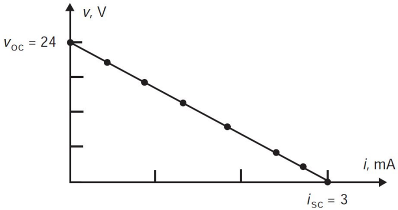


Under certain conditions $R_p = R_s$ and $v_s = R_s i_s$, the nonideal voltage source and the nonideal current source are equivalent to each other.

Source Transformations

We connect a resistor having resistance R to the terminals of the test circuit and measure the resistor voltage v and resistor current i.





$$v = (-\frac{v_{oc}}{i_{sc}})i + V_{oc} \qquad (1)$$

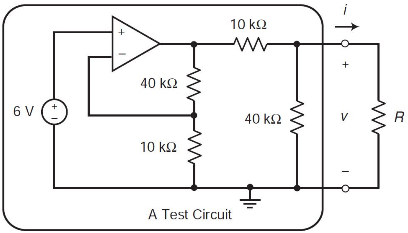
$$v = -Ri + V_{oc} \qquad (2)$$

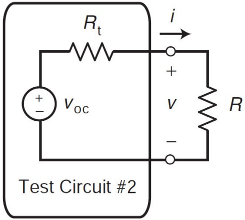
Next, we change the resistor and measure the new values of the resistor voltage and current.

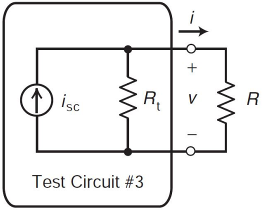
R, kΩ	0	1	2	5	10	20	50	∞
i, mA	3	2.667	2.4	1.846	1.33	0.857	0.414	0
v, V	0	2.667	4.8	9.231	13.33	17.143	20.69	24

Source Transformations

The second and third test circuits have names. They are called the "Thévenin equivalent circuit" and "Norton equivalent circuit" of the first test circuit.







$$v = -Ri + V_{oc} \tag{3}$$

$$v = -R_t i + V_{oc} \tag{4}$$

$$v = -R_t i + R_t i_{sc} \tag{5}$$

Source Transformations

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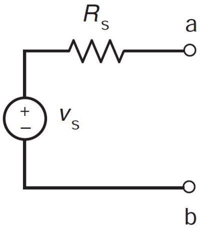
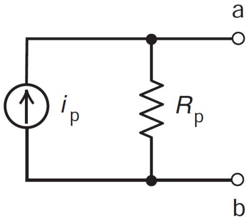




$$v_s = R_p i_p \text{ and } R_s = R_p \qquad (7)$$

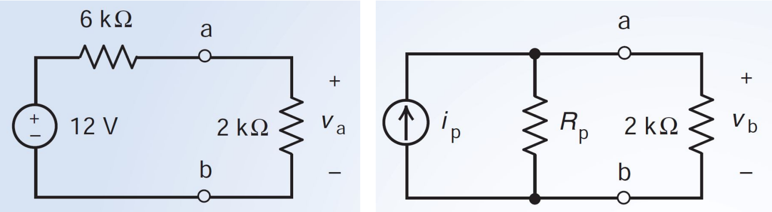
$$i_p = \frac{V_s}{R_s} \text{ and } R_p = R_s \qquad (8)$$

$$v = -R_s i + v_s \qquad (6)$$

$$v = -R_p i + R_p i_p \qquad (9)$$

Source Transformations

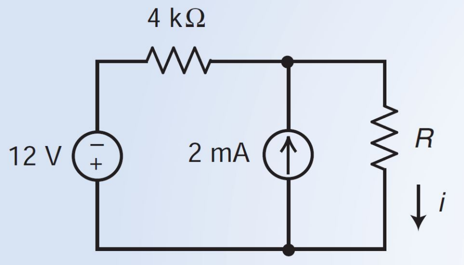
EXAMPLE 5.2-2 - First, determine the values of i_p and R_p that cause the part of the circuit connected to the 2-kV resistor. Next, determine the values of v_a and v_b.



Answer: $R_p = 6k\Omega, i_p = 2mA, v_a = 3V \; and \; v_b = 3V$

Source Transformations

EXAMPLE 5.2-3 - Use a source transformation to determine a relationship between the resistance R and the resistor current i.



