

*Aristotle*



*Gottlob Frege*



*Kurt Goedel*

# PHIL 50 - Introduction to Logic

**Marcello Di Bello, Stanford University, Spring 2014**

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*Week 1 – Monday Class*

# \* SYSTÈME FIGURÉ DES CONNOISSANCES HUMAINES.

## ENTENDEMENT.

### MEMOIRE.

SACRÉE. (HISTOIRE DES PROPHÉTIES.

ECCLESIASTIQUE.

CIVILE, } HIST. CIVILE, proprement dite, } MEMOIRES.

ANC. } ET MO- } HISTOIRE LITTERAIRE. } ANTIQUITÉS.

DERNE. } HISTOIRE COMPLETE.

HISTOIRE CELESTE.

UNIFORMITÉ DE LA NATURE. } DES METEORES.  
HISTOIRE } DE LA TERRE ET DE LA MER.  
DES MINERAUX.  
DES VÉGÉTAUX.  
DES ANIMAUX.  
DES ÉLÉMENS.

PRODIGES CELESTES.

METEORES PRODIGIEUX.

PRODIGES SUR LA TERRE ET LA MER.

MINERAUX MONSTRUEUX.

VÉGÉTAUX MONSTRUEUX.

ANIMAUX MONSTRUEUX.

PRODIGES DES ÉLÉMENS.

NATU-  
RELLE.

TRAVAIL ET USAGES DE L'OR.  
ET DE L'ARGENT. } MONNOEUR.  
FILEUR D'OR.  
TIREUR D'OR.  
ORFÈVRE.  
PLANEUR.  
METTEUR EN OUVRE, &c.

TRAVAIL ET USAGES DES PIERRES FINES ET PRÉCIEUSES. } LAPIDAIRE.  
DIAMENTAIRE.  
JOAILLER, &c.

TRAVAIL ET USAGES DU FER. } SERRURERIE.  
TAILLANDERIE.  
ARMURERIE.  
ARQUEBUSERIE, &c.

VERRERIES.

### RAISON.

MÉTAPHYSIQUE GÉNÉRALE, ou ONTOLOGIE, ou SCIENCE DE L'ETRE EN GÉNÉRAL, DE LA POSSIBILITÉ, DE L'EXISTENCE, DE LA DURÉE, &c.

SCIENCE DE DIEU. } THÉOLOGIE NATURELLE. } RELIGION,  
THÉOLOGIE RÉVÉLÉE. } D'où par abus,  
SCIENCE DES ESPRITS BIEN ET MAL FAISANS. } SUPERSTITIONS.  
DIVINATION.  
MAGIE NOIRE.

PNEUMATOLOGIE ou SCIENCE DE L'AME } RAISONNABLE.  
SENSITIVE.

ART DE PENSER.	APPREHENSION.	SCIENCE DES IDÉES.	
	JUGEMENT... RAISSONNEMENT.	SCIENCE DES PROPOSITIONS.	
	ET MÉTHODE...	INDUCTION, DÉMONSTRATION.	
		ANALYSE, SYNTHESE.	
ART DE RETENIR.	NATURELLE		
	MÉMOIRE... ARTIFI- CIELLE.	PRÉNOTION, EMBLEMSE.	
	SUPPLÉMENT DE LA MÉMOIRE.	ECRITURE, IMPRIMERIE.	
		ALPHABETH, CHIFFRES...	
LOGIQUE.	ARTS D'ÉCRIRE, D'IMPRIMER, DE LIRE, DE DÉCHIFFRER.	ORTHOGRAPHIE.	
		GESTE... SIGNES...	PANTOMIME, DÉCLAMATION.
		CHARACTÈRES	IDEAUX, HIÉROGLYPHIQUES, HERALDIQUES OU BLASON.
		SCIENCE DE L'INSTRUMENT DU DISCOURS.	PROSODIE, GRAMMAIRE, CONSTRUCTION, SYNTAXE, PHILOLOGIE, CRITIQUE.
ART DE COMMUNIQUER.	SCIENCE DES QUALITÉS DU DISCOURS.	PEDAGOGIQUE, CHOIX DES ÉTUDES, MANIÈRE D'ENSEIGNER.	
		RHÉTORIQUE.	
		MÉCHANIQUE DE LA POÉSIE OU VERSIFICATION.	

