PROBABILITY BASICS

MARCELLO DI BELLO

1. THE MEANING OF PROBABILITY

What do we mean when we say *there is x percent chance/probability that...*?

There are different interpretations of probability, such as:
classical
frequency
epistemic

$\boxed{\text{QUESTION:}}$ Which interpretation is the most fitting for gambling? Weather forecasting? Medical diagnoses? Trial verdicts?

2. THE MATHEMATICS OF PROBABILITY

P is a probability function provided:

(normality) $0 \leq P(A) \leq 1$, for any proposition A;

(certaintity) $P(\top) = 1$, with $\top$ any tautology; and

(additivity) $P(A \vee B) = P(A) + P(B)$, with A, B inconsistent propositions.

The *conditional probability* of A given some other proposition B is defined as follows:

(conditional probability) $P(A|B) = \frac{P(A \wedge B)}{P(B)}$.

Simple corollaries follow, such as:

(negation) $P(\neg A) = 1 - P(A)$;

(product) $P(A \wedge B) = P(A|B)P(B)$;

(product*) $P(A \wedge B) = P(A)P(B)$ if $P(A|B) = P(A)$;

$\boxed{\text{Notation:}}$ The symbol '$\neg$' stands for negation, '$\wedge$' for conjunction, and '$\vee$' for disjunction.

3. BAYES' THEOREM

Confusing $P(A|B)$ and $P(B|A)$ is known as the *inversion fallacy*. Bayes' theorem shows how the two probabilities are related, as follows:

$$P(A|B) = \frac{P(B|A)}{P(B)} P(A) = \frac{P(B|A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)} P(A).$$

Bayes' theorem allows us to calculate the *conditional* probability of A given B from:
(i) the probability $P(A)$ regardless of B;
(ii) the probability $P(B)$, where $P(B) = P(B|A)P(A) + P(B|\neg A)P(\neg A)$;
(iii) the *likelihood* $P(B|A)$, i.e. the probability of B given A.