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$\omega : \Sigma^* \times \Sigma^* \rightarrow \mathbb{N}$

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$w = a_1 \dots a_n \quad a_i \in \Sigma$

$$\text{eliminor}(w, i) = a_1 \dots a_{i-1} a_{i+1} \dots a_n$$

$$\omega : \Sigma^* \times \Sigma^* \rightarrow \mathbb{N}$$

$w = a_1 \dots a_n \quad a_i \in \Sigma$

$$\text{eliminator}(w, i) = a_1 \dots a_{i-1} a_{i+1} \dots a_n$$

$$\text{aggregator}(w, i, b) = a_1 \dots a_{i-1} b a_{i+1} \dots a_n$$

$$\ell\omega : \Sigma^* \times \Sigma^* \rightarrow \mathbb{N}$$

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$$\omega : \Sigma^* \times \Sigma^* \rightarrow \mathbb{N}$$

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$$\text{eliminor}(w, i) = q_1 \dots q_{i-1} q_{i+1} \dots q_n$$

$$\text{aggregor}(w, i, b) = q_1 \dots q_{i-1} b q_{i+1} \dots q_n$$

$$\text{aggregor}(w, 0, b) = bw$$

$\omega : \Sigma^* \times \Sigma^* \rightarrow \mathbb{N}$

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$\text{Combinor}(w, i, b)$

$$\omega : \Sigma^* \times \Sigma^* \rightarrow \mathbb{N}$$

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$$\omega : \Sigma^* \times \Sigma^* \rightarrow \mathbb{N}$$

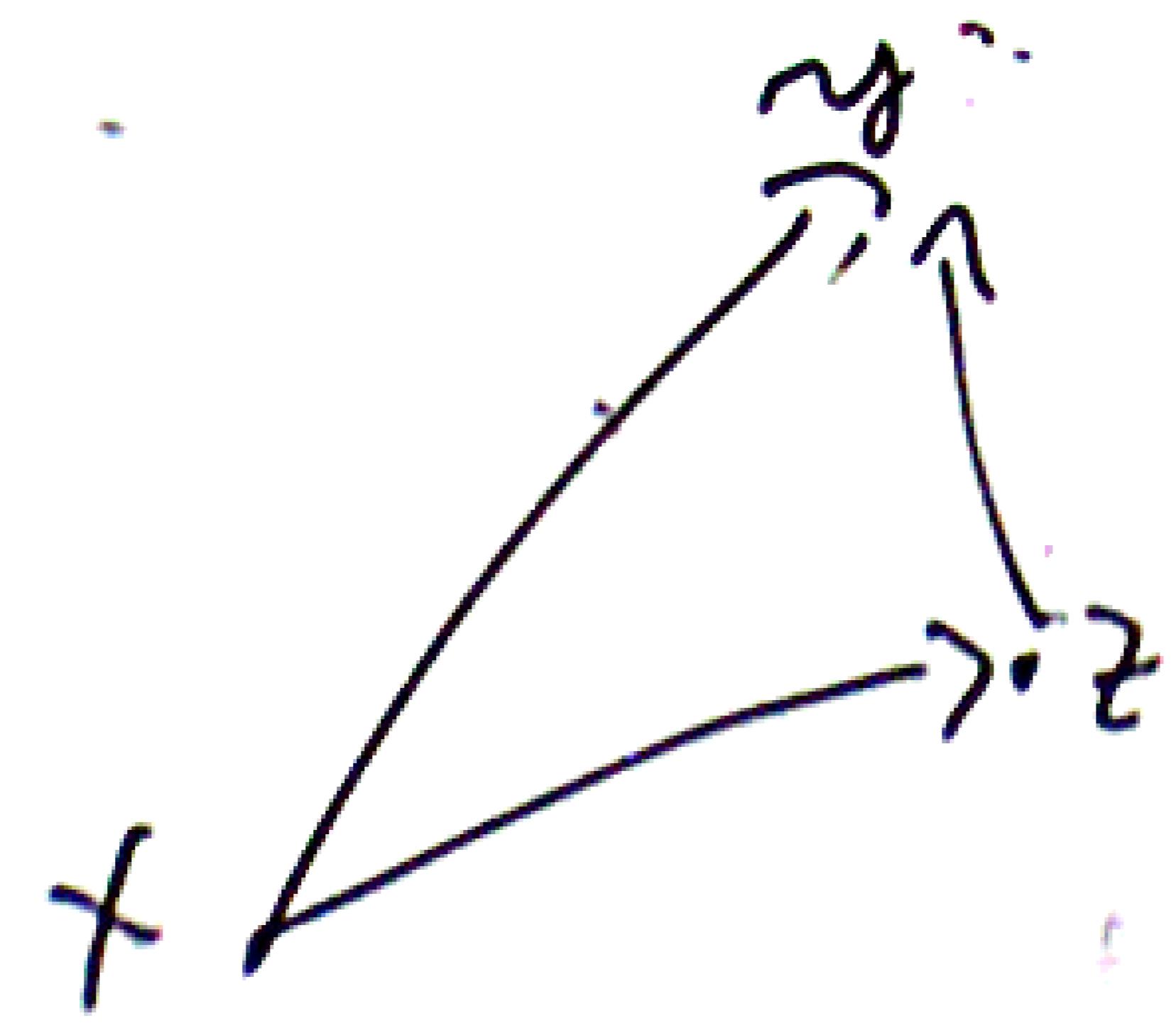
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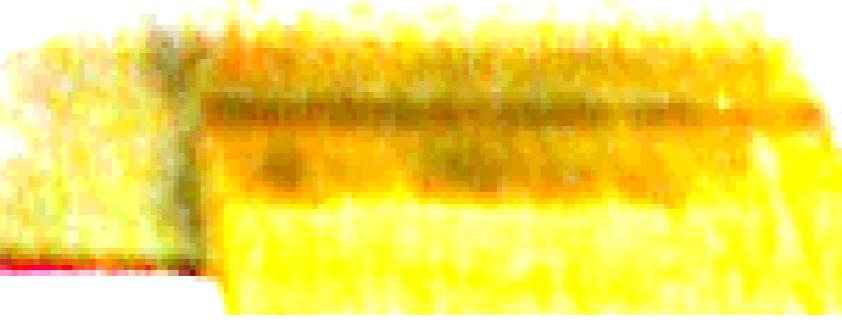


