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Lehman College CUNY

PHI 169 - Spring 2016

Solutions to Homework #1

Ellis Island

PROBLEM

Consider 1,000 immigrants such that:

1. each of the 1,000 is either Irish or Italian;
2. at least one of the 1,000 is Irish; and
3. for any pair of two different individuals randomly selected from the 1,000 immigrants at least one is Italian.

How many immigrants are Italian?

SOLUTION

there must be
999 Italian
immigrants

Intuitive Explanation

Try the same problem with 4 immigrants instead of 1,000.

Given immigrants A, B, C, and D, the pairs are:

AB, AC, AD, BC, BD, CD

The problem says that for **ANY** pair, at least one member of the pair must be Italian. If you run through all the pairs, you'll realize that there must be at least 3 Italians.

You can repeat the same problem with any number **n** and arrive at the result that the Italians must be at least **n-1**.

Still not convinced?

Do you want a more precise proof?

Suppose there are *fewer than* 999 Italians.
So, suppose there are 998 or fewer Italian immigrants. This implies there are *at least* 2 Irish immigrants. Which means there is a pair of 2 Irish immigrants, but this contradicts condition F3 that says that for ANY pair of immigrants at least one must be Italian. Therefore, we *cannot* suppose there are *less than* 999 immigrants, so there must be *at least* 999 Italian immigrants.

Further, since condition F2 says there must be *at least* one Irish, and condition F1 says that all immigrants must be Irish or Italian, there must be *exactly* 999 Italians.

This is an **argument by contradiction**. It begins with the assumption **X** and arrives at a contradiction, from which **non-X** follows.

Coloring an Numbering

PROBLEM: Draw a table with 4 rows and 9 columns. Color each cell and assign either 0 or 1 to each cell.

1. For any cell C in a given column, there is exactly one other cell C' in another column such that C and C' have the same color.
2. Cells with the same color (*twin cells*) must be assigned either both 1's or both 0's.
3. Each column has exactly one cell to which the number 1 is assigned, so the other three cells in the column are assigned 0.

SOLUTION: This cannot be done. (3) requires that there be exactly 9 cells numbered 1. Further, (2) and (1) require that each of these 9 cells numbered 1 have exactly one twin cell also numbered 1. So, (1) and (2) require there be an even number of cells numbered 1. That's impossible.