tarefa básica

MATRIZ INVERSA

① $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$ é a inversa de $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$ $x + y = ?$

$A \cdot A^{-1} = I_n$

$B \cdot A = I_n$

$$\begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} \cdot \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{cases} 3x - 5 = 1 & \text{I} \\ xy + 10 = 0 & \text{II} \end{cases}$$

I) $3x - 5 = 1$

$3x = 1 + 5$

$x = \frac{6}{3} = 2$

II) $xy + 10 = 0$

$2y + 10 = 0$

$2y = -10$

$y = \frac{-10}{2} = -5$

$x + y$

$2 - 5 = -3$

Alternativa c)

② $A = \begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{bmatrix}$ não admite inversa, então det = 0

$\det A = \begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{bmatrix} \begin{matrix} 1 & 0 \\ k & 1 \\ 1 & k \end{matrix}$

$1 + 3k + 0$ $\quad 3 + 0 + k^2$

$k^2 + 3 - (3k + 1) = 0$

$k^2 - 3k + 3 - 1 = 0$

$k^2 - 3k + 2 = 0$

$\Delta\ b^2 - 4 \cdot a \cdot c$

$\Delta\ (-3)^2 - 4 \cdot 1 \cdot 2$

$\Delta\ 9 - 8$

$\Delta\ 1$

$k = \frac{-b \pm \sqrt{1}}{2a}$

$k = \frac{3 \pm 1}{2}$

$k' = \frac{4}{2} = 2$ $\quad k'' = \frac{2}{2} = 1$

os valores de k são 1 e 2

Alternativa c)

/ /

③ $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ $B = A^{-1} \rightarrow B = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$ $\det A = 12 - 10$

$\det A = 2$

→ matriz de ordem 2

• Diagonal Principal: inverte a posição

• Secundária: muda o sinal

$B = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \div 2$

Divide a matriz pelo determinante de A.

$B = \begin{bmatrix} 4/2 & -5/2 \\ -2/2 & 3/2 \end{bmatrix} = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$

alternativa c)

④ $A = \begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix}$

→ inversível → $\det \neq 0$

$\det A = \begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix} \begin{matrix} x & 1 \\ 3 & 1 \\ 10 & 1 \end{matrix}$

$20 + 2x + 3x \quad x^2 + 20 + 6$

$x^2 + 26 - (20 + 5x) \neq 0$

$x^2 - 5x + 26 - 20 \neq 0$

$x^2 - 5x + 6 \neq 0$

$\Delta\, b^2 - 4 . a . c$

$\Delta\, (-5)^2 - 4 . 1 . 6$

$\Delta\, 25 - 24$

$\Delta\, 1$

$x \neq \dfrac{-b \pm \sqrt{1}}{2a}$

$x \neq \dfrac{5 \pm 1}{2}$

$x' \neq \dfrac{6}{2} \neq 3$

$x'' \neq \dfrac{4}{2} \neq 2$

$x \neq 3$ e $x \neq 2$

Alternativa A)

tilibra

5) $A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix}$ $\quad A + A^{-1} = ?$

Regras da matriz inversa
- calcular det ($\det \neq 0$)
- calcular A' (m. dos cofatores)
- calcular $\bar{A}$ (matriz adjunta)
- $A^{-1} = \bar{A} / \det A$

$$\det A = \left[\begin{array}{ccc} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{array}\right]\begin{array}{cc} -1 & -1 \\ 2 & 1 \\ 1 & 1 \end{array}$$

$2+2+2 = 6 \quad 1+2+4 = 7$

$7 - 6 = 1 \rightarrow \det A$

$$A' \rightarrow A = \begin{bmatrix} -1_{11} & -1_{12} & 2_{13} \\ 2_{21} & 1_{22} & -2_{23} \\ 1_{31} & 1_{32} & -1_{33} \end{bmatrix} \rightarrow A' = \begin{bmatrix} -1-(-2) & -2-(-2) & 2-1 \\ 1-2 & 1-2 & -1-(-1) \\ 2-2 & 2-4 & -1-(-2) \end{bmatrix}$$