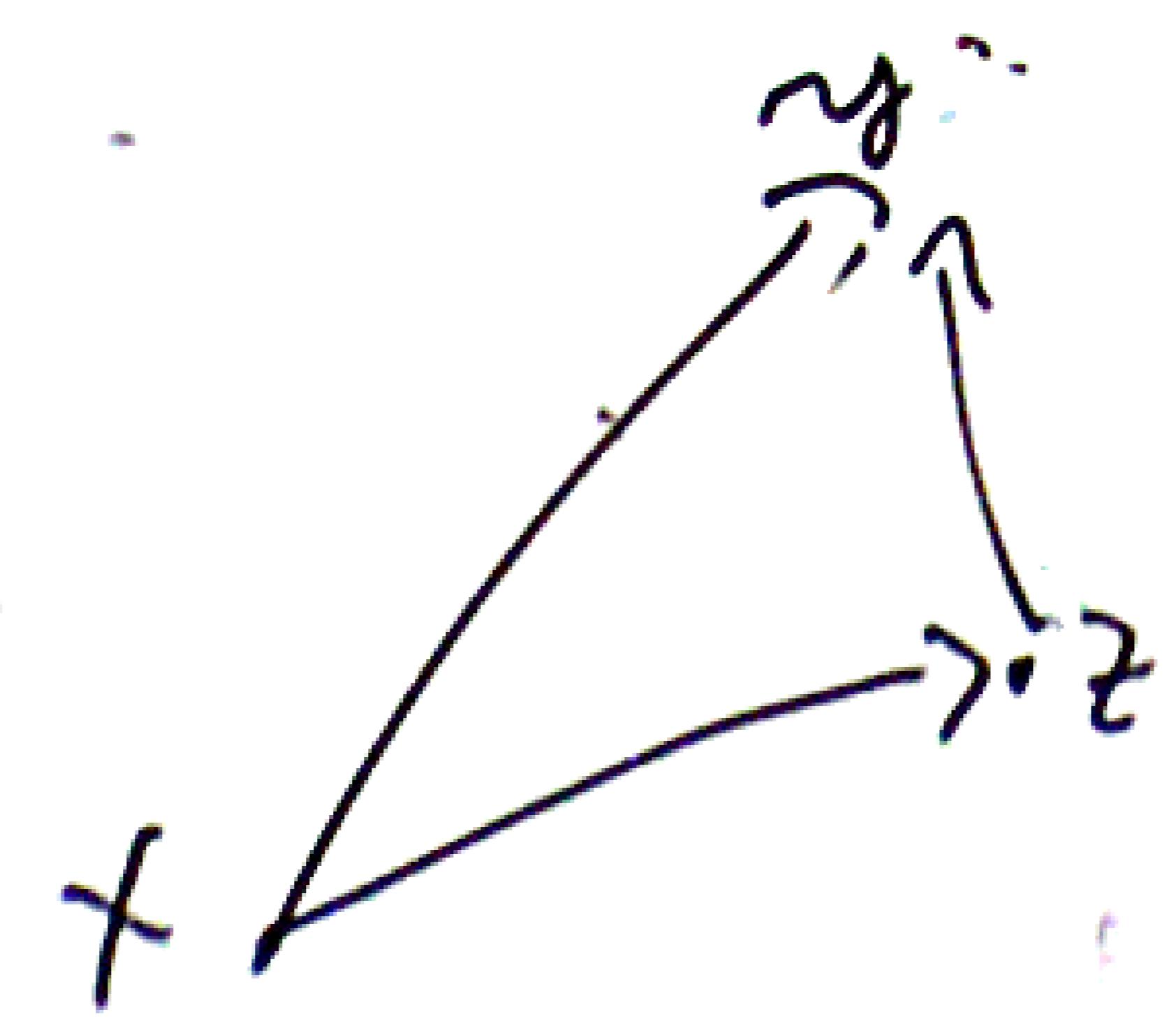
$$l_0 = \pi r^2$$

$$\frac{d}{dt} l_0 = \frac{d}{dt} (\pi r^2)$$

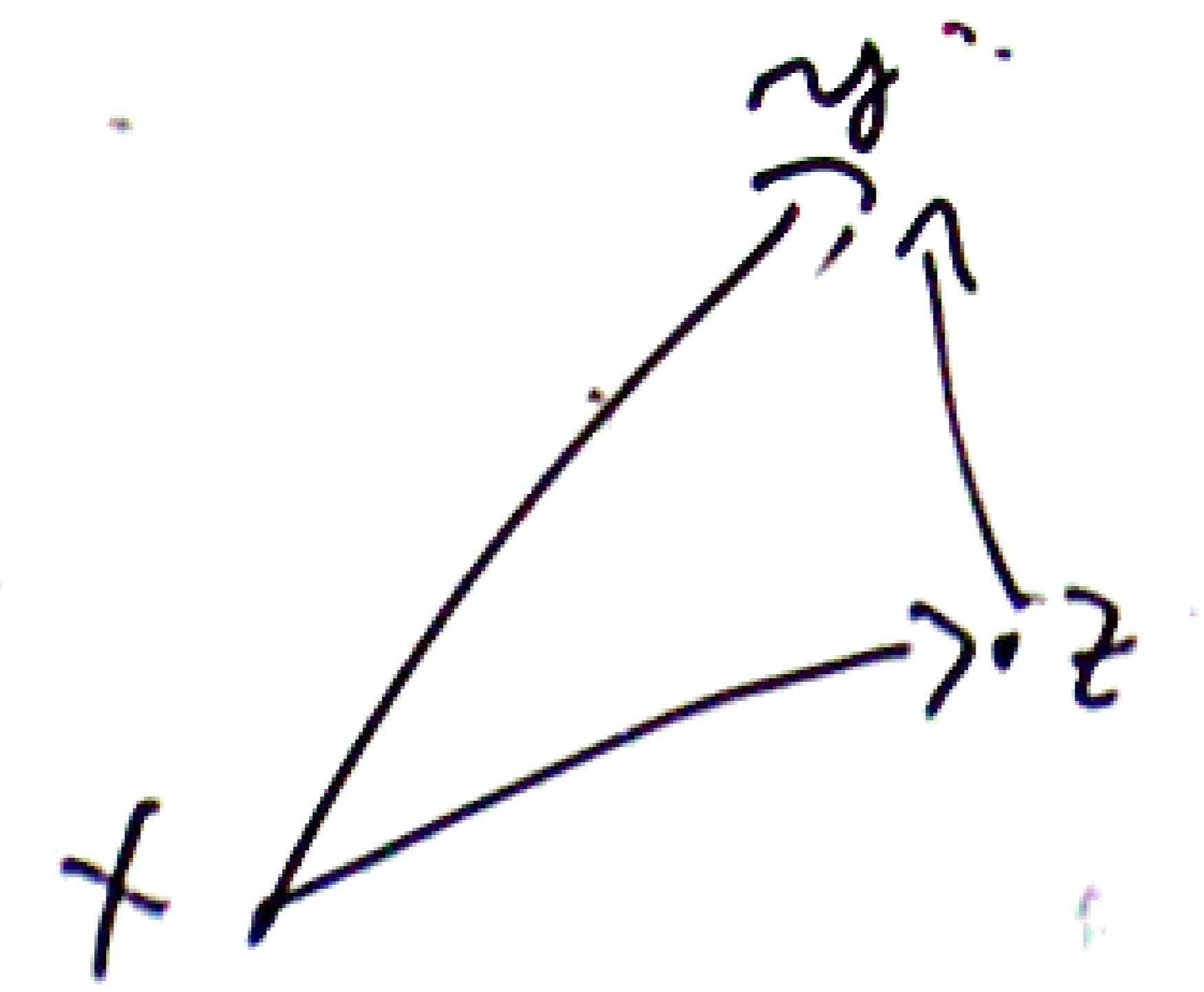
∴

$$\frac{d}{dt} r^2$$

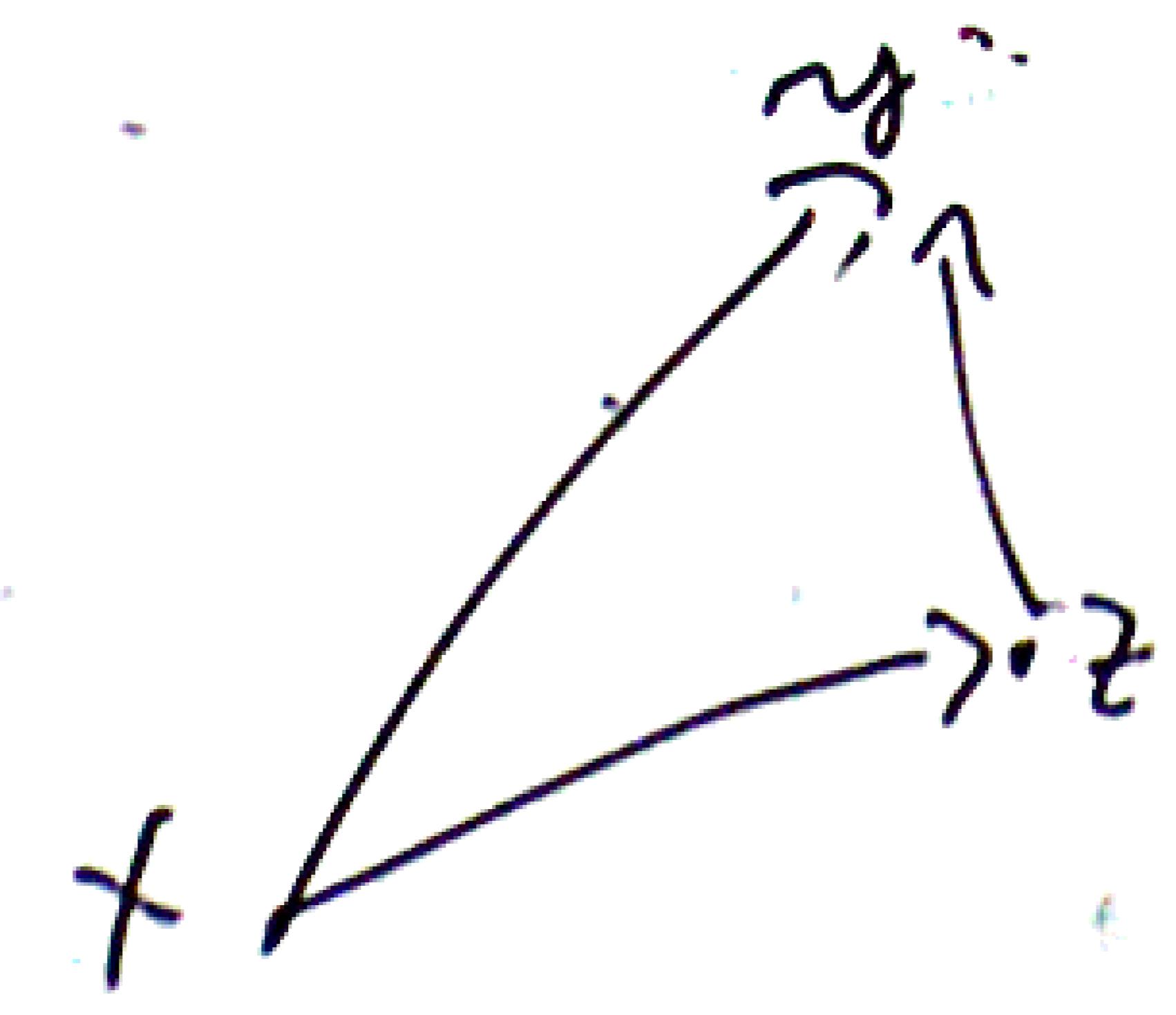




$$d(w_1, w_2) \leq d(w_1, w_3) + d(w_3, w_2)$$

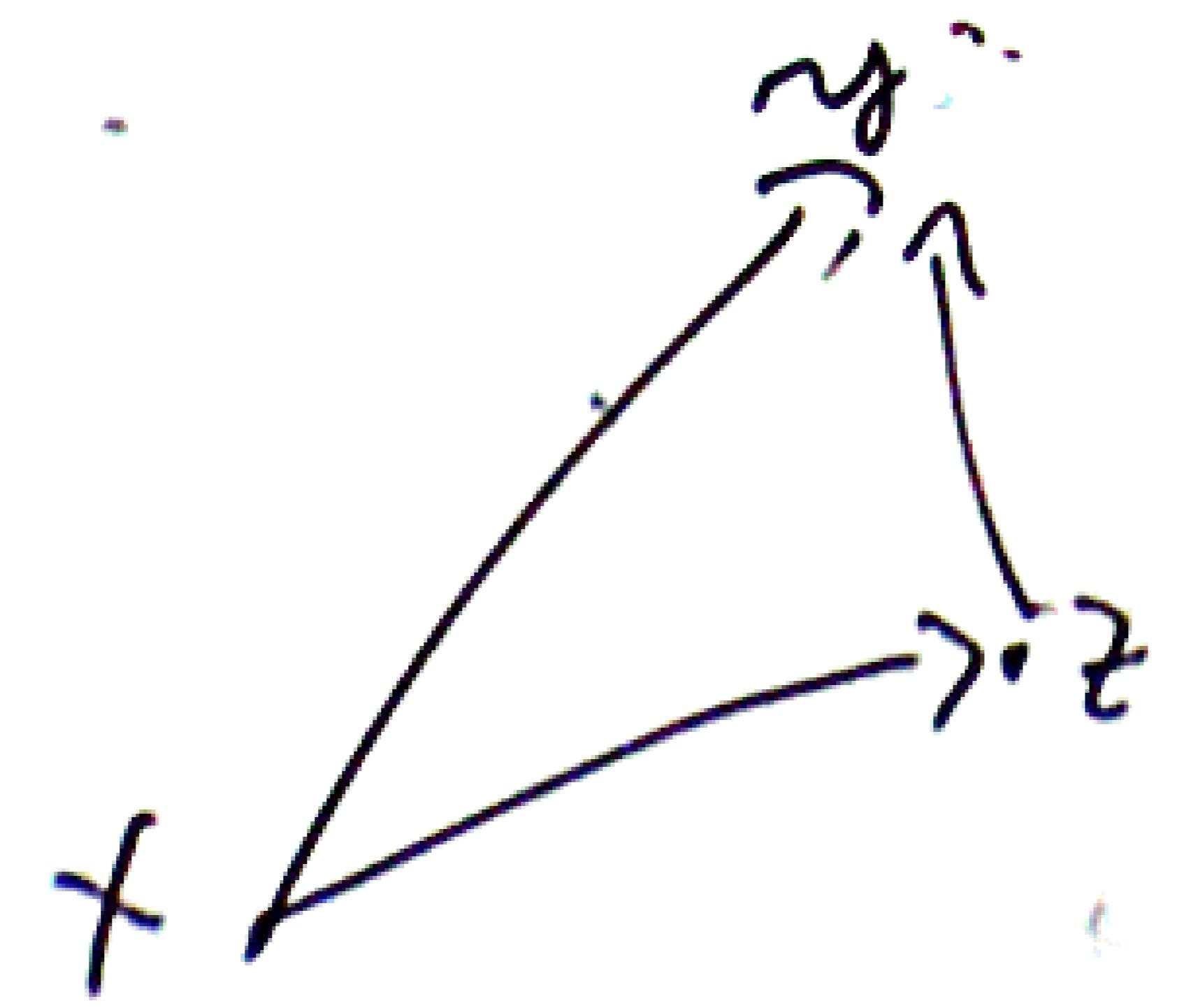


$$eb(w_1, w_2) \leq eb(w_1, w_3) + eb(w_3, w_2)$$



$$d(x,z) \leq d(x,y) + d(y,z)$$

$$d(x,z) \leq d(x,y) + d(y,z)$$

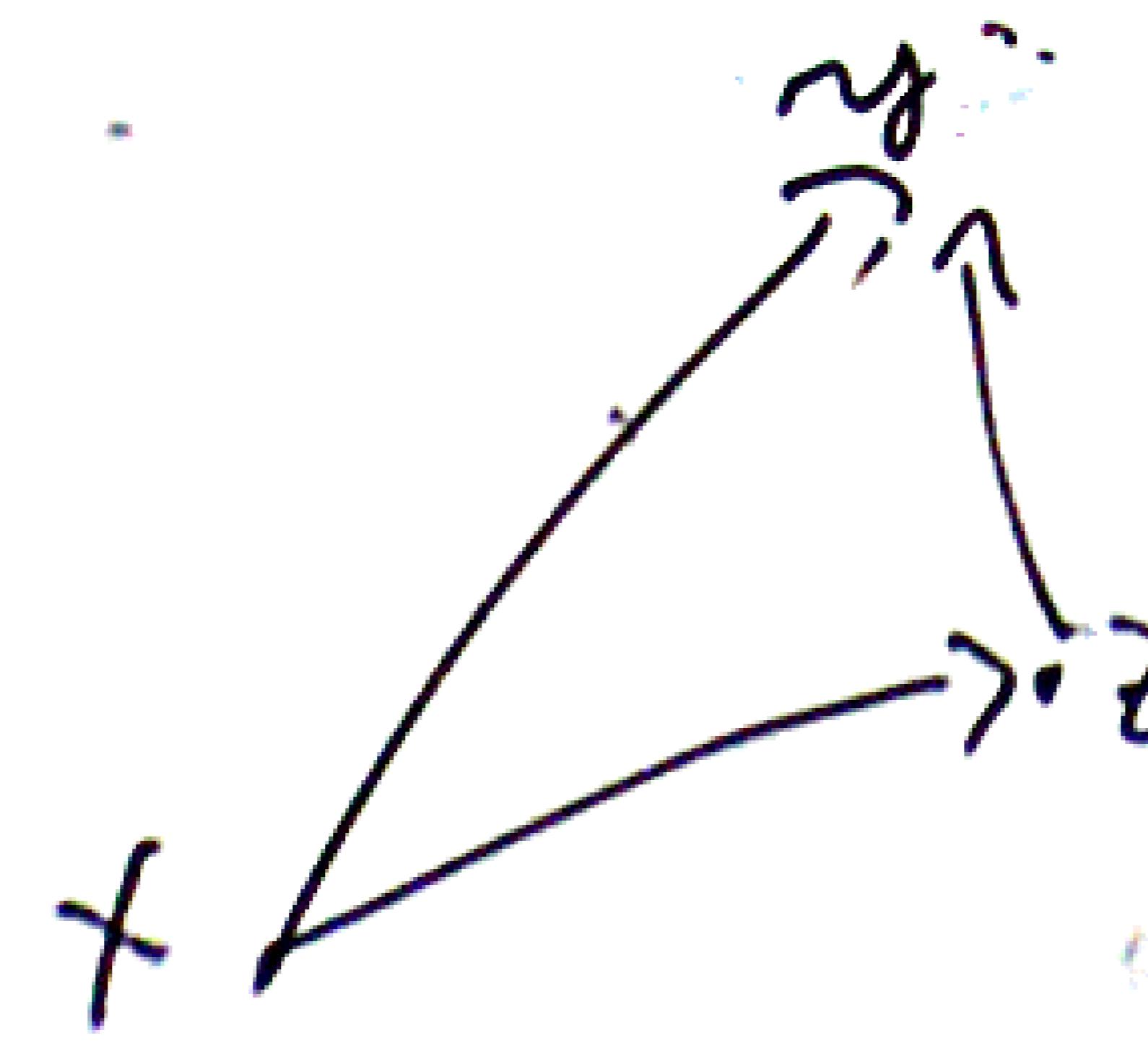


$$ed(w_1, w_2) \leq ed(w_1, w_3) + ed(w_3, w_2)$$

$$ld(w_1, w_2) \leq ld(w_1, w_3) + ld(w_3, w_2)$$

$$ed(w_1, w_2) < ld(w_1, w_2)$$

ab c d e f
b ~~c~~ d e f a

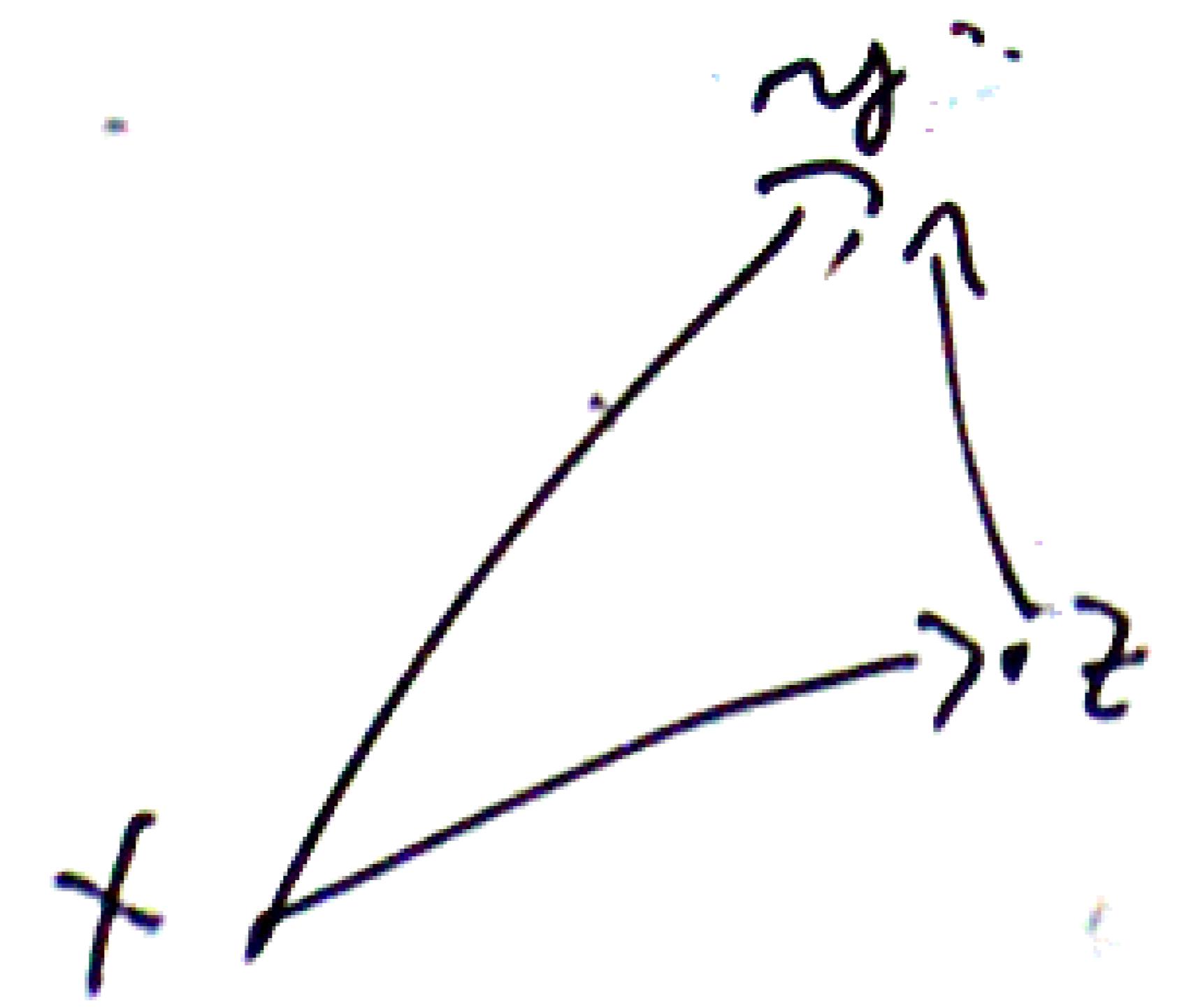


$$ed(w_1, w_2) \leq ed(w_1, w_3) + ed(w_3, w_2)$$

$$\omega(w_1, w_2) \leq hd(w_1, w_2)$$

$$ed(w_1, w_2) < hd(w_1, w_2)$$

ab c d e f
a b c d e f

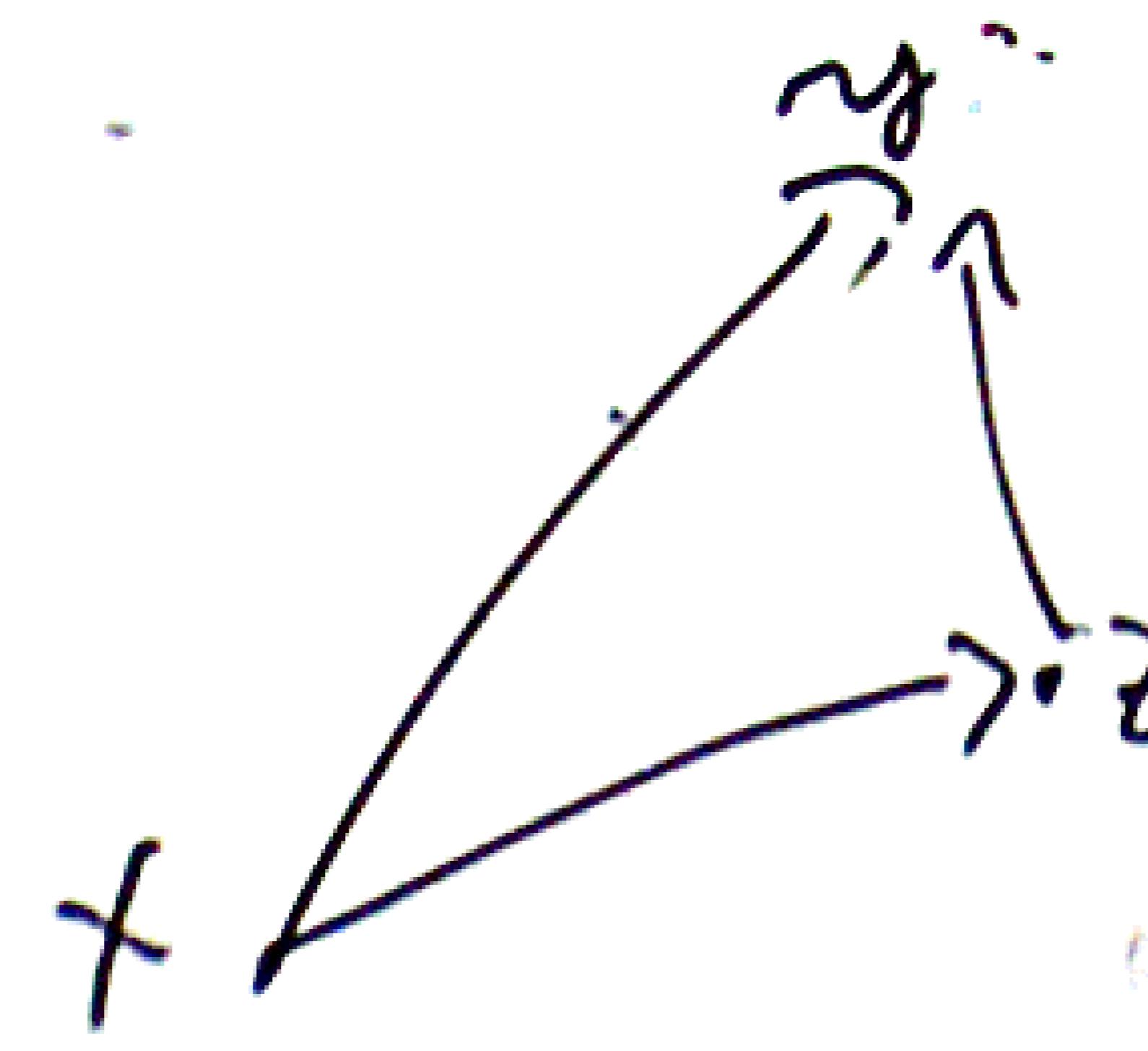


$$ed(w_1, w_2) \leq ed(w_1, w_3) + ed(w_3, w_2)$$

$$ld(w_1, w_2) \leq ld(w_1, w_3)$$

$$ld(w_1, w_2) < ld(w_1, w_3)$$

abcdef
ab**c**def



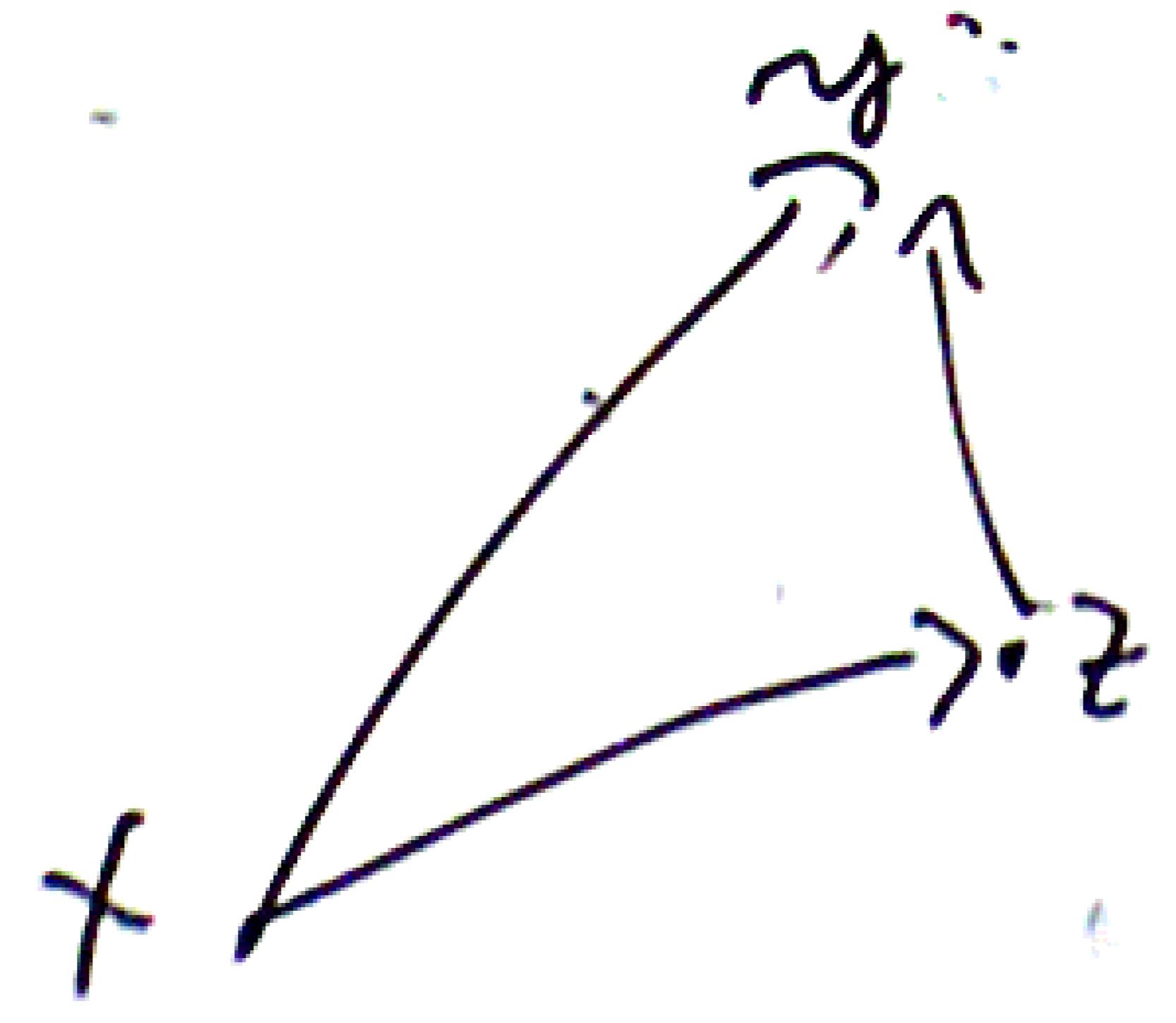
$$ed(w_1, w_2) \leq ed(w_1, w_3) + ed(w_3, w_2)$$

$$\omega(w_1, w_2) \leq hd(w_1, w_2)$$

$$ed(w_1, w_2) < hd(w_1, w_2)$$

~~abcdef~~
~~abcdef~~

ab c d e f
a b c d e f



$$\begin{aligned} \text{ed}(w_1, w_2) &\leq \text{ed}(w_1, w_3) + \text{ed}(w_3, w_2) \\ \omega(w_1, w_2) &\leq \text{hd}(w_1, w_2) \\ \text{ed}(w_1, w_2) &< \text{hd}(w_1, w_2) \\ w[i] \end{aligned}$$

$w_1 \xrightarrow{\quad \theta_1 \dots \theta_k \quad} w_2$

$$ld(\theta_1, \theta_2) = k$$

2!

2:

$\theta_1, \theta_2, \dots, \theta_k$

θ_1

θ_2

$\theta_3, \dots, \theta_k$

θ_1, θ_2

θ_3

θ_1

θ_2

θ_3

\dots

$$w_1 \xrightarrow{\quad \vdash \quad} w_2$$

$\vdash \theta_1 \dots \theta_k$

$0 \leq i \leq |w_1|$

$$ld(\theta_1, \theta_2) = k$$

2!

$w_1 \xrightarrow{\theta_1 \dots \theta_k} w_2$

$$0 \leq i \leq |w_1|$$

$w_1[1, i]$

$$\text{ld}(w_1, w_2) = k$$

2.

(w_2)

$\vdash \theta_1 \dots \theta_k$

$w_1 \xrightarrow{?} w_2$

$0 \leq i \leq |w_1|$

$w_1[1:i]$

$ld(w_1, w_2) = k$

2.

$|w_2|$

$w_1 \xrightarrow{\theta_1 \dots \theta_k} w_2$

$$0 \leq i \leq |w_1|$$

$w_1[1, i]$

$$\text{ld}(w_1, w_2) = k$$

2.

$|w_2|$

$\vdash \theta_1 \dots \theta_k$

$w_1 \xrightarrow{\quad} w_2$

$0 \leq i \leq |w_1|$

$w_1[1, i]$

$ld(w_1, w_2) = k$

2.

w_2

$w_1 \xrightarrow{\phi_1 \dots \phi_k} w_2$

$$0 \leq i \leq |w_1|$$

$w_1[1, i]$

$$\text{ld}(w_1, w_2) = k$$

$\phi_1 \dots \phi_{20}$

$w_1[1, 5]$

w_2

$w_1 \xrightarrow{\phi_1 \dots \phi_k} w_2$

$$0 \leq i \leq |w_1|$$

$w_1[1, i]$

a

$\phi_1 \dots \phi_i$

$$\text{ld}(w_1, w_2) = k$$

$w_2 \xrightarrow{\phi_1 \dots \phi_{20}} w_3$

$w_2[1, 5]$

w_3

w_1 $\xrightarrow{\quad \vdots \quad}$ w_2

$O_1 \dots O_k$

$0 \leq i \leq |w_1|$

$w_1[1, i]$

a

$ld(w_1, w_2) = k$

$O_1 \dots O_{20}$

$w_2[1, 5]$

$O_1 \dots O_7$

$w_2[1, 5]$

$w_1 \xrightarrow{\quad \cdot \quad} w_2$

$\vdots \quad \quad \quad \downarrow$

$o_1 \dots o_k$

$0 \leq i \leq |w_1|$

$w_1[1, i]$

a

$a \quad a \quad \dots \quad a \quad a$

$ld(w_1, w_2) = k$

$o_1 \dots o_{20}$

$w_1[1, 5]$

$o_1 \dots o_7$

CASA
~~COBO~~

$|w_2|$

$w_1 \xrightarrow{\phi_1 \dots \phi_k} w_2$

$$0 \leq i \leq |w_1|$$

$w_1[1, i]$

a

$$ld(w_1, w_2) = k$$

CASA
COSO

ϕ_1, \dots, ϕ_{20}

$w_1[1, 5]$

ϕ_1, \dots, ϕ_7

w_2

$\{ \quad \}$

ϕ_1

ϕ_2

$w_1 \xrightarrow{\phi_1 \dots \phi_k} w_2$

$$0 \leq i \leq |w_1|$$

$w_1[1, i]$

a

$\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta, \theta, \varphi, \psi, \omega$

$$ld(w_1, w_2) = k$$

ϕ_1, \dots, ϕ_{20}

$w_1[1, 5]$

ϕ_1, \dots, ϕ_7

CASA
OSD

w_2

$w_1 \xrightarrow{\quad} o_1 \dots o_k \xrightarrow{\quad} w_2$
 $0 \leq i \leq |w_1|$

$$w_1[1, i]$$

$$ld(w_1, w_2) = k$$

CASA
COSO

W / Γ_{152} λ_1 λ Γ_{152} Γ_{20}

wc1,52

1.  2. 

o_1, \dots, o_k

$w_1 \longrightarrow w_2$

o_1, \dots, o_{l-1}

$w_1[1, i]$

$s \in \{0, \dots, l-1\}$

o_{s+1}

$w_1[1, i]$

o_s

w_2

O_1, \dots, O_k

$w_1 \longrightarrow w_2$

O_1, \dots, O_{l-1}

$w_1[1, i]$

$s \in \{0, \dots, l-1\}$

O_{s+1}

$w_1[1, i]$

O_s

w_2

o_1, \dots, o_k

$w_1 \longrightarrow w_2$

o_1, \dots, o_{l-1}

$w_1[1, i]$

a

$s \in \{0, \dots, l-1\}$

o_{s+1}

$w_1[1, i]$

o_s

\vdots

w_2

O_1, \dots, O_k

$w_1 \longrightarrow w_2$

O_1, \dots, O_{l-1}

$w_1[1, i]$

$s \in \{0, \dots, l-1\}$

O_{s+1}

$w_1[1, i]$

O_s

\curvearrowleft

$O_s \quad O_{s+1}$

$\cancel{O_i}$

o_1, \dots, o_k

$w_1 \longrightarrow w_2$

o_1, \dots, o_{l-1}

$w_1[1, i]$

a

$s \in \{0, \dots, l-1\}$

o_{s+1}

$w_1[1, i]$

o_s

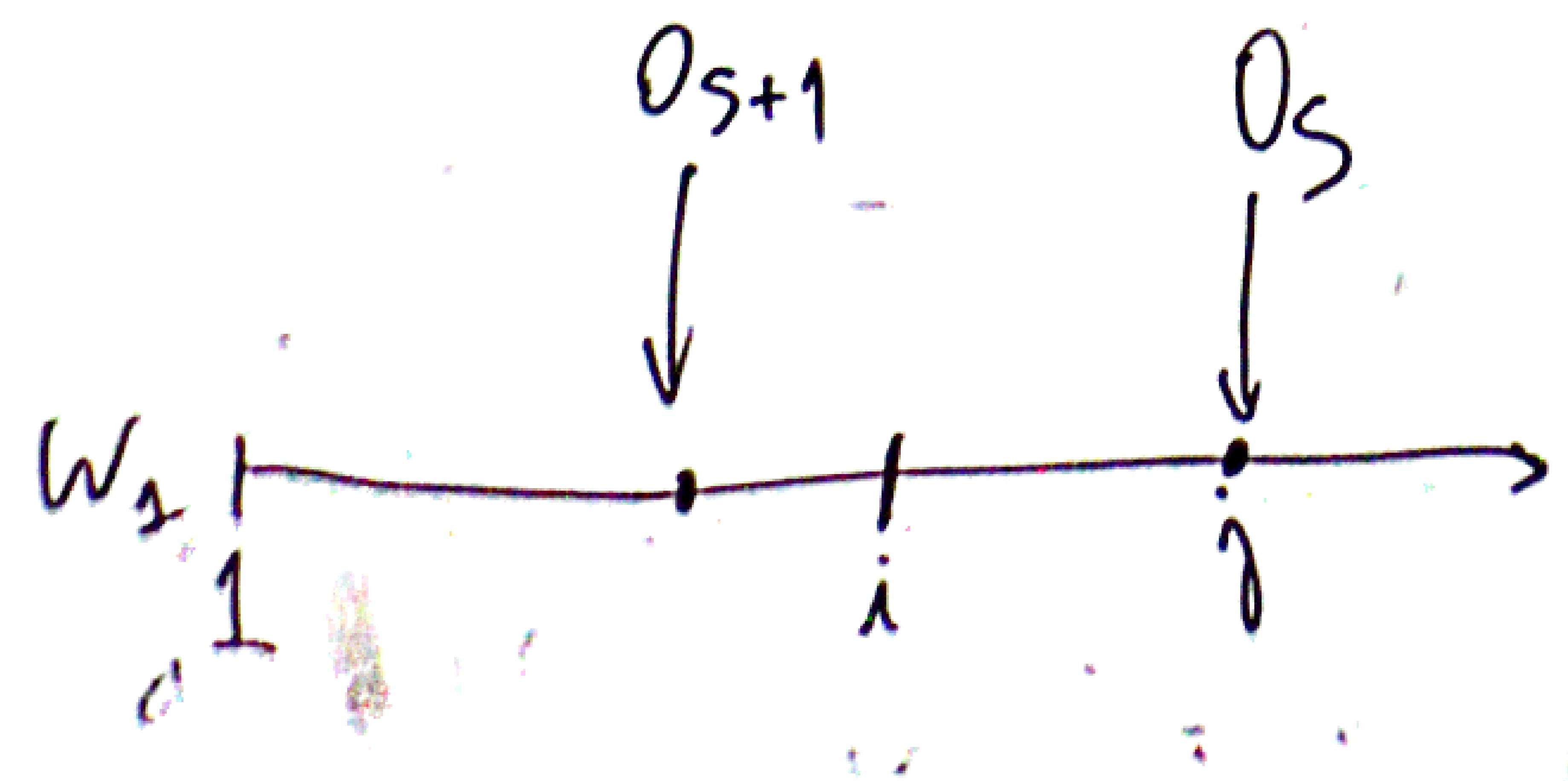
\curvearrowleft

$o_s \quad o_{s+1}$

~~(1, i)~~ (1, i)

