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$$n = 234421733$$

$$n-1 \quad O(n)$$

$$\text{degree } f \approx \log_{10}(n)$$

$$n \approx 10^{\log n}$$

$$O(10^{\log n})$$

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$x \cdot x \dots x$

$$(\quad)(\quad)(\quad)$$

$$(x+3)(2x+4) + \dots$$

$$P(x) = \sum_{i=1}^k \prod_{j=1}^{l_i} (a_{i,j}x + b_{i,j})$$

$$\begin{aligned} & 4 + 6 + 8 + \dots + 2(n-1) \\ & = 2 \cdot (2 + 3 + 4 + \dots + (n-1)) \\ & x \cdot x \dots x \end{aligned}$$

$$(x+1)(x+2)(x+3)\dots(x+n)$$

$$(x^2+2x+2)$$

1

$$(x+3)(2x+4) + \dots$$

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$$(x+3)(2x+4) + \dots$$

$$p(x) =$$

$$q(x)$$

$$\therefore p(a) = q(a)$$

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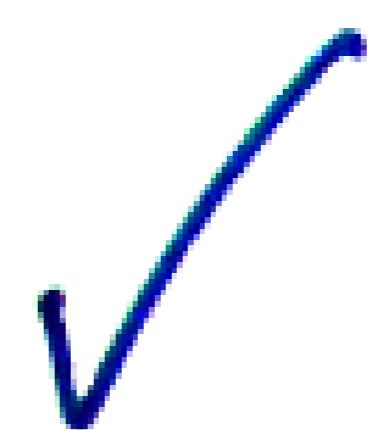
$$(x+3)(2x+4) + -$$

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$p(x), q(x)$  non equivalents



$p(x), q(x)$  no non equivalents

Pr<sub>a</sub>

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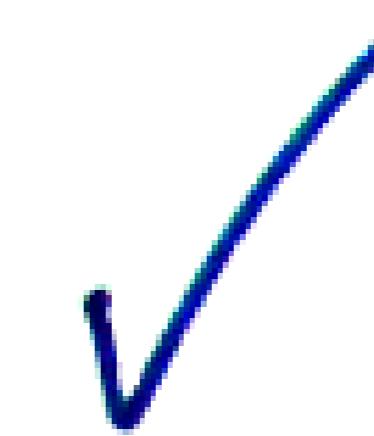
$$p(x) =$$

$$q(x)$$

$$p(a) = q(a)$$

$$r(x) = p(x) - q(x)$$

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$$\Pr_a (p(a) = q(a))$$

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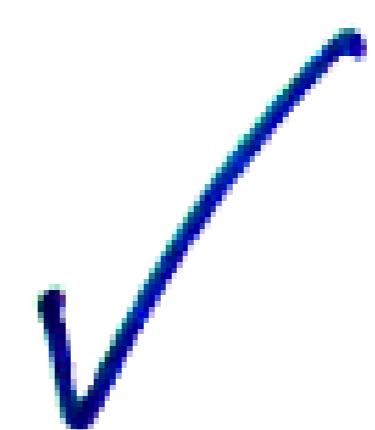
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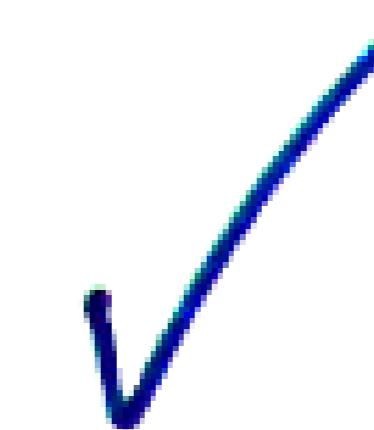
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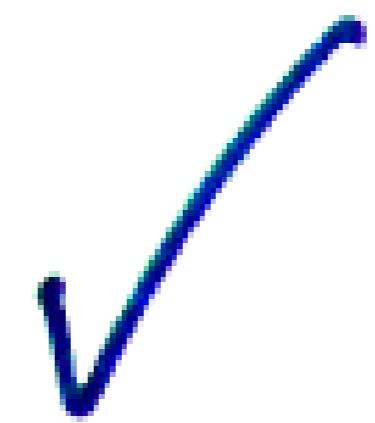
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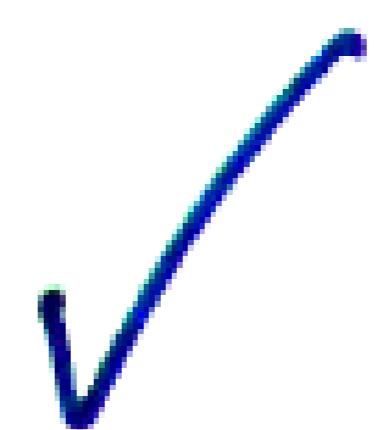
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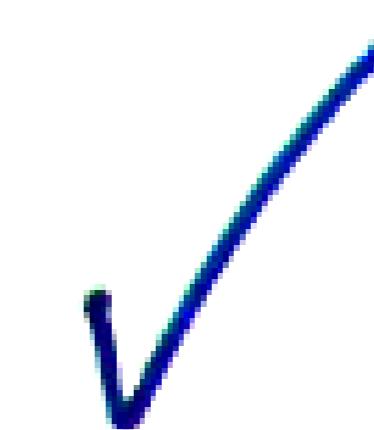
$$q(x)$$