MORALE } SCIENCE DU BIEN ET DU MAL EN GÉNÉRAL. DES DEVOIRS EN GÉNÉRAL.  
GENERALÉ } DE LA VERTU. DE LA NÉCESSITÉ D'ESTRE VERTUEUX, &c.

MORALE	NATURELLE.	
	SCIENCE DES LOIX, ou JURISPRUDENCE	ECONOMIQUE,
		POLITIQUE,
		COMMERCE INTÉRIEUR, EXTERIEUR, DE TERRE, DE MER.

MÉTAPHYSIQUE DES CORPS, ou PHYSIQUE GÉNÉRALE. DE L'ÉTENDUE,  
DE L'IMPÉNÉTRABILITÉ, DU MOUVEMENT, DU VUIDE, &c.

### IMAGINATION.

#### POÉSIE

SACRÉE. PROFANE.

NARRATIVE. TRAGEDIE.  
MADRIGAL. DRAMA-  
PIGRAMME TIQUE.  
ROMAN, &c. COMÉDIE.  
PEINTURE.  
SCULPTURE.  
ARCHITECTURE CIVILE.  
GRAVURE.  
OPERA.  
PASTORALES, &c.  
PARABOLIQUE.  
ALLEGORIES.

POÈME EPIQUE.  
MUSIQUE. THEORIQUE.  
PRATIQUE.  
INSTRUMENTALE.  
VOCALE.

PEINTURE.  
SCULPTURE.  
ARCHITECTURE CIVILE.  
GRAVURE.

# MAP of the SYSTEM of HUMAN KNOWLEDGE

## UNDERSTANDING

### MEMORY

SACRED (HISTORY OF PROPHETS)  
ECCLESIASTICAL  
CIVIL, ANCI-  
ENT &  
MOD-  
ERN } CIVIL HISTORY, (*properly*  
said) LITERARY HISTORY  
MEMORIES  
ANTIQUITIES  
COMPLETE HISTORIES

UNIFOR-  
MITY OF  
NATURE  
CELESTIAL HISTORY  
OF METEORS  
OF LAND AND SEA  
OF MINERALS  
OF PLANTS  
OF ANIMALS  
OF ELEMENTS

CELESTIAL WONDERS  
LARGE METEORS  
WONDERS ON LAND AND SEA  
MONSTROUS MINERALS  
MONSTROUS PLANTS  
MONSTROUS ANIMALS  
WONDERS OF THE ELEMENTS

NATU-  
RAL  
WORK AND USES OF GOLD AND SILVER  
MINTING  
GOLDSMITH  
GOLD SPINNING  
GOLDDRAWING  
SILVERS MITH  
PLANISHER etc.

WORK AND USES OF PRECIOUS STONES  
LAPIDARY  
DIAMOND CUTTING  
JEWELLER, etc.

WORK AND USES OF IRON  
LARGE FORGES  
LOCKSMITH  
TOOL MAKING  
ARMORER  
GUN-MAKING, etc.

WORK AND USES OF GLASS  
GLASS MAKING  
PLATE-GLASSMAKING  
MIRROR MAKING  
OPTICIAN

### REASON

GENERAL METAPHYSICS, or ONTOLOGY, or  
SCIENCE OF BEING IN GENERAL, OF POSSIBILITY,  
OF EXISTENCE, OF DURATION, etc.  
SCIENCE OF GOD  
NATURAL THEOLOGY  
REVEALED THEOLOGY  
SCIENCE OF GOOD AND EVIL SPIRITS  
RELIGION  
SUPERSTITION  
DIVINATION  
BLACK MAGIC

PNEUMATOLOGY OR SCIENCE OF THE SOUL  
ART OF THINKING  
APPREHEN-  
SION  
JUDGEMENT  
REASONING  
METHOD  
ART OF REMEM-  
BERING  
SUPPLEMENT TO MEMORY  
LOGIC  
ART OF COMMUN-  
ICATING  
SCIENCE OF THE INSTRUMENT OF DISCOURSE  
SCIENCE OF THE QUALITIES OF DISCOURSE  
GENERAL  
PARTI-  
CULAR  
ETHICS  
PURE