o_1, \dots, o_k

$w_1 \longrightarrow w_2$



$s \in \{0, \dots, l-1\}$

o_{s+1}

$w_1[1, i]$

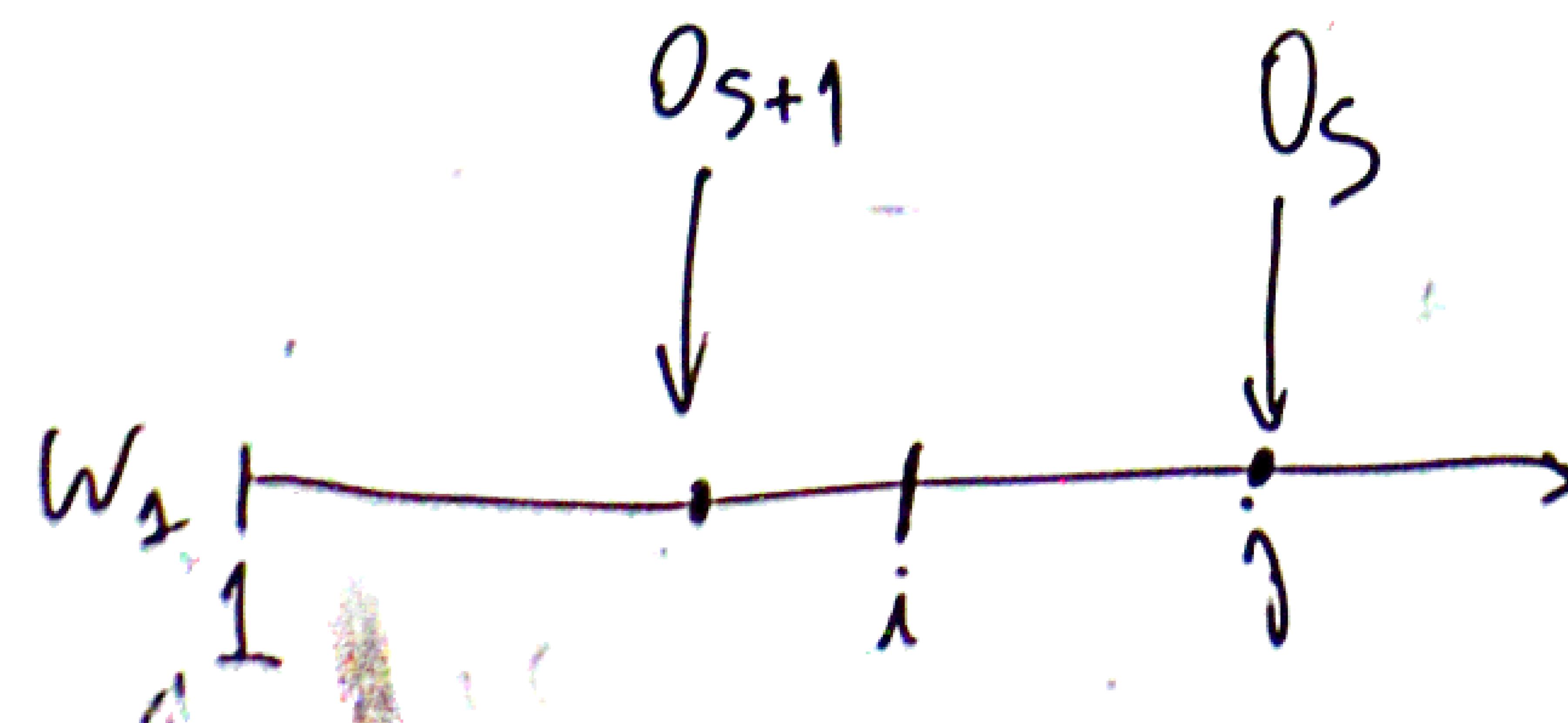
o_s

$o_s \quad o_{s+1}$

~~(1, i)~~ $(1, i)$

o_1, \dots, o_k

$w_1 \longrightarrow w_2$



$s \in \{0, \dots, l-1\}$

o_{s+1}

$w_1[1, i]$

o_s

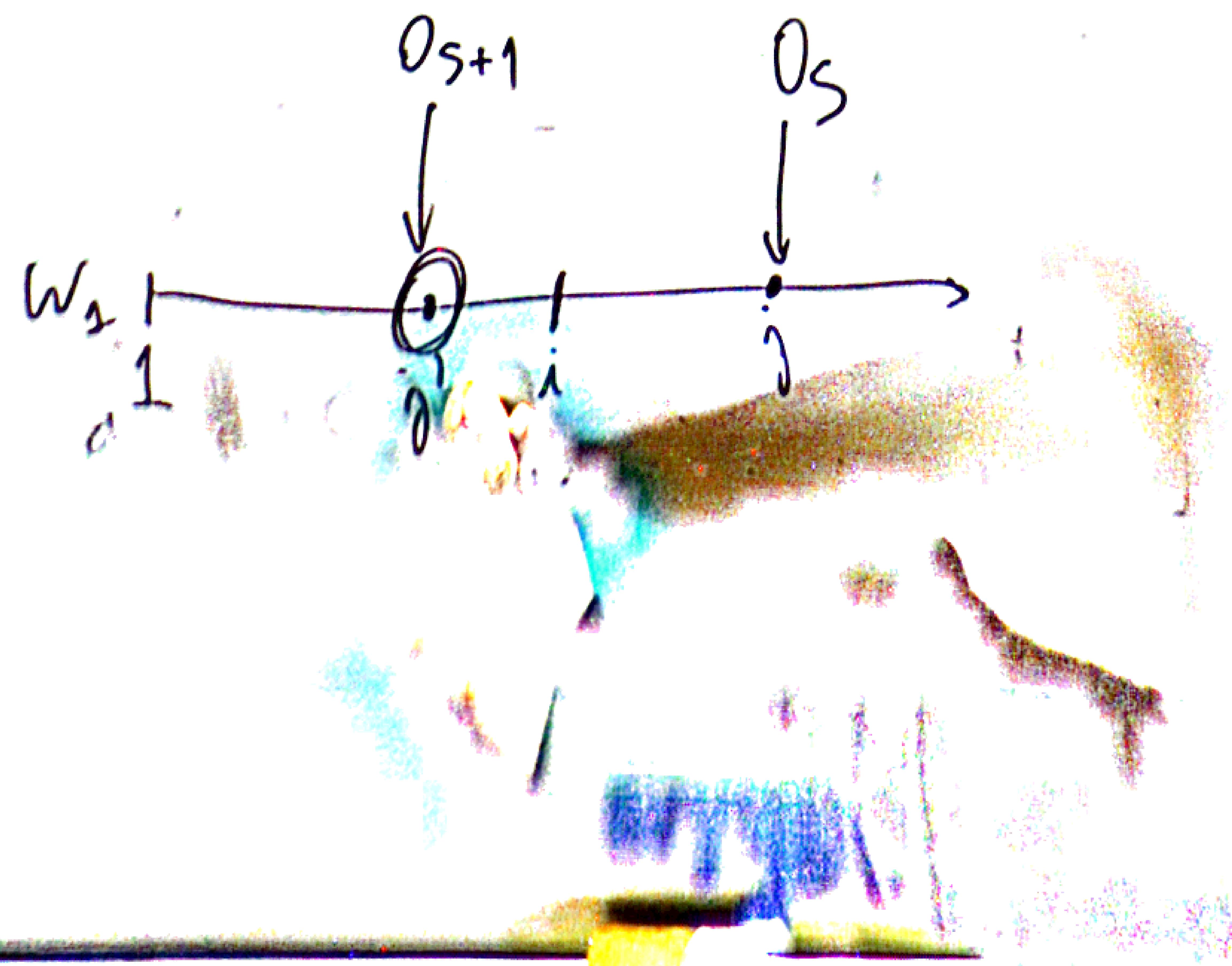
\curvearrowright

$o_s \quad o_{s+1}$

~~(i)~~ $(1, i)$

o_1, \dots, o_k

$w_1 \longrightarrow w_2$



$s \in \{0, \dots, l-1\}$

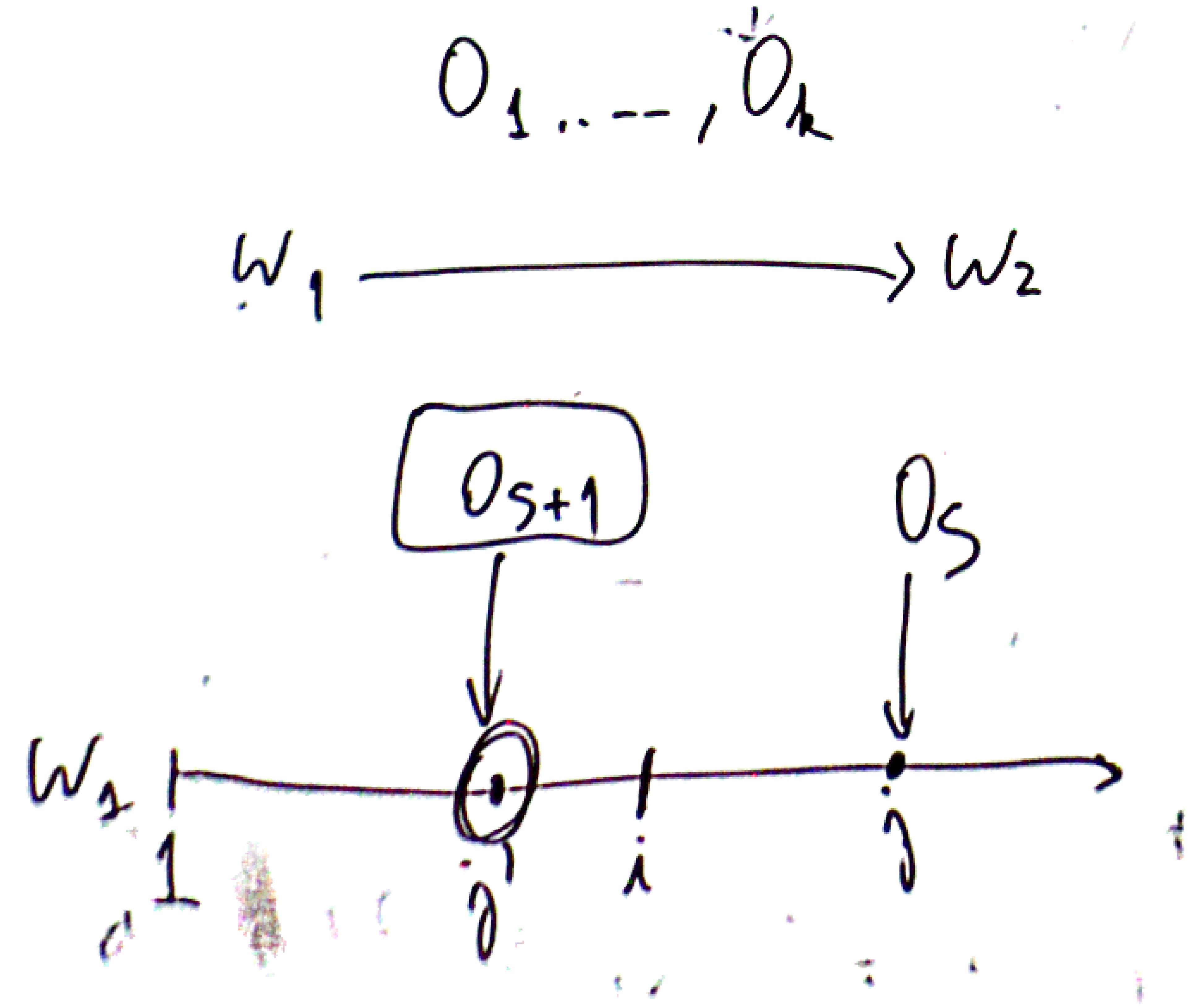
o_{s+1}

$w_1[1, i]$

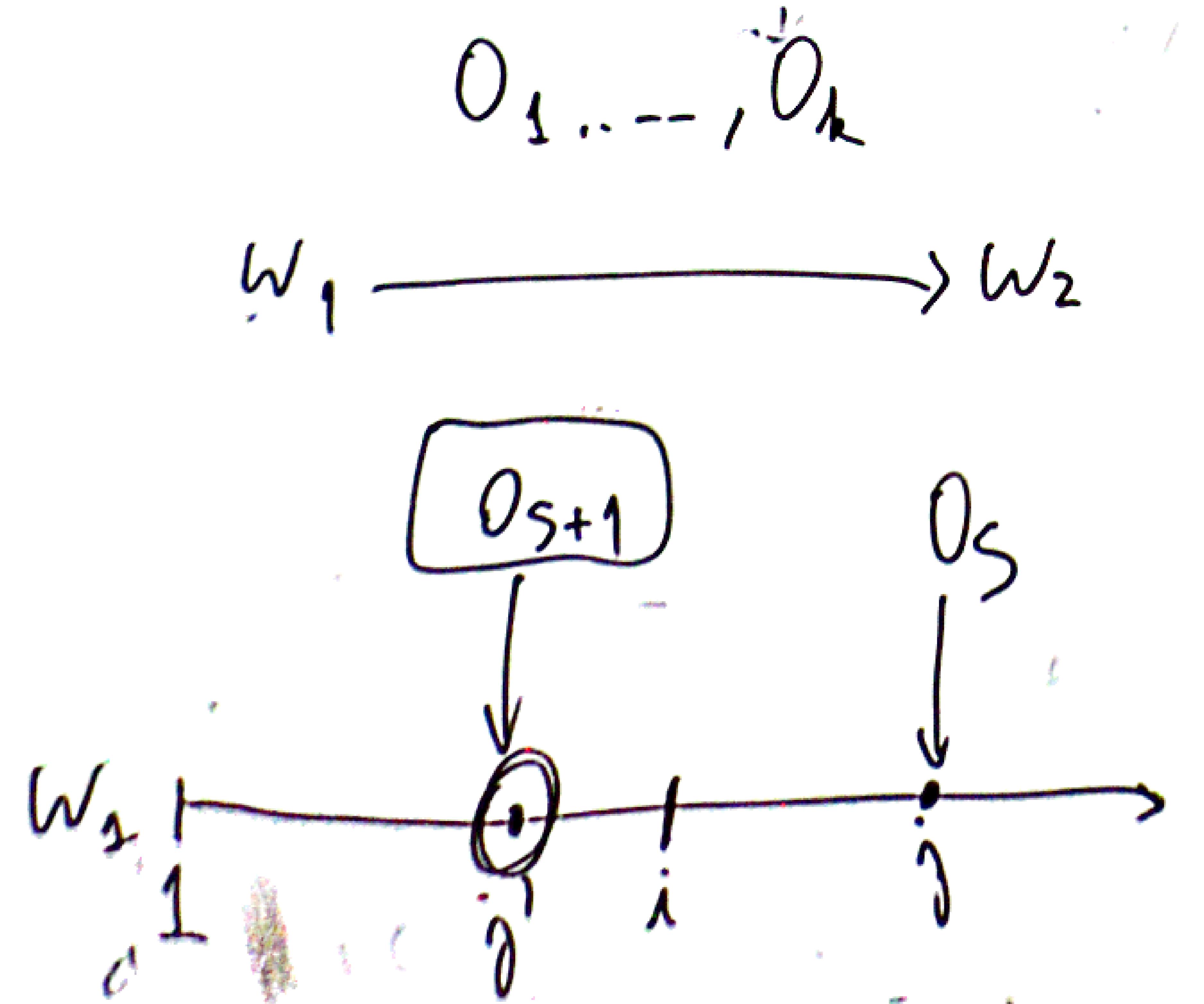
o_s

o_s o_{s+1}

~~(1, i)~~ $(1, i)$

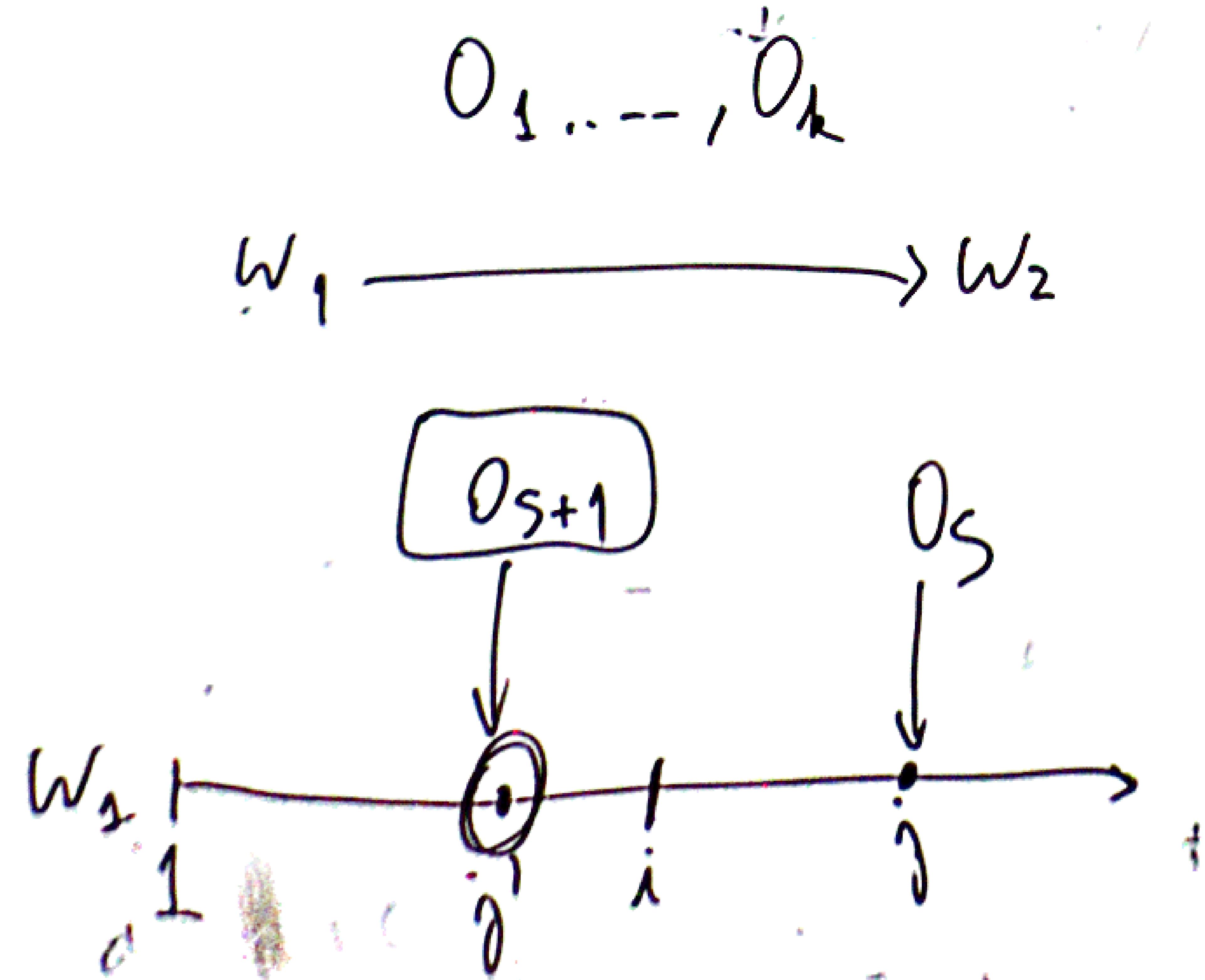


$w_1: a a a c d e$
 $w_2: a b a c e$
 $1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$



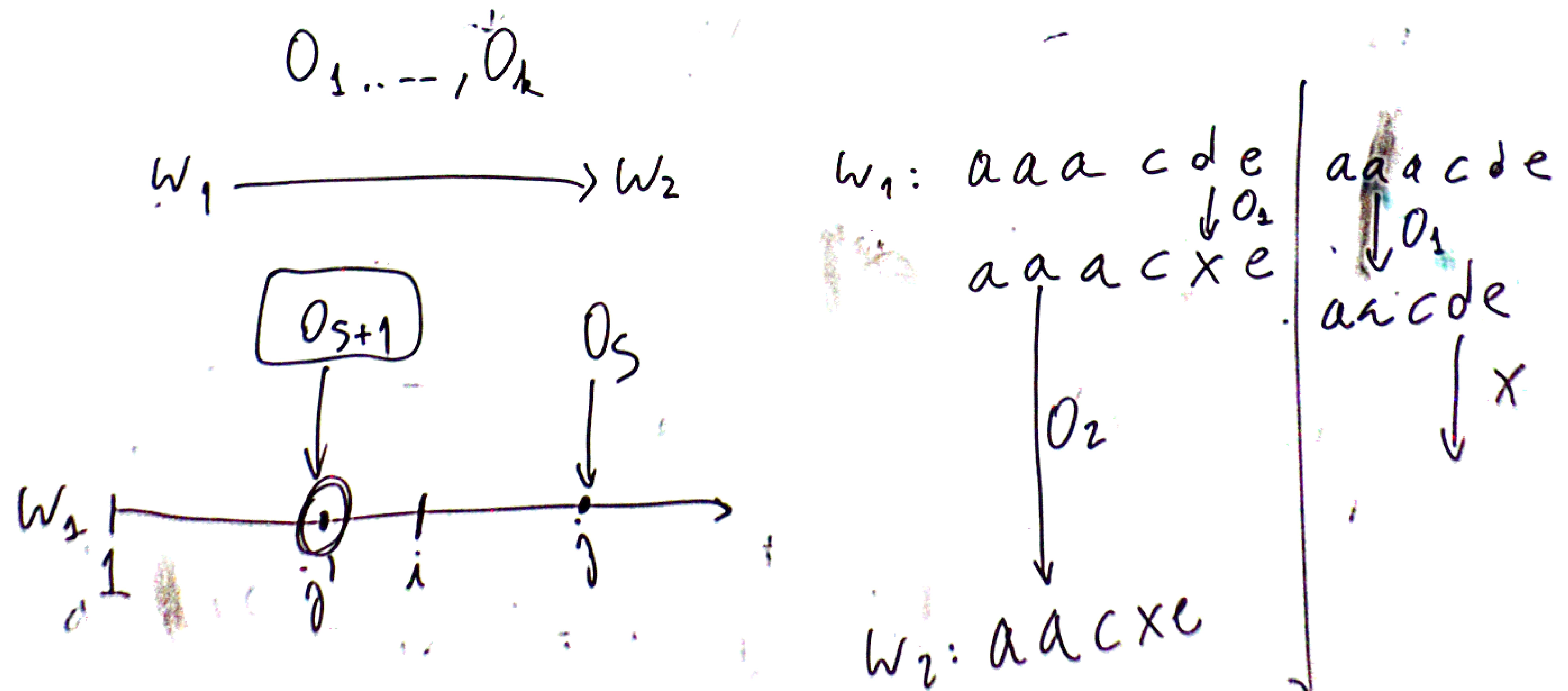
$w_1: a a a c d e$
 $\downarrow O_2$
 $a a a c x ?$
 $\downarrow O_2$
 $w_2: a a c x e$

