L :

R:

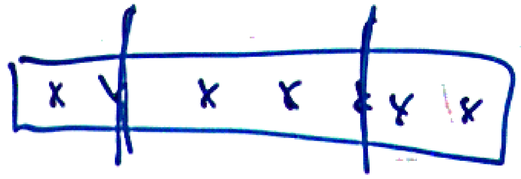
n

L :

R:

$n^{\frac{3}{4}}$

L :



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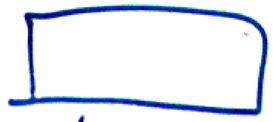

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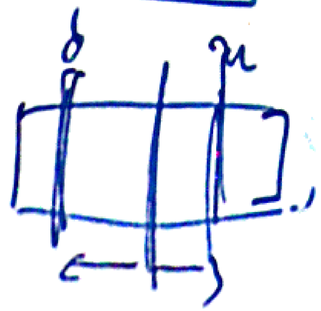

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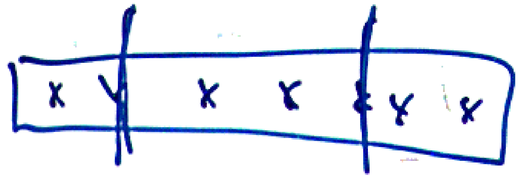

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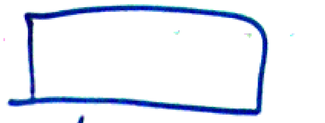
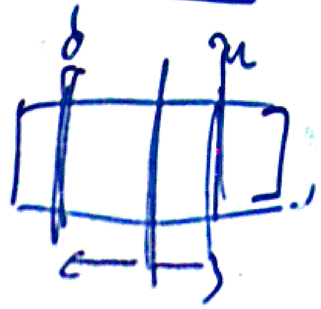
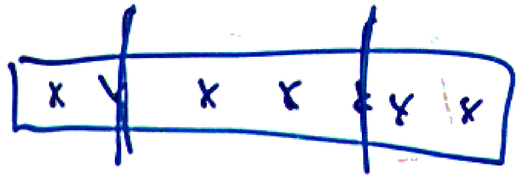


$n^{\frac{3}{4}}$



d, u

L :



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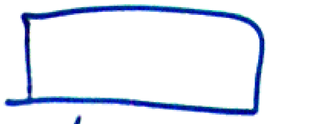
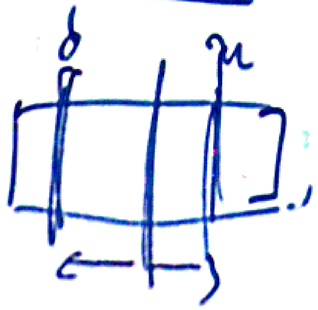


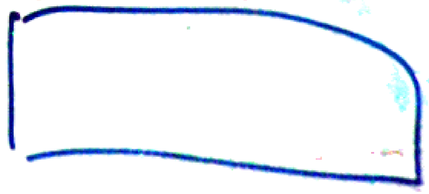
$n^{\frac{3}{4}}$

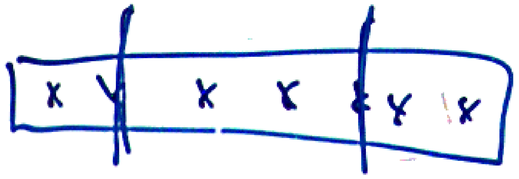


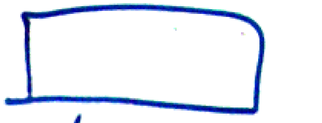
d, u

d u

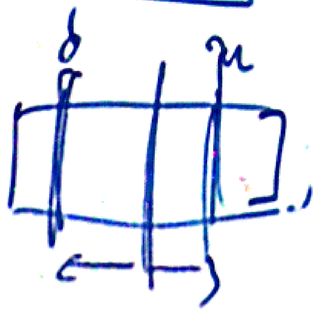
11



L : 

