

Solution to Taxi Cabs Here are some abbreviations:

B means “the taxi that hit the passerby belonged to Blue Cabs Inc.”

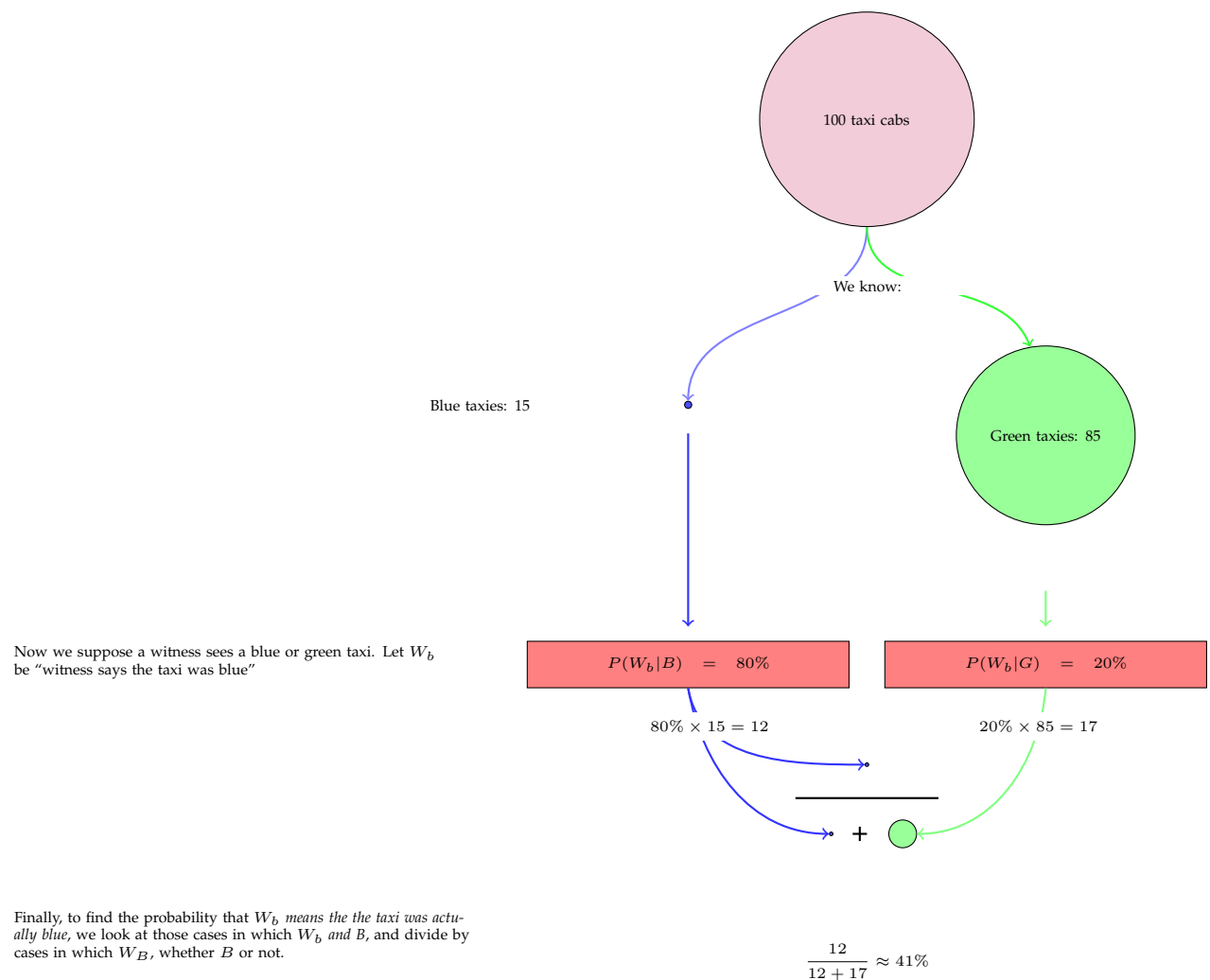
G means “the taxi that hit the passerby belonged to Green Cabs Inc.”

W_b means “the witness says the taxi that hit the passerby was blue”

W_g means “the witness says the taxi that hit the passerby was green”

We need to calculate $P(B|W_b)$. We are told that the witness is 80% reliable, that is, $P(W_b|B) = 80\%$ and $P(W_g|G) = 80\%$. So, $P(W_b|G) = 20\%$ and $P(W_g|B) = 20\%$. But, the problem does not give us information about how many taxis belonging to Green Cabs Inc. and Blue Cabs Inc. there are. We cannot calculate the required probability unless we have this information. The problem is not solvable.

To make the problem solvable, suppose that Green Cabs Inc. covers 85 % of the market and Blue Cabs Inc. covers the rest. Given this assumption, $P(B) = 85\%$ and $P(G) = 15\%$. We can now solve the problem by counting cases. Consider 100 taxi cabs, as follows:



We can arrive at the same result by Bayes’ theorem, as follows:

$$P(B|W_b) = \frac{P(W_b|B)P(B)}{P(W_b)} = \frac{P(W_b|B)P(B)}{P(W_b|B)P(B) + P(W_b|G)P(G)} = \frac{80\% \times 15\%}{80\% \times 15\% + 20\% \times 85\%} \approx 41\%.$$