Answer: $i = -\frac{4}{4000+R}$

Superposition

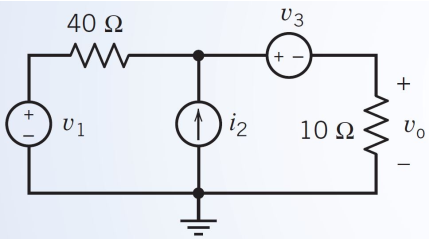
The output of a linear circuit can be expressed as a linear combination of its inputs. For example, consider any circuit having the following three properties:

- The circuit consists entirely of resistors and dependent and independent sources.
- The circuit inputs are the voltages of all the independent voltage sources and the currents of all the independent current sources.
- The output is the voltage or current of any element of the circuit.

Consequently, $v_o = a_1 v_1 + a_2 v_2 + a_3 v_3 + \ldots + a_n v_n$, where v_0 is the output of the circuit and $v_1, v_2, \ldots, v_n$ are the inputs to the circuit. The coefficients $a_1, a_2, \ldots, a_n$ of the linear combination are real constants called gains.

Superposition

EXAMPLE 5.3-1 - The circuit shown has one output, v_o, and three inputs, $v_1, i_2,$ *and* v_3. Express the output as a linear combination of the inputs.

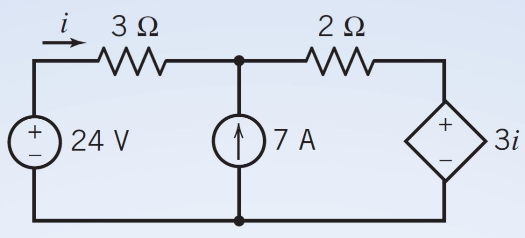


Because 0-V voltage sources are equivalent to short circuits and 0-A current sources are equivalent to open circuits, we replace the sources corresponding to the other inputs by short or open circuits.

Answer: $v_o = \frac{v_1}{5} + 8i_2 - \frac{v_3}{5}$

Superposition

EXAMPLE 5.3-2 - Find the current i for the circuit.

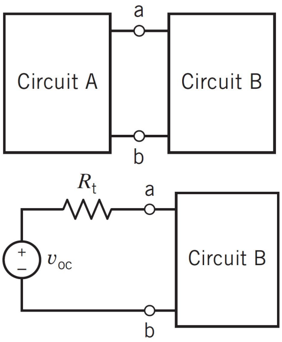


The output of a linear circuit due to several inputs working together is equal to the sum of the outputs due to each input working separately.

Answer: $i = \frac{5}{4}$

Thévenin's Theorem

In this section, we introduce the Thévenin equivalent circuit, based on a theorem developed by M. L. Thévenin, a French engineer, who first published the principle in 1883.



Finding the Thévenin equivalent circuit of circuit "A" involves three parameters:

- The open-circuit voltage, v_{oc}.
- The short-circuit current, i_{sc}.
- Thévenin resistance, R_t.

Thévenin's Theorem

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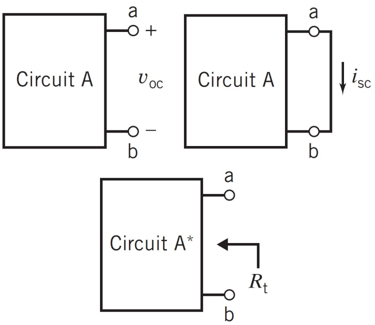
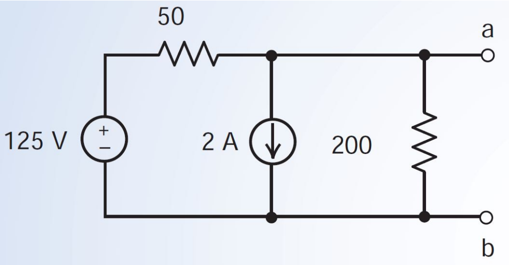


Thévenin's Theorem

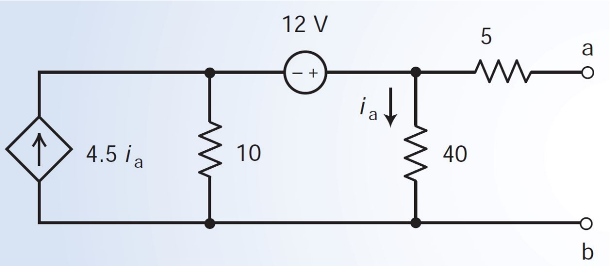
EXAMPLE 5.4-1 - Determine the Thevenin equivalent circuit for the circuit shown.



