

PROBABILITY TAKE HOME EXAM - CRITICAL REASONING - PHI 169

Answers the questions below, from (a) through (e2). The questions marked with * tend to be more difficult. They are optional. *Still, try to answer all of them and be perseverant!* Set aside a good chunk of time, at least one hour on Monday and one hour on Tuesday. *This take home exam is due Mon Nov 13 at the beginning of class.* You may work in groups, but please write up your answers individually. If you so decide, the grade for this take home exam can replace your grade for in-class exam #2.

- (a) What does it mean to say that event A is probabilistically dependent on event B ? *After giving the definition, please provide two illustrative examples.*
- (b*) If A is probabilistic dependent on B , does it follow that B is probabilistically dependent on A ? *Please explain with algebraic reasoning.*
- (c*) If $P(A) = 50\%$, and $P(B|A) = 1 - P(B|\neg A)$, does it follow that $P(A|B) = P(B|A)$? *Please explain with algebraic reasoning.*
- (d) Suppose you want to know whether women at a company were discriminated against during the hiring process for a highly paid position, call it \$\$ position. Please answer the following:
 - (d1) Researcher 1 collects data about people who were hired in the last two years, are currently working in the \$\$ position at the company, and are all equally qualified. The researcher finds out that, among such people, 95% are men and only 5% are female. What conditional probabilities do 95% and 5% stand for? Could this disparity be used to argue that the company discriminated against women during the hiring process for the \$\$ position? *Explain carefully.*
 - (d2) Researcher 2 collects data about a group of women and a group of men who had applied for the \$\$ position above at the company, both groups being equally qualified. In retrospect, the researcher finds out that 2% of the male applicants were hired for the \$\$ position, while only 0.2% of equally qualified female applicants were hired for the same position. What conditional probabilities do 2% and 0.2% stand for? Could this disparity be used to argue that the company discriminated against women during the hiring process for the \$\$ position? *Explain carefully.*
 - (d3*) Using the probabilities in (d1) and (d2), can you estimate what percentages among the applicants for the \$\$ position were male and what percentage were female? If you think you cannot estimate this percentage, please explain why not. *Hint: Use Bayes' theorem and assume, simplistically, that candidates can be either male or female, that is, $P(\text{MaleApplicant}) = 1 - P(\text{FemaleApplicant})$.*
- (e) Look up how many gold medals the US got in the last 2016 Olympics in Rio, and compare the numbers with the gold medals won by Germany. If you are a gold

medal holder, are you more likely to be American or German? Conversely, if you are American, are you more or less likely than a German to get a gold medal? Given these probabilities, which country do you think is better at producing athletes who win gold medals? *Explain carefully. Hint: you'll need to look up the population of the US and Germany.*

- (f) As you plan your solo backpacking adventure in King's Canyon National Park, you want to make sure to avoid bears as much as possible during your trip. You have read about signs to look for which indicate whether a bear is around or not. The signs are: a certain type of fresh excrements; broken trees; and particular sounds. These signs are not infallible, but your manual says that if there is a bear around, you'll see one of the signs with a 60% chance, and if there is no bear around, you could still see one of the signs with 20% chance. The manual also says that bears are rare. Data show that one could spot a bear with 5% chance in King's Canyon National Park. Given this information, please answer the following:
- (f1) You are hiking alone in the middle of King's Canyon, enjoying the stunning views. You see one of the "bear signs" you read about in the manual. How likely is it that a bear is around? Should you be worried and run away or should you keep enjoying the views?
- (f2*) What if—*immediately after!*—you find another of the signs indicating the presence of a bear? How likely is it that that a bear is nearby given that you found two such signs? Assume that the two signs are probabilistically independent.