$i + j =$ ímpar
muda o sinal

$$A^{-1} = \begin{bmatrix} -1_{11} & 0_{12} & 1_{13} \\ -1_{21} & -1_{22} & 0_{23} \\ 0_{31} & -2_{32} & 1_{33} \end{bmatrix} \quad A' = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\bar{A} = (A')^t = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

Então,

$$A^{-1} = \frac{\bar{A}}{\det A}$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \div 1 = \text{dá na mesma}$$

$A + A^{-1} = ?$

$$\begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$ alternativa B)

⑥ $(X.A)^t = B$

Sabemos que a matriz transposta de uma transposta resulta numa matriz normal:

$$((X.A)^t)^t = B^t$$

$$X.A = B^t$$

$$X.A.A^{-1} = B^t.A^{-1}$$

$$X = B^t \cdot A^{-1} \rightarrow \text{alternativa B)}$$

⑦ $B = \begin{bmatrix} x \\ y \end{bmatrix}$ e $C = \begin{bmatrix} 4x+5y \\ 5x+6y \end{bmatrix}$ $AB = C$

$$((X.A)^t)^t = B^t$$

$$X.A = B^t$$

$$X.A.A^{-1} = B^t.A^{-1}$$

$$X = B^t \cdot A^{-1} \rightarrow \text{alternativa B)}$$

7) $B = \begin{bmatrix} x \\ y \end{bmatrix}$ e $C = \begin{bmatrix} 4x+5y \\ 5x+6y \end{bmatrix}$ $AB = C$

$$A.\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x+5y \\ 5x+6y \end{bmatrix} \rightarrow A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} \quad det = 24 - 25 = -1$$

matriz de ordem 2

$$A^{-1} = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \div -1 \rightarrow A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix} \quad \text{alternativa D)}$$

) $A = \begin{bmatrix} 2 & K \\ \end{bmatrix}$ $\rightarrow = det\,A = det\,A^{-1}$

$\begin{bmatrix} y \end{bmatrix}$ $\begin{bmatrix} 4x+5y \\ 5x+6y \end{bmatrix} \rightarrow A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix}$ $\det = 24 - 25 = -1$

matriz de ordem 2

$A^{-1} = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \div -1 \rightarrow A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix}$ alternativa D)

(8) $A = \begin{bmatrix} 2 & k \\ -2 & 1 \end{bmatrix}$

$\det A = 2 + 2k$

$\rightarrow \det A = \det A^{-1}$

$\det A . \det A^{-1} = 1$

$(2+2k).(2+2k) = 1$

$4 + 4k + 4k + 4k^2 = 1$

$4k^2 + 8k + 3$

$\Delta\ b^2 - 4.a.c$

$\Delta\ 8^2 - 4.4.3$

$\Delta\ 64 - 48$

$\Delta\ 16$

$x = \frac{-b \pm \sqrt{16}}{2a}$

$x = \frac{-8 \pm 4}{8}$

$x' = \frac{-12^{:4}}{8^{:4}} = \frac{-3}{2}$

$x'' = \frac{-4^{:4}}{8^{:4}} = \frac{-1}{2}$

Soma dos K

$\frac{-3}{2} - \frac{1}{2} = \frac{-4}{2} = -2$

Alternativa B)

9 A e B são 2×2, com det $\neq 0$

A) $(A+B).(A-B)$

$$\boxed{A^2 - AB + AB - B^2} \to AB \neq BA$$

B) $(A+B)^2 = A^2 + 2AB + B^2$

$$\boxed{AB = BA}$$

c) $\dfrac{\det A}{\det(-A)}$ $\to \det(-A) = (-1)^2 \cdot \det A$

$\det(-A) = \det A$

$$\frac{\det A}{\det(-A)} = \frac{\det A}{\det A} = 1$$

b) $(A+B)^2 = A^2 + 2AB + B^2$

$$\boxed{AB = BA}$$

c) $\dfrac{\det A}{\det(-A)}$ $\rightarrow$ $\det(-A) = (-1)^2 \cdot \det A$

$\det(-A) = \det A$

$$\frac{\det A}{\det(-A)} = \frac{\det A}{\det A} = 1$$

D) $B = A^{-1}$ $\rightarrow$ $\det A \cdot \det B = 1$

$$\det B = \frac{1}{\det A}$$