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Special Theory of Relativity: Clocks and Rods

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Einstein based his special theory of relativity on two postulates, the principle of relativity and the light postulate. If we adopt these two principles, we already know that things cannot remain as they have been in classical Newtonian physics. Imagine a light signal flying past us; and an inertially moving spaceship that speeds after it. An immediate consequence of the light postulate is that observers in the inertially moving spaceship will not judge the light signal to have slowed, no matter how fast they are moving past us. That is impossible according to classical Newtonian physics.

If we are to retain both of Einstein's postulates, we will have to make systematic changes throughout our physics. Let us begin investigating these changes. They will overturn our classical presumptions about space and time.

A Light Clock

The first change we will investigate has to do with time. An inertially moving clock runs more slowly than one at rest.

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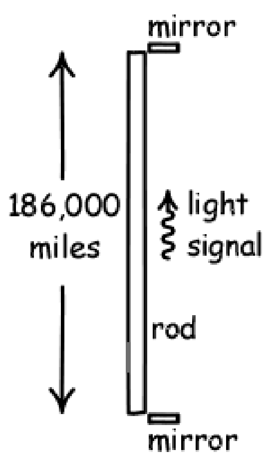
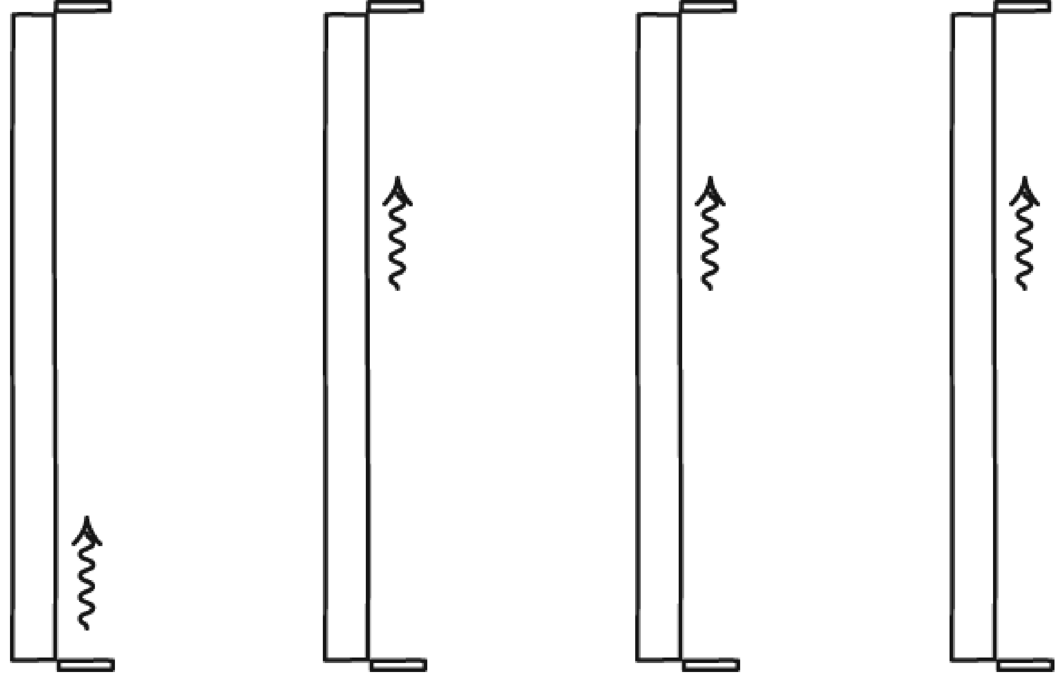


To see how this comes about, we could undertake a detailed analysis of a real clock, like a wristwatch, a pendulum clock or a mechanical wind-up clock. That would be difficult and complicated--and unnecessarily so. All we need is to demonstrate the effect for just one clock and that will be enough, as we shall see shortly, to give it to us for all clocks. So let us pick the simplest design of clock imaginable, one specifically chosen to make our analysis easy.

A light clock is an idealized clock that consists of a rod of length 186,000 miles with a mirror at each end. A light signal is reflected back and forth between the mirrors. Each arrival of the light signal at a mirror is a "tick" of the clock. Since light moves at 186,000 miles per second, it ticks once per second.