SCIENCE OF IDEAS  
SCIENCE OF PROPOSITIONS  
INDUCTION  
DEMONS-  
TRATION  
NATURAL  
ARTIFICIAL  
WRITING  
PRINTING  
GRAMMAR  
SCIENCE OF THE INSTRUMENT OF DISCOURSE  
SCIENCE OF THE QUALITIES OF DISCOURSE  
GENERAL SCIENCE OF GOOD AND EVIL, OF DUTIES IN GENERAL,  
OF VIRTUE, OF THE NECESSITY OF BEING VIRTUOUS, etc.  
NATURAL  
SCIENCE OF LAWS OF JURIS-  
PRUDENCE  
ECONOMIC  
POLITICAL  
INTERNAL AND EXTERNAL  
COMMERCE ON LAND AND SEA

METAPHYSICS OF BODIES or, GENERAL PHYSICS, OF EXTENT, OF  
IMPERMEABILITY, OF MOVEMENT, OF WORD, ETC.

ARITHMETIC  
ALGEBRA  
NUMERIC  
ELEMENTARY  
INFINITE-  
SINAL  
DIFFERENTIAL  
INTEGRAL

### IMAGINATION

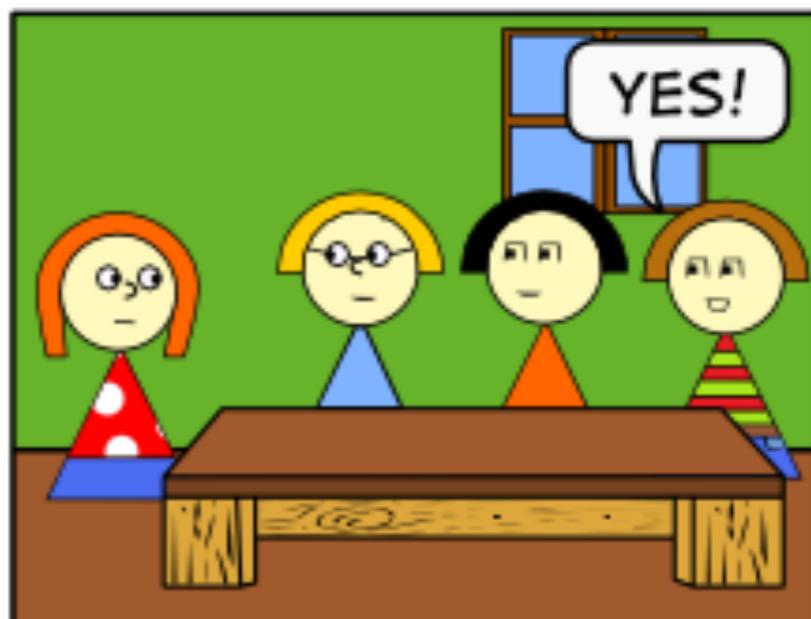
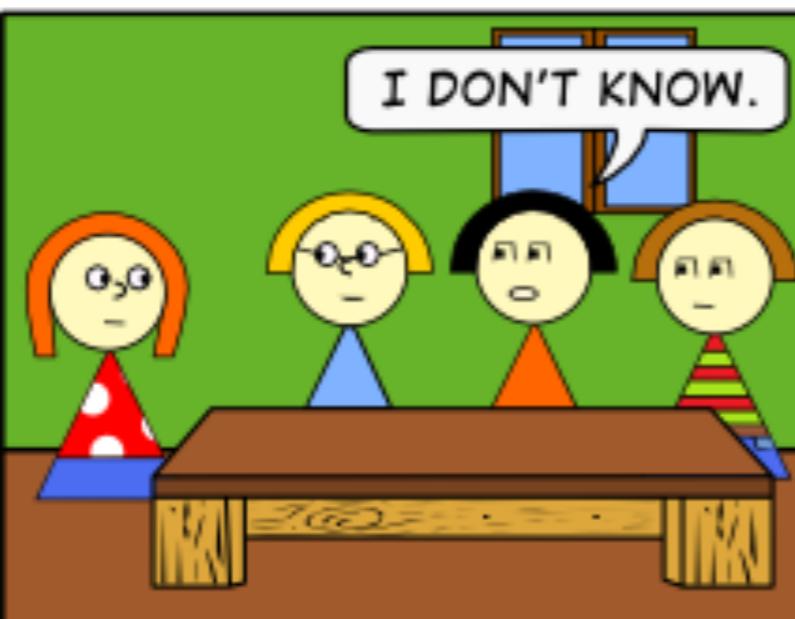
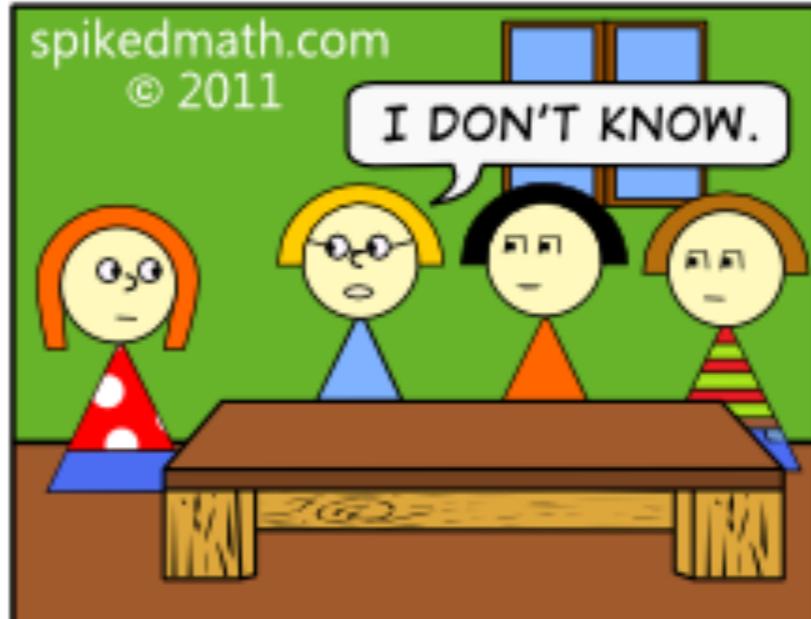
THEORY  
PRACTICE  
INSTRUMENTAL  
VOCAL  
MUSIC  
PAINTING  
SCULPTURE  
CIVIL  
ARCHITECTURE  
ENGRAVING