R: 

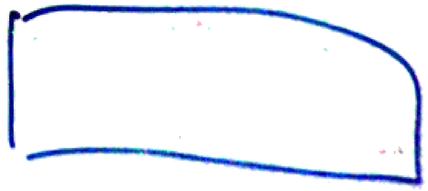
$n^{\frac{3}{4}}$

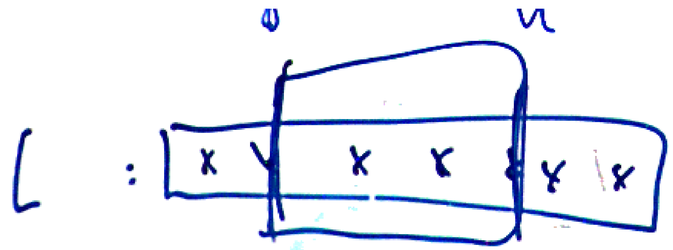


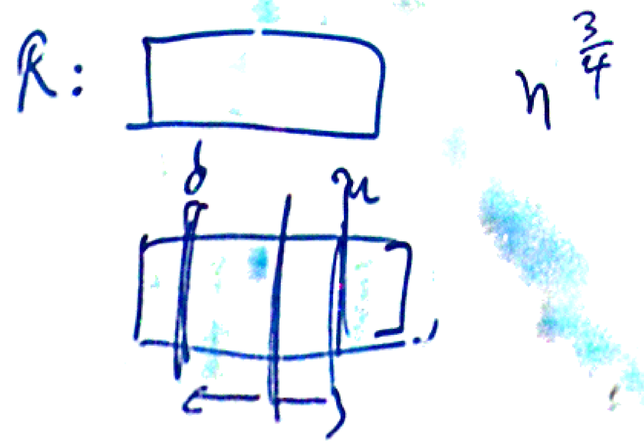
d u

d, u

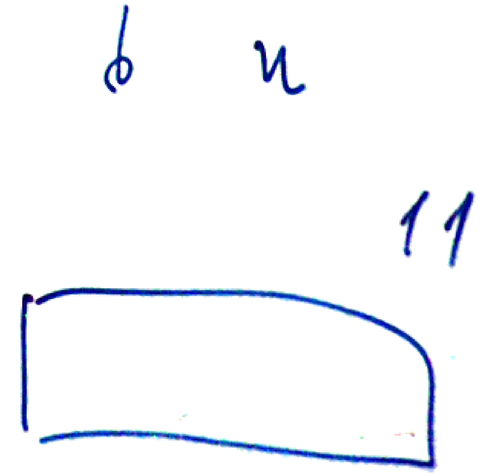
11

