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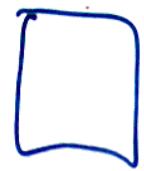
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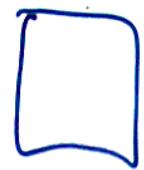
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$$q(x) = \sum_{i=}$$

$$q(x) = \sum_{i=0}^{n-1} q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (q_0, q_1, \dots, q_{\frac{n}{2}-1})$$

$$q(x) = \sum_{i=0}^{n-1} q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (q_0, q_2, \dots, q_{n-2}) \quad r \rightarrow (q_1, \dots, q_{n-1})$$

$$q(x) = \sum_{i=0}^{n-1} q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (q_0, q_2, \dots, q_{n-2}) \quad r \rightarrow (q_1, \dots, q_{n-1})$$

P



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$$q(x) = \sum_{i=0}^{n-1} q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (q_0, q_2, \dots, q_{n-2}) \quad r \rightarrow (q_1, \dots, q_{n-1})$$

(P)

$$w_n^0, w_n^1, \dots, w_n^{n-1}$$

l

$$q(x) = \sum_{i=0}^m q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (q_0, q_2, \dots, q_{\frac{n}{2}-2}) \quad r \rightarrow (q_1, \dots, q_{n-1})$$

$$p(w_n^i) = q((w_n^i)^2) + w_n^i \cdot r((w_n^i)^2)$$

$$w_n^0, w_n^1, \dots, w_n^{n-1}$$

$$q(x) = \sum_{i=0}^{n-1} q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (\underbrace{q_0, q_2, \dots, q_{\frac{n}{2}-2}}_{Q_0}) \quad r \rightarrow (\underbrace{q_1, \dots, q_{n-1}}_{Q_1})$$

$$[y_{0,0}, \dots, y_{0,\frac{n}{2}-1}] \quad [y_{1,0}, \dots, y_{1,\frac{n}{2}-1}]$$

$$q(x) = \sum_{i=0}^{n-1} q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (\underbrace{q_0, q_2, \dots, q_{\frac{n}{2}-2}}_{Q_0}) \quad r \rightarrow (\underbrace{q_1, \dots, q_{n-1}}_{Q_1})$$

$$[y_{0,0}, \dots, y_{0,\frac{n}{2}-1}] \quad [y_{1,0}, \dots, y_{1,\frac{n}{2}-1}]$$

$$\text{DFT}(\bar{x}) = [y_0, \dots, y_{n-1}]$$

$$q(x) = \sum_{i=0}^{n-1} q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (\underbrace{q_0, q_2, \dots, q_{\frac{n}{2}-2}}_{Q_0}) \quad r \rightarrow (\underbrace{q_1, \dots, q_{\frac{n}{2}-1}}_{Q_1})$$

$$[y_{0,0}, \dots, y_{0,\frac{n}{2}-1}] \quad [y_{1,0}, \dots, y_{1,\frac{n}{2}-1}]$$

$$\text{DFT}(\bar{x}) = [y_0, \dots, y_{n-1}]$$

$$k \in \{0, \dots, \frac{n}{2}-1\} \quad y_k =$$

$$q(x) = \sum_{i=0}^m q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (\underbrace{q_0, q_2, \dots, q_{\frac{n}{2}-2}}_{Q_0}) \quad r \rightarrow (\underbrace{q_1, \dots, q_{n-1}}_{Q_1})$$

$$[y_{0,0}, \dots, y_{0,\frac{n}{2}-1}] \quad [y_{1,0}, \dots, y_{1,\frac{n}{2}-1}]$$

$$\text{DFT}(\bar{x}) = [y_0, \dots, y_{n-1}]$$

$$k \in \{0, \dots, \frac{n}{2}-1\} \quad y_k = p(W^k)$$

$$q(x) = \sum_{i=0}^m q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^i)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (\underbrace{q_0, q_2, \dots, q_{\frac{n}{2}-2}}_{Q_0}) \quad r \rightarrow (\underbrace{q_1, \dots, q_{n-1}}_{Q_1})$$

$$[y_{0,0}, \dots, y_{0, \frac{n}{2}-1}] \quad [y_{1,0}, \dots, y_{1, \frac{n}{2}-1}]$$

$$DFT(\bar{x}) = [y_0, \dots, y_{n-1}]$$

$$k \in \{0, \dots, \frac{n}{2}-1\} \quad y_k = p(w_n^k) = q((w_n^k)^2) + w_n^k \cdot r((w_n^k)^2)$$

