AN INTRODUCTION TO

Probability and Inductive Logic

IAN HACKING
University of Toronto
Collège de France

CAMBRIDGE
UNIVERSITY PRESS

1 Logic

Logic is about good and bad reasoning. In order to talk clearly about reasoning, logicians have given precise meanings to some ordinary words. This chapter is a review of their language.

ARGUMENTS

Logicians attach a special sense to the word *argument*. In ordinary language, it usually takes two to argue. One dictionary defines an argument as:

1 A quarrel.
2 A discussion in which reasons are put forward in support of and against a proposition, proposal, or case.
3 A point or series of reasons presented to support a proposition which is the conclusion of the argument.

Definition (3) is what logicians mean by an argument.

Reasoning is stated or written out in arguments. So logicians study arguments (in sense 3).

An argument thus divides up into:

A point or series of reasons which are called *premises*,
and a *conclusion*.

Premises and conclusion are *propositions*, statements that can be either true or false. Propositions are "true-or-false."

GOING WRONG

The premises are supposed to be reasons for the conclusion. Logic tries to understand the idea of a good reason.

We find arguments convincing when we know that the premises are true, and when we see that they give a good reason for the conclusion.

So two things can go wrong with an argument:

- the premises may be false.
- the premises may not provide a good reason for the conclusion.

Here is an argument:

(*J) If James wants a job, then he will get a haircut tomorrow.
James will get a haircut tomorrow.
So:
James wants a job.

The first two propositions are the premises. The third proposition is the conclusion.

Someone might offer this argument, thinking the premises give a conclusive reason for the conclusion. They do not. The premises could be true and the conclusion false, for any number of reasons. For example:

James has a date with a girl who likes tidy men, and his hair is a mess.
He has to go home to his family, who would be disgusted by how he looks.
It is the third Monday of the month, and he always gets a haircut then.
No way does he want a job! Of course, if he did want a job, he'd get a haircut tomorrow.

Argument (*J), if offered as a conclusive argument, commits an error—a common error. That is why we labeled it with a "star" in front, as a warning that it is a bad argument.

Argument (*J) commits a *fallacy*. A fallacy is an error in reasoning that is so common that logicians have noted it. Sometimes they give it a name. Argument (*J) commits the fallacy called "affirming the consequent." The first premise in the argument is of the form:

If A, then C.

A is called the *antecedent* of this "if-then" proposition, and C is called the *consequent*.

The second premise of (*J) is of the form "C." So in stating this premise, we "affirm the consequent."

The conclusion is of the form "A." It is a fallacy to infer the antecedent A from the consequent C. That is the fallacy of affirming the consequent.

TWO WAYS TO CRITICIZE

Here is a conclusive argument that only looks a little like (*J):

(J) If James wants a job, then he will get a haircut tomorrow.
James wants a job.
So:
James will get a haircut tomorrow.

Here the premises do provide a conclusive reason for the conclusion. If the premises are true, then the conclusion must be true too.

But you might question the premises.

You might question the first premise if you knew that James wants a job as a rock musician. The last thing he wants is a haircut.

You might also question the second premise. Does James really want a job?

There are two basic ways to criticize an argument:

- Challenge the premises—show that at least one is false.
- Challenge the reasoning—show that the premises are not a good reason for the conclusion.

The two basic types of criticism apply to any kind of argument whatsoever. But logic is concerned only with reasoning. It cannot in general tell whether premises are true or false. It can only tell whether the reasoning is good or bad.

VALIDITY

Here is another conclusive argument:

(K) Every automobile sold by Queen Street Motors is rust-proofed.
Barbara's car was sold by Queen Street Motors.
Therefore:
Barbara's car is rust-proofed.

If the two premises of (K) are true, then the conclusion must be true too. The same goes for (J) above. But not for (*J)!

This idea defines a valid argument. It is logically impossible for the conclusion to be false given that the premises are true.

Validity is best explained in terms of logical form. The logical form of arguments (J) and (K) is:

1. If *A*, then *C*.	4. Every *F* is *G*.
2. *A*.	5. *b* is *F*.
So:	Therefore:
3. *C*.	6. *b* is *G*.

Whenever an argument of one of these forms has true premises, then the conclusion is also true. That is a definition of a valid argument form.

