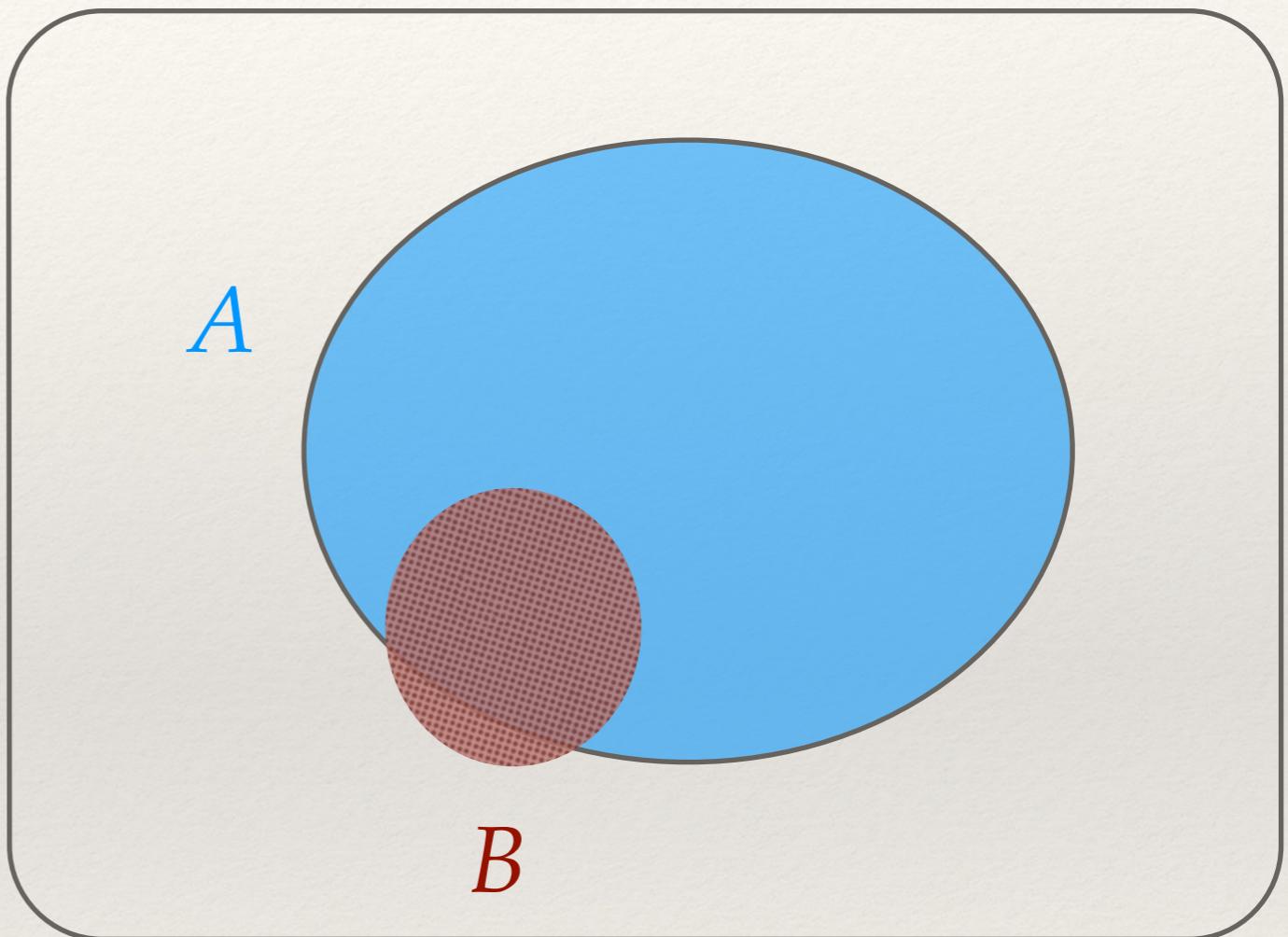


The Prosecutor's Fallacy

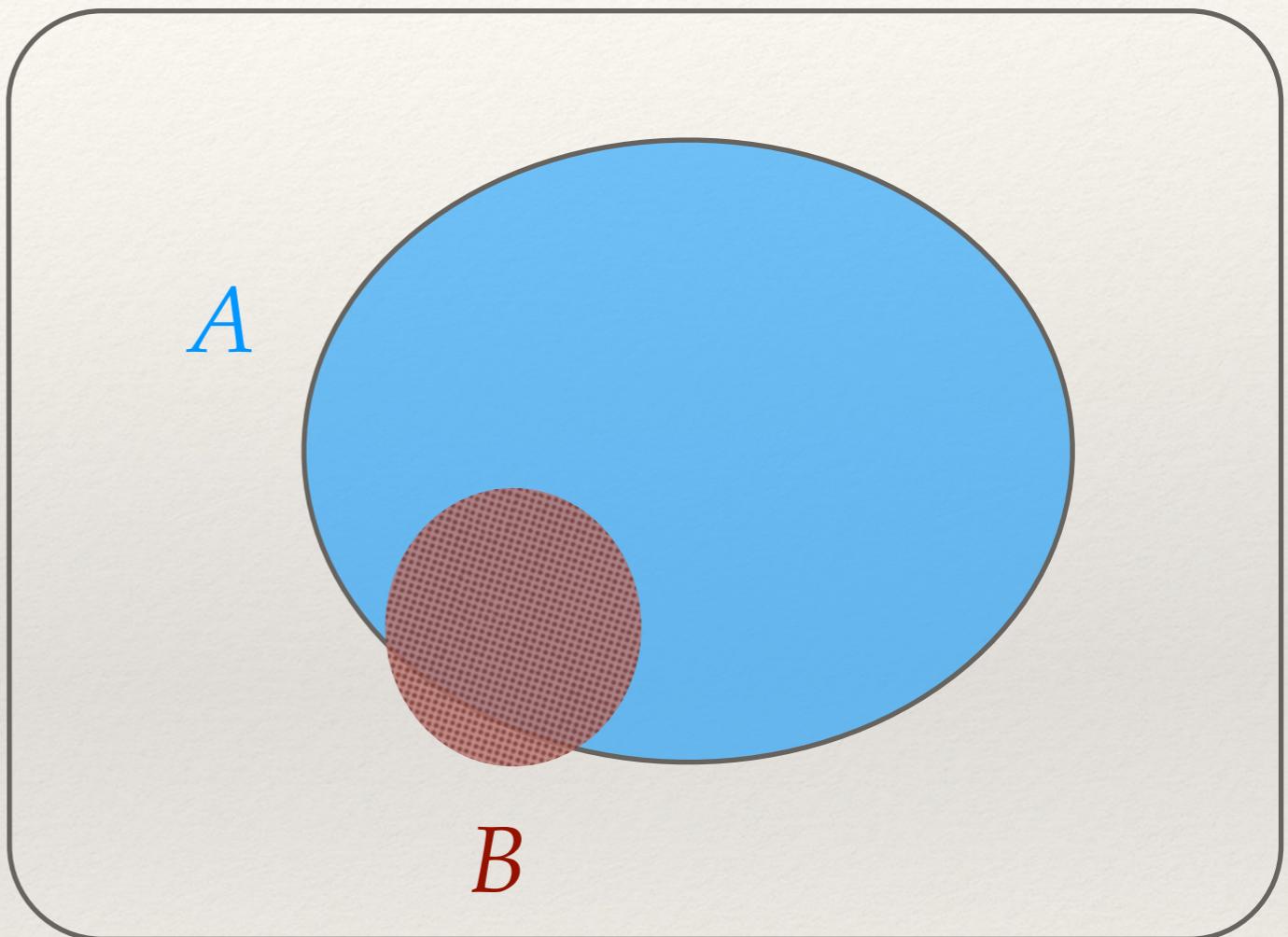
The Inversion Fallacy

Confusing
 $Pr(A | B)$
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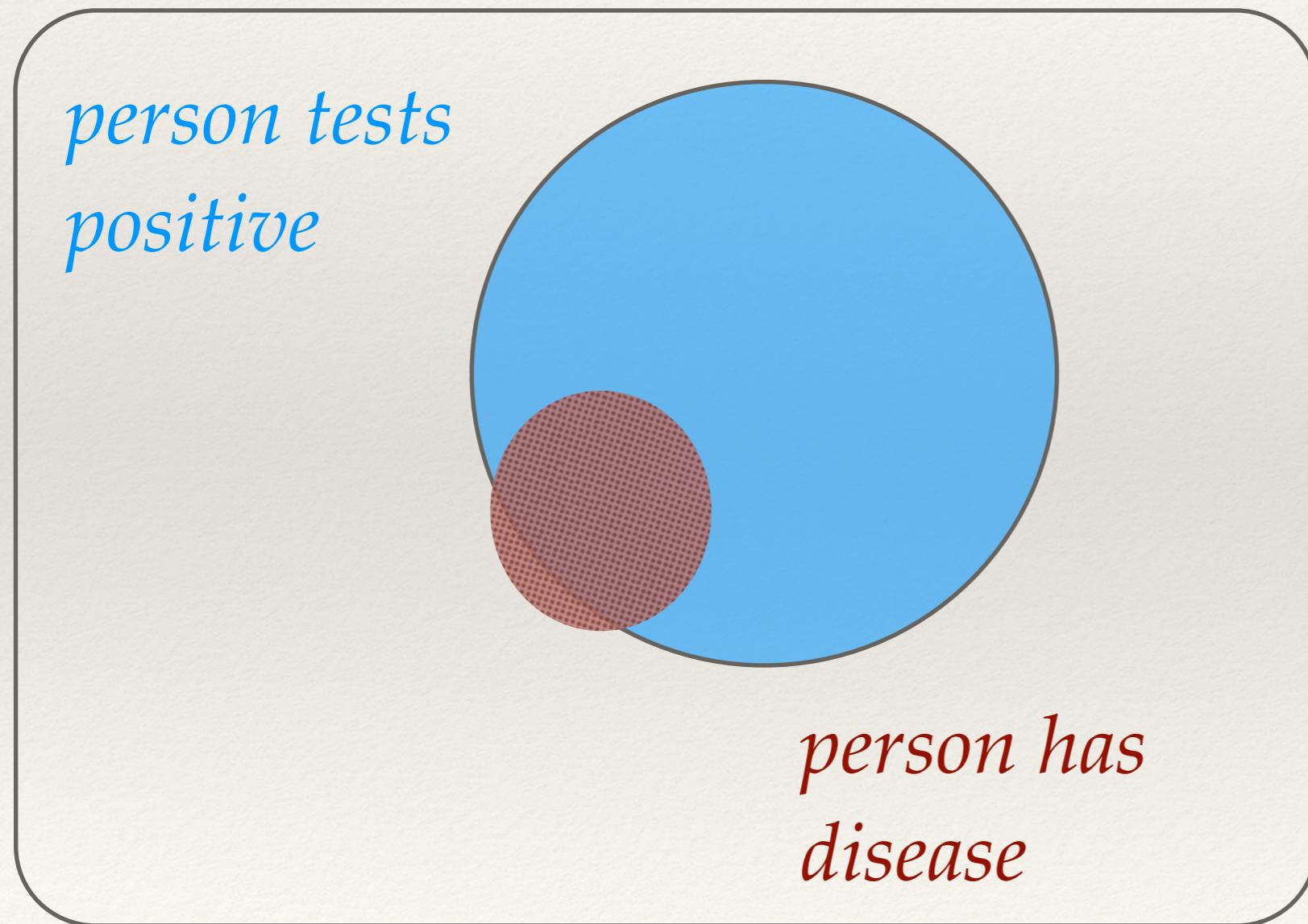
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$Pr(A | B)$ is HIGH

$Pr(B | A)$ is LOW

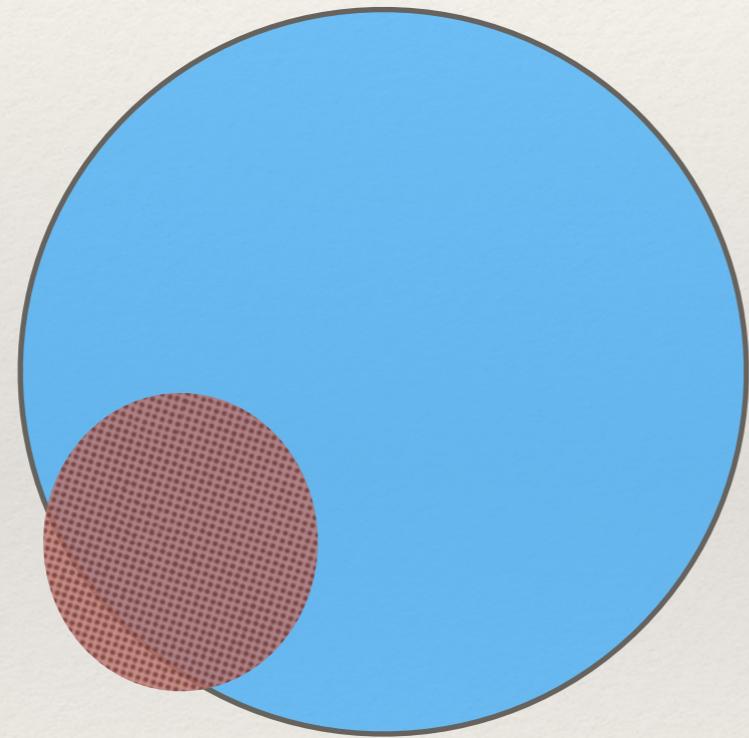
Example



Example

$\Pr(\text{test positive} \mid \text{disease})$ is **HIGH**
 $\Pr(\text{disease} \mid \text{test positive})$ is **LOW**

*person tests
positive*



*person has
disease*

“Inversion Fallacy”

Committed in a Criminal Trial Is

“Prosecutor’s Fallacy”

Lucia de Berk

- ❖ Lucia de Berk was a nurse in different children hospitals
- ❖ A high number of children died while she was on duty
- ❖ Could this be a coincidence?
- ❖ First degree trial and appellate court thought it was not
- ❖ Lucia was convicted
- ❖ Supreme Court reversed



Could it Be a Coincidence?

Shifts at Juliana's hosp.	without incidents	with incident
without Lucia	887	0
with Lucia	134	8

$$\frac{\binom{r}{x} p^x (1-p)^{r-x} \binom{n-r}{k-x} p^{k-x} (1-p)^{n-r-k+x}}{\binom{n}{k} p^k (1-p)^{n-k}}$$

Assuming Lucia was not the cause of the deaths (i.e. *coincidence hypothesis*), what is the probability of getting such extreme statistics? This is similar to a **p-value**.



Expert Henk Efflers calculated this probability was in **1 in 342 million or 0.000000292397 %**.

The Conclusion of the Court

“The court is of the opinion that the probabilistic calculations given by Dr H. Elffers in his report of May 29, 2002, entail that

it must be considered extremely improbable that the suspect experienced all incidents mentioned in the indictment coincidentally.

These calculations consequently show that

it is highly probable that there is a connection between the presence of the suspect and the occurrence of an incident.”

Quoted from (Meester et al., 2007, p. 241)

The Court's Reasoning

*It is improbable
— 1 in 342 million — that
the suspect experienced all
incidents as a result of
coincidence*

$\Pr(\text{statistics} \mid \text{coincidence})$
low "p-value"

*It is
improbable that
it was just a
coincidence*

$\Pr(\text{coincidence} \mid \text{statistics})$

*It is
probable that it
was not a
coincidence*



What Would Be the
Right Reasoning?

*Strength of evidence – depends
on $Pr(\text{statistics} \mid \text{coincidence})$*

*Prior probability –
i.e. $\Pr(\text{coincidence})$*

*Strength of evidence – depends
on $\Pr(\text{statistics} \mid \text{coincidence})$*

Bayes' Theorem:

Prior \times Strength of evidence = Posterior

*Prior probability –
i.e. $Pr(\text{coincidence})$*

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*Posterior probability – i.e.
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Upshot: If the prior probability of “*coincidence*” is high, its posterior might still be relatively high even if the evidence against “*coincidence*” is strong

Why Should We Be Worried About the Prosecutor's Fallacy?

