$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i \qquad (a_0, \dots, a_{n-1})$$

$$q(x) = \sum_{i=0}^{n-1} b_i \cdot x^i \qquad (b_0, \dots$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i \qquad (a_0, \ldots, a_{n-1})$$

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$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i \qquad \boxed{(a_0, \ldots, a_{n-1})}$$

$$q(x) = \sum_{i=0}^{n-1} b_i \cdot x^i \qquad (b_0, \ldots, b_{n-1})$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i$$

$$q(x) = \sum_{i=0}^{n-1} b_i \cdot x^i$$

$\boxed{(a_0, \ldots, a_{n-1})}$

$(b_0, \ldots, b_{n-1})$

$\to (a_0, \ldots, a_{n-1}, \underline{0, \ldots, 0})$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i$$

$$q(x) = \sum_{i=0}^{n-1} b_i \cdot x^i$$

$\boxed{(a_0, \dots, a_{n-1})}$

$(b_0, \dots, b_{n-1})$

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$$p(\omega_0), \ldots, p(\omega_{n-1})$$

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$$p(\omega_0), \ldots, p(\omega_{n-1})$$

$$e^{ix} = \cos x + i \sin x$$

$$x^5 - 1$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i$$

$$p(\omega_0), \ldots, p(\omega_{n-1})$$

$$x^n - 1 \qquad 1$$

$$e^{ix} = \cos x + i \sin x$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i$$

$$p(\omega_0), \ldots, p(\omega_{n-1})$$

$$e^{ix} = \cos x + i \sin x$$

$$x^n - 1 \qquad 1$$

$$e^{2\pi i}$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i$$

$$p(\omega_0), \ldots, p(\omega_{n-1})$$

$$X^n - 1 \qquad 1$$

$$e^{ix} = \cos x + i \sin x$$

$$\omega_n = e^{\frac{2\pi i}{n}}$$

$$\omega_n^n = \left(e^{\frac{2\pi i}{n}}\right)^n = e^{2\pi i} = 1$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i$$

$$p(\omega_0), \dots, p(\omega_{n-1})$$

$$x^n - 1 \qquad 1$$

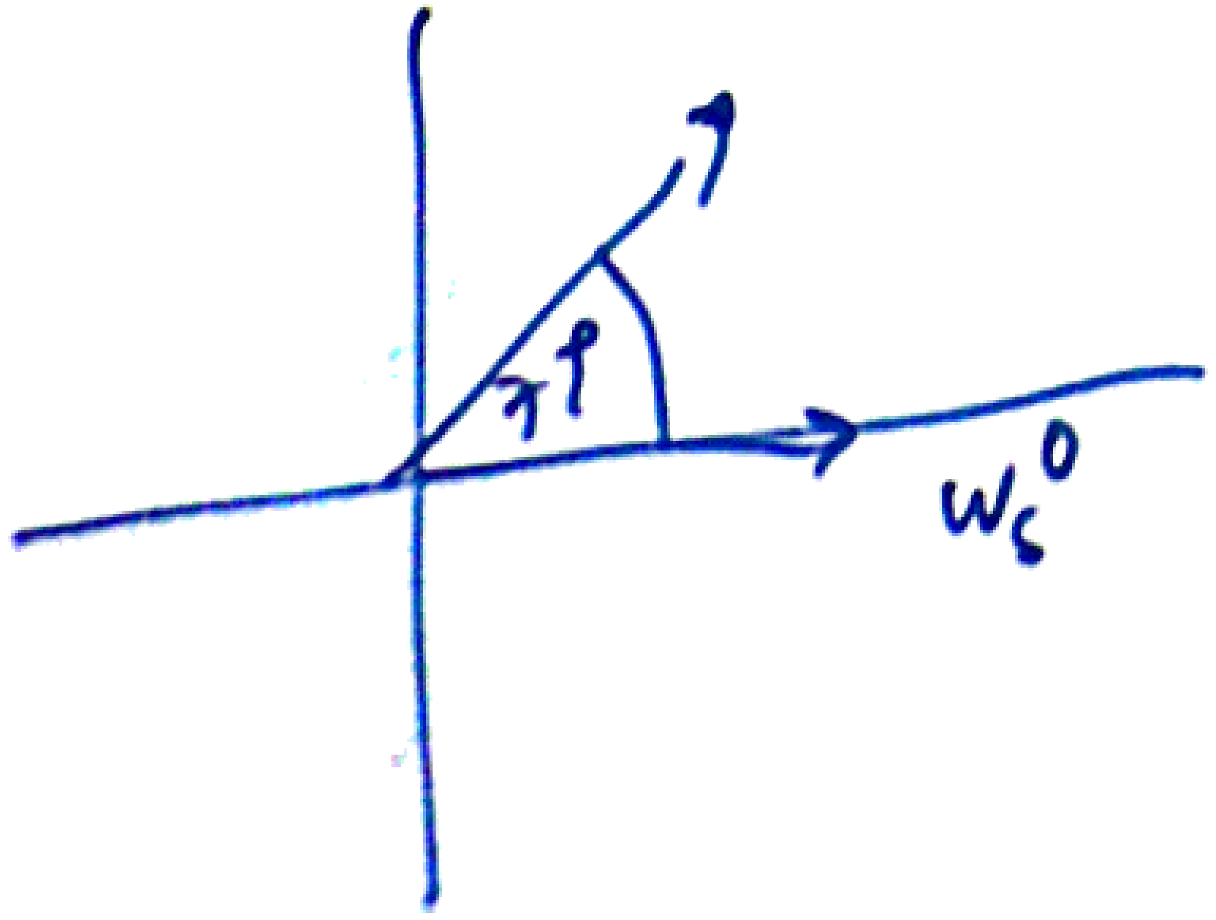
$$e^{ix} = \cos x + i \sin x$$

$$\omega_n = e^{\frac{2\pi i}{n}}$$

$$\omega_n^n = \left(e^{\frac{2\pi i}{n}}\right)^n = e^{2\pi i} = 1 \qquad \omega_n^k = e^{\frac{2\pi i}{n}\cdot k}$$

$$\left(\omega_n^k\right)^n = \left(e^{\frac{2\pi i}{\cancel{n}}\cdot k}\right)^{\cancel{n}} = \left(\underbrace{e^{2\pi i}}_{1}\right)^k$$

$$w_5^1 = e^{\frac{2\pi i}{5}}$$



$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i$$

$$p(w_0), \ldots, p(w_{n-1})$$

$$x^n - 1 \qquad 1$$

$$e^{ix} = \cos x + i \sin x$$

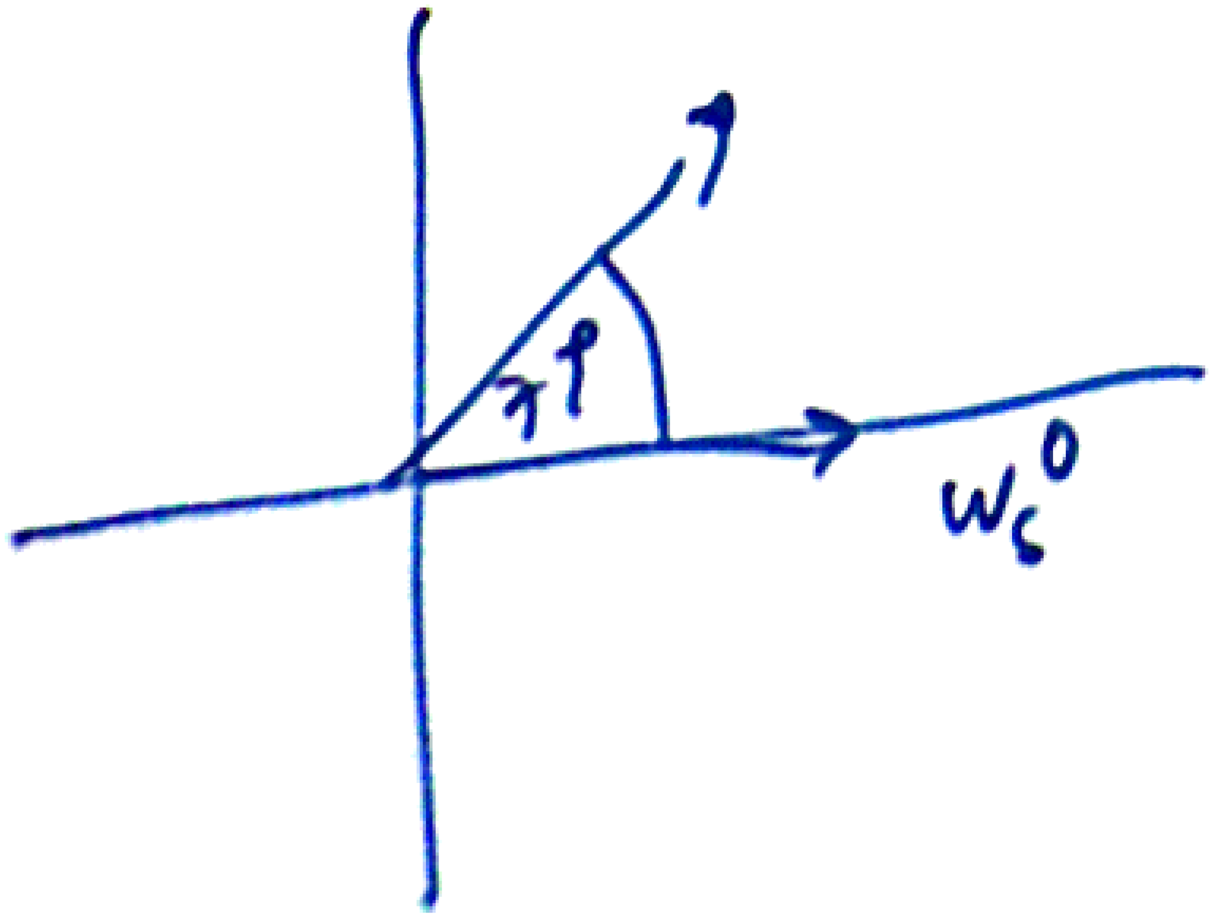
$$w_n = e^{\frac{2\pi i}{n}}$$

$$w_n^n = \left(e^{\frac{2\pi i}{n}}\right)^n = e^{2\pi i} = 1$$

$$w_n^k = e^{\frac{2\pi i}{n} \cdot k}$$

$$(w_n^k)^n = \left(e^{\frac{2\pi i}{\cancel{n}} \cdot k}\right)^{\cancel{n}} = \big(\underbrace{e^{2\pi i}}_{1}\big)^k$$

$$w_5^1 = e^{\frac{2\pi i}{5}}$$



$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i$$

$$p(w_0), \dots, p(w_{n-1})$$

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$$w_n = e^{\frac{2\pi i}{n}}$$

$$w_n^n = \left(e^{\frac{2\pi i}{n}}\right)^n = e^{2\pi i} = 1$$

$$w_n^k = e^{\frac{2\pi i}{n}\cdot k}$$

$$\left(w_n^k\right)^n = \left(e^{\frac{2\pi i}{\cancel{n}}\cdot k}\right)^{\cancel{n}} = \Big(\underbrace{e^{2\pi i}}_{1}\Big)^k$$

$$w_5^1 = e^{\frac{2\pi}{5} i}$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i$$

$$p(w_0), \ldots, p(w_{n-1})$$

$$x^n - 1$$

$$w_n = e^{\frac{2\pi i}{n}}$$

$$w_n^0, w_n^1, w_n^2, \ldots, w_n^{n-1}$$

$$w_n^k$$

$$W_5^1 = e^{\frac{2\pi i}{5}}$$

$$P(x) = \sum_{i=0}^{n-1} a_i \cdot x^i$$

$$P(w_0), \ldots, P(w_{n-1})$$

$$x^n - 1$$

$$W_n = e^{\frac{2\pi i}{n}}$$

$$W_n^0, W_n^1, W_n^2, \ldots, W_n^{n-1}$$

$$W_n^k$$

$$w_5^1 = e^{\frac{2\pi}{5} i}$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i$$

$$p(w_0), \ldots, p(w_{n-1})$$

$$x^n - 1$$

$$w_n = e^{\frac{2\pi i}{n}}$$

$$w_n^0, w_n^1, w_n^2, \ldots, w_n^{n-1}$$

$$w_n^k$$

$$W_n^{\ell} = W_n^{k} \Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{\ell} = \left(e^{\frac{2\pi i}{n}}\right)^{k}$$