$$q(x) = \sum_{i=0}^{n-1} q_i \cdot x^i$$

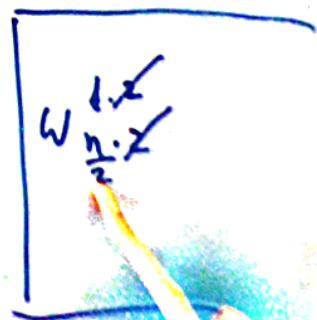
$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (\underbrace{q_0, q_2, \dots, q_{\frac{n}{2}-2}}_{Q_0}) \quad r \rightarrow (\underbrace{q_1, \dots, q_{n-1}}_{Q_1})$$

$$[y_{0,0}, \dots, y_{0, \frac{n}{2}-1}] \quad [y_{1,0}, \dots, y_{1, \frac{n}{2}-1}]$$

$$DFT(\bar{x}) = [y_0, \dots, y_{n-1}]$$

$$k \in \{0, \dots, \frac{n}{2}-1\} \quad y_k = p(w_n^k) = q((w_n^k)^2) + w_n^k \cdot r((w_n^k)^2)$$



$$q(x) = \sum_{i=0}^m q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (\underbrace{q_0, q_2, \dots, q_{\frac{n}{2}-2}}_{Q_0}) \quad r \rightarrow (\underbrace{q_1, \dots, q_{n-1}}_{Q_1})$$

$$[y_{0,0}, \dots, y_{0, \frac{n}{2}-1}] \quad [y_{1,0}, \dots, y_{1, \frac{n}{2}-1}]$$

$$DFT(\bar{x}) = [y_0, \dots, y_{n-1}]$$

$$k \in \{0, \dots, \frac{n}{2}-1\} \quad y_k = p(w_n^k) = q((w_n^k)^2) + w_n^k \cdot r((w_n^k)^2) \\ = q(w_n^k) + w_n^k \cdot r(w_n^k)$$

$$\boxed{w_{\frac{n}{2}, k}}$$

$$q(x) = \sum_{i=0}^{n-1} q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (\underbrace{q_0, q_2, \dots, q_{\frac{n}{2}-2}}_{Q_0}) \quad r \rightarrow (\underbrace{q_1, \dots, q_{\frac{n}{2}-1}}_{Q_1})$$

$$\boxed{[y_{0,0}, \dots, y_{0,\frac{n}{2}-1}]}$$

$$[y_{1,0}, \dots, y_{1,\frac{n}{2}-1}]$$

$$\boxed{w_{\frac{n}{2},2}}$$

$$DFT(\bar{x}) = [y_0, \dots, y_{n-2}]$$

$$k \in \{0, \dots, \frac{n}{2}-1\}$$

$$y_k = p(w_n^k) = q((w_n^k)^2) + w_n^k \cdot r((w_n^k)^2) \\ = q(w_n^k) + w_n^k \cdot r(w_n^k)$$

$$q(x) = \sum_{i=0}^m q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q \rightarrow (\underbrace{q_0, q_2, \dots, q_{\frac{n}{2}-2}}_{Q_0}) \quad r \rightarrow (\underbrace{q_1, \dots, q_{n-1}}_{Q_1})$$

$$\left[\underbrace{y_{0,0}, \dots, y_{0,\frac{n}{2}-1}}_{Q_0} \right]$$

$$\left[y_{1,0}, \dots, y_{1,\frac{n}{2}-1} \right]$$

$$\boxed{w_{\frac{n}{2}, k}}$$

$$DFT(\bar{x}) = [y_0, \dots, y_{n-1}]$$

$$k \in \{0, \dots, \frac{n}{2}-1\}$$

$$\begin{aligned} y_k &= p(w_n^k) = q((w_n^k)^2) + w_n^k \cdot r((w_n^k)^2) \\ &= q(w_n^k) + w_n^k \cdot r(w_n^k) \\ &\approx y_{0,k} \end{aligned}$$

$$q(x) = \sum_{i=0}^m q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$Q_0 + Q_1 x + Q_2 x^2 + Q_3 x^3$$

$$(Q_0, Q_1, Q_2, Q_3, 0, 0, 0, 0) \xrightarrow{q} (\underbrace{Q_0, Q_2, \dots, Q_{\frac{n}{2}-2}}_{Q_0}, \underbrace{Q_1, \dots, Q_{n-1}}_{Q_1})$$