Interpretation of Statistical Evidence in Criminal Trials

The Prosecutor's Fallacy and the Defense Attorney's Fallacy*

William C. Thompson† and Edward L. Schumann†

INTERPRETATION OF STATISTICAL EVIDENCE

171

The prosecutor's misguided judgmental strategy (which we shall call the Prosecutor's Fallacy) could lead to serious error, particularly where the other evidence in the case is weak and therefore the prior probability of guilt is low. Suppose, for example, that one initially estimates the suspect's probability of guilt to be only .20, but then receives additional evidence showing that the defendant and perpetrator match on a blood type found in 10% of the population. According to Bayes theorem, this new evidence should increase one's subjective probability of guilt to .71, not .90.³

Crime

suspects (Saferstein, 1977; Schroeder, 1977; Giannelli, 1983). Laboratory tests

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Second Example

DNA Evidence



DNA Evidence

- ❖ Traces are found on the crime scene



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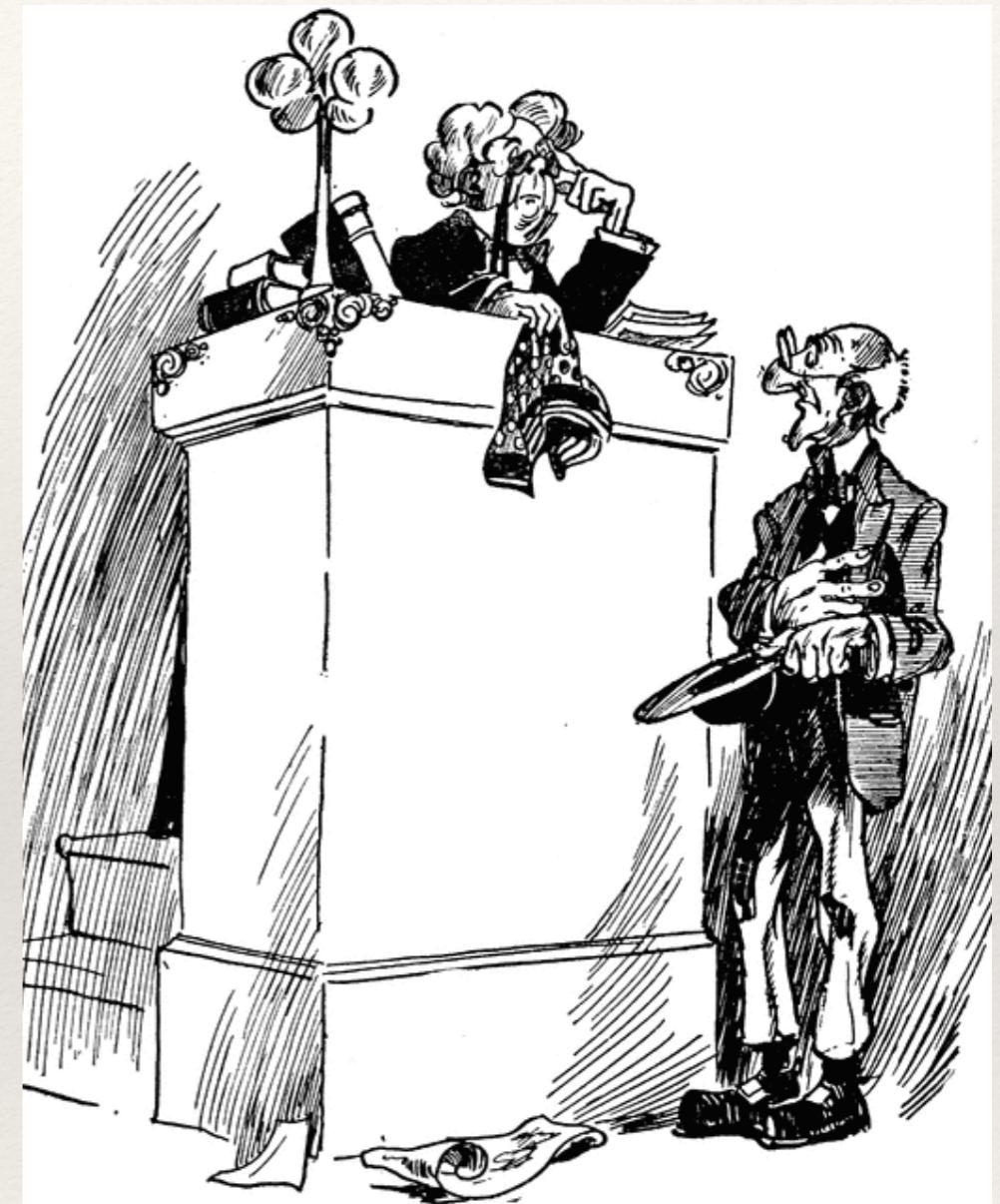
$Pr(\text{Innocence} \mid \text{Match})$
depends on suspect poll



Judge Hardwick - Missouri Ct. App.

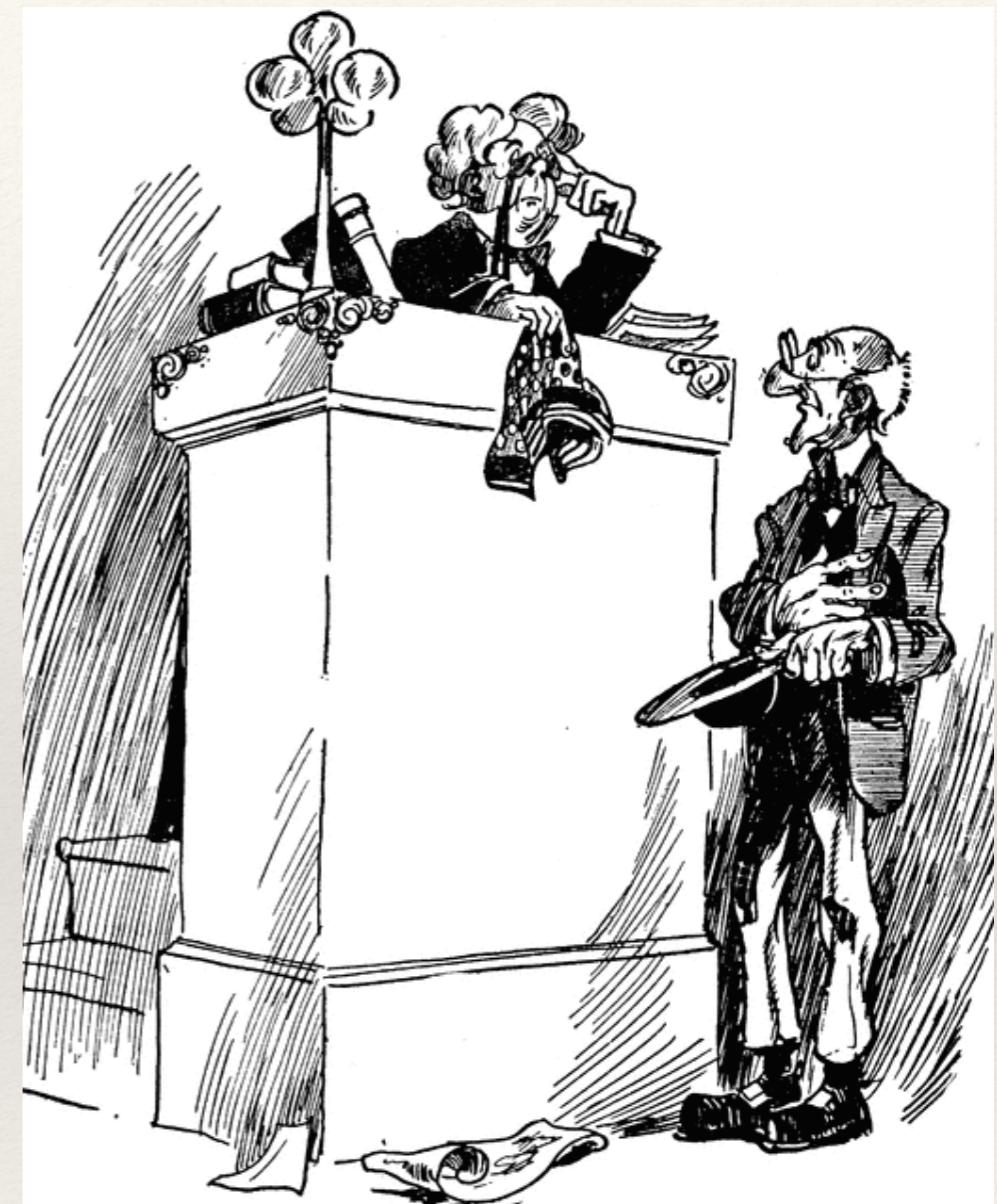
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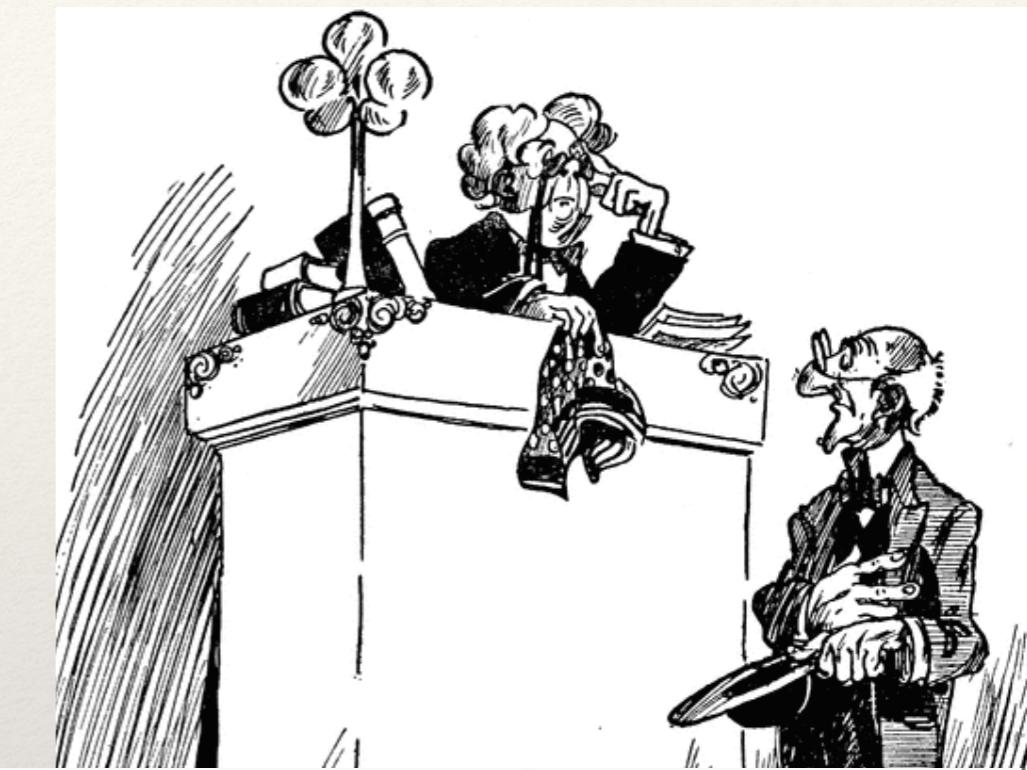
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Which probability was the judge referring to?

$Pr(\text{Match} \mid \text{Innocence})$

$Pr(\text{Innocence} \mid \text{Match})$

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low $Pr(\text{Innocence} \mid Evidence)$

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Why Is
High $Pr(Guilt | Evidence)$
or
Low $Pr(Ignocence | Evidence)$
the Wrong Guide?

First Problem:

How Should the Guilt Priors be Estimated?

Geographical Information

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Proposal 1

$Pr(Guilt)$ should incorporate geographical data.

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This is unfair. Uneven burdens of proof would be applied to defendants.

Suspect Population

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Proposal 2

$Pr(Guilt)=1/n$, where n is suspect population.

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The proposals lacks accuracy — $Pr(Guilt)$ is going to be higher or lower depending on the choice of n

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Proposal 1 based on
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But it seems unfair.

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Proposal 2 — that is, $1/n$ —
is fair because its priors are
equal across defendants.

But it seems inaccurate.

Second Problem:

How Is the Risk of Mistaken Conviction
for Innocent Defendants Kept Low?

Guilt Probability v. Risk of Mistaken Conviction

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How probable is the defendant's guilt given the evidence?

$$Pr(Guilt | Evidence)$$

guilt probability
on the evidence

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How probable is the conviction of an innocent defendant?