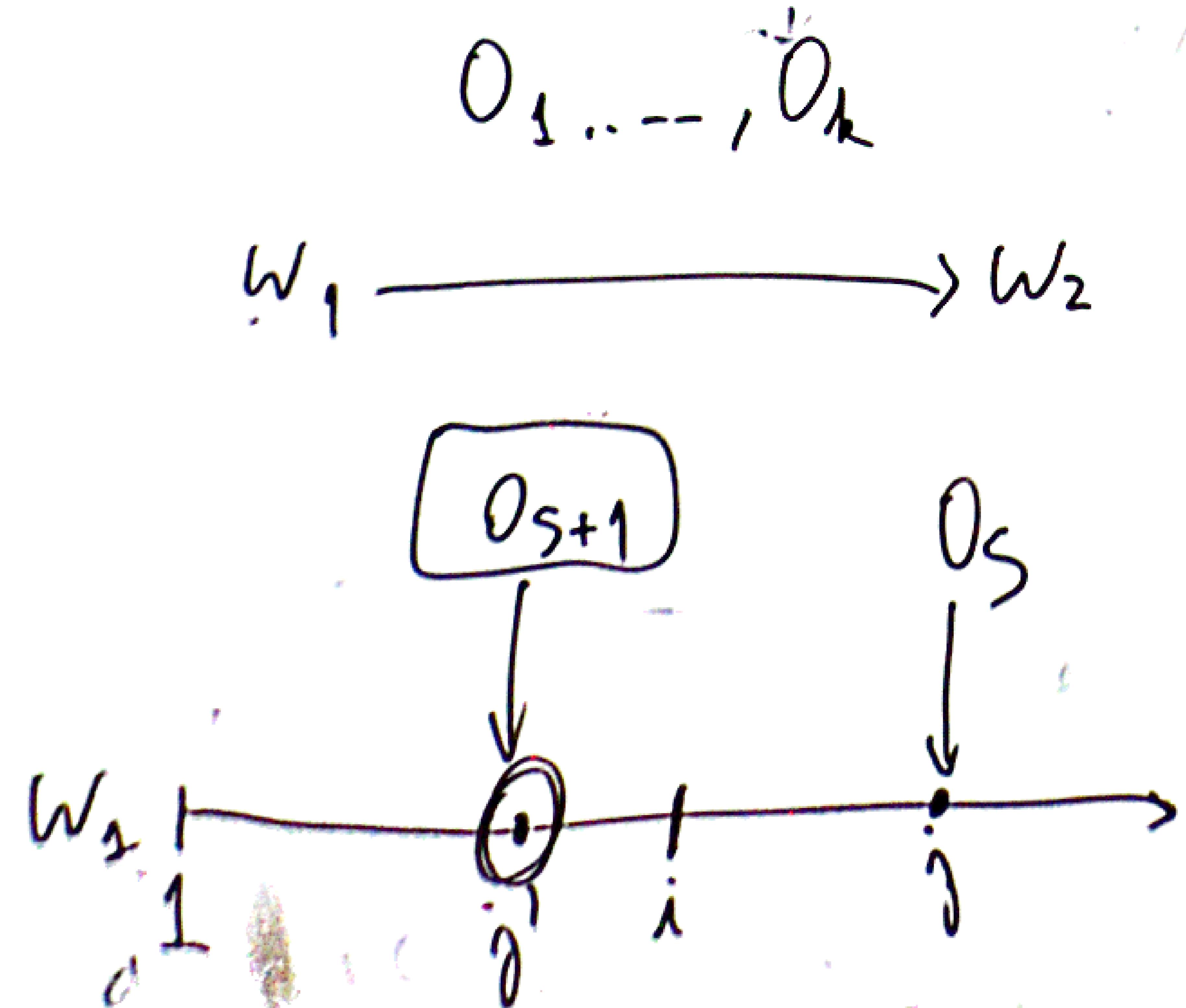
a a a c d e
 ↓
 a a c d e
 ↓
 a a c d e



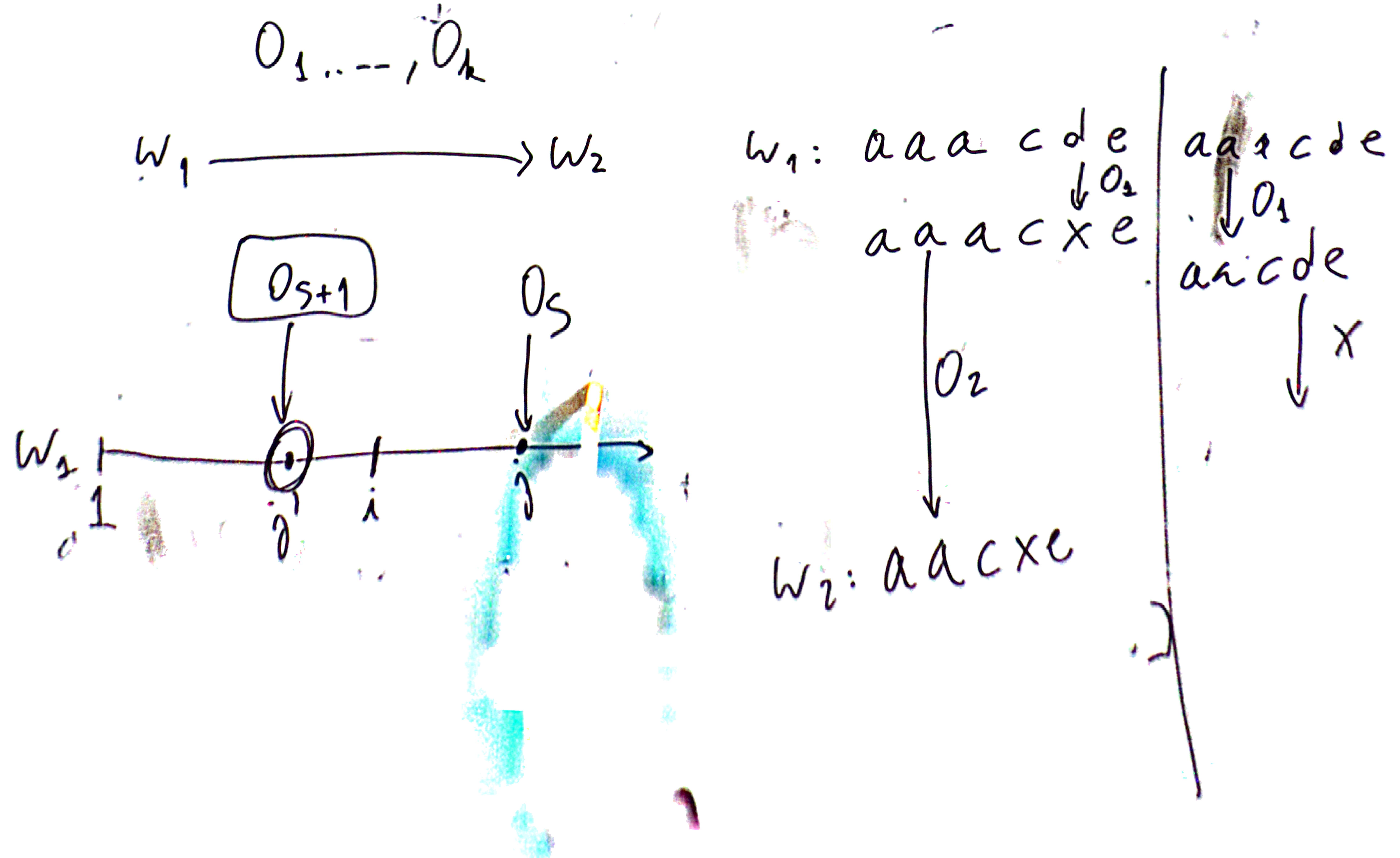
$w_1: a a a c d e$
 $\downarrow O_1$
 $a a a c x e$
 $\downarrow O_2$
 $w_2: a a c x e$

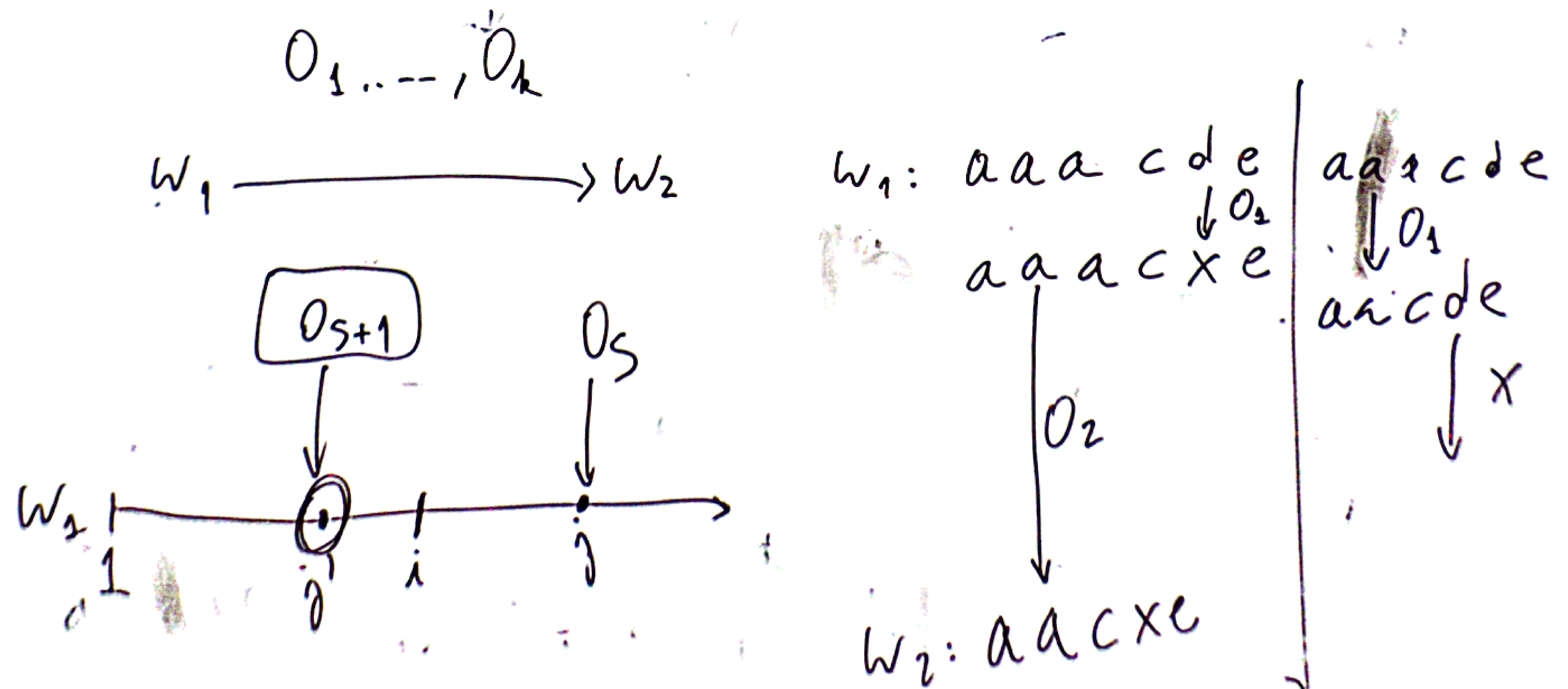
$a a a c d e$
 \downarrow
 $a a c d e$

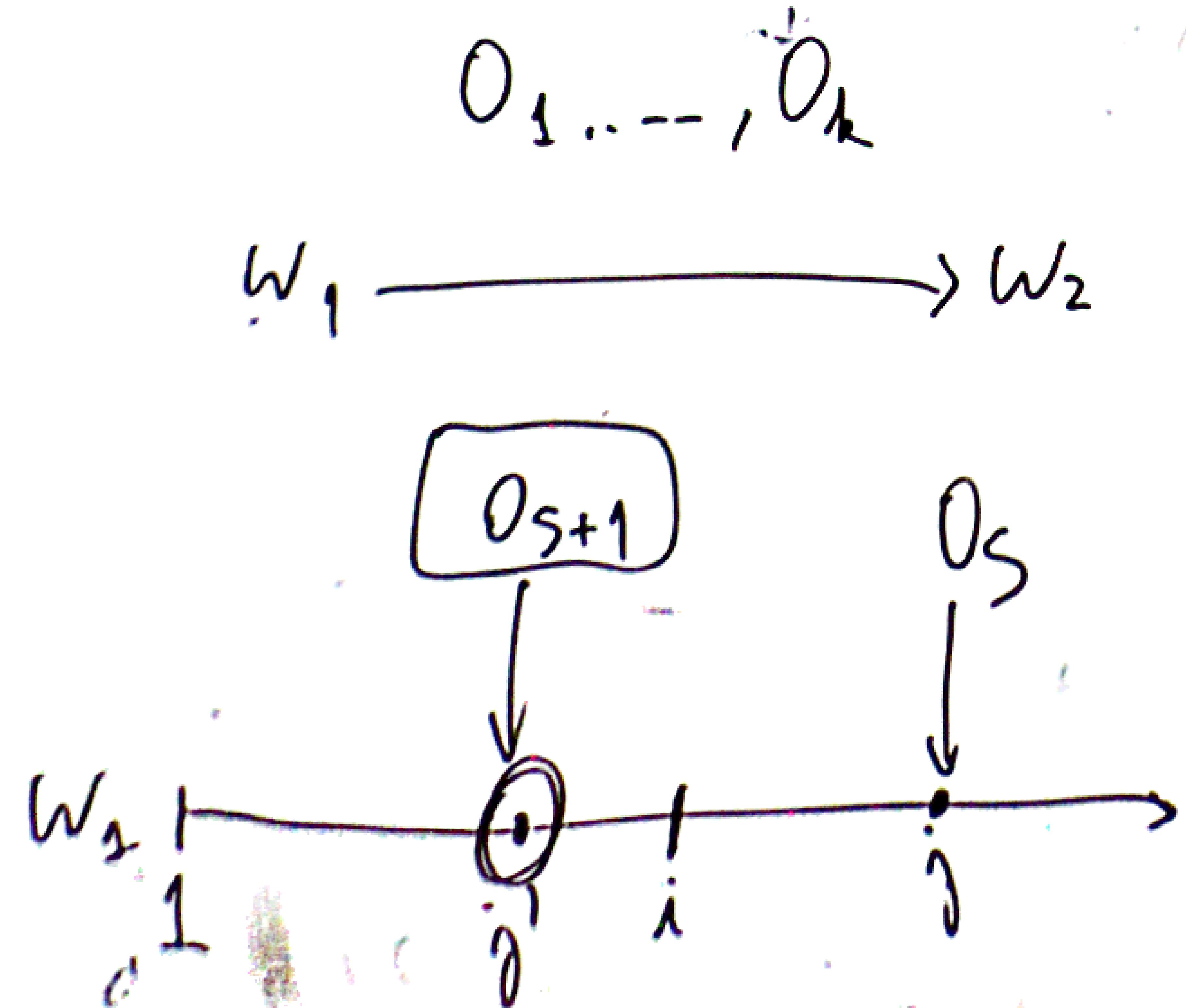




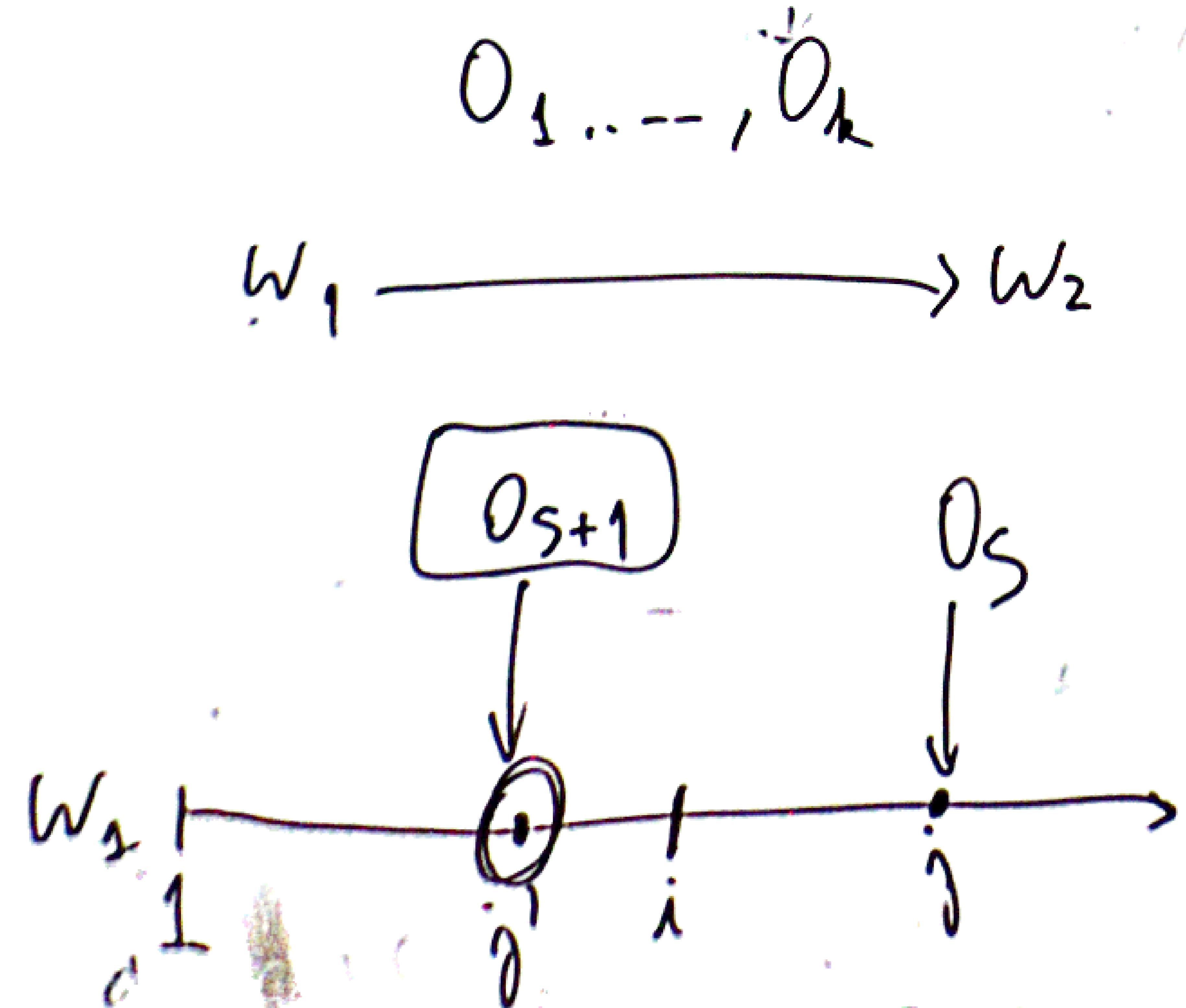
$w_1: a a a c d e$
 $\downarrow O_1$
 $a a a c x e$
 $\downarrow O_2$
 $w_2: a a c x e$
 $a a a c d e$
 $\downarrow O_1$
 $a a c d e$
 $\downarrow X$



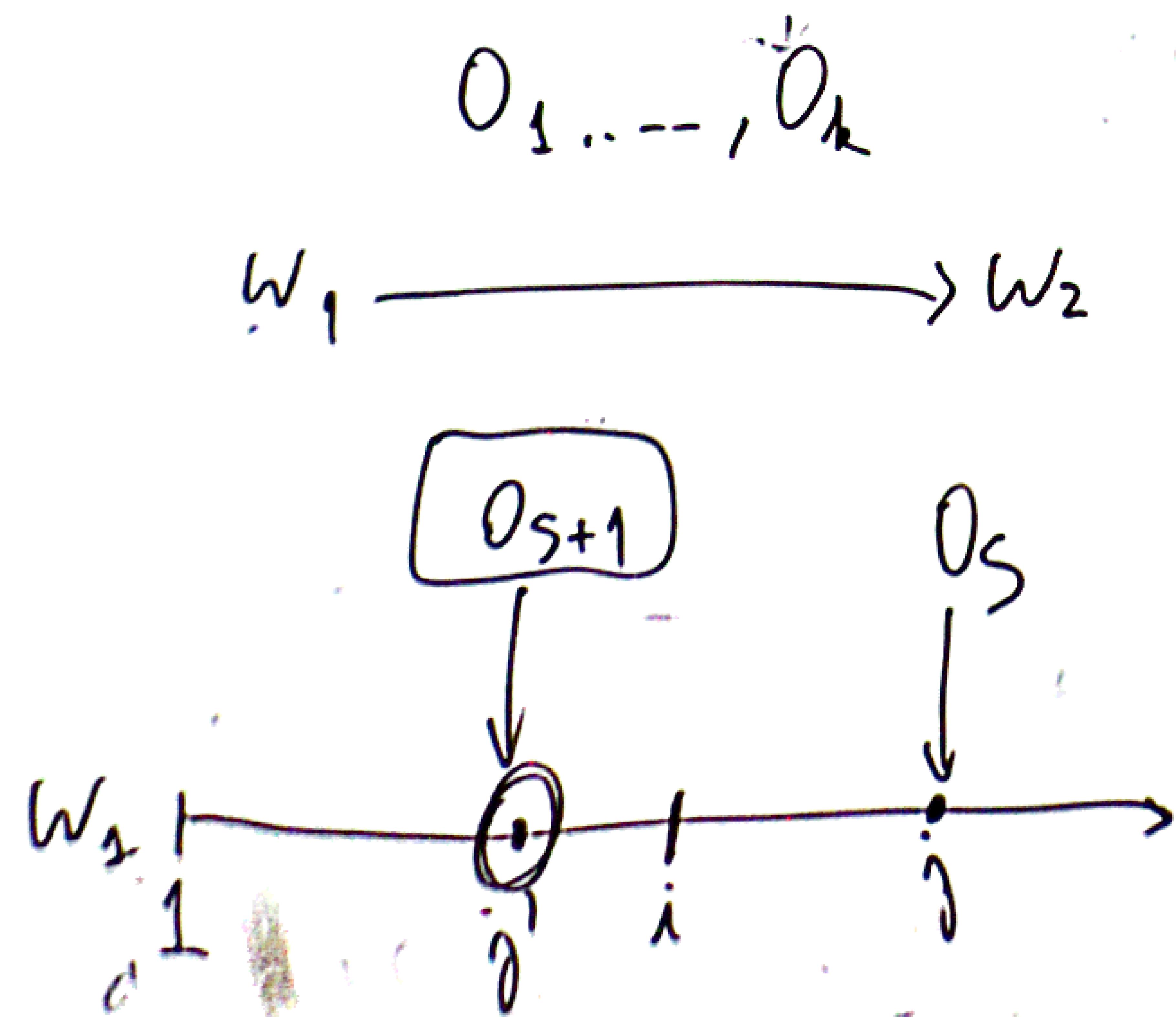




$w_1: a a a c d e$
 $\downarrow O_1$
 $a a a c x e$
 $\downarrow O_2$
 $w_2: a a c x e$
 \downarrow
 $a a s c d e$
 $\downarrow O_1$
 $a n c d e$
 $\downarrow X$

