$\mathcal{E}_n$ $L \in \mathcal{E}_n$

$|\mathcal{E}_n| = n!$ $Pr_n(L) = \frac{1}{n!}$

X_n

$\mathcal{E}_n$ $\qquad$ $L \in \mathcal{E}_n$

$|\mathcal{E}_n| = n!$ $\qquad$ $Pr_n(L) = \frac{1}{n!}$

$X_n(L)$

$\mathcal{E}_n$ $\quad L \in \mathcal{E}_n$

$|\mathcal{E}_n| = n!$ $\quad Pr_n(L) = \frac{1}{n!}$

$X_n(L)$

$E(X_n)$

$\mathcal{E}_n$ $\qquad L \in \mathcal{E}_n$

$|\mathcal{E}_n| = n!$ $\qquad Pr_n(L) = \frac{1}{n!}$

$X_n(L)$

$E(X_n)$

$\mathcal{E}_n$

$L \in \mathcal{E}_n$

$[3, 2, 1]$

$|\mathcal{E}_n| = n!$

$Pr_n(L) = \frac{1}{n!}$

$X_n(L)$

$Y_{i,j}(L)$

$Y_{2,3}$

$E(X_n)$

$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$X_n(L)$

$E(X_n)$

$L \in \mathcal{E}_n$

$Pr_n(L) = \frac{1}{n!}$

$Y_{i,j}(L)$

$X_n(L) =$

$[3, 2, 1]$

$Y_{2,3}$

$\mathcal{E}_n$

$L \in \mathcal{E}_n$

$[3, 2, 1]$

$|\mathcal{E}_n| = n!$

$Pr_n(L) = \frac{1}{n!}$

$X_n(L)$

$Y_{i,j}(L)$

$Y_{2,3}$

$E(X_n)$

$$X_n(L) = \sum_{i=1}^{n} \sum_{j=i}^{n} Y_{i,j}(L)$$

$\mathcal{E}_n$

$L \in \mathcal{E}_n$

$[3, 2, 1]$

$|\mathcal{E}_n| = n!$

$Pr_n(L) = \frac{1}{n!}$

$X_n(L)$

$Y_{i,j}(L)$

$Y_{2,3}$

$Y_{i,i}(L) = 0$

$E(X_n)$

$$X_n(L) = \sum_{i=1}^{n} \sum_{j=i}^{n} Y_{i,j}(L)$$

$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$X_n(L)$

$E(X_n)$

$L \in \mathcal{E}_n$

$Pr_n(L) = \frac{1}{n!}$

$Y_{i,j}(L)$

$X_n(L) = \sum_{i=1}^{n} \sum_{j=i}^{n} Y_{i,j}(L)$

$[3, 2, 1]$

$Y_{2,3}$

$Y_{i,i}(L) = 0$

$\mathcal{E}_n$

$L \in \mathcal{E}_n$

$[3, 2, 1]$

$|\mathcal{E}_n| = n!$

$Pr_n(L) = \frac{1}{n!}$

$X_n(L)$

$Y_{i,j}(L)$

$Y_{2,3}$

$Y_{i,i}(L) = 0$

$E(X_n)$

$$X_n(L) = \sum_{i=1}^{n} \sum_{j=i}^{n} Y_{i,j}(L)$$

$\mathcal{E}_n$

$L \in \mathcal{E}_n$

$[3, 2, 1]$

$|\mathcal{E}_n| = n!$

$Pr_n(L) = \frac{1}{n!}$

$X_n(L)$

$Y_{i,j}(L)$

$Y_{2,3}$

$Y_{i,i}(L) = 0$

$E(X_n)$

$$X_n(L) = \sum_{i=1}^{n} \sum_{j=i}^{n} Y_{i,j}(L)$$

Σ_n

$|\Sigma_n| = n!$

$X_n(L)$

$E(X_n)$

$Y_{1,n}(L)$

$[3, 2, 1]$

$Y_{2,3}$

$Y_{i,i}(L) = 0$

Σ_n

$|\Sigma_n| = n!$

$X_n(L)$

$E(X_n)$

$Y_{1,n}(L)$

[ ]

[3, 2, 1]

$Y_{2,3}$

$Y_{i,i}(L) = 0$

$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$X_n(L)$

$E(X_n)$

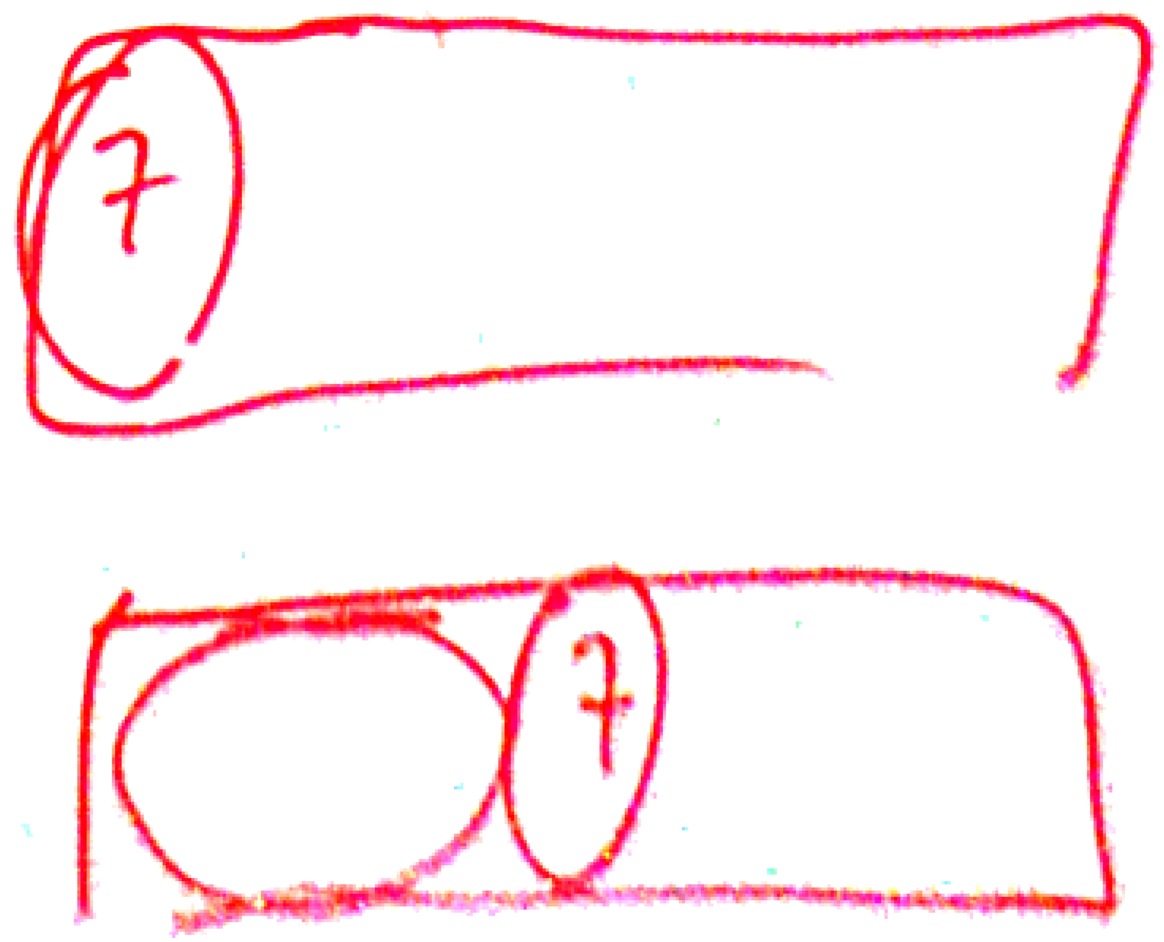
$Y_{1,n}(L)$

$[1 \quad\quad]$

$[3, 2, 1]$

$Y_{2,3}$

$Y_{i,i}(L) = 0$



Σ_n

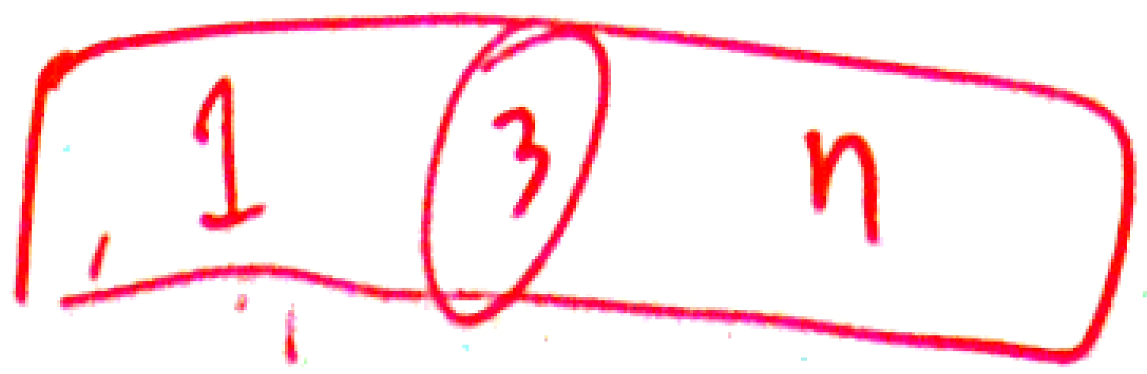
$|\Sigma_n| = n!$

$X_n(L)$

$E(X_n)$

$Y_{1,n}(L)$

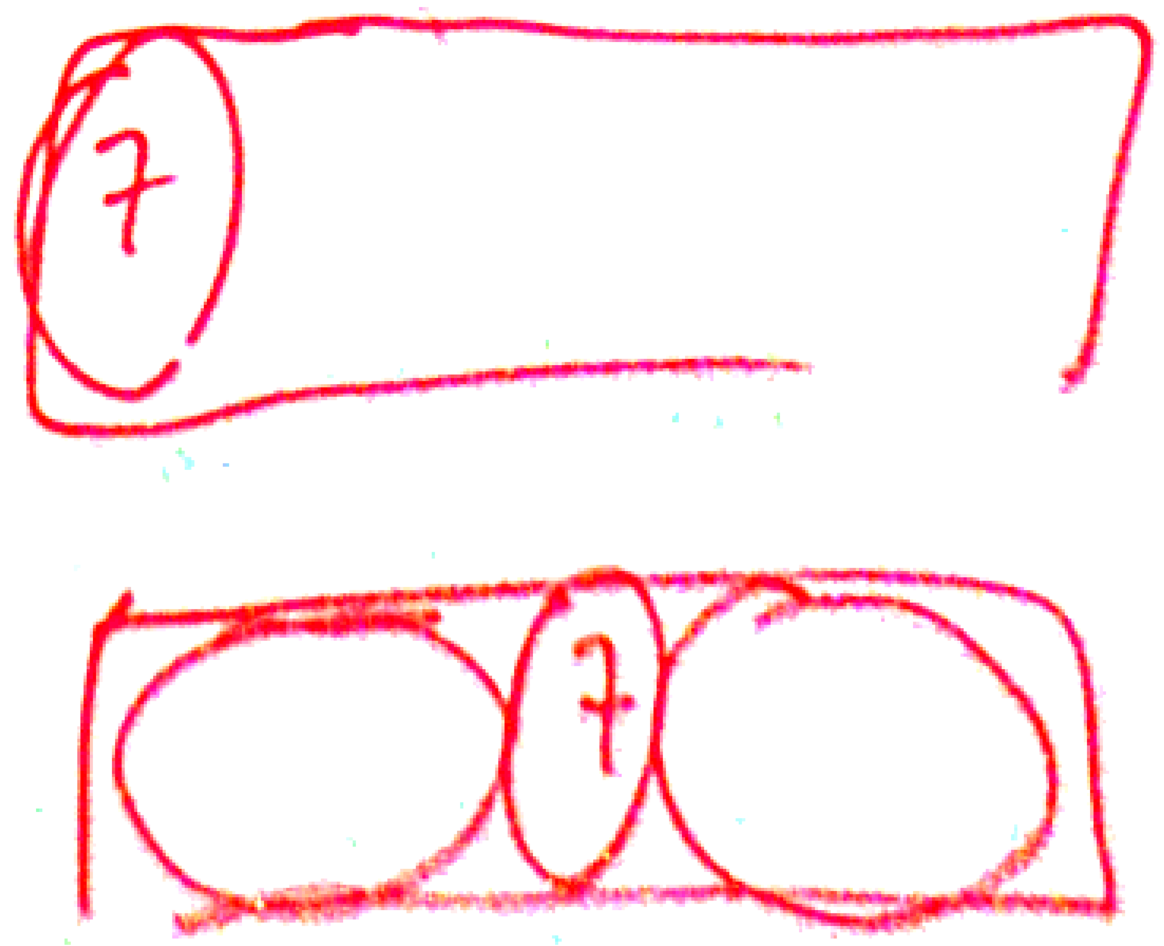
$[3 \quad 1 \quad n]$



$[3, 2, 1]$

$Y_{2,3}$

$Y_{i,i}(L) = 0$



Σ_n

$|\Sigma_n| = n!$

$X_n(L)$

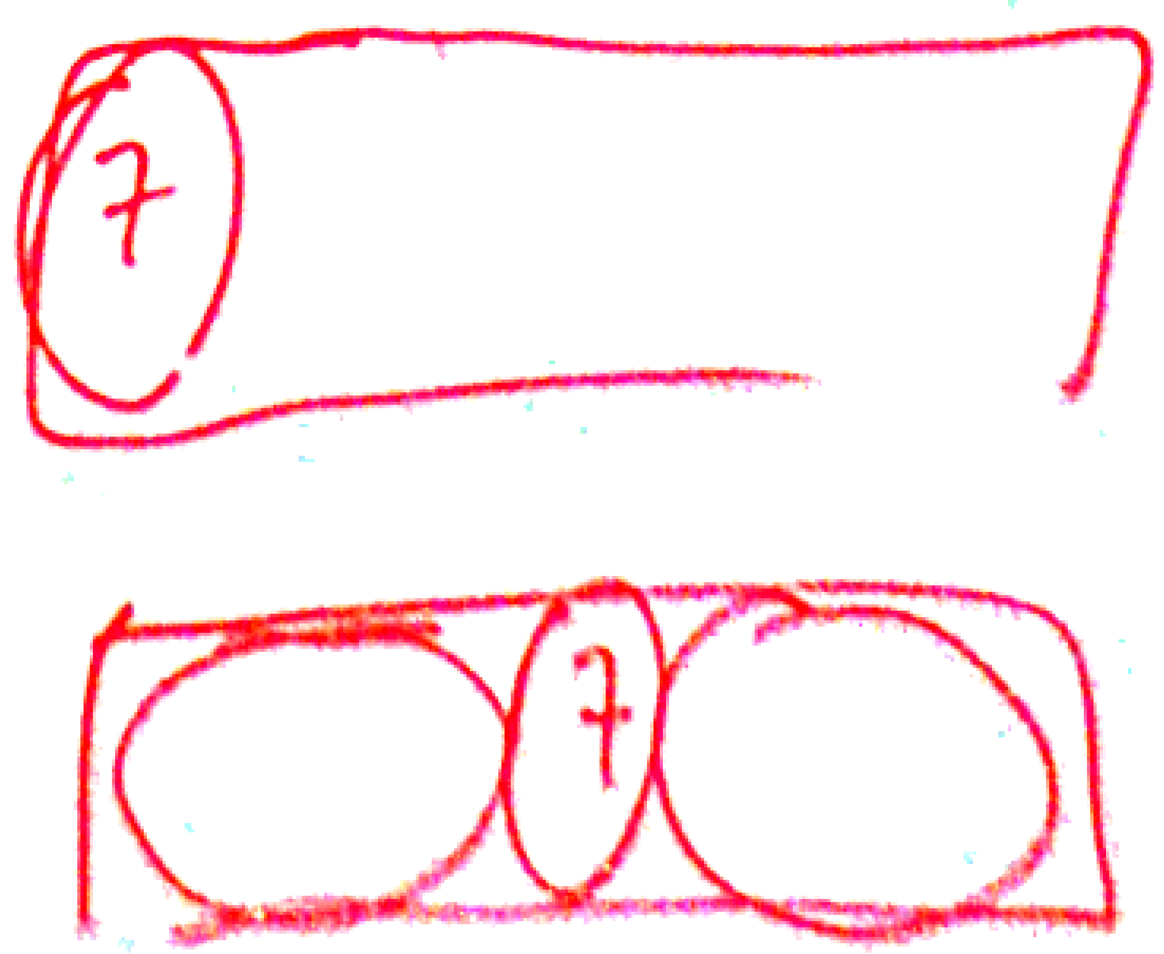
$E(X_n)$

$$Y_{1,n}(L) = \begin{cases} 1 \\ 0 \end{cases}$$

$$Pr(Y_{1,n}(L) = 1) = \frac{2}{n}$$

$[3, 2, 1]$

$Y_{2,3}$ $\quad Y_{i,i}(L) = 0$



$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$Y_{1,n}(L) = \begin{cases} 1 \\ 0 \end{cases}$

$[3, 2, 1]$

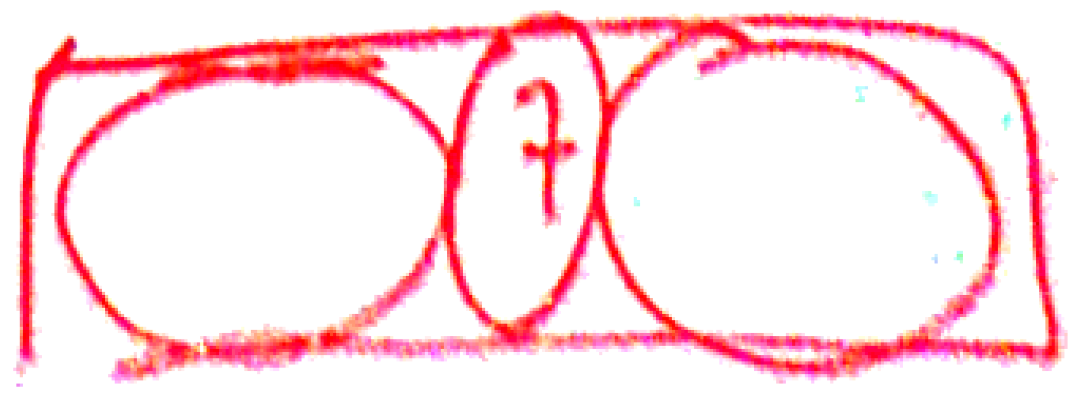
$X_n(L)$

$Y_{2,3}$ $\quad Y_{i,i}(L) = 0$

$E(X_n)$

$Pr(Y_{1,n}(L) = 1) = \frac{2}{n}$

$\frac{2(n-1)!}{n!} = \frac{2}{n}$



$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$Y_{1,n}(L) = \begin{cases} 1 \\ 0 \end{cases}$

$[3, 2, 1]$

$X_n(L)$

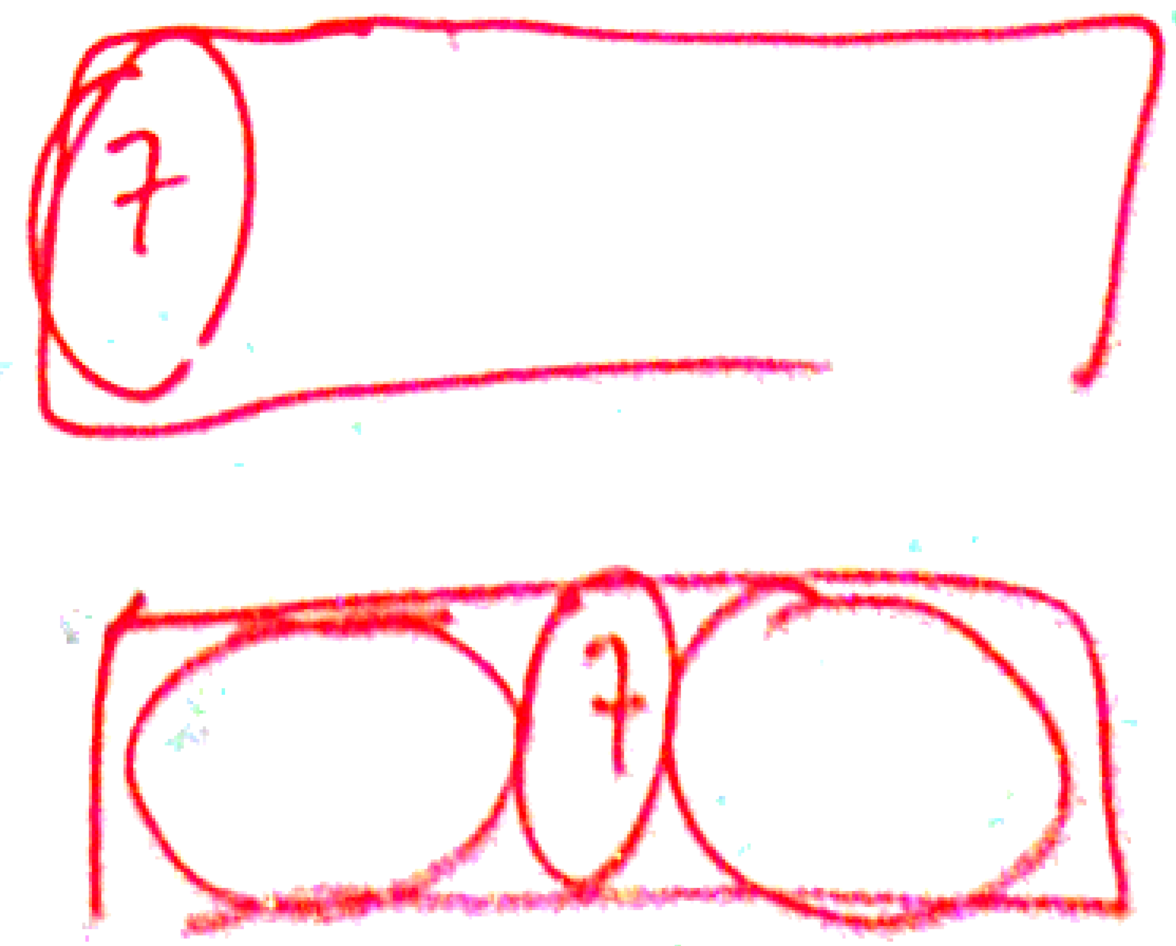
$Y_{2,3}$

$Y_{i,i}(L) = 0$

$E(X_n)$

$Pr(Y_{1,n}(L) = 1) = \frac{2}{n}$

$\frac{2(n-1)!}{n!} = \frac{2}{n}$



$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$$Y_{1,n}(L) = \begin{cases} 1 \\ 0 \end{cases}$$

