More Algorithm Analysis

Computer Science Department
University of Central Florida

COP 3502 – Computer Science I

Big-O Notation

- What is Big O?
  - Big O comes from Big-O Notation
    - In C.S., we want to know how efficient an algorithm is…how "fast" it is
    - More specifically…we want to know **how the performance of an algorithm responds to changes in problem size**
    - The goal is to provide a *qualitative* insight on the # of operations for a problem size of n elements.
    - And this total # of operations can be described with a mathematical expression in terms of n.
      - This expression is known as Big-O

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- Examples of Analyzing Code:
  - We now go over many examples of code fragments
  - Each of these functions will be analyzed for their runtime in terms of the variable n
  - Utilizing the idea of Big-O,
    - determine the Big-O running time of each

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- Example 1:
  - Determine the Big O running time of the following code fragment:

```
for (k = 1; k <= n/2; k++) {
       sum = sum + 5;
}
for (j = 1; j <= n*n; j++) {
       delta = delta + 1;
}
```

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- Example 1:
  - So look at what's going on in the code:
    - We care about the total number of REPETITIVE operations.
      - Remember, we said we care about the running time for LARGE values of n
      - So in a for loop with n as part of the comparison value determining when to stop `for (k=1; k<=n/2; k++)`
      - Whatever is INSIDE that loop will be executed a LOT of times
      - So we examine the code within this loop and see how many operations we find
        - When we say operations, we're referring to mathematical operations such as +, -, *, /, etc.

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- Example 1:
  - So look at what's going on in the code:
    - The number of operations executed by these loops is the sum of the individual loop operations.
    - We have 2 loops,
      - The first loop runs n/2 times
      - Each iteration of the first loop results in one operation
        - The + operation in: `sum = sum + 5;`
      - So there are n/2 operations in the first loop
      - The second loop runs n^2 times
      - Each iteration of the second loop results in one operation
        - The + operation in: `delta = delta + 1;`
      - So there are n^2 operations in the second loop.

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- Example 1:
  - So look at what's going on in the code:
    - The number of operations executed by these loops is the sum of the individual loop operations.
    - The first loop has n/2 operations
    - The second loop has n^2 operations
    - They are NOT nested loops.
      - One loop executes AFTER the other completely finishes
    - So <u>we simply ADD their operations</u>
    - The total number of operations would be $n/2 + n^2$
    - In Big-O terms, we can express the number of operations as $O(n^2)$

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- Example 2:
  - Determine the Big O running time of the following code fragment:

```
int func1(int n) {
        int i, j, x = 0;
        for (i = 1; i <= n; i++) {
                for (j = 1; j <= n; j++) {
                        x++;
                }
        }
        return x;
}
```

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- Example 2:
  - So look at what's going on in the code:
    - We care about the total number of REPETITIVE operations
    - We have two loops
      - AND they are NESTED loops
    - The outer loop runs n times
      - From i = 1 up through n
      - How many operations are performed at each iteration?
        - Answer is coming…
    - The inner loop runs n times
      - From j = 1 up through n
      - And only one operation (x++) is performed at each iteration

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- Example 2:
  - So look at what's going on in the code:
    - Let's look at a couple of iterations of the OUTER loop:
      - When i = 1, what happens?
        - The inner loop runs n times
        - Resulting in n operations from the inner loop
      - Then, i gets incremented and it becomes equal to 2
      - When i = 2, what happens?
        - Again, the inner loop runs n times
        - Again resulting in n operations from the inner loop
    - We notice the following:
      - For EACH iteration of the OUTER loop,
      - The INNER loop runs n times
        - Resulting in n operations

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- Example 2:
  - So look at what's going on in the code:
    - And how many times does the outer loop run?
      - n times
    - So the outer loop runs n times
    - And for each of those n times, the inner loop also runs n times
      - Resulting in n operations
    - So we have n operations per iteration of OUTER loop
    - And outer loop runs n times
    - Finally, we have n*n as the number of operations
    - We approximate the running time as O(n^2)

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- Example 3:
  - Determine the Big O running time of the following code fragment:

```
int func3(int n) {
        int i, x = 0;
        for (i = 1; i <= n; i++)
                x++;
        for (i = 1; i<=n; i++)
                x++;
        return x;
}
```

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- Example 3:
  - So look at what's going on in the code:
    - We care about the total number of REPETITIVE operations
    - We have two loops
      - They are NOT nested loops
    - The first loop runs n times
      - From i = 1 up through n
      - only one operation (x++) is performed at each iteration
    - How many times does the second loop run?
      - Notice that i is indeed reset to 1 at the beginning of the loop
      - Thus, the second loop runs n times, from i = 1 up through n
      - And only one operation (x++) is performed at each iteration