$$[y_{0,0}, \dots, y_{0,\frac{n}{2}-1}]$$

$$[y_{1,0}, \dots, y_{1,\frac{n}{2}-1}]$$

$$\boxed{w_{\frac{n}{2}, k}}$$

$$DFT(\bar{x}) = [y_0, \dots, y_{n-1}]$$

$$k \in \{0, \dots, \frac{n}{2}-1\}$$

$$\begin{aligned} y_k &= p(w_n^k) = q((w_n^k)^2) + w_n^k \cdot r((w_n^k)^2) \\ &= q(w_n^k) + w_n^k \cdot r(w_n^k) \\ &\approx y_{0,k} + w_n^k \cdot y_{1,k} \end{aligned}$$

$$q(x) = \sum_{i=0}^m q_i \cdot x^i$$

$$p(x) = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$Q_0 + Q_1 x + Q_2 x^2 + Q_3 x^3$$

$$(Q_0, Q_1, Q_2, Q_3, 0, 0, 0, 0) \xrightarrow{q} (\underbrace{Q_0, Q_2, \dots, Q_{\frac{n}{2}-2}}_{Q_0}) \quad \xrightarrow{r} (\underbrace{Q_1, \dots, Q_{n-1}}_{Q_1})$$

$$[y_{0,0}, \dots, y_{0,\frac{n}{2}-1}]$$

$$[y_{1,0}, \dots, y_{1,\frac{n}{2}-1}]$$

$$\boxed{w_{\frac{n}{2}, k}}$$

$$DFT(\bar{x}) = [y_0, \dots, y_{n-1}]$$

$$k \in \{0, \dots, \frac{n}{2}-1\}$$

$$y_k = p(w_n^k) = q((w_n^k)^2) + w_n^k \cdot r((w_n^k)^2)$$

$$= q((w_n^k)^2) + w_n^k \cdot r(w_n^k)$$

$$\approx y_{0,k} + w_n^k \cdot y_{1,k}$$

$$q(x) = \sum_{i=0}^m q_i \cdot x^i$$

$$\left(w_n^{\frac{n}{2}+k} \right)^2 = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$\left(w_n^{\frac{n}{2}+k} \right)^2$$

$$w_n = w_n^k \cdot w_n^{2k}$$

$$q \rightarrow (\underbrace{q_0, q_2, \dots, q_{\frac{n}{2}-2}}_{Q_0}) \quad r \rightarrow (\underbrace{q_1, \dots, q_{n-1}}_{Q_1})$$

$$[y_{0,0}, \dots, y_{0,\frac{n}{2}-1}]$$

$$[y_{1,0}, \dots, y_{1,\frac{n}{2}-1}]$$

$$w_n^{\frac{n}{2}+k}$$

$$DFT(\bar{x}) = [y_0, \dots, y_{n-1}]$$

$$k \in \{0, \dots, \frac{n}{2}-1\}$$

$$\begin{aligned} y_k &= p(w_n^k) = q((w_n^k)^2) + w_n^k \cdot r((w_n^k)^2) \\ &= q(w_n^k) + w_n^k \cdot r(w_n^k) \\ &\approx y_{0,k} + w_n^k \cdot y_{1,k} \end{aligned}$$

$$q(x) = \sum_{i=0}^n q_i \cdot x^i$$

$$\left(w_n^{\frac{n}{2}+k} \right)^2 = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$q(w_n^{\frac{k}{2}}) + w_n^{\frac{n}{2}+k} \cdot r(w_n^{\frac{k}{2}})$$

$y_{0,k}$

$$\begin{aligned} w_n \\ = & (w_n^{\frac{n}{2}} \cdot w_n^{\frac{n}{2}+k})^2 \\ = & w_n^k \end{aligned}$$

$$\boxed{w_n^{\frac{k}{2}+k}}$$

$$q(x) = \sum_{i=0}^n q_i \cdot x^i$$

$$\left(w_n^{\frac{n}{2}+k} \right)^2 = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$\begin{aligned} & w_n \\ &= (w_n^{\frac{n}{2}} \cdot w_n^{\frac{n}{2}+k})^2 \\ &= w_n^{\frac{n}{2}+k} \end{aligned}$$

