



## tarefa básica

### SISTEMAS LINEARES

① a)  $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} \rightarrow 6 - 1$$

$$DX = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} \rightarrow 6 - 3$$

$$DY = \begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix} \rightarrow -6 - (-2)$$

$$X = \frac{DX}{D} = \frac{3}{5}$$

$$-2 \quad -6$$

$$Y = \frac{DY}{D} = \frac{4}{5}$$

Resposta:  $V = \left\{ \left( \frac{3}{5}, \frac{4}{5} \right) \right\}$

b)  $\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 4 \end{cases}$

$$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} \rightarrow 10 - 13$$

$$0 + 9 + 4 = 13 \quad 0 - 12 + 2 = 10$$

$$DX = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} \rightarrow 1 - 1$$

$$DY = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 4 & -2 \end{vmatrix} \rightarrow 3 - 1$$

$$DZ = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 4 \end{vmatrix} \rightarrow 3 - 1$$

$$0 + 3 - 2 = 1 \quad 0 - 21 - 1 = -22 \quad -22 - 1 = -23$$

$$-4 + 63 - 4 = 55 \quad 6 + 12 + 14 = 32 \quad 32 - 55 = -23$$

$$0 - 3 - 14 = -17 \quad 0 + 4 + 2 = 6 \quad 6 - (-17) = 23$$

$$X = \frac{DX}{D} = \frac{-23}{-23} = 1$$

$$Y = \frac{DY}{D} = \frac{-23}{-23} = 1$$

$$Z = \frac{DZ}{D} = \frac{-23}{-23} = -1$$

$$V = \{(1, 1, -1)\}$$

$$\textcircled{2} \quad \begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases} \quad D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix} \rightarrow 61 - 31$$

$-5 - 12 + 48 = 31 \quad 15 + 8 + 8 = 61$

$$D_y = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix} \quad y = \frac{D_y}{D}$$

$4 - 12 + 2 = 4 \rightarrow 48 - 48$

$y = \frac{30}{30} = 1$

$-12 + 48 + 12 = 48 \quad 108 + 2 - 32 = 48$

alternativa A)

$$\textcircled{3} \quad \begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases} \quad D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix} \rightarrow -36 - (-34)$$

$2 - 33 - 6 = -34 \quad -1 - 44 + 9 = -36$

$$D_x = \begin{vmatrix} 1 & 2 & 1 \\ -2 & 1 & -11 \\ 1 & 3 & -1 \end{vmatrix} \quad D_y = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -2 & -11 \\ 2 & 1 & -1 \end{vmatrix} \quad D_z = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

$1 - 33 + 4 = -28 \quad -1 - 22 - 6 = -29 \quad -4 - 11 - 3 = -18 \quad 2 - 22 + 3 = -17 \quad 2 - 6 + 6 = 2 \quad 1 - 8 + 9 = 2$

$-29 - (-28) = -1 \quad -17 - (-18) = 1 \quad 2 - 2 = 0$

$$x = \frac{x}{1} = -1 \quad y = \frac{1}{1} = 1 \quad z = \frac{0}{1} = 0$$

a                          b                          c

$$v = \{(-1, 1, 0)\}^{\top} \quad a + b + c = 0 \quad \text{alternativa c)}$$

$-1 + 1 + 0 = 0$

$$\textcircled{4} \quad \begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases} \quad D = \left| \begin{array}{ccc|cc} 1 & 2 & -3 & 1 & 2 \\ 1 & 3 & 2 & 1 & 3 \\ 1 & -1 & -2 & 1 & -1 \end{array} \right| \rightarrow 1 - (-15) \\ -9 - 2 - 4 = -15 \quad -6 + 4 + 3 = 1 \\ 116 \end{array}$$

$$\begin{array}{l} D_x = \left| \begin{array}{ccc|cc} 29 & 2 & -3 & 29 & 2 \\ 4 & 3 & 2 & 4 & 3 \\ 8 & -1 & -2 & 8 & -1 \end{array} \right| \quad D_y = \left| \begin{array}{ccc|cc} 1 & 29 & -3 & 1 & 29 \\ 1 & 4 & 2 & 1 & 4 \\ 1 & 8 & -2 & 1 & 8 \end{array} \right| \\ -12 \cdot 58 - 16 = -146 \quad -174 + 32 + 12 = -130 \quad -12 + 16 - 58 = -54 \quad -8 + 58 - 24 = 26 \\ -130 - (-146) = 16 \quad 26 - (-54) = 80 \end{array}$$