$$p(a) = q(a)$$

$$r(x) = p(x) - q(x)$$

$$\frac{1}{10^{100}}$$

$p(x), q(x)$  non equivalents



$p(x), q(x)$  no non equivalents

$$\Pr_a (p(a) = q(a))$$

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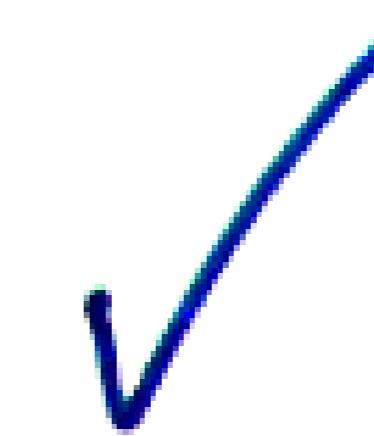
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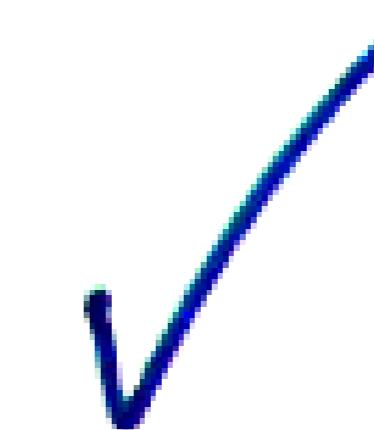
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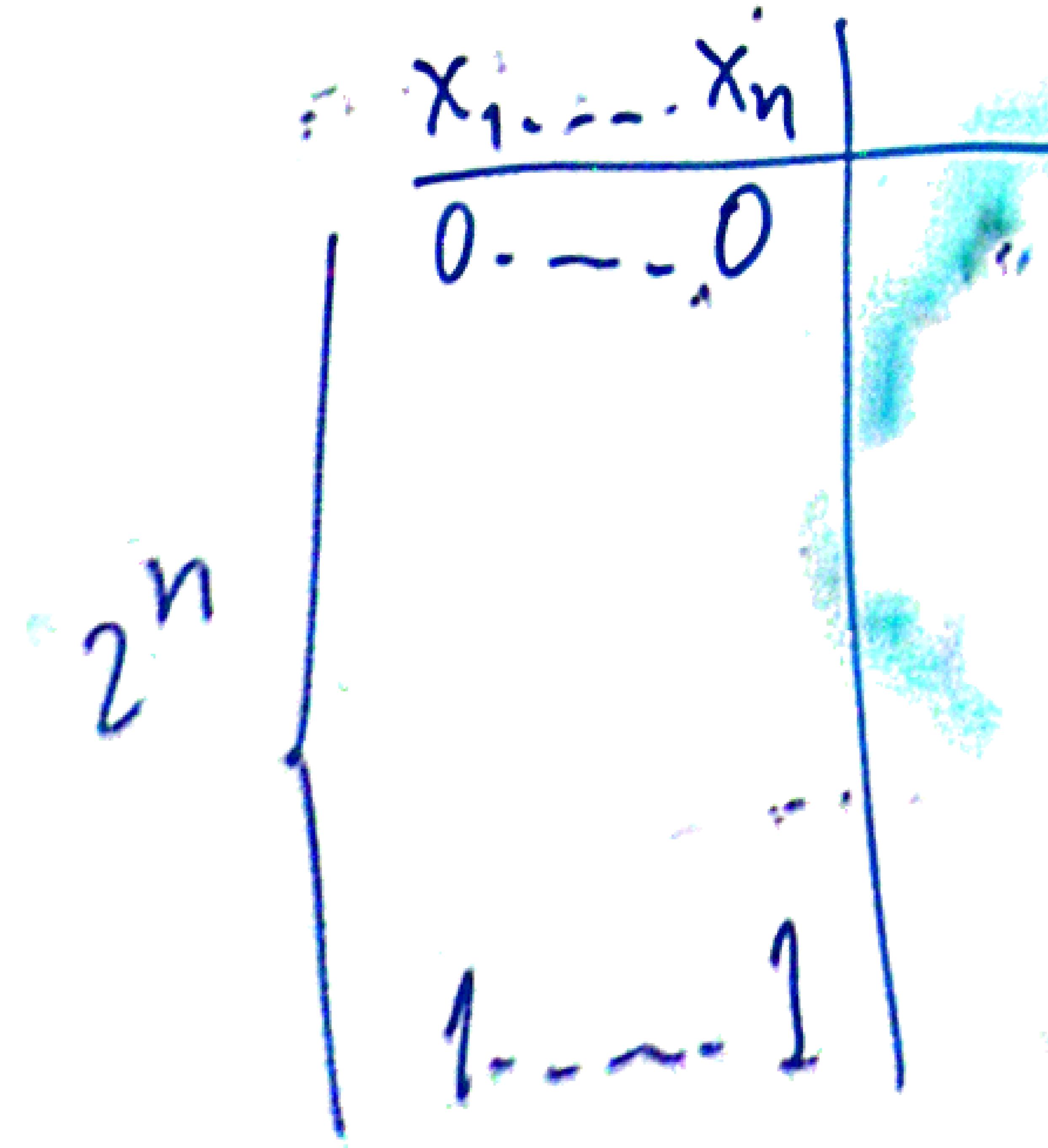
$p(x), q(x)$  non equivalents