$$q(w_n^{\frac{n}{2}+k}) + w_n^{\frac{n}{2}+k} \cdot r(w_n^{\frac{n}{2}+k})$$

$$y_{0,k} + w_n^{\frac{n}{2}} \cdot w_n^{\frac{n}{2}+k} r(w_n^{\frac{n}{2}+k})$$

$w_n^{\frac{n}{2}+k}$

$$q(x) = \sum_{i=0}^n q_i \cdot x^i$$

$$\left(w_n^{\frac{n}{2}+k} \right)^2 = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$\begin{aligned} & w_n \\ &= (w_n^{\frac{n}{2}} \cdot w_n^{\frac{n}{2}+k})^2 \\ &= w_n^{n+k} \end{aligned}$$

$$q(w_n^{\frac{n}{2}}) + w_n^{\frac{n}{2}+k} \cdot r(w_n^{\frac{n}{2}})$$

$$y_{0,k} + w_n^{\frac{n}{2}} \cdot w_n^k r(w_n^{\frac{n}{2}})$$

$$= \left(e^{\frac{2\pi i}{h} k} \right)^{\frac{n}{2}}$$

$w_n^{\frac{n}{2}+k}$

$$q(x) = \sum_{i=0}^n q_i \cdot x^i$$

$$\left(w_n^{\frac{n}{2}+k} \right)^2 = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$\begin{aligned} & w_n \\ &= (w_n^{\frac{n}{2}} \cdot w_n^{\frac{n}{2}+k})^2 \\ &= w_n^{n+k} \end{aligned}$$

$$q(w_n^{\frac{n}{2}}) + w_n^{\frac{n}{2}+k} \cdot r(w_n^{\frac{n}{2}})$$

$$q_0 + w_n^{\frac{n}{2}} \cdot w_n^k r(w_n^{\frac{n}{2}})$$

$$= \left(e^{\frac{2\pi i}{n} k} \right)^{\frac{n}{2}} = -1$$

$w_n^{\frac{n}{2}+k}$

$$q(x) = \sum_{i=0}^n q_i \cdot x^i$$

$$\left(w_n^{\frac{n}{2}+k} \right)^2 = \sum_{i=0}^{\frac{n}{2}-1} q_{2i}(x^2)^i + x \sum_{i=0}^{\frac{n}{2}-1} q_{2i+1} \cdot (x^2)^i = q(x^2) + x r(x^2)$$

$$w_n^{\frac{n}{2}} = w_n^{\frac{n}{2}} \cdot w_{\frac{n}{2}}^{\frac{n}{2}}$$

$$= w_{\frac{n}{2}}^k$$

$$q(w_{\frac{n}{2}}^k) + w_n^{\frac{n}{2}+k} \cdot r(w_{\frac{n}{2}}^k)$$

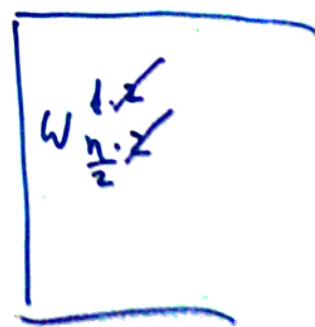
$$m_{0,k} + w_n^{\frac{n}{2}} \cdot w_n^k r(w_{\frac{n}{2}}^k)$$

$$m_1 = m_{0,k} + w_n^{\frac{n}{2}} \cdot m_{1,k} : \left(e^{\frac{2\pi i}{n} k} \right)^{\frac{n}{2}} = -1$$

w_{n/2}^k

1

(Q₀)



1

(Q₀)

$w_{\frac{n+1}{2}}$

1

(a_0)

$$\text{DFT}((a_0)) = [a_0]$$

$X=1$

① ②

(a_0, a_1)

$$p(x) = a_0 + a_1 x$$

$$p(1), p(-1)$$

$$\boxed{W_{\frac{n}{2}, k}}$$

1

(Q_0)

$$DFT((Q_0)) = [a_0]$$

$X=1$

① ②

(Q_0, Q_1)