Here are some light clocks ticking:



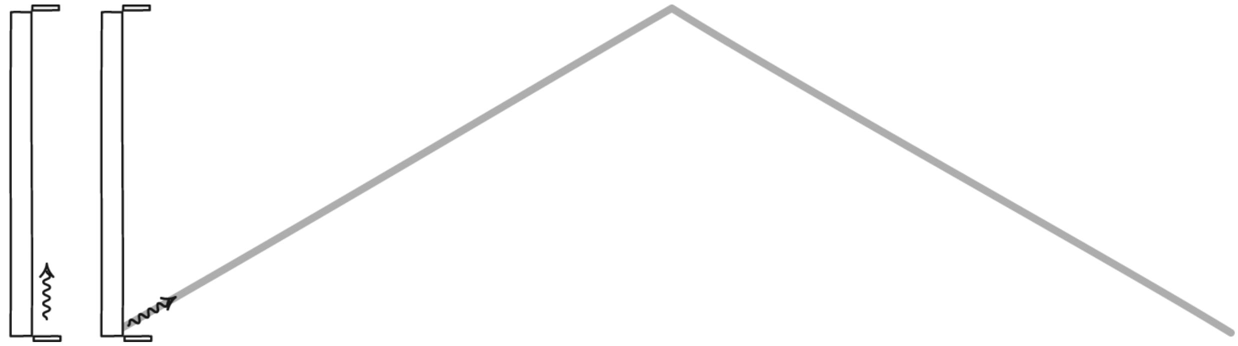
Light Clocks are Slowed by Motion

What happens when a light clock is set into rapid motion, close to the speed of light? It is easy to see without doing any sums that the light clock will be slowed down. That is, it will be slowed down in the judgment of someone who does not move with the light clock.

First, we will take the simple case of a light clock whose motion is perpendicular to the rod. The light clock will function as before. But now there is an added complication. The light signal leaves one end of the rod and moves toward the other end. But since the rod is moving rapidly, the light signal must chase after the other end as it flees. As a result, the light signal requires more time to reach the other end of the rod. That means that the moving light clock ticks more slowly than one at rest.

Remember the light postulate. It tells us that the light always travels at the same speed in any inertial frame of reference. That the rod along which it bounces is moving rapidly will not alter the speed of the light.

Here's an animation that shows a light clock at rest and a second light clock that moves perpendicular to its rod. The light signal in the moving clock chases after the rod. To reach the other end, it covers more distance and, as a result, requires more time.



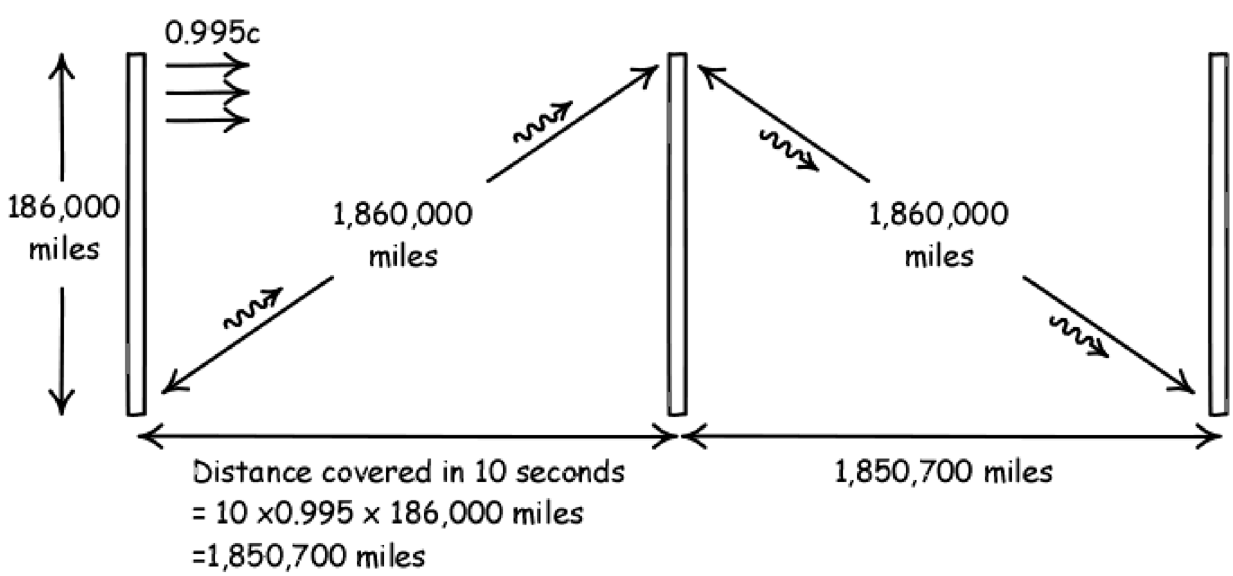
Here's the same animation in larger size in case you have a big screen.