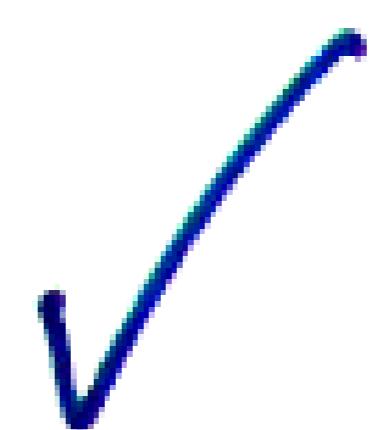
$p(x), q(x)$  no non equivalents

$$\Pr_a (p(a) = q(a))$$

$$(x+3)(2x+4) + \dots$$



$p(x), q(x)$  non-equivalents



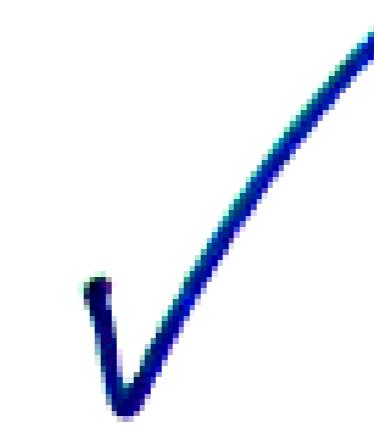
$p(x), q(x)$  no non-equivalents

$$\Pr_a (p(a) = q(a))$$

$$(x+3)(2x+4) + \dots$$

$x_1, \dots, x_n$	$0, \dots, 0$	$0$
$1, \dots, 1$	$0$	$0$

$p(x), q(x)$  non equivalents



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$$\Pr_a (p(a) = q(a))$$

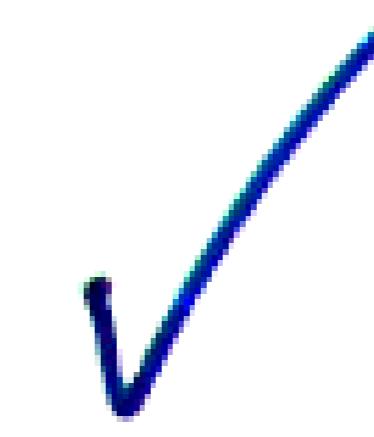
$$(x+3)(2x+4) + \dots$$

$x_1, \dots, x_n$	$0, \dots, 0$	$0, \dots, 0$
$2^n$	$\vdots$	$\vdots$
$1, \dots, 1$	$0$	$0$

$$2^{2^n}$$

$$\frac{1}{2^{2^n}}$$

$p(x), q(x)$  non equivalents



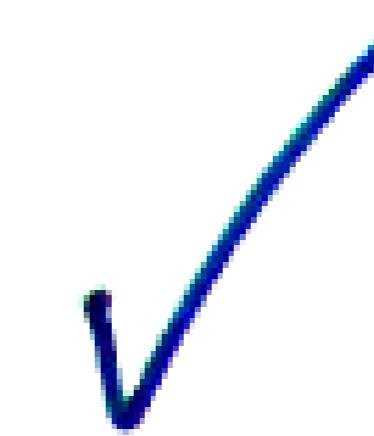
$p(x), q(x)$  no non equivalents

$$\Pr_a (p(a) = q(a))$$

$$(x+3)(2x+4) + \dots$$

$x_1, \dots, x_n$	$0, \dots, 0$	$0 \rightarrow 2$
		$1 \rightarrow 2$
$1, \dots, 1   0$		
		$\sqcup$
		$\{ 1, 2 \}^{\# \text{filters}} = 2^n$