$$P(X) = Q_0 + Q_1 X$$

$w_2^0 \cdot w_2^1$

$[P(1), P(-1)]$

$$DFT((Q_0, Q_1)) =$$

$[Q_0+Q_1, Q_0-Q_1]$

$$w_2 = e^{\frac{2\pi i}{N}} = -1$$

$w_{\frac{N}{2}, 2}$

$$1 \quad (Q_0)$$

$$\text{DFT}((Q_0)) = [a_0]$$

$X=1$

$$\textcircled{1} \quad \textcircled{2} \quad (Q_0, Q_1)$$

$$P(X) = Q_0 + Q_1 X$$

$$W_2^0 \cdot W_2^1$$

$$[P(1), P(-1)]$$

$$\text{DFT}((Q_0, Q_1)) =$$

$$[Q_0 + Q_1, Q_0 - Q_1]$$

$$W_2 = e^{\frac{j\pi i}{2}} = -1$$

$$W_{\frac{n}{2}, k}$$

$$1 \quad (Q_0)$$

$$X=1 \quad \therefore \quad DFT(Q_0) = [a_0]$$

$$\textcircled{1} \quad \textcircled{2} \quad (Q_0, Q_1)$$

$$P(X) = a_0 + a_1 X$$

$$W_2^0, W_2^1$$

$$[P(1), P(-1)]$$

$$W_2 = e^{\frac{j\pi i}{2}} = -1$$

$$DFT(Q_0, Q_1) =$$

$$[Q_0 + Q_1, Q_0 - Q_1]$$

$$\boxed{W_{\frac{n+2}{2}}}$$

1

(Q_0)

$X=1$

$$DFT((Q_0)) = [a_0]$$

① ②

(Q_0, Q_1)

$$P(X) = Q_0 + Q_1 X$$

$w_2^0 \cdot w_2^1$

$$[P(1), P(-1)]$$

$$DFT((Q_0, Q_1)) =$$

$$[Q_0 + Q_1, Q_0 - Q_1]$$

$$w_2 = e^{\frac{2\pi i}{2}} = -1$$

$$\boxed{w_{\frac{n+2}{2}}}$$

$$1 \quad (Q_0)$$

$$\text{DFT}((Q_0)) = [a_0]$$

$X=1$

$$\textcircled{1} \quad \textcircled{2} \quad (Q_0, Q_1)$$

$$P(X) = a_0 + a_1 X$$

$$W_2^0, W_2^1$$

$$[P(1), P(-1)]$$

$$\text{DFT}((Q_0, Q_1)) =$$

$$[Q_0 + Q_1, Q_0 - Q_1]$$

$$W_2 = e^{\frac{j\pi i}{2}} = -1$$

$$\boxed{W_{\frac{n+1}{2}}}$$

$$1 \quad (Q_0)$$

$$\text{DFT}((Q_0)) = [a_0]$$

$X=1$

$$\textcircled{1} \quad \textcircled{2} \quad (Q_0, Q_1)$$

$$P(X) = Q_0 + Q_1 X$$

$$W_2^0, W_2^1$$

$$[P(1), P(-1)]$$

$$\text{DFT}((Q_0, Q_1)) =$$

$$[Q_0 + Q_1, Q_0 - Q_1]$$

$$W_2 = e^{\frac{j\pi i}{2}} = -1$$

$$\boxed{W_2^{\frac{1+j}{2}}$$

$$1 \quad (Q_0)$$

$$\text{DFT}((Q_0)) = [a_0]$$

$X=1$

$$\textcircled{1} \quad \textcircled{2} \quad (Q_0, Q_1)$$

$$P(X) = a_0 + a_1 X$$

$$W_2^0, W_2^1$$

$$[P(1), P(-1)]$$

$$\text{DFT}((Q_0, Q_1)) =$$

$$[Q_0 + Q_1, Q_0 - Q_1]$$

$$W_2 = e^{\frac{j\pi i}{2}} = -1$$

$$W_{\frac{n+1}{2}}$$

$$1 \quad (Q_0)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + C \cdot n$$

$$DFT(Q_0) = [a_0]$$

$$\Theta(n \log n)$$

$$\textcircled{1} \quad \textcircled{2} \quad (Q_0, Q_1)$$

$$P(x) = a_0 + a_1 x$$

$$W_2^0 \cdot W_2^1$$

$$[P(1), P(-1)]$$

$$DFT(Q_0, Q_1) =$$

$$[Q_0 + Q_1, Q_0 - Q_1]$$

$$W_{\frac{n}{2}, k}$$

$$W_2 = e^{\frac{j\pi i}{2}} = -1$$

1

(Q₀)

$$T(n) = 2T\left(\frac{n}{2}\right) + C \cdot n$$

$$\text{DFT}(Q_0) = [a_0]$$

$$2^n < k < 2^{n+1}$$

$$O(2^{n+1})$$

$$\Theta(n \log n)$$

$$W_{\frac{n}{2}, k}$$

1

 (Q_0)