Answer: $v_{oc} = 20V, I_{sc} = 0.5A \text{ and } R_t = 40\Omega$

Thévenin's Theorem Circuit of a Circuit Containing a Dependent Source

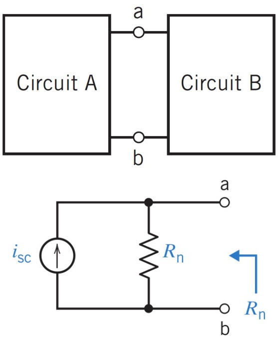
EXAMPLE 5.4-2 - Determine the Thevenin equivalent circuit for the circuit shown.



Answer: $v_{oc} = 96V, I_{sc} = 1.1294A \ and \ R_t = 85\Omega$

Norton's Theorem

E. L. Norton, proposed an equivalent circuit using a current source and an equivalent resistance.



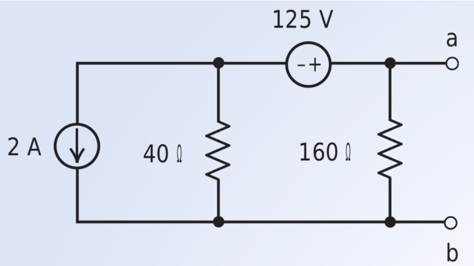
Finding the Norton equivalent circuit of circuit "A" involves two parameters:

- ♣ The short-circuit current, i_{sc}.
- ♣ Norton resistance, R_n.

The Norton equivalent is simply the source transformation of the Thévenin equivalent ($R_n = R_t$ *and* $v_{oc} = R_t i_{sc}$).

Norton's Theorem

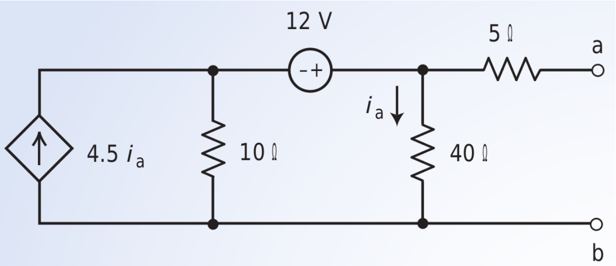
EXAMPLE 5.5-1 - Determine the Norton equivalent circuit for the circuit shown.



Answer: $i_{sc} = 1.125A \; and \; R_n = 32\Omega$

Norton Equivalent Circuit of a Circuit Containing a Dependent Source

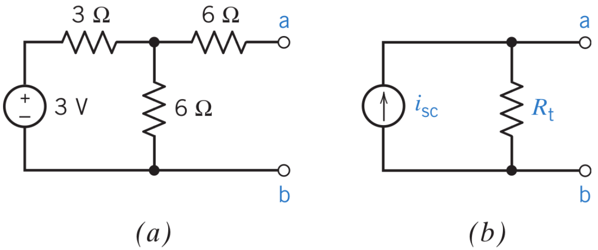
EXAMPLE 5.5-2 - Determine the Norton equivalent circuit for the circuit shown.



Answer: $v_{oc} = 96, i_{sc} = 1.13A \; and \; R_t = 85\Omega$

Norton Equivalent Circuit of a Circuit Containing a Dependent Source

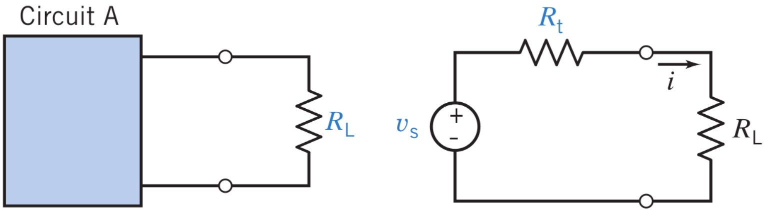
EXERCISE 5.5-1 - Determine values of R_t and i_{sc} that cause the circuit shown in Figure a to be the Norton equivalent circuit of the circuit in Figure b.



Answer: $i_{sc} = 0.25A \; and \; R_t = 8\Omega$

Maximum Power Transfer

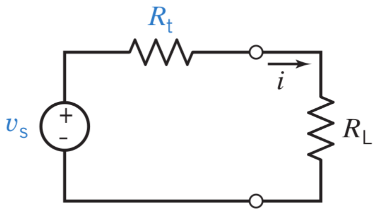
Many applications of circuits require the maximum power available from a source to be transferred to a load resistor R_L.



