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# *Solutions to Homework #1*



# Ellis Island

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## PROBLEM

Consider 1,000 immigrants such that:

1. each of the 1,000 is either Irish or Italian;
2. at least one of the 1,000 is Irish; and
3. for any pair of two different individuals randomly selected from the 1,000 immigrants at least one is Italian.

**How many immigrants are Italian?**

## SOLUTION

there must be  
**999** Italian  
immigrants



# Intuitive Explanation

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Try the same problem with 4 immigrants instead of 1,000.

Given immigrants A, B, C, and D, the pairs are:

AB, AC, AD, BC, BD, CD

The problem says that for **ANY** pair, at least one member of the pair must be Italian. If you run through all the pairs, you'll realize that there must be at least 3 Italians.

You can repeat the same problem with any number  $n$  and arrive at the result that the Italians must be at least  $n-1$ .



Still not convinced?

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# Do you want a more precise proof?

Suppose there are *fewer than* 999 Italians.  
So, suppose there are 998 or fewer Italian immigrants. This implies there are *at least* 2 Irish immigrants. Which means there is a pair of 2 Irish immigrants, but this contradicts condition F3 that says that for ANY pair of immigrants at least one must be Italian. Therefore, we *cannot* suppose there are *less than* 999 immigrants, so there must be *at least* 999 Italian immigrants.

Further, since condition F2 says there must be *at least* one Irish, and condition F1 says that all immigrants must be Irish or Italian, there must be *exactly* 999 Italians.

This is an **argument by contradiction**. It begins with the assumption **X** and arrives at a contradiction, from which **non-X** follows.



# Coloring an Numbering

**PROBLEM:** Draw a table with 4 rows and 9 columns. Color each cell and assign either 0 or 1 to each cell.

1. For any cell  $C$  in a given column, there is exactly one other cell  $C'$  in another column such that  $C$  and  $C'$  have the same color.
2. Cells with the same color (*twin cells*) must be assigned either both 1's or both 0's.
3. Each column has exactly one cell to which the number 1 is assigned, so the other three cells in the column are assigned 0.

**SOLUTION:** This cannot be done. (3) requires that there be exactly 9 cells numbered 1. Further, (2) and (1) require that each of these 9 cells numbered 1 have exactly one twin cell also numbered 1. So, (1) and (2) require there be an even number of cells numbered 1. That's impossible.