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- Example 3:
  - So look at what's going on in the code:
    - Again, the loops are NOT nested
    - So they execute sequentially (one after the other)
  - Therefore:
    - Our total runtime is on the order of n+n
    - Which of course equals 2n
  - Now, in Big O notation
    - We approximate the running time as O(n)

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- Example 4:
  - Determine the Big O running time of the following code fragment:

```
int func4(int n) {
        while (n > 0) {
                printf(“%d”, n%2);
                n = n/2;
        }
}
```

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- Example 4:
  - So look at what's going on in the code:
    - We have one while loop
      - You can't just look at this loop and say it iterates n times or n/2 times
      - Rather, it continues to execute as long as n is greater than 0
      - The question is: **<u>how many iterations will that be?</u>**
    - Within the while loop
      - The last line of code divides the input, n, by 2
      - So n is halved at each iteration of the while loop
    - If you remember, we said this ends up running in log n time
    - Now let's look at how this works

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- Example 4:
  - So look at what's going on in the code:
    - For the ease of the analysis, we make a new variable
      - originalN:
        - originalN refers to the value originally stored in the input, n
        - So if n started at 100, originalN will be equal to 100
    - The first time through the loop
      - n gets set to originalN/2
        - If the original n was 100, after one iteration n would be 100/2
    - The second time through the loop
      - n gets set to originalN/4
    - The third time through the loop
      - n gets set to originalN/8

Notice:
After **three** iterations, n gets set to originalN/2^3

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- Example 4:
  - So look at what's going on in the code:
    - In general, after k iterations
      - n gets set to $originalN/2^k$
    - The algorithm ends when $originalN/2^k = 1$, approximately
    - We now solve for k
    - Why?
      - Because we want to find the **total # of iterations**
    - Multiplying both sides by 2^k, we get $originalN = 2^k$
    - Now, using the definition of logs, we solve for k
      - k = log originalN
    - So we approximate the running time as O(log n)

Brief Interlude: Human Stupidity

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- Example 5:
  - Determine the Big O running time of the following code fragment:

```
int func5(int** array, int n)  {
        int i = 0, j = 0;
        while (i < n) {
                while (j < n && array[i][j] == 1)
                        j++;
                i++;
        }
        return j;
}
```

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- Example 5:
  - So look at what's going on in the code:
    - At first glance, we see two NESTED loops
    - This can often indicate an $O(n^2)$ algorithm
      - But we need to look closer to confirm
    - Focus on what's going on with i and j

```
int func5(int** array, int n)  {
        int i = 0, j = 0;
        while (i < n) {
                while (j < n && array[i][j] == 1)
                        j++;
                i++;
        }
```

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- Example 5:
  - So look at what's going on in the code:
    - Focus on what's going on with i and j
      - i and j clearly increase (from the j++ and i++)
      - BUT, they never decrease
      - AND, neither ever gets reset to 0

```
int func5(int** array, int n)  {
        int i = 0, j = 0;
        while (i < n) {
                while (j < n && array[i][j] == 1)
                        j++;
                i++;
        }
```

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- Example 5:
  - So look at what's going on in the code:
    - And the OUTER while loop ends once i gets to n
    - So, what does this mean?
      - The statement i++ can never run more than n times
      - And the statement j++ can never run more than n times

```
int func5(int** array, int n)  {
        int i = 0, j = 0;
        while (i < n) {
                while (j < n && array[i][j] == 1)
                        j++;
                i++;
        }
```

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- Example 5:
  - So look at what's going on in the code:
    - The MOST number of times these two statements can run (combined) is 2n times
    - So we approximate the running time as O(n)

```
int func5(int** array, int n)  {
        int i = 0, j = 0;
        while (i < n) {
                while (j < n && array[i][j] == 1)
                        j++;
                i++;
        }
```

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- Example 6:
  - Determine the Big O running time of the following code fragment:
    - What's the one big difference here???

```
int func6(int** array, int n)  {
       int i = 0, j;
       while (i < n) {
              j = 0;
              while (j < n && array[i][j] == 1)
                     j++;
              i++;
       }
       return j;
}
```

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- Example 6:
  - So look at what's going on in the code:
    - The difference is that we RESET j to 0 a the beginning of the OUTER while loop

```
int func6(int** array, int n)  {
       int i = 0, j;
       while (i < n) {
              j = 0;
              while (j < n && array[i][j] == 1)
                     j++;
              i++;
       }
       return j;
}
```