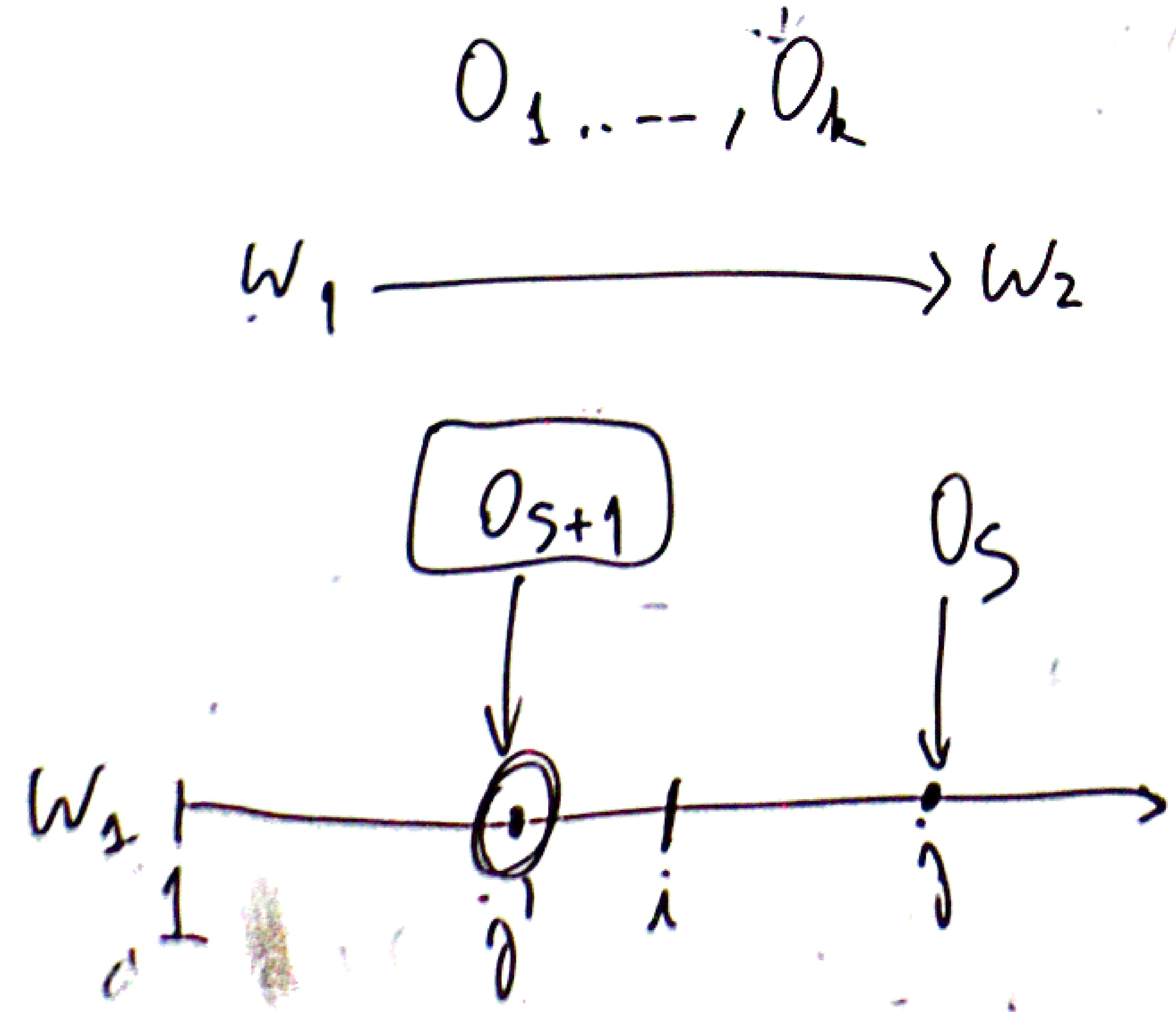


$w_1: a a a c d e$
 $\downarrow O_2$
 $a a a c x e$
 $\downarrow O_2$
 $w_2: a a c x e$
 $\downarrow X$
~~a a s c d e~~
 $\downarrow O_1$
~~a r i c d e~~
 $\downarrow X$



$w_1: a a a c d e$
 $\downarrow O_1$
 $a a a c x e$
 $\downarrow O_2$
 $w_2: a a c x e$

 $a a c d e$
 $\downarrow O_1$
 $a c d e$
 $\downarrow X$



$w_1: a a a c d e$
 $\downarrow O_1$
 $a a a c x e$
 $\downarrow O_2$
 $w_2: a a c x e$
 $\downarrow X$
 a a s c d e
 $\downarrow O_1$
 a n c d e
 $\downarrow X$

$w_2[3, i]$

$$\begin{array}{c} \downarrow \\ O_1, \dots, O_l \\ O'_1, \dots, O'_m \\ m < l \end{array}$$

 $w_2[1, j]$ w_1

$$\begin{array}{c} \downarrow \\ O_1, \dots, O_k \end{array}$$

 w_2

$$\begin{array}{c} \cancel{O_1, \dots, O_l, O_{l+1}, \dots, O_k} \\ O'_1, \dots, O'_m, O_{l+m}, \dots, O_k \end{array}$$

$w_1[1, i]$

$$\begin{array}{c} \downarrow \\ O_1, \dots, O_l \\ O'_1, \dots, O'_m \\ m < l \end{array}$$

 $w_2[1, j]$ w_1

$$O_{l+1}, \dots, O_k$$

 w_2

$$\begin{array}{c} \cancel{O_1, \dots, O_l, O_{l+1}, \dots, O_k} \\ O'_1, \dots, O'_m, O_{l+1}, \dots, O_k \end{array}$$

w_1



w_2

$w_1[1, i, j]$



$w_2[1, i]$

w_1



w_2

$w_2[1, i]$



$w_2[1, j]$

w_1

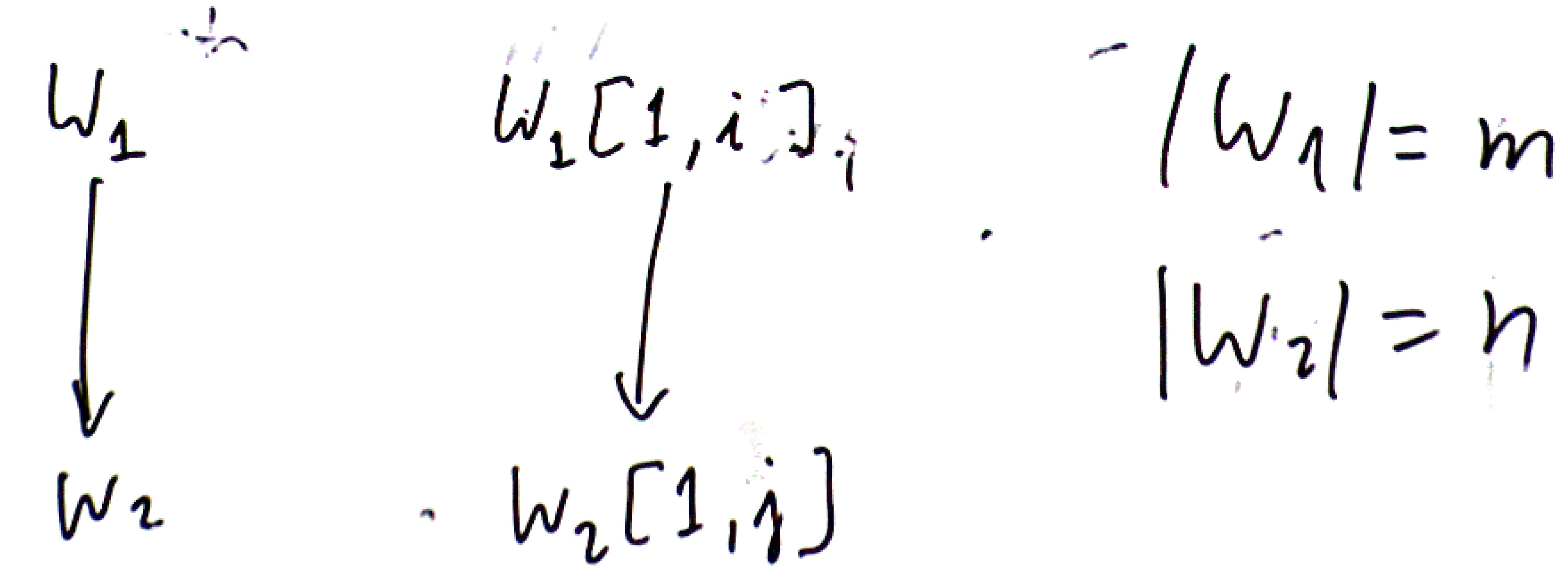


w_2

$w_2[1, i, j]$



$w_2[1, i]$



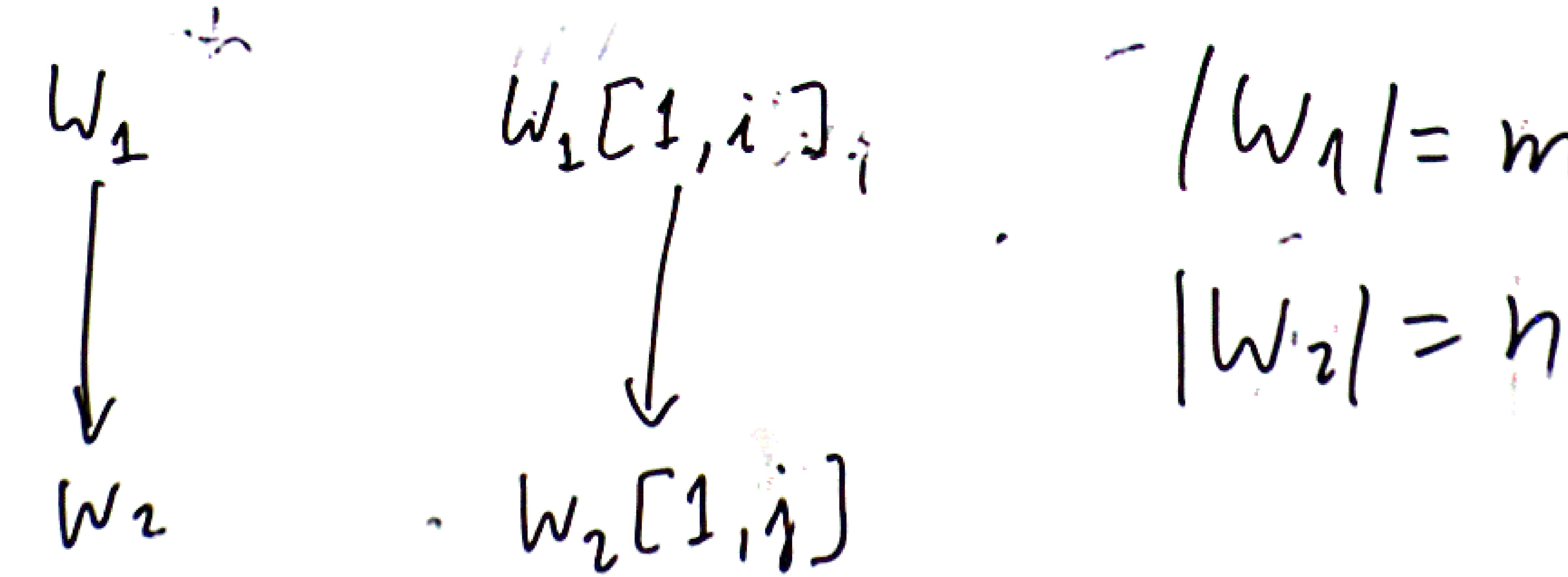
$$|w_1| = m$$

$$|w_2| = n$$

$$\begin{array}{c} w_1 \\ \downarrow \\ w_2 \end{array} \quad \begin{array}{c} w_1[1,i] \\ \downarrow \\ w_2[1,j] \end{array} \quad \begin{array}{l} |w_1| = m \\ |w_2| = n \end{array}$$

$\ell\ell(i,j) =$

$\ell\ell(w_1[1,i], w_2[1,j])$



$$|w_1| = m$$

$$|w_2| = n$$

$$ed(i, j) =$$

$$ed(w_1[1, i], w_2[1, j])$$

$$w(i,j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned}|W_1| &= m \\ |W_2| &= n\end{aligned}$$

$$\begin{aligned}w(i,j) &= \\ w(W_1[1,i], W_2[1,j]) &\end{aligned}$$

$$ld(i, j) = \begin{cases} i = 0 \\ \end{cases}$$

$i=0$

$$|W_1| = m$$

$$|W_2| = n$$

$$ld(i, j) =$$

$$ld(W_1[1:i], W_2[1:j])$$

$$W_1[1:0]$$

$$\parallel \varepsilon$$

$$ld(i, j) = \begin{cases} j & i=0 \\ i & j=0 \\ & i>0, j>0 \end{cases}$$

$$i=0$$

$$j=0$$

$$i>0, \\ j>0$$

$$|w_1|=m$$

$$|w_2|=n$$

$$ld(i, j) =$$

$$ld(w_1[1:i], w_2[1:j])$$

$$w_1[1:0]$$

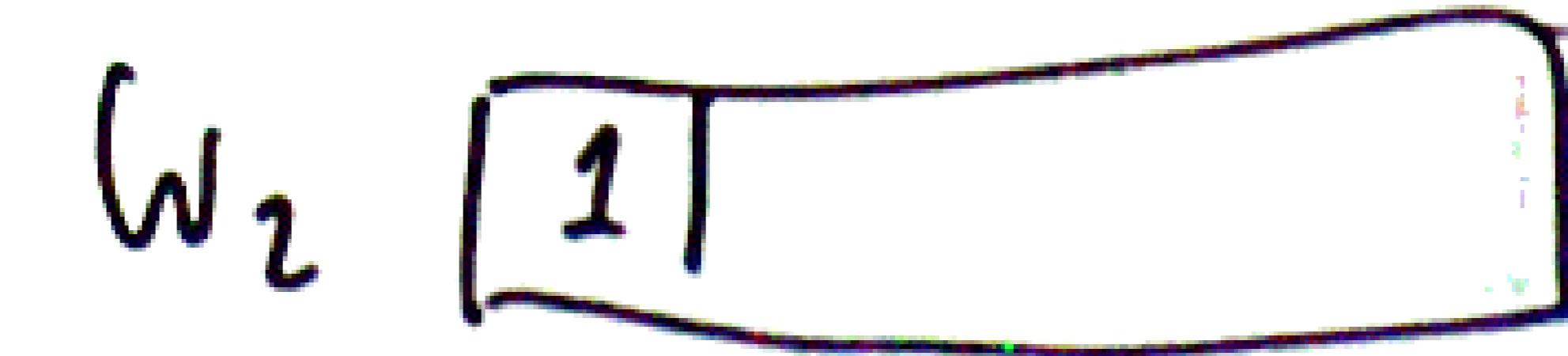
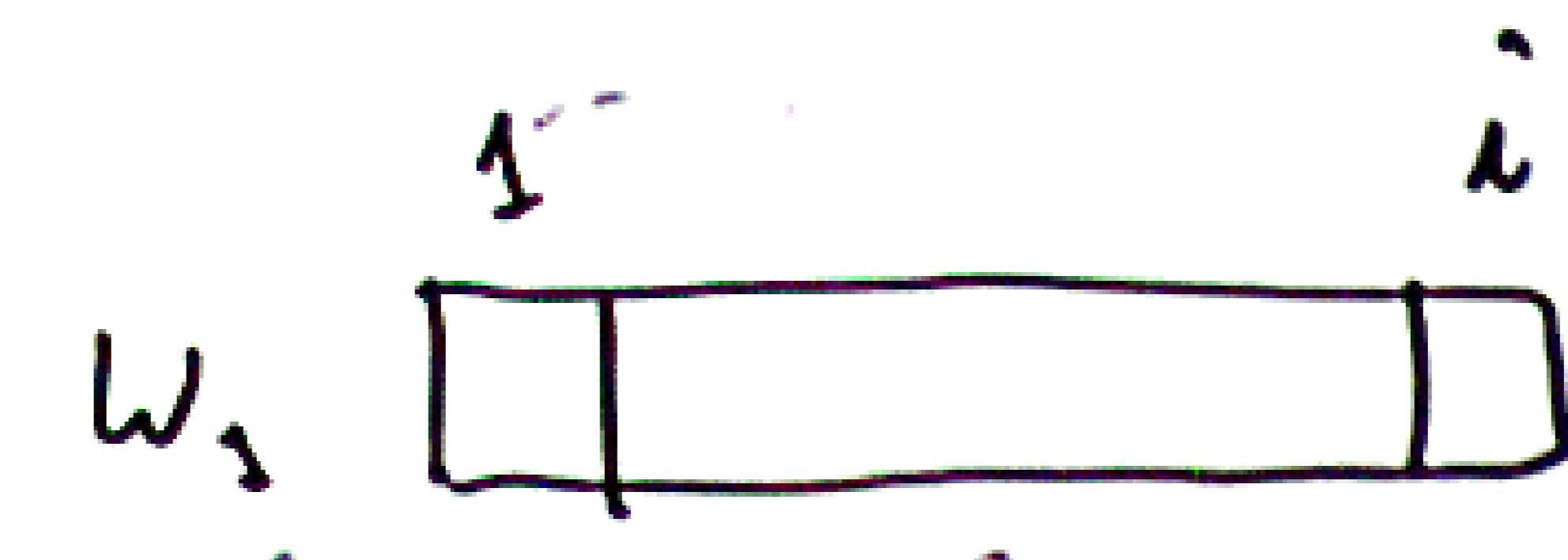
$$\parallel \varepsilon$$

$$ld(i, j) = \begin{cases} i & j \\ i & i \\ & \vdots \\ & i \end{cases}$$

$i=0$

$j=0$

$i>0$,
 $j>0$



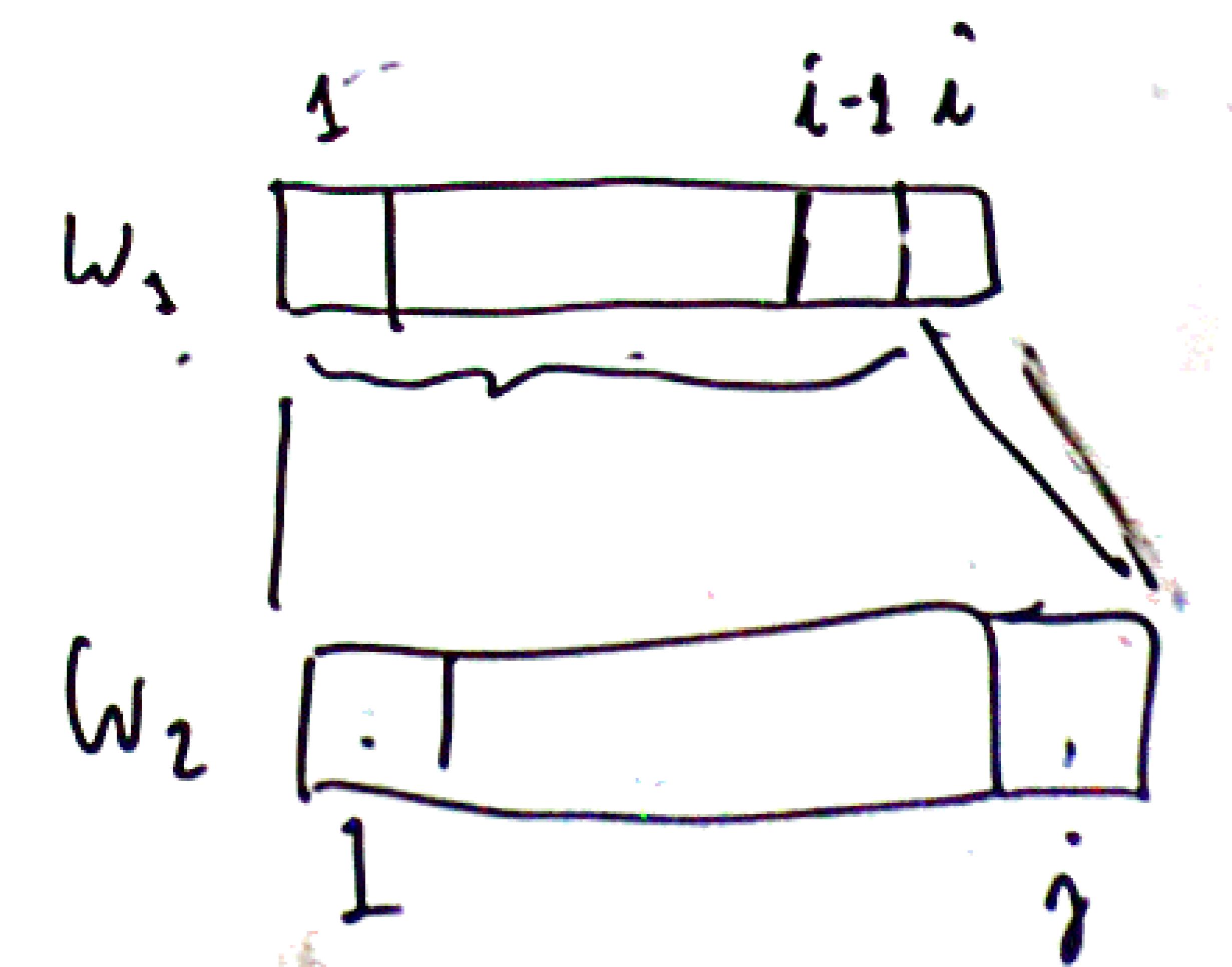
$$ld(i, j) = \begin{cases} i & j = 0 \\ n & i = 0 \\ \min\{i, j\} & i > 0, j > 0 \end{cases}$$

$$\lambda = 0$$

$$i = 0$$

$$i > 0$$

100

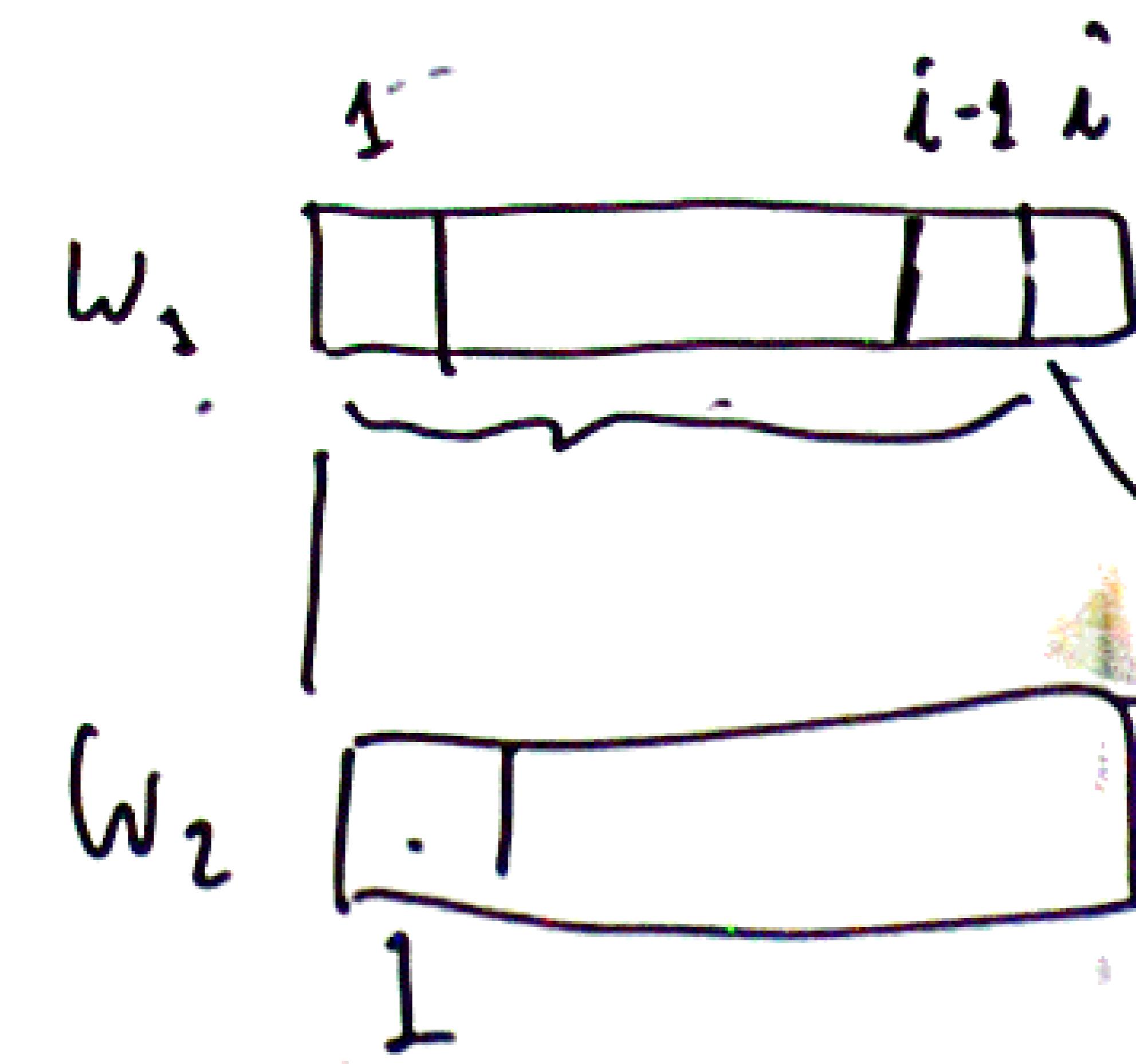


$$ld(i, j) = \begin{cases} i & j \\ i & i \\ & i \\ & i > 0 \\ & j > 0 \end{cases}$$