EPIC POETRY  
MADRIGAL  
PICTURE  
NOVEL, etc.  
TRAGEDY  
COMEDY  
PASTORAL,  
etc.  
PARABLE  
ALLEGORY

NARRATIVE  
PICTURE  
NOVEL, etc.  
TRAGEDY  
COMEDY  
PASTORAL,  
etc.  
PARABLE  
ALLEGORY

POETRY  
PROFANE  
SACRED  
DRAMA-  
TIC  
PARABLE

## THREE LOGICIANS WALK INTO A BAR...



What Should the Waiter Conclude?

# The Waiter's Reasoning

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- ✿ The question was “*Does everyone want beer?*”
- ✿ If the first guest did not want beer, he would have answered “No.”
- ✿ If the second guest did not want beer, he would have answered “No.”
- ✿ The third guest answered “Yes”, so **everybody must want beer.**

# What to Expect from this Course

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Learn about  
**propositional,**  
**predicate, modal,**  
**and inductive logic**

Learn some **history**  
and **philosophy of**  
**logic** along the way

Learn how to write  
**formal proofs**, both  
semantic and  
syntactic proofs

Learn about  
**logical puzzles** and  
**paradoxes**

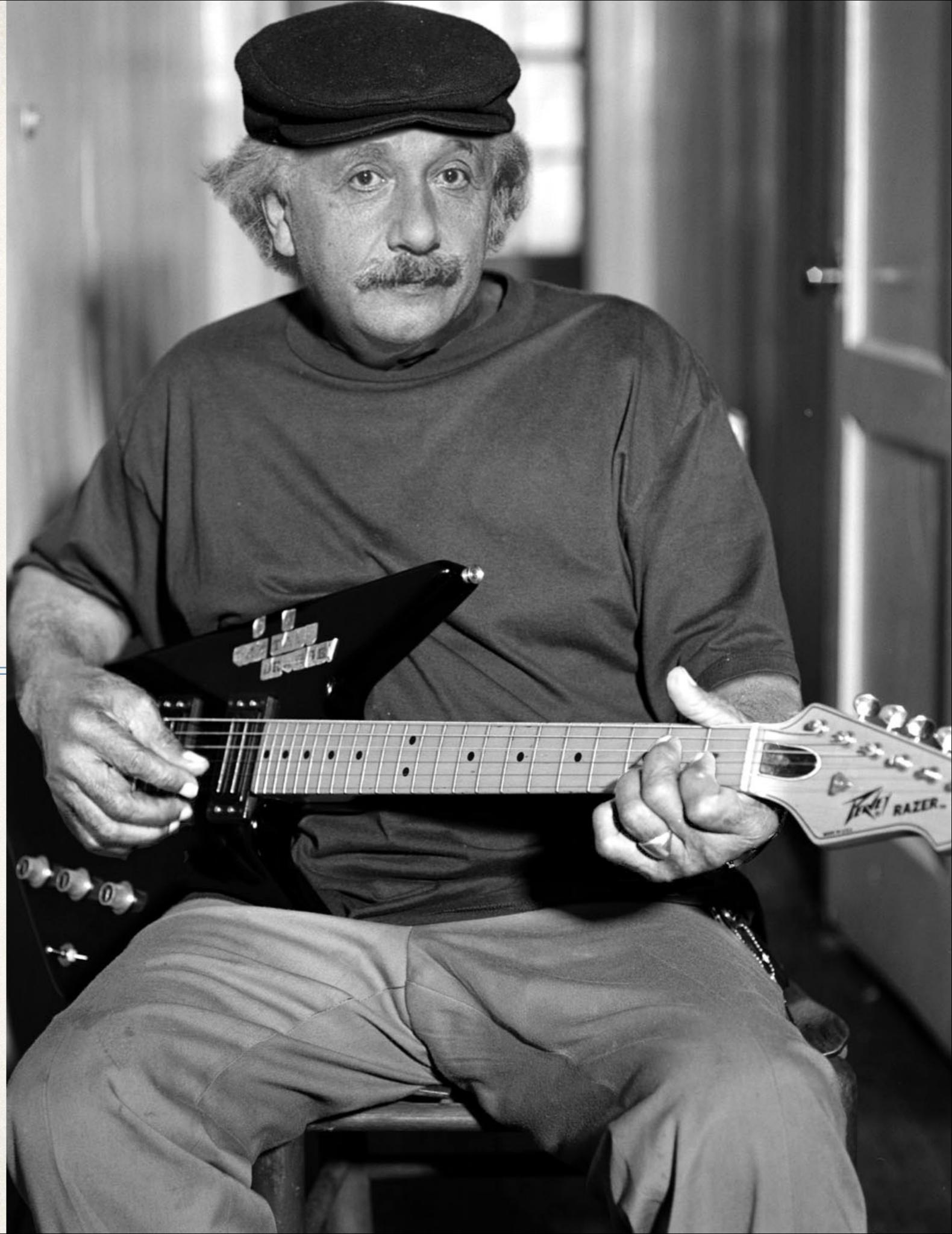
# What's Logic Good for?

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“Logic will bring  
you from A to B.  
Imagination will  
bring you  
everywhere.”