Valid is a technical term of deductive logic. The opposite of *valid* is *invalid*. In ordinary life, we talk about a valid driver's license. We say someone is making a valid point if there is a basis for it, or even if it is true. But we will stick to the special, logicians' meaning of the word. Arguments are valid or invalid.

Argument (*J) above was invalid. Here is another invalid argument:

(*K) Every automobile sold by Queen Street Motors is rust-proofed.
Barbara's car is rust-proofed.
Therefore:
Barbara's car was sold by Queen Street Motors.

This is invalid because the conclusion could be false, even when the premises are true. Many companies sell rust-proofed cars, so Barbara need not have bought hers at Queen Street Motors.

TRUE VERSUS VALID

Be careful about *true* and *valid*. In logic:

> Propositions are true or false.
> Arguments are valid or invalid.

You should also distinguish the argument (K) about Barbara's car from an "if-then" or conditional proposition like this:

If Barbara's car was sold by Queen Street Motors, and if every automobile sold by Queen Street Motors is rust-proofed, then Barbara's car is rust-proofed.

This is a true proposition of the form,

If p and if q, then r.

Or, in finer detail,

If b is F, and if every F is G, then b is G.

Argument (K), on the other hand, is of the form:

4. p.	Or, in finer detail,	4. Every F is G.
5. q.		5. b is F.
So:		Therefore:
6. r.		6. b is G.

To every argument there is a corresponding conditional proposition "if-then." An argument is valid if and only if the corresponding conditional proposition is a truth of logic.

METAPHORS

There are many ways to suggest the idea of validity:

The conclusion follows from the premises.
Whenever the premises are true, the conclusion *must* be true too.
The conclusion is a logical consequence of the premises.
The conclusion is implicitly contained in the premises.
Valid argument forms are truth-preserving.

"Truth-preserving" means that whenever you start out with true premises, you will end up with a true conclusion.

When you reason from true premises using a valid argument, you never risk drawing a false conclusion. When your premises are true, there is no risk that the conclusion will be false.

Textbooks on deductive logic make precise sense of these metaphors. For the purposes of this book, one metaphor says best what matters for validity:

> Valid arguments are risk-free arguments.

SOUND

A *valid* argument never takes you from true premises to a false conclusion.

But, of course, the argument might have a false premise.

We say an argument is *sound* when:

- all the premises are true, and
- the argument is valid.

Thus an argument may be unsound because:

- A premise is false.
- The argument is invalid.

Validity has to do with the logical connection between premises and conclusion, and *not* with the truth of the premises or the conclusion.

Soundness for deductive logic has to do with *both* validity *and* the truth of the premises.

LIKE BUILDING A HOUSE

Making a deductive argument is like building a house.

- It may be built on sand, and so fall down, because the foundations are not solid. That is like having a false premise.

- Or it may be badly built. That is like having an invalid argument.
- And, of course, a house built on sand with bad design may still stay up. That is like an invalid argument with false premises and a true conclusion.

There are two ways to criticize a contractor who built a house. "The foundations are no good!" Or, "The house is badly built!" Likewise, if someone shows you a deduction you can make two kinds of criticism. "One of your premises is false." Or, "The argument is invalid." Or both, of course.

VALIDITY IS NOT TRUTH!

A valid argument can have a *false premise* but a *true conclusion*. Example:

(R) Every famous philosopher who lived to be over ninety was a mathematical logician.
Bertrand Russell was a famous philosopher who lived to be over ninety.
So:
Bertrand Russell was a mathematical logician.

This argument is valid. The conclusion is true.

But the first premise is false. Thomas Hobbes, the famous political philosopher, lived to be over ninety, but he was not a mathematical logician.

Likewise an argument with *false premises* and a *false conclusion* could be valid. Validity is about the connection between premises and conclusion, not about truth or falsehood.

INVALIDITY IS NOT FALSEHOOD!

An invalid argument can have *true premises* and a *true conclusion*. Example:

(*R) Some philosophers now dead were witty and wrote many books.
Bertrand Russell was a philosopher, now dead.
So:
Bertrand Russell was witty and wrote many books.

Both premises are true. The conclusion is true. But the argument is invalid.

TWO WAYS TO CRITICIZE A DEDUCTION

Both (R) and (*R) are unsound, but for quite different reasons.

You can tell that (*R) is unsound because it is invalid. You can tell it is invalid without knowing *anything* about Bertrand Russell (except that "Bertrand Russell" was someone's name).