$i=0$

$j=0$

$i>0$
 $j>0$



$$w_1[1, i-1] \rightarrow w_2[1, i]$$

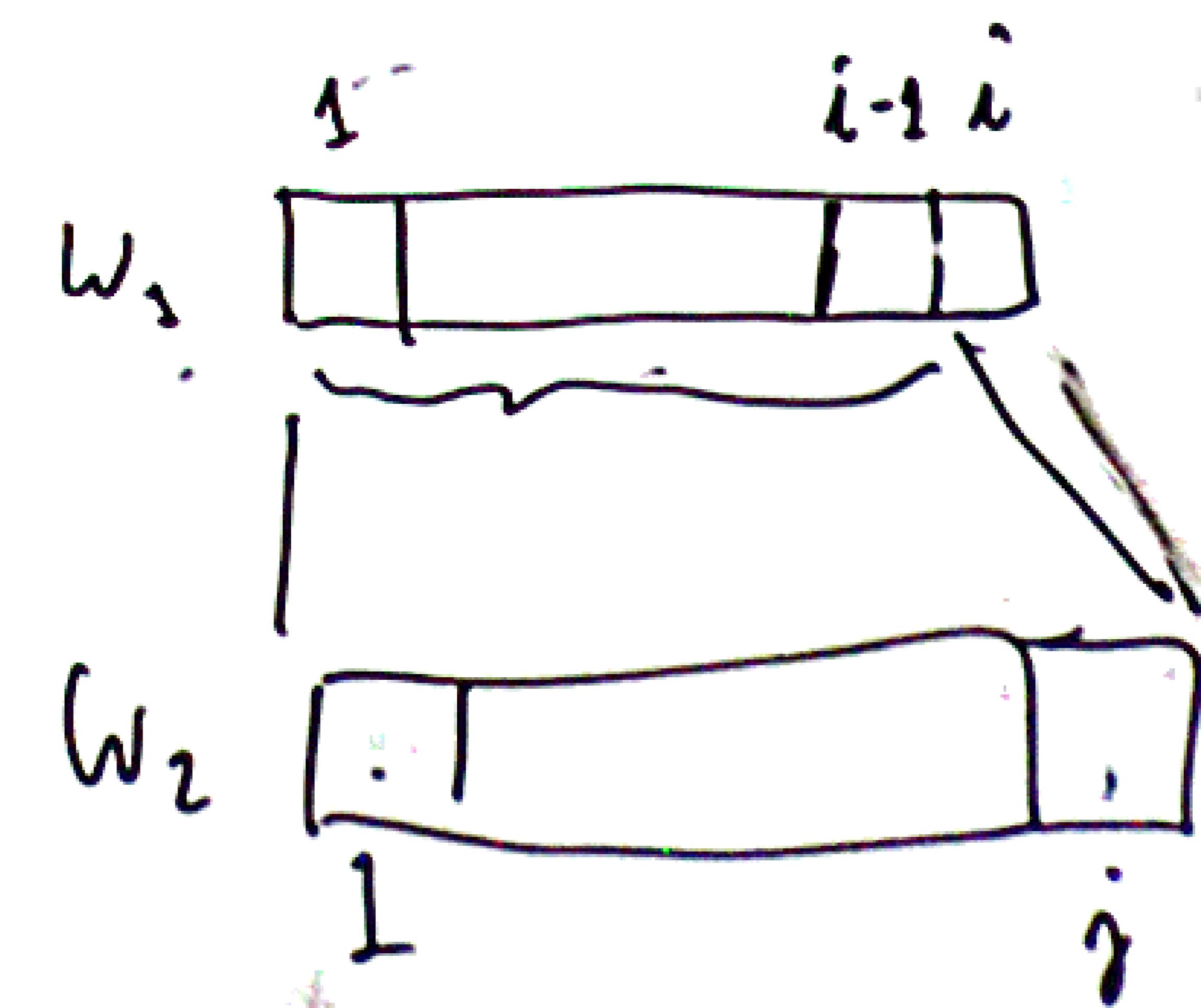
+ 1

$$ld(i, j) = \begin{cases} i & j \\ i & i \\ & i \\ & i > 0 \\ & j > 0 \end{cases}$$

$i=0$

$j=0$

$i > 0$
 $j > 0$



$$ld(i-1, j) + 1$$

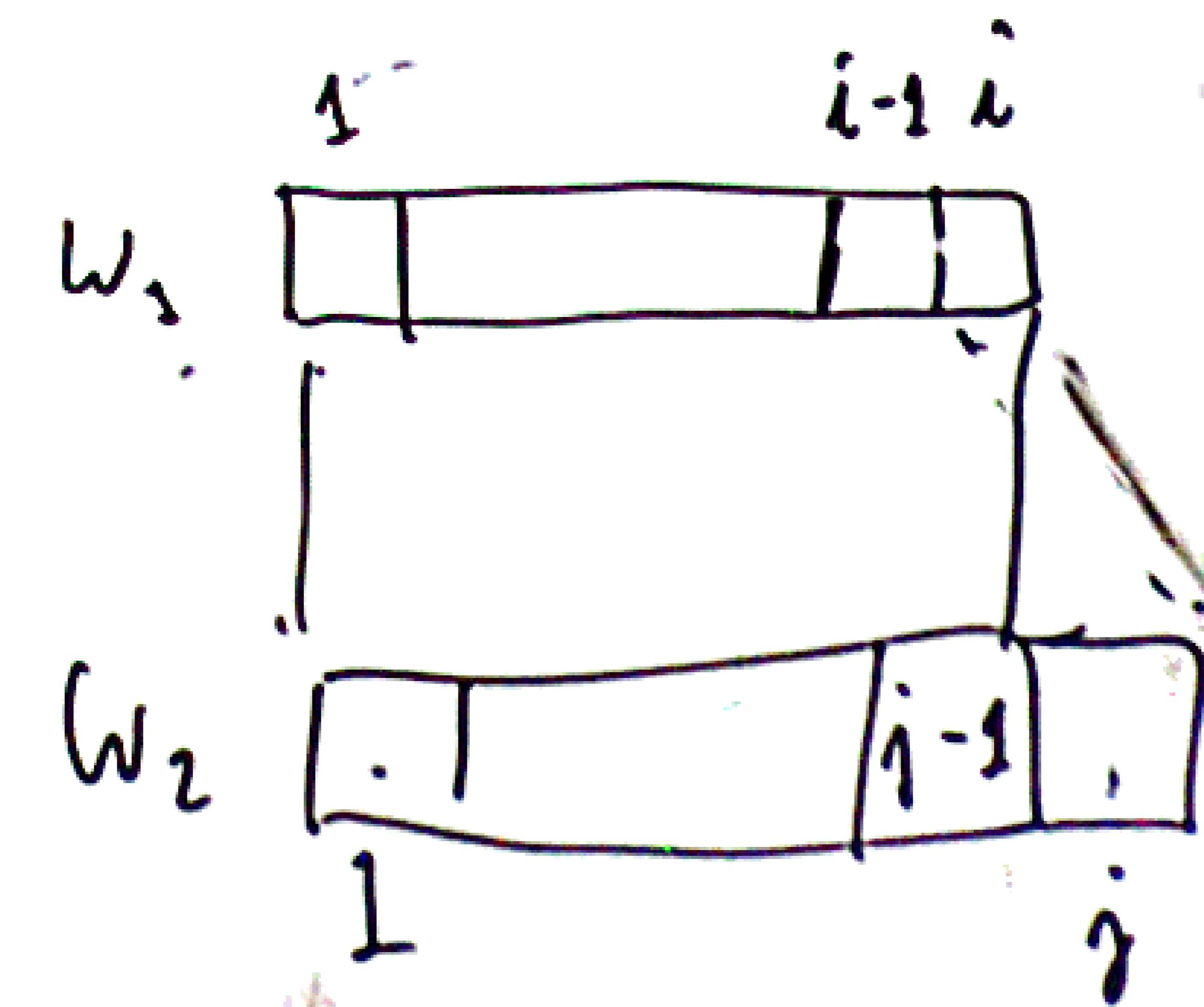
$$ld(i, j-1)$$

$$ld(i, j) = \begin{cases} i & j = 0 \\ j & i = 0 \\ i > 0, \\ j > 0 & \end{cases}$$

$i=0$

$j=0$

$i > 0,$
 $j > 0$



$ld(i-1, j) + 1$

$ld(i, j-1)$

$$ld(i, j) = \begin{cases} i & j = 0 \\ j & i = 0 \\ i + j & i > 0, j > 0 \end{cases}$$

$$\lambda = 0$$

$$i = 0$$

$$i > 0$$

The diagram illustrates the construction of a sequence w_2 from w_1 . The top part shows w_1 as a sequence of rectangles labeled 1^* , $i-1$, and i . The bottom part shows w_2 as a sequence of rectangles labeled 1 , $i-1$, and i . A vertical line connects the first rectangle of w_1 to the first rectangle of w_2 . Horizontal lines connect the right side of the first rectangle of w_1 to the second rectangle of w_2 , and the right side of the second rectangle of w_1 to the third rectangle of w_2 .

$$\omega_j(i_{\lambda-1} \dots i_1) + 1$$

$$\text{ad}(\lambda_{10^{-1}}) \neq 1$$

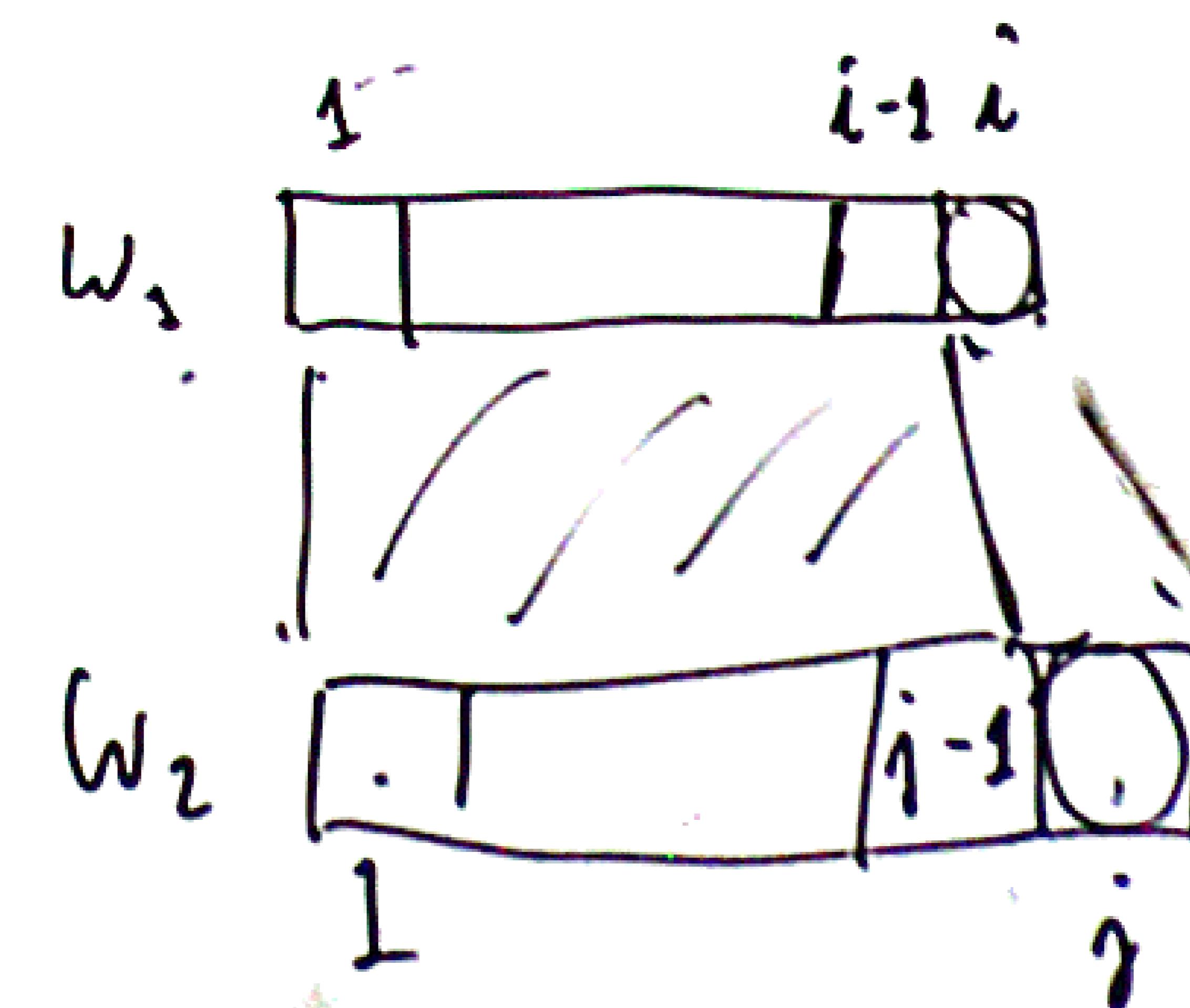
$$\text{ad}(i^{-1}i^{i-1})^+$$

$$ed(i, j) = \begin{cases} i & j = 0 \\ j & i = 0 \\ i > 0, \\ j > 0 & \end{cases}$$

$i=0$

$j=0$

$i>0,$
 $j>0$



$$ed(i-1, j) + 1$$

$$ed(i, j-1) + 1$$

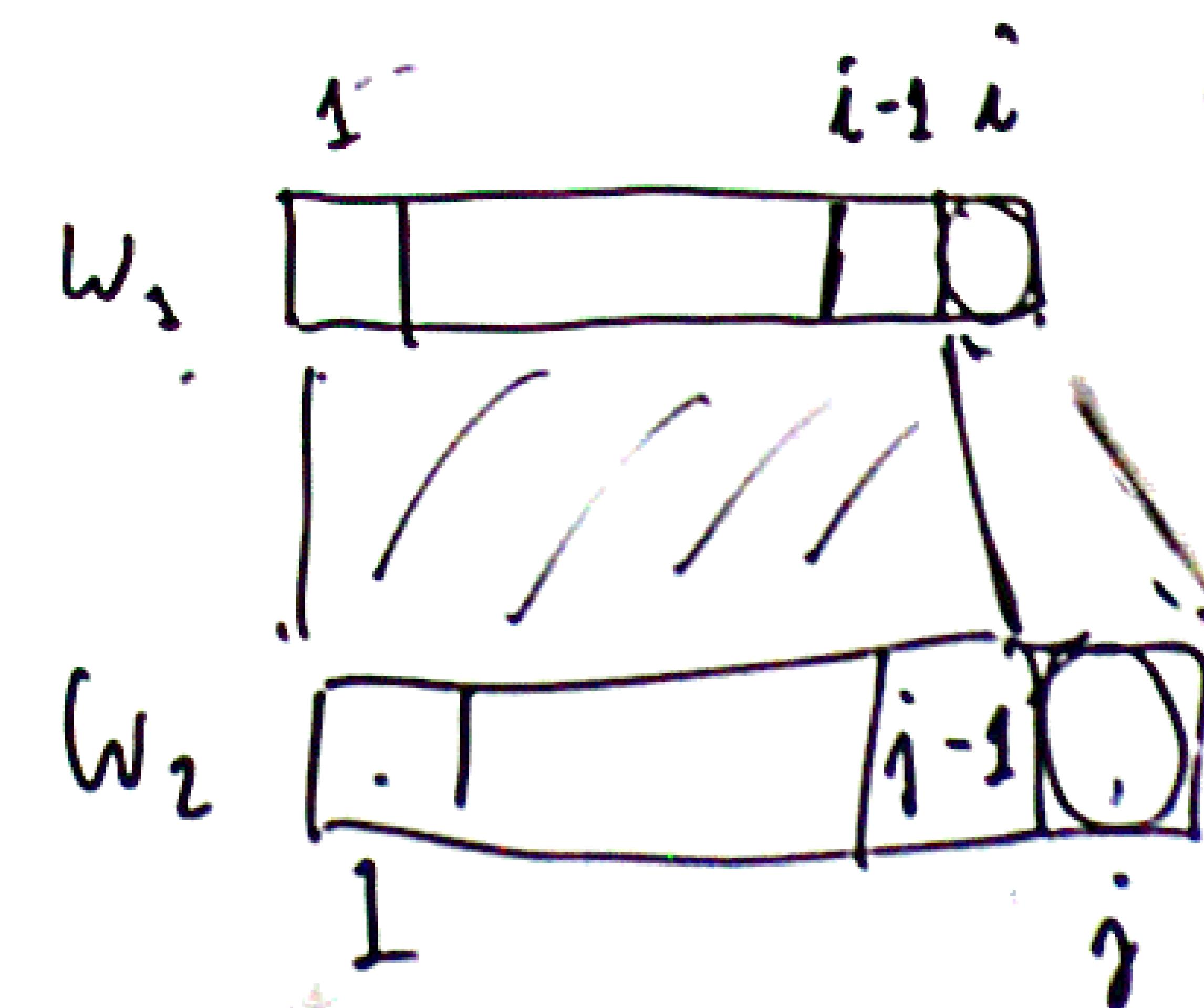
$$ed(i-1, j-1) +$$

$$ed(i, j) = \begin{cases} i & j = 0 \\ j & i = 0 \\ i > 0, \\ j > 0 & \end{cases}$$

$i=0$

$j=0$

$i>0,$
 $j>0$



$$ed(i-1, j) + 1$$

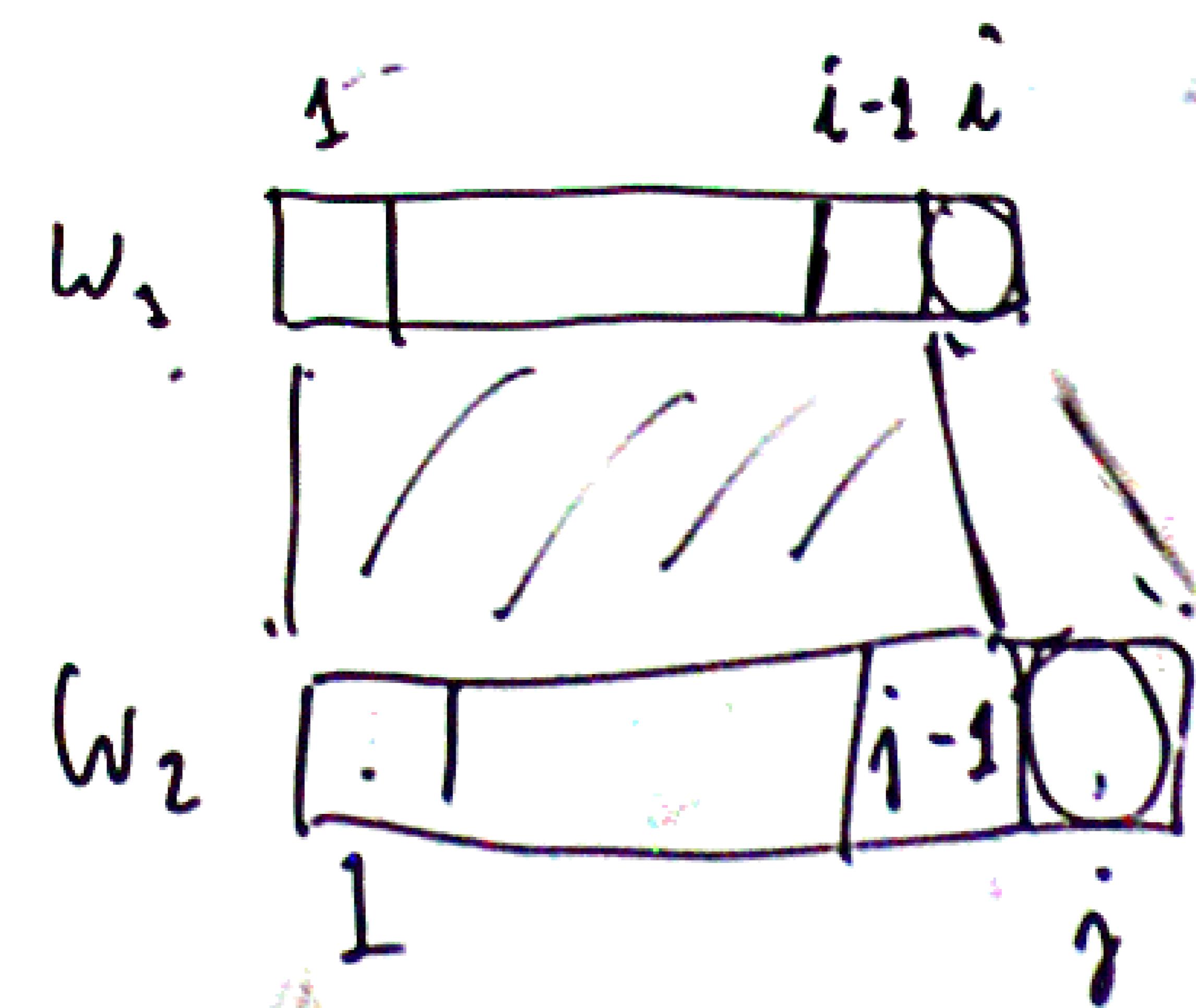
$$ed(i, j-1) + 1$$

$$ed(i-1, j-1) +$$

$ld(i,j) =$

$$\lambda = 0$$

$$i \leq 0$$



$$\omega_{\lambda}(i-1, \gamma) + 1$$

$$\text{lid}(\lambda_{10^{-1}}) + 1$$

$$\text{ad}(\lambda^{-1}, \theta^{-1}) +$$

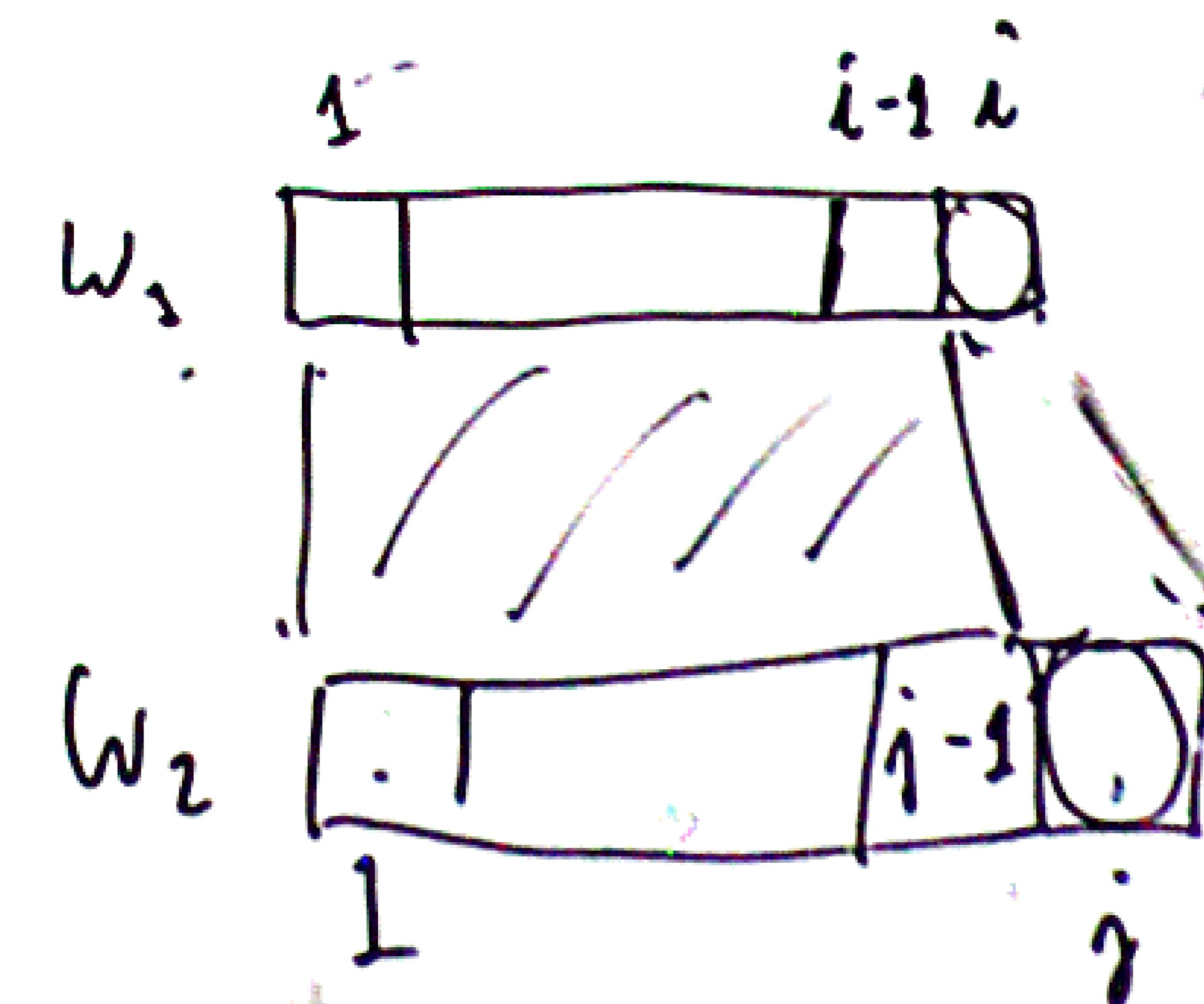
$$ed(i, j) = \begin{cases} i & j \\ j & i \end{cases}$$