If you watch the animation carefully, you will see that the moving light clock ticks at exactly half the speed of the resting clock. That is because the light signal of the moving clock has to cover twice the distance to go from one end of the rod to the other.

To get this doubling of the distance takes a careful adjustment of the speed of the moving clock. It turns out that the moving clock has to be traveling at 86.6% the speed of light.

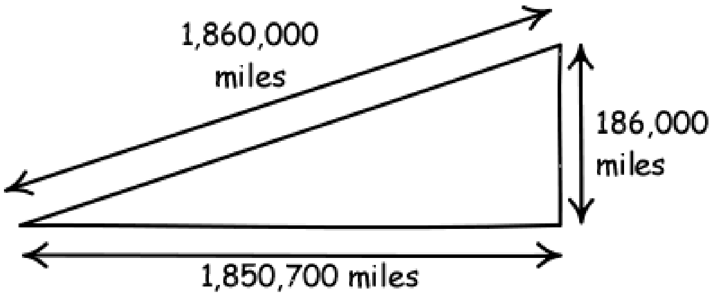
Just how much slowing do we get for some particular speed? That question turns out to be easy to answer with a little geometry. The trick is to figure out how much distance the light signal has to travel to reach the other end of the rod. Once we know that distance, we know the time taken, since light always travels at 186,000 miles per second.

To make things interesting, let's take a very high speed: 99.5% the speed of light. (We'll write this compactly as "0.995c.") An observer traveling with the clock will still find that the light signal bounces backwards and forwards between the mirrors as before. This process unfolds quite differently from the perspective of an observer who stays behind and does not move with the clock. The path traveled by the light will now be like this:



That observer at rest will agree with one that moves with the rod: a light signal leaves one end of the rod and arrives at the other end. But the observer at rest judges that end to be rushing away from the light signal at 99.5% the speed of light. A quick calculation shows that that the signal will now take 10 seconds to reach the other end of the rod.

To see this, note that in ten seconds the rod will move 1,850,700 miles, as shown in the figure above. So to get to the end of the rod, the light signal must traverse the diagonal path shown. A little geometry tells us that a right angle triangle with sides 186,000 miles and 1,850,700 miles will have a diagonal of 1,860,000 miles.



Pythagoras' theorem tells us the diagonal is 1,860,000 miles since

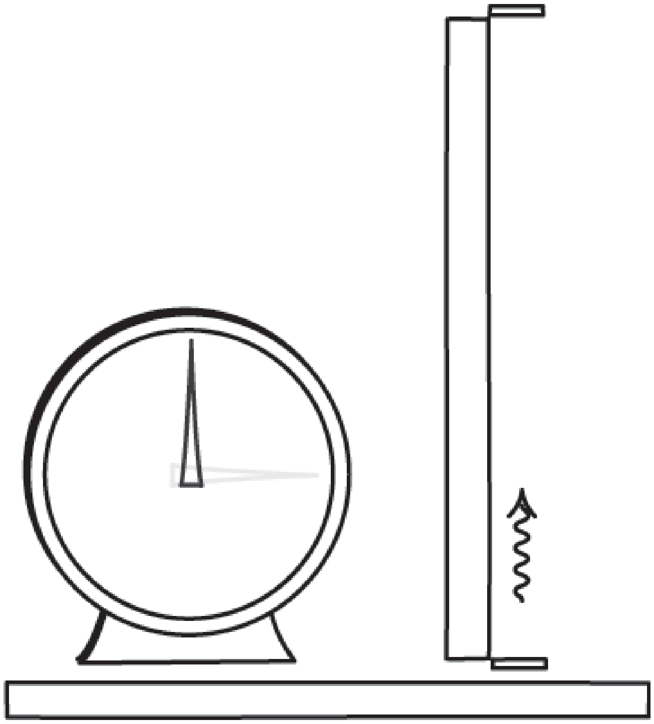
$$1{,}860{,}000 \text{ miles}^2 = 1{,}850{,}700 \text{ miles}^2 + 186{,}000 \text{ miles}^2$$

Since light moves at 186,000 miles per second, it will need ten seconds to traverse the diagonal.

Setting the arithmetic aside, the result is simple. Since the light signal must travel so much farther to traverse the rod of a moving clock, it takes much longer to do it. Hence a moving light clock ticks slower. In this case, for a clock moving at 99.5% the speed of light, it ticks once each ten seconds instead of once each second.

All Moving Clocks Are Slowed by Motion

All *light* clocks slow when they move rapidly. What about other clocks? You might be tempted to say that light clocks are exceptional. They slow because they depend on light; and we are learning that light does odd things. So why not just say that light clocks slow when they are set in motion and that shows that light clocks are not good clocks after all? Why not just say that other, real clocks are not slowed? It is tempting to say this, but it does not work.



