

tarefa básica

SISTEMAS LINEARES

1a)
$$\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} \rightarrow \begin{vmatrix} 6 & -1 \\ 5 \end{vmatrix}$$

$$Dx = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} \rightarrow \begin{vmatrix} 6 & -3 \\ 3 \end{vmatrix}$$

$$Dy = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} \rightarrow \begin{vmatrix} -6 & -(-2) \\ -4 \end{vmatrix}$$

$$x = \frac{Dx}{D} = \frac{3}{5}$$

$$y = \frac{Dy}{D} = \frac{4}{5}$$

Resposta: $v = \left(\frac{3}{5}, \frac{4}{5} \right)$

b)
$$\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 4 \end{cases}$$

$$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} \rightarrow \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} \rightarrow \begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix}$$

$$0 + 9 + 4 = 13 \quad 0 - 12 + 2 = -10$$

$$Dx = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} \rightarrow \begin{vmatrix} 4 & 1 \\ 4 & 1 \end{vmatrix}$$

$$Dy = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 4 & -2 \end{vmatrix} \rightarrow \begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} \rightarrow \begin{vmatrix} 4 & 4 \\ 4 & 4 \end{vmatrix}$$

$$Dz = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 4 \end{vmatrix} \rightarrow \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} \rightarrow \begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix}$$

$$0 + 3 - 2 = 1 \quad 0 - 21 - 1 = -22$$

$$-4 + 63 - 4 = 55 \quad 6 + 12 + 14 = 32$$

$$0 - 3 - 14 = -17 \quad 0 + 4 + 2 = 6$$

$$-22 - 1 = -23$$

$$32 - 55 = -23$$

$$6 - (-17) = 23$$

$$x = \frac{Dx}{D} = \frac{-23}{-23} = 1$$

$$y = \frac{Dy}{D} = \frac{-23}{-23} = 1$$

$$z = \frac{Dz}{D} = \frac{23}{-23} = -1$$

$$v = (1, 1, -1)$$

$$\textcircled{2} \begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases} \quad D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix} \begin{vmatrix} 3 & 4 \\ 4 & 5 \\ 1 & -2 \end{vmatrix}$$

$$-5 - 12 + 48 = 31 \quad 15 + 8 + 8 = 61$$

$$D_y = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix} \begin{vmatrix} 3 & 1 \\ 4 & 12 \\ 1 & 8 \end{vmatrix}$$

$$y = \frac{D_y}{D}$$

$$y = \frac{30}{30} = \textcircled{1}$$

$$-12 + 48 + 12 = 48 \quad 108 + 2 - 32 = 78$$

alternativa A)

$$\textcircled{3} \begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases} \quad D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{vmatrix}$$

$$2 - 33 - 6 = -37 \quad -1 - 44 + 9 = -36$$

$$D_x = \begin{vmatrix} 1 & 2 & 1 \\ -2 & 1 & -11 \\ 1 & 3 & -1 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ -2 & 1 \\ 1 & 3 \end{vmatrix}$$

$$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -2 & -11 \\ 2 & 1 & -1 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 3 & -2 \\ 2 & 1 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{vmatrix}$$

$$\begin{array}{l} 1 - 33 + 4 = -28 \quad -1 - 22 - 6 = -29 \quad -4 - 11 - 3 = -18 \quad 2 - 22 + 3 = -17 \quad 2 - 6 + 6 = 2 \quad 1 - 8 + 9 = 2 \\ -29 - (-28) = -1 \quad -17 - (-18) = 1 \quad 2 - 2 = 0 \end{array}$$

$$x = \frac{-1}{1} = -1$$

$$y = \frac{1}{1} = 1$$

$$z = \frac{0}{1} = 0$$

a

b

$$v = \{(-1, 1, 0)\}^T$$

$$a + b + c$$

$$-1 + 1 + 0 = 0 \quad \text{alternativa c)}$$

④
$$\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases} \quad D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{vmatrix} \rightarrow 1 - (-15) = 16$$

$$-9 - 2 - 4 = -15 \quad -6 + 4 + 3 = 1$$

$$D_x = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix} \begin{vmatrix} 29 & 2 \\ 4 & 3 \\ 8 & -1 \end{vmatrix} \quad D_y = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} \begin{vmatrix} 1 & 29 \\ 1 & 4 \\ 1 & 8 \end{vmatrix}$$