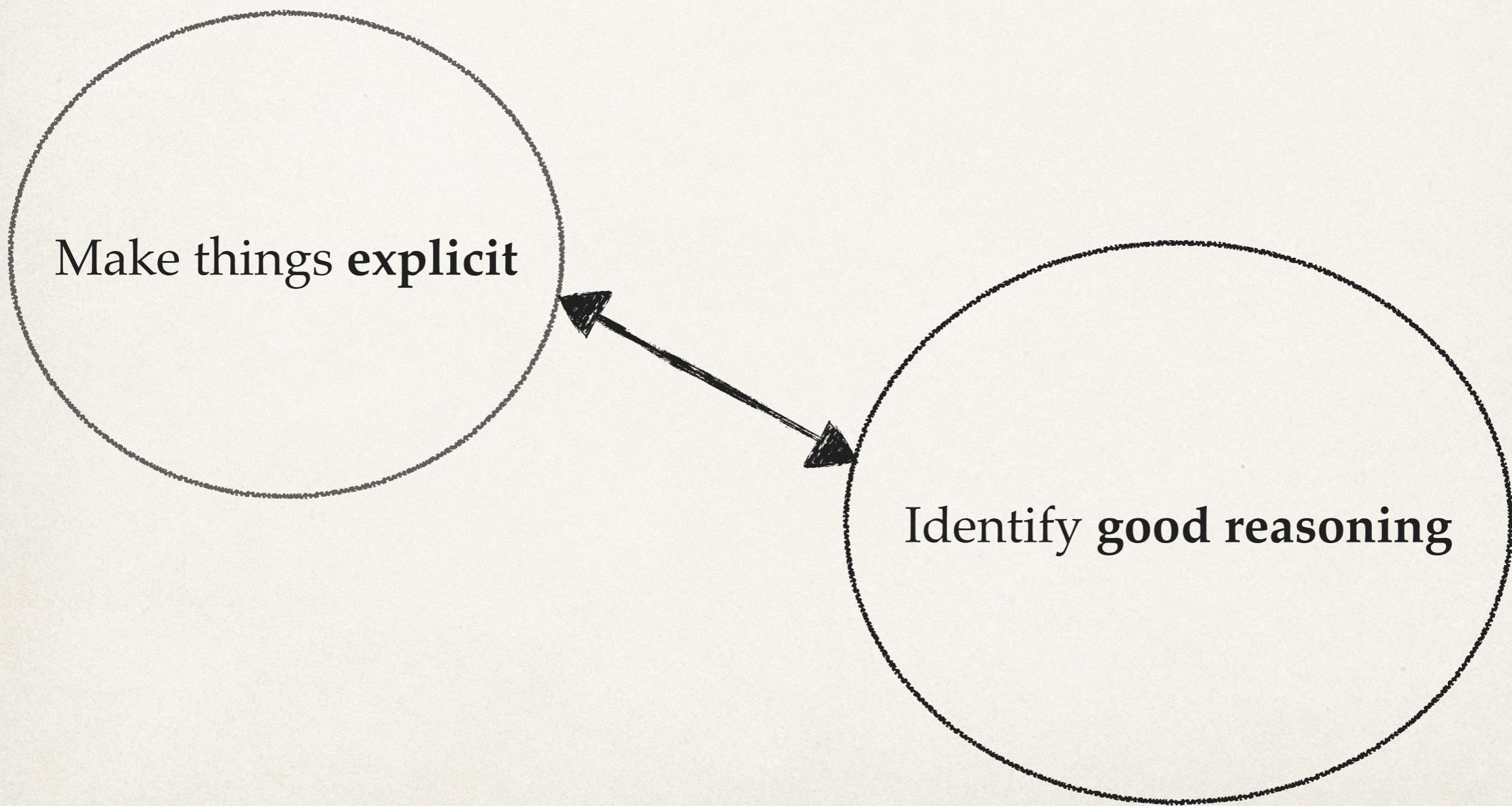
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Albert Einstein



# So...What's Logic Good For?

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# Making Things Explicit

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# How Difficult Can it Be to Crack an Egg?

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# A Standard Problem in Artificial Intelligence