$[3, 2, 1]$

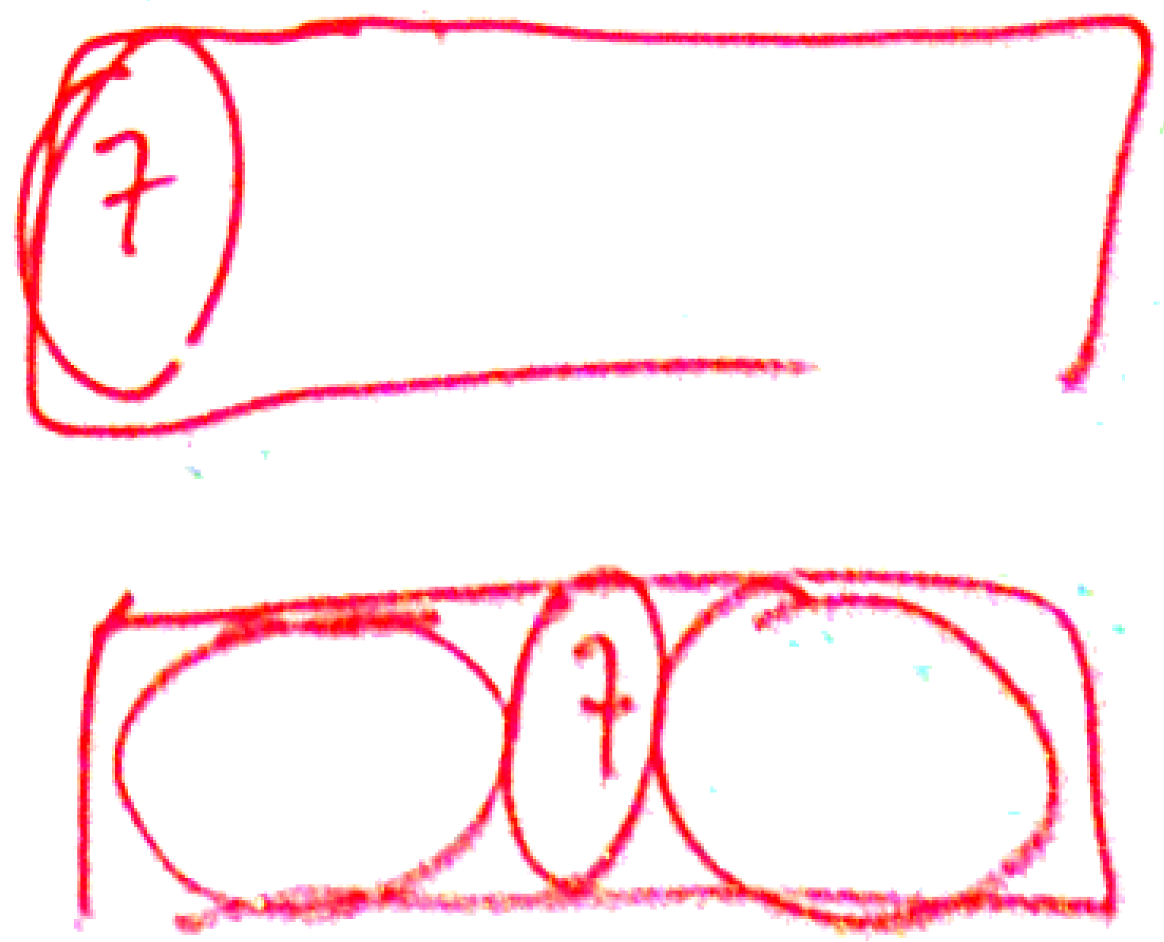
$X_n(L)$

$Y_{2,3}$ $\quad Y_{i,i}(L) = 0$

$E(X_n)$

$$Pr(Y_{1,n}(L) = 1) = \frac{2}{n}$$

$$\frac{2(n-1)!}{n!} = \frac{2}{n}$$



$$\mathcal{E}_n$$

$$|\mathcal{E}_n| = n!$$

$$Y_{1,n}(L) = \begin{cases} 1 \\ 0 \end{cases}$$

$$E(X) = \overset{\to 0}{\cancel{0 \cdot Pr(X=0)}} + 1 \cdot \underbrace{Pr(X=1)}_{p} = p$$

$$X_n(L)$$

$$E(X_n)$$

$$Pr(Y_{1,n}(L) = 1) = \frac{2}{n}$$

$$\frac{2(n-1)!}{n!} = \frac{2}{n}$$

$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$$Y_{1,n}(L) = \begin{cases} 1 \\ 0 \end{cases}$$

$$E(X) = 0 \cdot \overset{\to 0}{\cancel{Pr(X=0)}} + 1 \cdot \underbrace{Pr(X=1)}_{p} = p$$

$X_n(L)$

$E(X_n)$

$$Pr(Y_{1,n}(L) = 1) = \frac{2}{n}$$

$$\frac{2(n-1)!}{n!} = \frac{2}{n}$$

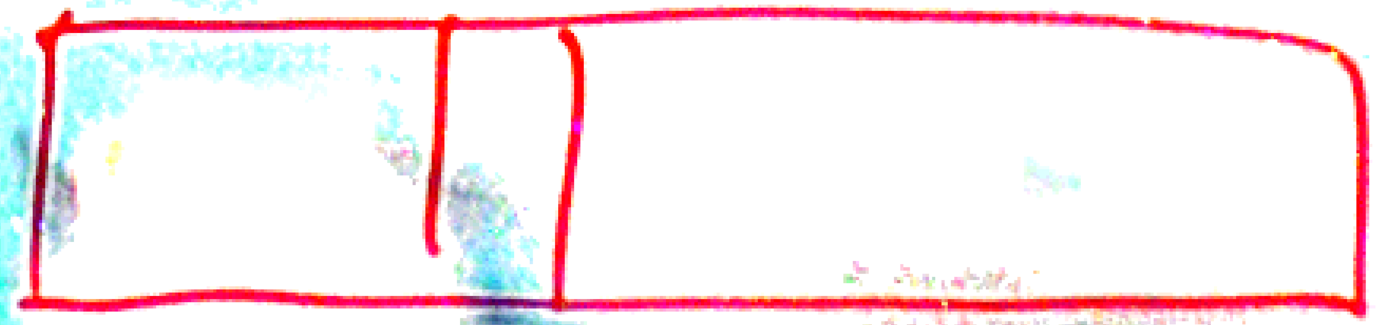
$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$1 \le i < j \le n$

$Y_{i,j}(L)$

$E(X) = 0 \cdot \overset{>0}{Pr(X=0)} + 1 \cdot \underset{p}{Pr(X=1)} = p$



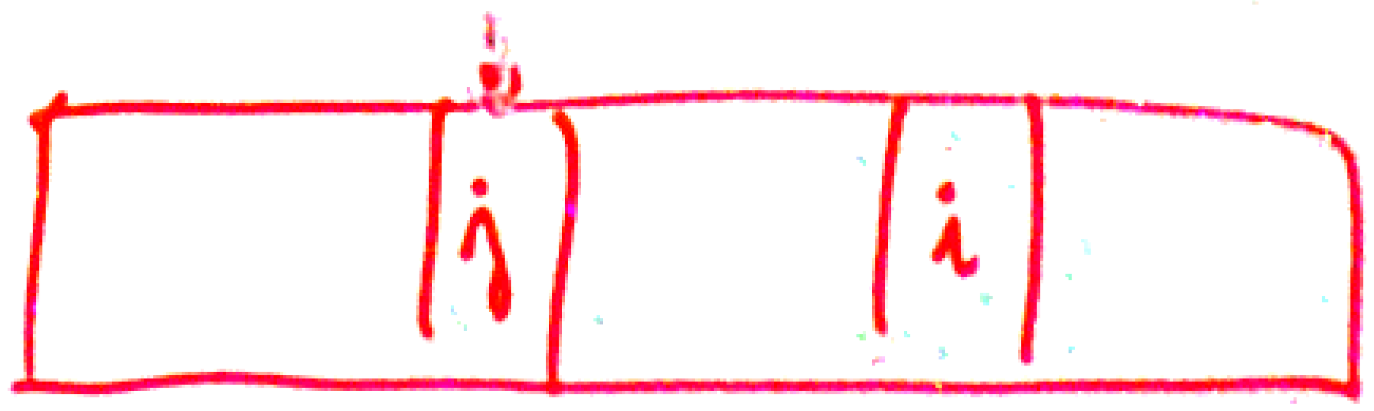
$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$1 \le i < j \le n$

$Y_{i,j}(L)$

$E(X) = 0 \cdot Pr(X=0) + 1 \cdot Pr(X=1) = p$



$\mathcal{E}_n$

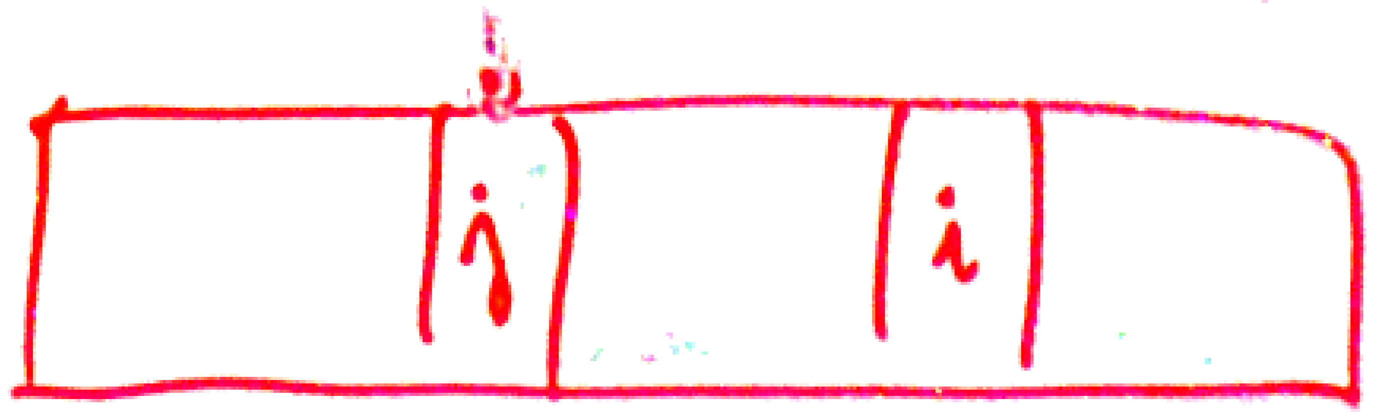
$|\mathcal{E}_n| = n!$

$1 \le i < j \le n$

$Y_{i,j}(L)$

$Pr(Y_{i,j} = 1)$

$E(X) = 0 \cdot \cancel{Pr(X=0)} + 1 \cdot Pr(X=1) = p$



Σ_n

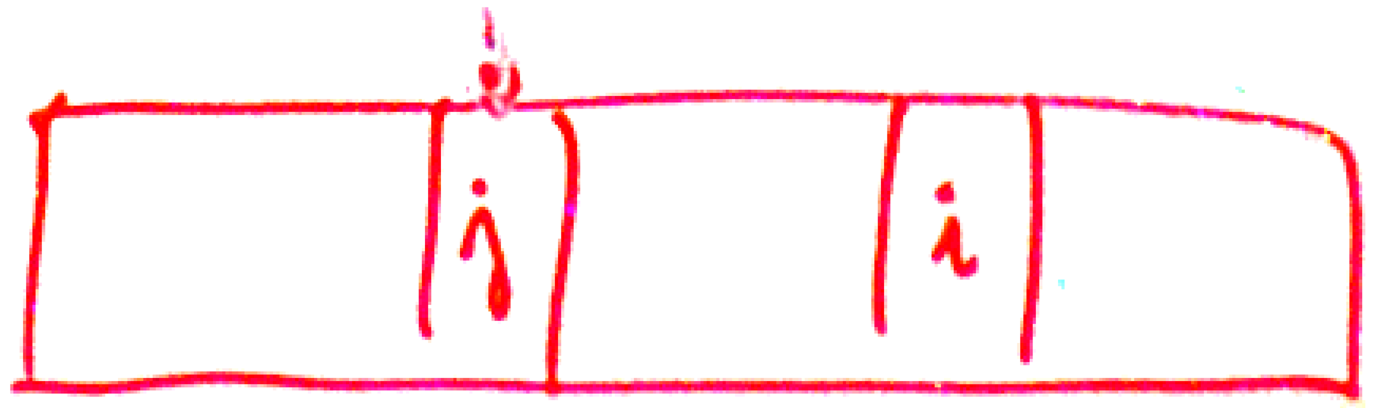
$|\Sigma_n| = n!$

$1 \le i < j \le n$

$Y_{i,j}(L)$

$Pr(Y_{i,j} = 1)$

$E(X) = \underbrace{0 \cdot Pr(X=0)}_{\to 0} + 1 \cdot \underbrace{Pr(X=1)}_{p} = p$



$\mathcal{E}_n$

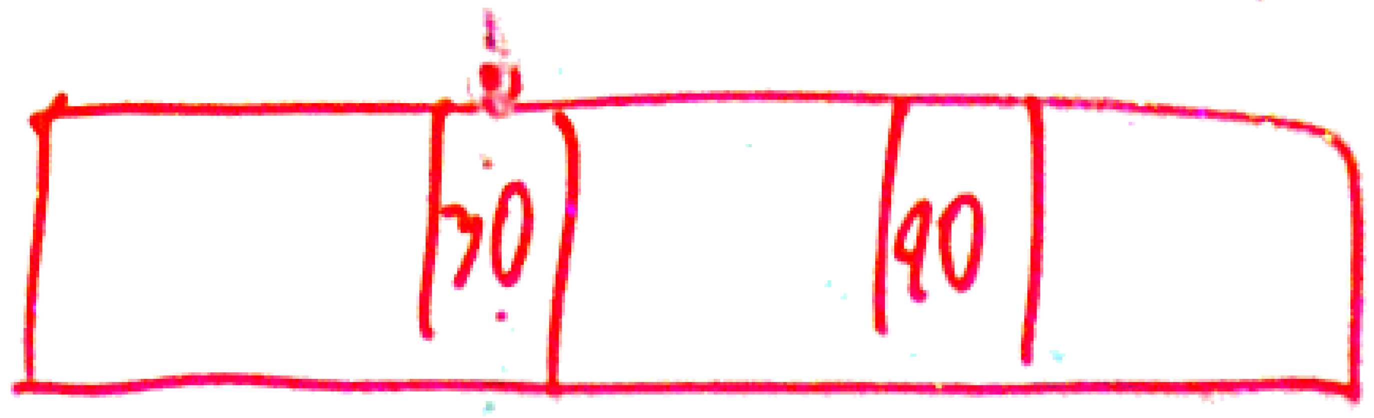
$|\mathcal{E}_n| = n!$

$1 \le i < j \le n$

$Y_{i,j}(L)$

$\Pr(Y_{i,j} = 1) =$

$E(X) = \overset{\to 0}{\cancel{0 \cdot \Pr(X=0)}} + 1 \cdot \Pr(\underset{p}{\cancel{X=1}}) = p$



$\mathcal{E}_n$

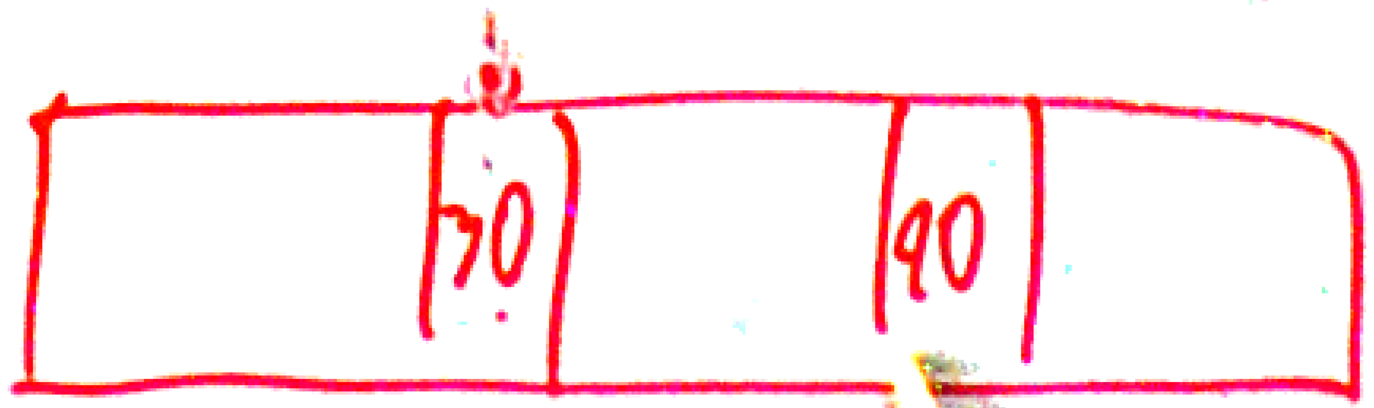
$|\mathcal{E}_n| = n!$

$1 \le i < j \le n$

$Y_{i,j}(L)$

$$\Pr(Y_{i,j} = 1) = \frac{2}{j-i+1}$$

$$E(X) = 0 \cdot \Pr(X=0) + 1 \cdot \Pr(X=1) = p$$



$\mathcal{E}_n$

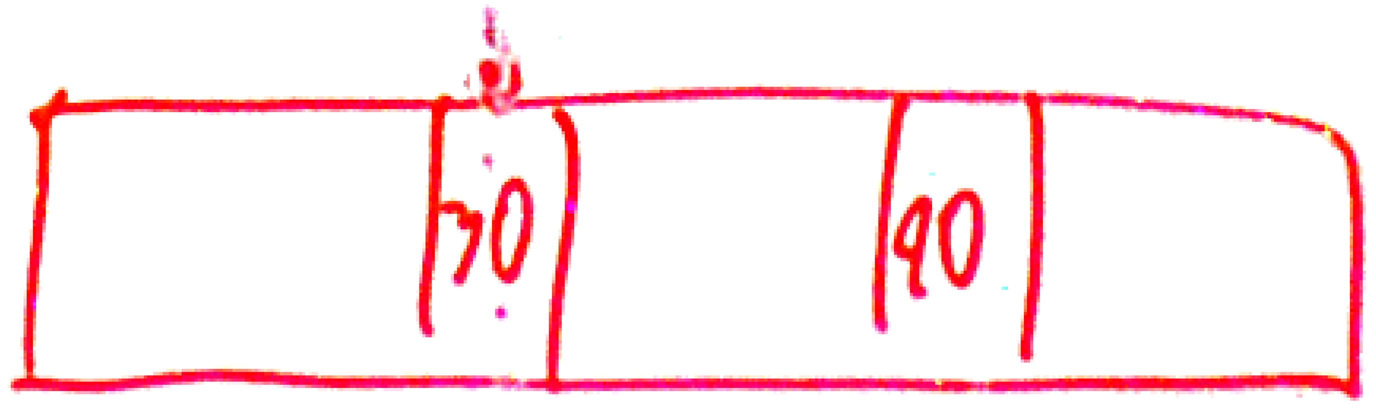
$|\mathcal{E}_n| = n!$

$1 \le i < j \le n$

$Y_{i,j}(L)$

$Pr(Y_{i,j} = 1) = \dfrac{2(j-i)!}{(j-i+1)!}$

$E(X) = 0 \cdot Pr(X=0) + 1 \cdot Pr(X=1) = p$



Σ_n

$|\Sigma_n| = n!$

$1 \le i < j \le n$

$Y_{i,j}(L)$

$$Pr(Y_{i,j} = 1) = \frac{2(j-i)!}{(j-i+1)!}$$

$$E(X) = \overset{\to 0}{\cancel{0 \cdot Pr(X=0)}} + 1 \cdot Pr(\underbrace{X=1}_{p}) = p$$