$$-42 - 58 - 16 = -116 \quad -174 + 32 + 12 = -130 \quad -12 + 16 - 58 = -54 \quad -8 + 58 - 24 = 26$$

$$-130 - (-116) = -14 \quad 26 - (-54) = 80$$

$$D_z = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{vmatrix} \quad x = \frac{-14}{16} = -\frac{7}{8} \quad z = \frac{-96}{16} = -6$$

$$87 - 4 + 16 = 99 \quad 24 + 8 - 29 = 3$$

$$3 - 99 = -96$$

$$y = \frac{80}{16} = 5$$

$$x + y + z$$

$$-\frac{7}{8} + 5 - 6 = -\frac{1}{8}$$

Alternative A)

⑤
$$\begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 4 \end{cases} \quad D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{vmatrix} \rightarrow 4 - 4 = 0$$

$$0 + 4 + 0 = 4 \quad 4 + 3 + 0 = 7$$

$$D_x = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 4 & 2 & 1 \end{vmatrix} \begin{vmatrix} 5 & 1 \\ 3 & 2 \\ 4 & 2 \end{vmatrix} \quad D_y = \begin{vmatrix} 2 & 5 & 0 \\ 0 & 3 & 1 \\ 3 & 4 & 1 \end{vmatrix} \begin{vmatrix} 2 & 5 \\ 0 & 3 \\ 3 & 4 \end{vmatrix} \quad D_z = \begin{vmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 3 & 2 & 4 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{vmatrix}$$

$$0 + 10 + 3 = 13 \quad 10 + 4 + 0 = 14 \quad 0 + 14 + 0 = 14 \quad 6 + 15 + 0 = 21 \quad 30 + 12 + 0 = 42 \quad 28 + 9 + 0 = 37$$

$$14 - 13 = 1$$

$$21 - 14 = 7$$

$$37 - 42 = -5$$

$$x = \frac{1}{3}, \quad y = \frac{7}{3}, \quad z = \frac{-5}{3}$$

Alternative D)

$$\textcircled{6} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix} \rightarrow \begin{cases} x = 3 \\ 2x + y = 4 \\ -x + 2y + 2z = -1 \end{cases} \rightarrow \begin{cases} x = 3 \\ y = ? \\ z = ? \end{cases}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \begin{array}{l} 1 \ 0 \\ 2 \ 1 \\ -1 \ 2 \end{array} \quad D_z = \begin{array}{cc|cc} 1 & 0 & 3 & 1 & 0 \\ 2 & 1 & 4 & 2 & 1 \\ -1 & 2 & -1 & -1 & 2 \end{array} \quad D_y = \begin{array}{ccc|cc} 1 & 3 & 0 & 1 & 3 \\ 2 & 4 & 0 & 2 & 4 \\ -1 & -1 & 2 & -1 & -1 \end{array}$$

$$0+0+0=0 \quad 2+0+0=2 \quad -3+14+0=11 \quad -1+0+12=11 \quad 0+0+12=12 \quad 14+0+0=14$$

$$2-0=2 \quad 11-11=0 \quad 14-12=2$$

$$x=3, \quad y = \frac{2}{2} = 1, \quad z = \frac{0}{2} = 0 \rightarrow \text{Alternativa E) } z=0$$

▷ Parte 2

$$\textcircled{1} \begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases} \rightarrow S = \begin{cases} x - 5z = 3 \\ x + 3y - z = 11 \\ 2x - y - 3z = -5 \end{cases}$$

$$\begin{array}{l} -2 \ -1 \\ \downarrow \\ \left(\begin{array}{ccc|c} 1 & 0 & -5 & 3 \\ 1 & 3 & -1 & 11 \\ 2 & -1 & -3 & -5 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -5 & 3 \\ 0 & 3 & 4 & 8 \\ 0 & -1 & 7 & -11 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -5 & 3 \\ 0 & 3 & 4 & 8 \\ 0 & 0 & 25 & -25 \end{array} \right) \end{array}$$

$$x - 5z = 3$$

$$x - 5(-1) = 3$$

$$x = 3 - 5$$

$$x = -2$$

$$3y + 4z = 8$$

$$3y + 4(-1) = 8$$

$$3y = 8 + 4$$

$$y = \frac{12}{3}$$

$$y = 4$$

$$25z = -25$$

$$z = -1$$

$$25$$

$$z = -1$$

$$v = d(-2, 4, -1)$$

$$\textcircled{2} \begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases} \rightarrow \begin{cases} x = 2y \\ z = \frac{2y}{3} \end{cases} \quad y = ?$$