Likewise, you can tell that (R) is valid without knowing anything about Bertrand Russell.

But to know whether the premises are true, you have to know something about the world, about history, about philosophers, about Bertrand Russell and others.

Maybe you did not know that Bertrand Russell was witty or that Thomas Hobbes was a famous political philosopher who lived to be over ninety. Now you do.

You need not know anything special about the world to know whether an argument is valid or invalid. But you need to know some facts to know whether a premise is true or false.

There are two ways to criticize a deduction:

- A premise is false.
- The argument is invalid.

So there is a *division of labor*.

Who is an expert on the truth of premises?

Detectives, nurses, surgeons, pollsters, historians, astrologers, zoologists, investigative reporters, you and me.

Who is an expert on validity?

A logician.

Logicians study the relations between premises and conclusions, but, as logicians, are not especially qualified to tell whether the premises are true or false.

EXERCISES

1 *Propositions.* The premises and conclusion of an argument are propositions. Propositions are expressed by statements that can be either true or false. For brevity, we say that propositions are true-or-false.

The headline of a newspaper story is:

SEIZED SERPENTS MAKE STRANGE OFFICE-FELLOWS
SHIPPING ERROR LANDS OFFICIAL WITH PYTHONS

There was a bizarre mix-up. A man who runs a tropical fish store in Windsor, Ontario, was delivered a box of ball pythons from a dealer in California. The newspaper tells us that:

The ball python is a central African ground dweller that can grow to more than a meter on a diet of small mammals.

(a) Is that true-or-false?
(b) Do you know whether it is true?
(c) Is it what logicians call a proposition? [You should give the same answer to (c) as to (a).]

The newspaper goes on to tell us that:

The ball python is named for its tendency to curl up into a ball.

(d) Is that true-or-false?
(e) Do you know whether it is true?

The story continues:

> The shipment of tropical fish intended for Windsor went to a snake dealer in Ohio.

(f) Is that a proposition?

In logic, propositions express matters of fact that can be either true or false. Judgments of personal taste, such as "avocados are delicious," are not strictly matters of fact. Avocados taste good to some people and taste slimy and disgusting to others. The proposition that avocados are delicious is not strictly speaking true-or-false. But if I say "avocados taste delicious to me," I am stating something about me, which happens to be true.

Joe, the man who owns the fish store, is quoted as saying:

> Ball pythons are very attractive animals.

(g) Is that true-or-false? Is it a proposition?

Suppose that he had said,

> I think ball pythons are very attractive animals.

(h) Is that true-or-false? Is it a proposition?

The newspaper begins the story by saying "It is not so nice to share your office with a box of snakes for two months." Then it adds, as a full paragraph:

> Especially when it was all a result of being soft-hearted.

(i) Is that a proposition?

Joe has to feed the snakes a lot of live mice. According to the reporter, Joe said,

> I'm not really too thrilled to hear baby mice squeaking and screaming behind me while I'm on the telephone.

(j) Is that a proposition?

Then Joe said,

> Thank God they don't eat every day!

(k) Is that a proposition?

He next asked,

> Do you know any zoos or schools who might want these snakes?

(l) Is that a proposition?

Joe phoned Federal Express, the shipper who had mixed up the deliveries, saying:

> You owe me for my expenses, my trouble, and your mistake.

(m) Is that a proposition?

The story ended happily:

> On Wednesday Federal Express bargained a $1000 payment to Joe.

(n) Is that a proposition?

2 *False all over.* State two arguments—they can be silly ones—in which the premises and conclusion are all false, and such that one argument is (a) valid and the other is (b) invalid.

3 *Unsound.* Is either of your answers to question 2 a sound argument?

4 *Combinations.* Only one of the following eight combinations is impossible. Which one?
(a) All premises true. Conclusion true. Valid.
(b) All premises true. Conclusion false. Valid.
(c) One premise false. Conclusion true. Valid.
(d) One premise false. Conclusion false. Valid.
(e) All premises true. Conclusion true. Invalid.
(f) All premises true. Conclusion false. Invalid.
(g) One premise false. Conclusion true. Invalid.
(h) One premise false. Conclusion false. Invalid.

5 *Soundness.* Which of the combinations just listed are sound arguments?

6 *Conditional propositions.* Which of the following is true-or-false? Which is valid-or-invalid? Which is an argument? Which is a conditional proposition?

