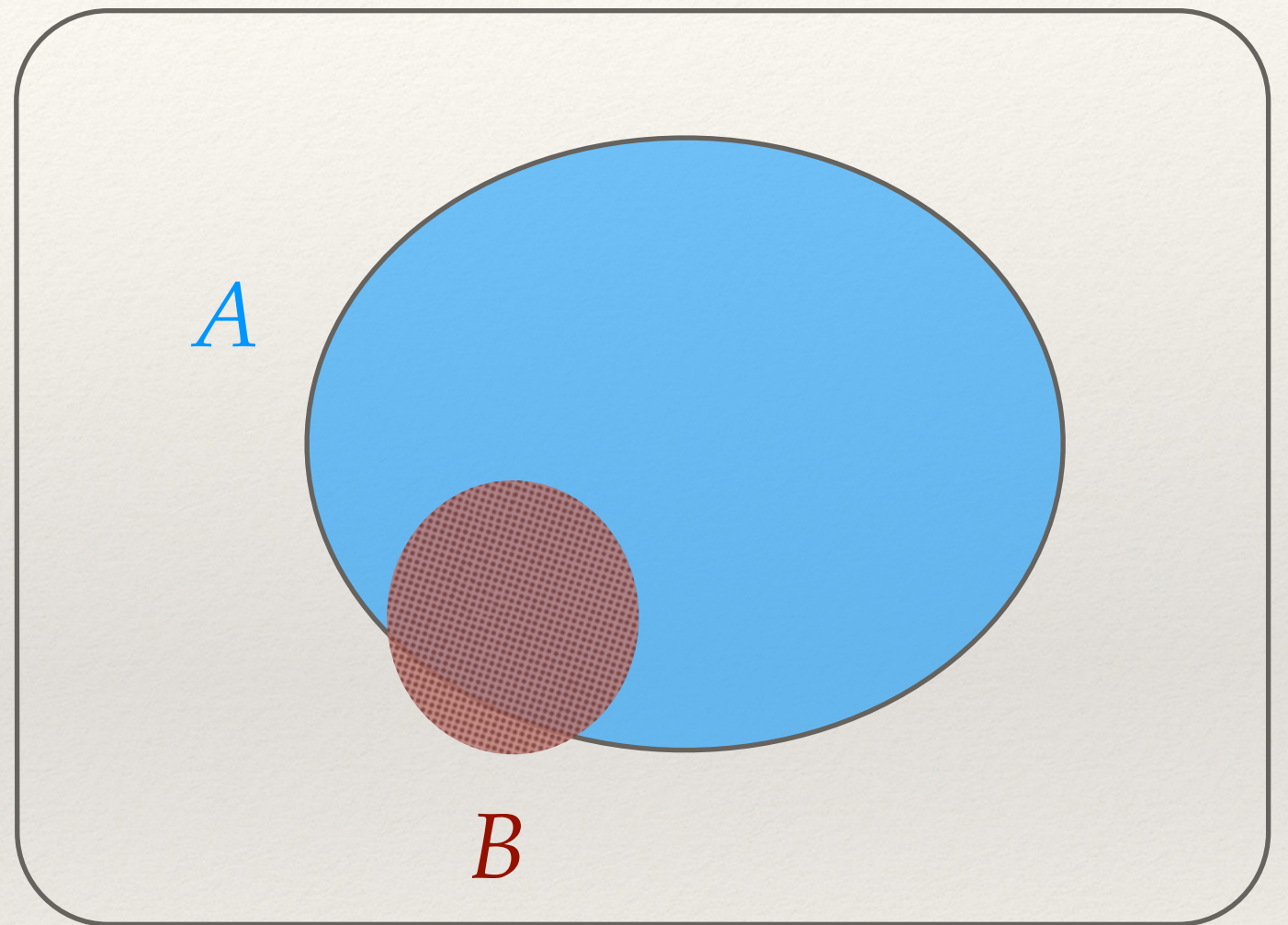


The Prosecutor's Fallacy

The Inversion Fallacy

Confusing
 $Pr(A | B)$
with
 $Pr(B | A)$



$Pr(A | B)$ is **HIGH**

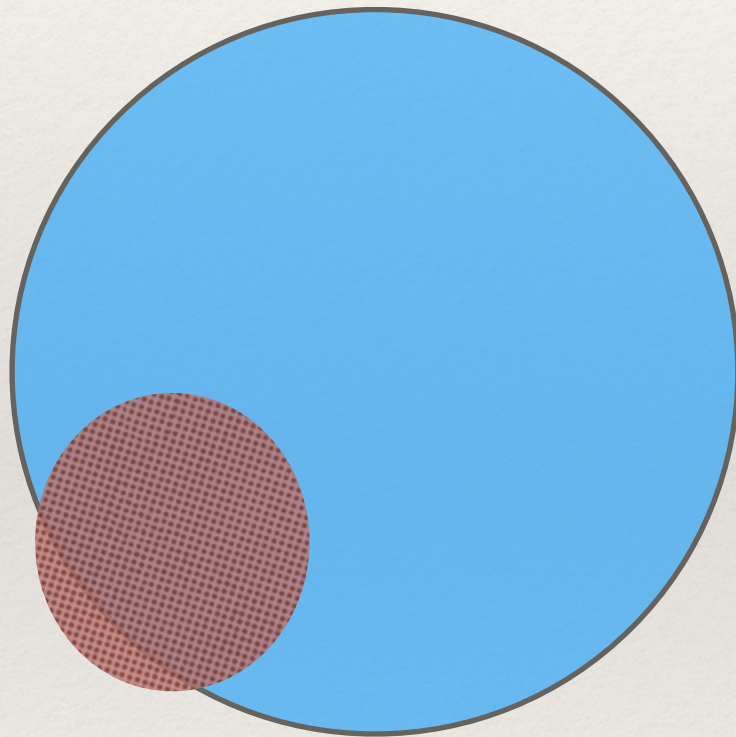
$Pr(B | A)$ is **LOW**

Example

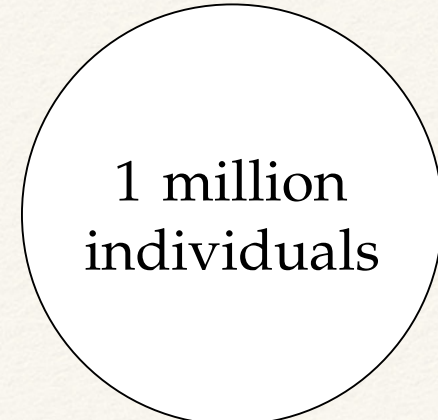
$Pr(\text{test positive} \mid \text{disease})$ is **HIGH**

$Pr(\text{disease} \mid \text{test positive})$ is **LOW**

*person tests
positive*



*person has