$$x + y + z = 11$$

$$2y + y + \frac{2y}{3} = 11$$

$$11y = 33$$

$$y = \frac{33}{11} = 3$$

$$\frac{6y}{3} + \frac{3y}{3} + \frac{2y}{3} = \frac{33}{3}$$

$$x = 2y$$

$$x = 2 \cdot 3$$

$$x = 6$$

$$y = 3$$

$$z = \frac{2y}{3}$$

$$z = \frac{2 \cdot 3}{3}$$

$$z = 2 \cdot 3 / 3$$

$$z = \frac{6}{3}$$

$$z = 2$$

$$x + 2y + 3z$$

$$6 + 2 \cdot 3 + 3 \cdot 2$$

$$6 + 6 + 6 = 18 \rightarrow \text{Alternative B)}$$

$$\textcircled{3} \begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{cases}$$

$$\begin{aligned} & -2 \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & 0 & -15 & -30 \end{array} \right) \\ & \quad \quad \quad 6 \quad 3 \end{aligned}$$

$$\rightarrow -15z = -30$$

$$z = \frac{-30}{-15}$$

$$z = 2$$

Alternative D)

$$\boxed{z = 2}$$

$$④ \quad A + B + C = 68$$

$$B + 20C = A$$

$$C + 20A = 3B$$

$$\rightarrow \begin{cases} A + B + C = 68 \\ A - B - 1/5C = 0 \\ 1/5A - 3B + C = 0 \end{cases}$$

$$\begin{aligned} & \xrightarrow{-1/5} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 68 \\ 1 & -1 & -1/5 & 0 \\ 1/5 & -3 & 1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 68 \\ 0 & -2 & -6/5 & -68 \\ 0 & -16/5 & 4/5 & -68/5 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 68 \\ 0 & -2 & -6/5 & -68 \\ 0 & -16 & 4 & -68 \end{array} \right) \\ & \xrightarrow{-8/5} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 68 \\ 0 & -2 & -6/5 & -68 \\ 0 & -16 & 4 & -68 \end{array} \right) \end{aligned}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 68 \\ 0 & -2 & -6/5 & -68 \\ 0 & 0 & 68/25 & 476/5 \end{array} \right)$$

$$\frac{68C}{25} = \frac{476}{5}$$

$$C = \frac{476 \cdot 25}{5 \cdot 68}$$

$$\boxed{C = 35}$$

$$-2B - 6 \cdot 35 = -68$$

$$-2B = -68 + 42$$

$$-2B = -26$$

$$B = \frac{-26}{-2} = 13$$

$$A + B + C = 68$$

$$A + 13 + 35 = 68$$

$$A = 68 - 48$$

$$\boxed{A = 20}$$

$$Ali = R\$ 20,00$$

$$Bia = R\$ 13,00$$

$$Caco = R\$ 35,00$$

Alternativa A)

/ /

$$\textcircled{5} \quad A = \begin{pmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad A \cdot X = \begin{pmatrix} 134 \\ 115 \\ 48 \end{pmatrix} \rightarrow \begin{cases} 3y + 4z = 134 \\ x + 5z = 115 \\ 2x + y = 48 \end{cases}$$

$$\begin{array}{l} -3 \rightarrow \left(\begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 2 & 1 & 0 & 48 \end{array} \right) \sim \left(\begin{array}{ccc|c} -6 & 0 & 4 & -10 \\ 1 & 0 & 5 & 115 \\ & & & \end{array} \right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 34 & 680 \\ & & & \\ & & & \end{array} \right) \end{array}$$

$$2x + y = 48$$

$$2 \cdot 15 + y = 48$$

$$30 + y = 48$$

$$y = 48 - 30$$

$$\boxed{y = 18}$$

$$\rightarrow x + 5z = 115$$

$$x + 5 \cdot 20 = 115$$

$$x + 100 = 115$$

$$x = 115 - 100$$

$$\boxed{x = 15}$$

$$\rightarrow 34z = 680$$

$$z = \frac{680}{34}$$

$$34$$

$$\boxed{z = 20}$$

$$x + y + z$$

$$15 + 18 + 20 = 53$$

Alternativa A)