

# Is Special Relativity Paradoxical?

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- [What's the Problem?](#)
- [The Car and the Garage](#)
- [Relativity of Simultaneity...](#)
- [...Solves the Problem](#)
- [Relativity of Simultaneity and the Measurement of Lengths](#)
- [Relativity of Simultaneity and the Measurement of the Rates of Clocks](#)
- [Are the Relativistic Effects Illusory Artefacts of Measurement?](#)
- [Separating Real from Unreal](#)
- [What You Need to Know](#)

*Background reading:* J. Schwartz and M. McGuinness, *Einstein for Beginners*. New York: Pantheon.. pp. 109 - 116.

## What's the Problem?

Relativity theory tells us that a moving clock is slowed down and a moving rod is shrunk in the direction of its motion. If I am an inertial observer, I will find the effect to come about for the clocks and rods of a spaceship moving past at rapid speed. But if that spaceship is moving inertially, then, by the principle of relativity, the spaceship's observer must find the same thing for my clocks and rods. Relative to that observer, my clocks and rods move past at great speed. So that observer would find my clocks to be slowed and my rods to be shrunk in the direction of my motion.

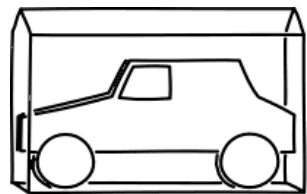
Each finds the other's clocks slowed and rods shrunk. How can both be possible? Is there an inconsistency in the theory? If I am bigger than you, then you must be smaller than me. You cannot also be bigger than me. That's the problem.

## The Car and the Garage

That each finds the other's clocks slowed and rods shrunk is troubling. But is it a real paradox in the sense of there being a logical contradiction? If I walk away from you, simple perspective effects make it look to each of us that the other is getting smaller. I judge you to grow smaller; and you judge me to grow smaller. No one should think that this is a paradox.



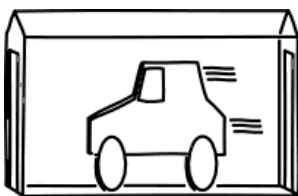
That perspectival effect should not worry anyone. The car in the garage problem is an **attempt** to show that the relativistic effects are more serious than this simple perspectival effect. There is, it tries to show, a real contradiction; and we should not tolerate contradictions in a physical theory.



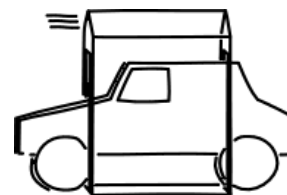
Here is how we might try to get a contradiction out of the relativistic effect of each observer judging the other to have shrunk. Imagine a car that fits perfectly into a garage. The garage is a small free standing shed that is just as long as the car. There is a door at the right and a door at the left of the garage. The car fits exactly--as long as it is at rest.

Now imagine that we drive the car at 86.6% speed of light through the garage from right to left. The doors have been opened at the right and the left of the garage to allow passage of the car. There is a garage attendant, who stands at rest with respect to the garage. Can the garage attendant close both doors so that, at least for a few brief moments, the car is fully enclosed within the garage?

According to the garage attendant, there is no problem achieving this. At 86.6% the speed of light, the car has shrunk to half of its length at rest. It fits in the garage handily. The garage attendant can close both doors and trap the car inside.



According to the car driver, however, matters are quite different. The car is at rest and the garage moves. The garage approaches the car at 86.6% the speed of light. So the car driver finds that it is the garage and not the car that has shrunk to half its length. The garage is now half as long as the car. The car driver says that there is no way the garage attendant can shut both doors and trap the car fully inside.



Now this is a serious problem. Either the car can or cannot be trapped fully within the garage, but not both. (Or so it would seem.)

More formally, we have a true paradox. The term paradox has multiple

meanings. It might just designate something that is unexpected in an amusing way; or something so unexpected as to be unbelievable. In its strongest form it is the appearance of a **logical contradiction** in a system we thought free of contradiction. (We have such a contradiction when we can deduce both some proposition A and its negation not-A.) That seems to be what is happening here. We seem to be able to deduce both of:

It IS the case that car is fully trapped within the garage.

It IS NOT the case that the car is fully trapped within the garage.

