

PHI 169 – CRITICAL REASONING - FALL 2017 – PROBABILITY QUIZ #2

QUIZ

Consider the statistics by the Federal Bureau of Justice Statistics. As convenient notation, let WM , BM and HM stand for “a person is a white male”, “a person is a black male” and “a person is a hispanic male”, respectively. Let I stand for “a person, male or female, is incarcerated”. Calculate the following probabilities or answer the following questions:

- (a) $P(I)$ in 2001 and $P(I)$ in 1974. Did the probability of being incarcerated increase or decrease? If yes, by how much?
(NB: Look up on-line the total US population in 2001 and 1974 to answer this question.)
- (b) $P(WM|I)$ in 2001.
- (c) $P(BM|I)$ in 2001.
- (d) If one is incarcerated in 2001, is this person more probably a white male or a black male?
- (e) What are people saying—exactly—when they say that blacks are disproportionately represented in the incarcerated population? Express this using the language of conditional probabilities. *Hint*: think about the difference between $P(I|BM)$ and $P(BM|I)$ and compare them with $P(I|WM)$ and $P(WM|I)$.

SOLUTIONS

- (a) By looking up on Google, one finds:

- total US population in 1974 was 213.9 million
- total US population in 2001 was 285 million

By looking at the statistics from FBJS, one finds:

- US population incarcerated in 1974 was 1.819 million
- US population incarcerated in 2001 was 5.618 million

The required probabilities can now be calculated, keeping in mind that $P = (\dots) = \frac{\text{PART}}{\text{WHOLE}}$

- in 1974 $P(I) = \frac{1.819}{213.9} = 0.0085039731 \approx 0.8\%$
- in 2001 $P(I) = \frac{5.618}{285} = 0.0197122807 \approx 2\%$

The probability increased by a factor of $\frac{0.02}{0.008} = 2.5$

- (b) The rule is $P(A|B) = \frac{P(A \& B)}{P(B)}$. Our B is I (“a person, male or female, is incarcerated”). Similarly, $A \& B$ in this case is $WM \& I$. So, we have
- $P(WM|I) = \frac{P(WM \& I)}{P(I)}$.

We know that in 2001 $P(I) = \frac{5.618}{285} = 0.0197122807 \approx 2\%$. We need to calculate $P(WM \& I)$. From the statistics by the FBJS, one finds that in 2001 white males incarcerated, i.e. $WM \& I$, were 1.978 million. So, in 2001 $P(WM \& I) = \frac{1.978}{285} = 0.00694035087 \approx 0.7\%$

By putting everything together, we get

$$P(WM|I) = \frac{P(WM \& I)}{P(I)} = \frac{\frac{1.978}{285}}{\frac{5.618}{285}} = \frac{0.00694035087}{0.0197122807} \approx 0.35 \approx 35\%.$$

You could have done this faster, that is, $P(WM|I) = \frac{1.978}{5.618} \approx 0.35 \approx 35\%$.

- (c) Follow the same procedure but replace WM with BM . From the FBJIS we know that in 2001 white males incarcerated, i.e. $WM \& I$, were 1.936 million.

$$\text{So, } P(BM|I) = \frac{P(WM \& I)}{P(I)} = \frac{\frac{1.936}{285}}{\frac{5.618}{285}} = \frac{0.00679298245}{0.0197122807} \approx 0.34 \approx 34\%.$$

You could have done this faster, that is, $P(BM|I) = \frac{1.936}{5.618} \approx 0.34 \approx 34\%$.

- (d) The person is more likely to be white male even though by a slight margin.
- (e) The claim about racial disproportion is $P(I|BM) > P(I|WM)$, although $P(BM|I) < P(WM|I)$. Black make up 12% of the US total population. Half should be black males.

$$\text{So, } P(I|BM) = \frac{P(I \& BM)}{P(BM)} = \frac{\frac{1.936}{285}}{0.06} = \frac{0.00679298245}{0.06} = 0.11321637416 \approx 11\%$$

Non-hispanic white in the US are 62% of the population. Half should be white males.

$$\text{So, } P(I|WM) = \frac{P(I \& WM)}{P(WM)} = \frac{\frac{1.978}{285}}{0.31} = \frac{0.00694035087}{0.31} = 0.02238822861 \approx 2\%$$

In 2011 black males were more likely to be incarcerated than white males by a factor of $\frac{11}{2} = 5.5$. This is what is meant by racial disproportion among incarcerated people.