A simple application of the principle of relativity shows that all clocks must be slowed by motion, not just light clocks. We set a clock of any construction next to a light clock at rest in an inertial laboratory.

We notice that they both tick at the same rate.

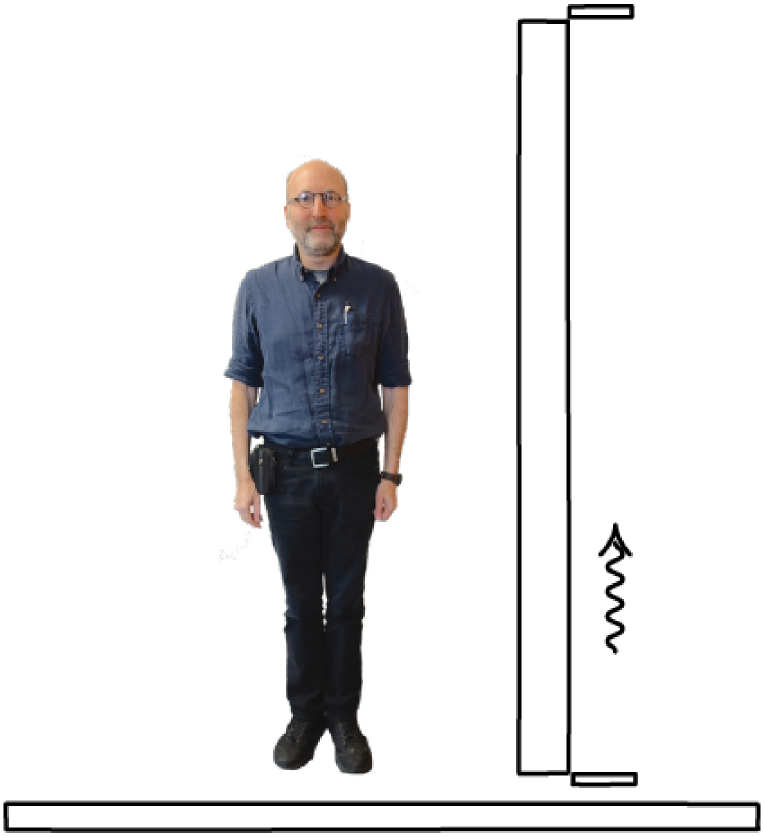
That must remain true when we set the laboratory into a different state of inertial motion.

But since the light clock has slowed with the motion, the other clock must also slow if it is to keep ticking at the same rate as the light clock.

You might still be tempted to say that the other clock would not keep pace with the light clock. Instead, you might say, it would keep running at just the same speed as it did when at rest. But then you would have devised a device that detects absolute motion, in contradiction with the principle relativity. That device would pick out absolute rest as the only state of motion in which the two clocks run at the same rate. Whenever the light clock ran slower than the other clock, we would know we are moving absolutely and we could use the difference of rates to determine just how fast that motion is. This cannot be; the principle of relativity prohibits it.

At first this might seem too easy. How can the principle of relativity force this slowing on all possible clocks?! Recall that the principle of relativity restricts all physical laws. It says that none of them are allowed a notion of absolute rest in space. This requirement forces systematic adjustments in almost every branch of physics. These adjustments will have the overall effect of forcing every clock that we might construct to slow when it is set in motion. For example, a mechanical clock keeps time by counting the oscillations of a mass bouncing on the end of a spring. A digital watch keeps time by counting the electromagnetically induced oscillations of a crystal. The physics of all of these systems will be altered slightly by the principle of relativity and in a way that will slow these clocks by the same amount as a light clock.

Perhaps the most extreme example of a clock is a human metabolic system. I can use my pulse to check that the light clock is running roughly at one tick per second. That check must give the same result in every inertial frame of reference. Hence my metabolism must slow in concert with the slowing of the light clock. That it should do this is fully to be expected. My metabolism is governed by complex laws of biochemistry. These laws must also respect the principle of relativity. So these laws also will be adjusted in a way that ensures that our metabolic clocks slow by just the same amount as a light clock, when both move at the same speed.



The outcome is that observers who move with a clock will not discern its slowing. For the observer's metabolism is slowed exactly as much as the clock. This unobservability of the slowing is required by principle of relativity. For, as we just saw, if I use my pulse beat to measure the rate of a clock, I must get the same outcome for the experiment if it is repeated with me and the clock set into rapid, uniform motion.

Rods Moving Parallel to the Direction of Their Motion...