disease*



We know:

Individuals with
disease 10,000



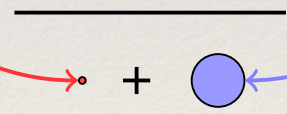
$Pr(\text{test positive} \mid \text{disease}) = 90\%$

$P(T[\text{positive}] \mid D) = 90\%$

$P(T[\text{positive}] \mid \neg D) = 10\%$

$90\% \times 10,000 = 9,000$

$10\% \times 990,000 = 99,000$



$Pr(\text{disease} \mid \text{test positive}) = 8.3\%$

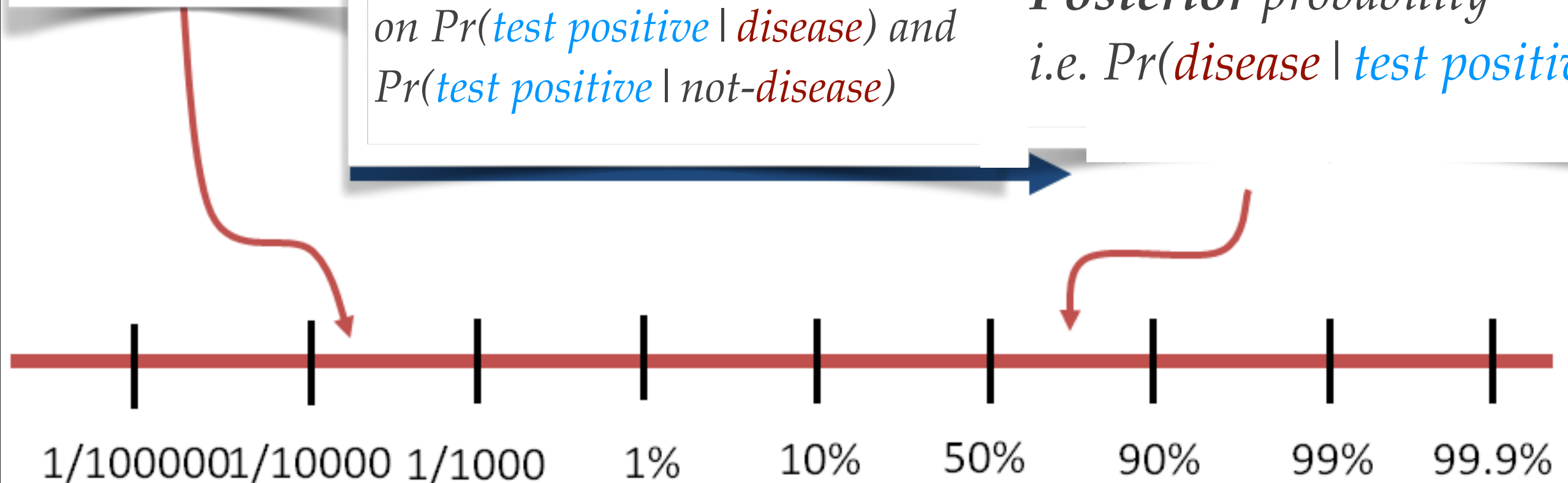
$\frac{9,000}{9,000 + 99,000} \approx 8.3\%$