It is usually taken to be a **fatal problem** when a theory is shown to harbor contradictions. I say "usually" since there are exceptions. We shall see later that, when quantum theory first emerged, it harbored a highly visible logical contradiction. Dealing with it was an urgent problem.

## Relativity of Simultaneity...

There is a solution. It depends upon our remembering that there is more in special relativity than the slowing of clocks and the shrinking of rods. We have already seen the [relativity of simultaneity](#) which will take on greater and greater importance in our assessment of the theory. It tells us that observers in relative motion can disagree on the timing of spatially separated events.

Note that an "event" in the context of relativity theory has a narrow meaning. It is something that happens at one place and at one time. Events are not spread out in space and time as might be the sort of events that we talk about in everyday talk. In relativity theory, an event happens at just one moment and one spot.

## ...Solves the Problem

The possibility of that disagreement is the key to the problem of the car and the garage. A judgment of the simultaneity of events is essential to any judgment of **whether the car was trapped** in the garage by the closing of doors. The car driver and the garage attendant disagree on whether the car is ever fully enclosed in the garage simply because they disagree on the time order of two events.

The garage attendant says:

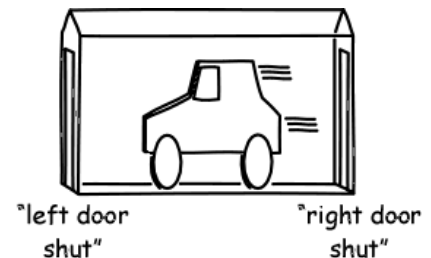
There are two events:

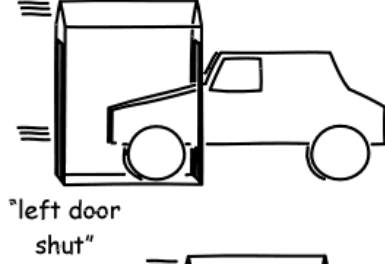
"Left door shut": I closed the left door before the car struck it.

"Right door shut": I closed the right door after the car passed.

*And these events happened at the same time.*

Therefore the car was fully enclosed.





The car driver says:

"There are two events.

"Left door shut": You closed the left door before the car struck it.

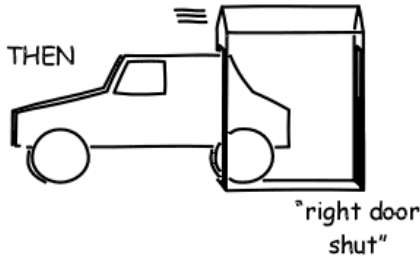
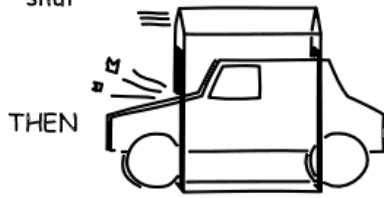
"Right door shut": You closed the right door after the car passed.

*But these events **did not** happen at the same time.*

You closed the left door first.

Then--later--you closed the right door after the front of the car had already burst through the closed left door.

Therefore the car was **never** fully enclosed.



Both agree that the two events "left door shut" and "right door shut" happened. They *disagree* on the time order in which they happened. But that time order is what is needed to decide whether the car was fully enclosed in the garage. In a nutshell:

- The car can only be said to have been fully enclosed in the garage if both doors were shut at the same time.
- There is no observer independent fact of the matter as to timing of these events.
- Therefore there is no observer independent fact as to whether the car was ever fully enclosed in the garage.

## Relativity of Simultaneity and the Measurement of Lengths

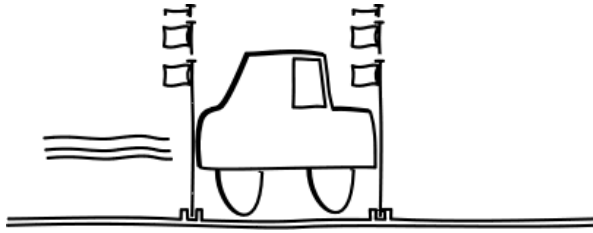
The problem of the car and the garage shows how judgments of lengths are entangled with judgments of simultaneity. This entanglement runs throughout special relativity. Indeed, one can understand **all the odd kinematical effects** as derived from it; for this reason, it was the first effect Einstein discussed in his 1905 paper.