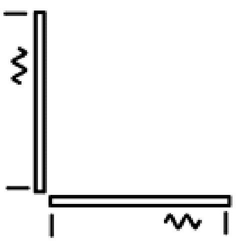
So far, we have considered a light clock whose rod is perpendicular to the direction of its motion. If we now consider a light clock whose rod is oriented parallel to the direction of motion, we will end up concluding that its rod must shrink in the direction of its motion. To get this result, we proceed by reasoning just as we have before. Once again we have a light signal on a moving clock, chasing after an end of a rod that flees rapidly. We now add in the extra complication that the rod is parallel to the direction of motion of the clock. That extra complication will force us to conclude that the rod has shrunk.

Getting to this result uses no new ideas or methods. It is just messier, so if you are not too bothered by details piling up, work through what follows. Or, if you are not so brave, you can skip to the end and just read the final result.

To get to the result, we need two steps:

1

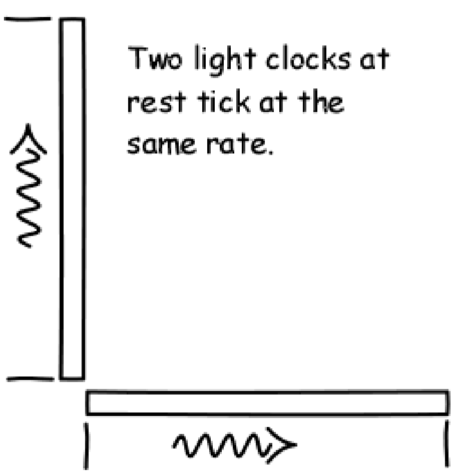
Light clocks oriented perpendicular to one another run at the same speed.

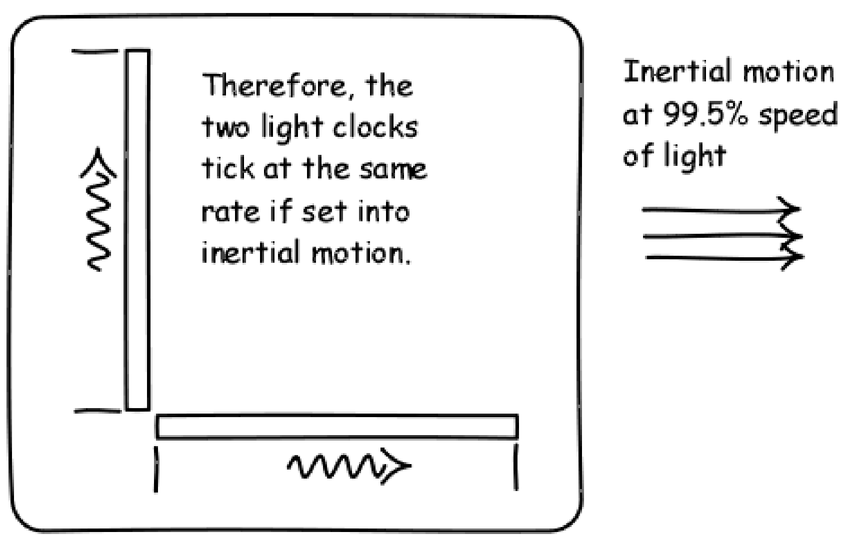


Take the light clock considered above. Image a second, identical light clock with its rod oriented parallel to the direction of the motion. Once again the principle of relativity requires that both clocks run at the same speed. We could just leave it at that--an application of the earlier result. However it is reassuring to go through it from scratch.

To begin, we don't need the principle of relativity to see that the clocks at rest run at the same rate. They will run at the same rate simply because they are the same clocks oriented in different directions. That just follows from the isotropy of space. All directions in space are physically equivalent. So the orientation of the clock cannot affect its speed.

Now imagine that we take the entire system of the two clocks and set it into rapid motion at, say, 99.5% the speed of light, in the direction of one of the light clocks.





An observer moving with the two light clocks must find them to continue to run at the same rate. We now do need the principle of relativity to establish this. Our earlier isotropy argument doesn't work anymore, since the two directions of the clocks are

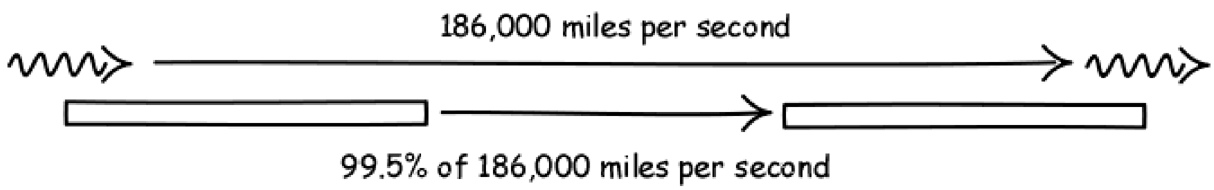
We could detect absolute motion just by taking two light clocks perpendicular to each other and checking if they run at the same rate. Only when we are rest would they run at the same rate. If they do not run at the same rate we intrinsically different. One is perpendicular to the direction of motion; the other is parallel to it. The principle of relativity requires that they run at the same rate. For, if they ran at different rates, the device would be an experiment that could detect absolute motion.

would know we are moving absolutely. The principle of relativity prohibits an experiment that can do this. So the two clocks must run at the same rate.