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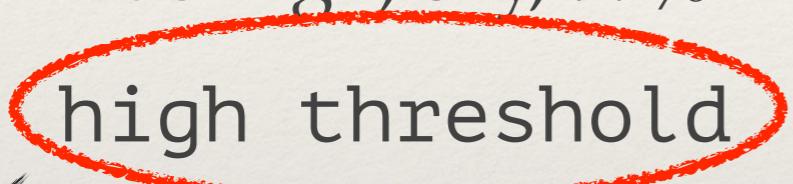
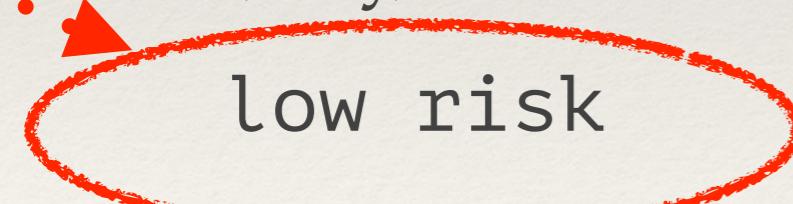
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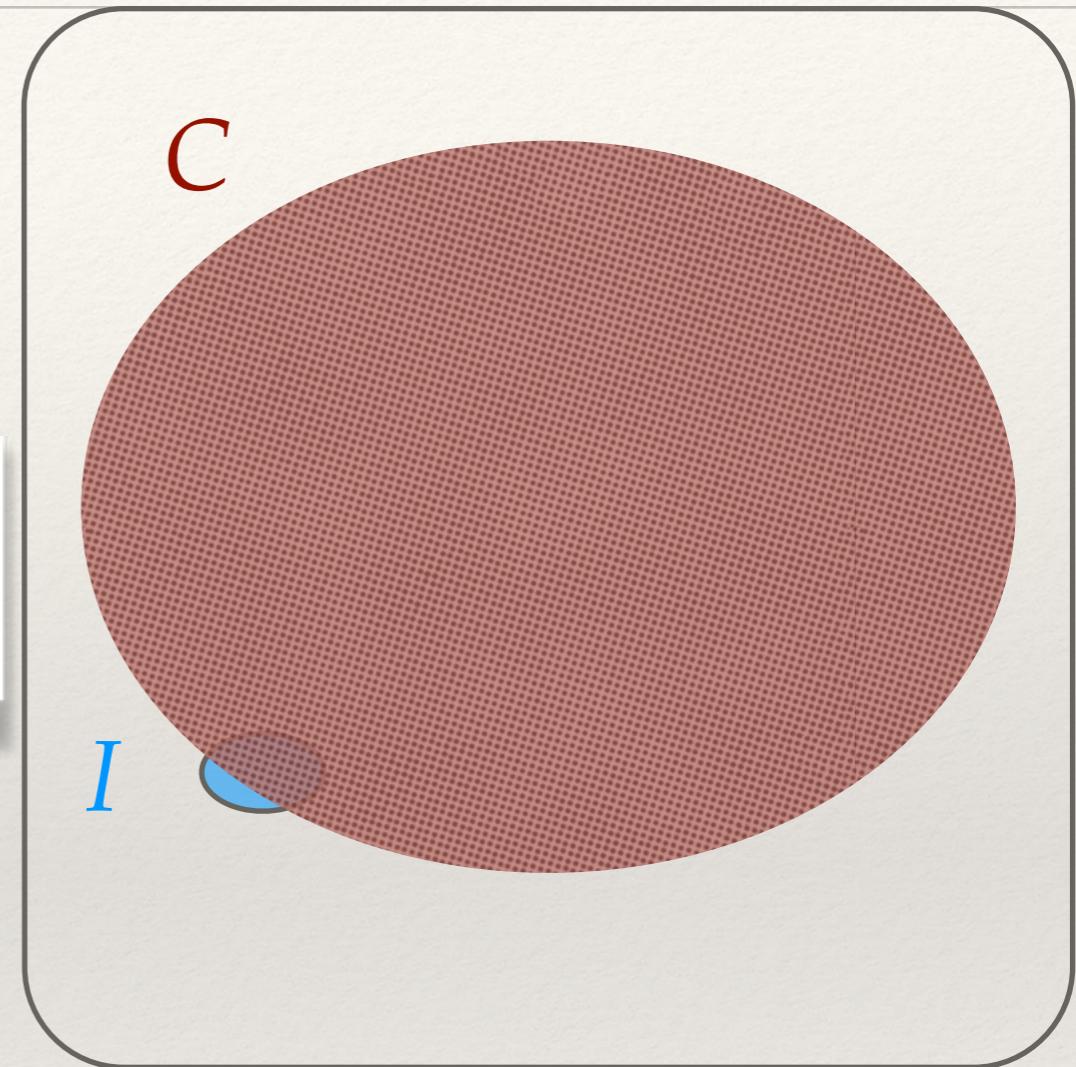
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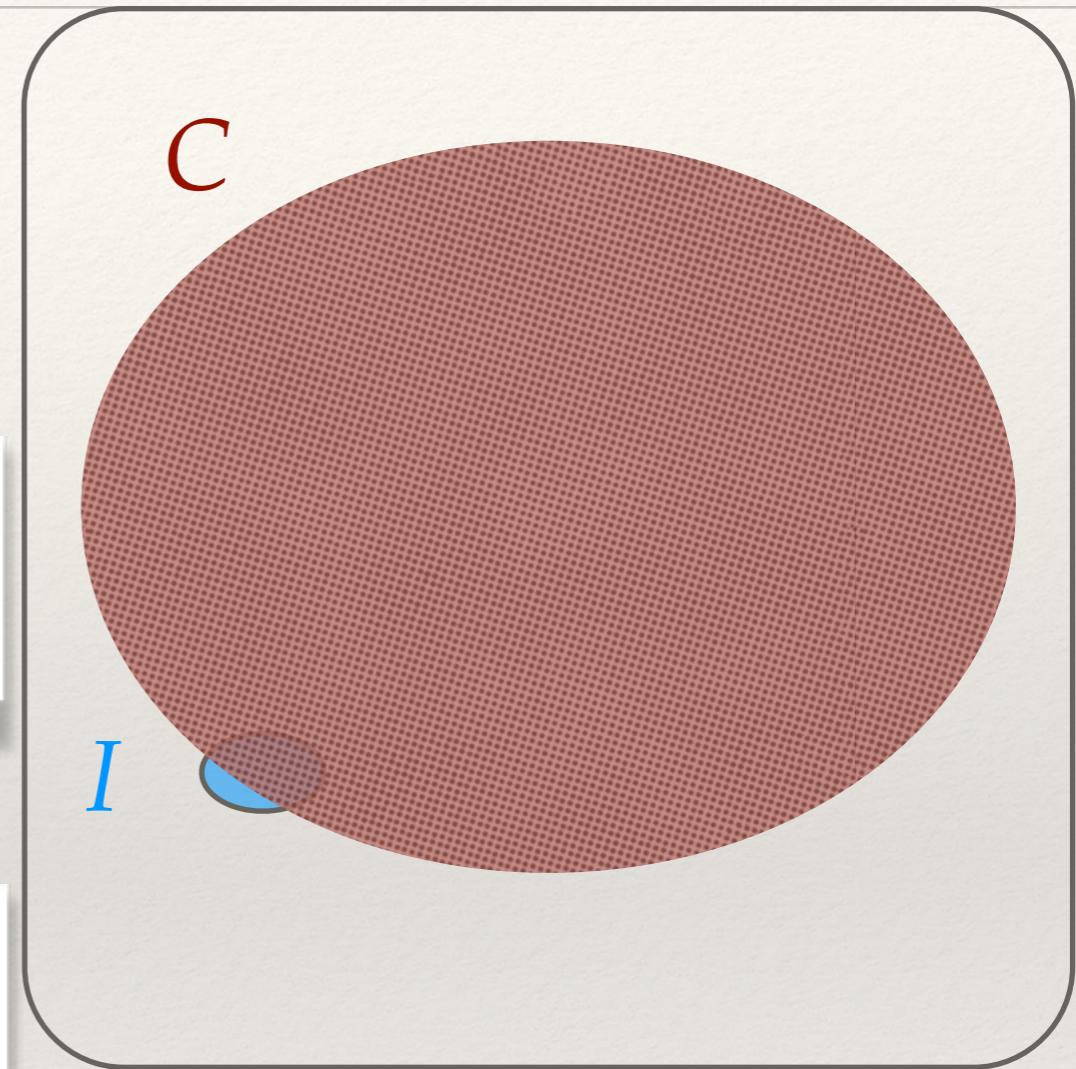
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What Is the Alternative to a Decision Rule Based on $Pr(Guilt | Evidence)$?

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What about focusing instead on the “p-value” $Pr(Evidence | Innocence)$ and requiring it to be suitably low?

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Is this proposal any better than a suitably high guilt probability threshold?

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What About Lucia de Berk?

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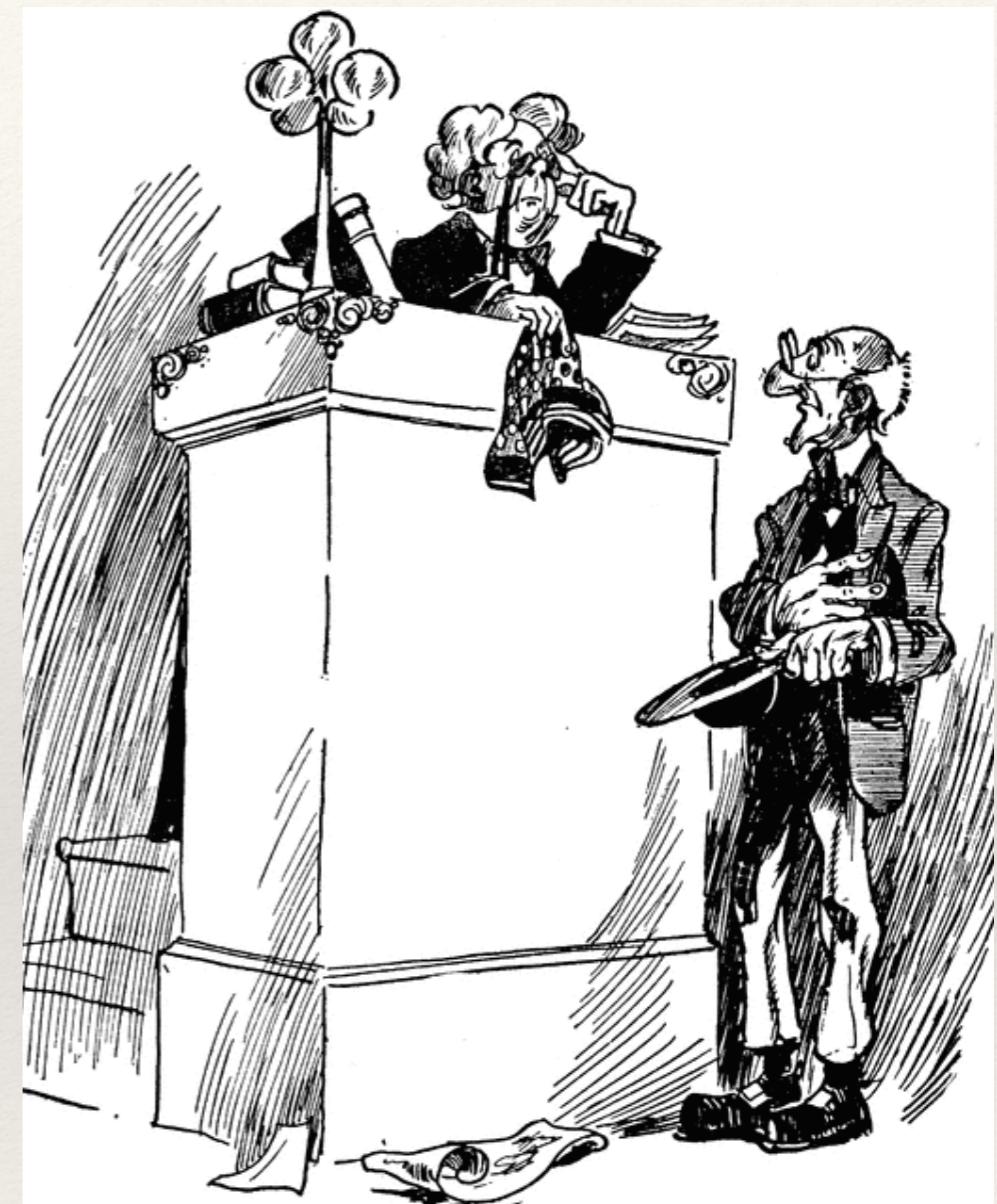
Lucia should be convicted

Thank you!

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... but if we accept that a low $Pr(\text{Match} \mid \text{Innocence})$ is the guiding principle, her conclusion is still correct.

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My claim: If $\Pr(\text{Statistics} \mid \text{Coincidence})$ is
suitably low and “Coincidence” = “Innocence”,
then Lucia should be convicted.

Why Low $Pr(Evidence | Innocence)$
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$$Pr(C | I \& Ev) * Pr(Ev | I)$$

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=

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NB: The equivalence holds given
only one type of evidence
available, say, DNA evidence which
could be a positive or negative match.

If there are more pieces of evidence,
the calculations are more complicated.

$Pr(Conviction | Innocence)$

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What About the Likelihood Ratio?

Convict *if and only if*
the likelihood ratio
 $\Pr(Ev \mid G)/\Pr(Ev \mid I)$ is
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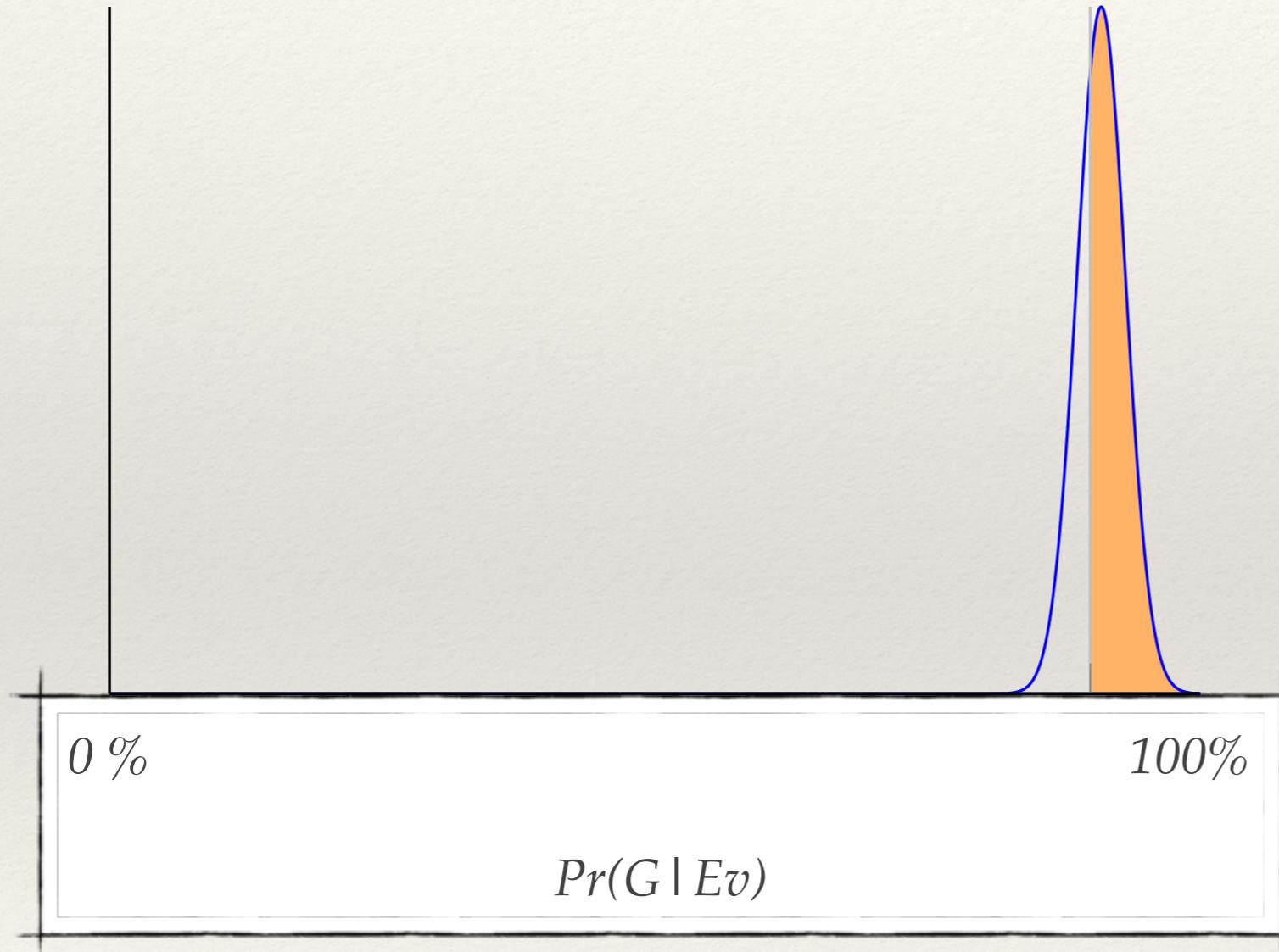
Convict *if and only if*
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If Ev does not appear very regularly given G , the likelihood ratio would be low. Still, there should be no reason to discard Ev as the basis for a conviction provided $Pr(Ev | I)$ is low and Ev is incriminating.