For example, the relativity of simultaneity lies behind relativistic length contraction. To see this, consider how we might measure the length of a moving object. Take a car moving along a freeway at fancifully high speeds, so that relativistic effects come into play. I am standing by the roadside and want to know the car's length--or at least its length relative to me.

I cannot just hold up a measuring rod and proceed in the normal way: that is, check which marks on the rod align with each end of the car. For the car is zooming past. By the time I have noted the alignment of the front of the car with, say, the 0 mark on the measuring rod, the car has long since zoomed off into distance. I will have had no chance to check where the rear of car aligned. I need a more refined procedure.

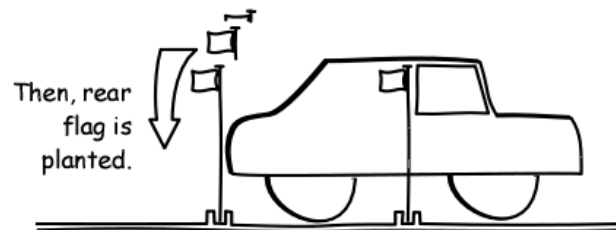
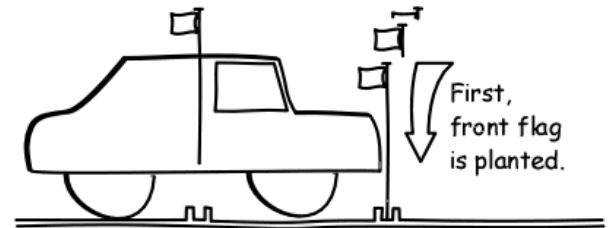
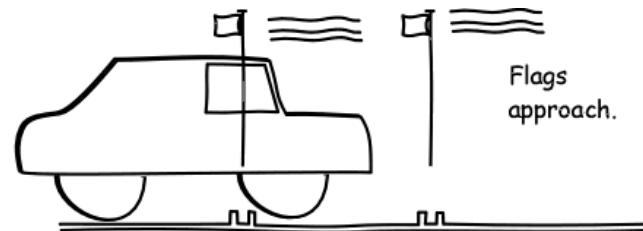
Here's one: as the car zooms by, I stand with a friend at the roadside, each of us holding a raised flag, ready to plant into the roadside. As the front of the car

passes, I plant my flag into the roadside; as the rear of the car passes my friend, my friend plants his flag into the roadside. The car zooms away. But that doesn't matter anymore. I have the information I need in the locations of the flags. I can use my measuring rod to determine the distance between the flags. That is the length of the moving car.

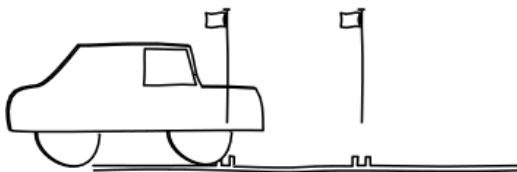


What is essential to this procedure is that I and my friend plant our flags *at the same time*. Otherwise the distance between the two marks will not properly reflect the length of the car.

But there's the catch. The car driver will disagree with my judgments of which events are simultaneous. The car driver will agree, of course, that there are two events, the planting of the two flags. But the car driver will not agree that I and my friend placed the marks simultaneously. Rather the car driver will find my friend and I to be rushing toward the car and the two flag plantings to have happened at *different* times. As the figure shows, the car driver will judge the planting of my flag at the front to have happened first; and the planting of my friend's flag at the rear to have happened later.



Here's an animated version of this process.



Since my friend delayed the planting of the flag at the rear (in the car driver's judgment), the rear of the car advanced for some short time after I'd planted my flag at the front. Therefore (in the car driver's judgment) the distance we staked out with the flags is shorter than the length of the car and our determination of the length of the car is wrong. Hence we end up disagreeing about the length of the car.