$i=0$

$j=0$

$i>0$

$j>0$



$$ed(i-1, j) + 1$$

$$ed(i, j-1) + 1$$

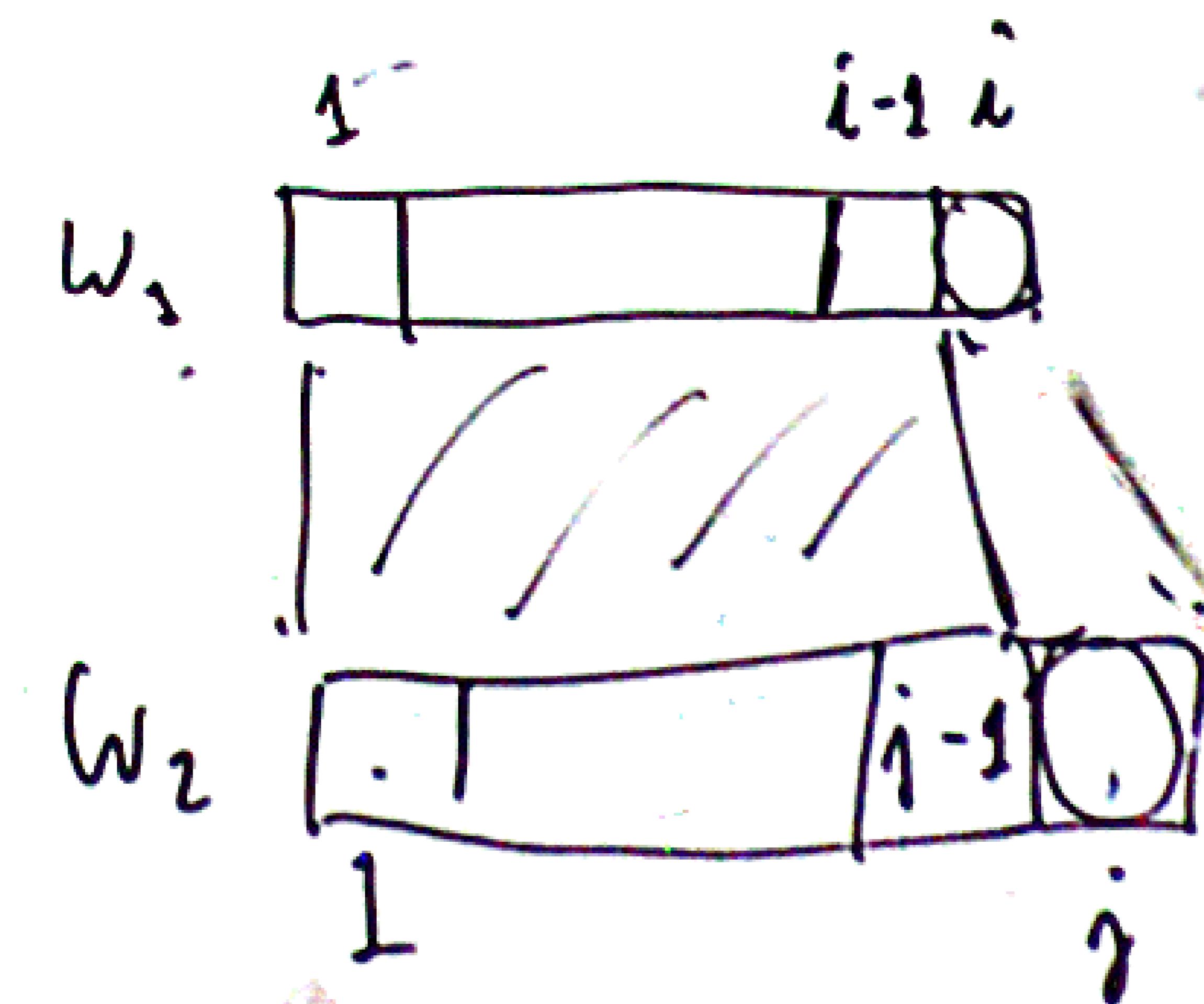
$$ed(i-1, j-1) +$$

$$ed(i, j) = \begin{cases} i & j \\ i & j=0 \\ i > 0, & j > 0 \end{cases}$$

$i=0$

$j=0$

$i > 0,$
 $j > 0$



$$ed(i-1, j) + 1$$

$$ed(i, j-1) + 1$$

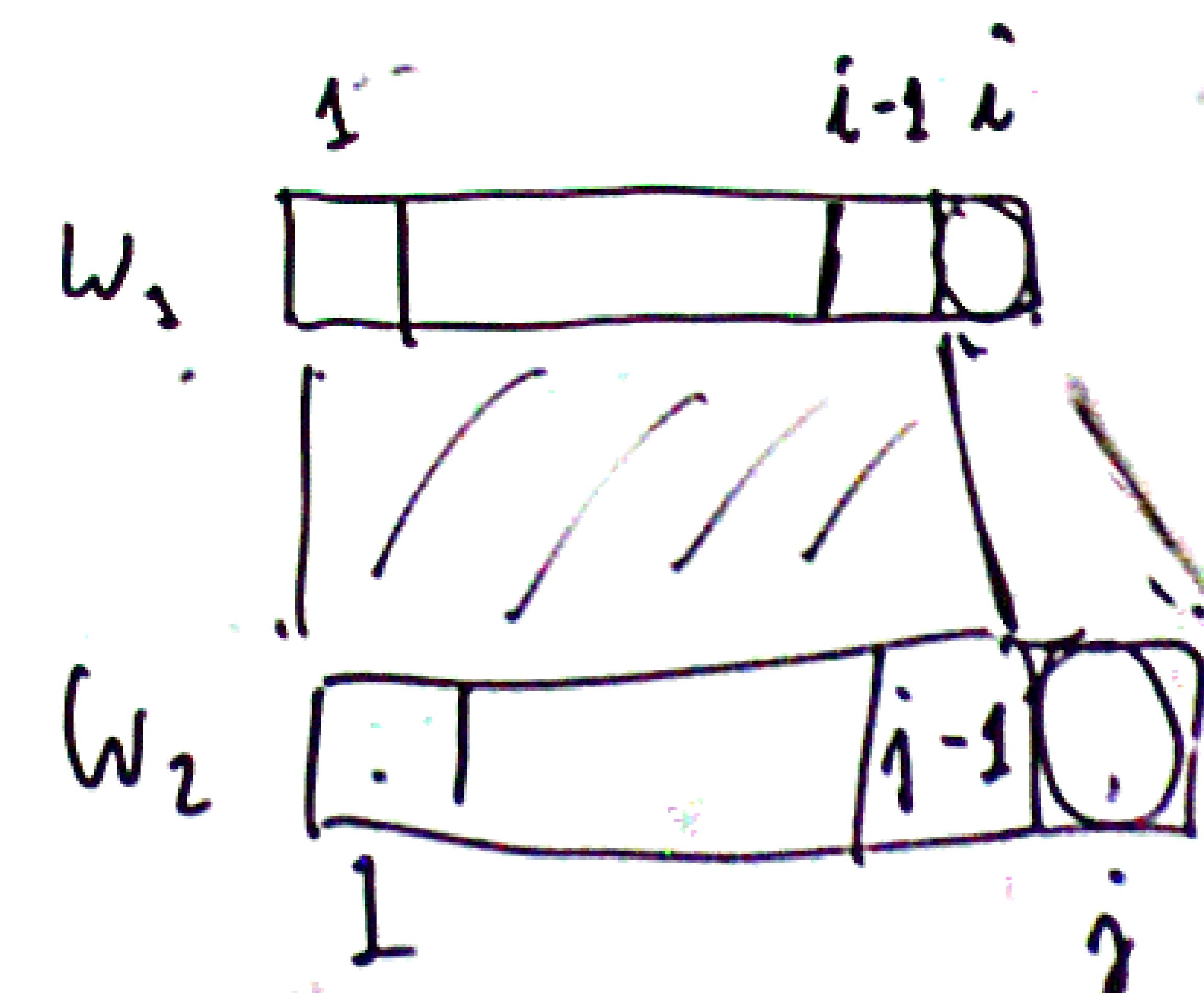
$$ed(i-1, j-1) +$$

$$ed(i, j) = \begin{cases} i & j = 0 \\ j & i = 0 \\ i > 0, \\ j > 0 & \end{cases}$$