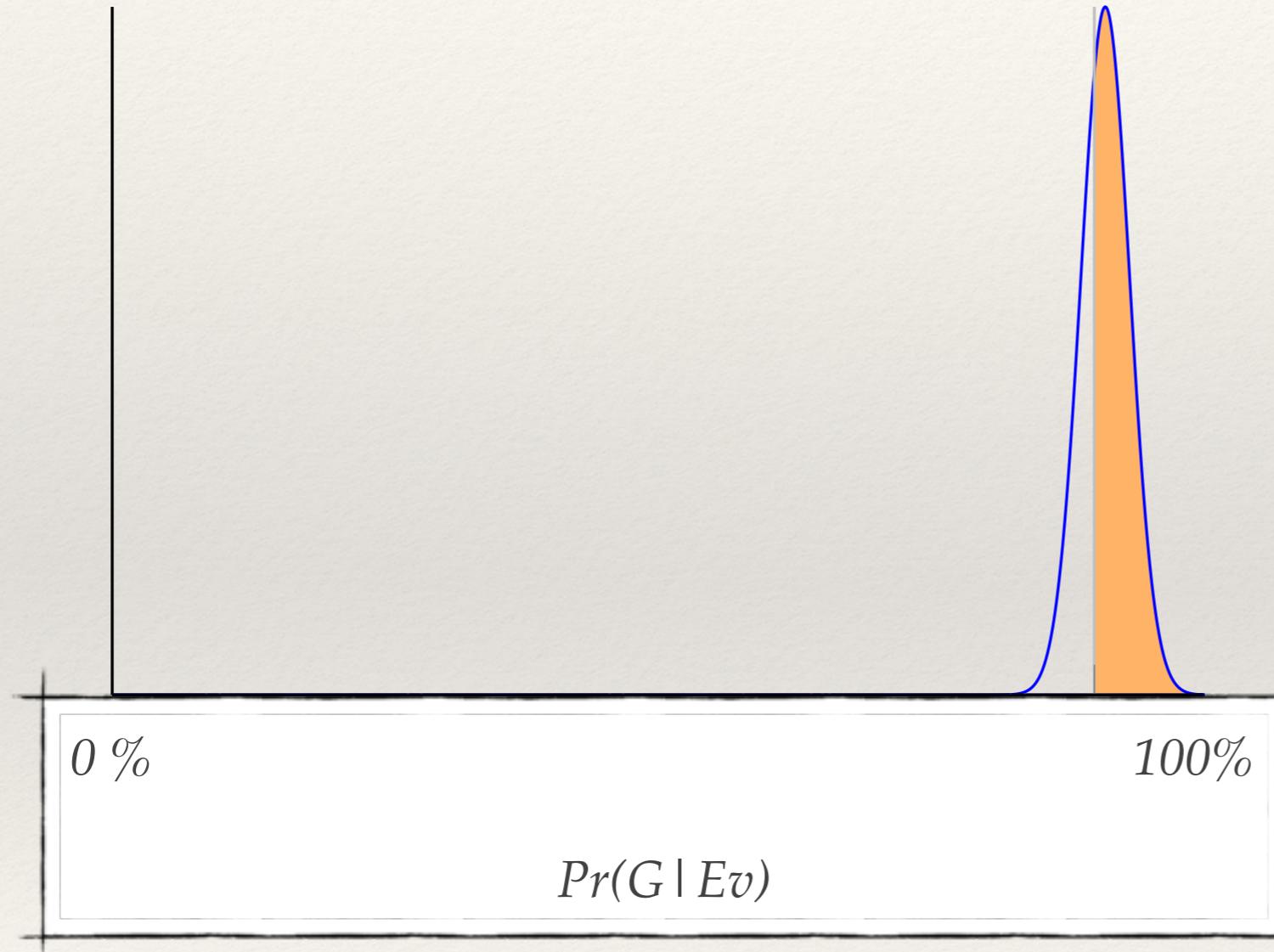
Likelihood ratio rule is more stringent than one based on $Pr(Ev | I)$.

Open question: does a high likelihood ratio guarantee low error risk $Pr(C | I)$?

High Threshold – High Risk



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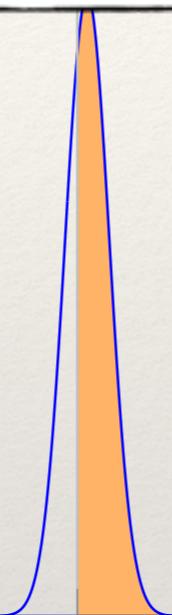


Given a certain distribution of innocent defendants (as in the graph), the risk of mistaken conviction could still be above 50% even with a high probability threshold

(1) Are The Two Decision Rules That Different?

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Convict iff $\Pr(G \mid Ev)$ is high



0%

100%

$\Pr(Guilt \mid Ev)$

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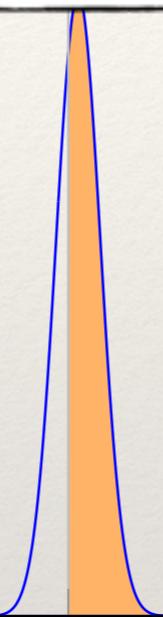
0%

$Pr(Ev \mid Innocence)$

(2) Are the Two Decision Rules That Different?

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0%

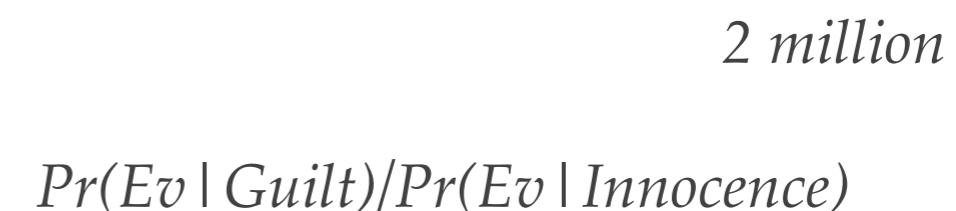
100%

$\Pr(Guilt \mid Ev)$

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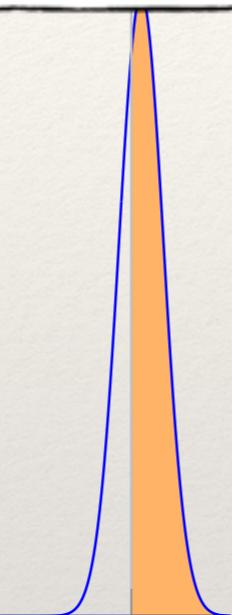
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0%

100%

$Pr(Guilt | Ev)$



1

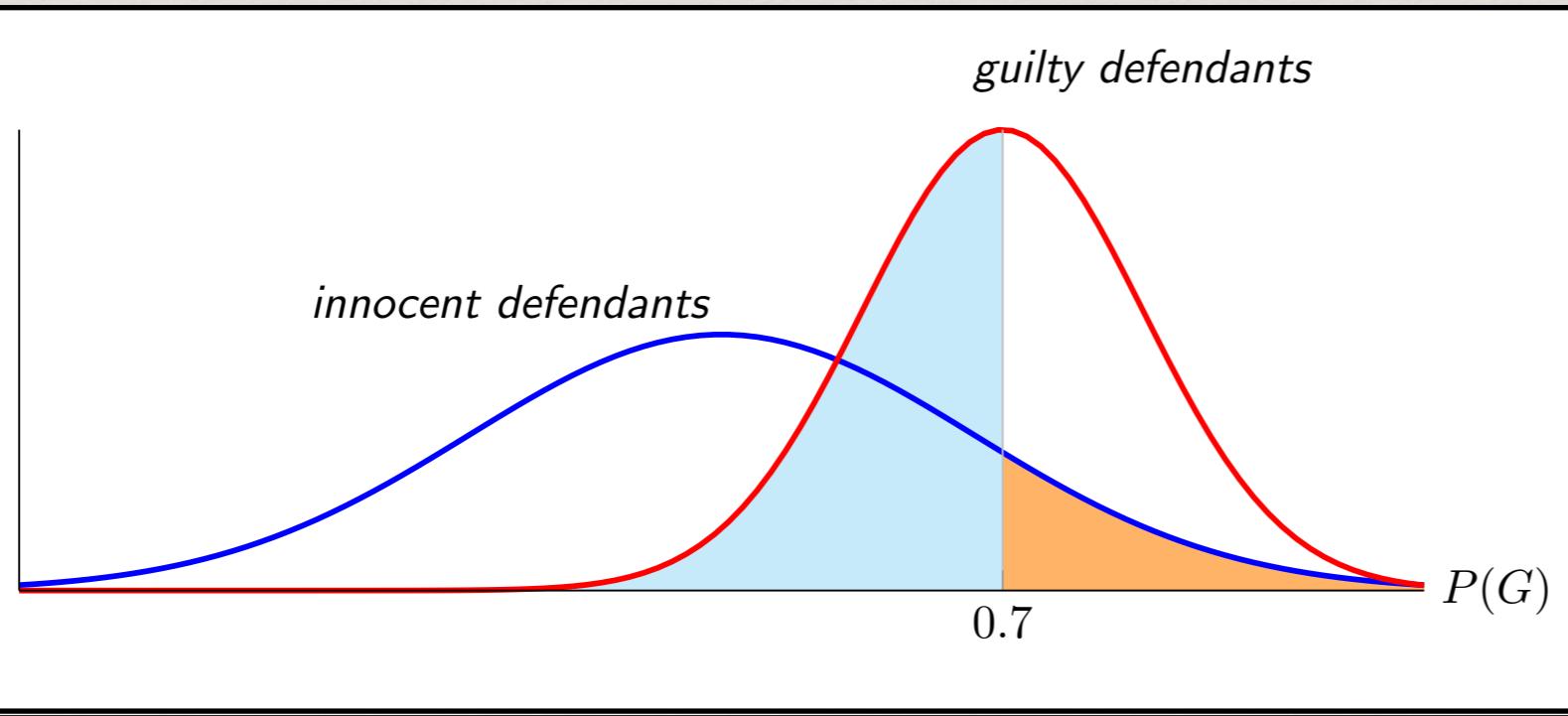
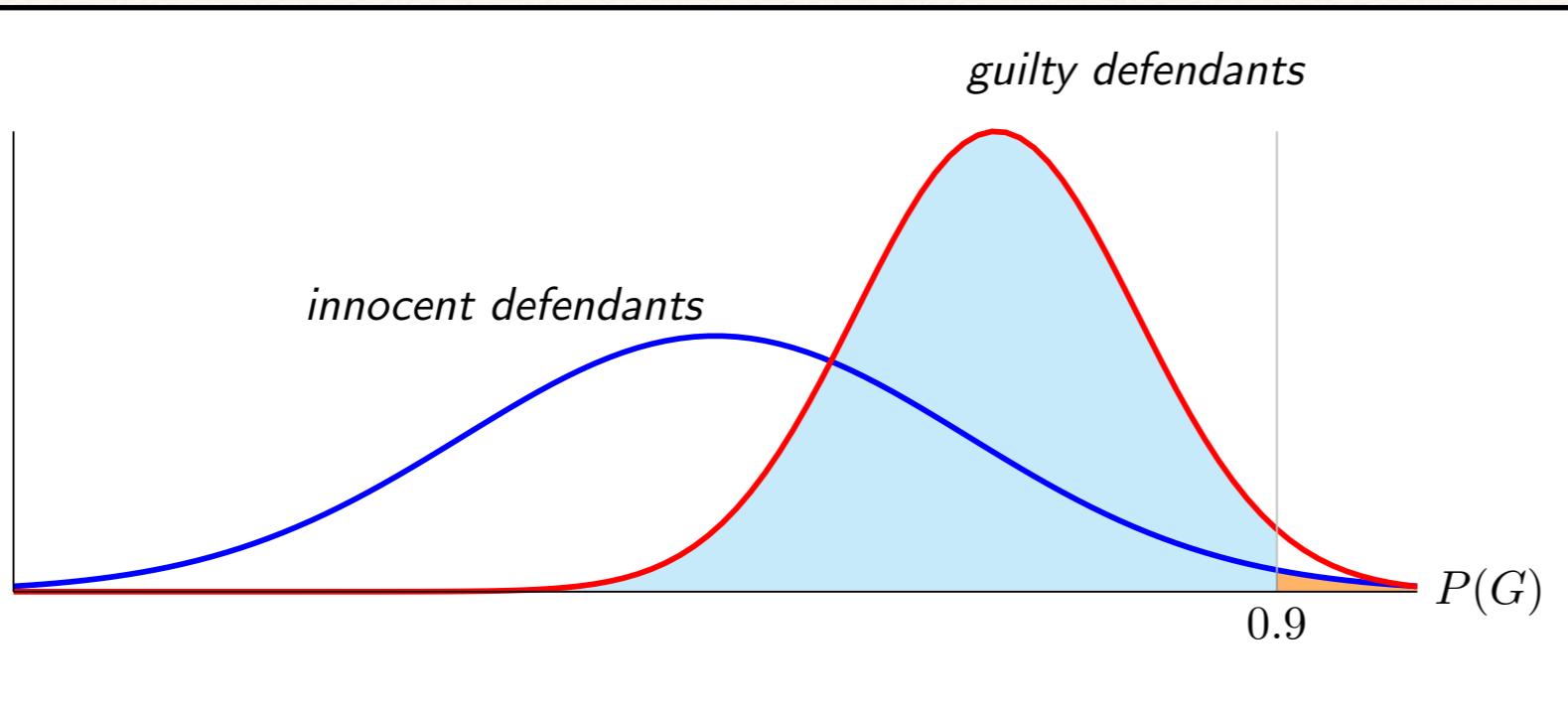
2 million

$Pr(Ev | Guilt)/Pr(Ev | Innocence)$

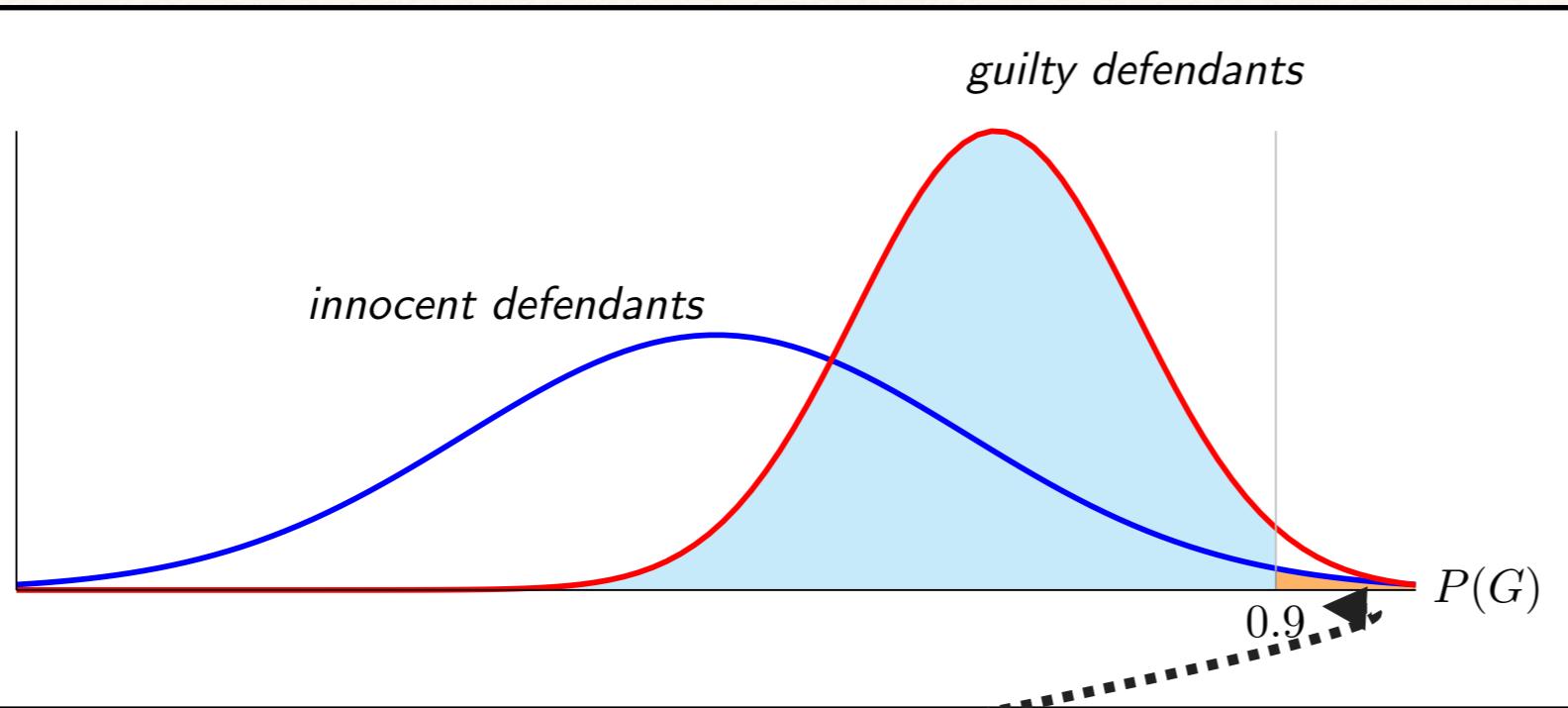
Upshot – Any threshold-based rule of decision is unable to keep the error risk low, regardless of whether the threshold applies to the **guilt probability**, the **p-value** or the **likelihood ratio**