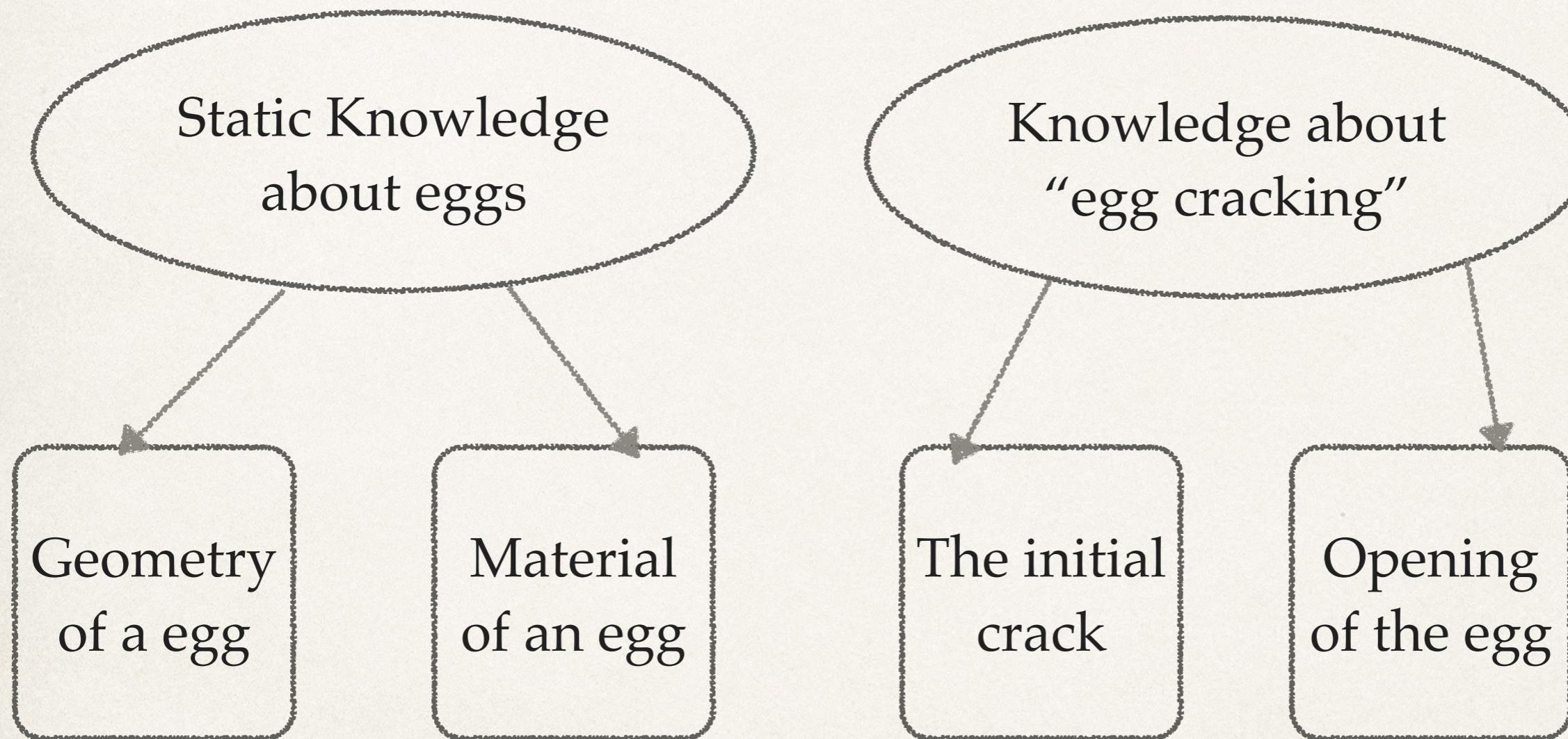
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*“A cook is cracking a raw egg against a glass bowl. Properly performed, the impact of the egg against the edge of the bowl will crack the eggshell in half. Holding the egg over the bowl, the cook will then separate the two halves of the shell with his fingers, enlarging the crack, and the contents of the egg will fall gently into the bowl. The end result is that the entire contents of the egg will be in the bowl, with the yolk unbroken, and that the two halves of the shell are held in the cook’s fingers.”*

**Source:** <http://www-formal.stanford.edu/leora/commonsense/>

# Cracking an Egg Isn't That Easy!

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By making this knowledge explicit, we can instruct a computer

# Identifying Good Reasoning

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# Good Reasoning (1)

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*Premise 1:* If you take the medication, then you will get better

*Premise 2:* You are taking the medication

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*Conclusion:* You will get better

*Modus Ponens:*

If A, then B

A

---

B

# Good Reasoning (2)

Premise 1: If you take the medication, then you will get better

Premise 2: You are NOT getting better

Conclusion: You are NOT taking the medication

*Modus Tollens:*

If A, then B  
not-B

\_\_\_\_\_  
not-A

**Good reasoning is**  
reasoning that conforms to a **good**  