Bayes' Theorem (graphical representation)

Prior
probability —
i.e. $\Pr(\text{disease})$

Strength of evidence — depends
on $\Pr(\text{test positive} \mid \text{disease})$ and
 $\Pr(\text{test positive} \mid \text{not-disease})$

Posterior probability —
i.e. $\Pr(\text{disease} \mid \text{test positive})$



$$\text{Prior} \times \text{Strength of evidence} = \text{Posterior}$$

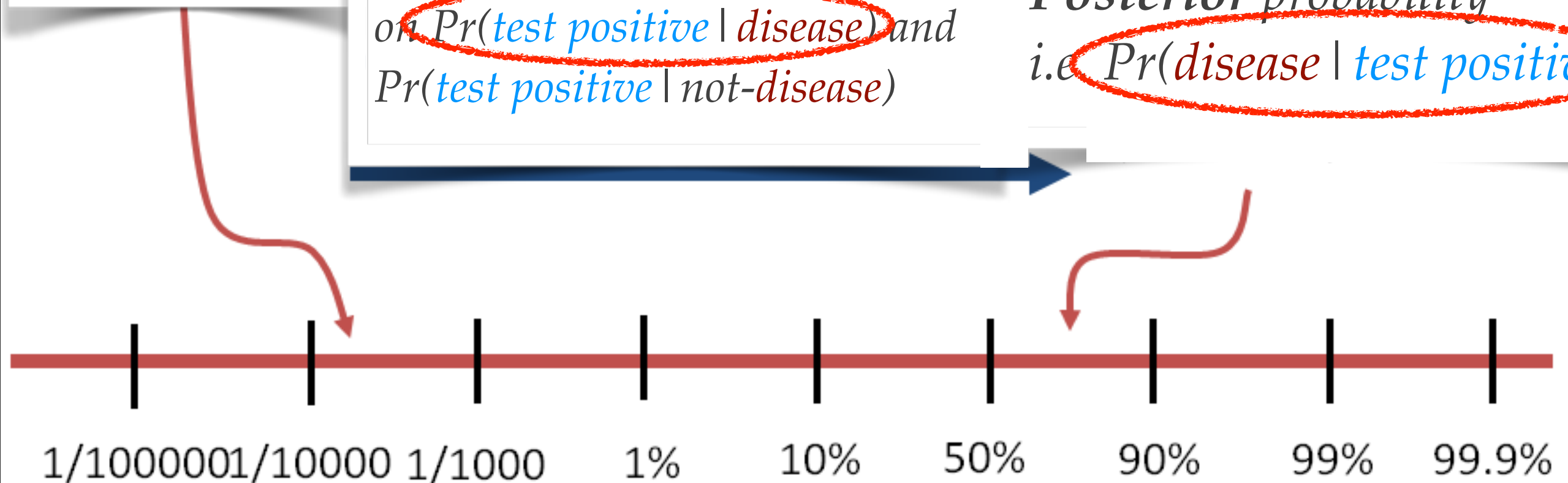
Bayes' Theorem

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 $\Pr(\text{test positive} \mid \text{not-disease})$

Posterior probability —
i.e. $\Pr(\text{disease} \mid \text{test positive})$

Upshot: even if the test is good, the posterior probability of having the disease given a positive test result, could still be low if the priors are low



$$\text{Prior} \times \text{Strength of evidence} = \text{Posterior}$$

“Inversion Fallacy”

Committed in a Criminal Trial Is

“Prosecutor’s Fallacy”

Why Should We Be Worried About the Prosecutor's Fallacy?