2

The rod oriented in the direction of motion must shrink.

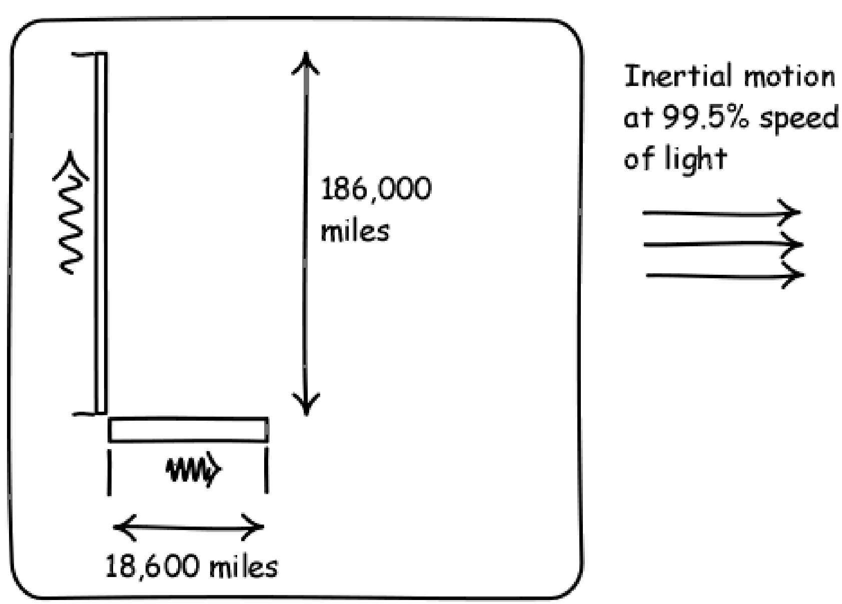
We know from the earlier analysis that a light clock (indeed any clock) moving at 99.5% the speed of light is slowed, so that it ticks only once in ten seconds. So now we know that the light clock oriented parallel to the direction of motion must tick once each ten seconds. But that cannot happen if everything is just as we describe it. Imagine the outward journey of the light signal.



The light signal has to go from one end to the other of a 186,000 mile rod. The light moves at 186,000 miles per second. But the rod is also moving in the same direction at 99.5% the speed of light. So the light has to chase after a rapidly fleeing end and will need much more than a second to catch it. With a little arithmetic it turns out that the light will need 200 seconds to make the trip.

How do I get this? If you *have* to know, here are the details. Think about the interval of space between the light signal and the end of the rod. At one boundary is a light signal moving at c. At the other boundary is the end of the rod moving just slightly slower at 99.5%c. It follows that the distance between the two boundaries diminishes at 100%c - 99.5%c = 0.5%c, which is 930 miles per second. The distance is initially 186,000 miles, so it takes 186,000/930 = 200 seconds to shrink to zero. That is when the light signal arrives at the rod's end.

But the light clock has to tick once every ten seconds! Something has gone badly wrong. What has gone wrong is our assumption that the rod parallel to the direction of motion retains its length. That is incorrect. That rod actually shrinks to 10% of original length, so the moving pair of clocks really looks more like:



Now the light signal has time to get from one end of the rod to the other and keep the clock ticking at once each ten seconds as expected. The signal just has far less distance to travel so now it can maintain the rate of ticking expected.

There are more details in this last calculation that I don't want to bother you with. But since some of you will ask, here they are--but only for those who really want them.

Overall it will turn out that the light signal now needs 20 seconds to complete the journey from the trailing end of the rod to the front and then back. That is what we expect. The round trip journal is "two ticks" and should take 2x10=20 seconds. The catch is that virtually all of the 20 seconds will be spent in the forward trip and virtually none of it in the rearward trip. This effect actually figures in the relativity of simultaneity which we will discuss at some length later.