Higher Threshold / Lower Risk - *Right?*

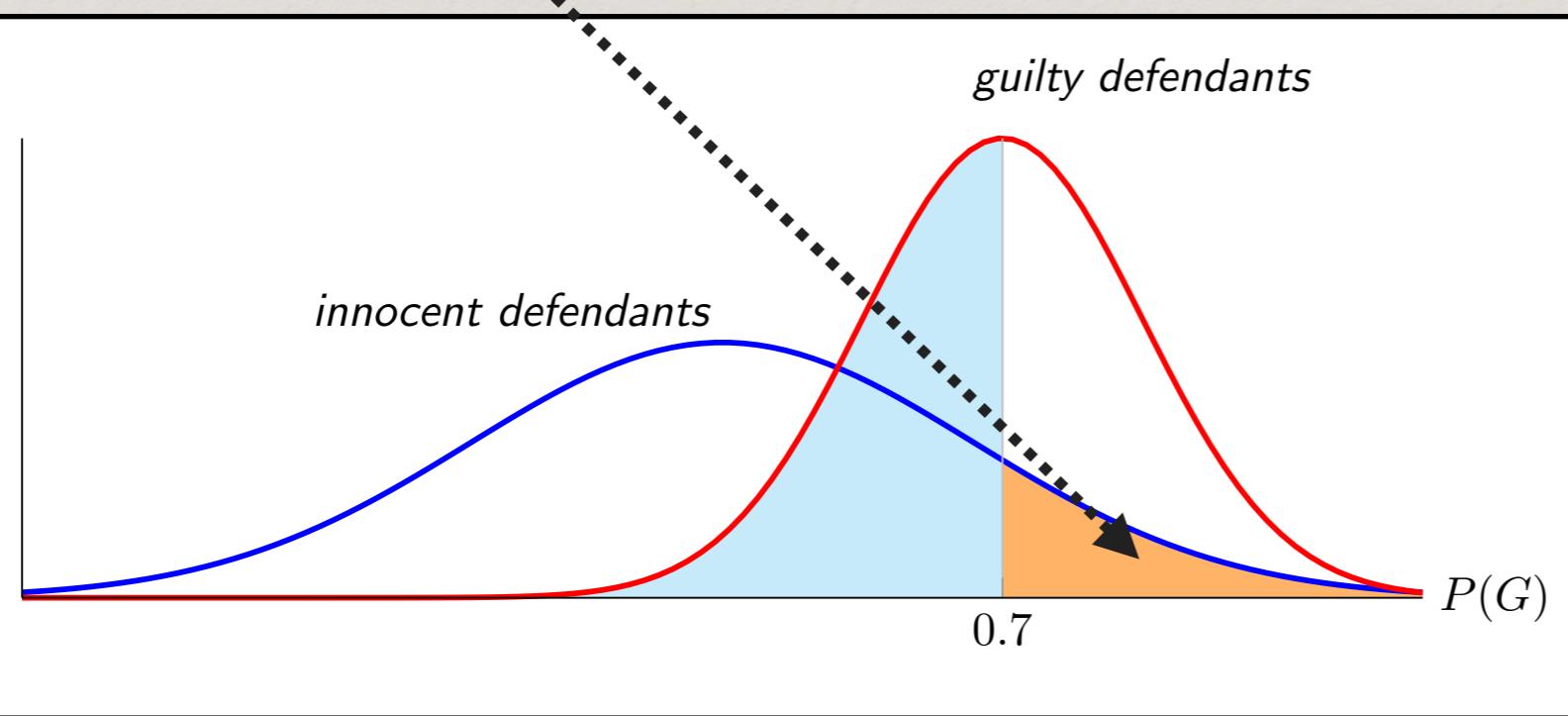
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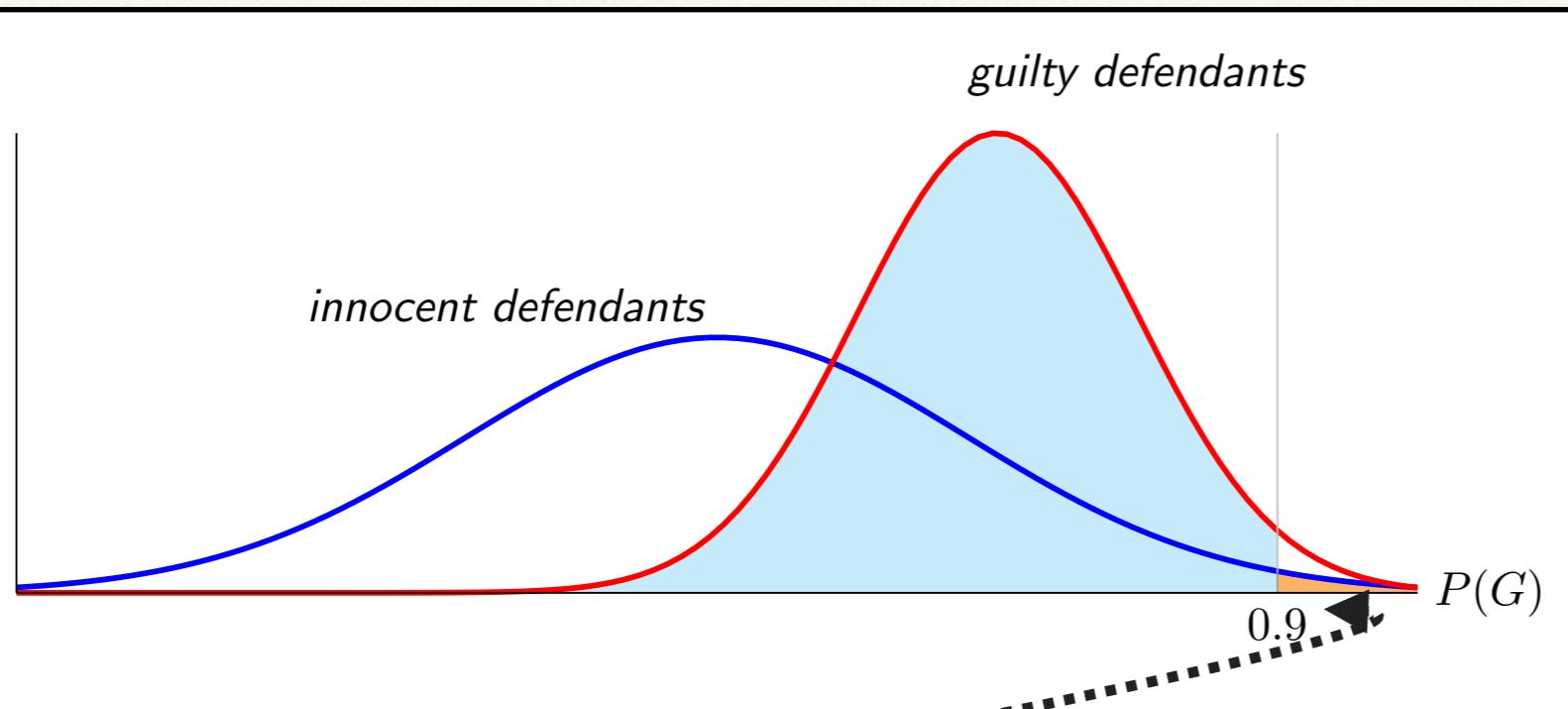
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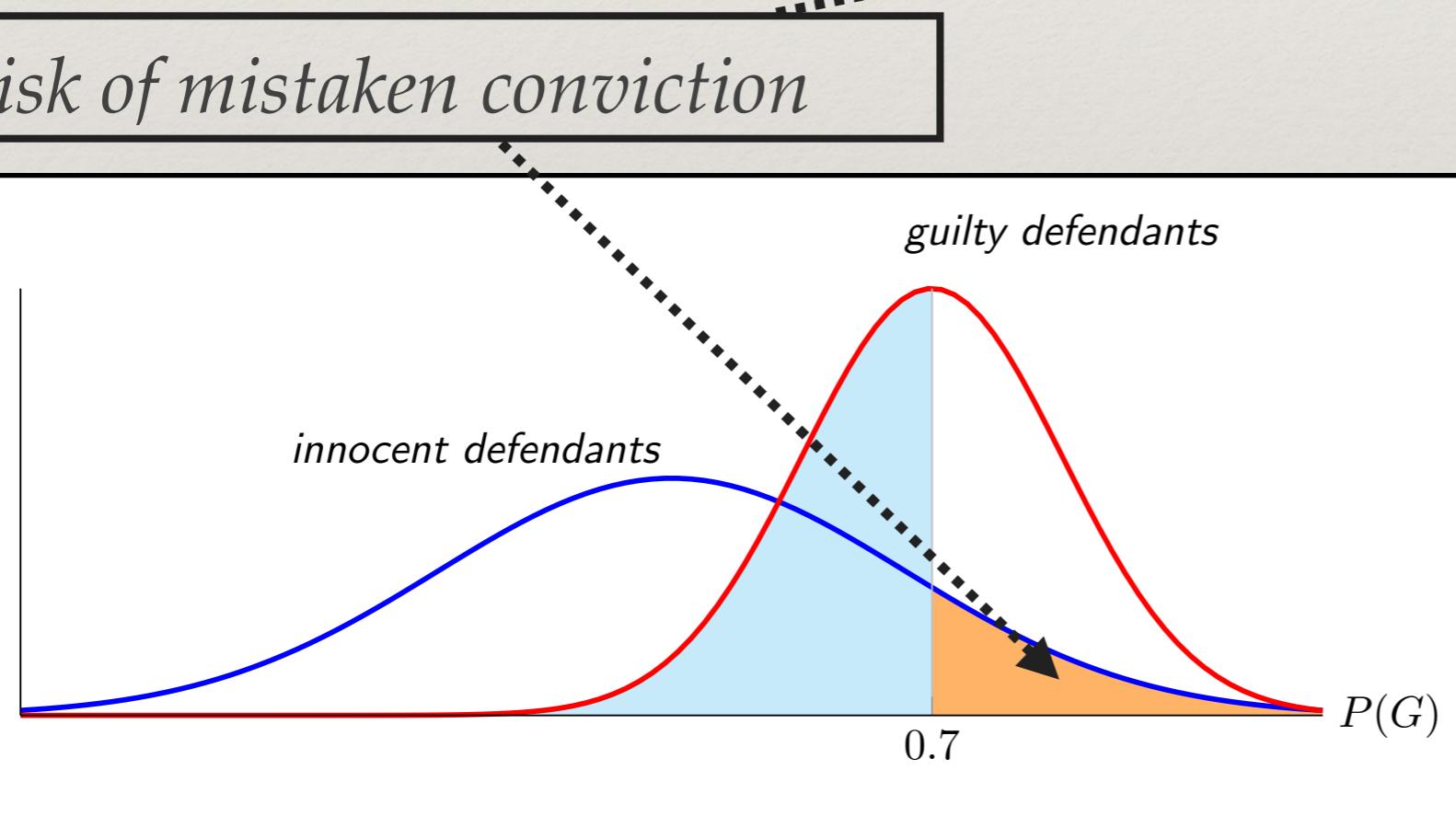
Risk of mistaken conviction