The general problem of power transfer can be discussed in terms of efficiency and effectiveness. Power utility systems are designed to transport the power to the load with the greatest efficiency by reducing the losses on the power lines.

Maximum Power Transfer

Let us consider the general circuit of Figure.



$$p = i^2 R_L, i = \frac{v_s}{R_L + R_t} \; and \; p = (\frac{v_s}{R_L + R_t})^2 R_L \tag{10}$$

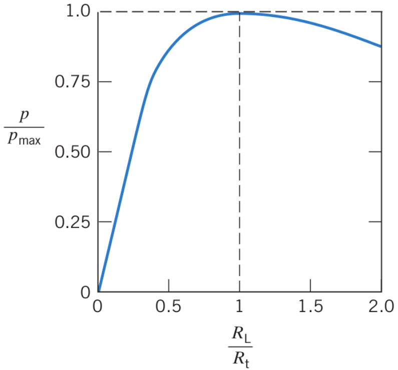
$$\frac{dp}{dR_L} = v_s^2 \frac{(R_t + R_L)^2 - 2(R_t + R_L)R_L}{(R_L + R_t)^4} \tag{11}$$

$$(R_t + R_L)^2 - 2(R_t + R_L)R_L = 0 \tag{12}$$

$$(R_t + R_L)(R_t + R_L - 2R_L) = 0 \tag{13}$$

$$Solving\ (13), we\ obtain\ R_t = R_L. \tag{14}$$

Maximum Power Transfer

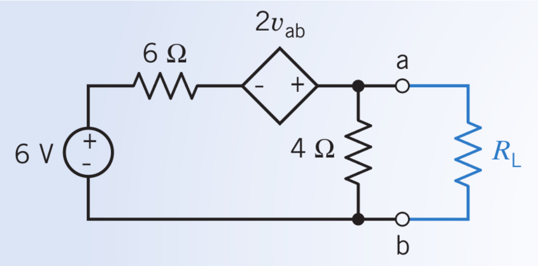


The **maximum power transfer** theorem states that the maximum power delivered to a load by a source is attained when the load resistance, R_L , is equal to the Thévenin resistance, R_t, of the source.

$$p_{max} = \frac{v_s^2 R_t}{(2R_t)^2} = \frac{v_s^2}{(4R_t)} \qquad (15)$$

Maximum Power Transfer

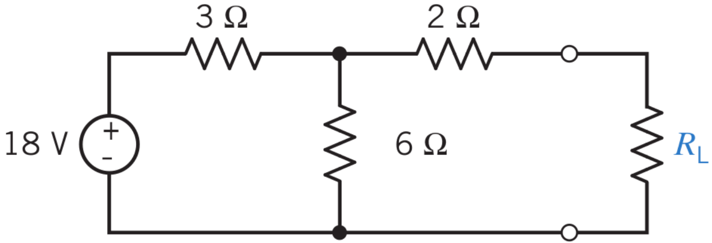
EXEMPLE 5.6-2 - Find the load R_L that will result in maximum power delivered to the load of the circuit. Also, determine p_{max} delivered.



Answer: $R_L = R_t = 12\Omega \; and \; p_{max} = 3W$

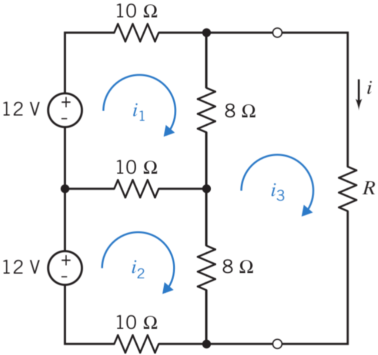
Maximum Power Transfer

EXERCISE 5.6-1 - Find the maximum power that can be delivered to R_L for the circuit of Figure, using a Thévenin equivalent circuit.



Answer: $9\ W\ when\ R_L = 4\Omega$

Using MATLAB to Determine the Thévenin Equivalent Circuit



MATLAB can be used to reduce the work required to determine the Thévenin equivalent of a circuit such as the one shown in Figure.

$$\begin{bmatrix} 28 & -10 & -8 \\ -10 & 28 & -8 \\ -8 & -8 & 16+R \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} R_a i_a \\ R_b i_b \end{bmatrix} = \begin{bmatrix} 1 & -i_a \\ 1 & -i_b \end{bmatrix} \begin{bmatrix} v_t \\ R_t \end{bmatrix}$$