The important point is that neither of us (driver and roadside observer) has made an error. There is no absolute fact as to which of us is really moving. Therefore there is **no absolute fact** as to which of our judgements of the timing of the two events is correct. Just as in the case of the car and the garage, we each judge the other as shrunk because

# Relativity of Simultaneity and the Measurement of the Rates of Clocks

Similar considerations arise in judgments of the slowing of moving clocks. To see how the relativity of simultaneity underlies the relativistic slowing of clocks, we attend to a procedure we might use to measure the effect.

To judge the rate of a clock that passes me I need to be able to compare its reading with my wristwatch now and then compare its reading again later with my wristwatch after some time has passed. If the clock is running slow, I'll notice that its rate lags behind my wristwatch.

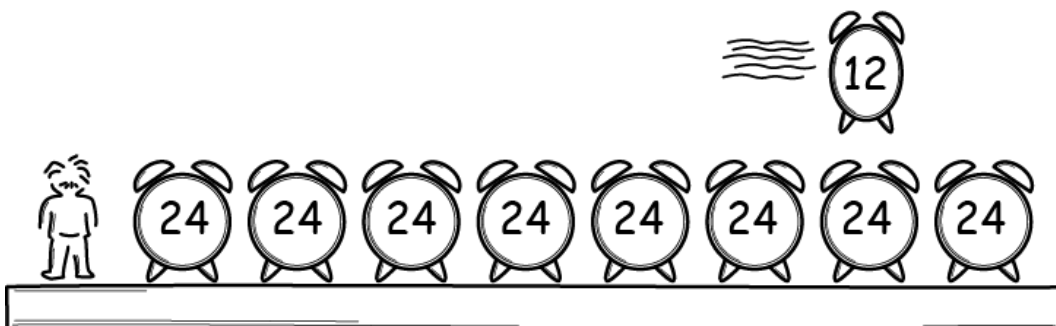
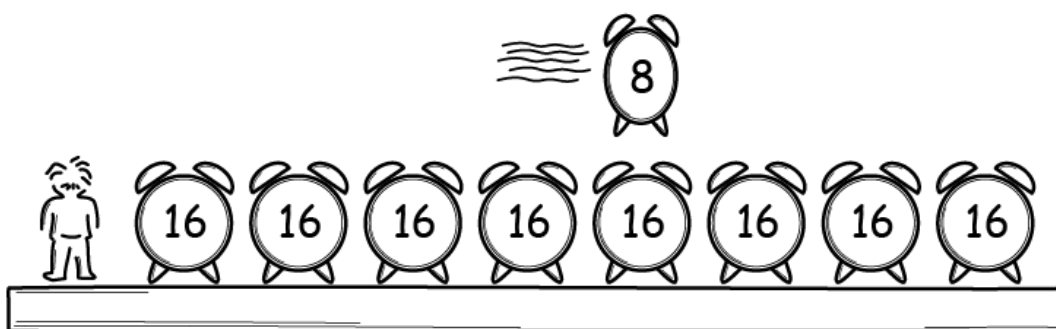
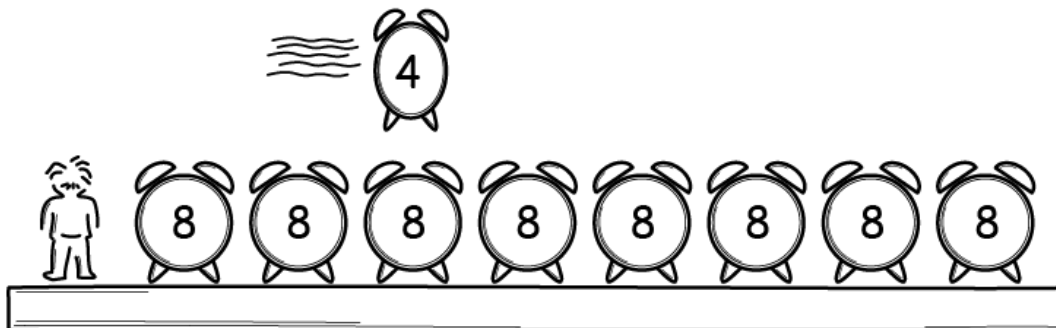
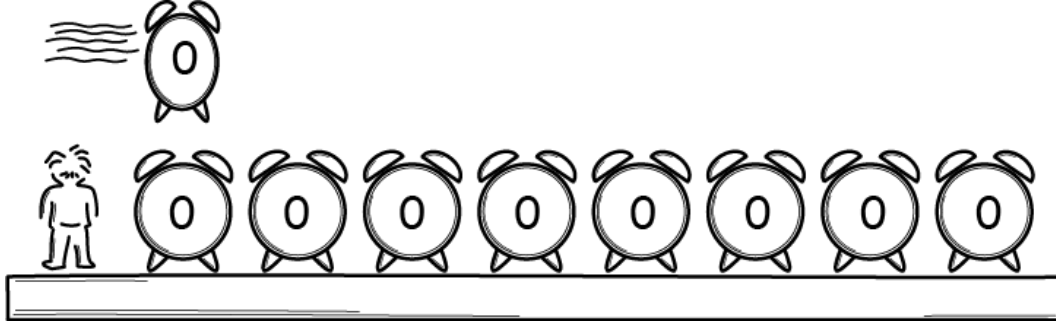
The **catch** in this simple procedure is that the clock is moving. I might find that both it and my wristwatch read the same time now, at the moment the clock passes. But the clock is moving rapidly. So after some time has elapsed, it has moved off into the distance.