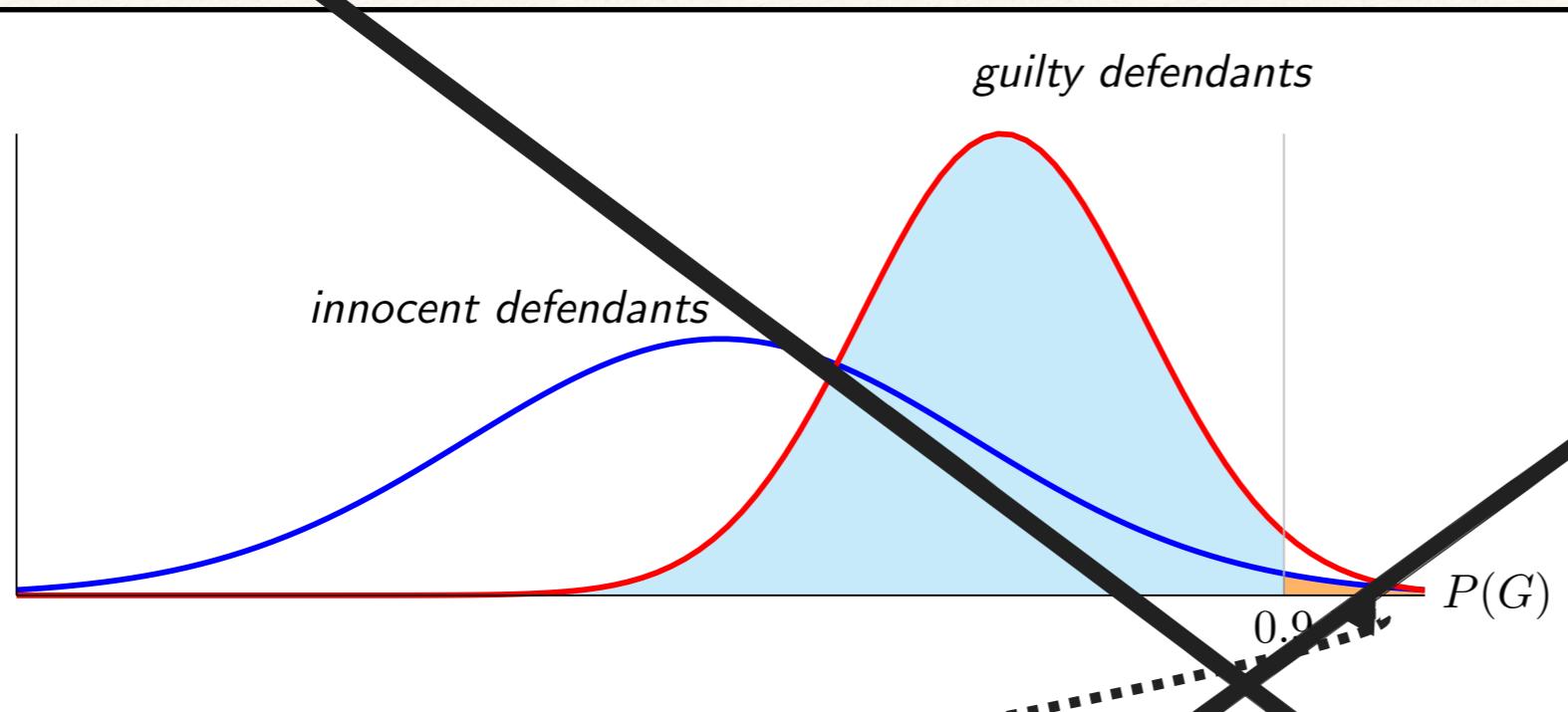
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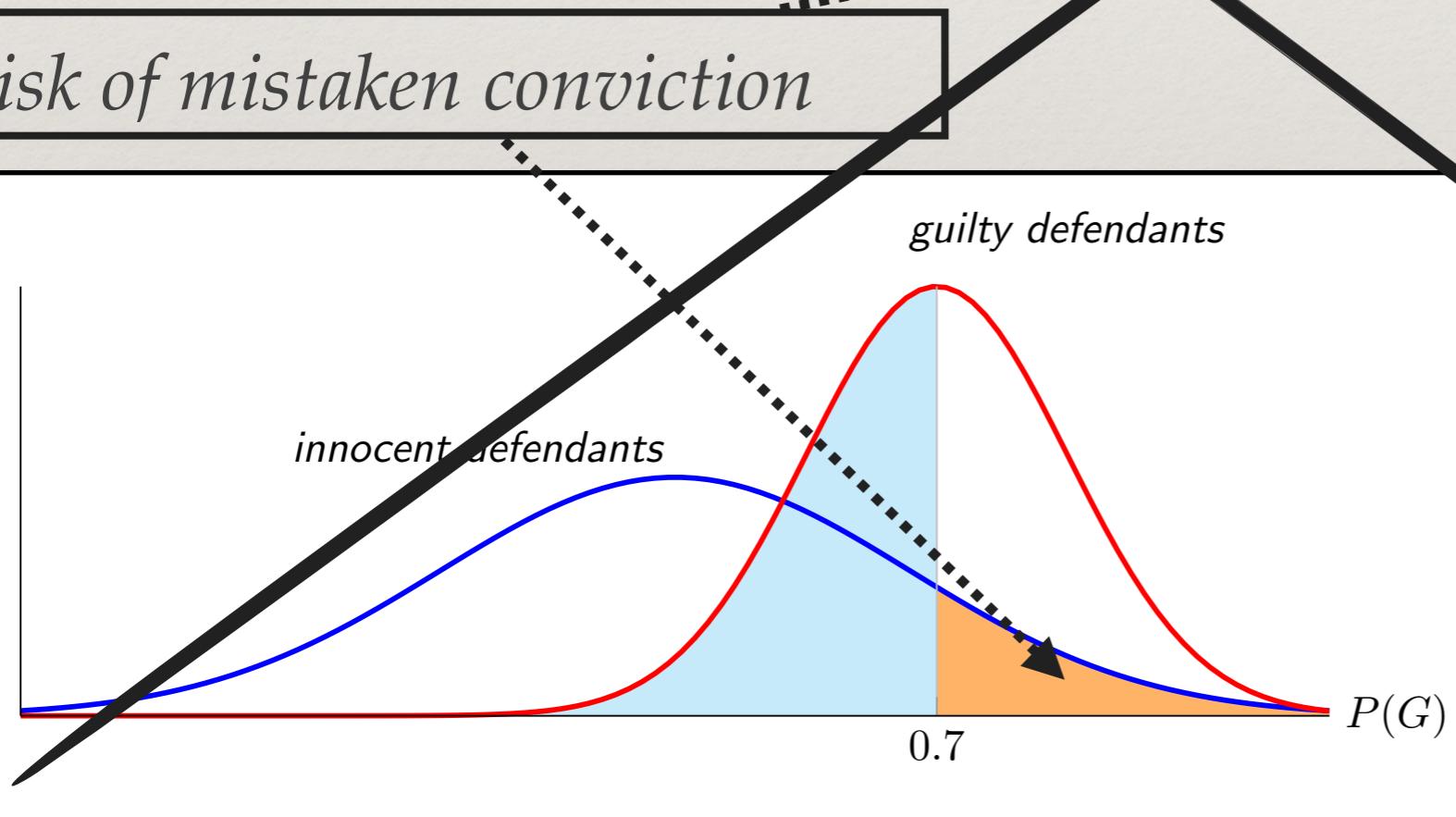
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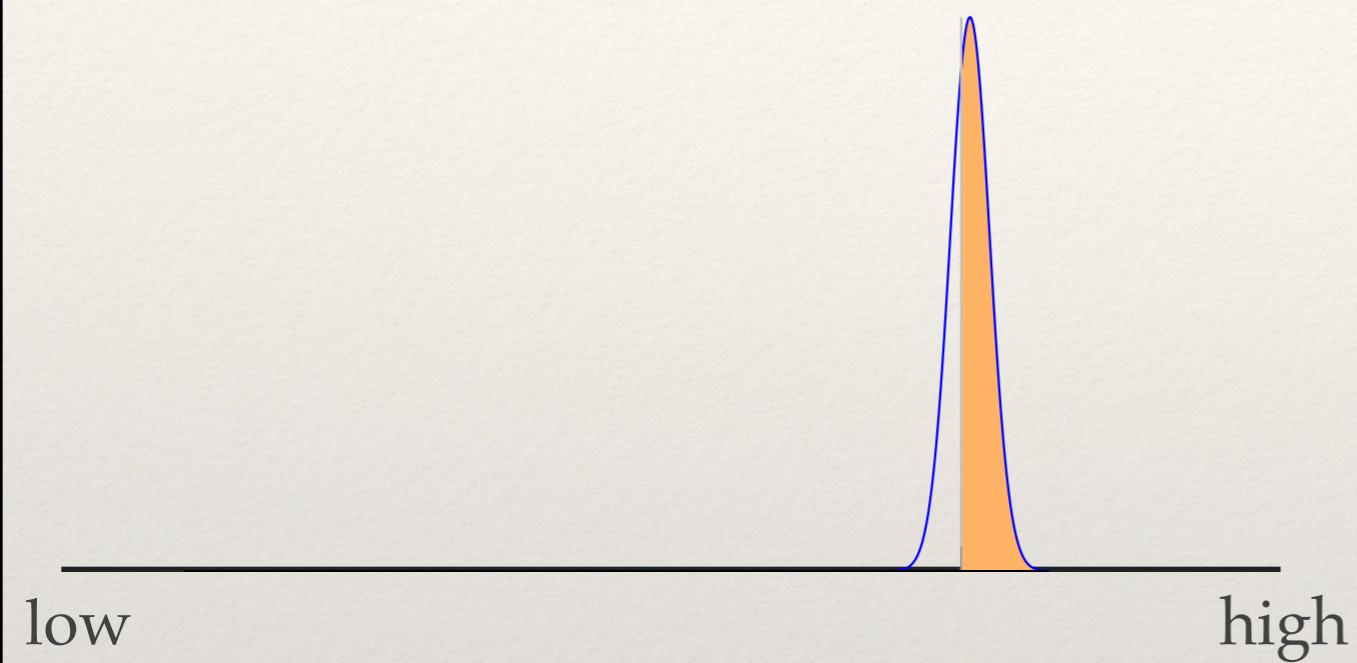


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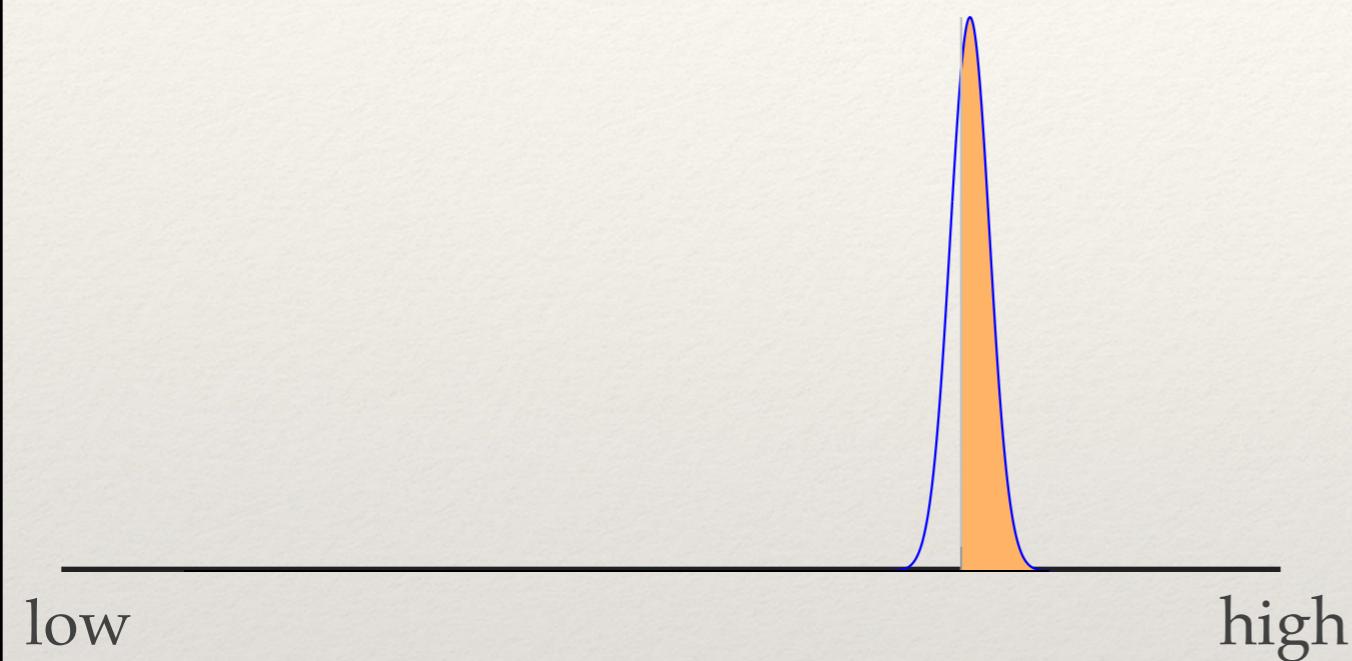


(1) Feedback Selection Mechanisms

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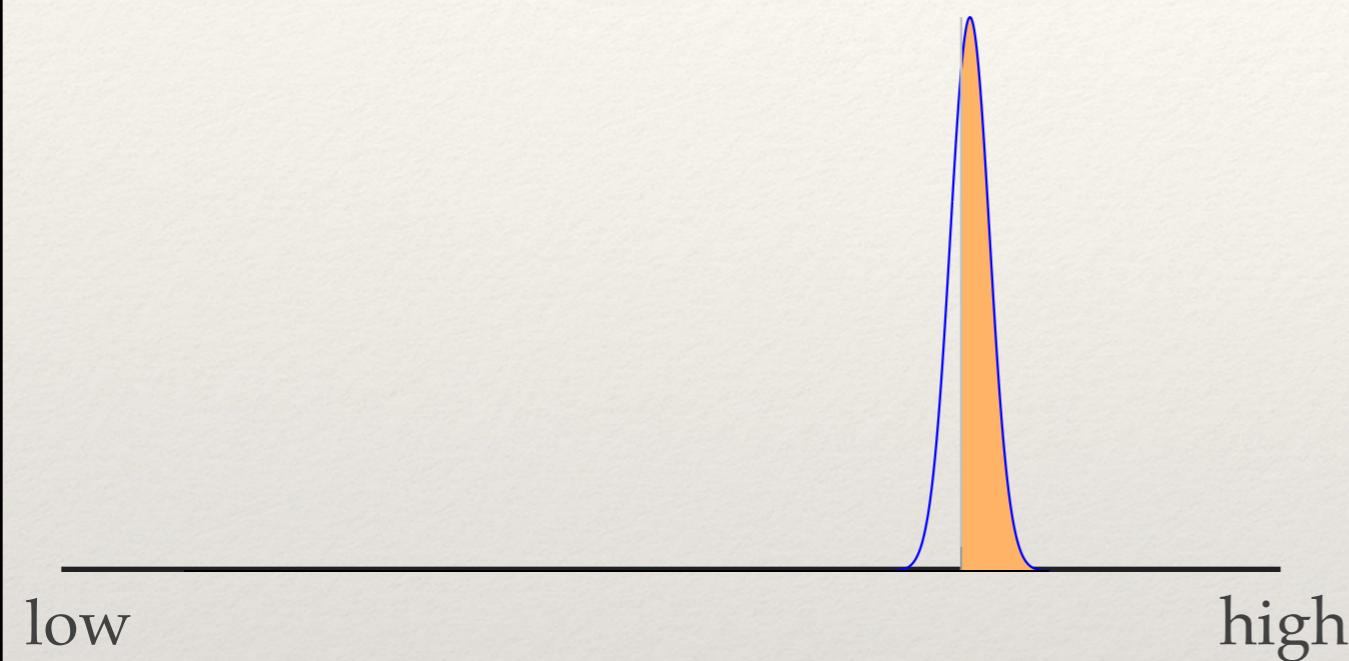


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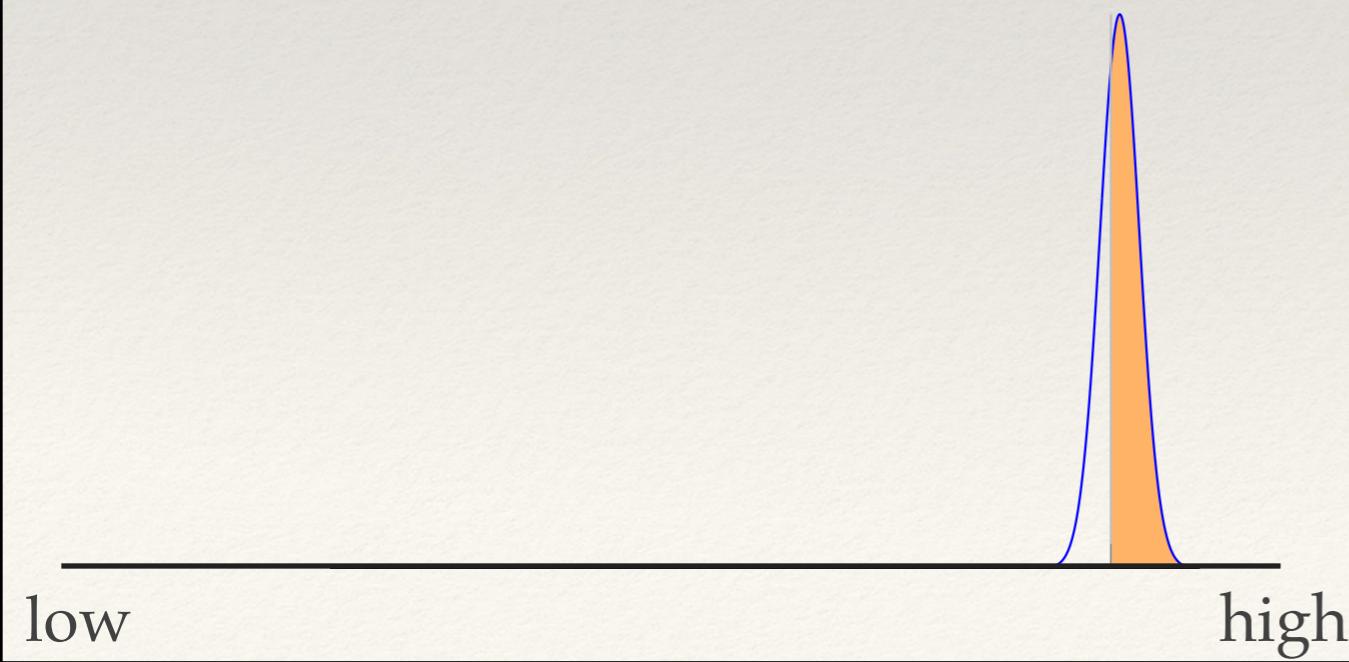


Given a higher threshold, prosecutors will bring to trial only those defendants who stand a good chance of being convicted (*feedback selection mechanisms*)