$i=0$

$j=0$

$i>0,$
 $j>0$



$$ed(i-1, j) + 1$$

$$ed(i, j-1) + 1$$

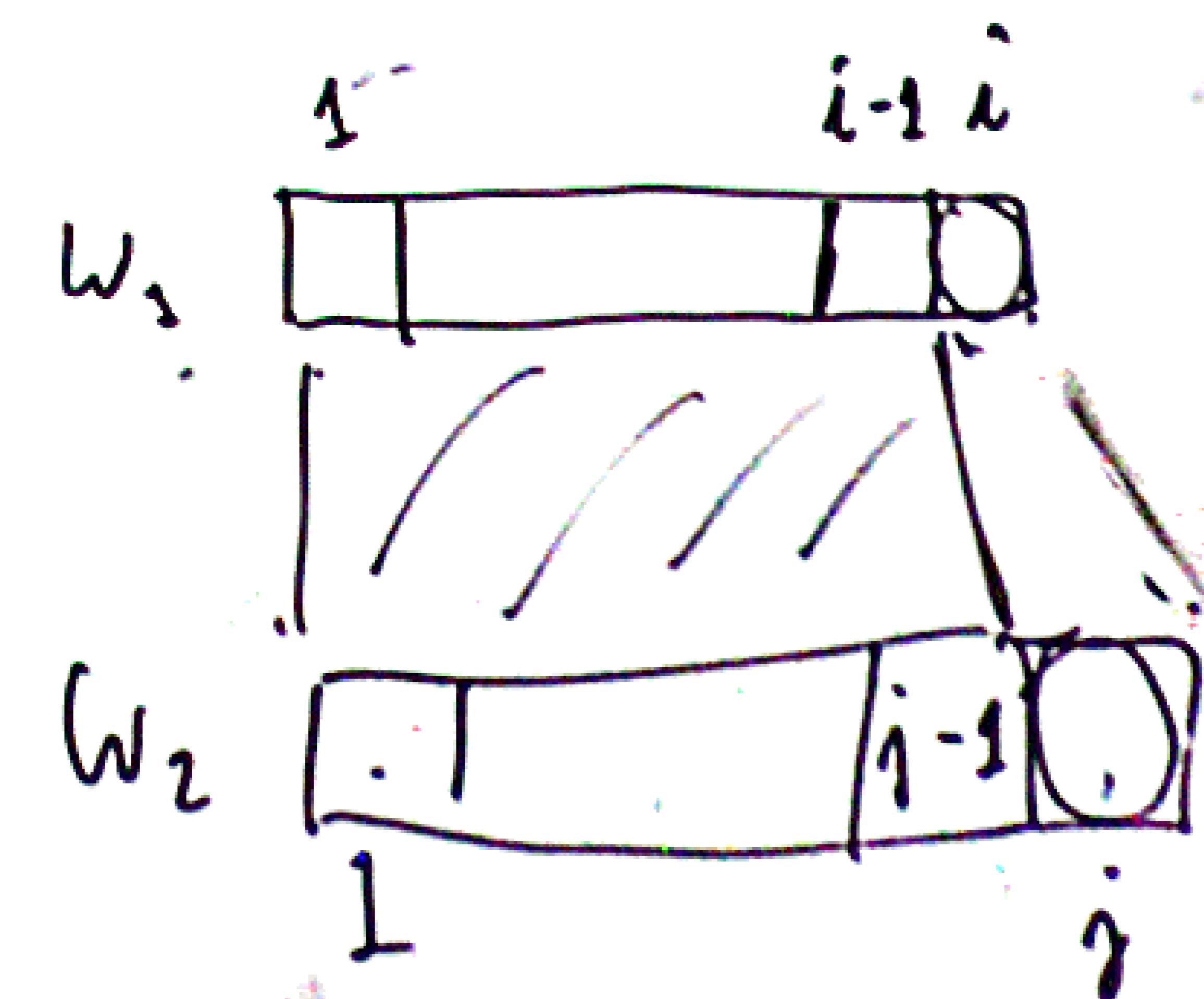
$$ed(i-1, j-1) +$$

$$ed(i, j) = \begin{cases} i & j = 0 \\ n & i > 0 \\ j & j > 0 \end{cases}$$

$$i = 0$$

$$j = 0$$

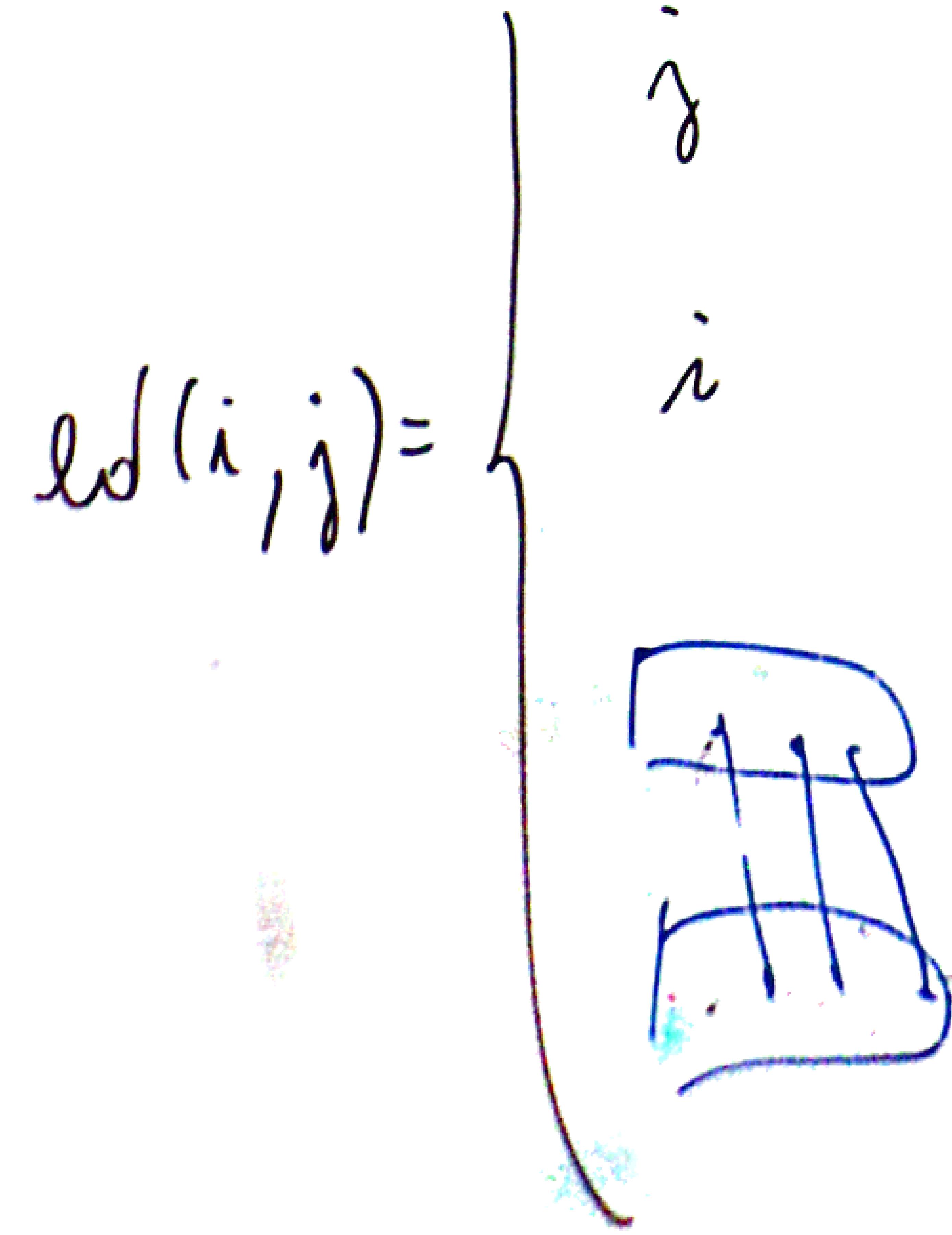
$$i > 0$$



$$\omega(i-1) + 1$$

$$ld(\lambda_{n_0-1}) \neq 1$$

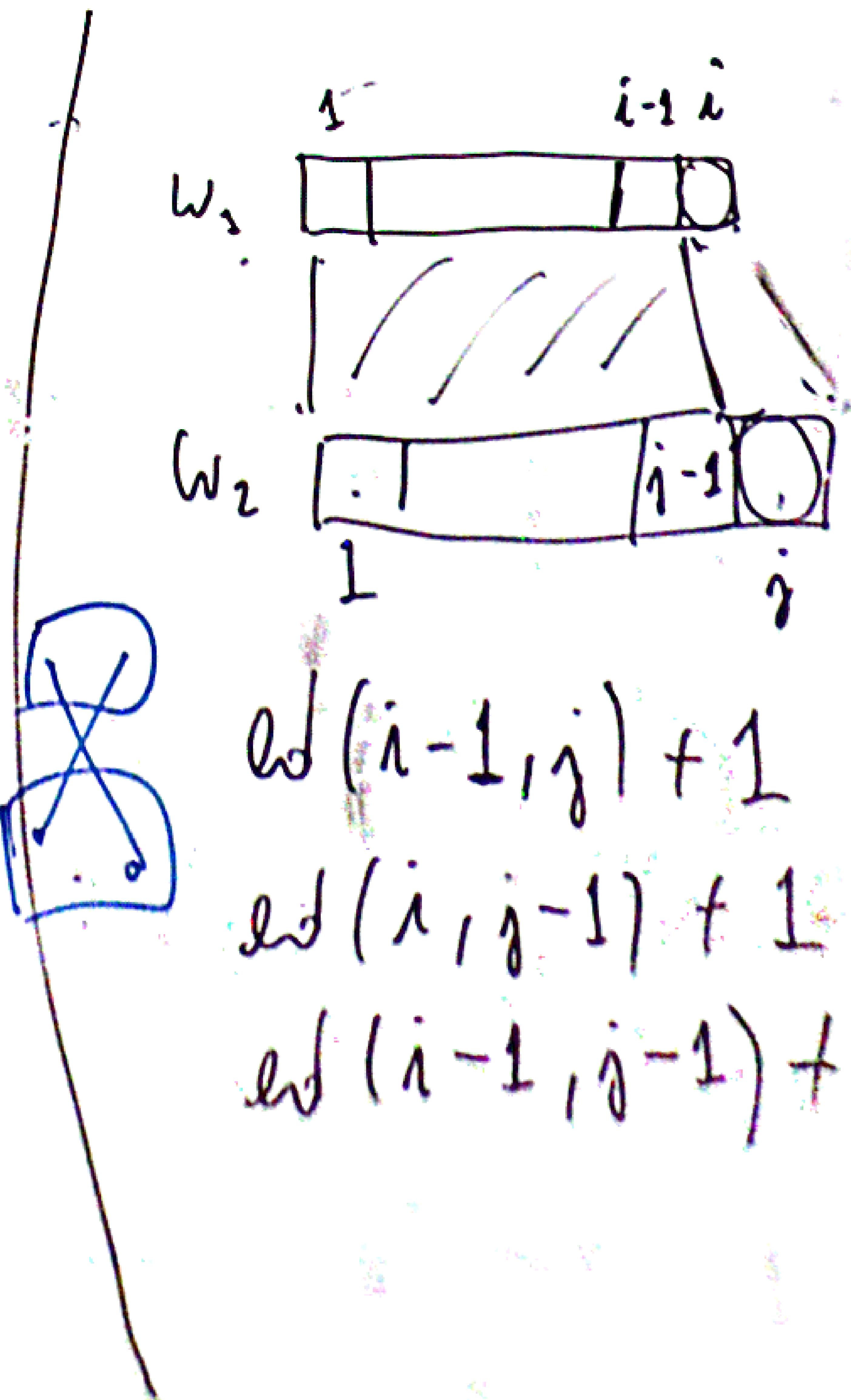
$$\omega(\lambda^{-1}, \theta^{-1})$$



$i=0$

$j=0$

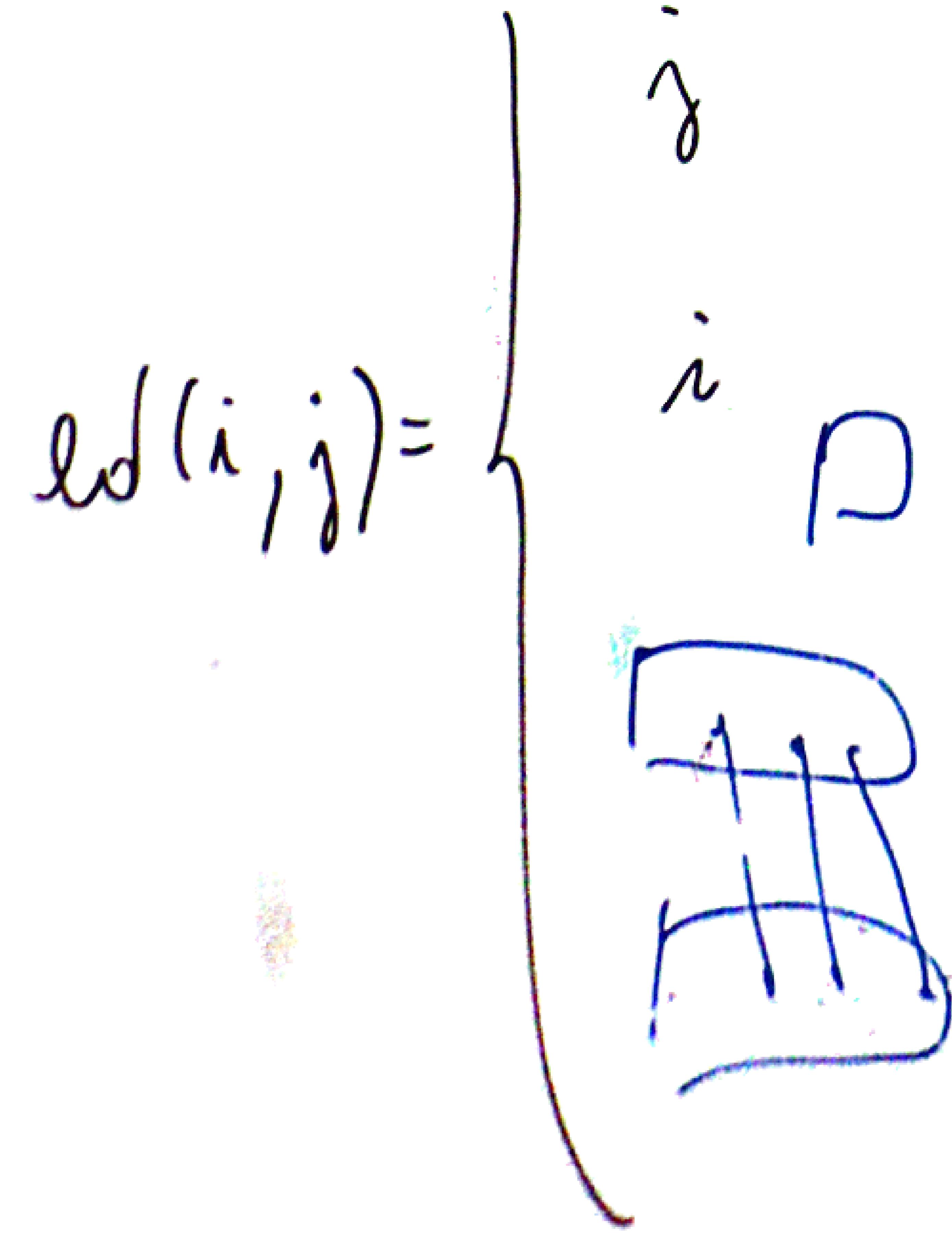
$i>0$
 $j>0$



$$ed(i-1, j) + 1$$

$$ed(i, j-1) + 1$$

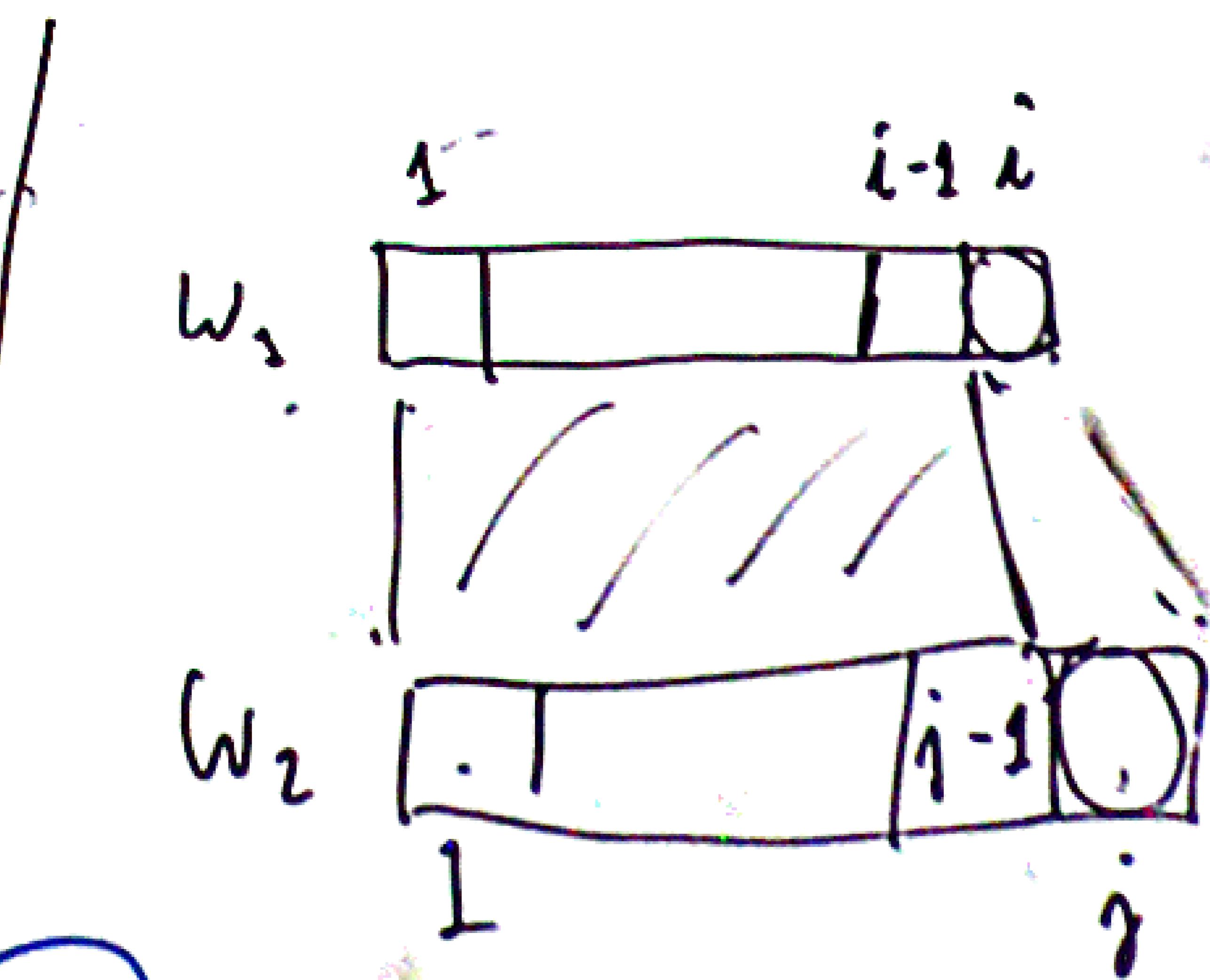
$$ed(i-1, j-1) +$$



$i=0$

$j=0$

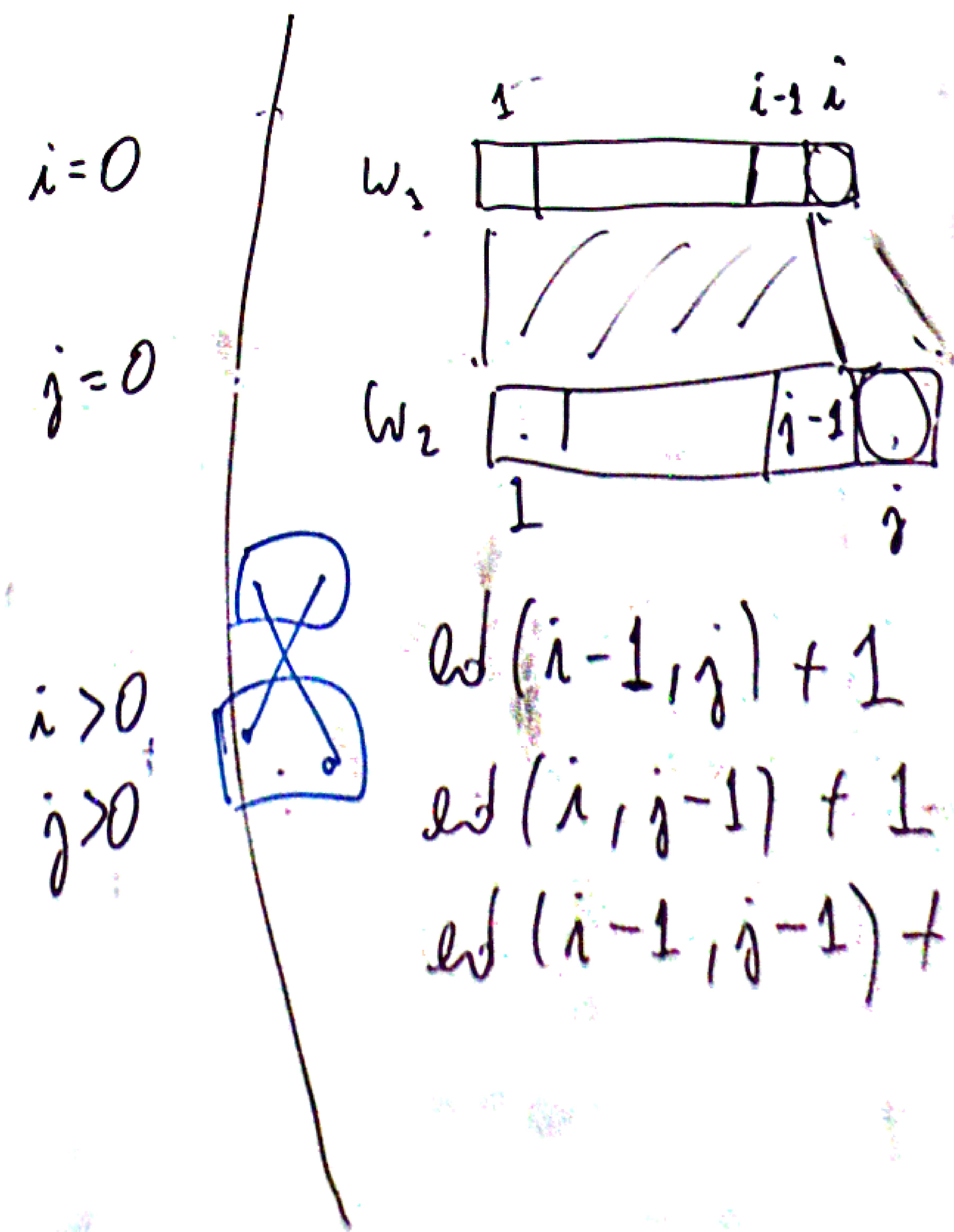
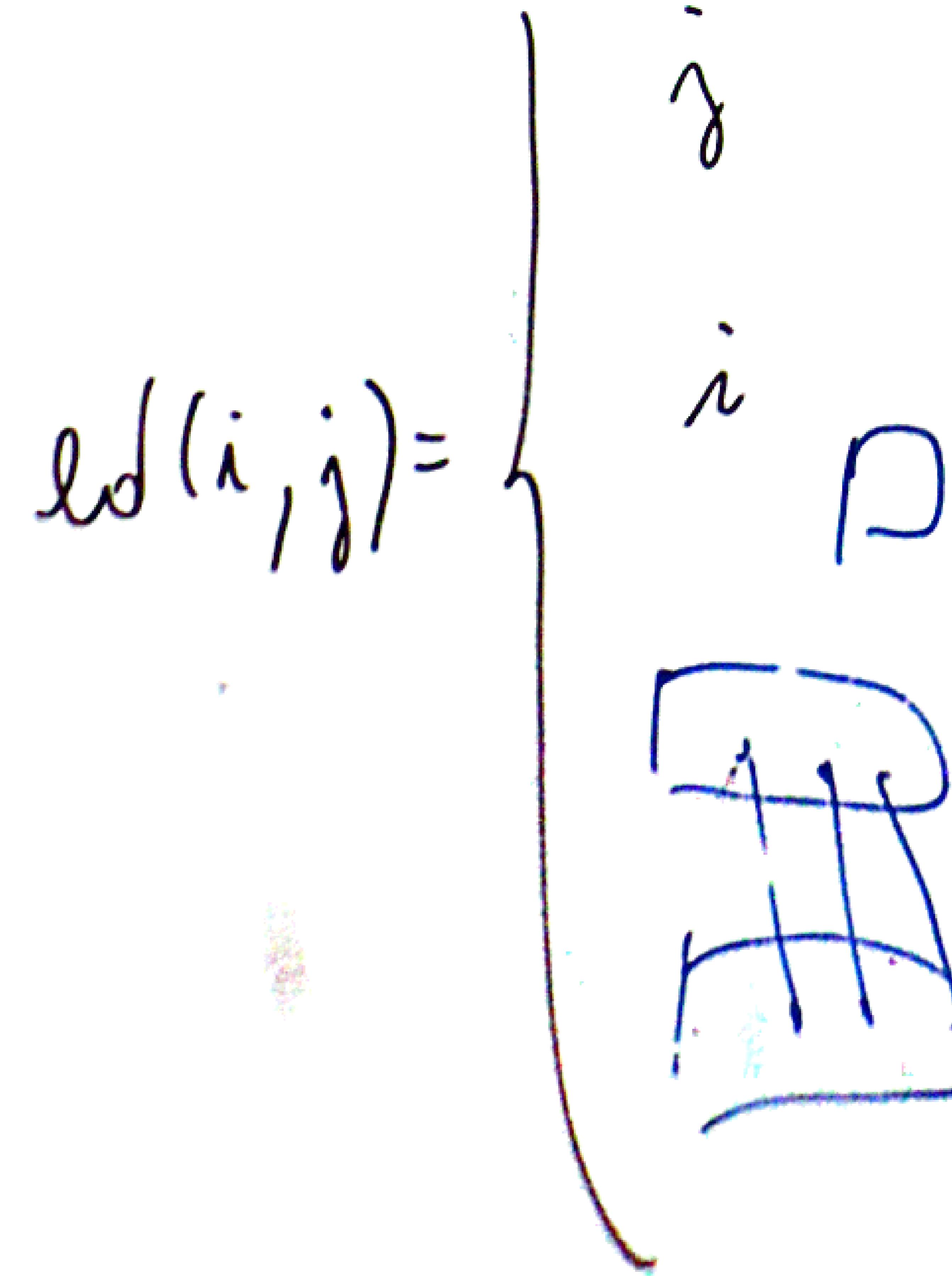
$i>0$,
 $j>0$



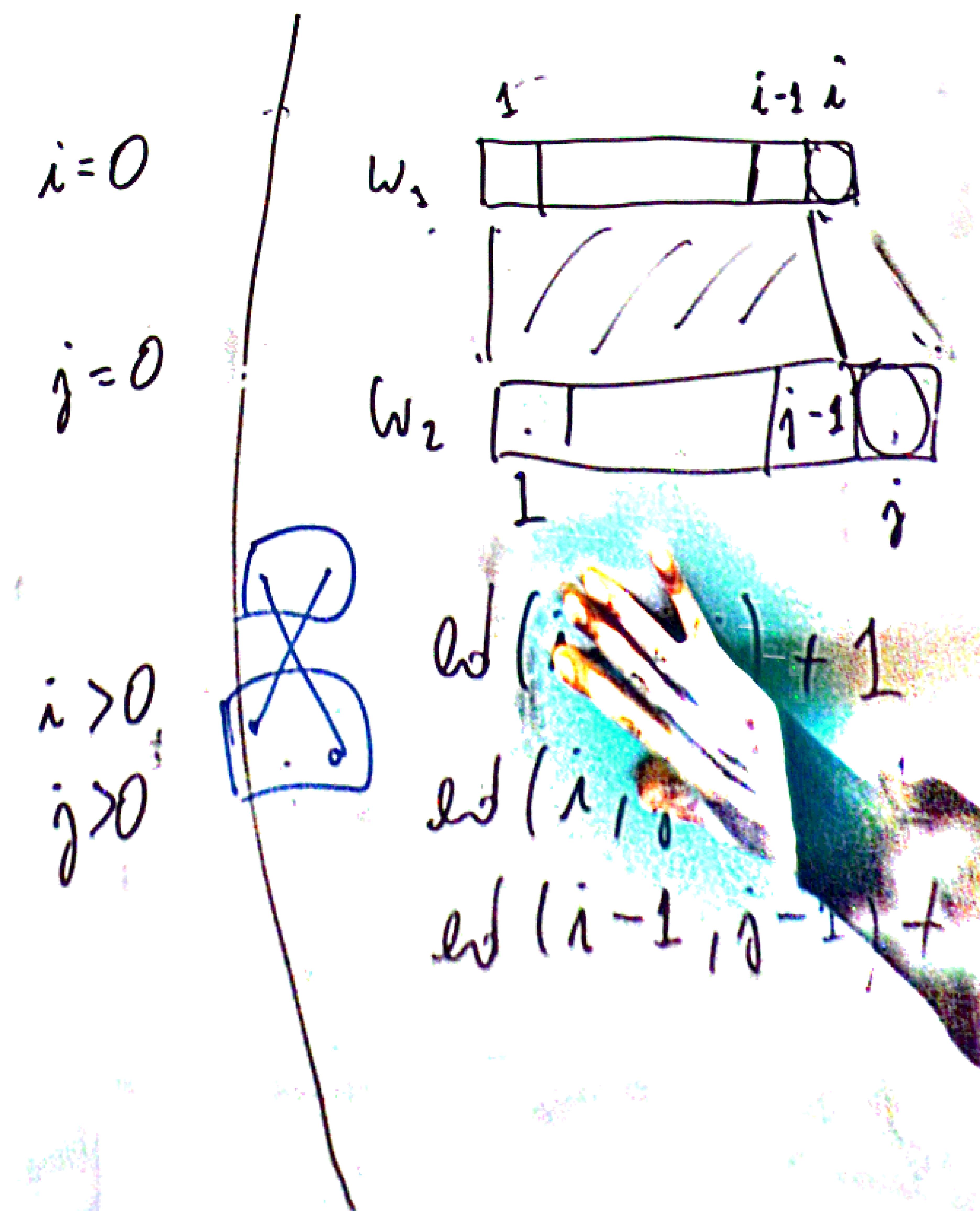
$$ed(i-1, j) + 1$$

$$ed(i, j-1) + 1$$

$$ed(i-1, j-1) +$$



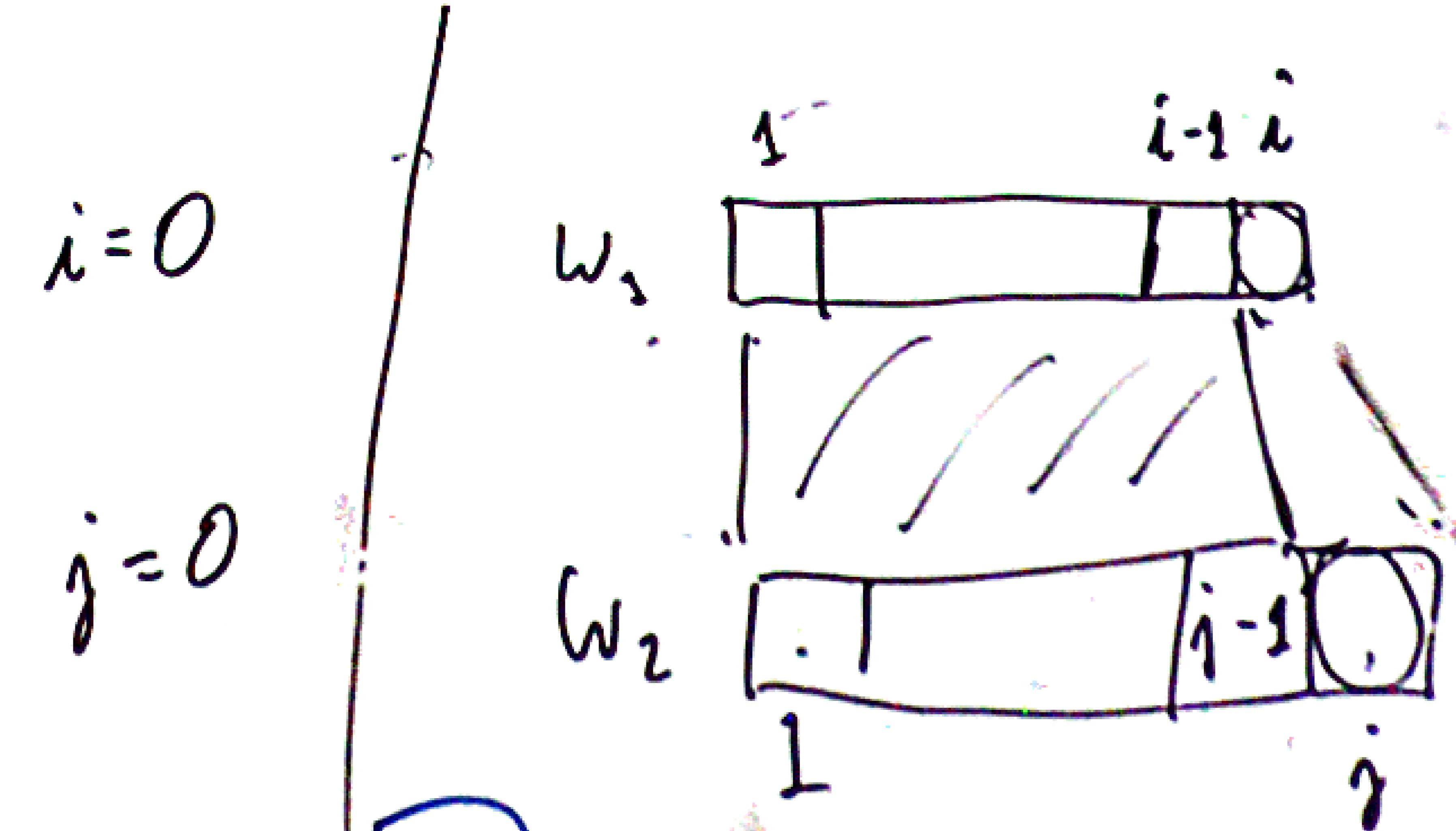
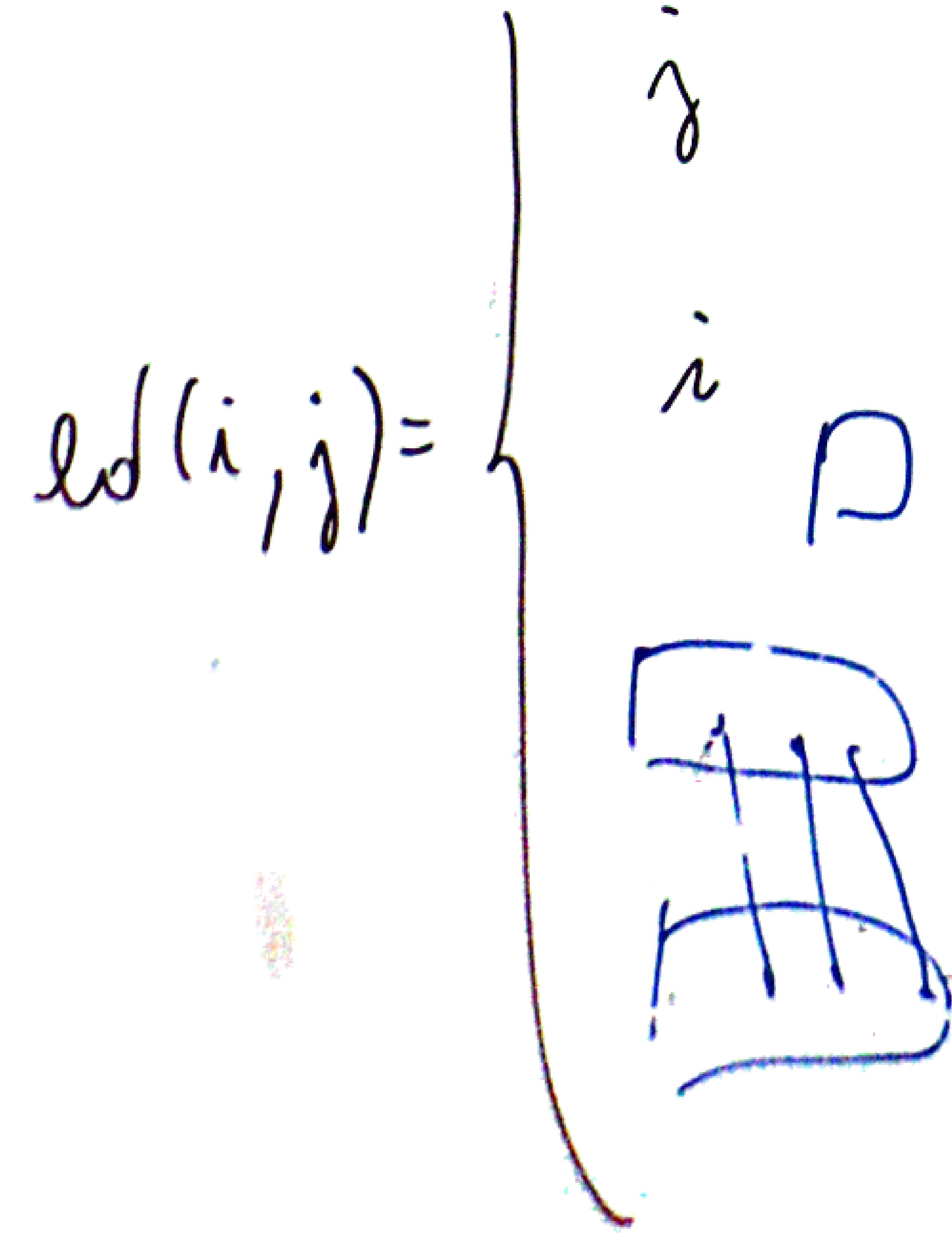
$$ed(i, j) = \begin{cases} i & j \\ i & P \\ \text{Diagram} & \end{cases}$$



$$ed(i, j) + 1$$

$$ed(i, 0)$$

$$ed(i-1, 0) + 1$$



$ed(i-1, j) + 1$
 $ed(i, j-1) + 1$
 $ed(i-1, j-1) +$