$$\Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{\ell - k} = 1$$

$$\Rightarrow e^{\frac{(\ell - k)}{n} 2\pi i} = 1$$

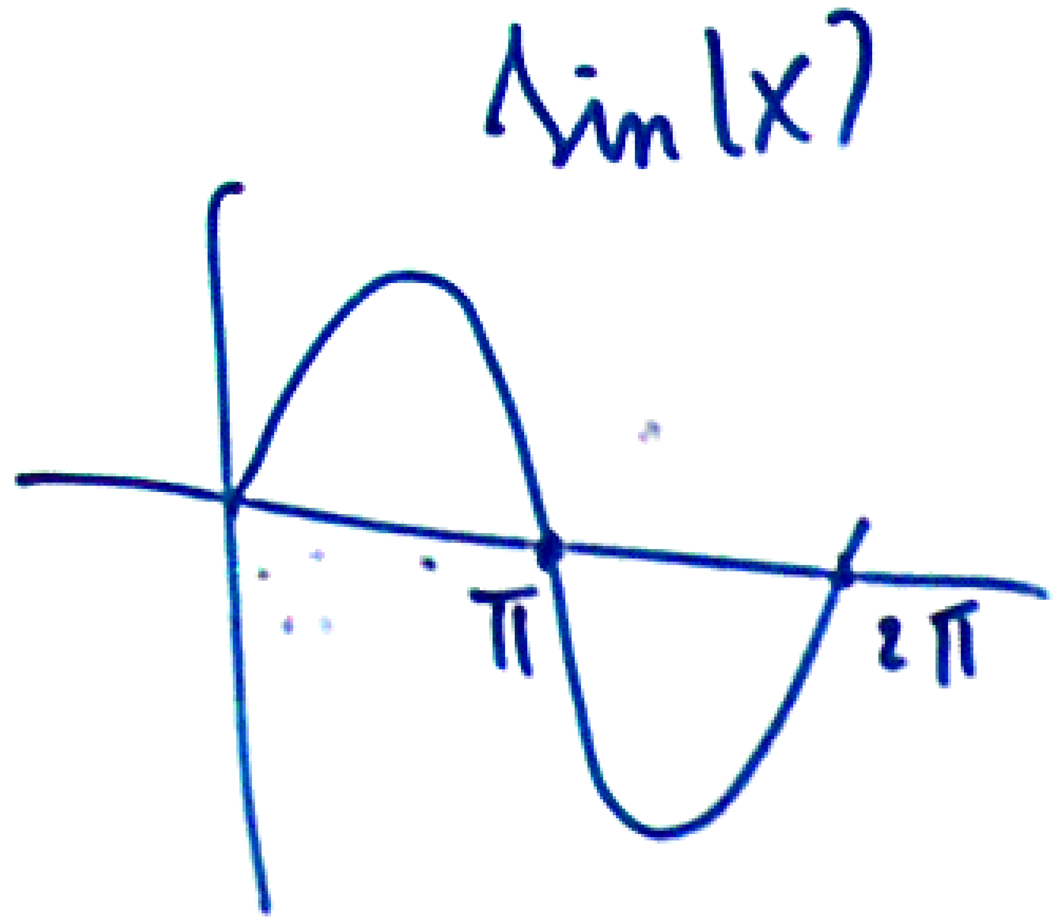
$$W_n^l = W_n^k \Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^l = \left(e^{\frac{2\pi i}{n}}\right)^k$$

$$\Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{l-k} = 1$$

$$\Rightarrow e^{\frac{(l-k)}{n} 2\pi i} = 1$$

$$\Rightarrow \cos\left(\frac{(l-k)}{n} \cdot 2\pi\right) + i \sin\left(\frac{(l-k)}{n} \cdot 2\pi\right) = 1$$

$$W_n^l = W_n^k \Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^l = \left(e^{\frac{2\pi i}{n}}\right)^k$$

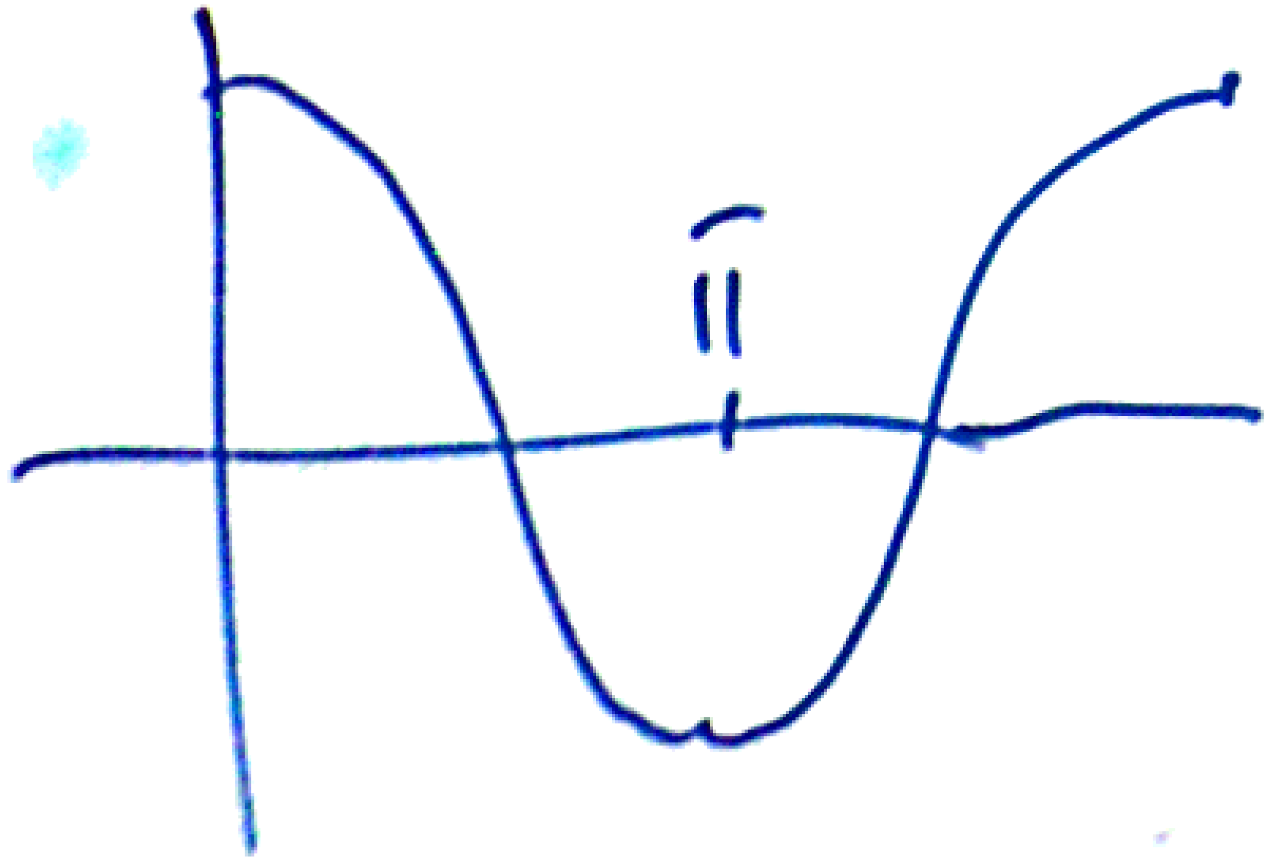


$$\Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{l-k} = 1$$

$$\Rightarrow e^{\frac{(l-k)2\pi i}{n}} = 1$$

$$\Rightarrow \cos\left(\frac{(l-k)}{n}\cdot 2\pi\right) + i \sin\left(\frac{(l-k)}{n}\cdot 2\pi\right) = 1$$

$$w_n^{l} = w_n^{k} \Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{l} = \left(e^{\frac{2\pi i}{n}}\right)^{k}$$



$$\Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{l-k} = 1$$

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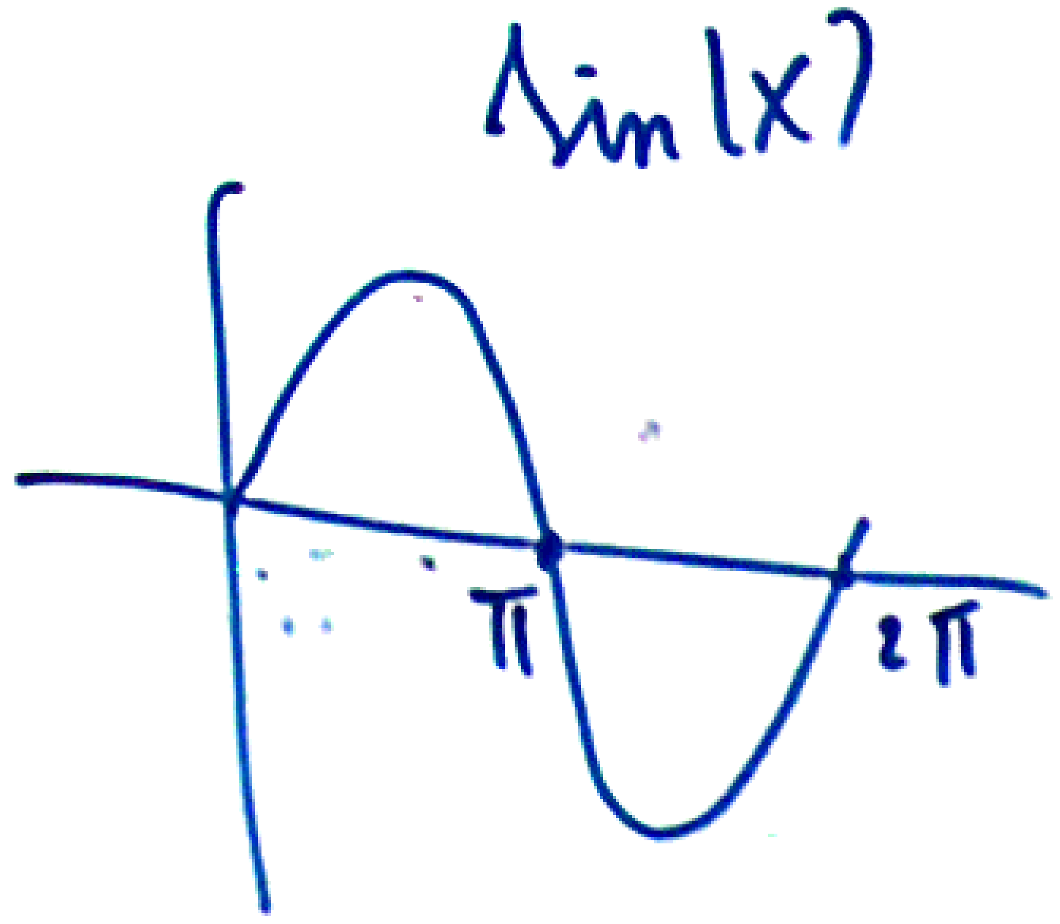
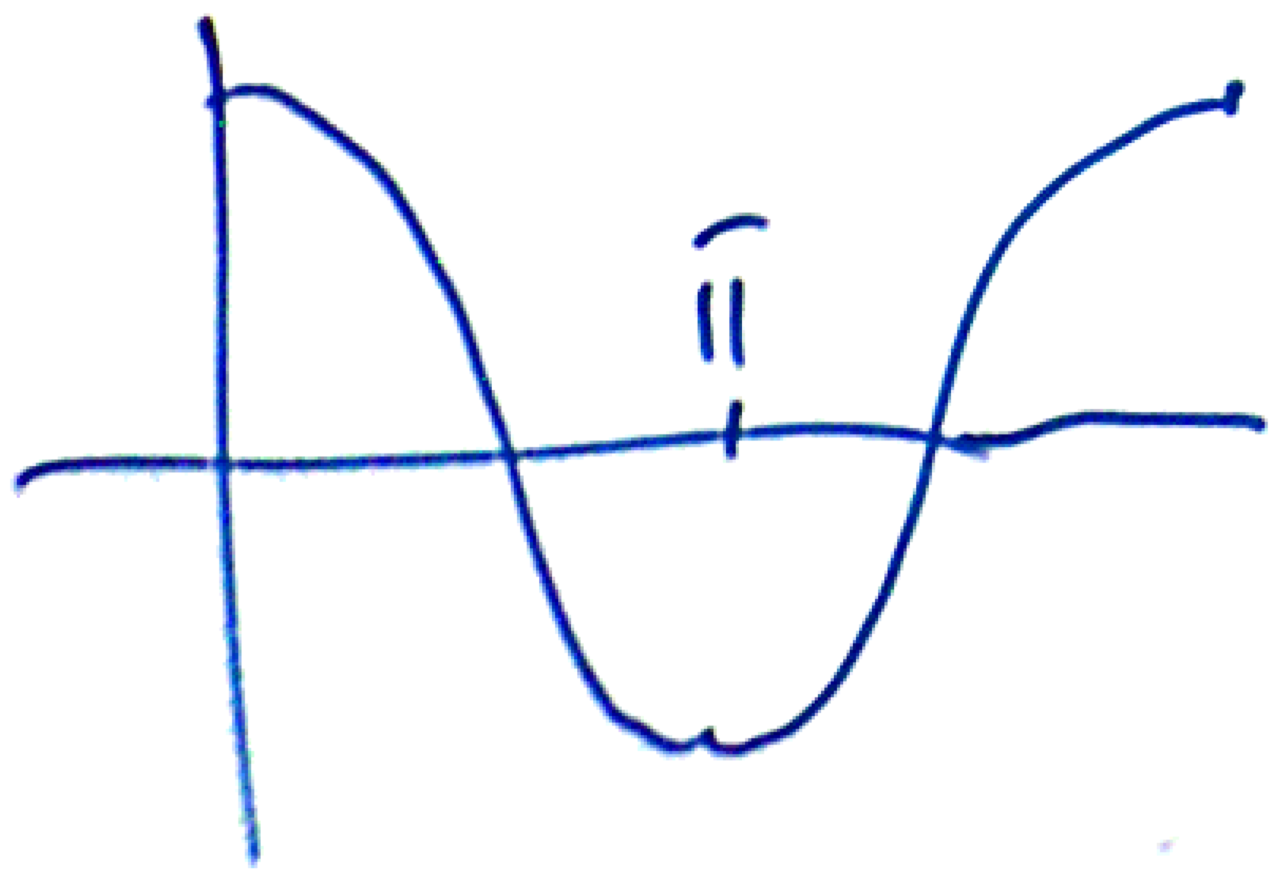


$$w_n^l = w_n^k \Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^l = \left(e^{\frac{2\pi i}{n}}\right)^k$$