(a) Tom, Dick, and Harry died.
So:
All men are mortal.

(b) If Tom, Dick, and Harry died, then all men are mortal.

7 *Chewing tobacco.* Which of these arguments are valid?

(a) I follow three major league teams. Most of their top hitters chew tobacco at the plate.
So:
Chewing tobacco improves batting average.

(b) The top six hitters in the National League chew tobacco at the plate.
So:
Chewing tobacco improves batting average.

(c) A study, by the American Dental Association, of 158 players on seven major league teams during the 1988 season, showed that the mean batting average for chewers was .238, compared to .248 for non users. Abstainers also had a higher fielding average.
So:
Chewing tobacco does not improve batting average.

(d) In 1921, every major league pitcher who chewed tobacco when up to bat had a higher batting average than any major league pitcher who did not.
So:
Chewing tobacco improves the batting average of pitchers.

8 *Inductive baseball.* None of the arguments (7a)–(7d) is valid. Invalid arguments are not conclusive. But some non-conclusive arguments are better than others. They are risky arguments. Each of the arguments (a)–(d) is risky. We have not done any inductive logic yet, but you probably think some of (7a)–(7d) are better arguments than others. Which is best? Which is worst?

KEY WORDS FOR REVIEW

Argument	Conclusion
Proposition	Valid
True-or-false	Sound
Premise	Conditional

2 What Is Inductive Logic?

Inductive logic is about risky arguments. It analyses inductive arguments using probability. There are other kinds of risky arguments. There is inference to the best explanation, and there are arguments based on testimony.

Valid arguments are risk-free. Inductive logic studies risky arguments. A risky argument can be a very good one, and yet its conclusion can be false, even when the premises are true. Most of our arguments are risky.

Begin with the big picture. The Big Bang theory of the origin of our universe is well supported by present evidence, but it could be wrong. That is a risk.

We now have very strong evidence that smoking causes lung cancer. But the reasoning from all that evidence to the conclusion "smoking causes lung cancer" is still risky. It might just turn out that people predisposed to nicotine addiction are also predisposed to lung cancer, in which case our inference, that smoking causes lung cancer, would be in question after all.

After a lot of research, a company concludes that it can make a profit by marketing a special left-handed mouse for personal computers. It is taking a risk.

You want to be in the same class as your friend Jan. You reason that Jan likes mathematics, and so will take another logic class. You sign up for inductive logic. You have made a risky argument.

ORANGES

Here are some everyday examples of risky arguments.

A small grocer sells her old fruit at half-price. I want a box of oranges, cheap. But I want them to be good, sweet, and not rotten. The grocer takes an orange from the top of a box, cuts it open, and shows it to me. Her argument is:

(A) This orange is good.
So:
All (or almost all) the oranges in the box are good.

The premise is evidence for the conclusion: but not very good evidence. Most of the oranges in the box may be rotten.

Argument (A) is not a valid argument. Even if the premise is true, the conclusion may be false. This is a risky argument.

If I buy the box at half-price on the strength of this argument, I am taking a big risk. So I reach into the box, pick an orange at random, and pull it out. It is good too. I buy the box. My reasoning is:

(B) This orange that I chose at random is good.
So:
All (or almost all) the oranges in the box are good.

This argument is also risky. But it is not as risky as (A).

Julia takes six oranges at random. One, but only one, is squishy. She buys the box at half-price. Her argument is:

(C) Of these six oranges that I chose at random, five are good and one is rotten.
So:
Most (but not all) of the oranges in the box are good.

Argument (C) is based on more data than (B). But it is not a valid argument. Even though five out of six oranges that Julia picked at random are fine, she may just have been lucky. Perhaps most of the remaining oranges are rotten.

SAMPLES AND POPULATIONS

There are many forms of risky argument. Arguments (A)–(C) all have this basic form:

Statement about a sample drawn from a given population.
So:
Statement about the population as a whole.

We may also go the other way around. I might know that almost all the oranges in this box are good. I pick four oranges at random to squeeze a big glass of orange juice. I reason:

All or almost all the oranges in this box are good.
These four oranges are taken at random from this box.
So:
These four oranges are good.

This too is a risky argument. I might pick a rotten orange, even if most of the oranges in the box are fine. The form of my argument is:

Statement about a population.
So:
Statement about a sample.