How can I find out what the moving clock reads an hour from now when it is no longer anywhere near me? Here's **one procedure**: an observer sets up many clocks at rest with respect to a long platform that extends throughout space. Then, while a moving clock passes each one of those clocks, a friend notes what the moving clock reads and what the local resting clock reads; and so on for each of the clocks passed. From the friends' reports, the platform observer can figure out whether the moving clock has slowed or not.

The figure shows the bare essentials of the moving clock and all the other clocks spread out along the platform. The moving clock agrees with the "0" reading of the leftmost clock as it passes by. As it passes clocks successively along the platform, here's the list of reports the platform observers make:

When the moving clock reads "0," it is passing a clock that reads "0."  
When the moving clock reads "4," it is passing a clock that reads "8."  
When the moving clock reads "8," it is passing a clock that reads "16."  
When the moving clock reads "12," it is passing a clock that reads "24."

In short, the moving clock is found to have slowed. It runs at half the speed of the clocks at rest on the platform.

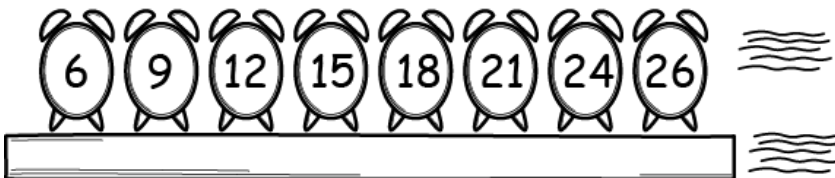


This procedure seems quite sound. So does that mean an observer who travels with the moving clock would agree and judge the moving clock to have slowed? No! We have seen that relativity theory requires that observer to judge the platform array of clocks to be running more slowly! How can that be?

By now you know the answer. An essential part of the procedure is that all the clocks laid out through space on the platform **must be synchronized**. A judgment of proper synchronization is a judgment of the simultaneity of events. Take the event at each clock when each of them reads "0." These must be simultaneous events. Otherwise the clocks will not be properly synchronized. The figure shows that, as far as the platform observer is concerned, the clock are properly synchronized.

The relativity of simultaneity tells us that an observer moving with the clock will disagree with the platform observer on the relative timing of the events on the platform. Thus, if the platform observer judges the clocks on the platform to be properly synchronized, an observer moving with the moving clock will disagree: that observer will find them not to be properly synchronized.

Here is how the successive instants will be judged to occur by an observer who moves with the clock:



The new, moving observer will judge the platform clocks *not* to be properly synchronized. Since trailing events are judged earlier and leading events later, the moving observer will judge the deviations from synchrony to be as shown in the figure. This mis-setting of the clocks then fully explains to the moving observer how clocks on the platform, passed successively, read double the reading of the clock of the moving observer. The clocks on the trailing end of the platform have set well ahead of those on the leading edge.

Otherwise, everything proceeds as the moving observer expects. The clocks on the moving platform actually run at half the speed of the observer's clock.