Interpretation of Statistical Evidence in Criminal Trials

The Prosecutor's Fallacy and the Defense Attorney's Fallacy*

William C. Thompson† and Edward L. Schumann†

INTERPRETATION OF STATISTICAL EVIDENCE

171

The prosecutor's misguided judgmental strategy (which we shall call the Prosecutor's Fallacy) could lead to serious error, particularly where the other evidence in the case is weak and therefore the prior probability of guilt is low. Suppose, for example, that one initially estimates the suspect's probability of guilt to be only .20, but then receives additional evidence showing that the defendant and perpetrator match on a blood type found in 10% of the population. According to Bayes theorem, this new evidence should increase one's subjective probability of guilt to .71, not .90.³

Crime... of cr...
suspects (Saferstein, 1977; Schroeder, 1977; Giannelli, 1983). Laboratory tests

* This research was supported by grants to the first author from the National Science Foundation (No. SES 86-05323) and the UCI Academic Senate Committee on Research. The authors wish to thank Karen Rook and Robyn M. Dawes for comments on earlier versions of this article and John Van-Vlear for help in collecting data.

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The prosecutor's fallacy is worrisome because it often leads to *exaggeration of the probability of guilt*

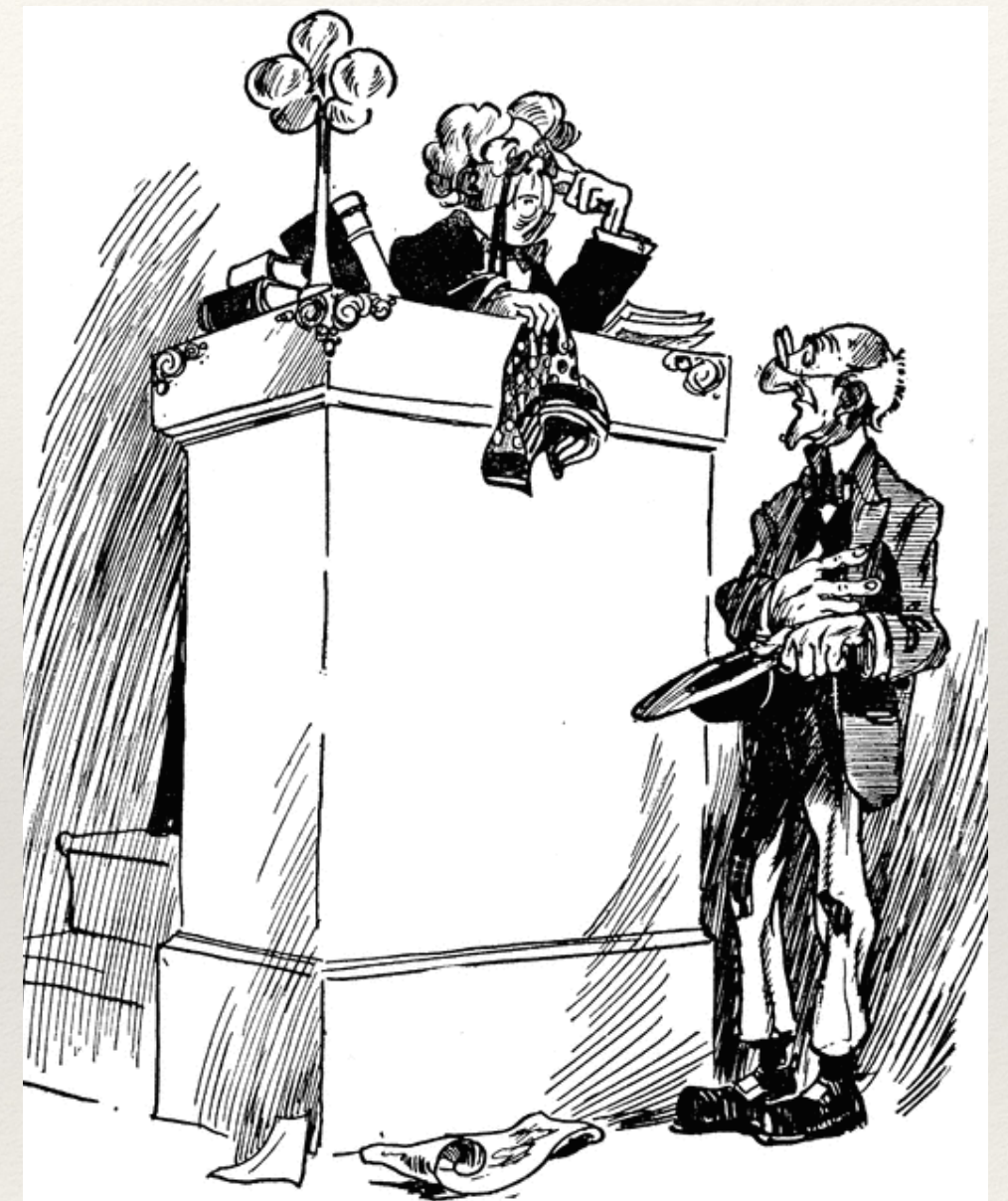
DNA Evidence

- ❖ Traces are found on the crime scene
- ❖ The traces conclusively establish that whoever left them is the perpetrator
- ❖ The question is, *WHO left the traces?*
 - ❖ One man's DNA profile **matches** with the traces found at the crime scene
 - ❖ The DNA profile has a **genotype probability of 1 in 1 million or 0.0001%**



Judge Hardwick - Missouri Ct. App.

“We conclude that where, as here, DNA material is found in a location, quantity, and type **inconsistent with casual contact** and there is one in **one quintillion likelihood** that some else was the source of the material, the evidence is **legally sufficient to support a guilty verdict**”

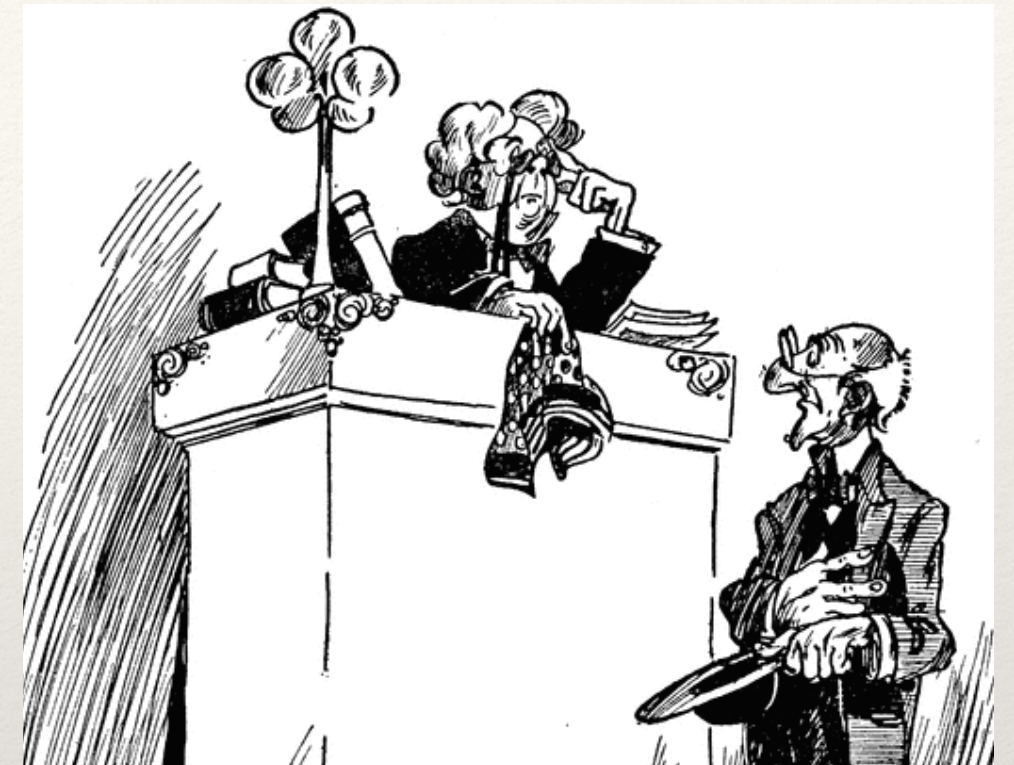


Missouri v. Abdelmalik, 273 S.W.3d, 61, 66
(Mo. Ct. App. 2008)

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Which probability was the judge referring to?

$Pr(\text{Match} \mid \text{Innocence})$

$Pr(\text{Innocence} \mid \text{Match})$

What We Should Rethink

Claim 1



confusing $Pr(A | B)$ with $Pr(B | A)$ is an error in probabilistic reasoning

Claim 2



the confusion leads to exaggeration of the probability of guilt given the evidence, that is, $Pr(Guilt | Evidence)$

Tacit Claim



decisions to convict should be guided by

low $Pr(Innocence | Evidence)$

or

high $Pr(Guilt | Evidence)$