	70		90		

$\mathcal{E}_n$

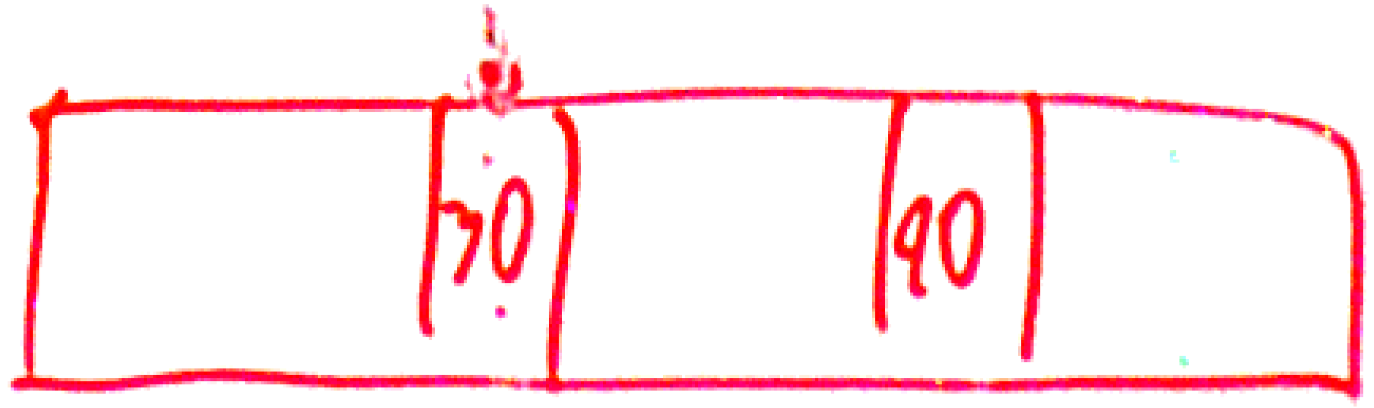
$|\mathcal{E}_n| = n!$

$1 \le i < j \le n$

$Y_{i,j}(L)$

$$\Pr(Y_{i,j} = 1) = \frac{2(j-i)!}{(j-i+1)!}$$

$$E(X) = \cancel{0 \cdot \Pr(X=0)}^{\to 0} + 1 \cdot \Pr(X=1) = p$$



$\mathcal{E}_n$

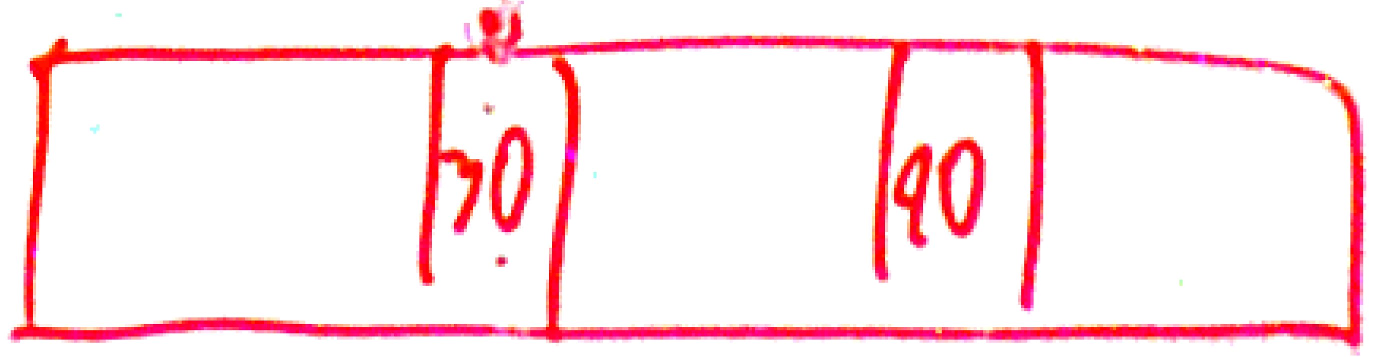
$|\mathcal{E}_n| = n!$

$1 \le i < j \le n$

$$E(X) = \cancel{0 \cdot Pr(X=0)}^{>0} + 1 \cdot Pr(X=1) = p$$

$$E(X_n) = \sum_{k=2}^{n} (n+1-k) \cdot \frac{2}{k}$$

$$= 2(n+1)\left(\sum_{k=2}^{n} \frac{1}{k}\right) - 2(n-1)$$



$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$1 \leq i < j \leq n$

$$\sum_{k=1}^{n} \frac{1}{k}$$

$$E(X_n) = \sum_{k=2}^{n} (n+1-k) \cdot \frac{2}{k}$$

$$= 2(n+1) \sum_{k=2}^{n} \frac{1}{k} - 2(n-1)$$

Σ_n

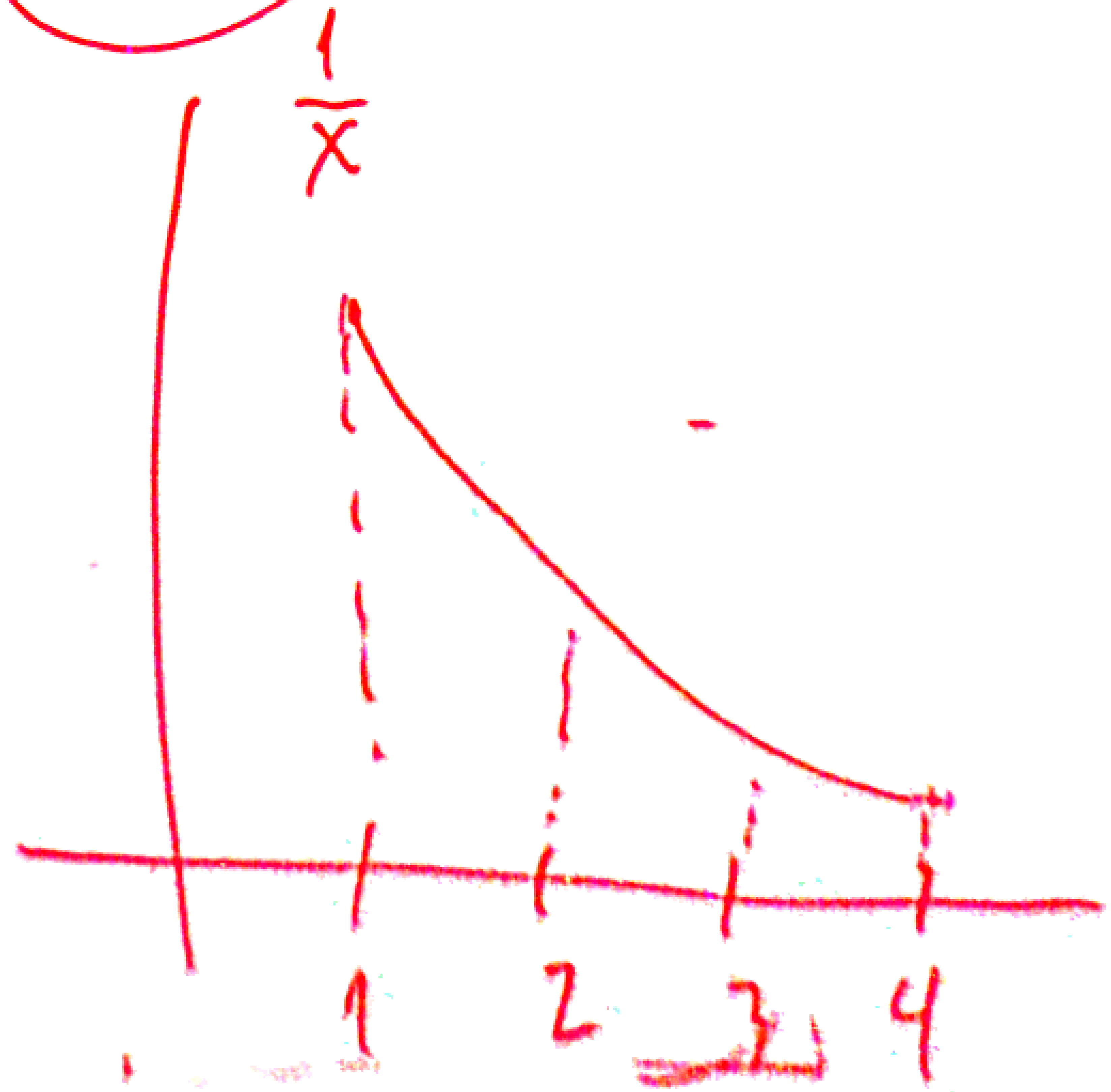
$|\Sigma_n| = n!$

$1 \le i < j \le n$

$$E(X_n) = \sum_{k=2}^{n} (n+1-k) \cdot \frac{2}{k}$$

$$= 2(n+1) \sum_{k=2}^{n} \frac{1}{k} - 2(n-1)$$

$$\sum_{k=1}^{n} \frac{1}{k}$$



$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$1 \le i < j \le n$

$$E(X_n) = \sum_{k=2}^{n} (n+1-k) \cdot \frac{2}{k}$$

$$= 2(n+1) \sum_{k=2}^{n} \frac{1}{k} - 2(n-1)$$

$$\int_1^4 \frac{1}{x}\,dx$$

$\mathcal{E}_n$

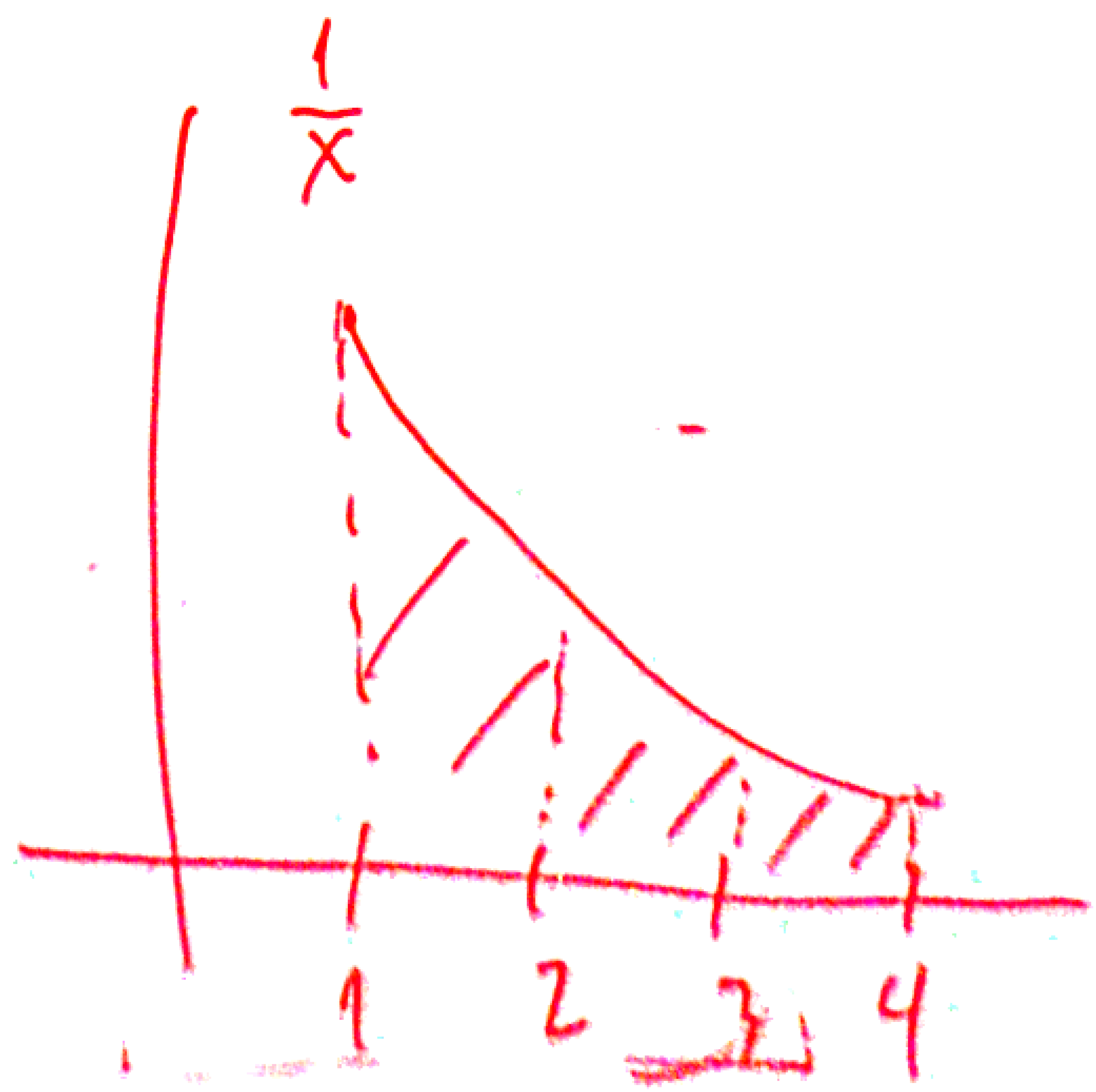
$|\mathcal{E}_n| = n!$

$1 \le i < j \le n$

$\int_1^4 \frac{1}{x}\,dx \le$

$$E(X_n) = \sum_{k=2}^{n} (n+1-k)\cdot\frac{2}{k}$$

$$= 2(n+1)\sum_{k=2}^{n}\frac{1}{k} - 2(n-1)$$



Σ_n

$|\Sigma_n| = n!$

$1 \le i < j \le n$

$E(X_n) = \sum_{k=2}^{n} (n+1-k) \cdot \frac{2}{k}$

$X_n(L)$

$= 2(n+1) \sum_{k=2}^{n} \frac{1}{k} - 2(n-1)$

$\int_1^4 \frac{1}{x}\, dx \le$

$\mathcal{E}_n$

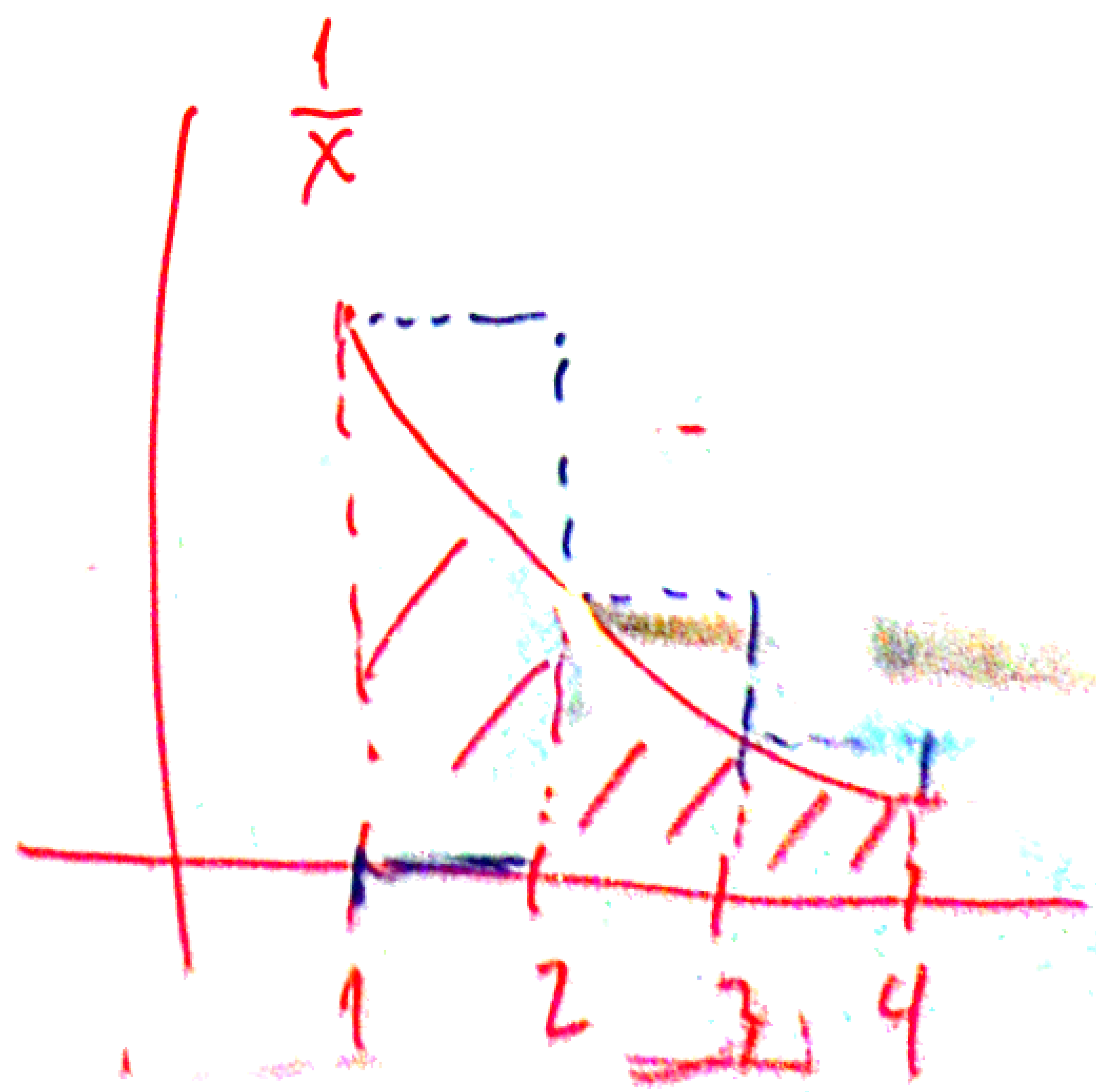
$|\mathcal{E}_n| = n!$ $\quad 1 \leq i < j \leq n$

$$\int_1^4 \frac{1}{x}\,dx \leq \sum_{k=1}^{3} \frac{1}{k}$$

$$E(X_n) = \sum_{k=2}^{n} (n+1-k)\cdot\frac{2}{k}$$

$X_n(L)$

$$= 2(n+1)\sum_{k=2}^{n}\frac{1}{k} - 2(n-1)$$



Σ_n

$|\Sigma_n| = n!$

$1 \le i < j \le n$

$$E(X_n) = \sum_{k=2}^{n} (n+1-k) \cdot \frac{2}{k}$$

$X_n(L)$

$$= 2(n+1) \sum_{k=2}^{n} \frac{1}{k} - 2(n-1)$$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n-1} \frac{1}{k}$$

Σ_n

$|\Sigma_n| = n!$

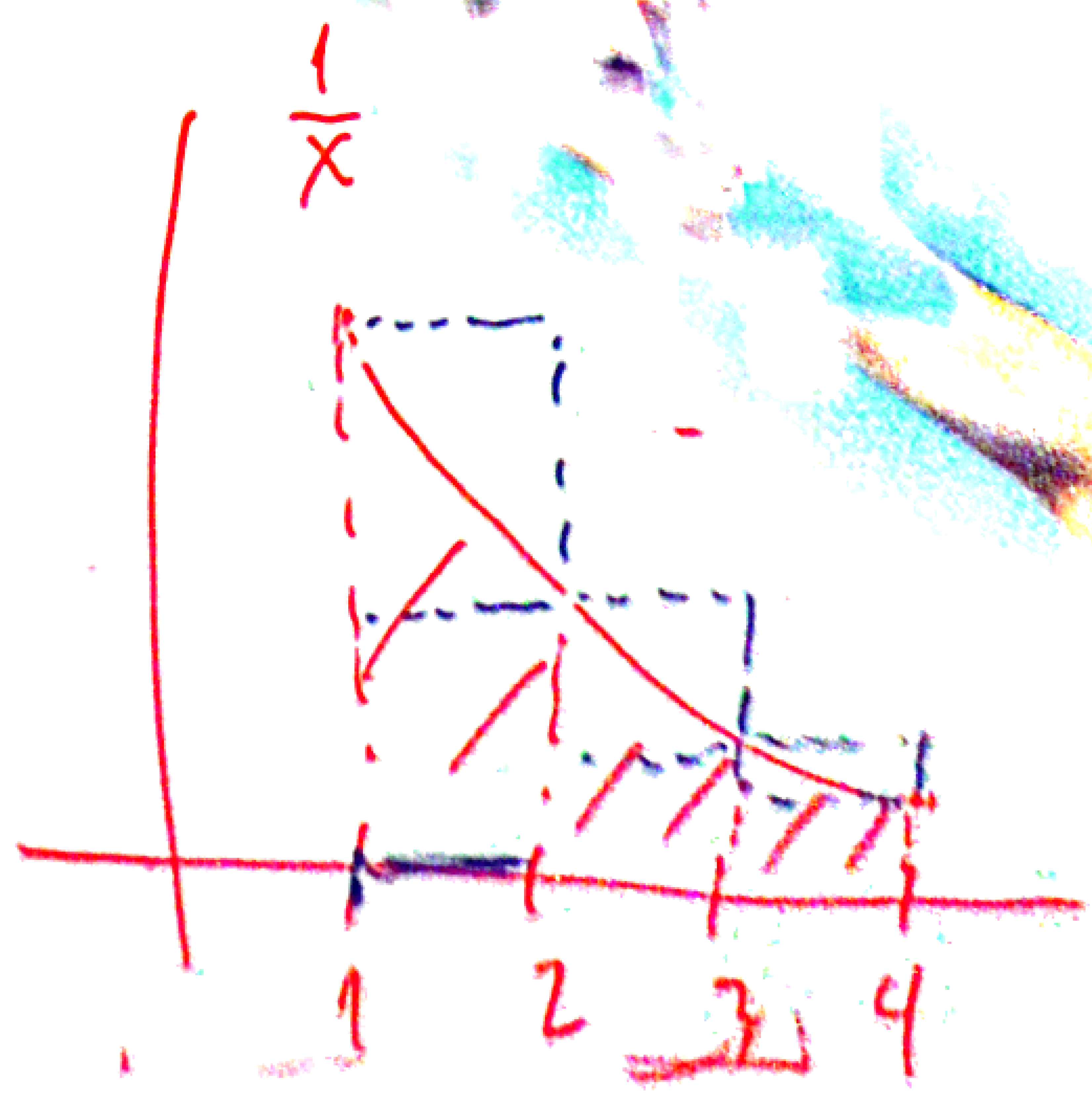
$1 \le i < j \le n$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$

$$E(X_n) = \sum_{k=2}^{n} (n+1-k)\cdot \frac{2}{k}$$

$X_n(L)$

$$= 2(n+1)\sum_{k=2}^{n} \frac{1}{k} - 2(n-1)$$



Σ_n

$|\Sigma_n| = n!$

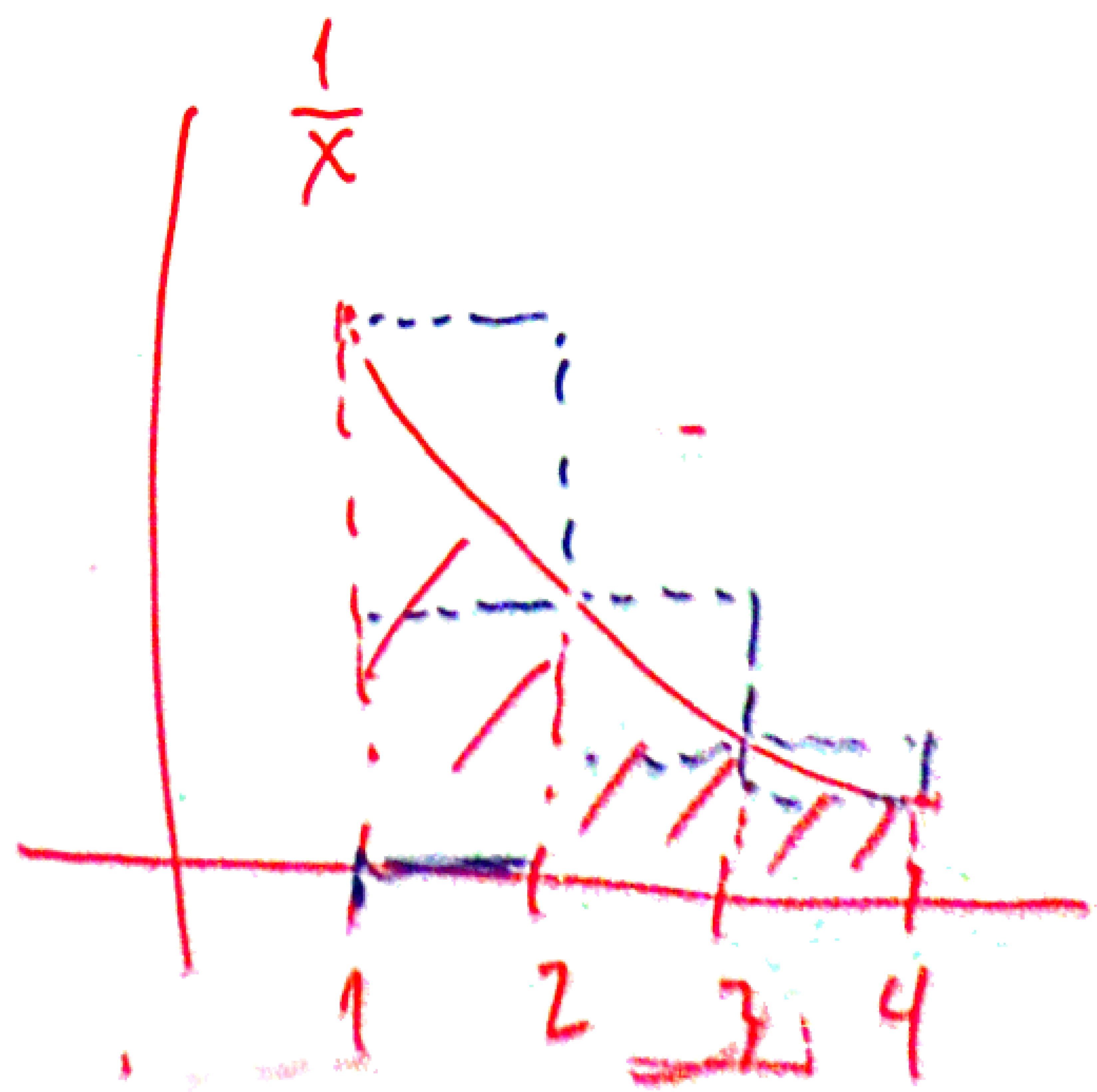
$1 \le i < j \le n$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$