We can also go from sample to sample:

These four oranges that I chose at random are good.
So:
The next four oranges that I draw at random will also be good.

The basic form of this argument is:

Statement about a sample.
So:
Statement about a new sample.

PROPORTIONS

We can try to be more exact about our arguments. These are small juice oranges, 60 to the box. A cautious person might express "almost all" by "90%," and then the argument would look like this:

These four oranges, that I chose at random from a box of 60 oranges, are good.
So:
At least 90% (or 54) of the oranges in the box are good.

At least 90% (or 54) of the oranges in this box are good. These four oranges are taken at random from this box.
So:
These four oranges are good.

PROBABILITY

Most of us are happy putting a "probably" into these arguments:

These four oranges, that I chose at random from a box of 60 oranges, are good.
So, probably:
At least 90% (or 54) of the oranges in the box are good.

At least 90% (or 54) of the oranges in this box are good.
These four oranges are taken at random from this box.
So, probably:
These four oranges are good.

These four oranges, that I chose at random from a box of 60 oranges, are good.
So, probably:
The next four oranges that I draw at random will also be good.

Can we put in numerical probability values? That would be one way of telling which arguments are riskier than others. We will use ideas of probability to study risk.

Probability is a fundamental tool for inductive logic.

We will only do enough probability calculations to make ideas clear. *The focus in this book is on the ideas, not on the numbers.*

DEDUCING PROBABILITIES

Inductive logic uses probabilities. *But not all arguments using probabilities are inductive.* Not all arguments where you see the word "probability" are risky. Probability can be made into a rigorous mathematical idea. Mathematics is a deductive science. We make deductions using probability. In chapter 6 we state basic laws, or axioms, of probability. We *deduce* other facts about probability from these axioms.

Here is a simple deduction about probabilities:

> This die has six faces, labeled 1, 2, 3, 4, 5, 6.
> Each face is equally probable. (Each face is as likely as any other to turn up on a roll of the die.)
> So,
> The probability of rolling a 4 is 1/6.

This argument is valid. You already know this. Even if you have never studied probability, you make probabilities add up to 1.

You intuitively know that when the events are *mutually exclusive*—the die can land only one face up on any roll—and *exhaustive*—the die must land with one of the six faces up—then the probabilities add up to 1.

Why is the argument valid? Given the basic laws of probability, whenever the premises of an argument of this form are true, then the conclusion must be true too.

Here is another valid argument about probability.

> This die has six faces, labeled 1, 2, 3, 4, 5, 6.
> Each face is equally probable.
> So:
> The probability of rolling a 3 or a 4 is 1/3.

Even if you have never studied probability, you know that probabilities add up. If two events are *mutually exclusive*—one or the other can happen, but not both at the same time—then the probability that one or the other happens is the sum of their probabilities.

Given the basic laws of probability, whenever the premises of an argument of this form are true, then the conclusion must be true too. So the argument is valid.

The two arguments just stated are both valid. Notice how they differ from this one:

> This die has six faces, labeled 1, 2, 3, 4, 5, 6.
> In a sequence of 227 rolls, a 4 was rolled just 38 times.
> So:
> The probability of rolling a 4 with this die is about 1/6.

That is a risky argument. The conclusion might be false, even with true premises. The die might be somewhat biased against 4. The probability of rolling a 4 might be 1/8. Yet, by chance, in the last 227 rolls we managed to roll 4 almost exactly 1/6 of the time.

ANOTHER KIND OF RISKY ARGUMENT

Probability is a fundamental tool for inductive logic. But we have just seen that:

- There are also deductively valid arguments about probability.

Likewise:

- Many kinds of risky argument need not involve probability.

There may be more to a risky argument than inductive logic. Inductive logic does study risky arguments—but maybe not every kind of risky argument. Here is a new kind of risky argument. It begins with somebody noticing that:

> It is very unusual in our university for most of the students in a large elementary class to get As. But in one class they did.

That is odd. It is something to be explained. One explanation is that the instructor is an easy marker.

> Almost all the students in that class got As.
> So:
> The instructor must be a really easy marker.

Here we are *not* inferring from a sample to a population, or from a population to a sample.

We are offering a *hypothesis* to explain the observed facts. There might be other explanations. Almost all the students in that class got As,

> So:
> That was a very gifted class.