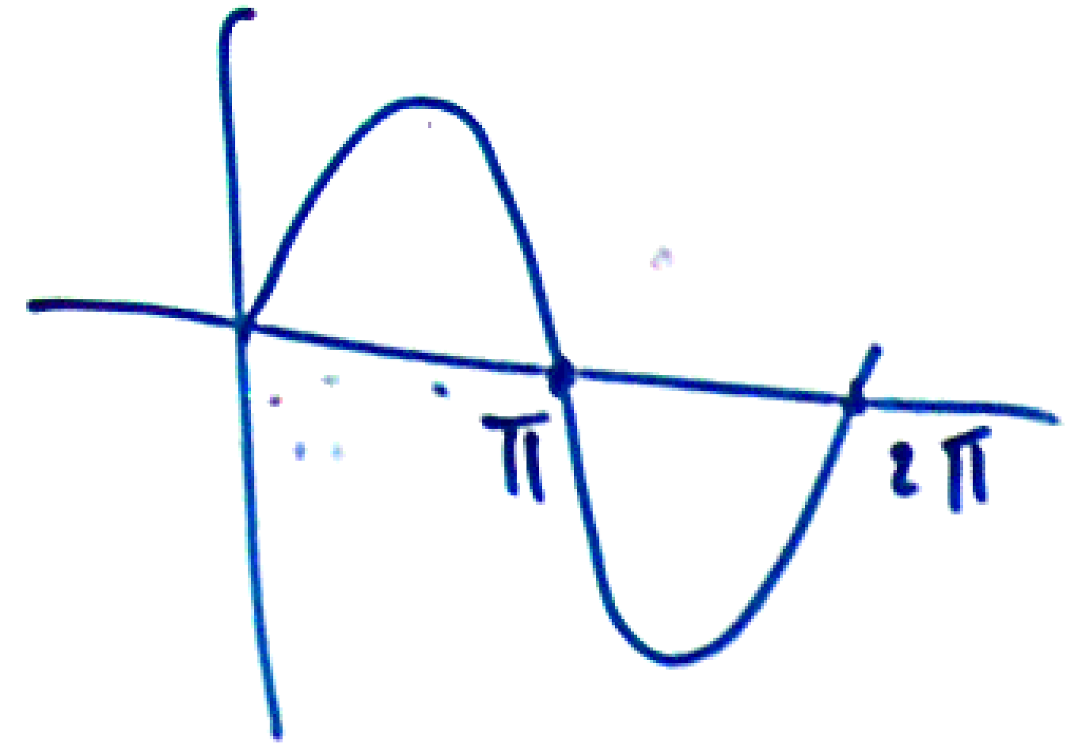


$$\Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{l-k} = 1$$

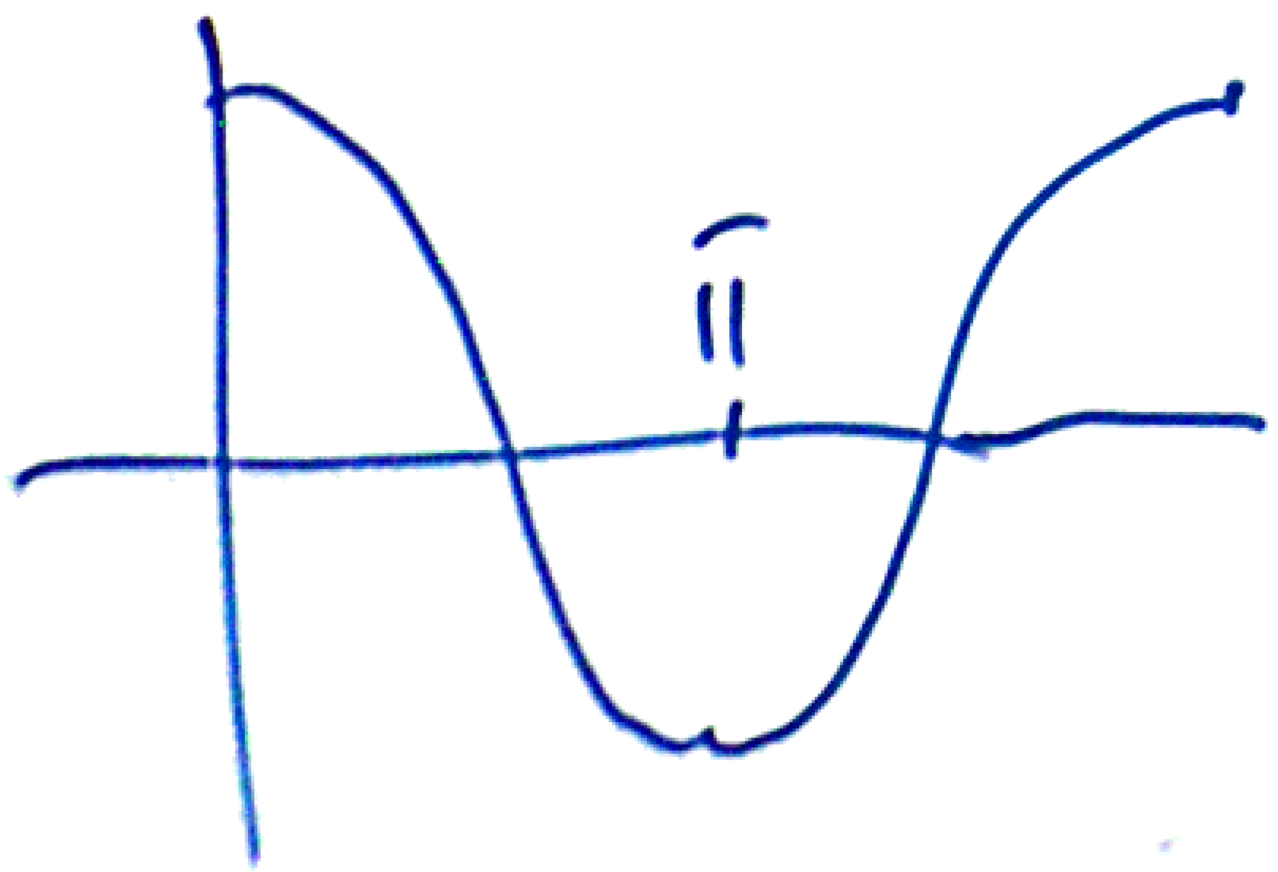
$$\Rightarrow e^{\frac{(l-k)2\pi i}{n}} = 1$$

$$\Rightarrow \cos\left(\frac{(l-k)}{n}\cdot 2\pi\right) + i \sin\left(\frac{(l-k)}{n}\cdot 2\pi\right) = 1$$

sin(x)



$$w_n^l = w_n^k \Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^l = \left(e^{\frac{2\pi i}{n}}\right)^k$$

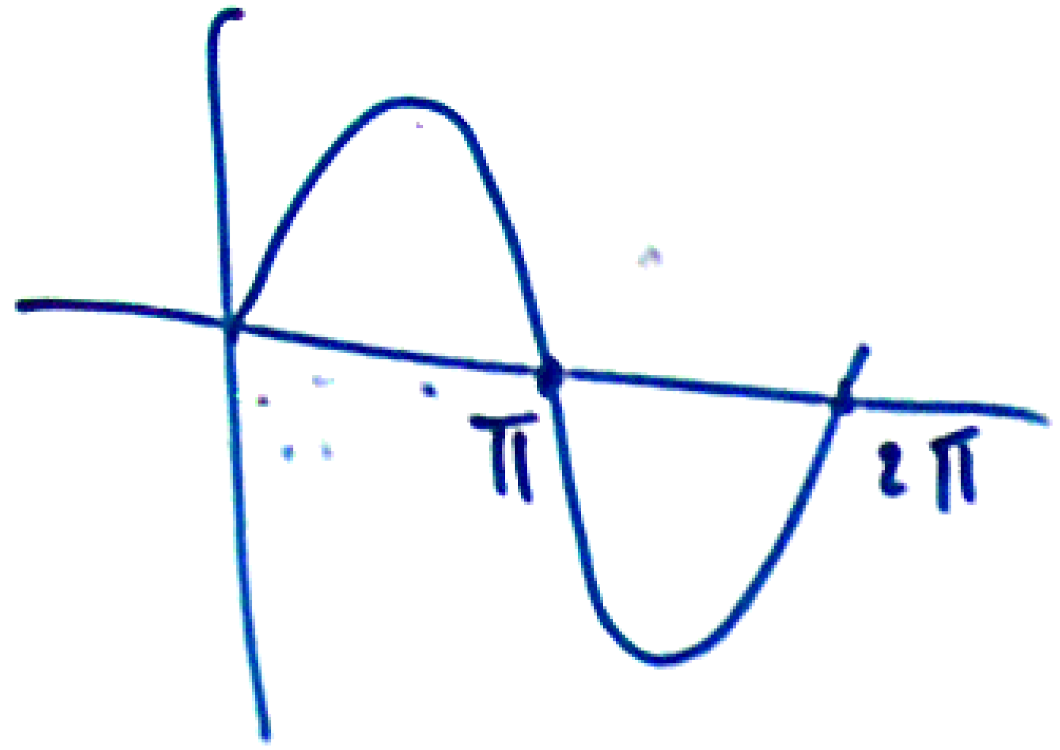


$$\Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{l-k} = 1$$

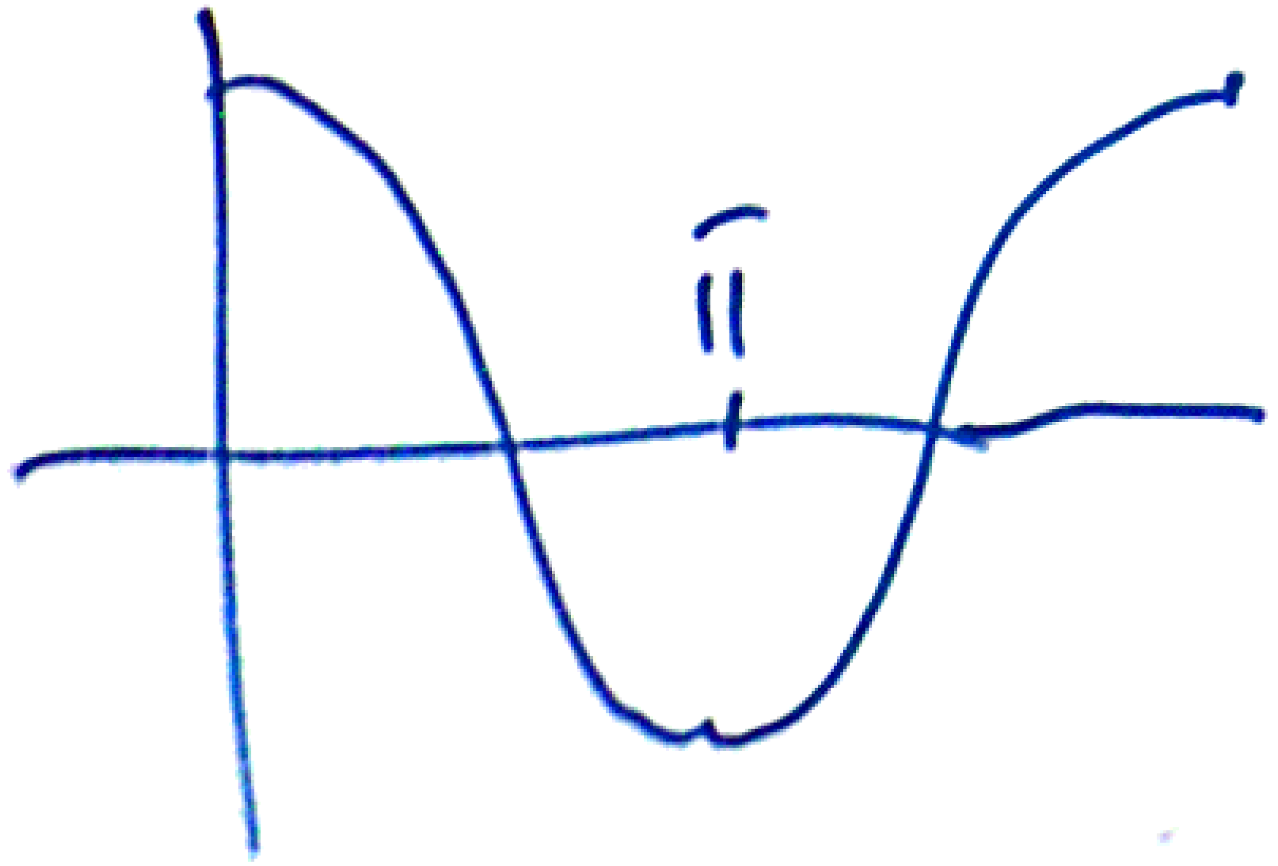
$$\Rightarrow e^{\frac{(l-k)}{n}2\pi i} = 1$$

$$\Rightarrow \cos\left(\frac{(l-k)}{n}\cdot 2\pi\right) + i\sin\left(\frac{(l-k)}{n}\cdot 2\pi\right) = 1$$