$$E(X_n) = \sum_{k=2}^{n} (n+1-k) \cdot \frac{2}{k}$$

$X_n(L)$

$$= 2(n+1) \sum_{k=2}^{n} \frac{1}{k} - 2(n-1)$$



$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

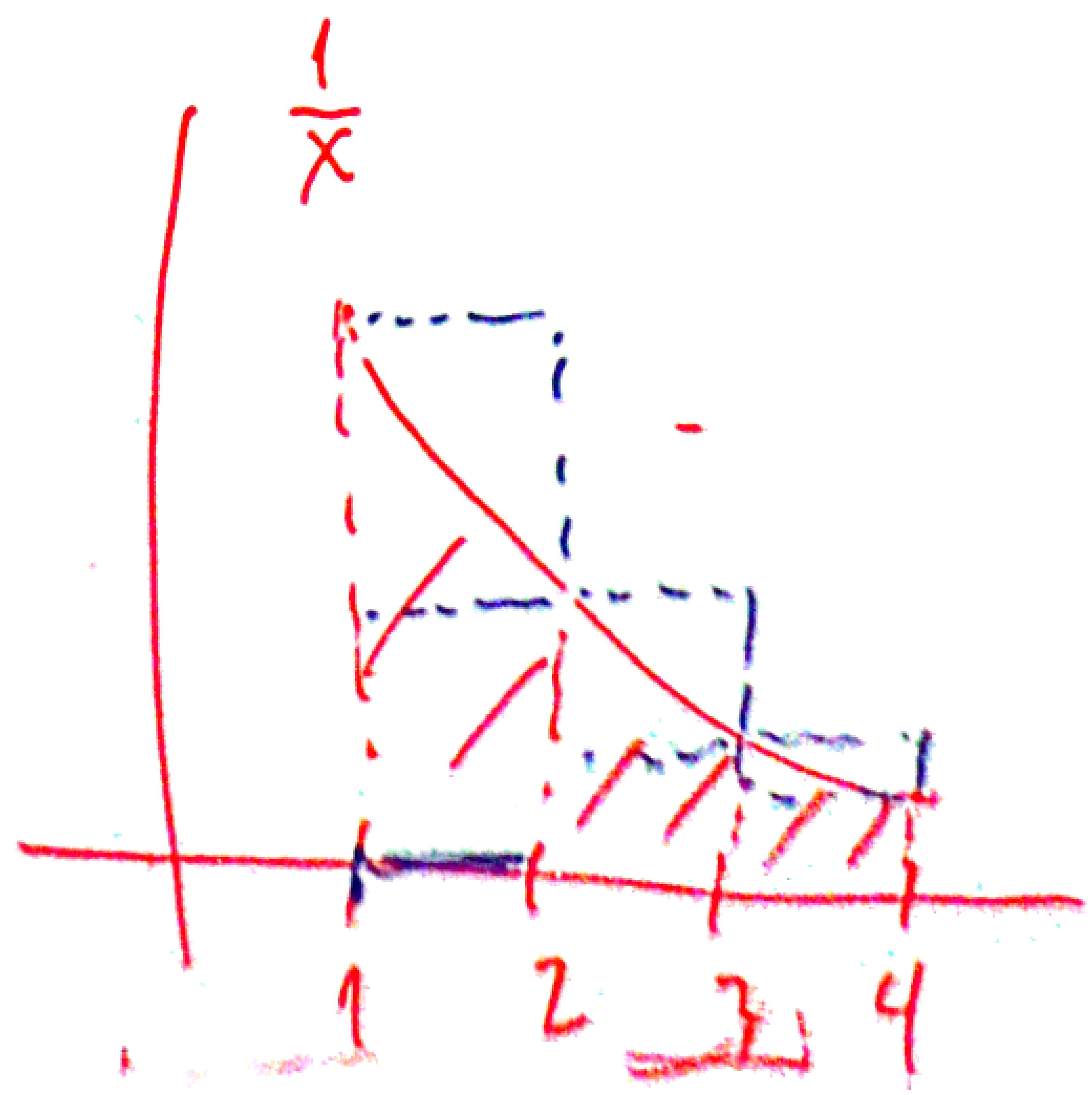
$1 \le i < j \le n$

$E(X_n) = \sum_{k=2}^{n} (n+1-k) \cdot \frac{2}{k}$

$X_n(L)$

$= 2(n+1)\left(\sum_{k=2}^{n} \frac{1}{k}\right) - 2(n-1)$

$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$



Σ_n

$|\Sigma_n| = n!$

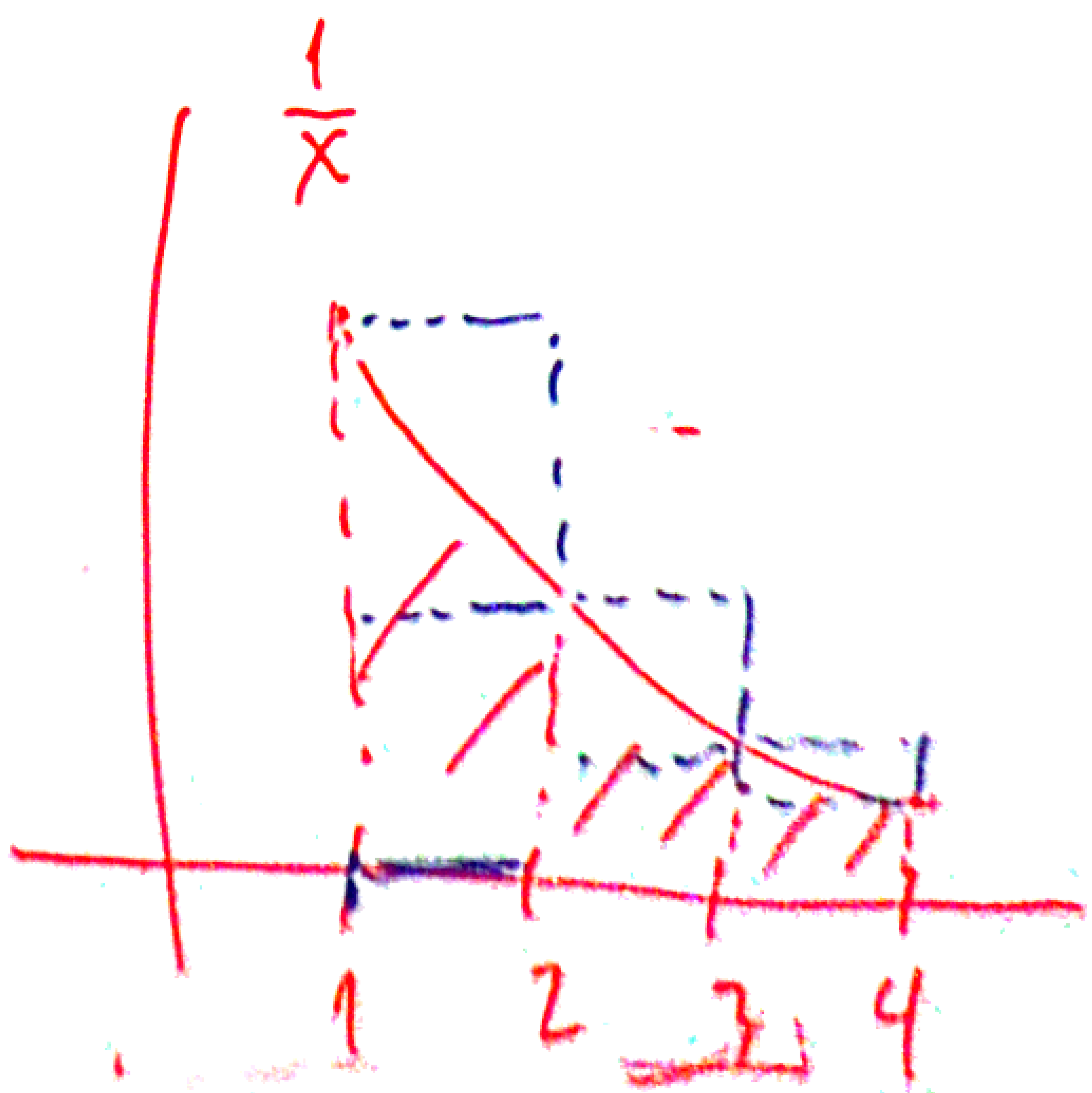
$1 \leq i < j \leq n$

$$\sum_{k=2}^{n} \frac{1}{k} \leq \int_1^n \frac{1}{x}\,dx \leq \sum_{k=1}^{n} \frac{1}{k}$$

$$E(X_n) = \sum_{k=2}^{n} (n+1-k) \cdot \frac{2}{k}$$

$X_n(L)$

$$= 2(n+1) \sum_{k=2}^{n} \frac{1}{k} - 2(n-1)$$



$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$1 \le i < j \le n$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$

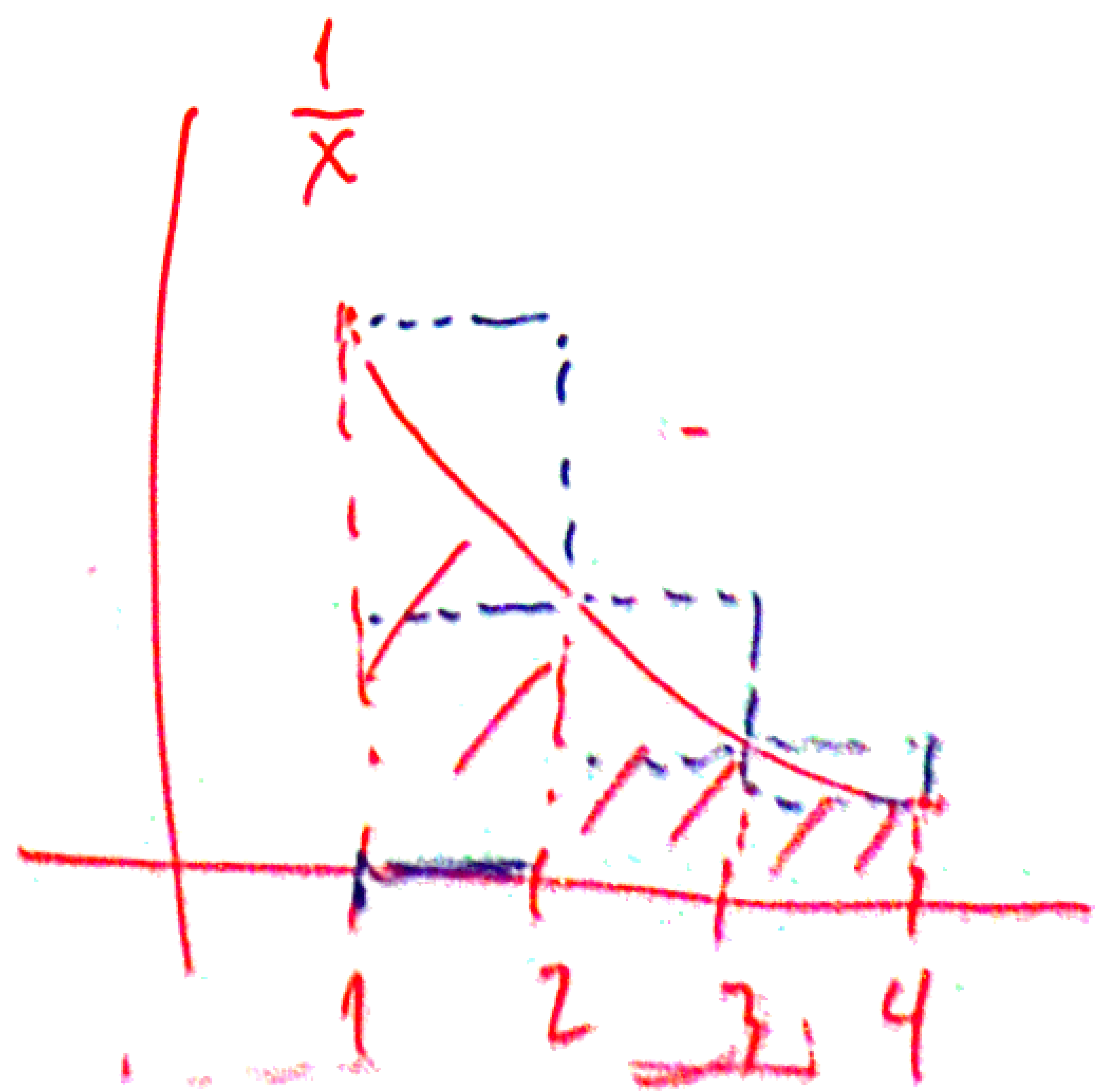
$$E(X_n) = \sum_{k=2}^{n} (n+1-k)\cdot\frac{2}{k}$$

$X_n(L)$

$$= 2(n+1)\sum_{k=2}^{n} \frac{1}{k} - 2(n-1)$$

X_{10}

X_{12}



$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$1 \le i < j \le n$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$

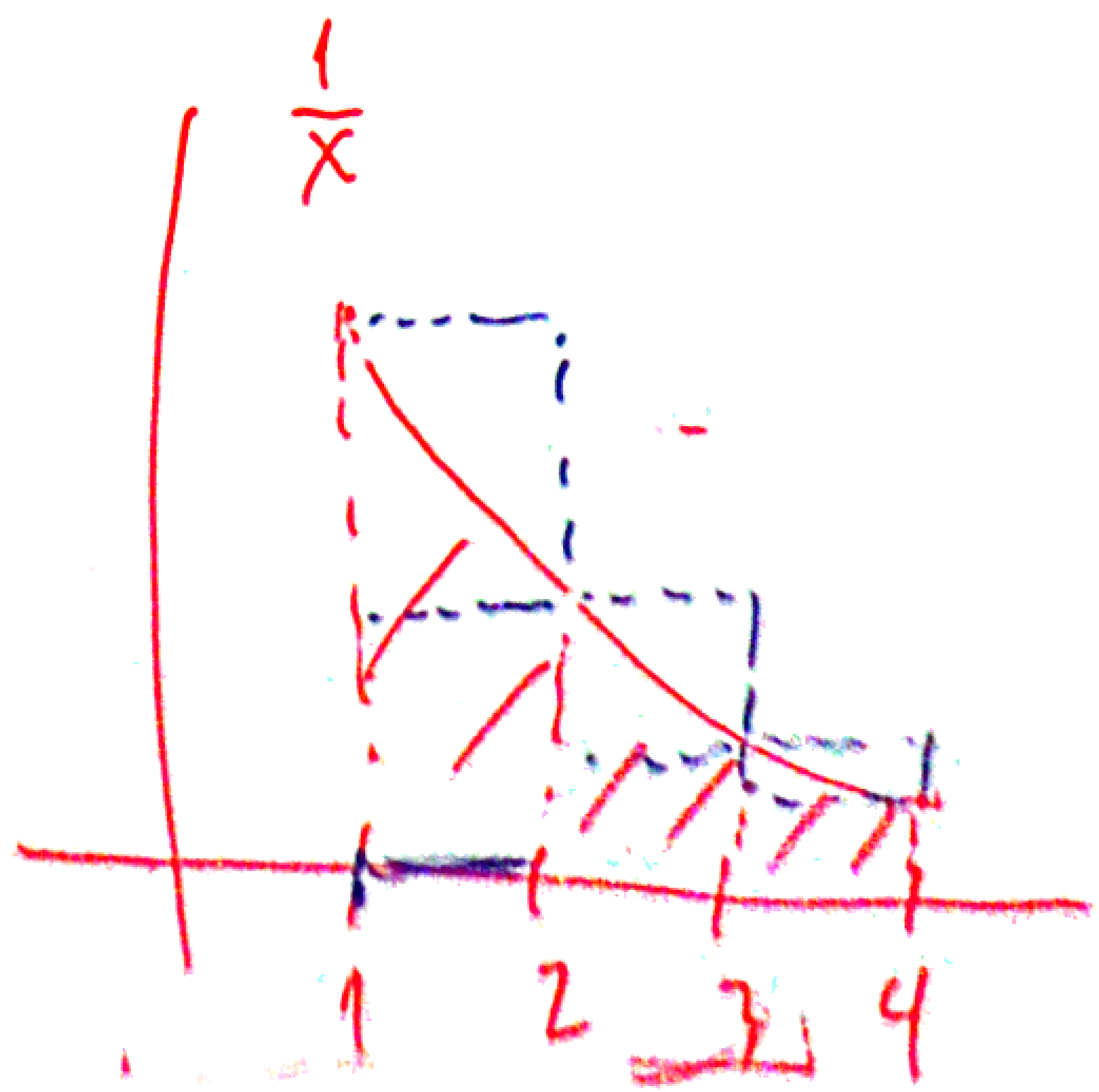
$$E(X_n) = \sum_{k=2}^{n} (n+1-k) \cdot \frac{2}{k}$$

$X_n(L)$

$$= 2(n+1) \sum_{k=2}^{n} \frac{1}{k} - 2(n-1)$$

X_{10}

X_{12}



Σ_n

$|\Sigma_n| = n!$

$1 \le i < j \le n$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$

$$E(X_n) = \sum_{k=2}^{n} (n+1-k) \cdot \frac{2}{k}$$

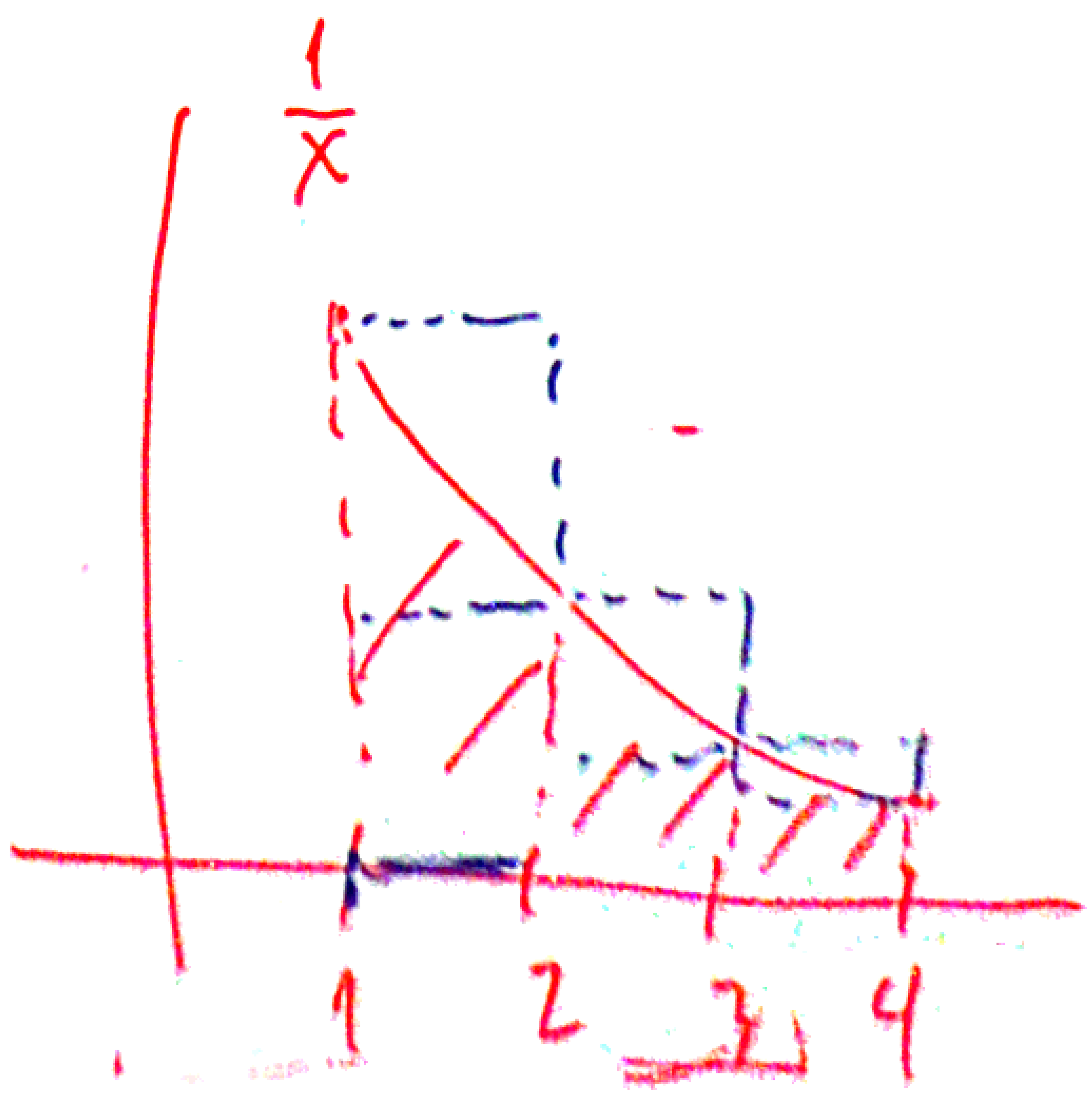
$X_n(L)$

$$= 2(n+1) \sum_{k=2}^{n} \frac{1}{k} - 2(n-1)$$

X_{10}

X_{12}

!