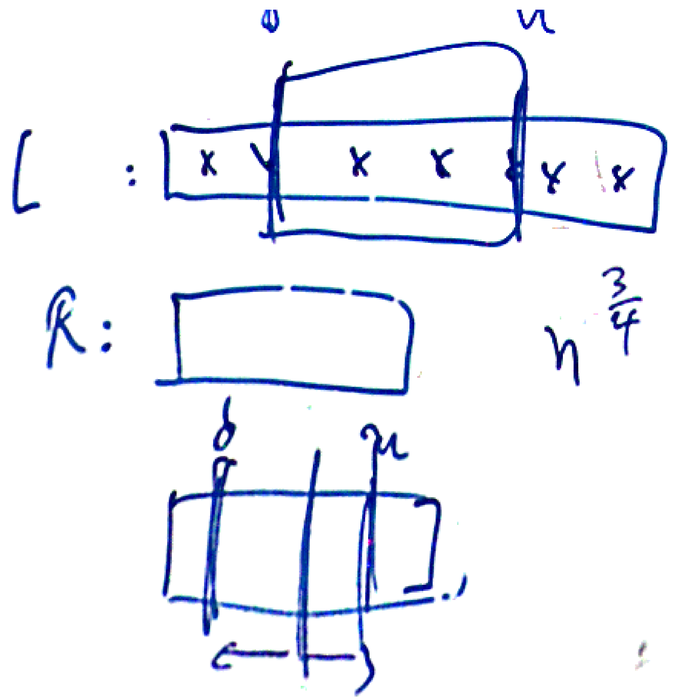






d, u





d u

d, u

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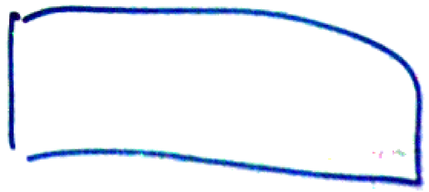
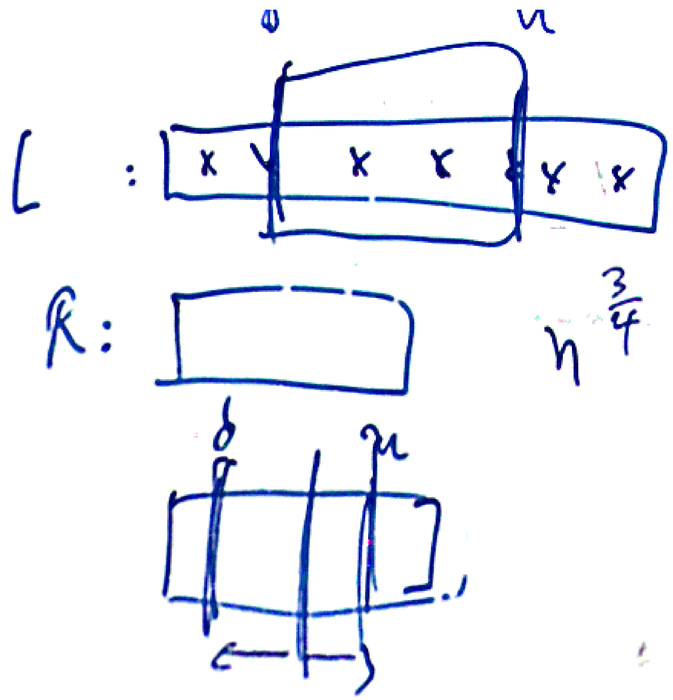
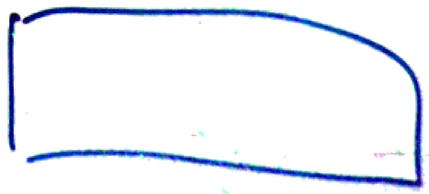