$\sin(x)$



$$w_n^{l} = w_n^{k} \Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{l} = \left(e^{\frac{2\pi i}{n}}\right)^{k}$$



$$\Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{l-k} = 1$$

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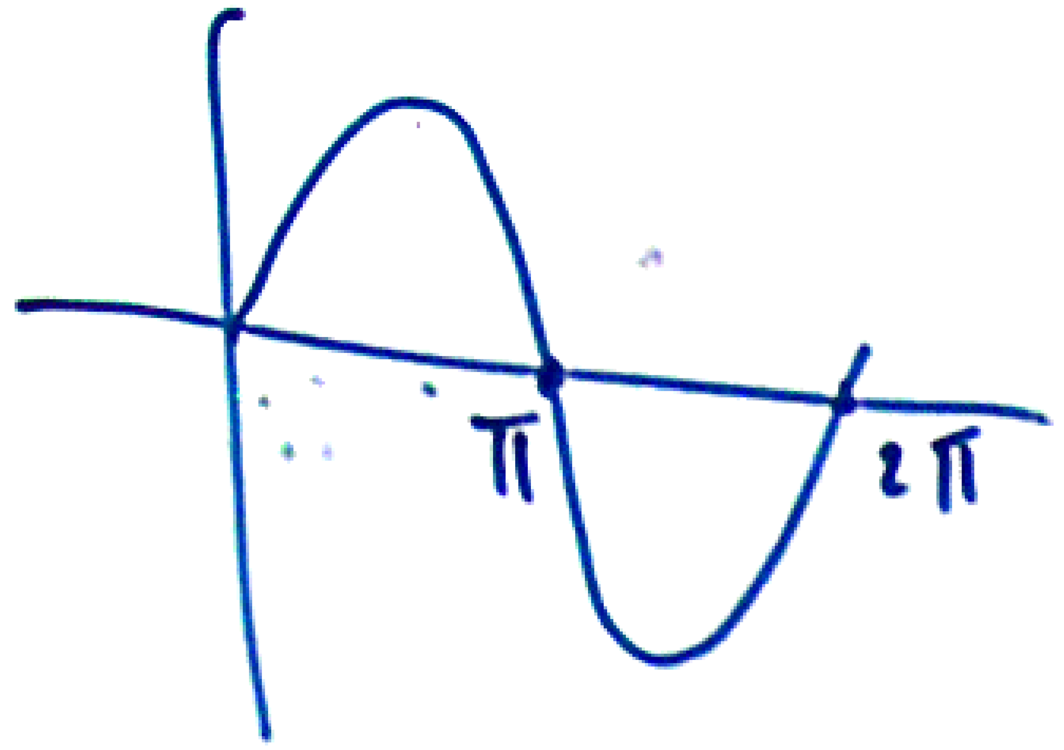
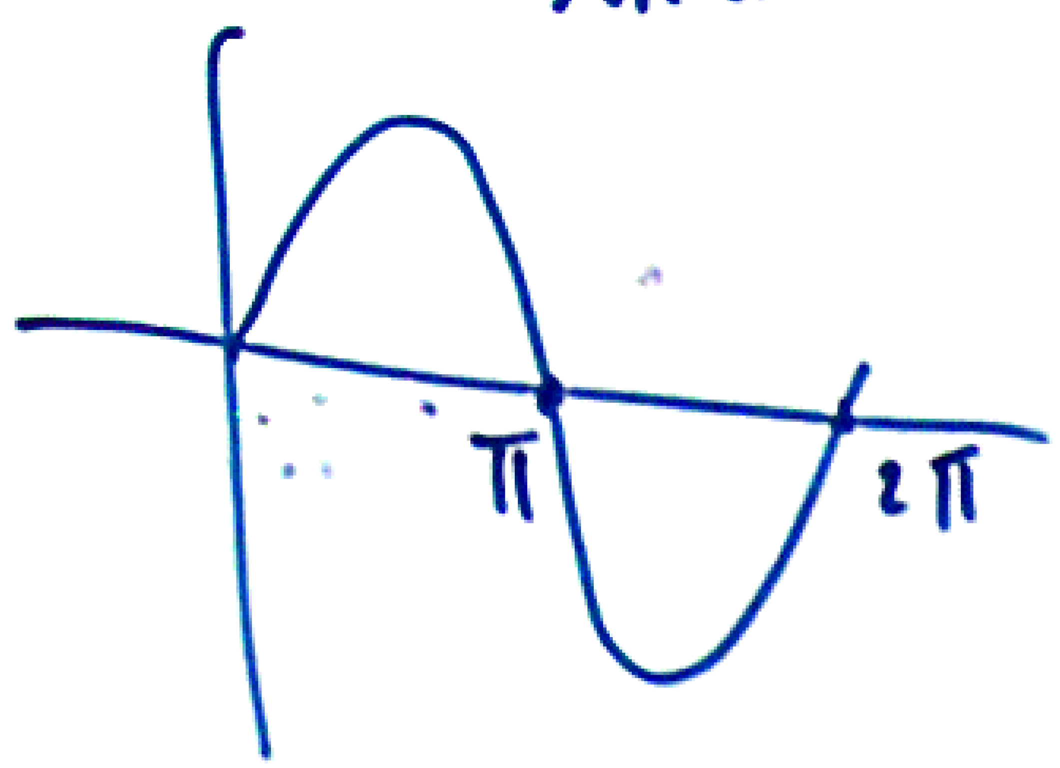
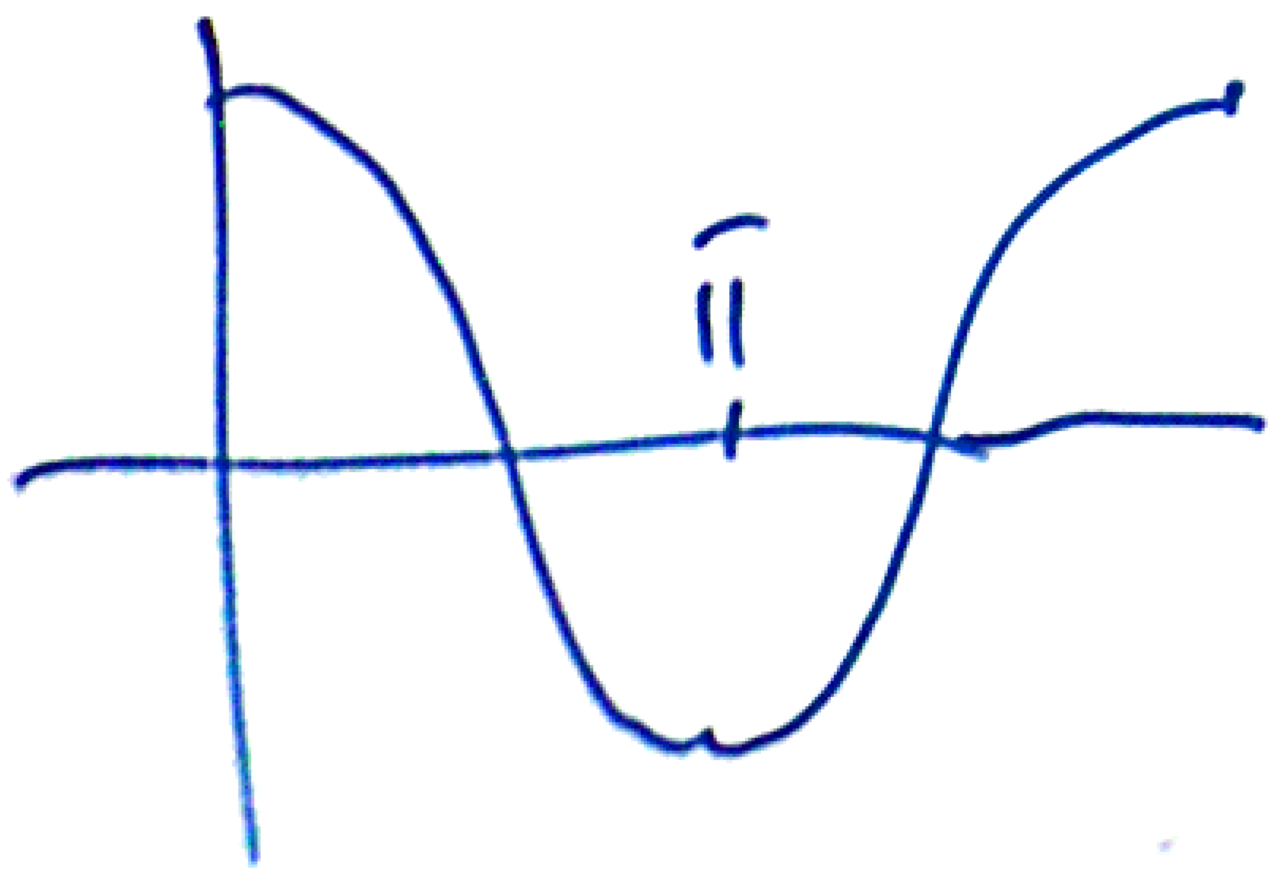
sin(x)



$$w_4 = e^{\frac{2\pi i}{4}} = e^{\frac{\pi}{2} i}$$

$$W_n^l = W_n^k \Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^l = \left(e^{\frac{2\pi i}{n}}\right)^k$$