As before, there is no absolute fact as to whether the clocks are properly synchronized. Therefore there is no absolute fact as to whether the moving clock slows with respect to the platform clocks; or whether the platform clocks slow with respect to the moving clock.



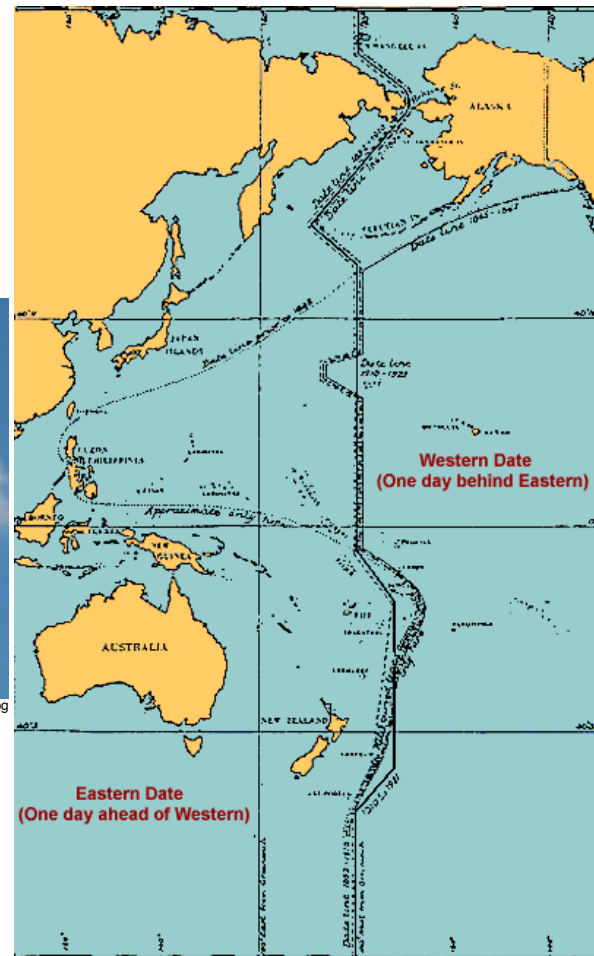
# Are the Relativistic Effects Illusory Artefacts of Measurement?

Once you recognize how fully the relativity of simultaneity is bound up in the relativistic length contraction and clock slowing effects, it is easy to fall into a new misunderstanding. One might think that the effects are not really part of the world at all, but that they somehow come about solely because of the way we set our clocks.

**An analogy:** it is possible to board a transpacific flight in Sydney, Australia, on one day and, after 16 hours of travel, disembark in Los Angeles the day before! Is this time travel? Of course not. During the flight, you crossed the international date line. That the calendar reads a day earlier in Los Angeles is purely an artefact of how we set our clocks and calendars across the world.



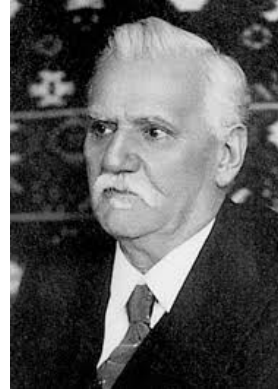
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[http://aa.usno.navy.mil/faq/docs/international\\_date.php](http://aa.usno.navy.mil/faq/docs/international_date.php) Historical positions of the International Date Line from "Notes on the History of the Date or Calendar Line," in The New Zealand Journal of Science and Technology, Vol. XI, pp. 385 - 388

geometry of a rotating disk. in 1911, Vladimir Varicak offered the following diagnosis of the origin of relativistic length contraction:

It "is only an illusory, subjective appearance, caused by the manner of our regulation of clocks and measurement of length" and "a psychological and not physical effect."



The rotating disk has some odd properties. Its circumference is relativistically contracted but its radius is not, resulting in "Ehrenfest's paradox." But this is a topic for another time.



Einstein's reply of the same year read:

"The question of whether the Lorentz contraction really exists or not is misleading. ...[it is] not real in so far as it does not exist for a co-moving observer. ...[it is] real in so far as it can be demonstrated in principle by physical means by an observer that is not co-moving"

Einstein's reply is terse. What I think he is getting at is this. He is accusing Varicak of conflating two distinctions:

Real  
versus  
unreal

Observer independent  
versus  
observer dependent

That we age is real. That we travel backwards in time when flying from Sydney to Los Angeles is unreal.