**reasoning pattern**

# Monetarist Politics

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*Let's assume, given our knowledge of the US economy, that if the money supply were to increase at less than 5%, the rate of inflation would come down. Now, since the money supply is increasing at a rate well above 10%, we must conclude that inflation will not come down.*

Is this a good piece of reasoning?

# Good Reasoning?

Premise 1: If the money supply increases by less than 5%, inflation will decrease  
Premise 2: The money supply does NOT increase by less than 5%

Conclusion: Inflation will NOT decrease

If A, then B

not-A

\_\_\_\_\_

not-B

This reasoning does **not**  
seem to conform to a **good**  
**reasoning pattern**

# Which Patterns Are Good? (1)

---

If A, then B

A

B

If A, then B

not-B

not-A

If A, then B

not-A

not-B

If A, then B

B

A

Good

(Typically  
considered)  
**Bad**

# Which Patterns Are Good? (2)

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If A, then B  
If B, then C  
If C, then D  
A  
\_\_\_\_\_  
D

If A, then B  
If C, then D  
A and C  
\_\_\_\_\_  
B and D

If A, then B  
If C, then D  
not-B and not-D  
\_\_\_\_\_  
not-A and not-C

There is an infinite number of good reasoning patterns.  
*We cannot list them all!*

# Another Way to Identify Good Reasoning

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# A Murder Case

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- A1 At least one of them is guilty.
- A2 Not all of them are guilty.
- A3 If Mrs White is guilty, then Colonel Mustard helped her (he is guilty too).
- A4 If Miss Scarlet is innocent then so is Colonel Mustard.