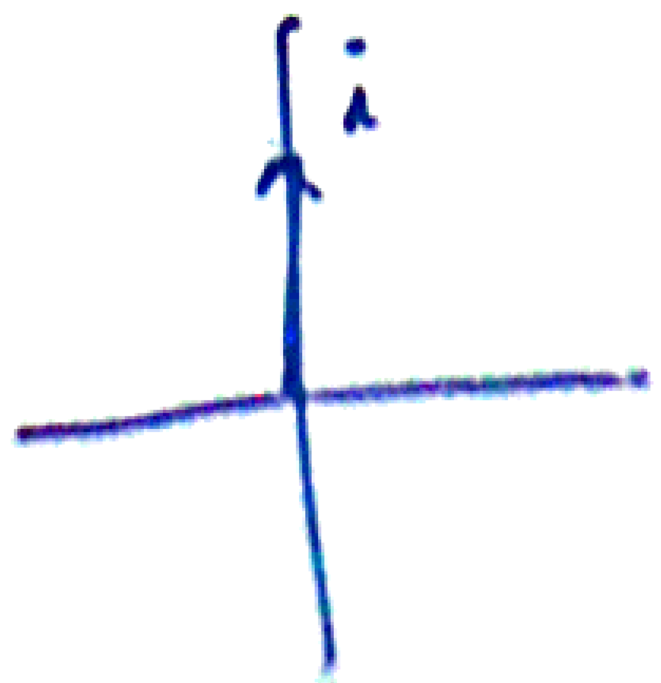




$$\Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{l-k} = 1$$

$$\Rightarrow e^{\frac{(l-k)2\pi i}{n}} = 1$$

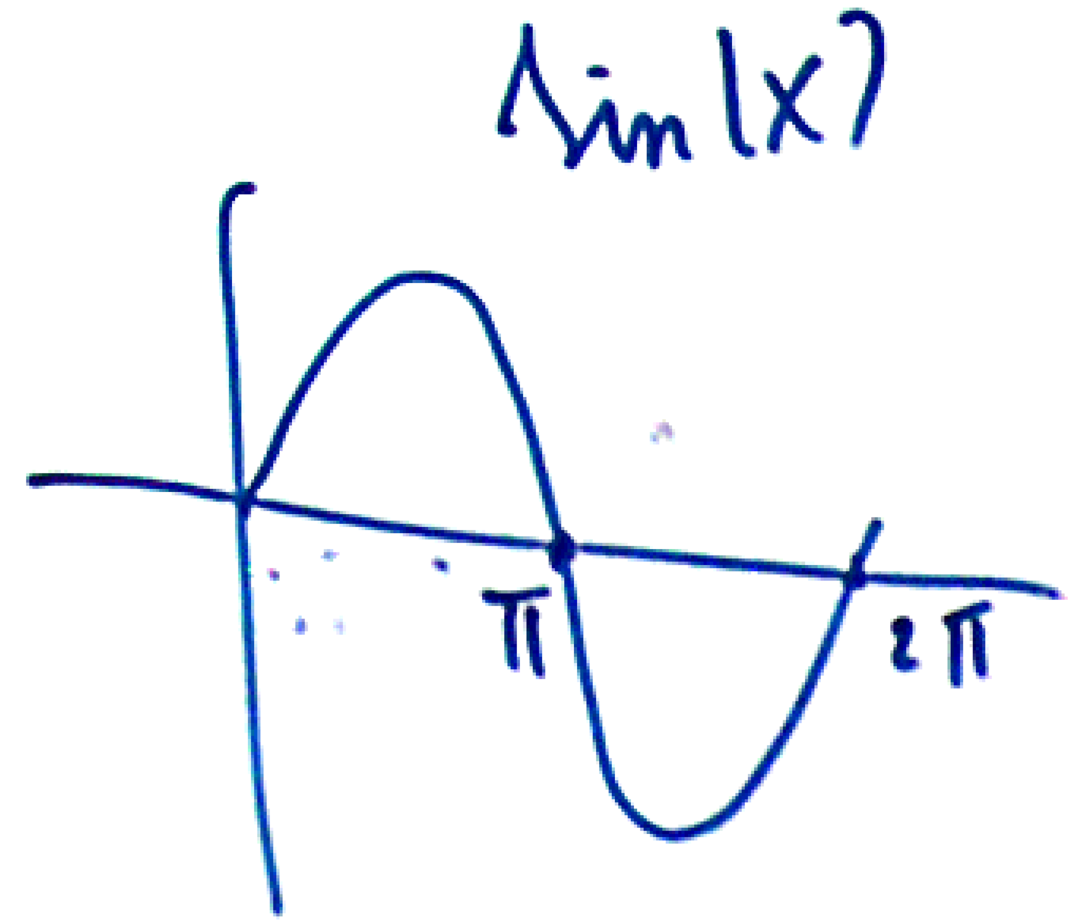
$$\Rightarrow \cos\left(\frac{(l-k)}{n}\cdot 2\pi\right) + i \sin\left(\frac{(l-k)}{n}\cdot 2\pi\right) = 1$$

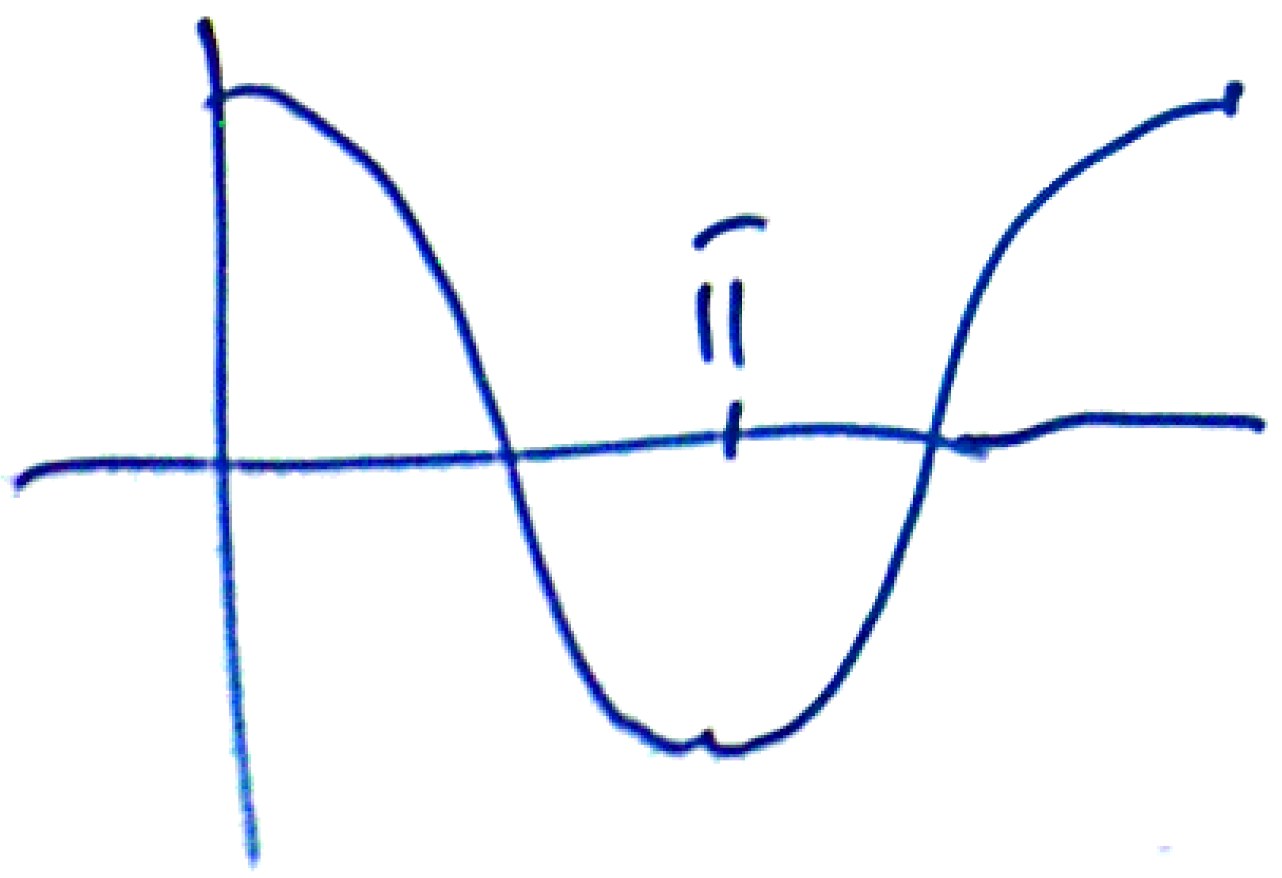


$$W_4 = e^{\frac{2\pi i}{4}} = e^{\frac{\pi}{2} i}$$

$$\boxed{i}$$

$$w_n^{l} = w_n^{k} \Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{l} = \left(e^{\frac{2\pi i}{n}}\right)^{k}$$





$$\Rightarrow \left(e^{\frac{2\pi i}{n}}\right)^{l-k} = 1$$

$$\Rightarrow e^{\frac{(l-k)2\pi i}{n}} = 1$$

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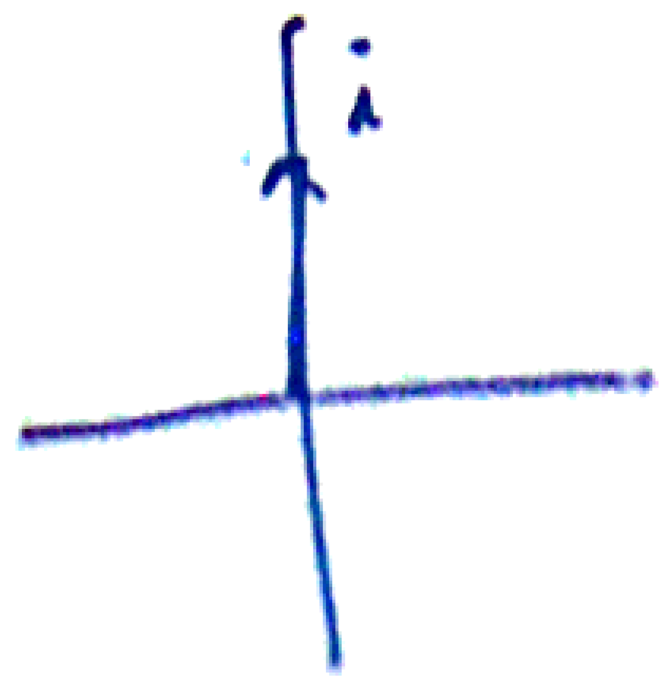

$$w_4 = e^{\frac{2\pi i}{4}} = e^{\frac{\pi}{2} i}$$

$$\boxed{i}$$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \overline{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$

$DFT(\overline{a})$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$

$P(w_n^0), \ldots, P(w_n^{n-1})$ $\qquad y_i = P(w_n^i)$

$DFT(\bar{a}) = [y_0, \ldots, y_{n-1}]$

$DFT(\bar{a})$

$$
\begin{tikzcd}
 & \bar{a},\ n \arrow[dl] \arrow[dr] & \\
\bar{a}_0 \quad \frac{n}{2} & & \bar{a}_1 \quad \frac{n}{2}
\end{tikzcd}
$$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

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$$DFT(\bar{a})$$

$$\begin{tikzcd}[column sep=small]
& \bar{a},\ n \arrow[dl] \arrow[dr] & \\
\bar{a}_0,\ \frac{n}{2} & & \bar{a}_1,\ \frac{n}{2}
\end{tikzcd}$$

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right)$$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$

$p(w_n^0), \ldots, p(w_n^{n-1})$

$y_i = p(w_n^i)$

$DFT(\bar{a}) = [y_0, \ldots, y_{n-1}]$

$DFT(\bar{a})$

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + c \cdot n$$

$$\begin{tikzcd}
 & \bar{a} \,/\, n \arrow[dl] \arrow[dr] & \\
\bar{a}_0 \;\; \frac{n}{2} & & \bar{a}_1 \;\; \frac{n}{2}
\end{tikzcd}$$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$

$p(w_n^0), \ldots, p(w_n^{n-1})$

$y_i = p(w_n^i)$

$DFT(\bar{a}) = [y_0, \ldots, y_{n-1}]$

$\bar{a} \; n$

$\bar{a}_0 \quad \bar{a}_1$

$\frac{n}{2} \quad \frac{n}{2}$

$DFT(\bar{a})$

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + c \cdot n$$

$(a+bi)^{1/k}$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \overline{a}$

$p(w_n^0), \ldots, p(w_n^{n-1})$

$y_i = p(w_n^i)$

$DFT(\overline{a}) = [y_0, \ldots, y_{n-1}]$

$\overline{a}, n$

$DFT(\overline{a})$

$\overline{a}_0$ $\frac{n}{2}$

$\overline{a}_1$ $\frac{n}{2}$

$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + c \cdot n$

$(a+bi)^k$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

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$$DFT(\bar{a})$$

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + c \cdot n$$

$$\begin{tikzcd} & \bar{a} \;/\; n \arrow[dl] \arrow[dr] & \\ \bar{a}_0 \;\; \frac{n}{2} & & \bar{a}_1 \;\; \frac{n}{2} \end{tikzcd}$$

$$(a+bi)^k$$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$

$y_i = p(w_n^i)$

$DFT(\bar{a}) = [y_0, \dots, y_{n-1}]$

$DFT(\bar{a})$

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + c \cdot n$$

$$\bar{a} \,/\, n \;\rightarrow\; \bar{a}_0 \;\; \frac{n}{2}, \qquad \bar{a}_1 \;\; \frac{n}{2}$$

$(a+bi)^{1/k}$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$

$y_i = p(w_n^i)$

$\mathrm{DFT}(\bar{a}) = [y_0, \dots, y_{n-1}]$

$\bar{a}, n$

$$\begin{array}{cc} \bar{a}_0 & \bar{a}_1 \\ \frac{n}{2} & \frac{n}{2} \end{array}$$

$(a+bi)^{1/k}$

$\mathrm{DFT}(\bar{a})$

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + c \cdot n$$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

$$p(w_n^0), \dots, p(w_n^{n-1})$$

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$$\begin{tikzcd} & \bar{a} \,/\, n \arrow[dl] \arrow[dr] & \\ \bar{a}_0 \;\; \frac{n}{2} & & \bar{a}_1 \;\; \frac{n}{2} \end{tikzcd}$$

$$DFT(\bar{a})$$

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + c \cdot n$$

16

$$w_{16} = e^{\frac{2\pi i}{16}}$$

$$(a+bi)^k$$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$

$P(w_n^0), \ldots, P(w_n^{n-1})$

$y_i = P(w_n^i)$

$DFT(\bar{a}) = [y_0, \ldots, y_{n-1}]$

$DFT(\bar{a})$

$\bar{a}, n$

$\bar{a}_0 \quad \frac{n}{2}$

$\bar{a}_1 \quad \frac{n}{2}$

$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + c \cdot n$

$(a+bi)^k$

16

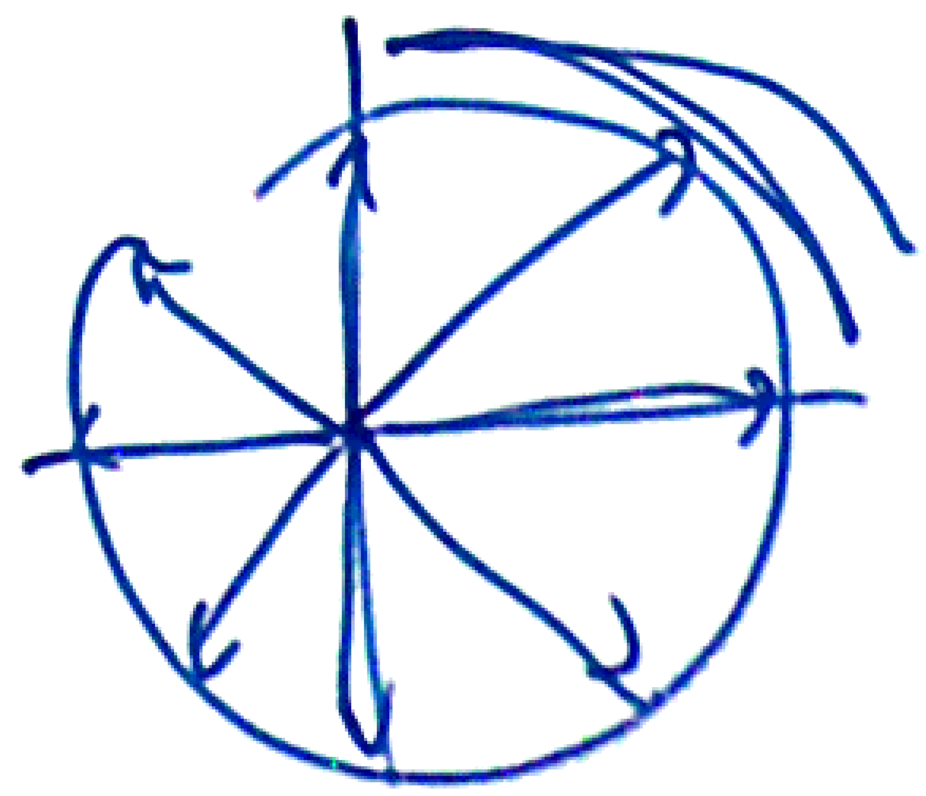
$W_{16} = e^{\frac{2\pi i}{16}}$

$8 \longrightarrow W_8 = e^{\frac{2\pi i}{8}}$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \overline{a}$