That an object spins on it axis is observer independent; it is verified by the presence of inertial forces. That an asteroid moves uniformly in space must be judged relative to another object.

What Varicak supposes that being real goes with being observer independent; and that being unreal goes with being observer dependent. Hence, when we find an observer dependent effect, we have found an unreal effect.

Einstein's response is that we should decouple the two distinctions. That an effect is observer dependent does not determine whether it is real or not.

The simplest way to see how this works is to consider an asteroid rapidly approaching us. That it is moving rapidly is an observer dependent effect. For an observer on the asteroid, the asteroid is at rest. The effect is observer dependent, but the motion of the asteroid is very real for us.

We should view relativistic length contraction in the same way. As the asteroid speeds past Earth--a fortunate near miss--we judge its length to be shortened. It is an observer dependent effect since it is not shared by another observer on the asteroid. However the contraction of its length is still a real effect.

## Separating Real from Unreal

Some observer dependent effects will be real; others will be unreal. What complicates our separation of the two cases is that changes in the observer's measurement procedures will create corresponding changes in the effect. We can use this complication to our advantage. We separate the real from the

unreal by checking whether changes in the observer's procedures can eradicate the effect or not.

Recall the appearance of time travel that arises when one flies across the Pacific Ocean and crosses the international date line. The appearance of **time travel *everywhere*** can be eradicated merely by setting our world clocks differently. If clocks in Sydney and Los Angeles are set to read the international standard of Greenwich time, then the arrival in Los Angeles would always come later than the departure. There would be no appearance of time travel for any flight anywhere in the world. Thus the effect is unreal.

Take the case of the relativistic slowing of clocks. Recall the earlier arrangement. A rapidly moving clock is judged to have slowed since it is found to show earlier times than the clocks it passes of the inertial frame of reference from which the effect is judged. We can eradicate the effect merely by introducing a non-standard synchronization of those frame clocks so that they agree with the readings of the rapidly moving clock as it passes each of them.

The slowing will be eradicated for this particular clock. But what of others? That's the catch. The slowing effect will **not be eliminated for all** clocks. Take a clock moving rapidly in the opposite direction at the same speed, where its speed is judged by the original synchronization of the frame clocks. It will be judged to slow at twice the rate of the original unconfounded effect.

This clock slowing effect is a real effect. It is manifested to an observer in an inertial frame by means of the clocks the observer has arranged through the frame. The mode of synchronization of those clocks will alter how the rate of slowing manifests. But the effect is not produced by the synchronization. For it cannot be removed completely by altering the mode of synchronization. We may eradicate it for one motion, but it comes back with greater strength for another. It is always there, even if in an obscured or confounded form.

You can see this just by thinking through how clocks must be reset to eradicate the slowing of the first clock. A faster way is to recall the "[twin effect](#)," to be discussed in a later chapter. A clock that moves away from us and returns to the same place will have slowed. The twin effect is measured at one place and thus is unaffected by how we might synchronize our clocks. We can eradicate the portion of the slowing that appears on the outward leg by non-standard synchronization of the frame clocks. But the total slowing must still appear for the full journey if we are to preserve the twin effect. Therefore there must be a double slowing on the return leg.

The literature on the thesis of the conventionality of simultaneity has explored what happens if we adopt some of these non-standard synchrony rules. This is discussed in a later chapter that covers the [conventionality of simultaneity](#).

It is helpful to compare this with the corresponding problem in ordinary Newtonian physics. There, rapidly moving clocks do not slow. However we could set up the clocks of some frame so as to give the illusion of a slowing of rapidly moving clocks. There would be a corresponding speeding up of clocks moving in the other direction. We would affirm that the whole effect is just an illusion. For by setting the clocks back to their normal synchronizations, we would find the entire effect of the speeding up and slowing down of clocks to have been eradicated.

## What You Need to Know

- The relativity of simultaneity
- How it solves the car in the garage problem.
- How the relativity of simultaneity is involved in judgments of the length of moving bodies and rates of clocks.
- Why this doesn't mean that the relativistic effects are illusions.