# Listing All the Possibilities



innocent	innocent	innocent
innocent	innocent	guilty
innocent	guilty	innocent
innocent	guilty	guilty
guilty	innocent	innocent
guilty	innocent	guilty
guilty	guilty	innocent
guilty	guilty	guilty

# Ruling Out Possibilities

- A1 At least one of them is guilty.
- A2 Not all of them are guilty.
- A3 If Mrs White is guilty, then Colonel Mustard helped her (he is guilty too).
- A4 If Miss Scarlet is innocent then so is Colonel Mustard.



innocent	innocent	innocent
innocent	innocent	guilty
innocent	guilty	innocent
innocent	guilty	guilty
guilty	innocent	innocent
guilty	innocent	guilty
guilty	guilty	innocent
guilty	guilty	guilty

# Good Reasoning

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$$\checkmark \quad \frac{A_1, A_2, A_3, A_4}{}$$

Miss Scarlet is guilty

In **every** situation in which  $A_1, A_2, A_3$  and  $A_4$  are all true, “**Miss Scarlet is guilty**” is true.

$$\checkmark \quad \frac{A_1, A_2, A_3, A_4}{}$$

Mrs White is innocent

In **every** situation in which  $A_1, A_2, A_3$  and  $A_4$  are all true, “**Mrs White is innocent**” is true.

# Not So Good

---

$$\cancel{x} \quad \frac{A_1, A_2, A_3, A_4}{\text{Colonel Mustard is guilty}}$$

There is a situation in which  $A_1, A_2, A_3$  and  $A_4$  are all true, but “Colonel Mustard is guilty” is false (there is a counter-example).

$$\cancel{x} \quad \frac{A_1, A_2, A_3, A_4}{\text{Colonel Mustard is innocent}}$$

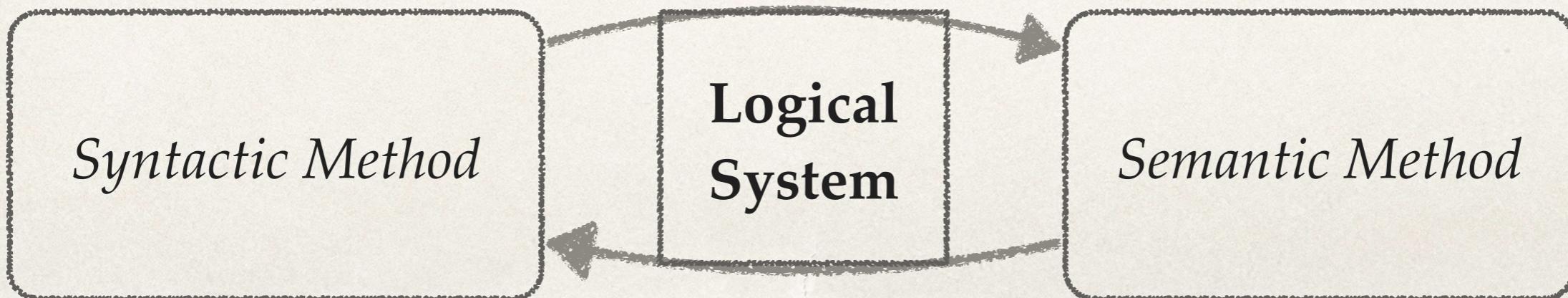
There is a situation in which  $A_1, A_2, A_3$  and  $A_4$  are all true, but “Colonel Mustard is innocent” is false (there is a counter-example).

# Two Ways to Identify Good Reasoning

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Check whether the proposed reasoning **conforms to a good reasoning pattern**.

List all the possibilities, rule out the possibilities that are excluded by the premises, and check whether **the putative conclusion is true in all the possibilities that are left**.



# From Lewis Carroll

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*There is no box of mine here that I dare open. My writing desk is made of rosewood. All my boxes are painted, except those that are here. There is no box of mine I dare not open, unless it is full of live scorpions. All my rosewood boxes are unpainted. Therefore...*

*Therefore...my desk is full of live scorpions!*