$p(w_n^0), \ldots, p(w_n^{n-1})$ $\qquad y_i = p(w_n^i)$

$DFT(\overline{a}) = [y_0, \ldots, y_{n-1}]$



$DFT(\overline{a})$

$T(n) = 2 \cdot T(\frac{n}{2}) + c \cdot n$

16

$W_{16} = e^{\frac{2\pi i}{16}}$

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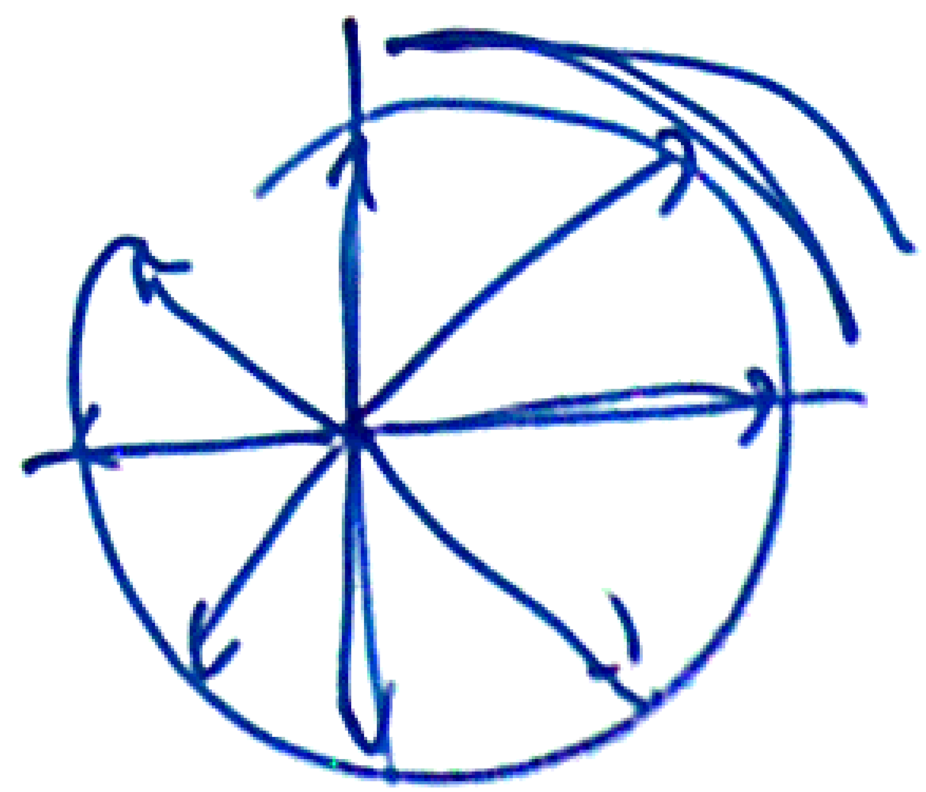
$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$

$y_i = p(w_n^i)$

$DFT(\bar{a}) = [y_0, \dots, y_{n-1}]$



$DFT(\bar{a})$

$T(n) = 2 \cdot T(\frac{n}{2}) + c \cdot n$

16

$w_{16} = e^{\frac{2\pi i}{16}}$

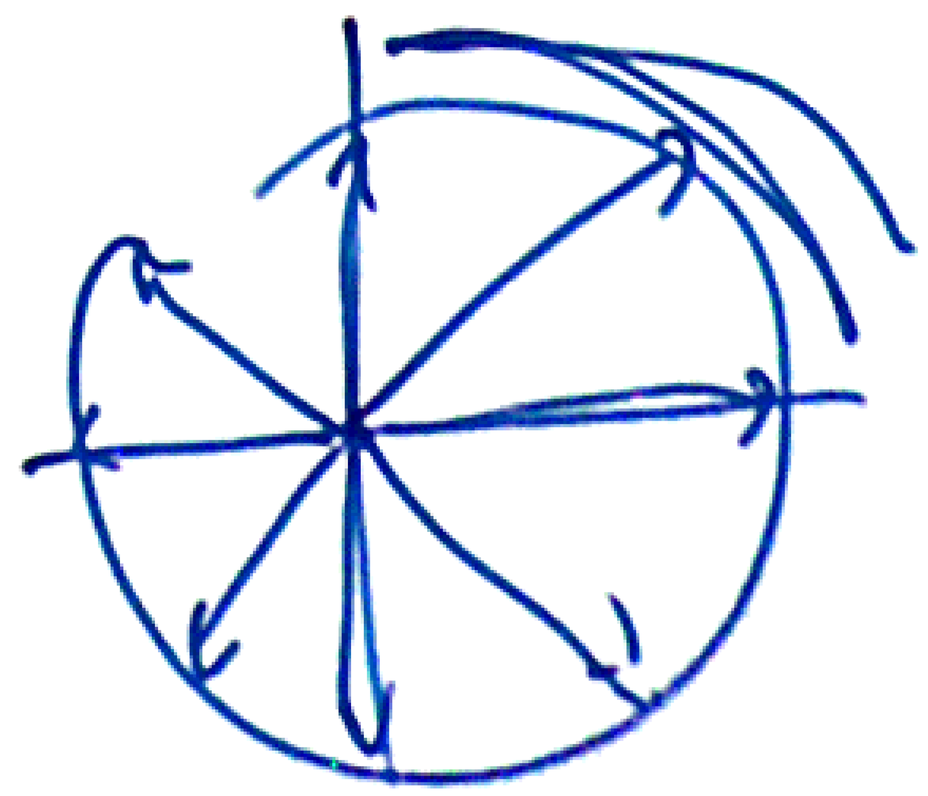
$8 \rightarrow w_8 = e^{\frac{2\pi i}{8}}$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \overline{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$

$y_i = p(w_n^i)$

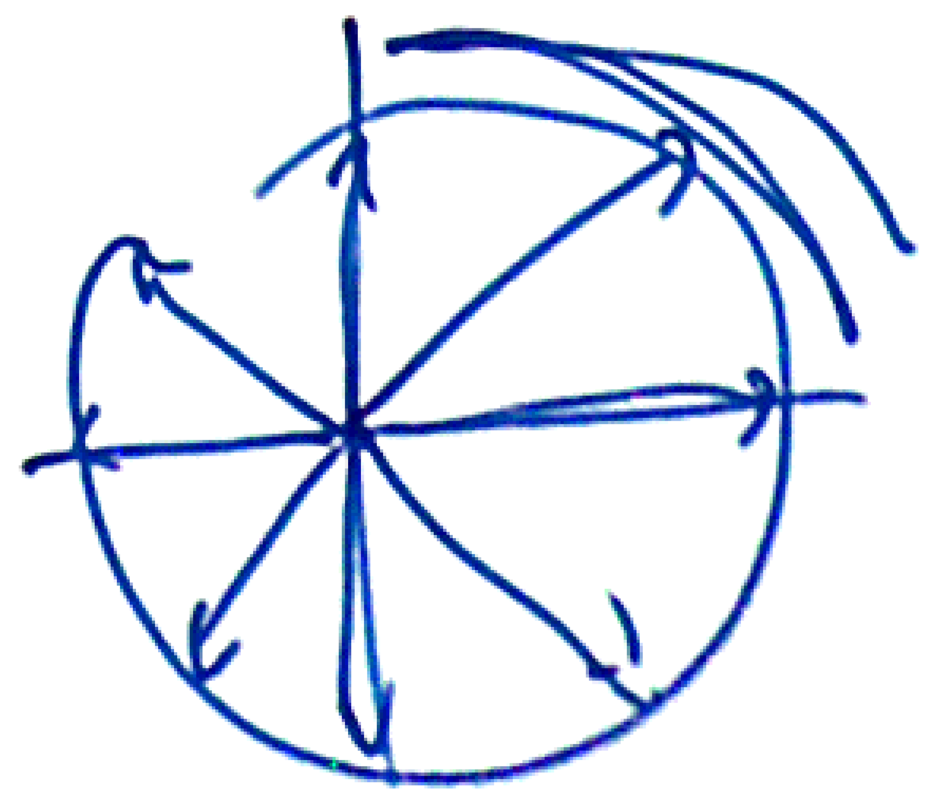
$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$



$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$

$p(w_n^0), \ldots, p(w_n^{n-1})$

$y_i = p(w_n^i)$



$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$$= \sum_{i=0}^{\frac{n}{2}-1} a_{2i} \cdot x^{2i} +$$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \overline{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$ $\qquad y_i = p(w_n^i)$

$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$$= \sum_{i=0}^{\frac{n}{2}-1} a_{2i} \cdot x^{2i} + \sum_{i=0}^{\frac{n}{2}-1} a_{2i+1} \cdot x^{2i+1}$$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \overline{a}$$

$$p(w_n^0), \dots, p(w_n^{n-1}) \qquad y_i = p(w_n^i)$$

$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$$= \sum_{i=0}^{\frac{n}{2}-1} a_{2i} \cdot x^{2i} + \sum_{i=0}^{\frac{n}{2}-1} a_{2i+1} \cdot x^{2i+1}$$

$$= \sum_{i=0}^{\frac{n}{2}-1} a_{2i} \cdot (x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} a_{2i+1} \cdot (x^2)^i$$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \overline{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$

$p(x) = q(x^2) + x \cdot r(x^2)$

$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$$= \sum_{i=0}^{\frac{n}{2}-1} a_{2i} \cdot x^{2i} + \sum_{i=0}^{\frac{n}{2}-1} a_{2i+1} \cdot x^{2i+1}$$

$$= \sum_{i=0}^{\frac{n}{2}-1} a_{2i} \cdot (x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} a_{2i+1} \cdot (x^2)^i$$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$

$p(w_n^0), \ldots, p(w_n^{n-1})$

$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$\frac{n}{2} - 1$

$p(x) = q(x^2) + x \cdot r(x^2)$

$DFT(\bar{a})$

$= [p(w_n^0), \ldots, p(w_n^{n-1})]$

$q \longrightarrow$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$

$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$\frac{n}{2} - 1$

$p(x) = q(x^2) + x \cdot r(x^2)$

$\mathrm{DFT}(\bar{a})$

$= [p(0), p(1), p(2)]$

$q \rightarrow q(0), q(1), q(4)$

$r \rightarrow r(0), r(1), r(4)$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$

$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$\frac{n}{2} - 1$

$p(x) = q(x^2) + x \cdot r(x^2)$

$\mathrm{DFT}(\bar{a})$

$= [p(w_n^0), \dots, p(w_n^{n-1})]$

$q \rightarrow q((w_n^0)^2), \dots, q((w_n^{n-1})^2)$

$r \rightarrow r((w_n^0)^2), \dots, r((w_n^{n-1})^2)$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$

$p(w_n^0), \ldots, p(w_n^{n-1})$

$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$\frac{n}{2} - 1$

$p(x) = q(x^2) + x \cdot r(x^2)$

$DFT(\bar{a})$

$= [p(w_n^0), \ldots, p(w_n^{n-1})]$

$q \rightarrow q((w_n^0)^2), \ldots, q((w_n^{n-1})^2)$

$r \rightarrow r((w_n^0)^2), \ldots, r((w_n^{n-1})^2)$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$

$$p(x) = \sum_{\substack{i=0 \\ n/2-1}}^{n-1} a_i x^i$$

$p(x) = q(x^2) + x \cdot r(x^2)$

$DFT(\bar{a})$

$= [p(w_n^0), \dots, p(w_n^{n-1})]$

$q \rightarrow q((w_n^0)^2), \dots, q((w_n^{n-1})^2)$

$r \rightarrow r((w_n^0)^2), \dots, r((w_n^{n-1})^2)$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$