$p(x), q(x)$  non equivalents



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$$\Pr_a (p(a) = q(a))$$

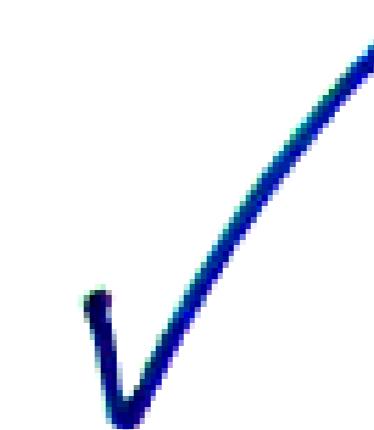
$$(x+3)(2x+4) + \dots$$

$x_1, \dots, x_n$	
0, ..., 0	$0 \rightarrow 2$
1, ..., 1	$1 \rightarrow 2$

$2^n$  {

$$\bigcup_{\ell=1}^{\# \text{filters}} = 2^{2^n}$$

$p(x), q(x)$  non equivalents

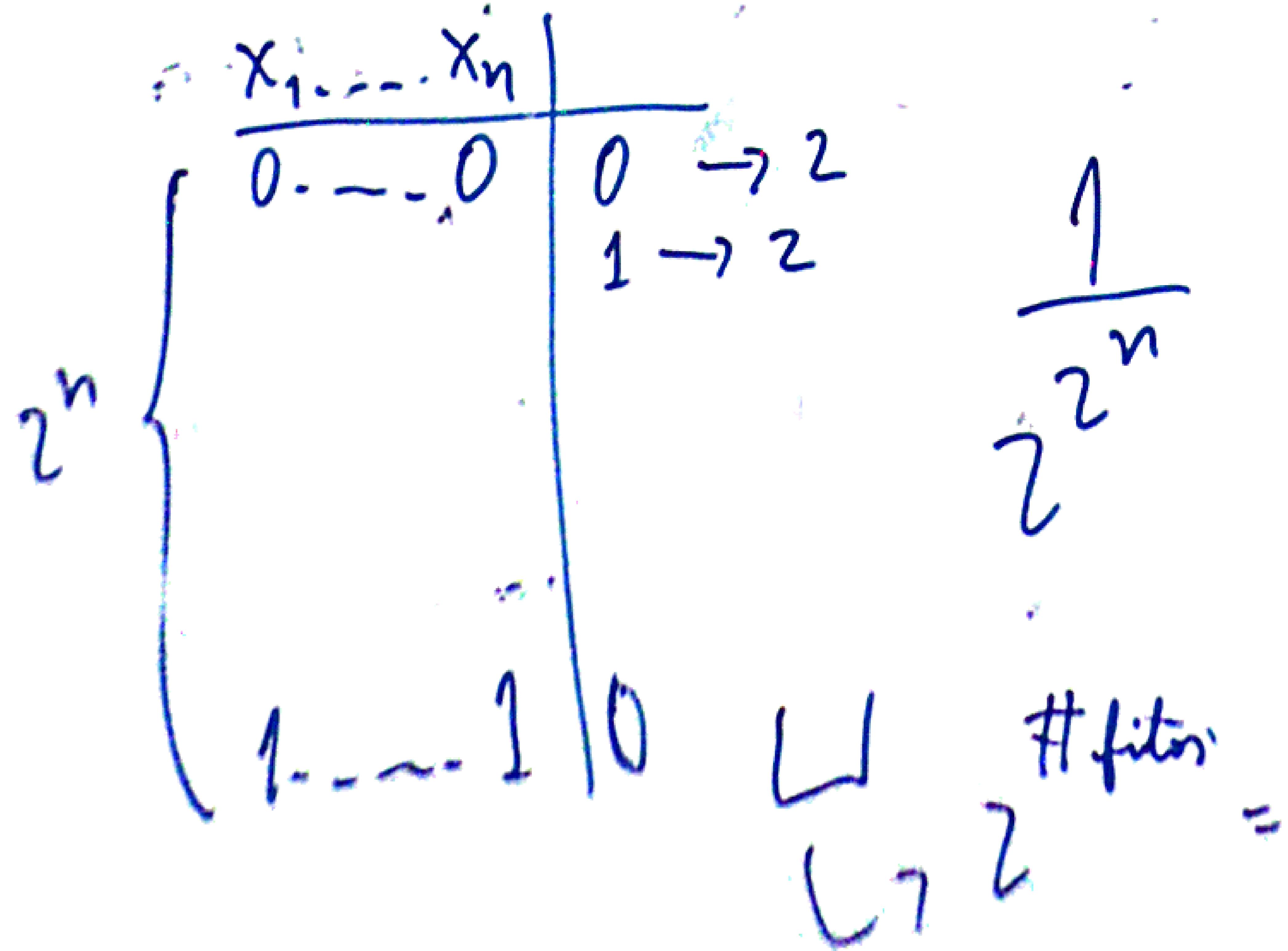


$p(x), q(x)$  no non equivalents