$$\begin{array}{l} D_z = \left| \begin{array}{ccc|cc} 1 & 2 & 29 & 1 & 2 \\ 1 & 3 & 4 & 1 & 3 \\ 1 & -1 & 8 & 1 & -1 \end{array} \right| \quad x = \frac{-16}{16} = 1 \quad z = \frac{-96}{16} = -6 \\ y = \frac{80}{16} = 5 \\ 8 + 4 + 16 = 99 \quad 24 + 8 - 29 = 31 \quad x + y + z \\ 31 - 99 = -68 \quad 1 + 5 - 6 = 0 \end{array}$$

x + y + z  
Alternative A)

$$\textcircled{5} \quad \begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 4 \end{cases} \quad D = \left| \begin{array}{ccc|cc} 2 & 1 & 0 & 2 & 1 \\ 0 & 2 & 1 & 0 & 2 \\ 3 & 2 & 1 & 3 & 2 \end{array} \right| \rightarrow 2 \cdot 4 \\ 0 + 4 + 0 = 4 \quad 4 + 3 + 0 = 7$$

$$\begin{array}{l} D_x = \left| \begin{array}{ccc|cc} 5 & 1 & 0 & 5 & 1 \\ 3 & 2 & 1 & 3 & 2 \\ 1 & 2 & 1 & 1 & 2 \end{array} \right| \quad D_y = \left| \begin{array}{ccc|cc} 2 & 5 & 0 & 2 & 5 \\ 0 & 3 & 1 & 0 & 3 \\ 3 & 4 & 1 & 3 & 4 \end{array} \right| \quad D_z = \left| \begin{array}{ccc|cc} 2 & 1 & 5 & 2 & 1 \\ 0 & 2 & 3 & 0 & 2 \\ 3 & 2 & 4 & 3 & 2 \end{array} \right| \\ 0 + 10 + 3 = 13 \quad 10 + 4 + 0 = 14 \quad 0 + 14 + 0 = 14 \quad 6 + 15 + 0 = 21 \quad 30 + 12 + 0 = 42 \quad 28 + 9 + 0 = 37 \\ 14 - 13 = 1 \quad 21 - 14 = 7 \quad 34 - 42 = -8 \end{array}$$

$$x = \frac{1}{3}, \quad y = \frac{7}{3}, \quad z = \frac{-5}{3}$$

Alternative D)

1 / 1

$$\textcircled{6} \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix} \rightarrow \begin{cases} x = 3 \\ 2x + y = 4 \\ -x + 2y + 2z = -1 \end{cases} \Rightarrow \begin{array}{l} x = 3 \\ y = ? \\ z = ? \end{array}$$

$$D = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} = 10 \quad D_x = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ -1 & 2 & -1 \end{vmatrix} = 10 \quad D_y = \begin{vmatrix} 1 & 3 & 0 \\ 2 & 4 & 0 \\ -1 & -1 & -1 \end{vmatrix} = 10$$

$$0+0+0=0 \quad 2+0+0=2 \quad -3+4+0=1 \quad -1+0+12=11 \quad 0+0+12=12 \quad 14+0+0=14$$

$$2-0=2 \quad 11-11=0 \quad 14-12=2$$

$$x = 3, \quad y = \frac{2}{2} = 1, \quad z = \frac{0}{2} = 0 \rightarrow \text{Alternativo E) } z = 0$$

### ► Parte 2

$$\textcircled{1} \quad S = \begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases} \rightarrow S = \begin{cases} x - 5z = 3 \\ x + 3y - z = 11 \\ 2x - y - 3z = -5 \end{cases}$$

$$\begin{array}{c} -2 \quad -1 \\ \downarrow \quad \downarrow \\ \begin{array}{|ccc|c} \hline 1 & 0 & -5 & 3 \\ 1 & 3 & -1 & 11 \\ 2 & -1 & -3 & -5 \\ \hline \end{array} \end{array} \sim \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \begin{array}{|ccc|c} \hline 1 & 0 & 3 & 8 \\ 0 & 1 & 4 & -11 \\ 0 & 0 & 25 & -25 \\ \hline \end{array} \sim \begin{array}{c} 1 \\ 0 \\ 0 \end{array}$$

$$x - 5z = 3$$

$$x - 5(-1) = 3$$

$$x = 3 - 5$$

$$x = -2$$

$$3y + 4z = 8$$

$$3y + 4(-1) = 8$$

$$3y = 8 + 4$$

$$y = 12$$

$$3$$

$$y = 4$$

$$25z = -25$$

$$z = -25$$

$$25$$

$$z = -1$$

$$v = \{(-2, 4, -1)\}$$

1 / 1

$$\textcircled{2} \quad \left\{ \begin{array}{l} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{array} \right. \quad \rightarrow \quad \begin{array}{l} x = 2y \\ z = 2y \\ 3 \end{array}$$