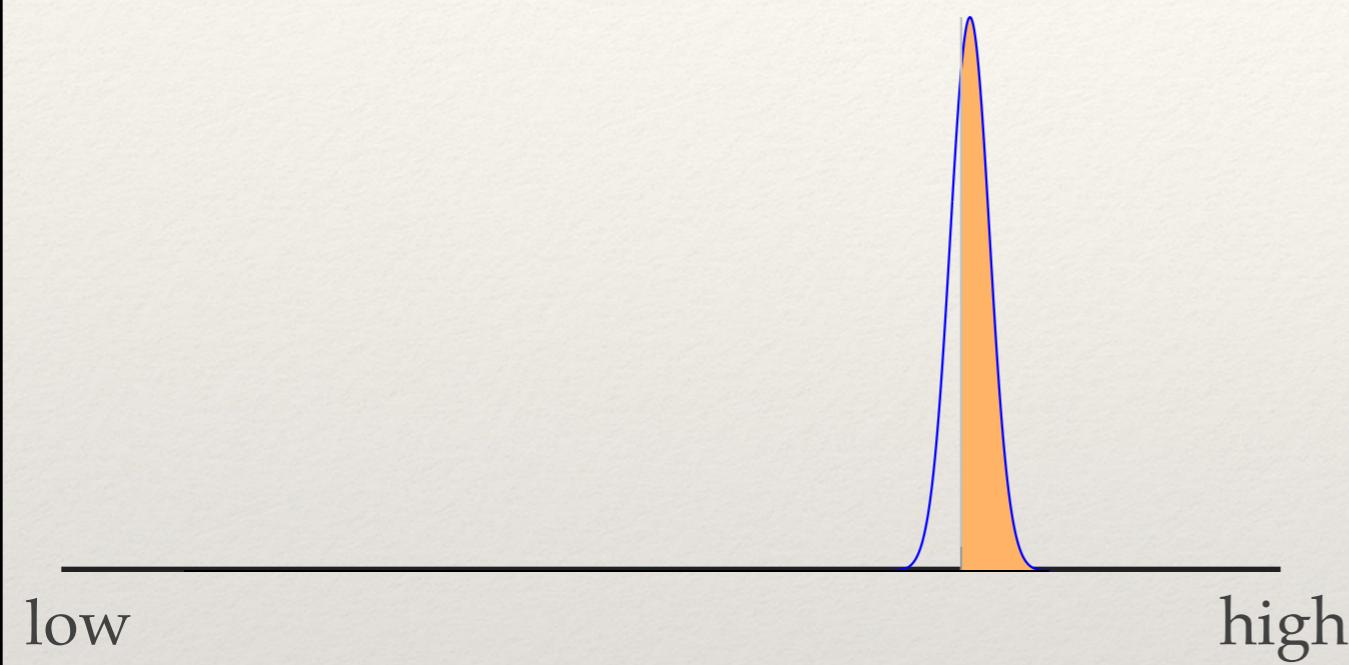
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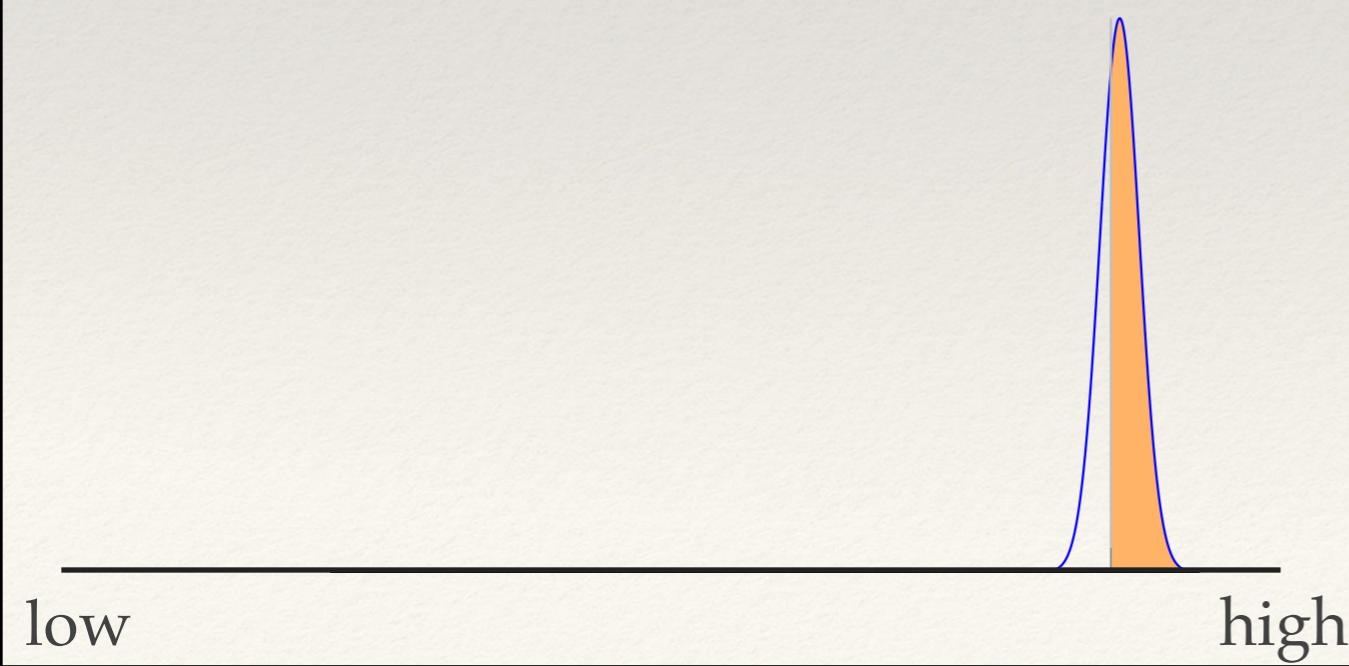
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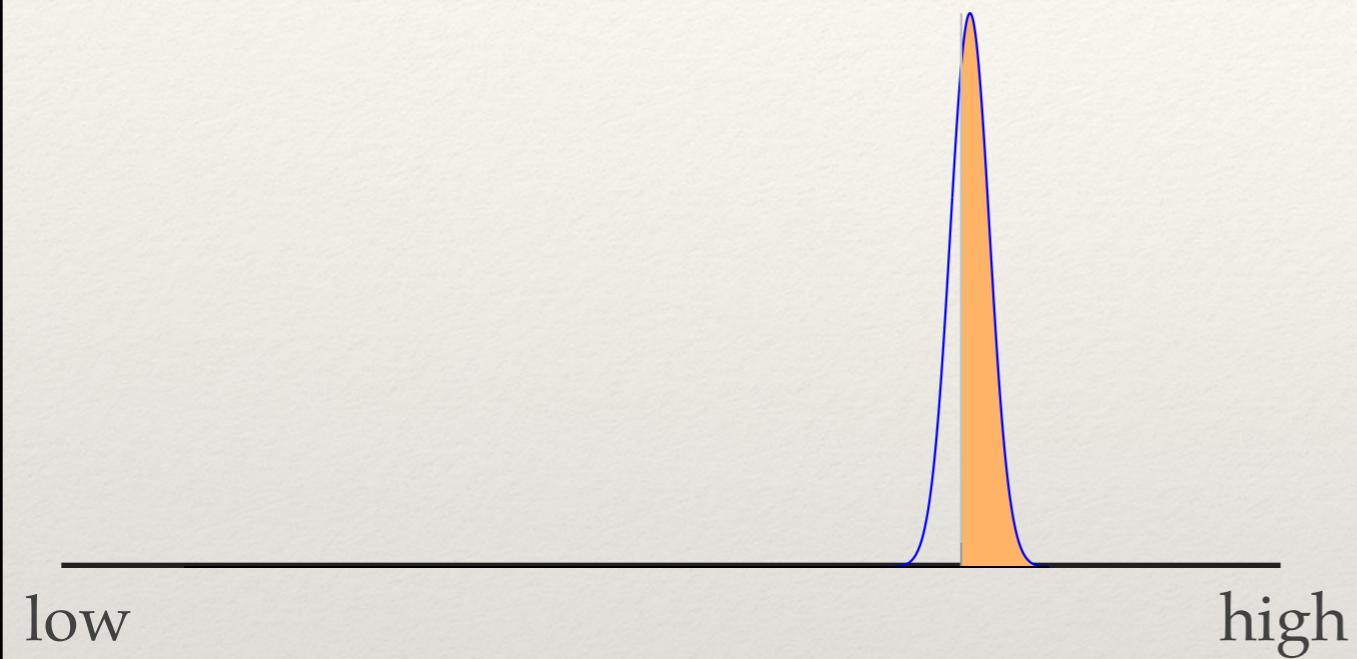
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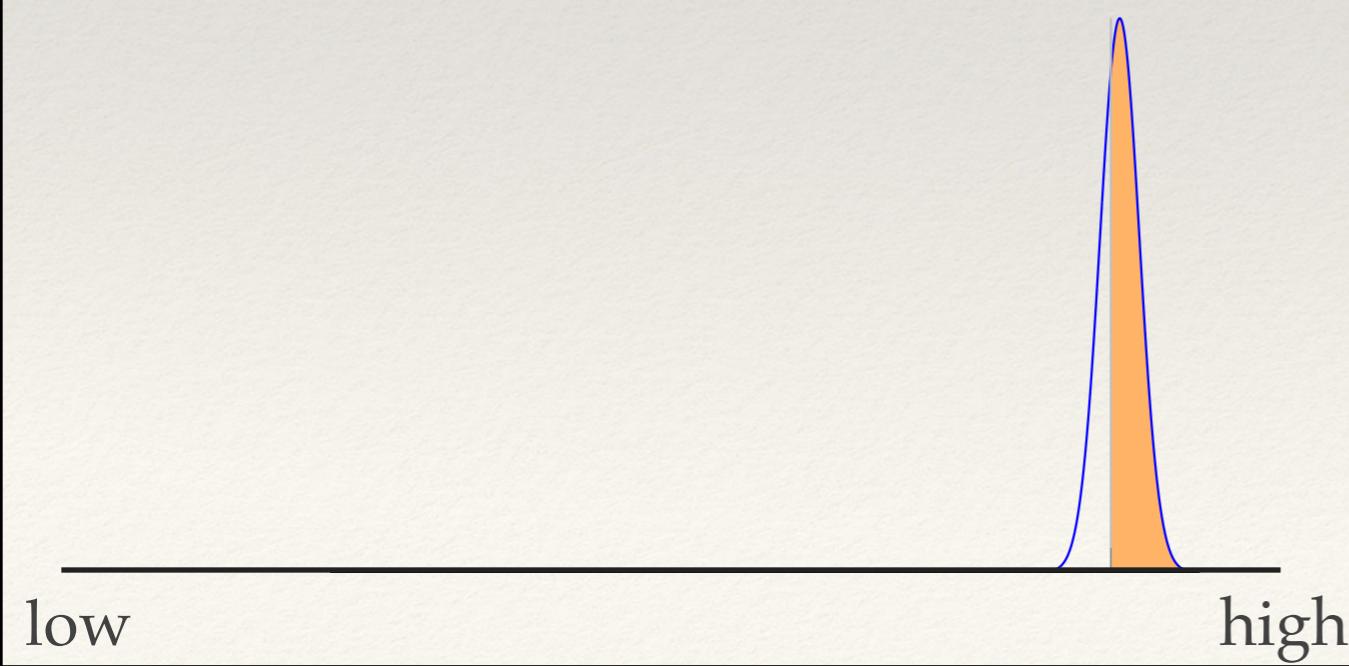
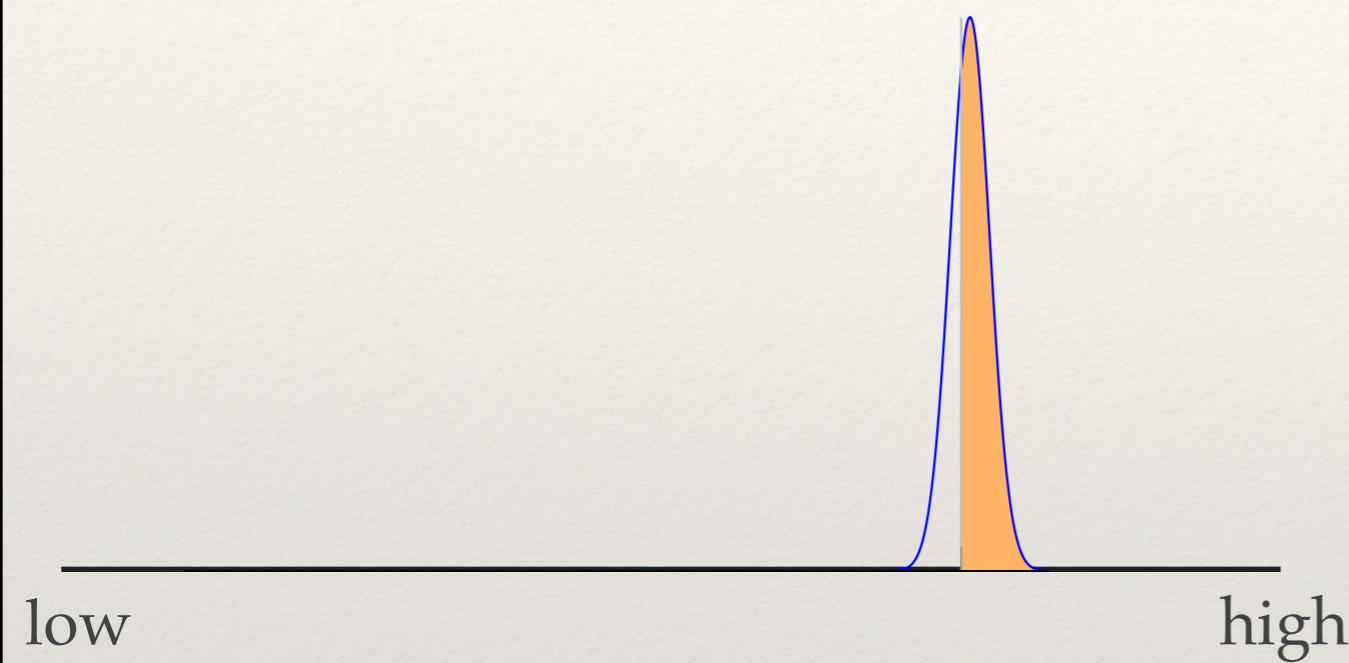
The distribution of innocent defendants will thus move along with the higher threshold, so the error risk will be constant