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$$(x+3)(2x+4) + \dots$$

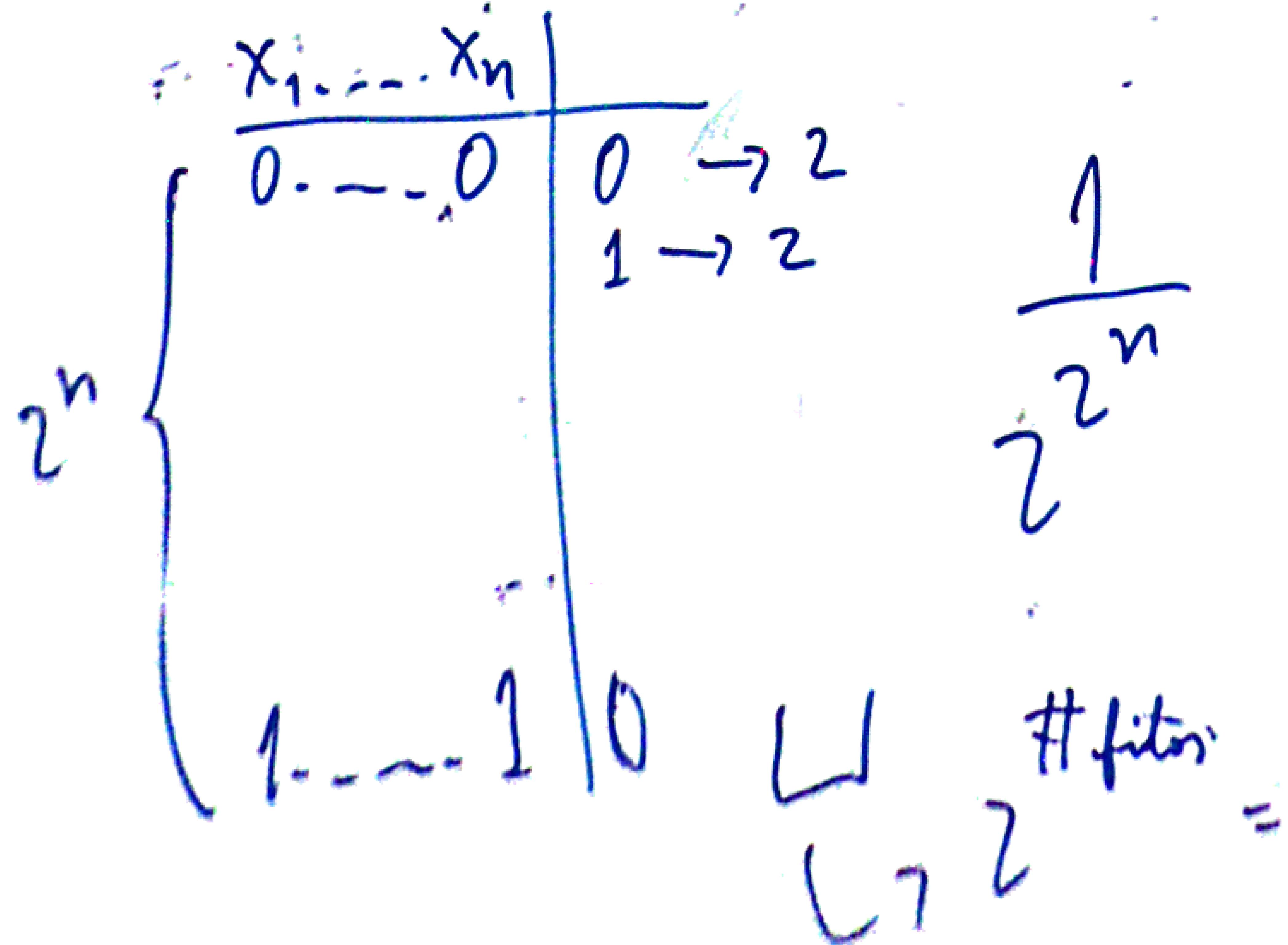
0, 1



100K

$\log(100K)$

$$(x+3)(2x+4) + \dots$$



0, 1

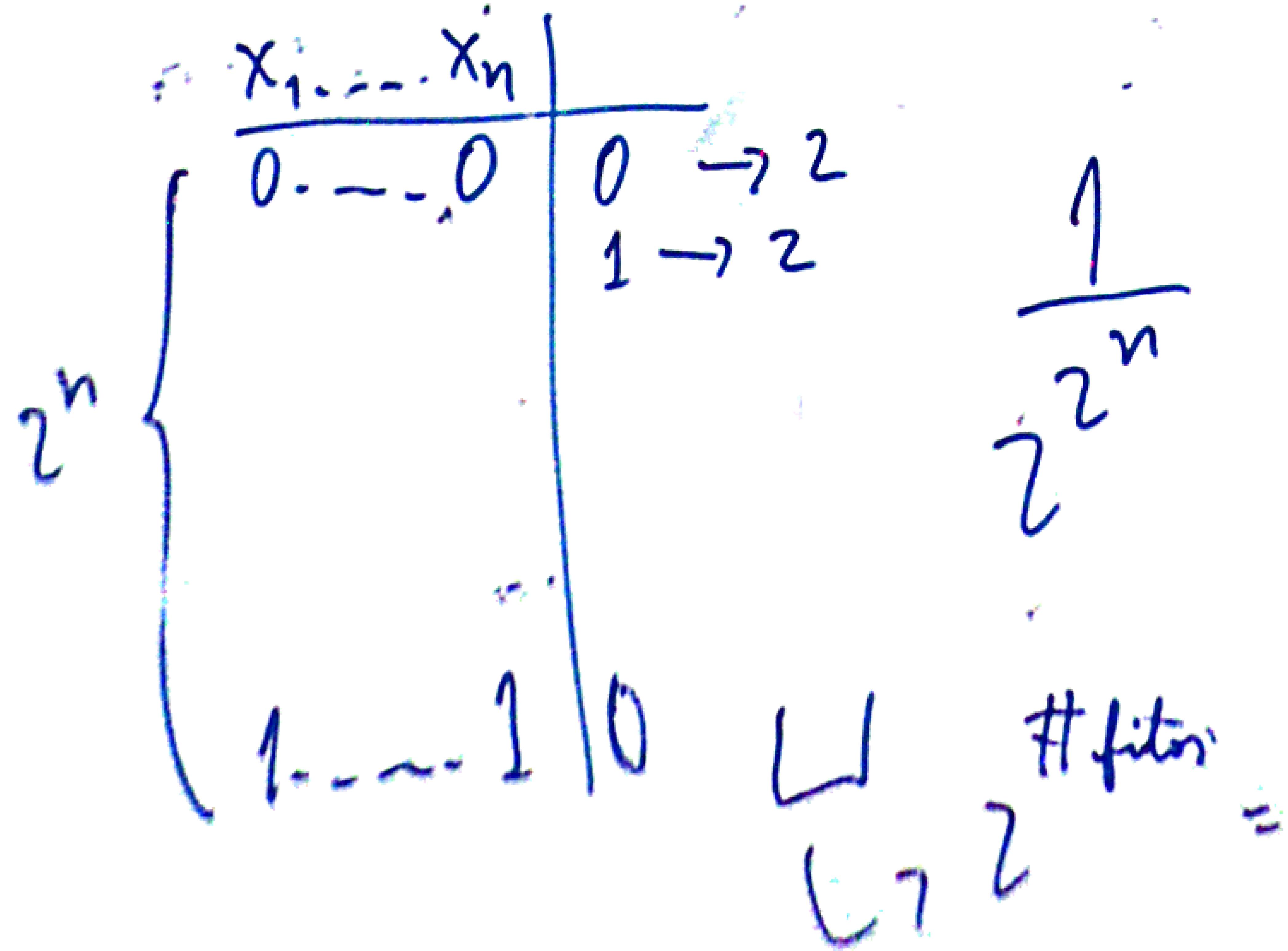
1

0 ↗  
n

100K

$\log(100K)$

$$(x+3)(2x+4) + \dots$$

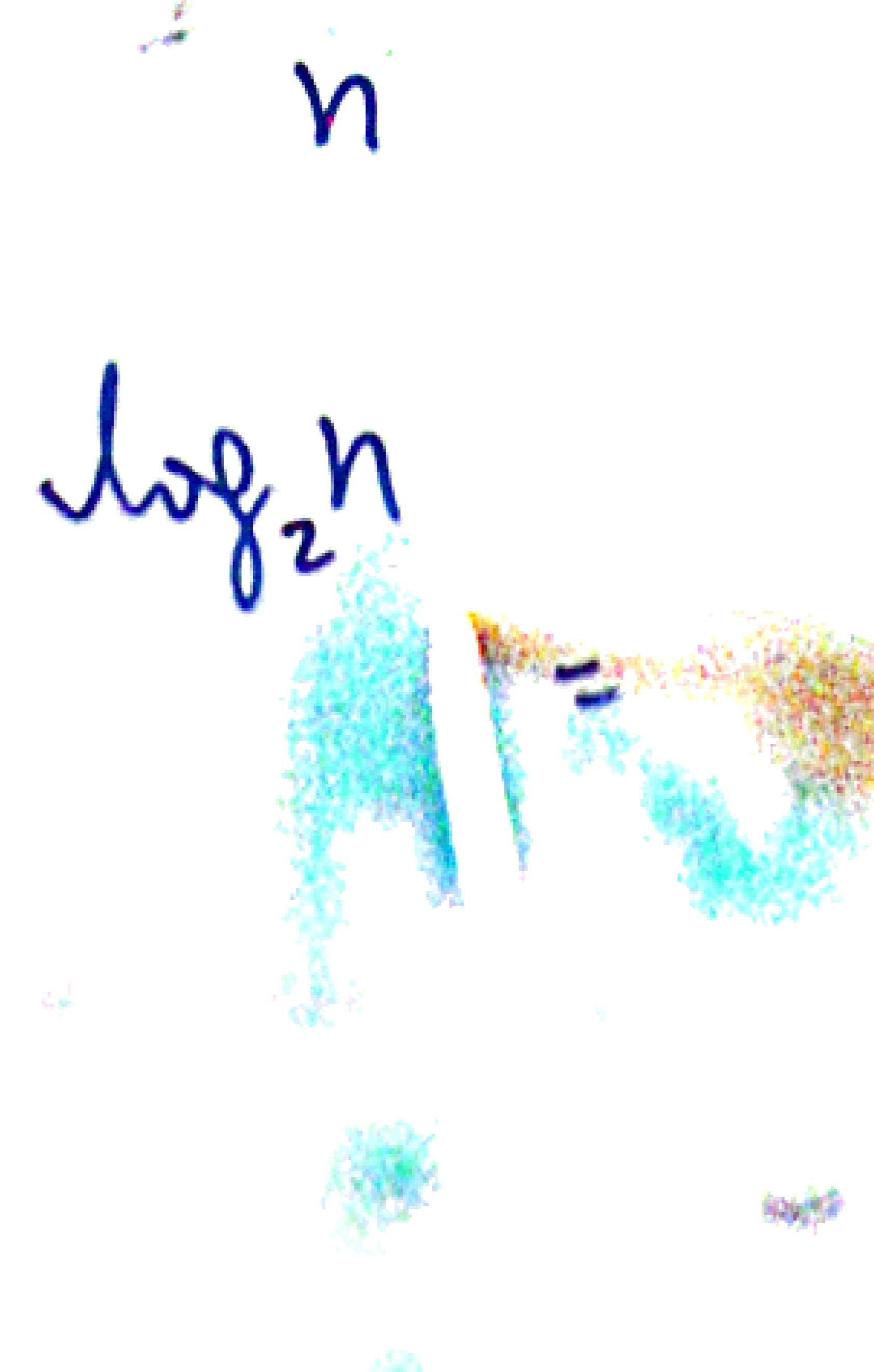


100K

$\log(100K)$

$$(x+3)(2x+4) + \dots$$

$x_1, \dots, x_n$	
0, ..., 0	$0 \rightarrow 2$
1, ..., 1	$1 \rightarrow 2$