$\mathcal{E}_n$

$|\mathcal{E}_n| = n!$

$1 \leq i < j \leq n$

$$\sum_{k=2}^{n} \frac{1}{k} \leq \int_1^n \frac{1}{x}\,dx \leq \sum_{k=1}^{n} \frac{1}{k}$$

$$E(X_n) = \sum_{k=2}^{n} (n+1-k) \cdot \frac{2}{k}$$

$X_n(L)$

$$= 2(n+1) \sum_{k=2}^{n} \frac{1}{k} - 2(n-1)$$

X_{10}

X_{12}

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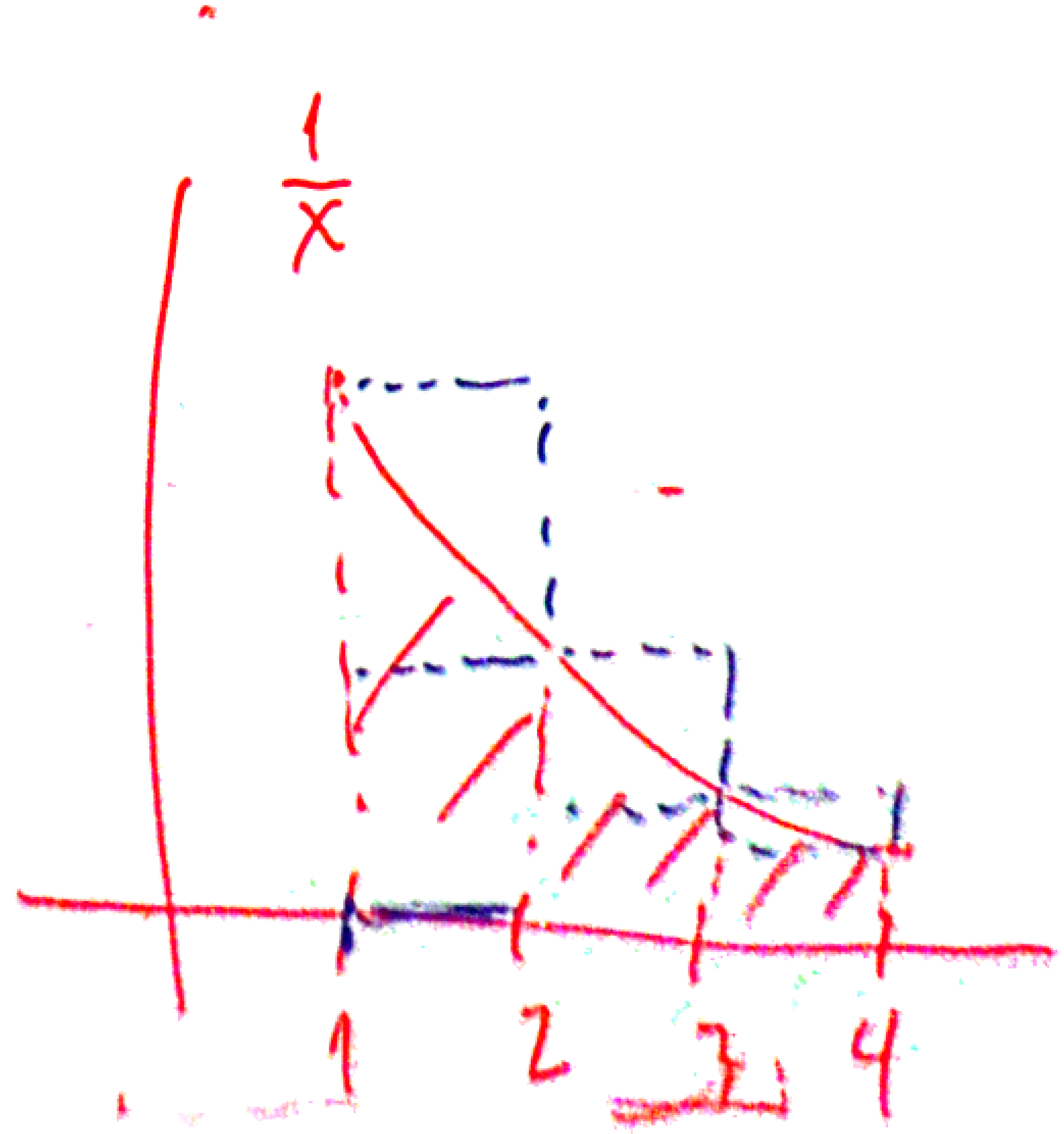
X_{12}



Σ_n

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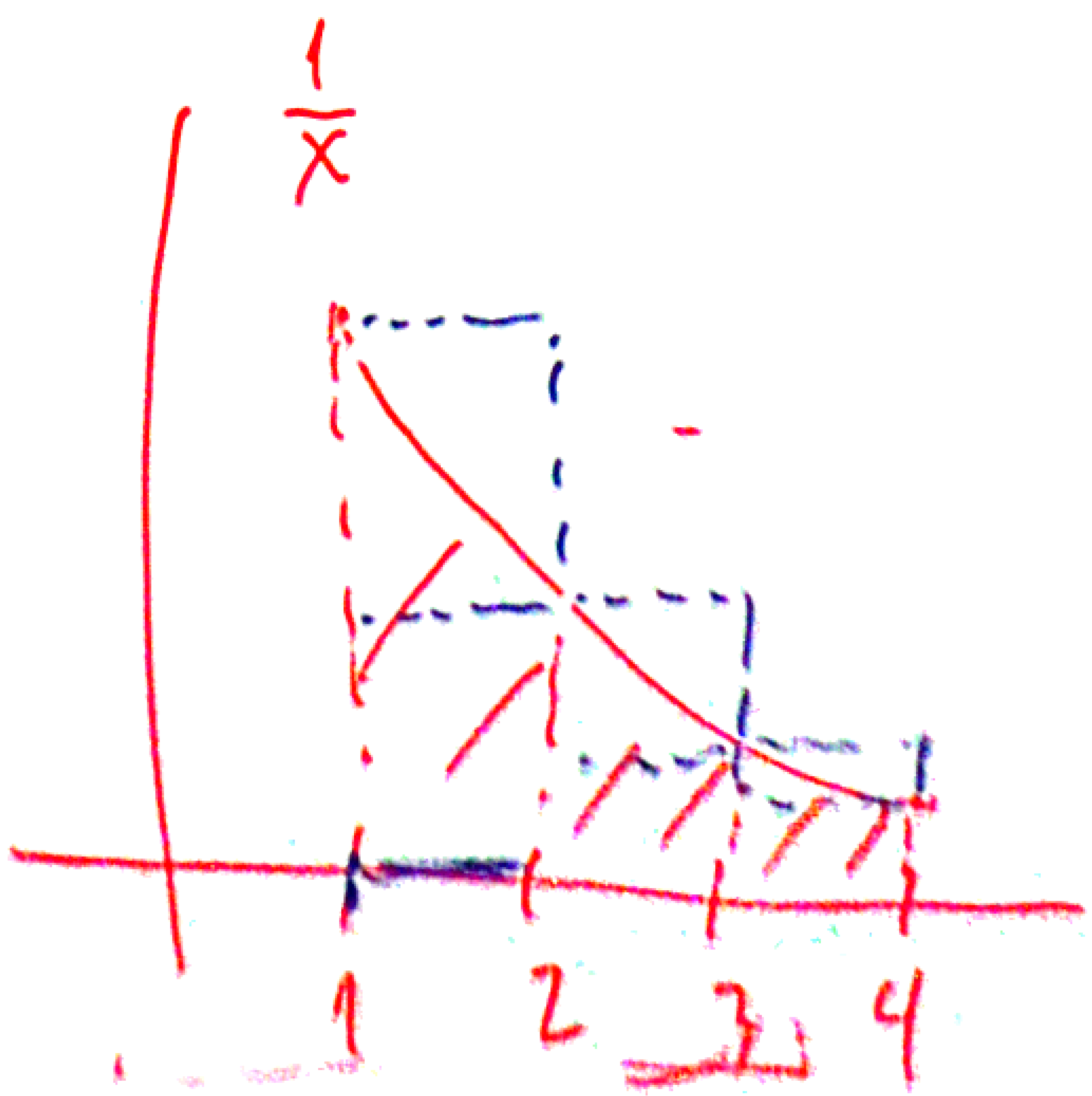

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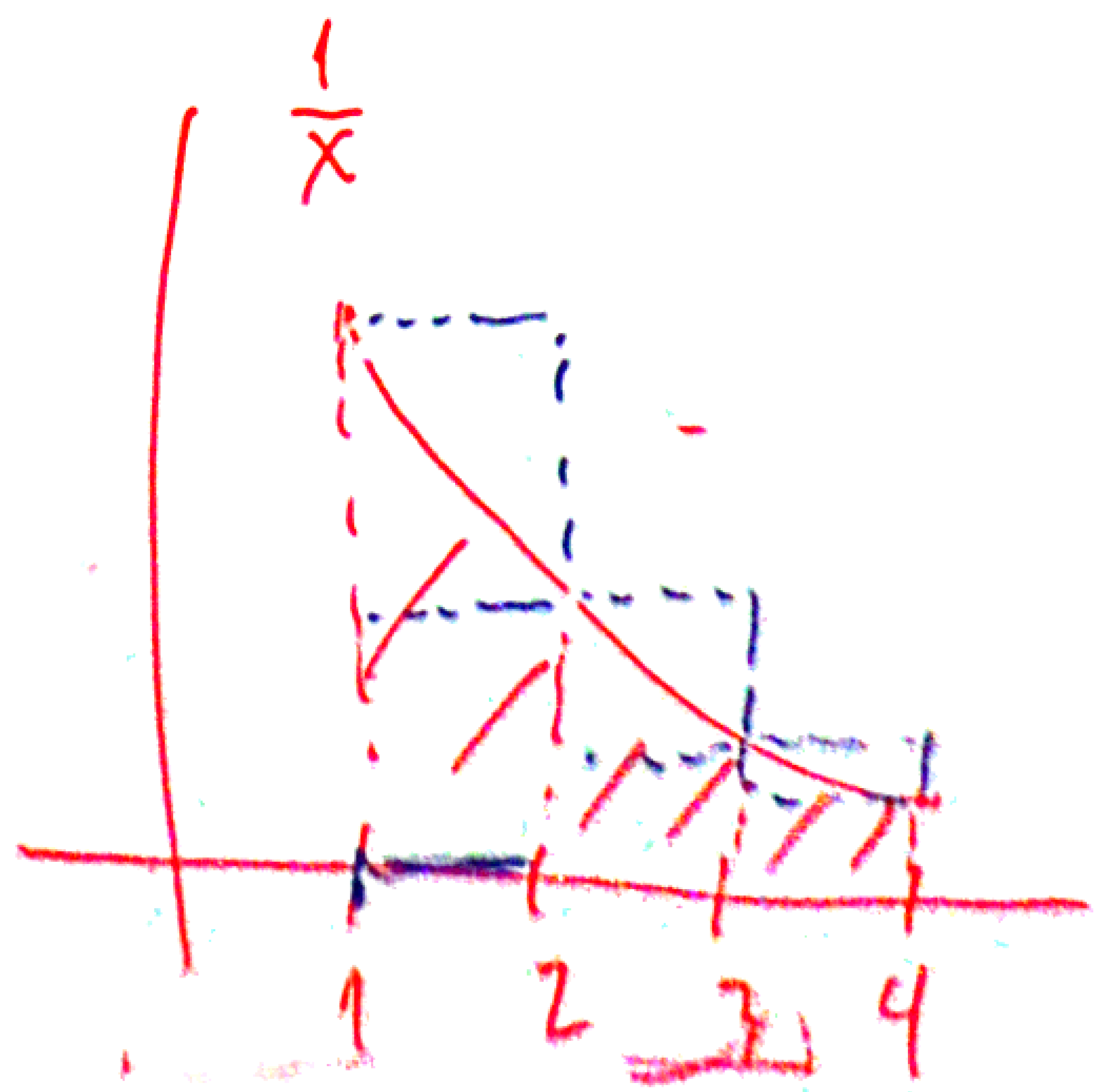
X_{12}



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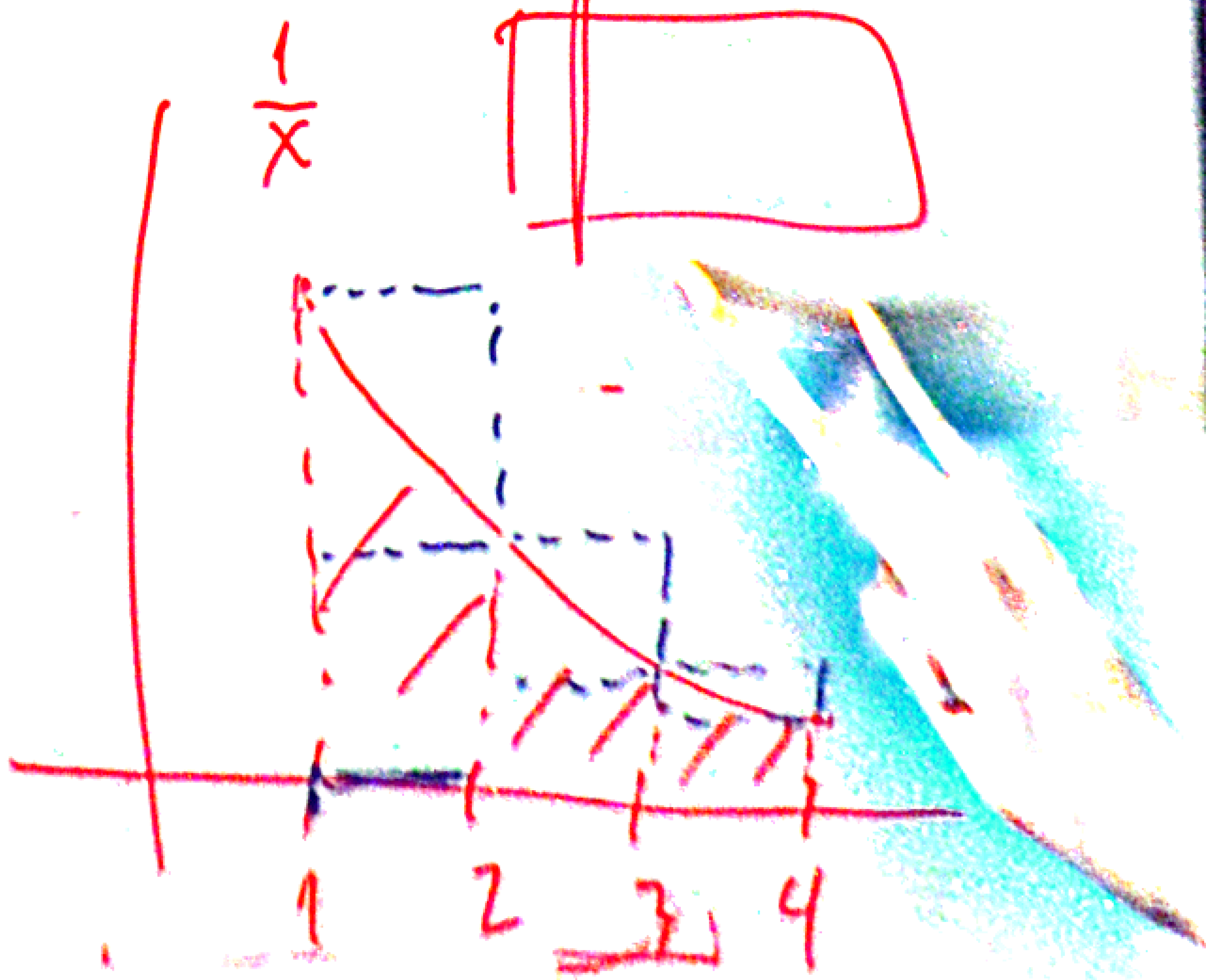

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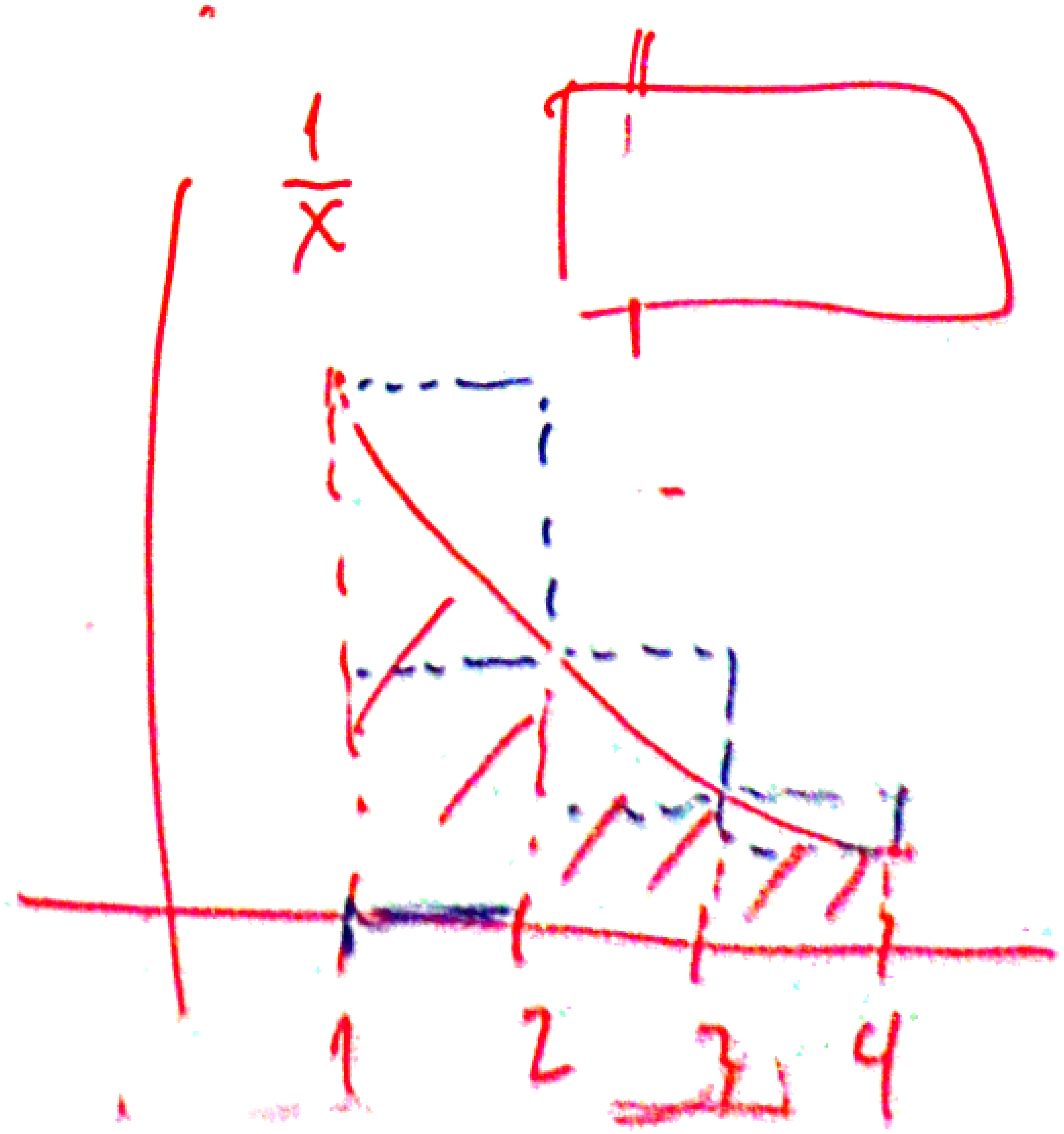

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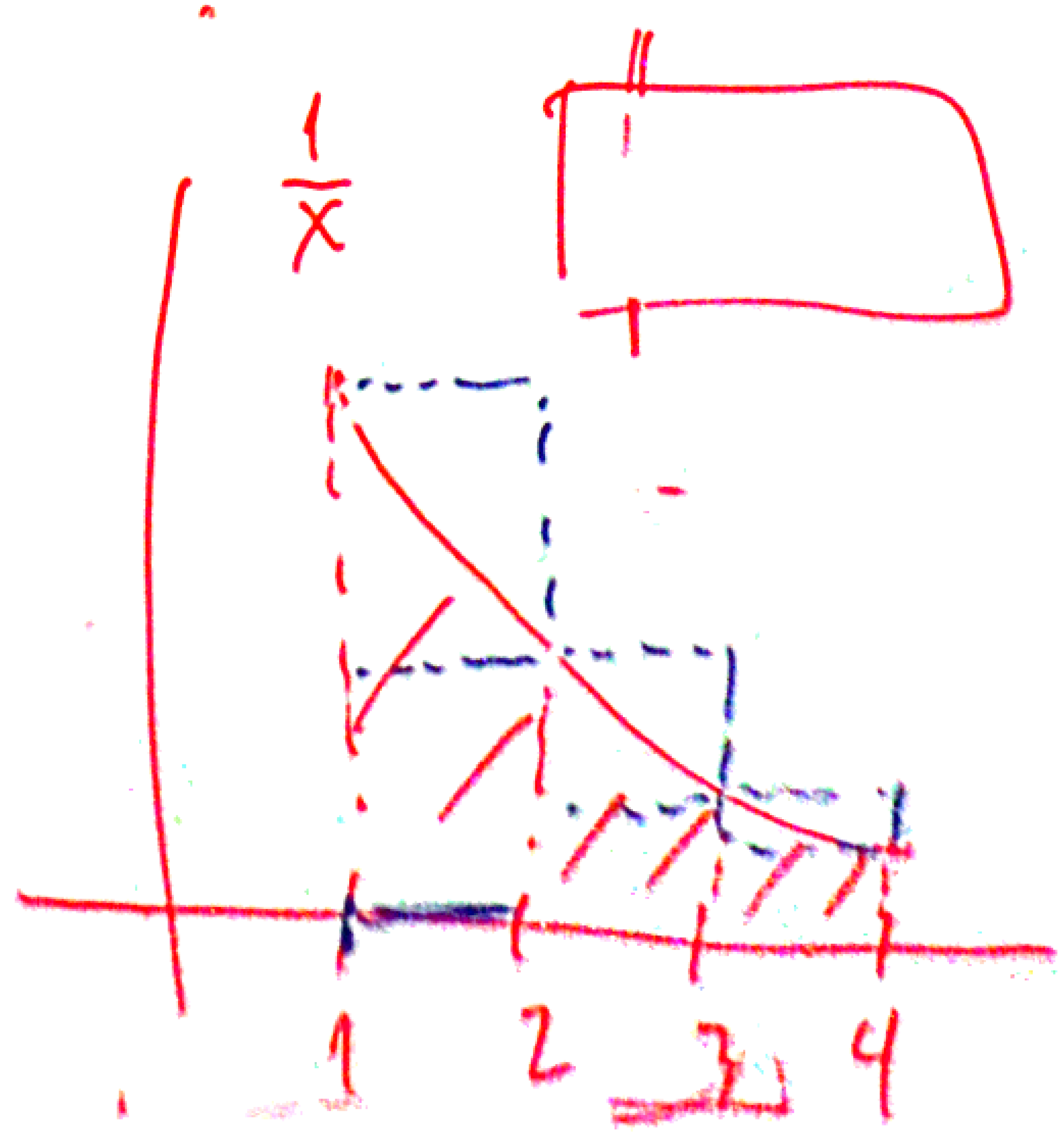