So:
The instructor is a marvelous teacher.
So:
The material in that course is far too easy for well-prepared students.

Each of these arguments ends with a *plausible explanation* of the curious fact that almost everyone in the class got an A grade.

Remember argument (*J) on page 2:

(*J) If James wants a job, then he will get a haircut tomorrow.
James will get a haircut tomorrow.
So:
James wants a job.

This is an invalid argument. It is still an argument, a risky argument. Let us have some more details. James gets his hair cut once in a blue moon. He is broke. You hear he is going to the barber tomorrow. Why on earth? Because he wants a job. The conclusion is a *plausible explanation.*

INFERENCE TO THE BEST EXPLANATION

Each of the arguments we've just looked at is an *inference to a plausible explanation.*

If one explanation is much more plausible than any other, it is an *inference to the best explanation.*

Many pieces of reasoning in science are like that. Some philosophers think that whenever we reach a theoretical conclusion, we are arguing to the best explanation. For example, cosmology was changed radically around 1967, when the Big Bang theory of the universe became widely accepted. The Big Bang theory says that our universe came into existence with a gigantic "explosion" at a definite date in the past. Why did people reach this amazing conclusion? Because two radio astronomers discovered that a certain low "background radiation" seems to be uniformly distributed everywhere in space that can be checked with a radio telescope. The best explanation, then and now, is that this background radiation is the result of a "Big Bang."

"ABDUCTION"

One philosopher who thought deeply about probability was Charles Sanders Peirce (1839–1914). Notice that it is spelled PEIrce. His name is not "Pierce." Worse still, his name is correctly pronounced "purse"! He came from an old New England family that spelled their name "Pers" or "Perse."

Peirce liked things to come in groups of three. He thought that there are three types of good argument: deduction, induction, and inference to the best explanation. Since he liked symmetries, he invented a new name for inference to the best explanation. He called it *abduction.* So his picture of logic is this:

	Deduction
Logic	Induction
	Abduction

Induction and abduction are, in his theory, two distinct types of risky argument.

Some philosophers believe that probability is a very useful tool in analyzing arguments to the best explanation. Other philosophers, like Peirce, do not think so. There is a debate about that. We leave that debate to philosophers of science. The issues are very interesting, but this book will not discuss inference to the best explanation.

TESTIMONY

Most of what you believe, you believe because someone told you so.

How reliable are your parents? Your psychology instructor? The evening news? Believing what they say involves risky arguments.

I know I was born on February 14, because my mother told me so.
So:
I was born on February 14.

My psychology instructor says that Freud was a fraud, and is a worthless guide to human psychology.
So:
Freud is a worthless guide to human psychology.

According to the evening news, the mayor is meeting with out-of-town officials to discuss the effect of the flood.
So:
The mayor is meeting with out-of-town officials to discuss the effect of the flood.

These are risky arguments. The evening news may be misinformed. Your psychology instructor may hate Freud, and be a very biased informant.

The argument about your birthday is the least risky. It is still risky. How do you know that your parents are telling the truth?

You look at your birth certificate. You can't doubt that! Well, maybe your parents lied by a day, so they could benefit from a new law about child benefits that took effect the day after you were born. Or maybe you were born on Friday the thirteenth, and they thought it would be better if you thought you were born on Valentine's Day. Or maybe you were born on a taxi ride to the hospital, and in the excitement no one noticed whether you were born before or after midnight . . .

All the examples are arguments based on the *testimony* of someone else: your family, your instructor, the evening news.

Some kinds of testimony can be analyzed using probability, but there are a lot of problems. Inductive logic touches on testimony, but there is a lot more to testimony than probability.

In this book we will *not* discuss inference to the best explanation, and we will *not* discuss testimony. But if you really want to understand risky arguments, you should think about testimony, and inference to the best explanation. In this book we study only one side of probability.

ROUGH DEFINITION OF INDUCTIVE LOGIC

> Inductive logic analyzes risky arguments using probability ideas.

DECISION THEORY

There is a whole other side to reasoning: *decision.* We don't just reason about what to believe.

We reason about what to do.

The probability theory of practical reasoning is called *decision theory,* and it is very close to inductive logic.

We decide what to do on the basis of two ingredients:

- What we think will probably happen (*beliefs*).
- What we want (*values*).