$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$\frac{n}{2} - 1$

$p(x) = q(x^2) + x \cdot r(x^2)$

$\mathrm{DFT}(\bar{a})$

$= [p(w_n^0), \dots, p(w_n^{n-1})]$

$q \to q((w_n^0)^2), \dots, q((w_n^{n-1})^2)$

$r \to r((w_n^0)^2), \dots, r((w_n^{n-1})^2)$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$

$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$\frac{n}{2} - 1$

$p(x) = q(x^2) + x \cdot r(x^2)$

$\mathrm{DFT}(\bar{a})$

$= [p(w_n^0), \dots, p(w_n^{n-1})]$

$q \rightarrow q((w_n^0)^2), \dots, q((w_n^{n-1})^2)$

$r \rightarrow r((w_n^0)^2), \dots, r((w_n^{n-1})^2)$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$

$p(w_n^0), \ldots, p(w_n^{n-1})$

$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$\frac{n}{2} - 1$

$p(x) = q(x^2) + x \cdot r(x^2)$

$DFT(\bar{a})$

$= [p(w_n^0), \ldots, p(w_n^{n-1})]$

$q \rightarrow \quad q((w_n^0)^2), \ldots, q((w_n^{n-1})^2)$

$r \rightarrow \quad r((w_n^0)^2), \ldots, r((w_n^{n-1})^2)$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$p(w_n^0), \dots, p(w_n^{n-1})$

$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$\frac{n}{2} - 1$

$p(x) = q(x^2) + x \cdot r(x^2)$

$DFT(\bar{a})$

$= [p(w_n^0), \dots, p(w_n^{n-1})]$

$q \longrightarrow q((w_n^0)^2), \dots, q((w_n^{n-1})^2)$

$r \longrightarrow r((w_n^0)^2), \dots, r((w_n^{n-1})^2)$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$

$p(w_n^0), \ldots, p(w_n^{n-1})$

$$p(x) = \sum_{i=0}^{n-1} a_i x^i$$

$\frac{n}{2} - 1$

$p(x) = q(x^2) + x \cdot r(x^2)$

$\mathrm{DFT}(\bar{a})$

$= [p(w_n^0), \ldots, p(w_n^{n-1})]$

$q \rightarrow q((w_n^0)^2), \ldots, q((w_n^{n-1})^2)$

$r \rightarrow r((w_n^0)^2), \ldots, r((w_n^{n-1})^2)$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$n \to w_n^0, \dots, w_n^{n-1}$

$\frac{n}{2} \to w_{\frac{n}{2}}^0, w_{\frac{n}{2}}^1, \dots, w_{\frac{n}{2}}^{\frac{n}{2}-1}$

$p(x) = q(x^2) + x \cdot r(x^2)$

$DFT(\bar{a})$

$= [p(w_n^0), \dots, p(w_n^{n-1})]$

$q \longrightarrow q((w_n^0)^2), \dots, q((w_n^{n-1})^2)$

$r \longrightarrow r((w_n^0)^2), \dots, r((w_n^{n-1})^2)$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$

$p(x) = q(x^2) + x \cdot r(x^2)$

$n \to \omega_n^0, \ldots, \omega_n^{n-1}$

$\mathrm{DFT}(\bar{a})$

$\frac{n}{2} \to \omega_{\frac{n}{2}}^0, \omega_{\frac{n}{2}}^1, \ldots, \omega_{\frac{n}{2}}^{\frac{n}{2}-1}$

$= [p(\omega_n^0), \ldots, p(\omega_n^{n-1})]$

$\omega_n^0, \ldots, \omega_n^{\frac{n}{2}-1}, \omega_n^{\frac{n}{2}}, \ldots, \omega_n^{n-1}$

$q \to q((\omega_n^0)^2), \ldots, q((\omega_n^{n-1})^2)$

$r \to r((\omega_n^0)^2), \ldots, r((\omega_n^{n-1})^2)$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

$$n \to w_n^0, \dots, w_n^{n-1}$$

$$\frac{n}{2} \to w_{\frac{n}{2}}^0, w_{\frac{n}{2}}^1, \dots, w_{\frac{n}{2}}^{\frac{n}{2}-1}$$

$$\underbrace{w_n^0, \dots, w_n^{\frac{n}{2}-1}}_{\frac{n}{2}}, \underbrace{w_n^{\frac{n}{2}}, \dots, w_n^{n-1}}_{\frac{n}{2}}$$

$$p(x) = q(x^2) + x \cdot r(x^2)$$

$$DFT(\bar{a})$$

$$\left(w_n^{\frac{n}{2}+k}\right)^2 = w_n^{n+2k}$$

$$= \underbrace{\cancel{w_n^n}}_{1} \cdot \left(w_n^k\right)^2$$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

$$p(x) = q(x^2) + x \cdot r(x^2)$$

$$n \to \omega_n^0, \dots, \omega_n^{n-1}$$

$$\mathrm{DFT}(\bar{a})$$

$$\frac{n}{2} \to \omega_{\frac{n}{2}}^0, \omega_{\frac{n}{2}}^1, \dots, \omega_{\frac{n}{2}}^{\frac{n}{2}-1}$$

$$\left(\omega_n^{\frac{n}{2}+k}\right)^2 = \omega_n^{n+2k}$$

$$= \cancel{\omega_n^n}\left(\omega_n^k\right)^2$$

$$1$$

$$\underbrace{\omega_n^0, \dots, \omega_n^{\frac{n}{2}-1}}_{\frac{n}{2}}, \underbrace{\omega_n^{\frac{n}{2}}, \dots, \omega_n^{n-1}}_{\frac{n}{2}}$$

$$\left(\omega_n^k\right)^2$$

$$\omega_{m\cdot\cancel{\ell}}^{k\cdot\cancel{\ell}}$$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$p(x) = q(x^2) + x \cdot r(x^2)$

$n \to \omega_n^0, \dots, \omega_n^{n-1}$

$\mathrm{DFT}(\bar{a})$

$\frac{n}{2} \to \omega_{\frac{n}{2}}^0, \omega_{\frac{n}{2}}^1, \dots, \omega_{\frac{n}{2}}^{\frac{n}{2}-1}$

$$\left(\omega_n^{\frac{n}{2}+k}\right)^2 = \omega_n^{n+2k}$$

$$= \omega_n^{n} \left(\omega_n^k\right)^2$$

$$1$$

$$\underbrace{\omega_n^0, \dots, \omega_n^{\frac{n}{2}-1}}_{\frac{n}{2}}, \underbrace{\omega_n^{\frac{n}{2}}, \dots, \omega_n^{n-1}}_{\frac{n}{2}}$$

$$\left(\omega_n^k\right)^2 = \omega_{\frac{n}{2}\cdot\cancel{2}}^{2k}$$

$$\omega_{m\cdot\cancel{d}}^{k\cdot\cancel{d}}$$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

$$n \to \omega_n^0, \dots, \omega_n^{n-1}$$

$$\frac{n}{2} \to \omega_{\frac{n}{2}}^0, \omega_{\frac{n}{2}}^1, \dots, \omega_{\frac{n}{2}}^{\frac{n}{2}-1}$$

$$\underbrace{\omega_n^0, \dots, \omega_n^{\frac{n}{2}-1}}_{\frac{n}{2}}, \underbrace{\omega_n^{\frac{n}{2}}, \dots, \omega_n^{n-1}}_{\frac{n}{2}}$$

$$p(x) = q(x^2) + x \cdot r(x^2)$$

$$DFT(\bar{a})$$

$$\left(\omega_n^{\frac{n}{2}+k}\right)^2 = \omega_n^{n+2k}$$

$$= \cancel{\omega_n^n} \left(\omega_n^k\right)^2$$

$$1$$

$$\left(\omega_n^k\right)^2 = \omega_{\frac{n}{2} \cdot \cancel{2}}^{\cancel{2}k}$$

$$\omega_{m \cdot \cancel{\ell}}^{k \cdot \cancel{\ell}}$$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$

$p(x) = q(x^2) + x \cdot r(x^2)$

$n \rightarrow w_n^0, \ldots, w_n^{n-1}$

$DFT(\bar{a})$

$\frac{n}{2} \rightarrow w_{\frac{n}{2}}^0, w_{\frac{n}{2}}^1, \ldots, w_{\frac{n}{2}}^{\frac{n}{2}-1}$

$\left(w_n^{\frac{n}{2}+k}\right)^2 = w_n^{n+2k}$

$= \cancel{w_n^n} \left(w_n^k\right)^2$

1

$\underbrace{w_n^0, \ldots, w_n^{\frac{n}{2}-1}}_{\frac{n}{2}}, \underbrace{w_n^{\frac{n}{2}}, \ldots, w_n^{n-1}}_{\frac{n}{2}}$

$\left(w_n^k\right)^2 = w_{\frac{n}{2} \cdot \cancel{2}}^{\cancel{2}k}$

$w_{m \cdot \cancel{l}}^{k \cdot \cancel{l}}$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

$$n \to \omega_n^0, \dots, \omega_n^{n-1}$$

$$\frac{n}{2} \to \omega_{\frac{n}{2}}^0, \omega_{\frac{n}{2}}^1, \dots, \omega_{\frac{n}{2}}^{\frac{n}{2}-1}$$

$$\underbrace{\omega_n^0, \dots, \omega_n^{\frac{n}{2}-1}}_{\frac{n}{2}}, \underbrace{\omega_n^{\frac{n}{2}}, \dots, \omega_n^{n-1}}_{\frac{n}{2}}$$

$$p(x) = q(x^2) + x \cdot r(x^2)$$

$$DFT(\bar{a}) = [y_0, \dots, y_{n-1}]$$

$$DFT(\bar{R}_0) = [$$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

$$n \rightarrow \omega_n^0, \dots, \omega_n^{n-1}$$

$$\frac{n}{2} \rightarrow \omega_{\frac{n}{2}}^0, \omega_{\frac{n}{2}}^1, \dots, \omega_{\frac{n}{2}}^{\frac{n}{2}-1}$$

$$\underbrace{\omega_n^0, \dots, \omega_n^{\frac{n}{2}-1}}_{\frac{n}{2}}, \underbrace{\omega_n^{\frac{n}{2}}, \dots, \omega_n^{n-1}}_{\frac{n}{2}}$$

$$p(x) = q(x^2) + x \cdot r(x^2)$$

$$DFT(\bar{a}) = [y_0, \dots, y_{n-1}]$$

$$DFT(\bar{a}_0) = [u_0, \dots, u_{\frac{n}{2}-1}]$$

$$DFT(\bar{a}_1) = [v_0, \dots, v_{\frac{n}{2}-1}]$$

$$y_0 =$$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

$$n \rightarrow \omega_n^0, \dots, \omega_n^{n-1}$$

$$\frac{n}{2} \rightarrow \omega_{\frac{n}{2}}^0, \omega_{\frac{n}{2}}^1, \dots, \omega_{\frac{n}{2}}^{\frac{n}{2}-1}$$

$$\underbrace{\omega_n^0, \dots, \omega_n^{\frac{n}{2}-1}}_{\frac{n}{2}}, \underbrace{\omega_n^{\frac{n}{2}}, \dots, \omega_n^{n-1}}_{\frac{n}{2}}$$

$$\boxed{p(x) = q(x^2) + x \cdot r(x^2)}$$

$$DFT(\bar{a}) = [y_0, \dots, y_{n-1}]$$

$$DFT(\bar{a}_0) = [u_0, \dots, u_{\frac{n}{2}-1}]$$

$$DFT(\bar{a}_1) = [v_0, \dots, v_{\frac{n}{2}-1}]$$

$$y_0 =$$

$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$

$n \rightarrow w_n^0, \ldots, w_n^{n-1}$

$\frac{n}{2} \rightarrow w_{\frac{n}{2}}^0, w_{\frac{n}{2}}^1, \ldots, w_{\frac{n}{2}}^{\frac{n}{2}-1}$