(2) Feedback Selection Mechanisms

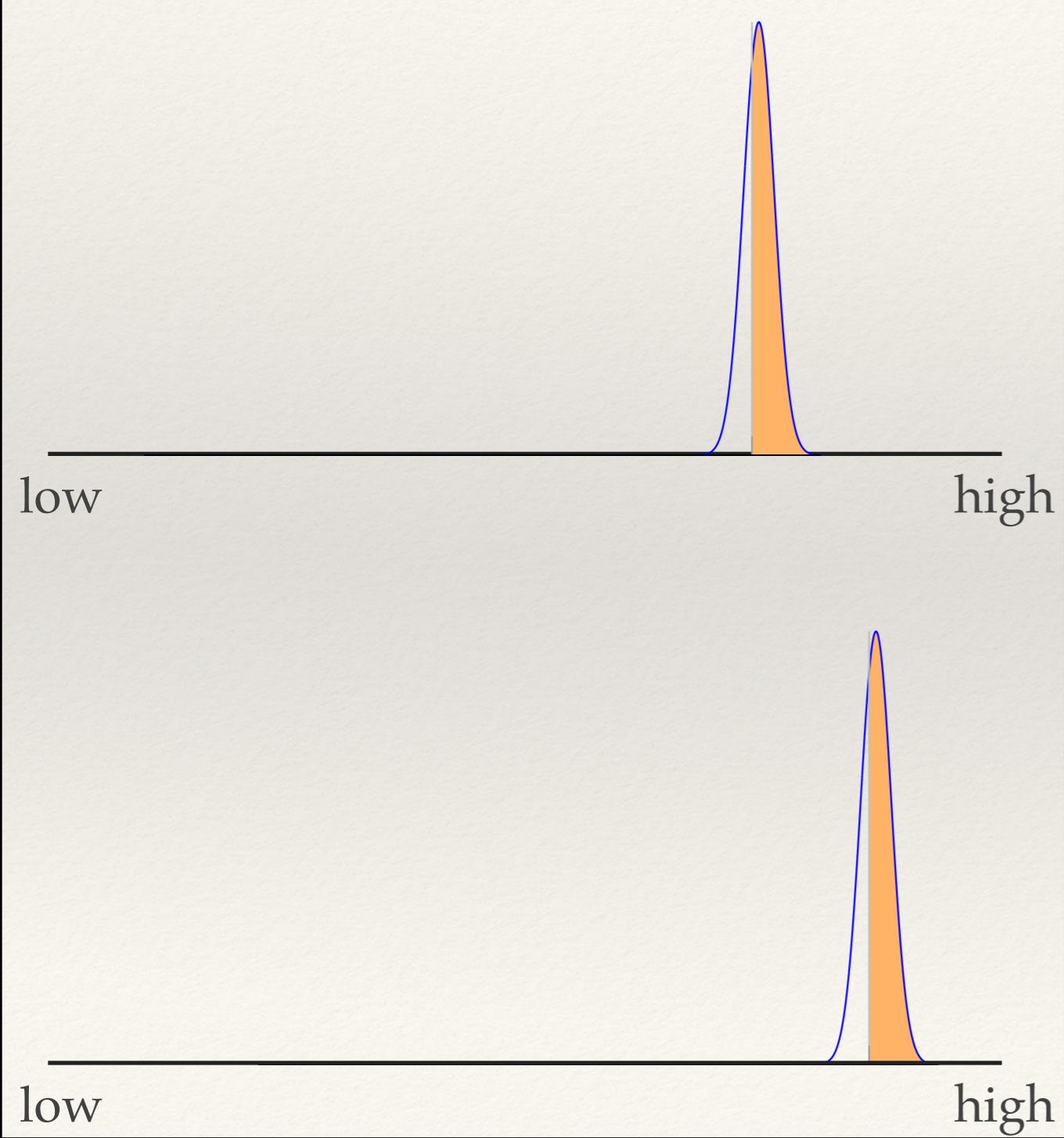
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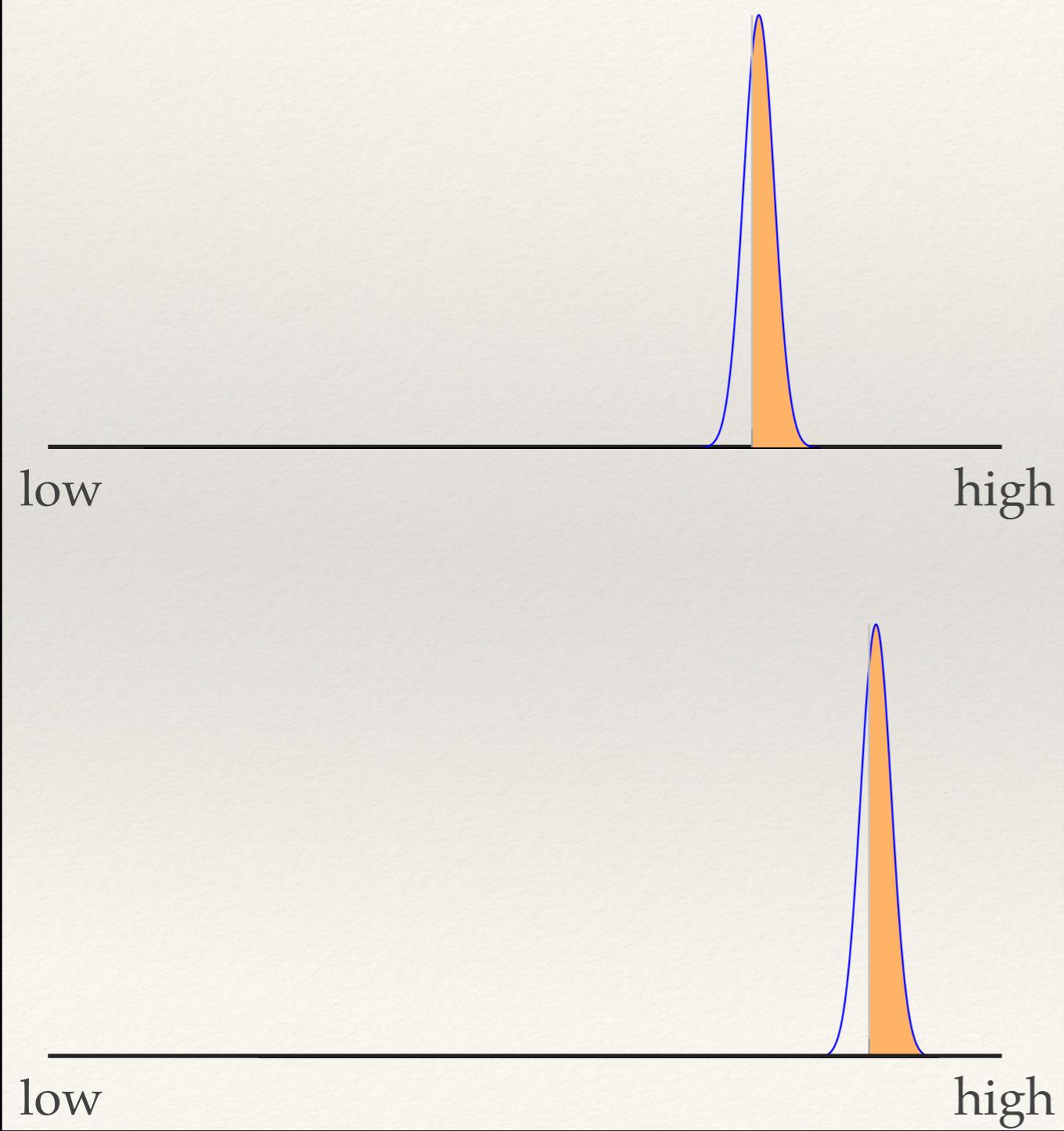


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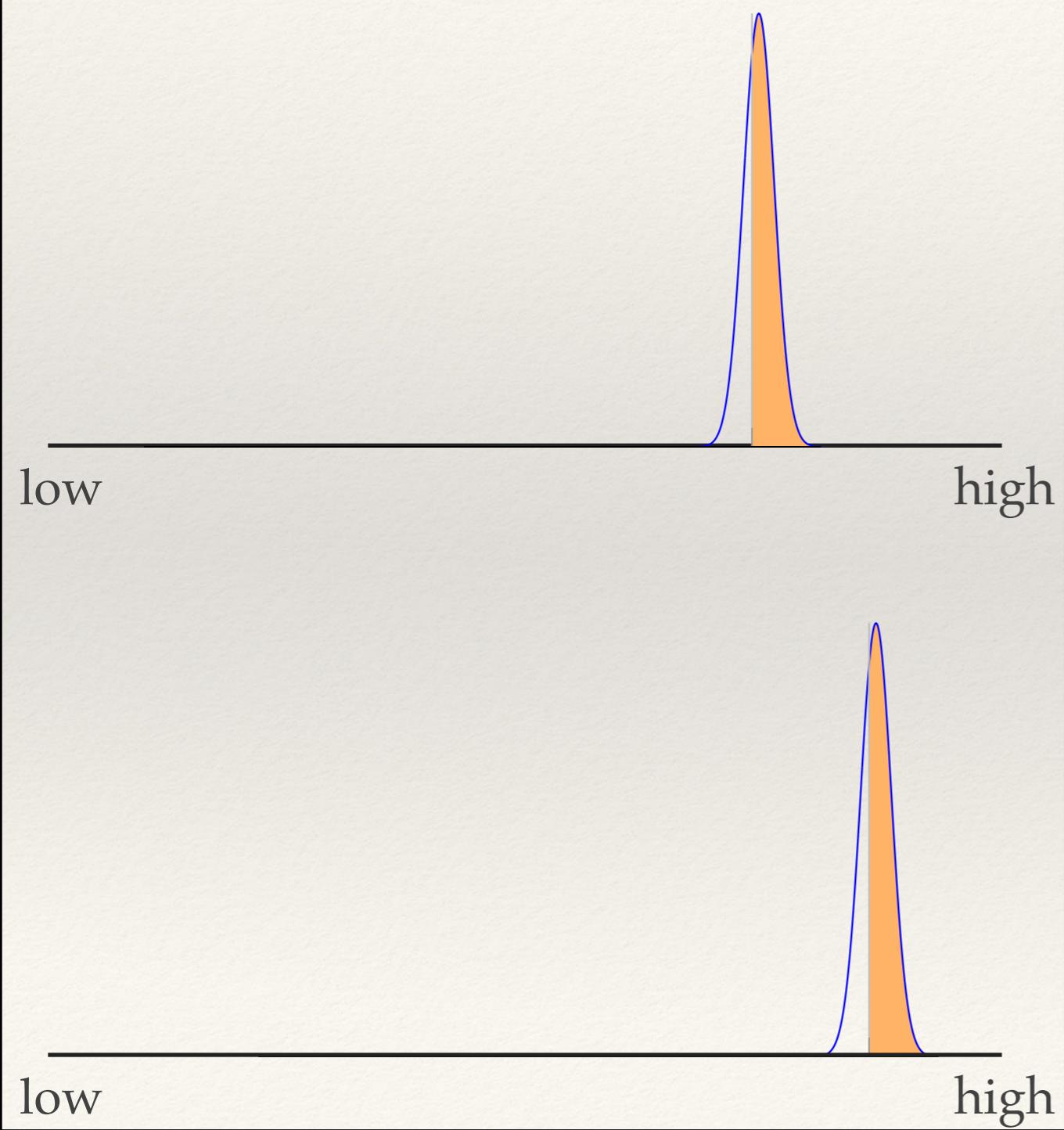
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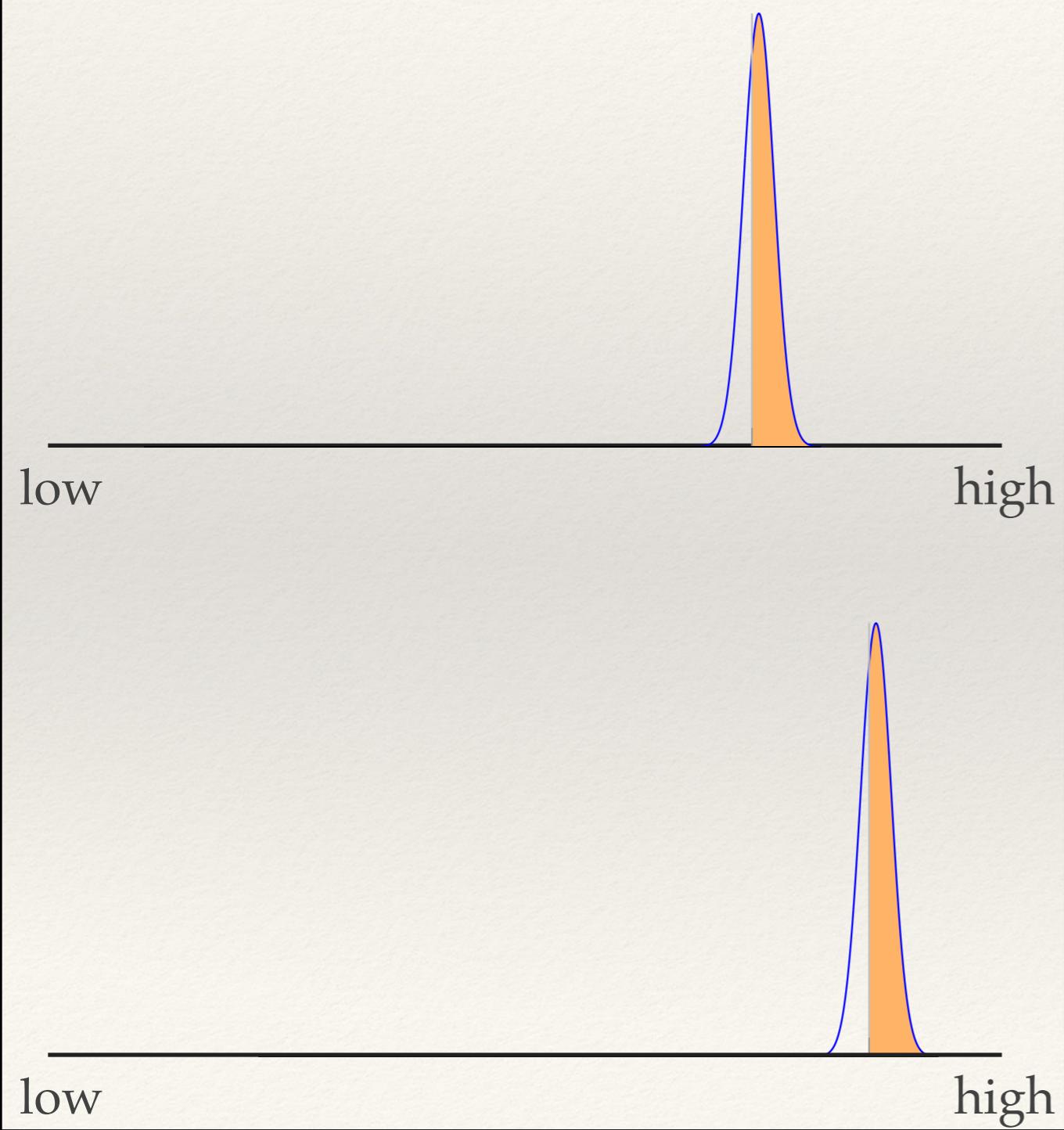


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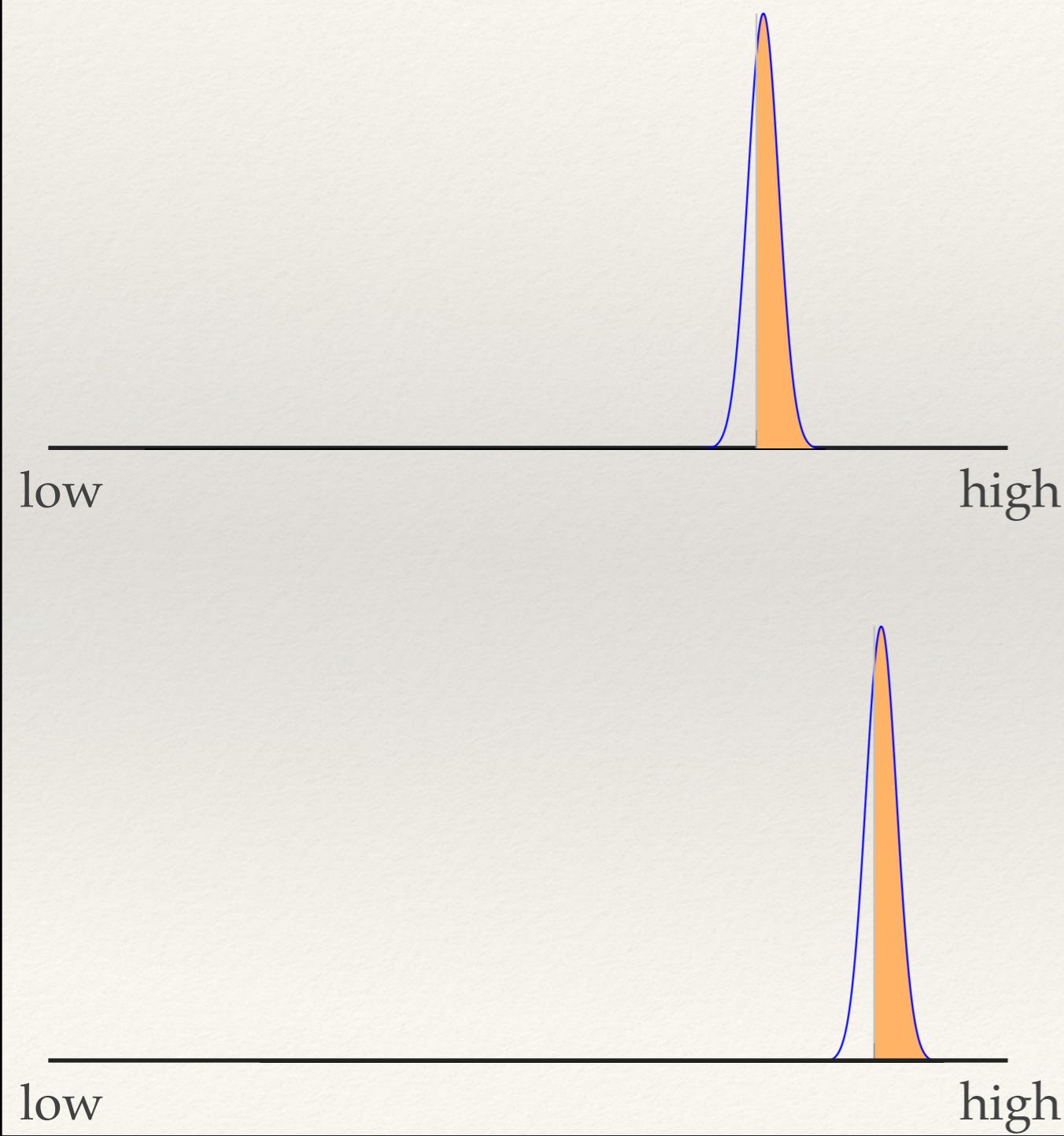
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What would that rule look like?