$$T(n) = 2T\left(\frac{n}{2}\right) + c \cdot n$$

$$DFT(Q_0) = [a_0]$$

$$\Theta(n \log n)$$

$$2^m < n < 2^{m+1}$$

$$O(2^{m+1} \cdot \log(2^{m+1}))$$

$$W_{\frac{n}{2}, k}$$

1

 (Q_0)

$$T(n) = 2T\left(\frac{n}{2}\right) + c \cdot n$$

$$DFT(Q_0) = [a_0]$$

$$2^m \leq n < 2^{m+1}$$

$$O(2^{m+1} \cdot \log(2^{m+1}))$$

$$2^{m+1} < 2n$$

$$\Theta(n \log n)$$

$$W_{\frac{n}{2}, 2}^{1, 2}$$

$$c \cdot 2^{m+1} \cdot \log(2^{m+1})$$

$$c \cdot 2n \cdot \log(2n)$$

1

 (Q_0)

$$T(n) = 2T\left(\frac{n}{2}\right) + c \cdot n$$

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$$W_{\frac{n}{2}, 2}$$

$$c \cdot 2^{m+1} \cdot \log(2^{m+1})$$

$$c \cdot 2n \cdot \log(2n)$$

$$w_0, w_1, \dots, w_{n-1}$$

$$T(n) = 2T\left(\frac{n}{2}\right) + C \cdot n$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i \quad p(1) = a_0 + a_1 + \dots + a_{n-1} = m_0$$

$$p(w_n) = a_0 + a_1 w_n + a_2 w_n^2 + \dots + a_{n-1} w_n^{n-1} = m_1$$

$$\Theta(n \log n)$$

$$\begin{pmatrix} & \begin{pmatrix} a_0 \\ \vdots \\ a_{n-1} \end{pmatrix} \\ & = \end{pmatrix} \begin{pmatrix} m_0 \\ m_1 \\ \vdots \\ m_{n-1} \end{pmatrix},$$

$$w_0, w_1, \dots, w_{n-1}$$

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$$\Theta(n \log n)$$

$$\begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & w_n & w_n^2 & \dots & w_n^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix},$$

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w_0, w_1, \dots, w_{n-1}

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$$\Theta(n \log n)$$

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$$T(n) = 2T\left(\frac{n}{2}\right) + C \cdot n$$

$\Theta(n \log n)$

$$\begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & w_n & w_n^2 & \dots & w_n^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix},$$

$$F_n \cdot \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = F_n^{-1} \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$w_0, w_n^1, \dots, w_n^{n-1}$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i \quad p(1) = a_0 + a_1 + \dots + a_{n-1} = y_0$$

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$\Theta(n \log n)$

$$\begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & w_n & w_n^2 & \dots & w_n^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix},$$

$$F_n \cdot \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \frac{1}{n} F_n \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$w_0, w_1, \dots, w_{n-1}$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i \quad p(1) = a_0 + a_1 + \dots + a_{n-1} = y_0$$

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$\Theta(n \log n)$

$$\begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & w_n & w_n^2 & \dots & w_n^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix},$$

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$$T(n) = 2T\left(\frac{n}{2}\right) + C \cdot n$$

$\Theta(n \log n)$

$$\begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & w_n & w_n^2 & \dots & w_n^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} m_0 \\ m_1 \\ \vdots \\ m_{n-1} \end{pmatrix},$$

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$$T(n) = 2T\left(\frac{n}{2}\right) + C \cdot n$$

$\Theta(n \log n)$

$$\begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & w_n & w_n^2 & \dots & w_n^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} m_0 \\ m_1 \\ \vdots \\ m_{n-1} \end{pmatrix},$$

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$$p(w_n) = a_0 + a_1 w_n + a_2 w_n^2 + \dots + a_{n-1} w_n^{n-1} = m_1$$

SHOR

$$\begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & w_n & w_n^2 & \dots & w_n^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} m_0 \\ m_1 \\ \vdots \\ m_{n-1} \end{pmatrix},$$

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$$\Theta(n \log n)$$

$$F_n \cdot \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} m_0 \\ m_1 \\ \vdots \\ m_{n-1} \end{pmatrix}$$