0, 1

1  
-  
-  
-  
0  
1

100K

$\log(100K)$

$$(x+3)(2x+4) + \dots$$

$x_1, \dots, x_n$	
0, ..., 0	$0 \rightarrow 2$
1, ..., 0	$1 \rightarrow 2$
1, ..., 1	0

$$\log_2 n =$$

0, 1

1  
0  
1

100K

$\log(100K)$

$$(x+3)(2x+4) + \dots$$

$x_1, \dots, x_n$	
0, ..., 0	$0 \rightarrow 2$
1, ..., 1	$1 \rightarrow 2$

$$\log_2 n$$

n

0, 1

1  
0  
1

100K

$\log(100K)$

$$(x+3)(2x+4) + \dots$$

$x_1, \dots, x_n$	
0, ..., 0	$0 \rightarrow 2$
1, ..., 1	$1 \rightarrow 2$

$2^n$

$$\log_2 n =$$

n

0, 1

1  
- ) 0  
- ) 1  
-

100K

$\log(100K)$

$$(x+3)(2x+4) + \dots$$

$x_1, \dots, x_n$	
0, ..., 0	0 $\rightarrow$ 2
1, ..., 0	1 $\rightarrow$ 2
1, ..., 1	0

$$\log_2 n =$$

n

0, 1

1  
0  
1

100K

$\log(100K)$

$$(x+3)(2x+4) + \dots$$

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1.

0, 1

1  
- ) 0  
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100K

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$x_1, \dots, x_n$	
0, ..., 0	$0 \rightarrow 2$
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$2^n$

$$\log_2 n$$

n

0, 1

1  
0  
1  
-

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0, ..., 0	$0 \rightarrow 2$
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$$\log_2 n =$$

n

0, 1

1  
-  
-  
-  
0  
-  
-  
1

100K

$\log(100K)$