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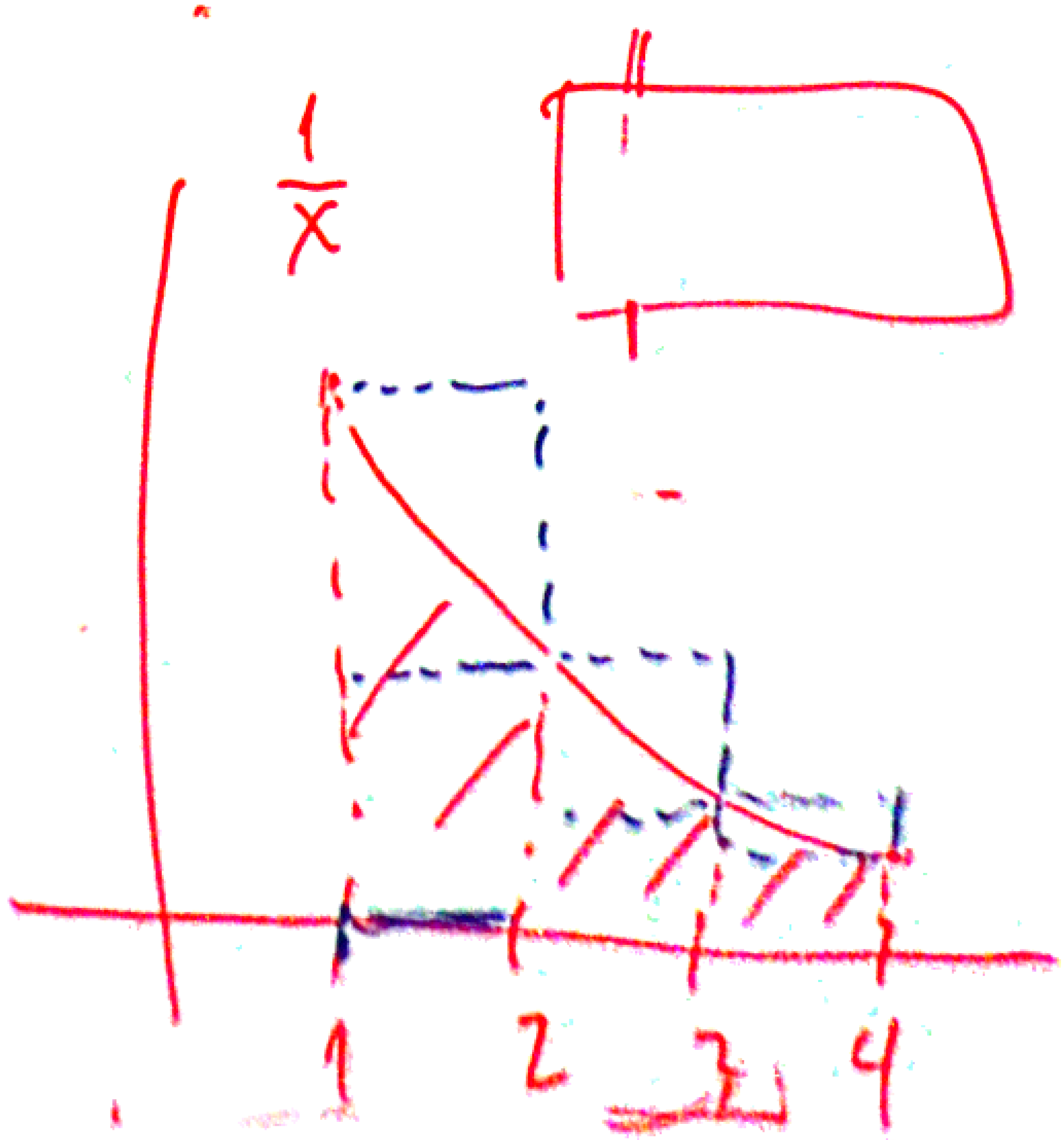

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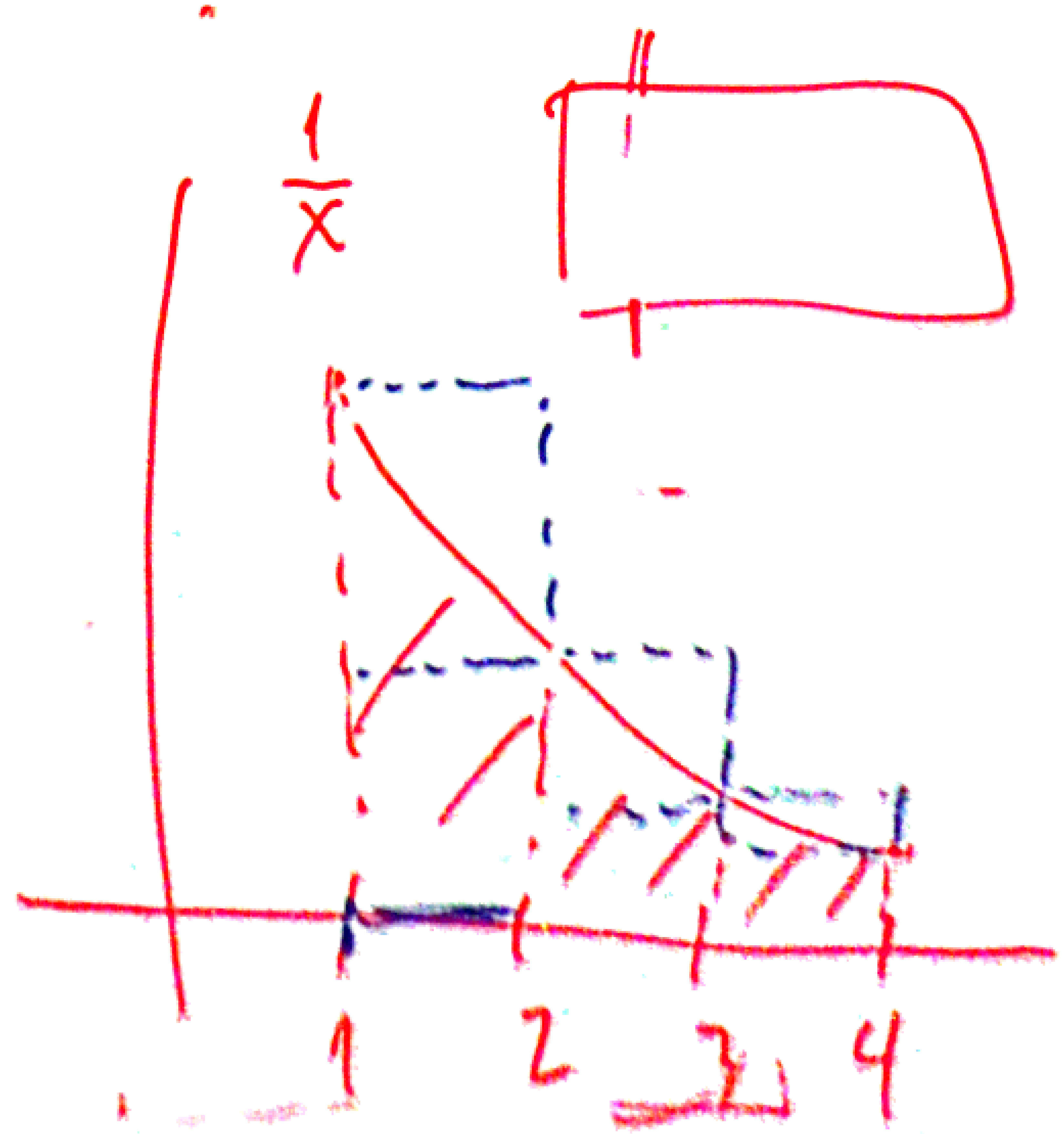

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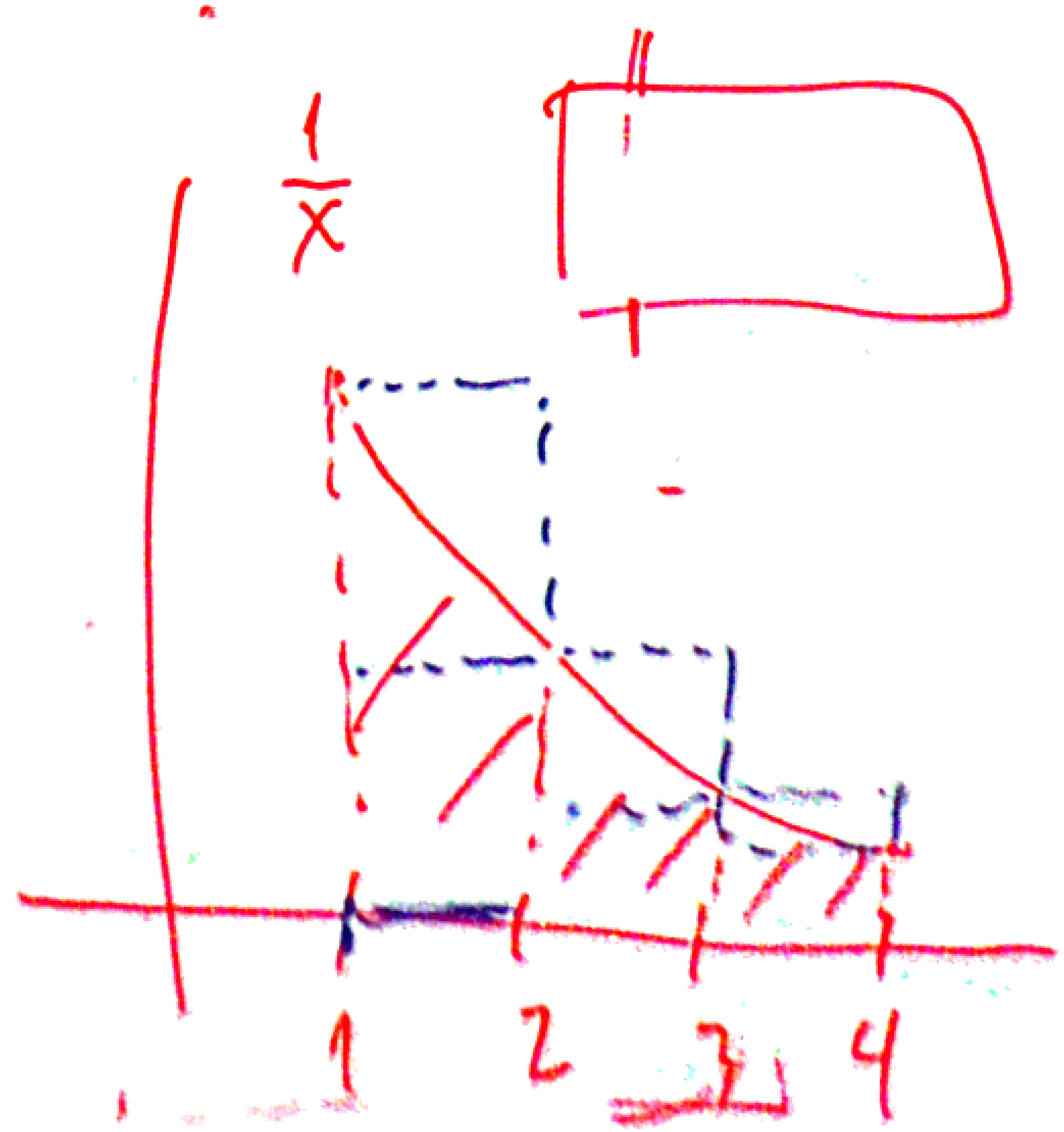

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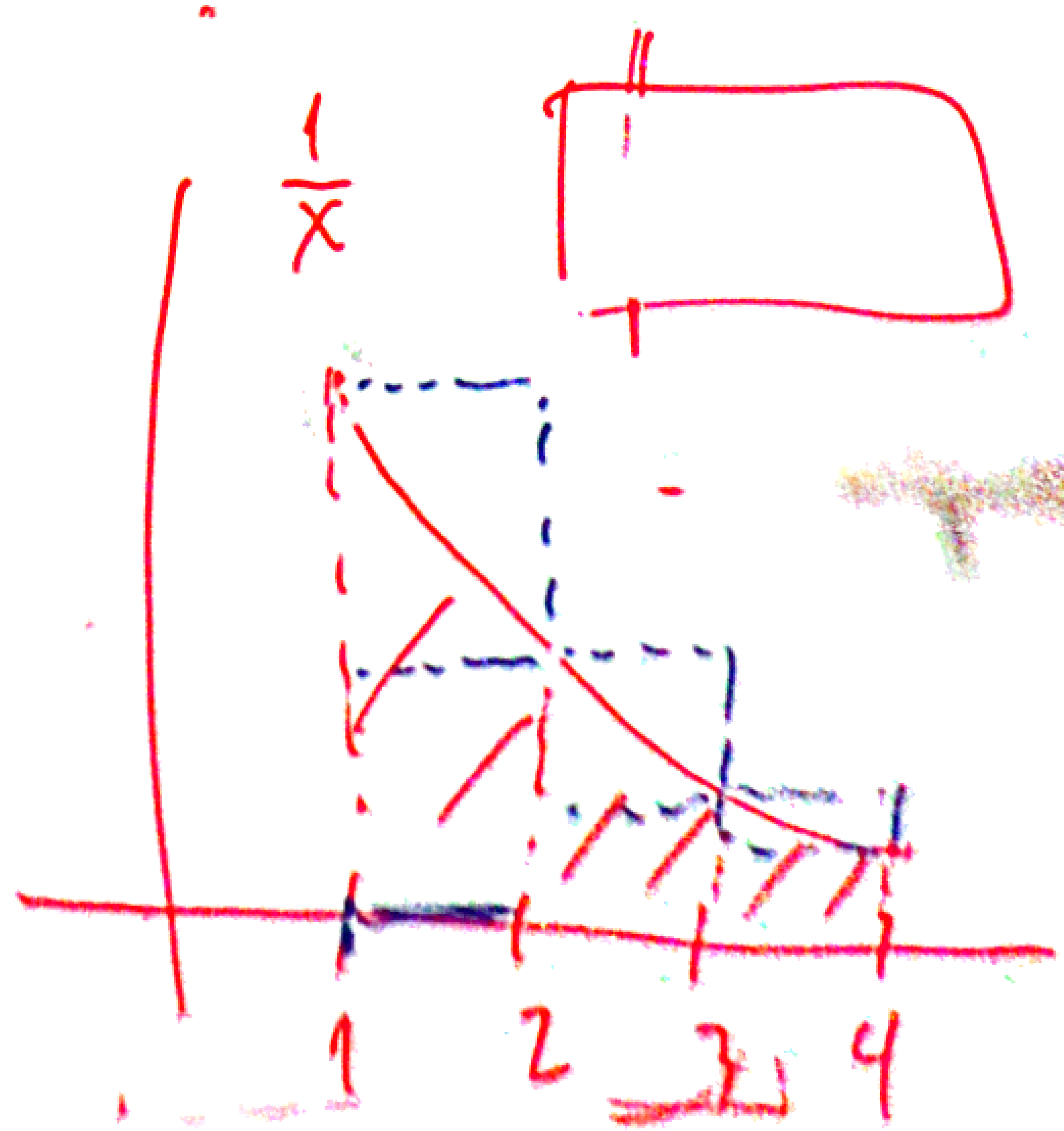

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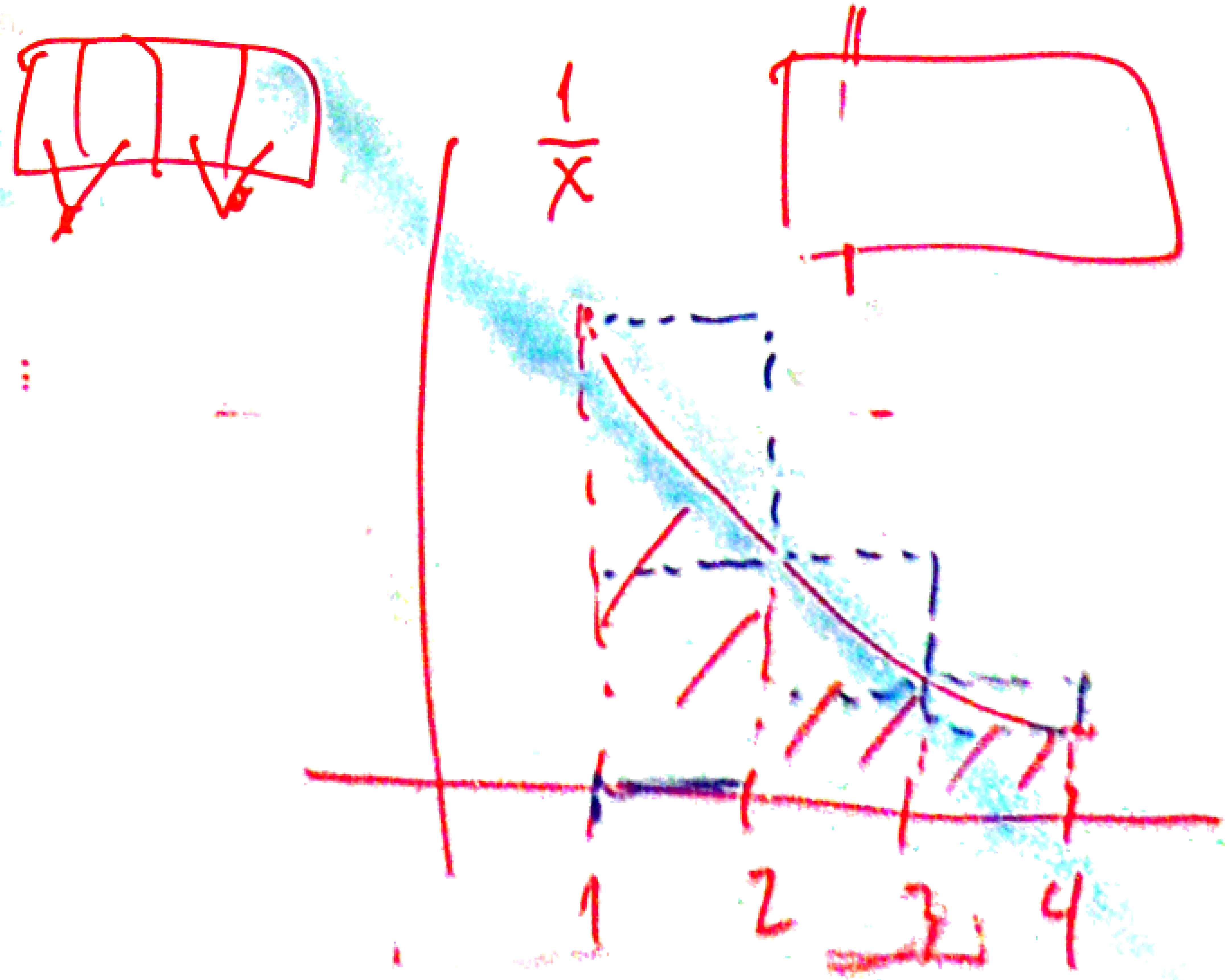


$u = |U|$

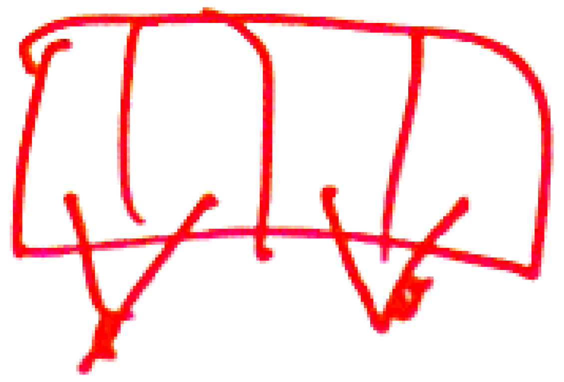
$g = |G|$

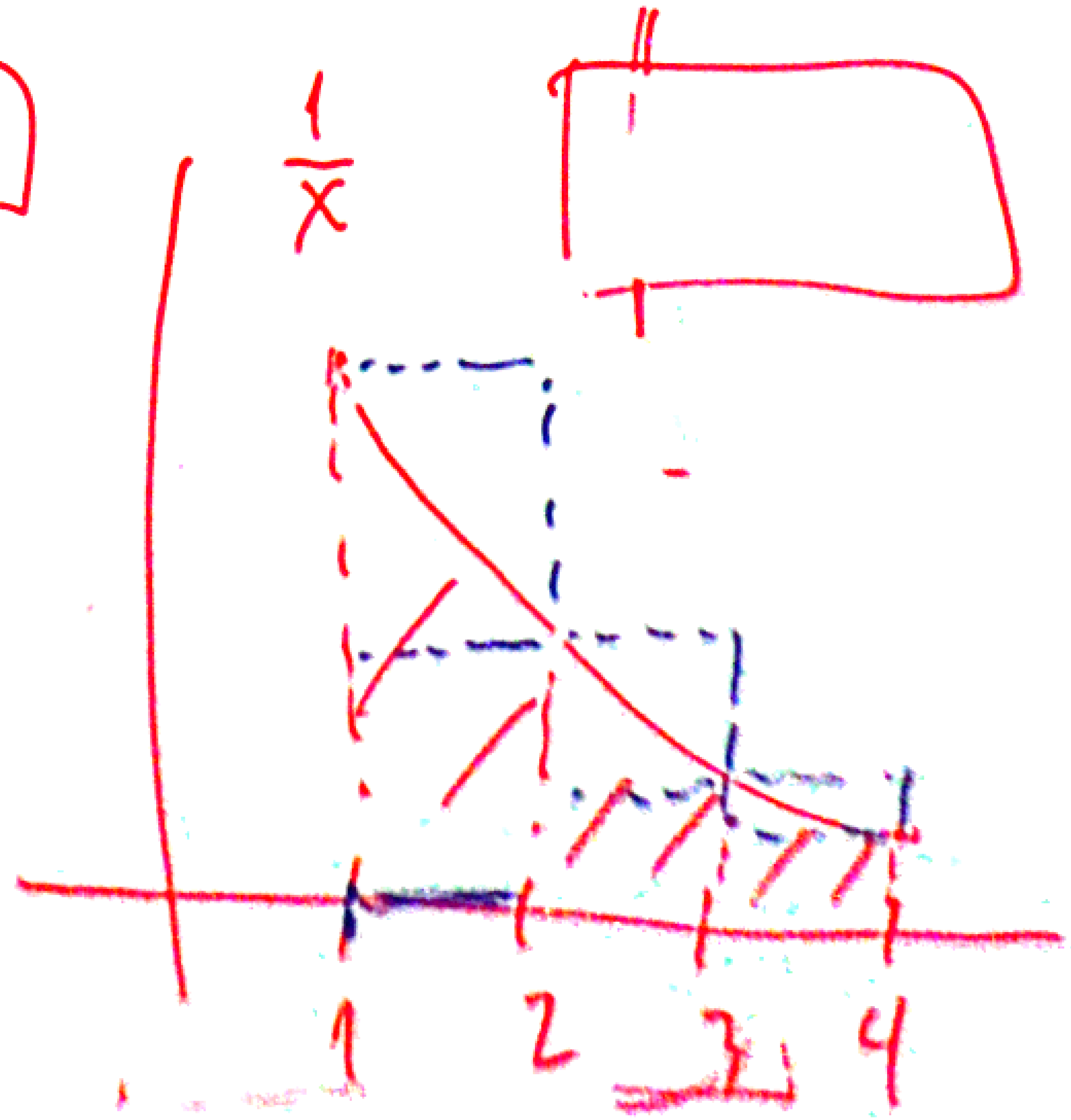
$p = |P|$

(u, g, p)

$u + g + p = n$

$$\sum_{k=2}^{n} \frac{1}{k} \leq \int_1^n \frac{1}{x}\,dx \leq \sum_{k=1}^{n} \frac{1}{k}$$