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$$w_0, w_1, \dots, w_{n-1}$$

$$T(n) = 2T\left(\frac{n}{2}\right) + C \cdot n$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i \quad p(1) = a_0 + a_1 + \dots + a_{n-1} = y_0$$

$$\Theta(n \log n)$$

$$F_n[i, j] \quad w_n$$

$$F_n \cdot \begin{pmatrix} a_0 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ \vdots \\ a_{n-1} \end{pmatrix} = \frac{1}{n} F_n \begin{pmatrix} y_0 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$(w_0, w_1, \dots, w_{n-1})$$

$$T(n) = 2T\left(\frac{n}{2}\right) + C \cdot n$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i \quad p(1) = a_0 + a_1 + \dots + a_{n-1} = n_0$$

$$\Theta(n \log n)$$

$$F_n[i, j] = (w_n^{i-1})^{j-1}$$

$$= w_n^{(i-1)(j-1)}$$

$$F_n \cdot \begin{pmatrix} a_0 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ i \\ a_{n-1} \end{pmatrix} = \frac{1}{n} F_n \begin{pmatrix} y_0 \\ i \\ y_{n-1} \end{pmatrix}$$

$$(w_0, w_1, \dots, w_{n-1})$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i \quad p(1) = a_0 + a_1 + \dots + a_{n-1} = n_0$$

$$\begin{aligned} F_n[i, j] &= (w_n^{i-1})^{j-1} \\ &= w_n^{(i-1)(j-1)} \end{aligned}$$

$$F_n[j, i]$$

$$T(n) = 2T\left(\frac{n}{2}\right) + C \cdot n$$

$$\Theta(n \log n)$$

$$F_n \cdot \begin{pmatrix} a_0 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} n_0 \\ \vdots \\ n_{n-1} \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ \vdots \\ a_{n-1} \end{pmatrix} = \frac{1}{n} F_n \begin{pmatrix} n_0 \\ \vdots \\ n_{n-1} \end{pmatrix}$$

$$(w_0, w_1, \dots, w_{n-1})$$

$$T(n) = 2T\left(\frac{n}{2}\right) + C \cdot n$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i \quad p(1) = a_0 + a_1 + \dots + a_{n-1} = n_0$$

$$\Theta(n \log n)$$

$$F_n[i, j] = (w_n^{i-1})^{j-1}$$

$$= w_n^{(i-1)(j-1)}$$

$$q + b_i$$

$$F_n \cdot \begin{pmatrix} q_0 \\ q_1 \\ \vdots \\ q_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$F_n[j, i]$$

$$\overline{q + b_i} = q - b_i$$

$$\begin{pmatrix} q_0 \\ q_1 \\ \vdots \\ q_{n-1} \end{pmatrix} = \frac{1}{n} F_n \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$(w_0, w_1, \dots, w_{n-1})$$

$$T(n) = 2T\left(\frac{n}{2}\right) + C \cdot n$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i \quad p(1) = a_0 + a_1 + \dots + a_{n-1} = n^0$$

$$\Theta(n \log n)$$

$$F_n[i, j] = (w_n^{i-1})^{j-1}$$

$$= w_n^{(i-1)(j-1)}$$

Q + bi

$$F_n \cdot \begin{pmatrix} a_0 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$F_n[j, i]$$

$$\overline{Q + bi} = Q - bi$$

$$\begin{pmatrix} a_0 \\ i \\ a_{n-1} \end{pmatrix} = \frac{1}{n} F_n \begin{pmatrix} y_0 \\ i \\ y_{n-1} \end{pmatrix}$$

$$(w_0^0, w_1^1, \dots, w_n^{n-1})$$

$$T(n) = 2T\left(\frac{n}{2}\right) + C \cdot n$$

$$p(x) = \sum_{i=0}^{n-1} a_i \cdot x^i \quad p(1) = a_0 + a_1 + \dots + a_{n-1} = n^0$$

$$\Theta(n \log n)$$

$$F_n[i, j] = (w_n^{i-1})^{j-1}$$

$$= w_n^{(i-1)(j-1)}$$

$$F_n \cdot \begin{pmatrix} a_0 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$F_n[j, i]$$

$$A^*[i, j] = \overline{A[j, i]} \quad \begin{pmatrix} a_0 \\ \vdots \\ a_{n-1} \end{pmatrix} = \frac{1}{n} F_n \begin{pmatrix} y_0 \\ \vdots \\ y_{n-1} \end{pmatrix}$$