Decision theory involves both probabilities and values. We measure values by what are called *utilities.*

ROUGH DEFINITION OF DECISION THEORY

> Decision theory analyzes risky decision -making using ideas of probability and utility.

EXERCISES

1 *Fees.* With a budgetary crisis, administrators at Memorial University state that they must either increase fees by 35% or increase class sizes and limit course offerings. Students are asked which option they prefer. There is a sharp difference of opinion.

Which of these risky arguments is from sample to population? From population to sample? From sample to sample?

(a) The student body as a whole is strongly opposed to a major fee increase.
65 students will be asked about the fee increase.
So:
Most of the 65 students will say that they oppose a major fee increase.

(b) A questionnaire was given to 40 students from all subjects and years.
32 said they were opposed to a major fee increase.
So:
Most students are opposed to a major fee increase.

(c) The student body as a whole is strongly opposed to a major fee increase.
So (probably):
The next student we ask will oppose a major fee increase.

(d) A questionnaire was a given to 40 students from all subjects and years.
32 said they were opposed to a major fee increase.
So (probably):
The next student we ask will oppose a major fee increase.

2 *More fees.* Which of these is an inference to a plausible explanation? Which is an inference based on testimony?

(a) The student body as a whole is strongly opposed to a major fee increase.
So:
They prefer to save money rather than get a quality education.

(b) The student body as a whole is strongly opposed to a major fee increase.
So:
Many students are so poor, and loans are so hard to get, that many students would have to drop out of school if fees went up.

(c) Duodecimal Research Corporation polled the students and found that 46% are living below the official government poverty line.
So:
The students at Memorial cannot afford a major fee increase.

3 Look back at the Odd Questions on pages xv–xvii. Each question will be discussed later on. But regardless of which answer is correct, we can see that any answer you give involves an argument.

3.1 *Boys and girls.* Someone argues:

About as many boys as girls are born in hospitals.
Many babies are born every week at City General.
In Cornwall, a country town, there is a small hospital where only a few babies are born every week.
An unusual week at a hospital is one where more than 55% of the babies are girls, or more than 55% are boys.
An unusual week occurred at either Cornwall or City General Hospital last week.
So:
The unusual week occurred at Cornwall Hospital.

Explain why this is a risky argument.

3.2 *Pia.* The premises are as stated in Odd Question 2.
Which is the riskier conclusion, given those premises?
(a) Pia is an active feminist.
(e) Pia is a bank teller and an active feminist who takes yoga classes.

3.3 *Lotteries.* Your aunt offers you as a present one of two free Lotto 6/49 tickets for next week's drawing. They are:

A. 1, 2, 3, 4, 5, and 6.
B. 39, 36, 32, 21, 14, and 3.

(a) Construct an argument for choosing (A). If you think it is stupid to prefer (A) over (B), then you can produce a bad or weak argument! But try to make it plausible.
(b) You decide to take (A). Is this a risky decision?

3.4 *Dice.*

Two dice are fair: each face falls as often as any other, and the number that falls uppermost with one die has no effect on the number that falls uppermost with the other die.

So:

It is more probable that 7 occurs on a throw of these two dice, than 6.

Is this a risky argument?

3.5 *Taxicabs.* Amos and Daniel are both jurors at a trial. They both hear the same information as evidence, namely the information stated in Odd Question 5. In the end, they have to make a judgment about what happened.

Amos concludes: So, the sideswiper was blue.
Daniel concludes: So, the sideswiper was green.

(a) Are these risky arguments?
(b) Could you think of them as risky decisions?

3.6 *Strep throat.* The physician has the information reported in Odd Question 6. She concludes that the results are worthless, and sends out for more tests. Explain why that is a risky decision.

4 *Ludwig van Beethoven.*
(a) What kind of argument is this? How good is it?

Beethoven was in tremendous pain during some of his most creative periods—pain produced by cirrhosis of the liver, chronic kidney stones (passing a stone is excruciatingly painful), and bouts of nonstop diarrhea.
Yet his compositions are profound and often joyous.
So:
He took both pain killers and alcohol, and these drugs produced states of elation when he did his composing.

(b) Give an example of a new piece of information which, when added to the premises, strengthens the argument.

Books on "critical thinking" teach you how to analyze real-life complicated arguments. Among other things, they teach you how to read, listen, and think critically about the things that people actually say and write. This is not a book for critical thinking, but it is worth looking at a few real-life arguments. All are taken from a daily newspaper.