$u = |U|$

$g = |G|$

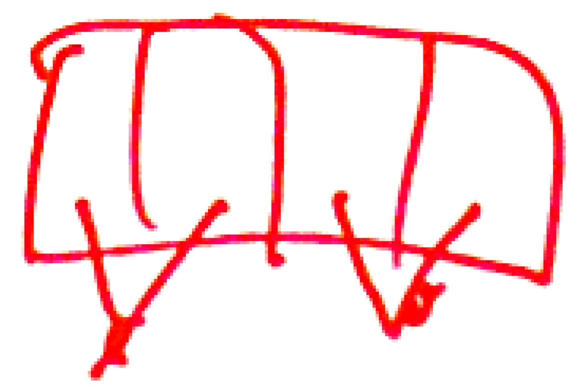
$p = |P|$

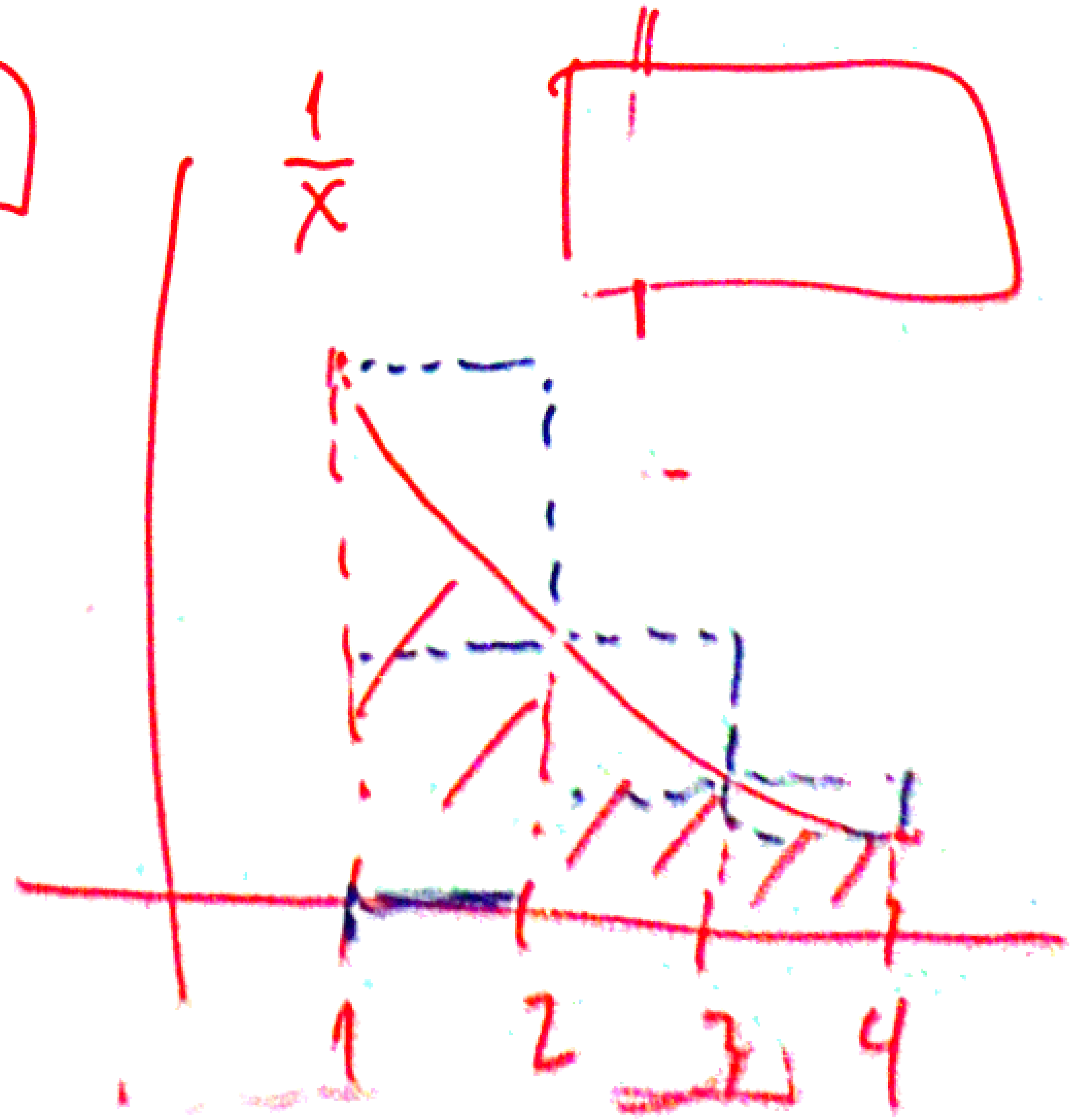
(u, g, p)

$u + g + p = n$

$(0, 1, n-1)$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_{1}^{n} \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$





$n = |U|$

$g = |G|$

$p = |P|$

$b \in U$

$g \in U$

(n, g, p)

$n + g + p = n$

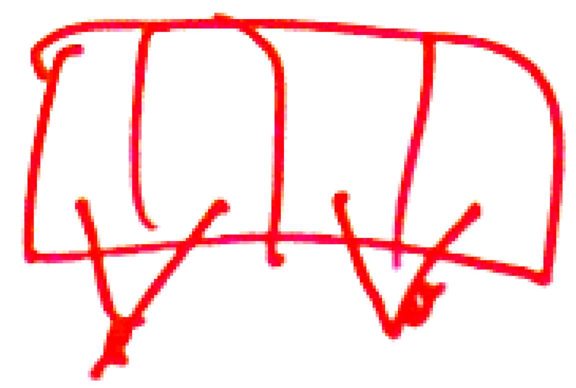
$(0, 1, n-1)$

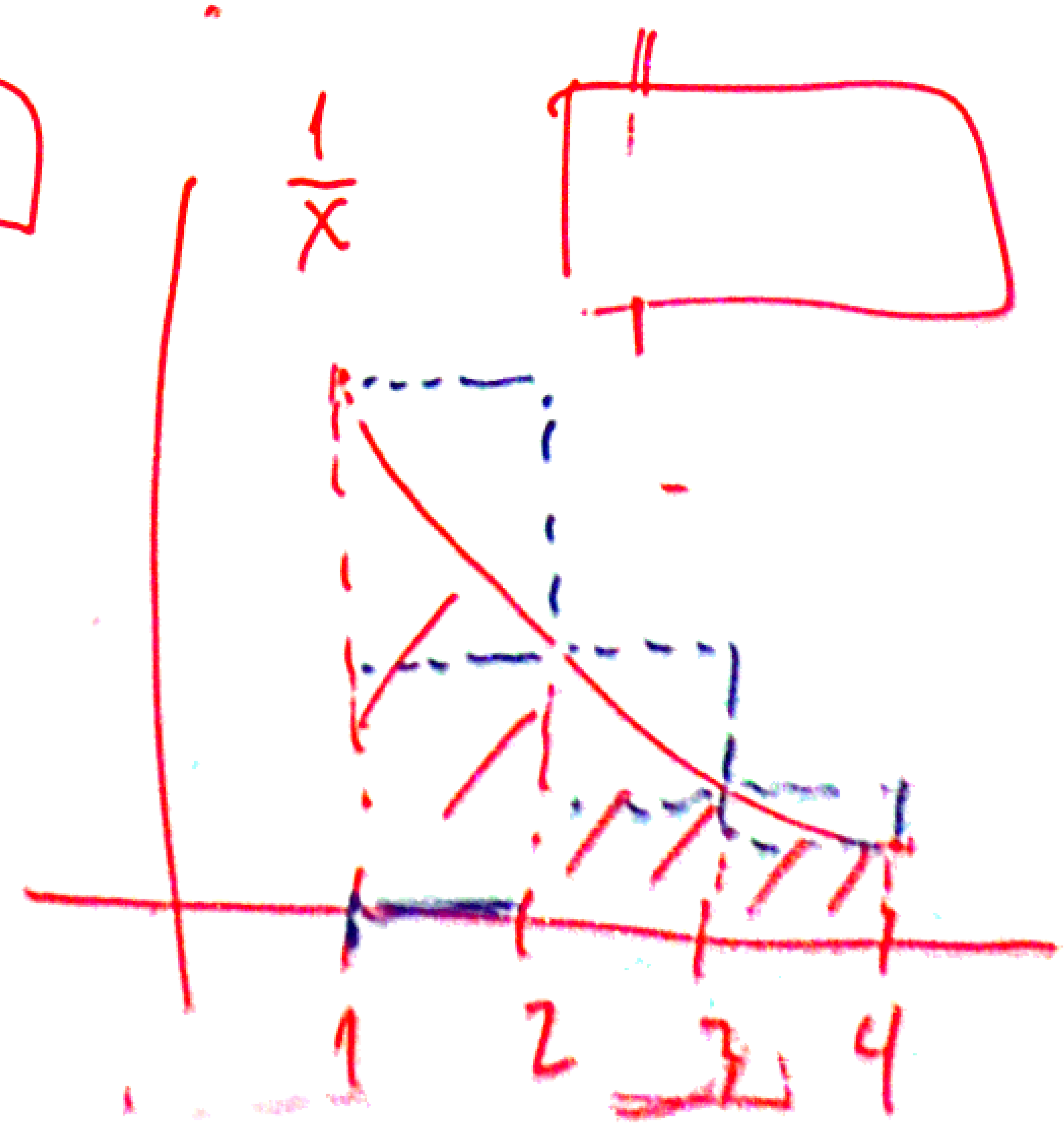
(n, g, p)

$\downarrow$

$(n-2, g+1, p+1)$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$





$u = |U|$

$g = |G|$

$p = |P|$

(u, g, p)

$u + g + p = n$

$(0, 1, n-1)$

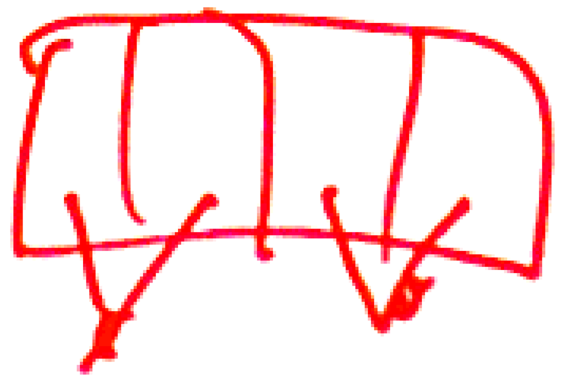
$b \in U$

$c \in G$

(u, g, p)

$(n-1$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$



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$b \in U$

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(u, g, p)

$u + g + p = n$

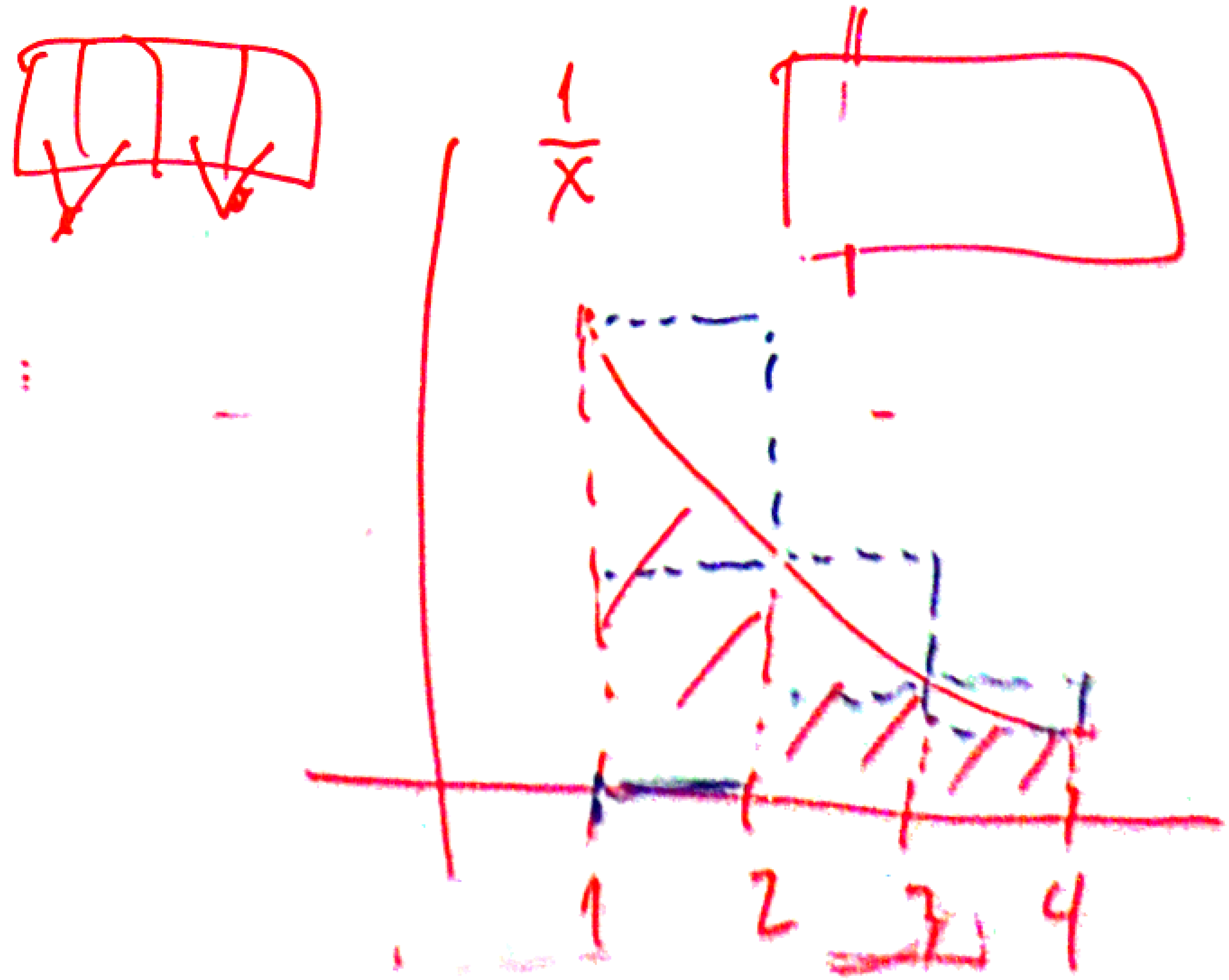
$(0, 1, n-1)$

(u, g, p)

$\downarrow$

$(u-1, g, p+1)$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$



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$p = |P|$

$b \in U$

$c \in G$

(n, g, p)

$n + g + p = n$

$(0, 1, n-1)$

(n, g, p)

$\downarrow$

$(n-1, g, p+1)$

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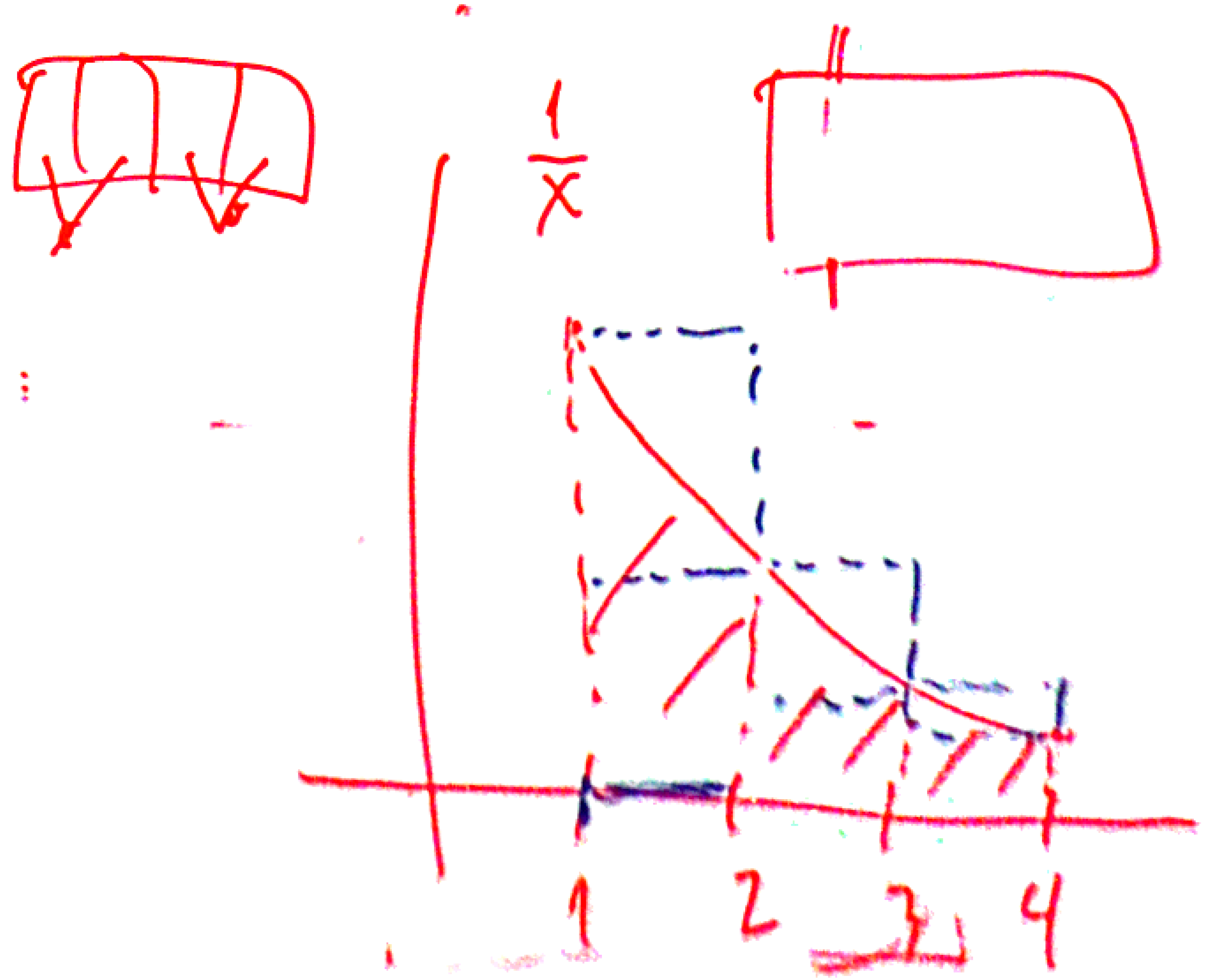


$u = |U|$

$g = |G|$

$p = |P|$

$b \in U$

$c \in P$

(u, g, p)

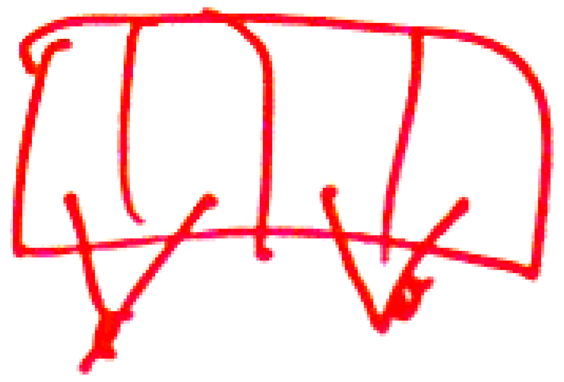
$u + g + p = n$

(u, g, p)

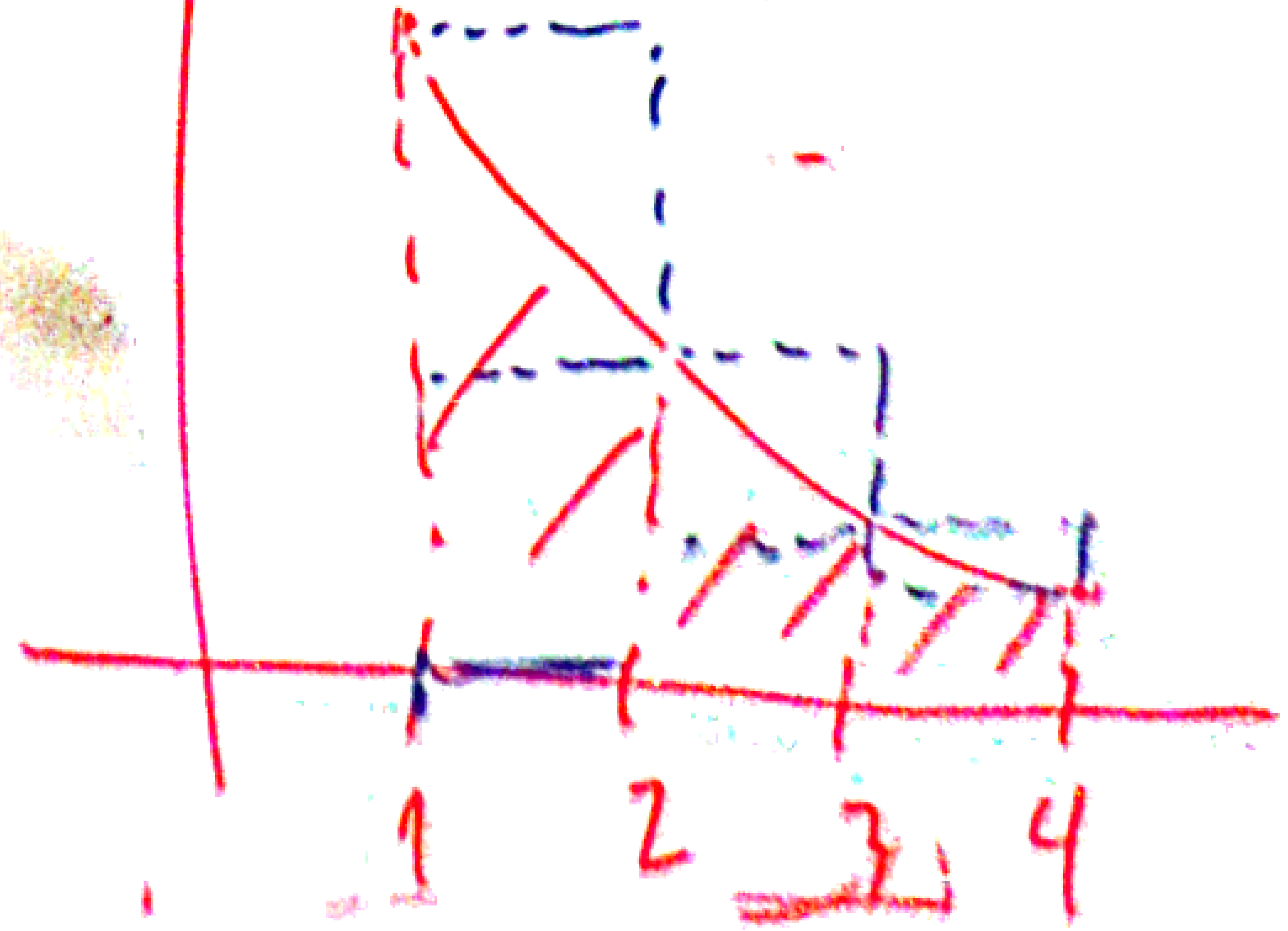
$b > c: (u-1, g+1, p)$

$b < c: (u-1, g, p+1)$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$



$\frac{1}{x}$



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(u, g, p)

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$c \in P$

(u, g, p)