$\underbrace{w_n^0, \ldots, w_n^{\frac{n}{2}-1}}_{\frac{n}{2}}, \underbrace{w_n^{\frac{n}{2}}, \ldots, w_n^{n-1}}_{\frac{n}{2}}$

$$\boxed{p(x) = q(x^2) + x \cdot r(x^2)}$$

$DFT(\bar{a}) = [y_0, \ldots, y_{n-1}]$

$DFT(\bar{a}_0) = [u_0, \ldots, u_{\frac{n}{2}-1}]$

$DFT(\bar{a}_1) = [v_0, \ldots, v_{\frac{n}{2}-1}]$

$p($

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

$$\boxed{p(x) = q(x^2) + x \cdot r(x^2)}$$

$$n \to W_n^0, \dots, W_n^{n-1}$$

$$\text{DFT}(\bar{a}) = [y_0, \dots, y_{n-1}]$$

$$\frac{n}{2} \to W_{\frac{n}{2}}^0, W_{\frac{n}{2}}^1, \dots, W_{\frac{n}{2}}^{\frac{n}{2}-1}$$

$$\text{DFT}(\bar{a}_0) = [u_0, \dots, u_{\frac{n}{2}-1}]$$

$$\text{DFT}(\bar{a}_1) = [v_0, \dots, v_{\frac{n}{2}-1}]$$

$$\underbrace{W_n^0, \dots, W_n^{\frac{n}{2}-1}}_{\frac{n}{2}}, \underbrace{W_n^{\frac{n}{2}}, \dots, W_n^{n-1}}_{\frac{n}{2}}$$

$$y_0 = p(W_n^0) = q((W_n^0)^2) + W_n^0 \cdot r((W_n^0)^2)$$

$$= q($$

$$(W_n^0)^2 = W_2^{2 \cdot 0}$$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

$$\boxed{p(x) = q(x^2) + x \cdot r(x^2)}$$

$$n \to \omega_n^0, \dots, \omega_n^{n-1}$$

$$DFT(\bar{a}) = [y_0, \dots, y_{n-1}]$$

$$\frac{n}{2} \to \omega_{\frac{n}{2}}^0, \omega_{\frac{n}{2}}^1, \dots, \omega_{\frac{n}{2}}^{\frac{n}{2}-1}$$

$$DFT(\bar{a}_0) = [u_0, \dots, u_{\frac{n}{2}-1}]$$

$$DFT(\bar{a}_1) = [v_0, \dots, v_{\frac{n}{2}-1}]$$

$$\underbrace{\omega_n^0, \dots, \omega_n^{\frac{n}{2}-1}}_{\frac{n}{2}}, \underbrace{\omega_n^{\frac{n}{2}}, \dots, \omega_n^{n-1}}_{\frac{n}{2}}$$

$$y_0 = p(\omega_n^0) = q((\omega_n^0)^2) + \omega_n^0 \cdot r((\omega_n^0)^2)$$

$$(\omega_n^0)^2 = \omega_{\frac{n}{2}}^{\cancel{2} \cdot 0}$$

$$= q(\omega_{\frac{n}{2}}^0)$$

$$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$$

$$\boxed{p(x) = q(x^2) + x \cdot r(x^2)}$$

$$n \to \omega_n^0, \ldots, \omega_n^{n-1}$$

$$\frac{n}{2} \to \omega_{\frac{n}{2}}^0, \omega_{\frac{n}{2}}^1, \ldots, \omega_{\frac{n}{2}}^{\frac{n}{2}-1}$$

$$\underbrace{\omega_n^0, \ldots, \omega_n^{\frac{n}{2}-1}}_{\frac{n}{2}}, \underbrace{\omega_n^{\frac{n}{2}}, \ldots, \omega_n^{n-1}}_{\frac{n}{2}}$$

$$DFT(\bar{a}) = [y_0, \ldots, y_{n-1}]$$

$$DFT(\bar{a}_0) = [u_0, \ldots, u_{\frac{n}{2}-1}]$$

$$DFT(\bar{a}_1) = [v_0, \ldots, v_{\frac{n}{2}-1}]$$

$$y_0 = p(\omega_n^0) = q((\omega_n^0)^2) + \omega_n^0 \cdot r((\omega_n^0)^2)$$

$$= q(\omega_{\frac{n}{2}}^0) + \omega_n^0 \cdot r(\omega_{\frac{n}{2}}^0)$$

$$= u_0 + \omega_n^0 \cdot v_0$$

$$(\omega_n^0)^2 = \omega^{\cancel{2} \cdot 0}_{\cancel{2} \cdot \frac{n}{2}}$$

$$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$$

$$n \rightarrow \omega_n^0, \ldots, \omega_n^{n-1}$$

$$\omega_{m\cdot l}^{k\cdot l} = \left(e^{\frac{2\pi i}{m\cdot l}}\right)^{k\cdot l}$$

$$= e^{\frac{2\pi i}{\cancel{m}\cdot l}\cdot k\cdot \cancel{l}}$$

$$= \left(e^{\frac{2\pi i}{l}}\right)^{k}$$

$$= \omega$$

$$\boxed{p(x) = q(x^2) + x\cdot r(x^2)}$$

$$DFT(\bar{a}) = [y_0, \ldots, y_{n-1}]$$

$$DFT(\bar{a}_0) = [u_0, \ldots, u_{\frac{n}{2}-1}]$$

$$DFT(\bar{a}_1) = [v_0, \ldots, v_{\frac{n}{2}-1}]$$

$$y_0 = p(\omega_n^0) = q((\omega_n^0)^2) + \omega_n^0 \cdot r((\omega_n^0)^2)$$

$$\omega_{\cancel{2}\cdot\frac{n}{2}}^{\cancel{2}\cdot 0}$$

$$= q(\omega_{\frac{n}{2}}^0) + \omega_n^0 \cdot r(\omega_{\frac{n}{2}}^0)$$

$$= u_0 + \omega_n^0 \cdot v_0$$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$n \rightarrow W_n^0, \dots, W_n^{n-1}$

$$W_{m \cdot l}^{k \cdot l} = \left(e^{\frac{2\pi i}{m \cdot l}}\right)^{k \cdot l}$$

$$= e^{\frac{2\pi i}{m \cdot \cancel{l}} \cdot k \cdot \cancel{l}}$$

$$= \left(e^{\frac{2\pi i}{m}}\right)^{k}$$

$$= W_m^k$$

$$W_{\cancel{2} \cdot \frac{n}{2}}^{\cancel{2} \cdot 0}$$

$$\boxed{p(x) = q(x^2) + x \cdot r(x^2)}$$

$DFT(\bar{a}) = [y_0, \dots, y_{n-1}]$

$DFT(\bar{a}_0) = [u_0, \dots, u_{\frac{n}{2}-1}]$

$DFT(\bar{a}_1) = [v_0, \dots, v_{\frac{n}{2}-1}]$

$$y_0 = p(W_n^0) = q((W_n^0)^2) + W_n^0 \cdot r((W_n^0)^2)$$

$$= q(W_{\frac{n}{2}}^0) + W_n^0 \cdot r(W_{\frac{n}{2}}^0)$$

$$= u_0 + W_n^0 \cdot v_0$$

$$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$$

$$n \rightarrow \omega_n^0, \ldots, \omega_n^{n-1}$$

$$\omega_{m\cdot l}^{k\cdot l} = \left(e^{\frac{2\pi i}{m\cdot l}}\right)^{k\cdot l} = e^{\frac{2\pi i}{m\cdot \cancel{l}}\cdot k\cdot \cancel{l}} = \left(e^{\frac{2\pi i}{m}}\right)^{k} = \omega_m^k$$

$$\boxed{p(x) = q(x^2) + x\cdot r(x^2)}$$

$$DFT(\bar{a}) = [y_0, \ldots, y_{n-1}]$$

$$DFT(\bar{a}_0) = [u_0, \ldots, u_{\frac{n}{2}-1}]$$

$$DFT(\bar{a}_1) = [v_0, \ldots, v_{\frac{n}{2}-1}]$$

$$y_0 = p(\omega_n^0) = q((\omega_n^0)^2) + \omega_n^0 \cdot r((\omega_n^0)^2)$$

$$\omega_{\cancel{2}\cdot\frac{n}{2}}^{\cancel{2}\cdot 0}$$

$$= q(\omega_{\frac{n}{2}}^0) + \omega_n^0 \cdot r(\omega_{\frac{n}{2}}^0)$$

$$= u_0 + \omega_n^0 \cdot v_0$$

$$p(x) \longrightarrow (a_0, \ldots, a_{n-1}) = \bar{a}$$

$$\boxed{p(x) = q(x^2) + x \cdot r(x^2)}$$

$$n \to \omega_n^0, \ldots, \omega_n^{n-1}$$

$$DFT(\bar{a}) = [y_0, \ldots, y_{n-1}]$$

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$$DFT(\bar{a}_1) = [v_0, \ldots, v_{\frac{n}{2}-1}]$$

$$\omega_{m \cdot \ell}^{k \cdot \ell} = \left(e^{\frac{2\pi i}{m \cdot \ell}}\right)^{k \cdot \ell} = e^{\frac{2\pi i}{m \cdot \cancel{\ell}} \cdot k \cdot \cancel{\ell}} = \left(e^{\frac{2\pi i}{m}}\right)^{k} = \omega_m^k$$

$$y_0 = p(\omega_n^0) = q((\omega_n^0)^2) + \omega_n^0 \cdot r((\omega_n^0)^2)$$

$$\omega_{\cancel{2} \cdot \frac{n}{2}}^{\cancel{2} \cdot 0}$$

$$= q(\omega_{\frac{n}{2}}^0) + \omega_n^0 \cdot r(\omega_{\frac{n}{2}}^0)$$

$$= u_0 + \omega_n^0 \cdot v_0$$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

$$n \rightarrow w_n^0, \dots, w_n^{n-1}$$

$$w_{m\cdot l}^{k\cdot l} = \left(e^{\frac{2\pi i}{m\cdot l}}\right)^{k\cdot l} = e^{\frac{2\pi i}{m\cdot \cancel{l}}\cdot k\cdot \cancel{l}} = \left(e^{\frac{2\pi i}{m}}\right)^{k} = w_m^k$$

$$w_{\cancel{2}\cdot\frac{n}{2}}^{\cancel{2}\cdot 0}$$

$$\boxed{p(x) = q(x^2) + x\cdot r(x^2)}$$

$$DFT(\bar{a}) = [y_0, \dots, y_{n-1}]$$

$$DFT(\bar{a}_0) = [u_0, \dots, u_{\frac{n}{2}-1}]$$

$$DFT(\bar{a}_1) = [v_0, \dots, v_{\frac{n}{2}-1}]$$

$$y_0 = p(w_n^0) = q((w_n^0)^2) + w_n^0 \cdot r((w_n^0)^2)$$

$$= q(w_{\frac{n}{2}}^0) + w_n^0 \cdot r(w_{\frac{n}{2}}^0)$$

$$= u_0 + w_n^0 \cdot v_0$$

$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$

$n \to \omega_n^0, \dots, \omega_n^{n-1}$

$$\omega_{m \cdot l}^{k \cdot l} = \left(e^{\frac{2\pi i}{m \cdot l}}\right)^{k \cdot l} = e^{\frac{2\pi i}{m \cdot \cancel{l}} \cdot k \cdot \cancel{l}} = \left(e^{\frac{2\pi i}{m}}\right)^{k} = \omega_m^k$$

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$$y_0 = p(\omega_n^0) = q((\omega_n^0)^2) + \omega_n^0 \cdot r((\omega_n^0)^2)$$

$$\omega_{\cancel{2} \cdot \frac{n}{2}}^{\cancel{2} \cdot 0}$$

$$= q(\omega_{\frac{n}{2}}^0) + \omega_n^0 \cdot r(\omega_{\frac{n}{2}}^0)$$

$$= u_0 + \omega_n^0 \cdot v_0$$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

$$n \to W_n^0, \dots, W_n^{n-1}$$

$$W_{m\cdot l}^{k\cdot l} = \left(e^{\frac{2\pi i}{m\cdot l}}\right)^{k\cdot l}$$

$$= e^{\frac{2\pi i}{m\cdot \cancel{l}}\cdot k\cdot \cancel{l}}$$

$$= \left(e^{\frac{2\pi i}{m}}\right)^{k}$$

$$= W_m^k$$

$$\boxed{p(x) = q(x^2) + x\cdot r(x^2)}$$

$$DFT(\bar{a}) = [y_0, \dots, y_{n-1}]$$

$$DFT(\bar{a}_0) = [u_0, \dots, u_{\frac{n}{2}-1}]$$

$$DFT(\bar{a}_1) = [v_0, \dots, v_{\frac{n}{2}-1}]$$

$$y_0 = p(W_n^0) = q((W_n^0)^2) + W_n^0 \cdot r((W_n^0)^2)$$

$$W_{\cancel{2}\cdot\frac{n}{2}}^{\cancel{2}\cdot 0}$$

$$= q(W_{\frac{n}{2}}^0) + W_n^0 \cdot r(W_{\frac{n}{2}}^0)$$

$$= u_0 + W_n^0 \cdot v_0$$

$$p(x) \longrightarrow (a_0, \dots, a_{n-1}) = \bar{a}$$

$$n \to w_n^0, \dots, w_n^{n-1}$$

$$w_{m\cdot l}^{k\cdot l} = \left(e^{\frac{2\pi i}{m\cdot l}}\right)^{k\cdot l} = e^{\frac{2\pi i}{m\cdot \cancel{l}}\cdot k\cdot \cancel{l}} = \left(e^{\frac{2\pi i}{m}}\right)^{k} = w_m^k$$

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$$y_0 = p(w_n^0) = q((w_n^0)^2) + w_n^0 \cdot r((w_n^0)^2)$$

$$w_{\cancel{2}\cdot\frac{n}{2}}^{\cancel{2}\cdot 0}$$

$$= q(w_{\frac{n}{2}}^0) + w_n^0 \cdot r(w_{\frac{n}{2}}^0)$$

$$= u_0 + w_n^0 \cdot v_0$$