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- Example 6:
  - So look at what's going on in the code:
    - The difference is that we RESET j to 0 a the beginning of the OUTER while loop
    - How does that change things?
      - Now j can iterate from 0 to n for EACH iteration of the OUTER while loop
        - For each value of i
      - This is similar to the 2nd example shown
    - So we approximate the running time as O(n^2)

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- Example 7:
  - Determine the Big O running time of the following code fragment:

```
int func7(int A[], int sizeA, int B[], int sizeB) {
        int i, j;
        for (i = 0; i < sizeA; i++)
                for (j = 0; j < sizeB; j++)
                        if (A[i] == B[j])
                                return 1;
        return 0;
}
```

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- Example 7:
  - So look at what's going on in the code:
    - First notice that the runtime here is NOT in terms of n
    - It will be in terms of sizeA and sizeB
    - And this is also just like Example 2
    - The outer loop runs sizeA times
    - For EACH of those times,
      - The inner loop runs sizeB times
    - So this algorithm runs sizeA*sizeB times
    - We approximate the running time as O(sizeA*sizeB)

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- Example 8:
  - Determine the Big O running time of the following code fragment:

```
int func8(int A[], int sizeA, int B[], int sizeB) {
      int i, j;
      for (i = 0; i < sizeA; i++) {
            if (binSearch(B, sizeB, A[i]))
                  return 1;
      }
      return 0;
}
```

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- Example 8:
  - So look at what's going on in the code:
    - Note: we see that we are calling the function binSearch
    - As discussed previously, a single binary search runs in O(log n) time
      - where n represents the number of items within which you are searching
  - Examining the for loop:
    - The for loop will execute sizeA times
    - For EACH iteration of this loop
      - a binary search will be run
    - We approximate the running time as O(sizeA*log(sizeB))

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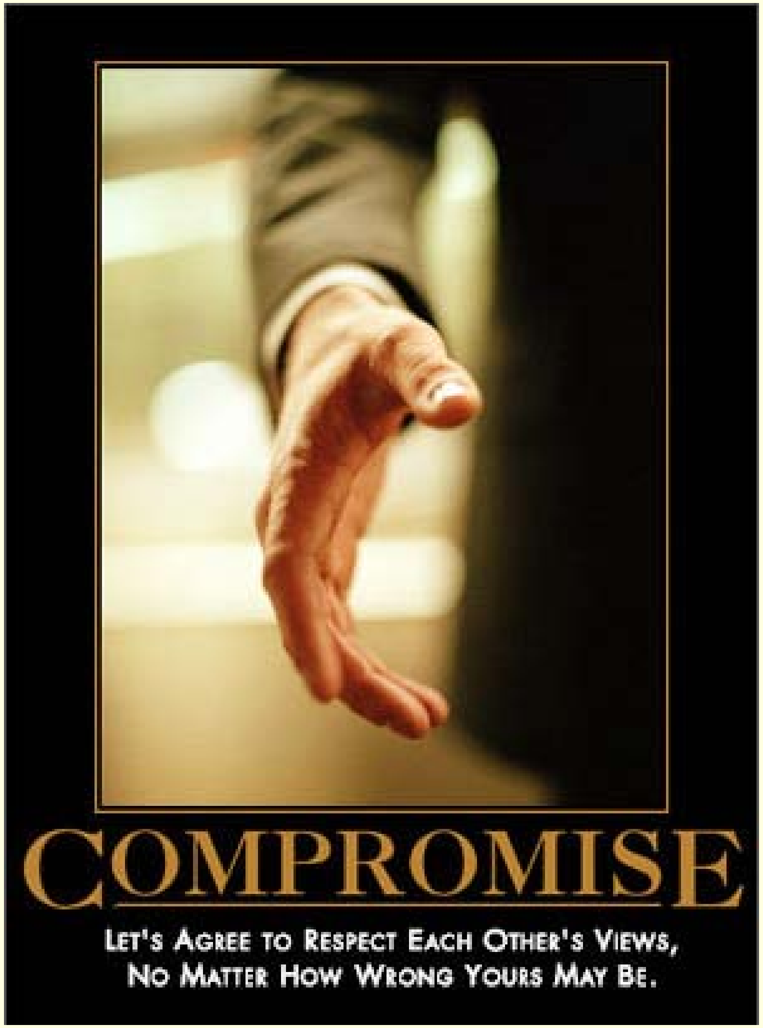
WASN'T THAT SWEET!

Daily Demotivator



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