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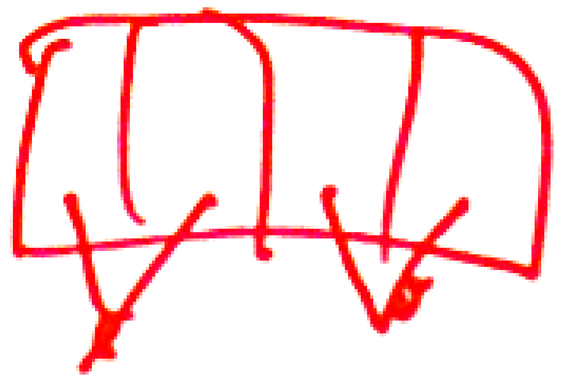

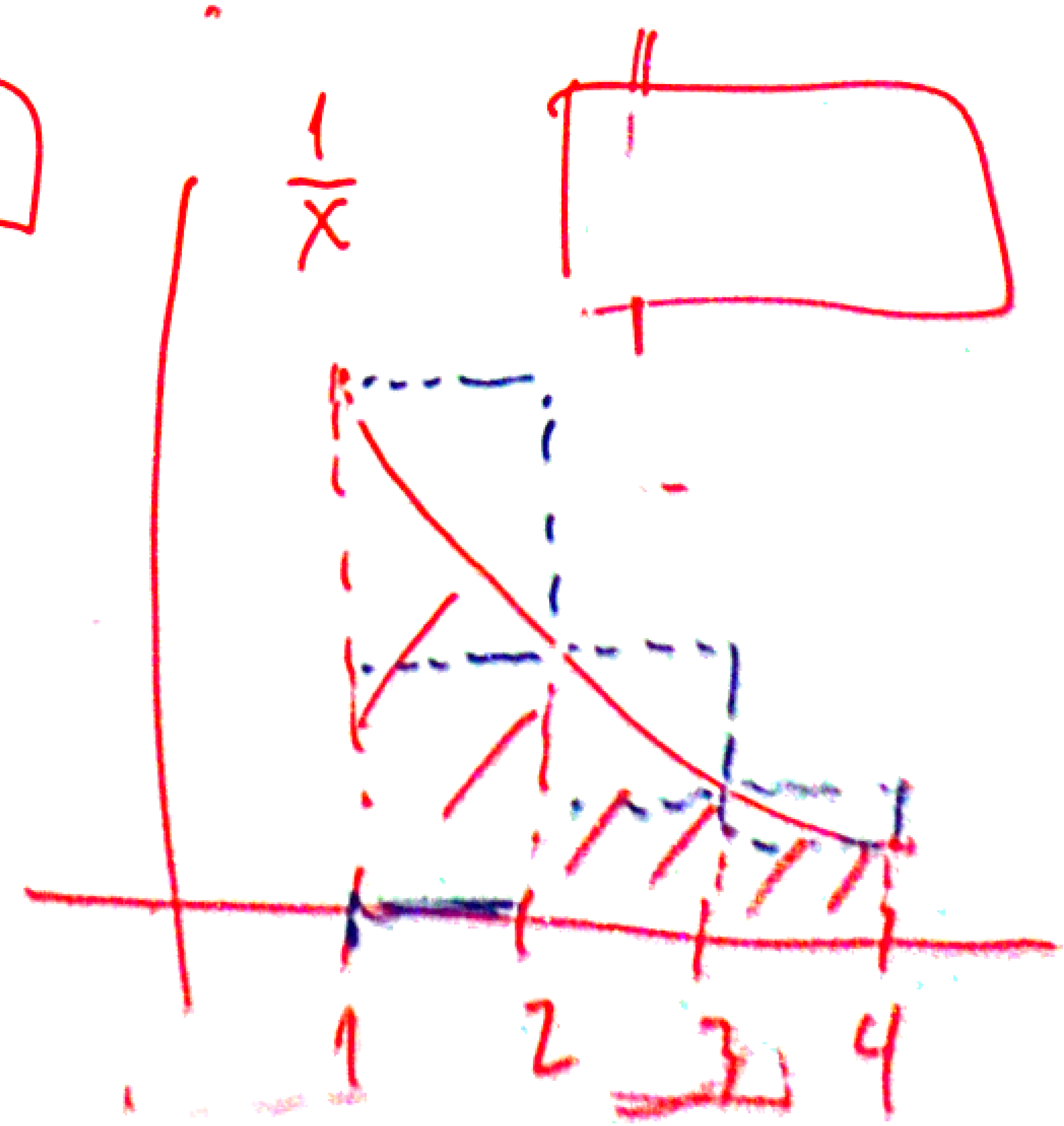

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$p = |P|$

(u, g, p)

$u + g + p = n$

$$\sum_{k=2}^{n} \frac{1}{k} \leq \int_1^n \frac{1}{x}\,dx \leq \sum_{k=1}^{n} \frac{1}{k}$$

$(n, 0, 0)$

$\downarrow$

$(0, 1, n-1)$

$b \in U$
$c \in P$

(u, g, p)

$b > c: (u-1, g+1, p)$

$b < c: (u-1, g, p+1)$

$u = |U|$

$g = |G|$

$p = |P|$

$b \in U$

$c \in P$

(u, g, p)

$u + g + p = n$

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$b > c: \ (u-1, g+1, p)$

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$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$

$$(n, 0, 0) \longrightarrow (0, 1, n-1)$$

$n-1$

$u = |U|$

$g = |G|$

$p = |P|$

(u, g, p)

$u + g + p = n$

(u, g, p)

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$c \in P$

$b > c: (u-1, g+1, p)$

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$$\sum_{k=2}^{n} \frac{1}{k} \leq \int_1^n \frac{1}{x}\,dx \leq \sum_{k=1}^{n} \frac{1}{k}$$

$(n, 0, 0)$

$\downarrow$

$(0, 1, n-1)$

$n-1$

$n-2$

$2n-3$

$u = |U|$

$g = |G|$

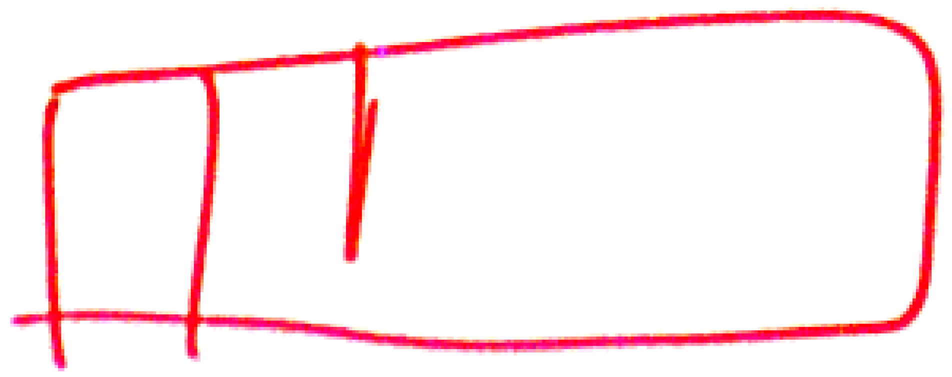
$p = |P|$

(u, g, p)

$u + g + p = n$

$b \in U$

$c \in P$



a

b

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$

$(n, 0, 0)$

$\downarrow$

$(0, 1, n-1)$

$n-1$

$n-2$

$2n-3$

$u = |U|$

$g = |G|$

$p = |P|$

(u, g, p)

$u + g + p = n$

$b \in U$

$c \in P$

a

b

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_{1}^{n} \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$

$(n, 0, 0)$

$\downarrow$

$(0, 1, n-1)$

$n-1$

$n-2$

$2n-3$

$u = |U|$

$g = |G|$

$p = |P|$

(u, g, p)

$u + g + p = n$

$b \in U$

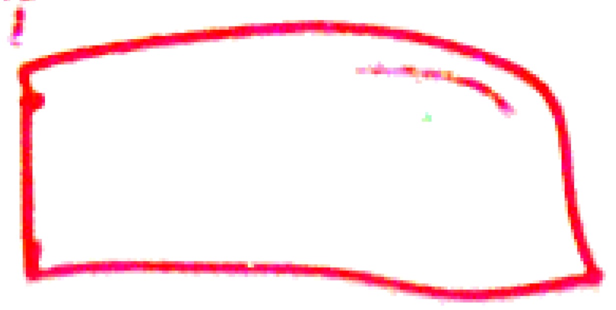
$c \in P$

$$\sum_{k=2}^{n} \frac{1}{k} \leq \int_{1}^{n} \frac{1}{x}\,dx \leq \sum_{k=1}^{n} \frac{1}{k}$$

$(n, 0, 0)$

$\downarrow$

$(0, 1, n-1)$



$$\begin{array}{r} n-1 \\ n-2 \\ \hline 2n-3 \end{array}$$

$(n, 0, 0)$

$u = |U|$

$g = |G|$

$p = |P|$

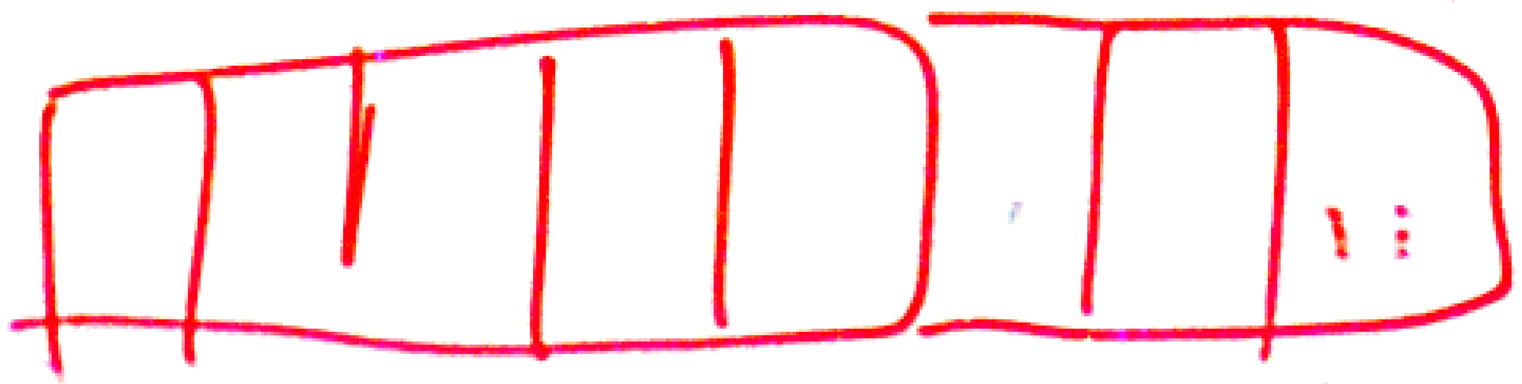
(u, g, p)

$u + g + p = n$

$b \in U$

$c \in P$

$$\sum_{k=2}^{n} \frac{1}{k} \leq \int_1^n \frac{1}{x}\,dx \leq \sum_{k=1}^{n} \frac{1}{k}$$



$(n, 0, 0)$

$\downarrow$

$(0, 1, n-1)$

u

u

$\frac{n}{2} + \left(\frac{n}{2} - 1\right)$

$+ \left(\frac{n}{2} - 1\right)$

$\rightarrow \frac{3n}{2} - 2 \rightarrow 2n-3$

$n-1$

$n-2$

$2n-3$

$u = |U|$

$g = |G|$

$p = |P|$

$b \in U$

$c \in P$

(u, g, p)

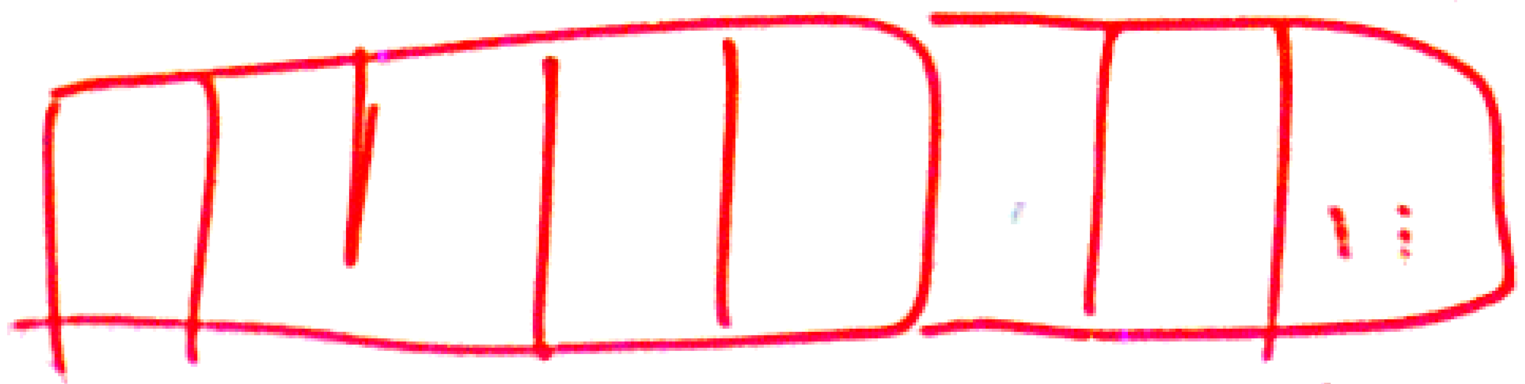
$u + g + p = n$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$

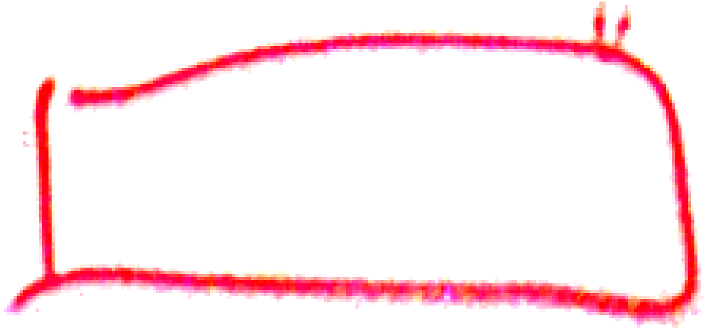
$(n, 0, 0)$

$\downarrow$

$(0, 1, n-1)$



4 

4 

$\frac{n}{2} + \left(\frac{n}{2} - 1\right)$

$+ \left(\frac{n}{2} - 1\right)$

$\frac{3n}{2} - 2$

$n-1$

$n-2$

$2n-3$

$u = |U|$

$g = |G|$

$p = |P|$

(u, g, p)

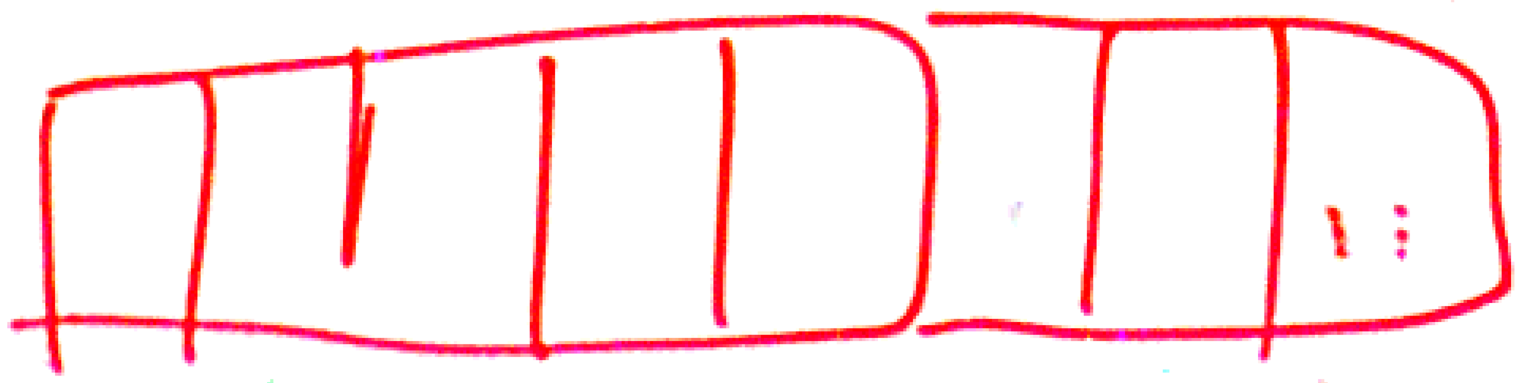
$u + g + p = n$

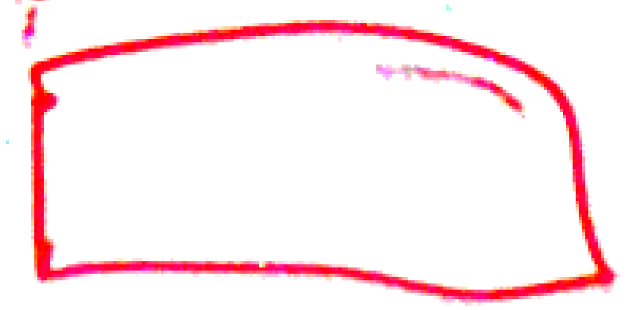
$b \in U$

$c \in P$

$$\sum_{k=2}^{n} \frac{1}{k} \leq \int_1^n \frac{1}{x}\,dx \leq \sum_{k=1}^{n} \frac{1}{k}$$

$(n, 0, 0)$



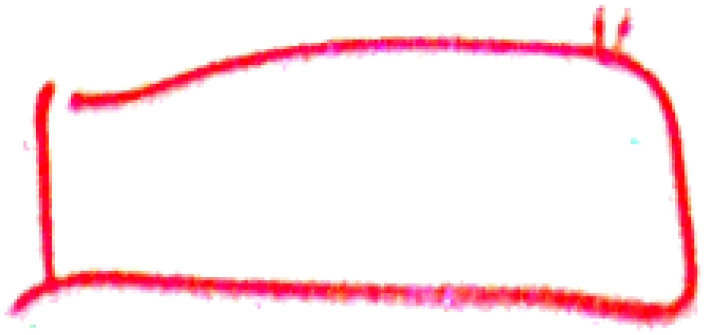


$\downarrow$

$(0, 1, n-1)$

4

4



$\frac{n}{2} + \left(\frac{n}{2} - 1\right) + \left(\frac{n}{2} - 1\right) \rightarrow \frac{3n}{2} - 2$

$n-1$

$n-2$

$2n-3$

$u = |U|$

$g = |G|$

$p = |P|$

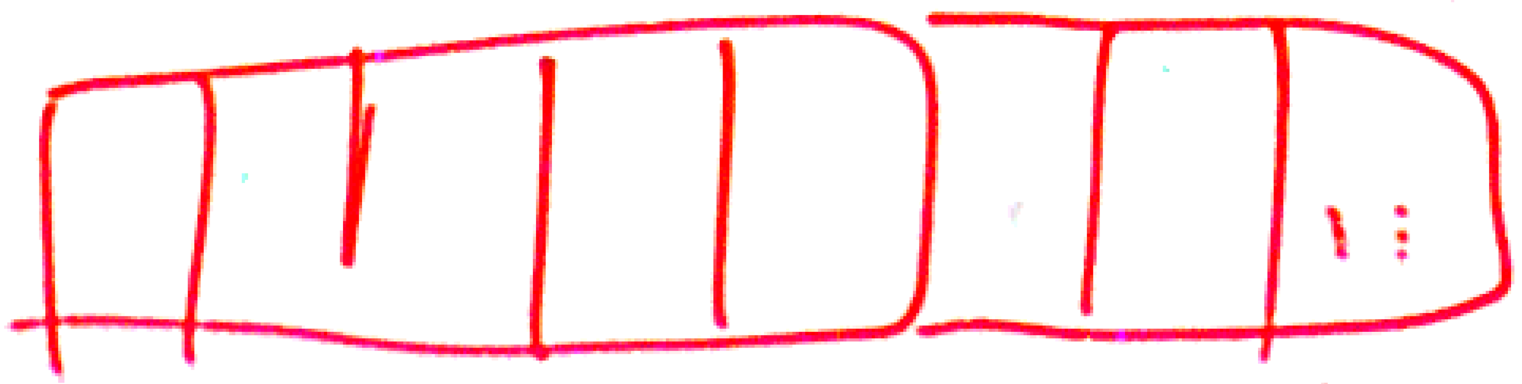
(u, g, p)

$u + g + p = n$

$b \in U$

$c \in P$

$$\sum_{k=2}^{n} \frac{1}{k} \leq \int_{1}^{n} \frac{1}{x}\,dx \leq \sum_{k=1}^{n} \frac{1}{k}$$



$(n, 0, 0)$

$\downarrow$

$(0, 1, n-1)$

4

$\frac{n}{2} + \left(\frac{n}{2} - 1\right)$

4

$+\left(\frac{n}{2} - 1\right) \to \frac{3n}{2} - 2$

$n-1$

$n-2$

$2n-3$

$u = |U|$

$g = |G|$

$p = |P|$

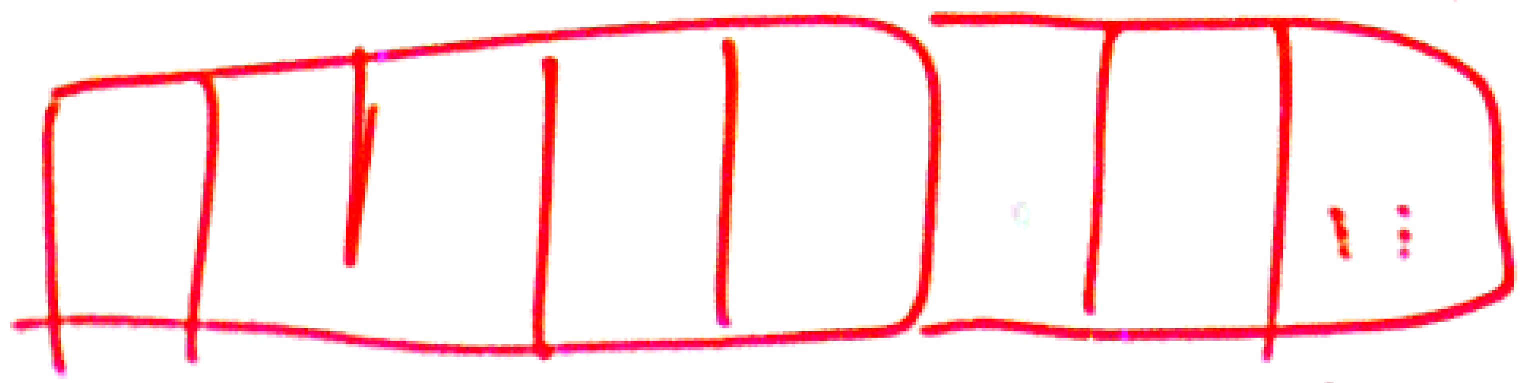
$b \in U$

$c \in P$

(u, g, p)

$u + g + p = n$

$$\sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n} \frac{1}{k}$$



4

4

$\frac{n}{2} + \left(\frac{n}{2} - 1\right)$

$+ \left(\frac{n}{2} - 1\right)$

$\frac{3n}{2} - 2$

$(n, 0, 0)$

$\downarrow$

$(0, 1, n-1)$

$$\begin{array}{r} n-1 \\ n-2 \\ \hline 2n-3 \end{array}$$