$$x + y + z = 11 \quad \rightarrow \quad 11 \cdot y = 33$$

$$\frac{2y + y + 2y}{3} = 11$$

$$\frac{6y}{3} + \frac{3y}{3} + \frac{2y}{3} = \frac{33}{3}$$

$$\begin{array}{l} x = 2y \\ y = 3 \\ z = 2y \end{array} \quad \left\{ \begin{array}{l} y = 3 \\ z = 2y \end{array} \right\} \quad \begin{array}{l} z = 6 \\ 3 \\ z = 2 \end{array}$$

$$x + 2y + 3z$$

$$6 + 2.3 + 3.2$$

$$6 + 6 + 6 = 18 \rightarrow \text{Alternativa B})$$

$$\textcircled{3} \quad \begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{cases}$$

$$* -2 \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 2 & -1 & -2 & | & 1 \\ 0 & 6 & 3 & | & -12 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & -3 & -4 & | & 1 \\ 0 & 6 & 3 & | & -12 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 1 & 4/3 & | & 1/3 \\ 0 & 0 & -15 & | & -30 \end{pmatrix}$$

$$\Rightarrow -15z = -30$$

$$\underline{z = -30} \\ -15$$

Alternative D )

$$z = 2$$

$$\textcircled{4} \quad \begin{aligned} A + B + C &= 68 \\ \frac{B + 2C}{100} &= A \end{aligned} \quad \rightarrow \quad \left\{ \begin{array}{l} A + B + C = 68 \\ A - B - 1/5C = 0 \\ 1/5A - 3B + C = 0 \end{array} \right.$$

$$C + \frac{20}{100} A = 3B$$

$$\begin{array}{c} \xrightarrow{-1/5} \\ \xrightarrow{\quad} \\ \xrightarrow{\quad} \end{array} \left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 68 \\ 1 & -1 & -1/5 & 0 & 0 \\ 1/5 & -3 & 1 & 0 & 0 \end{array} \right) \sim \left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 68 \\ 0 & -2 & -6/5 & -68 & 0 \\ 0 & -16/5 & 4/5 & -68/5 & 0 \end{array} \right) \sim \left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 68 \\ 0 & -2 & -6/5 & -68 & 0 \\ 0 & -16 & 4 & -68 & 0 \end{array} \right)$$

$$\sim \left( \begin{array}{ccc|c} & & & 1 \\ & & & 1 \\ 0 & 0 & \frac{68}{25} & \frac{476}{5} \end{array} \right) \quad \frac{68C}{25} = \frac{476}{5}$$

$$C = 35$$

$$-2B - \frac{6}{5}S = -68 \quad \left. \begin{array}{l} A + B + C = 68 \\ A + 13 + 3S = 68 \end{array} \right\}$$

$$-2B = -68 + 42$$

$$-2B = -26$$

$$B = \frac{-26}{-2} = 13$$

$$A + B + C = 68$$

$$A + 13 + 3S = 68$$

$$A = 68 - 48$$

$$A = 20$$

$$\text{Ali} = R\$ 20,00$$

$$\text{Bis} = R\$ 13,00$$

$$\text{Caco} = R\$ 35,00$$

Alternativa A)

$$\textcircled{5} \quad A = \begin{vmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{vmatrix} \quad X = \begin{vmatrix} x \\ y \\ z \end{vmatrix} \quad A \cdot X = \begin{vmatrix} 134 \\ 115 \\ 48 \end{vmatrix} \rightarrow \begin{cases} 3y + 4z = 134 \\ x + 5z = 115 \\ 2x + y = 48 \end{cases}$$

$$\xrightarrow{-3} \left( \begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 2 & 1 & 0 & 48 \end{array} \right) \sim \left( \begin{array}{ccc|c} -6 & 0 & 4 & -10 \\ 1 & 0 & 5 & 115 \\ 2 & 1 & 0 & 48 \end{array} \right) \sim \left( \begin{array}{ccc|c} 0 & 0 & 34 & 680 \\ 1 & & & \\ & & & \end{array} \right)$$

$$2x + y = 48$$

$$2 \cdot 15 + y = 48$$

$$30 + y = 48$$

$$y = 48 - 30$$

$$\boxed{y = 18}$$

$$\downarrow x + 5z = 115$$

$$x + 5 \cdot 20 = 115$$

$$x + 100 = 115$$

$$x = 115 - 100$$

$$\boxed{x = 15}$$

$$\downarrow 34z = 680$$

$$z = \underline{680}$$

$$34$$

$$\boxed{z = 20}$$

$$x + y + z$$

$$15 + 18 + 20 = 53$$

Alternativa A)