5 *The slender oarfish.*

A rare deep-sea creature, the slender oarfish, is helping Japanese scientists predict major earthquakes. In Japanese folklore, if an oarfish, which normally lives at depths of more than 200 meters, is landed in nets, then major tremors are not far behind.

Two slender oarfish were caught in fixed nets recently only days before a series of earthquakes shook Japan. This reminds us that one of these fish was caught two days before a major earthquake hit Nijima Island, near Tokyo, in 1963. Moreover, when shock waves hit Uwajima Bay in 1968, the same type of rare fish was caught.

The oarfish has a unique elongated shape, which could make it susceptible to underwater shock waves. It may be stunned and then float to the surface. Or the real reason could be that poisonous gases are released from the Earth's crust during seismic activity. At any rate, whenever an oarfish is netted, a geological upheaval is in progress or about to occur.

And, having just caught some slender oarfish, Japanese seismologists are afraid that another disaster is imminent.

(a) In the first paragraph, there is a statement based on testimony. What is it? On what testimony is it based?
(b) The third paragraph states one conclusion of the entire discussion. What is the conclusion?
(c) The second paragraph states some evidence for this conclusion. Would you say that the argument to the conclusion (b) is more like an argument from population to sample, or from sample to population?
(d) The third paragraph offers two plausible explanations for the facts stated in the second paragraph. What are they?
(e) There are several distinct arguments leading to the final conclusion in the fourth paragraph. Describe how the arguments fit together.

6 *Women engineers.*

Since 1986, only 11% of engineering school graduates have been women. That showing is particularly poor considering that in other formerly male-dominated fields there are signs of real progress. Some examples from 1986: law, 48%; commerce, 44%; medicine, 45%; and in the biological sciences, nearly 50% of the graduates are women.

(a) What is the conclusion? (b) What kind of argument is it? Valid? Inductive and risky? Inference to a plausible explanation?

7 *Plastic surgery.*

In her private counseling service for women, Martha Laurence, a professor of social work, tries to get behind the reasons women give for wanting plastic surgery. "Usually it is because they have a lack of confidence in who they are, the way they are," she said. "There is no simple answer, but the real problem is one of equity and of women's control over the self."

Her conclusion is that "the real problem is one of equity and of women's control over the self." What type of argument does she have for this conclusion?

8 *Manitoba marijuana.*

Basement operations are sprouting up in rural Manitoba to supply hydroponically grown marijuana for the Winnipeg market, police say. As Constable Duane Rhone of the rural Selkirk community of Winnipeg said in a recent interview, "It's cheap, it's easy to set up and there is a high return on investment. You can produce more marijuana of a better quality in a small amount of space," he said, adding that the necessary equipment is readily available. "It's become the thing to do. We've been seeing a lot more of this hydroponics marijuana in the last little while. There must be plenty more of these operators that we don't know about."

Conclusion: There are many as-yet undiscovered marijuana growers in rural Manitoba.

What kind of argument is Constable Rhone offering?

KEY WORDS FOR REVIEW

Population
Inference to the best explanation
Inductive logic
Sample
Testimony
Decision Theory

3 The Gambler's Fallacy

Most of the main ideas about probability come up right at the beginning. Two major ones are **independence** and **randomness**. Even more important for clear thinking is the notion of a **probability model**.

ROULETTE

A gambler is betting on what he thinks is a *fair* roulette wheel. The wheel is divided into 38 segments, of which:

- 18 segments are black.
- 18 segments are red.
- 2 segments are green, and marked with zeroes.

If you bet $10 on red, and the wheel stops at red, you win $20. Likewise if you bet $10 on black and it stops at black, you win $20. Otherwise you lose. The house always wins when the wheel stops at zero.

Now imagine that there has been a long run—a dozen spins—in which the wheel stopped at black. The gambler decides to bet on red, because he thinks:

> The wheel must come up red soon.
> This wheel is fair, so it stops on red as often as it stops on black.
> Since it has not stopped on red recently, it must stop there soon. I'll bet on red.

The argument is a risky one. The conclusion is, "The wheel must stop on red in the next few spins." The argument leads to a risky decision. The gambler decides to bet on red. There you have it, an argument and a decision. Do you agree with the gambler?

Since this chapter is called "the gambler's fallacy" there must be something wrong with the gambler's argument. Can you say what?