If you want to see this for yourself you should redo the calculations. If you do, you'll need to undo my rounding off. The rod is not contracted exactly 10%--I rounded things off to keep life simple. It is 9.987%. The ticks are not exactly 10 seconds apart, but 10.0125 seconds. The forward trip will take 19.9750 seconds. The rearward trip will take 0.05 seconds. That gives a total round trip of 20.025 seconds = 2x10.0125 as expected.

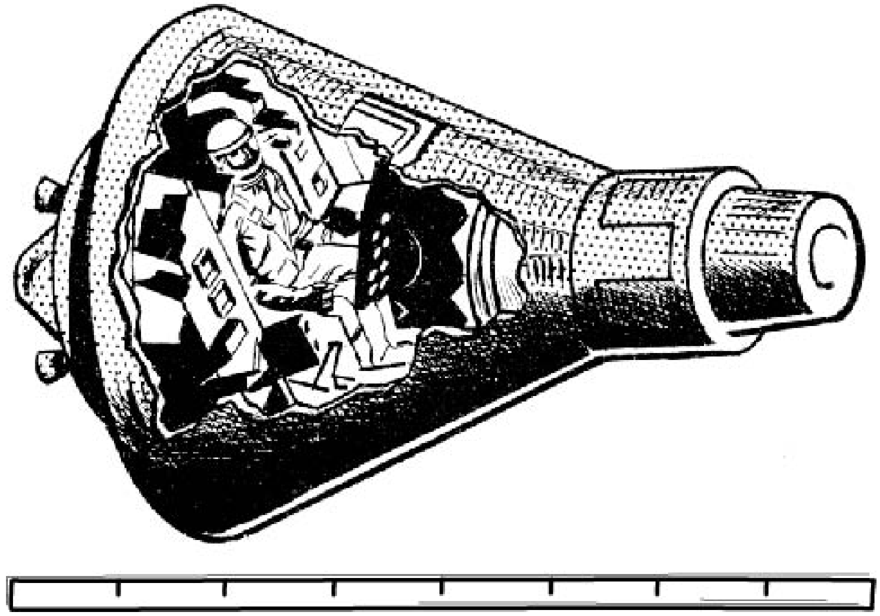
. . . Are Contracted in the Direction of their Motion

The analysis is now complete. A light clock oriented *perpendicular* to the direction of its motion slows by a factor of 10 when it moves at 99.5% the speed of light. A light clock oriented *parallel to* the direction of its motion must slow by the same amount when it moves at 99.5% the speed of light. It can do this if its rod shrinks in the direction of motion by a factor of ten.

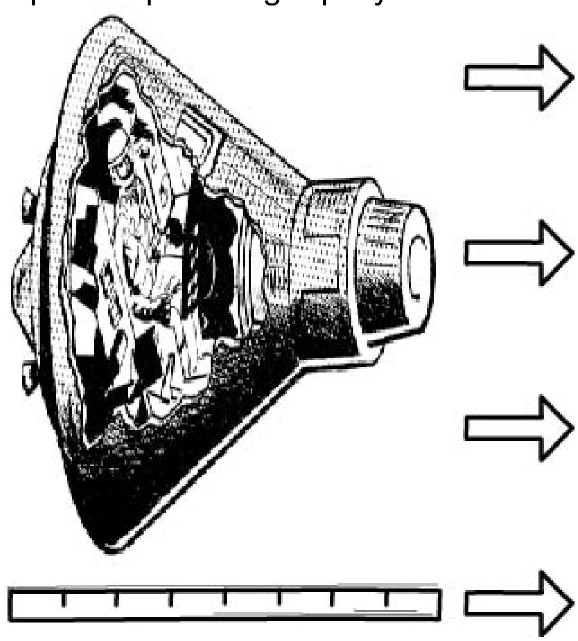
This length contraction result is quite general. Any object--such as a spaceship placed next to the rod--must also contract by same amount. If it did not, then we could use the difference in the contraction of the light clock rod and the object to reveal our absolute motion. We would have an absolute motion detector. But that is prohibited by the principle of relativity.

The effect is different in different directions: the object is shortened in the direction of its motion, but its size remains the same in a direction perpendicular to its motion. That is, the shapes of moving bodies are distorted from their resting shapes. Here is how that that distortion goes:

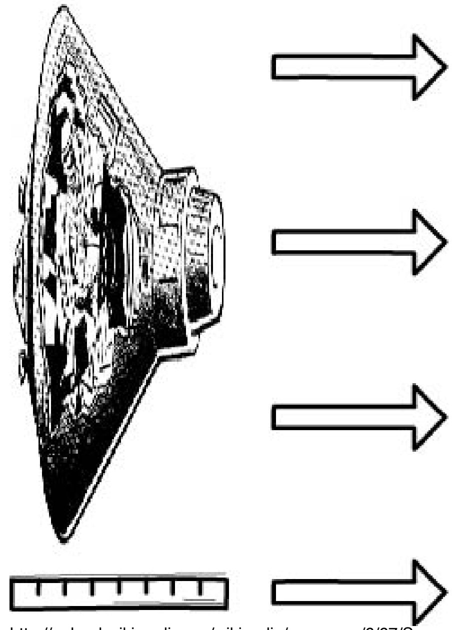
Spaceship at rest:



Spaceship moving rapidly:



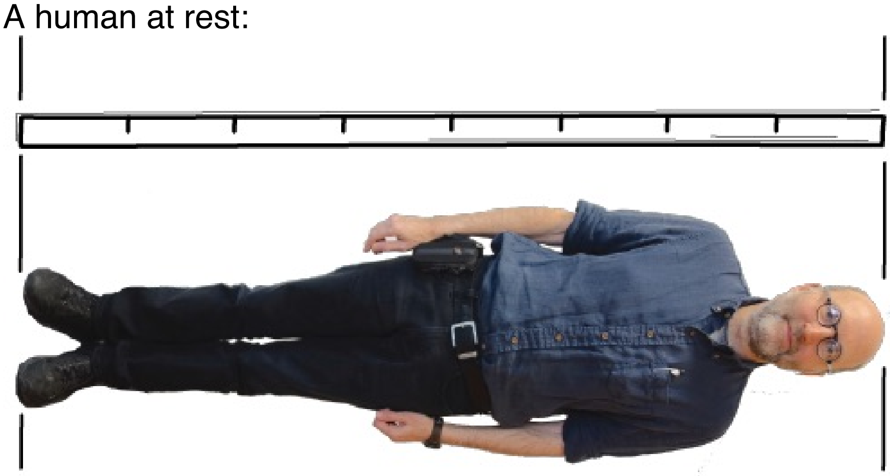
Spaceship moving even more rapidly:



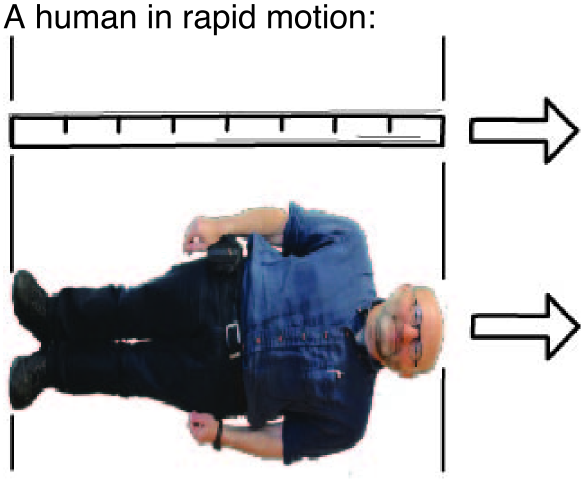
http://upload.wikimedia.org/wikipedia/commons/6/67/Spacecraft_%28PSF%29.png

The length contraction effect applies not just to ordinary mechanical objects like spaceships. It applies to living beings as well. When anything at all is set into motion, it contracts in the direction of its length.

A human at rest:



A human in rapid motion:



The principle of relativity assures us that, when we move, we have no direct awareness of this relativistic contraction effect. When we are at rest, every experiment can do would show us that we have our normal size. If I measure my height with a measuring rod, I will get the expected result. If I am now transported to a spaceship moving at 99.5% the speed of light, the experiments must come out the same. The principle of relativity requires it. This will be so, even though people back on earth would judge me to have shrunk by a factor of ten in the direction of my motion. And since I judge them to moving in the opposite direction, I would judge them to have shrunk correspondingly, while they have no measurement that can reveal it!

To Sum Up.

We have learned that a clock moving at 99.5% the speed of light, slows by a factor of ten. It ticks once each ten seconds instead of once each second. A rod, oriented in the direction of motion, shrinks to 10% of its length. Rods perpendicular to the direction of motion are unaffected.

These two effects have always been there. They were not noticed until Einstein's time because they are so very, very small as long as our speeds are far from that of light. They become marked when we get close to the speed of light. The closer we get the the speed of light, the closer clocks come to stopping completely and rods come to shrinking to no length in the direction of motion. For more details of how the effects depend on speed, see What Happens at High Speeds.

What you need to know:

- What a light clock is.
- How it is slowed by motion.
- How the principle of relativity now assures us all that all clocks are slowed by motion.
- Moving rods are shrunk in the direction of their motion.
- Why neither of these effects is measureable by observers moving with the rods and clocks.