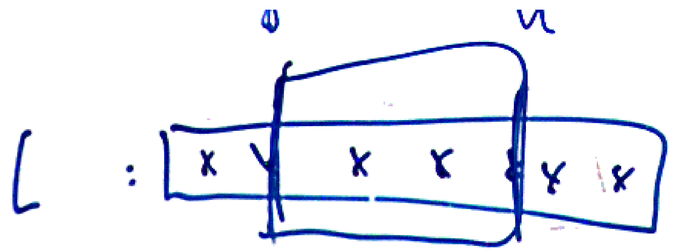


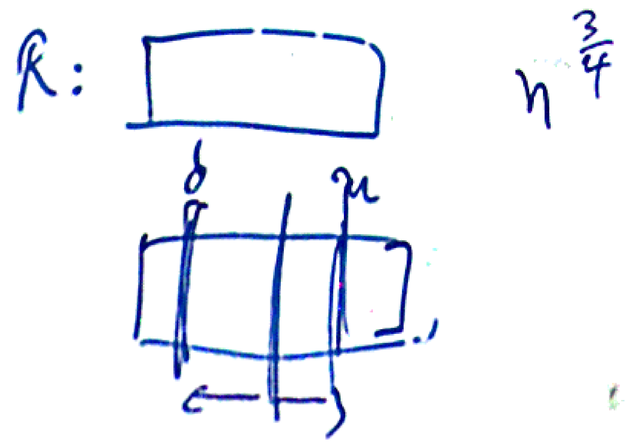
o u

o, u

11



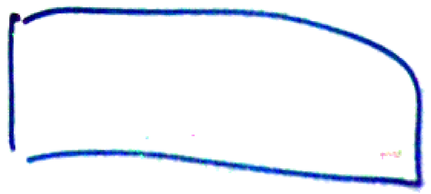


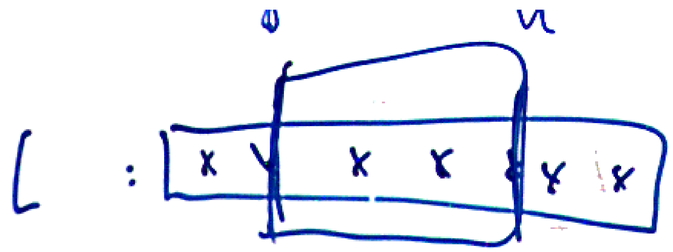


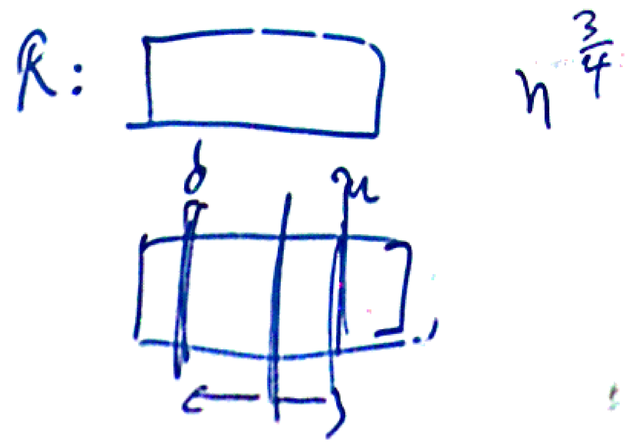
d, u

$d \quad u$

11



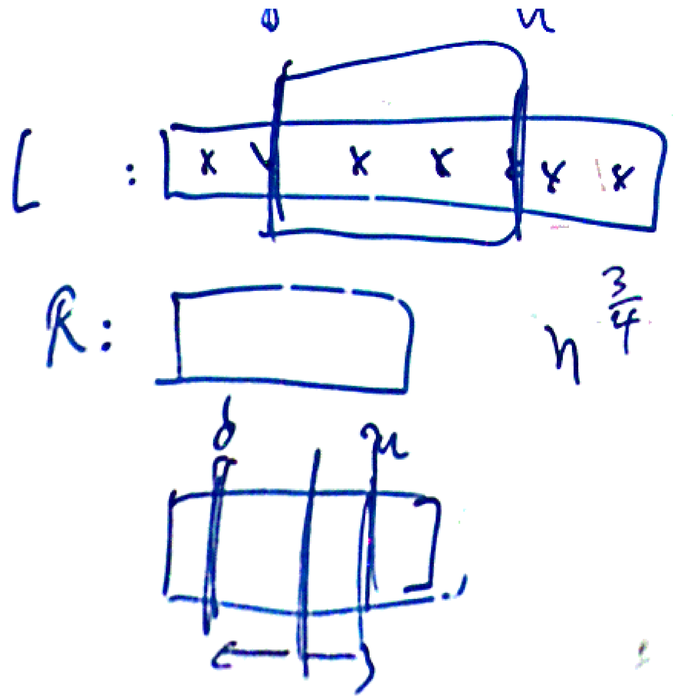




d, u

$d \quad u$

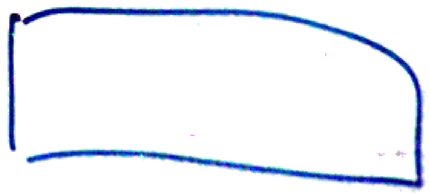
11

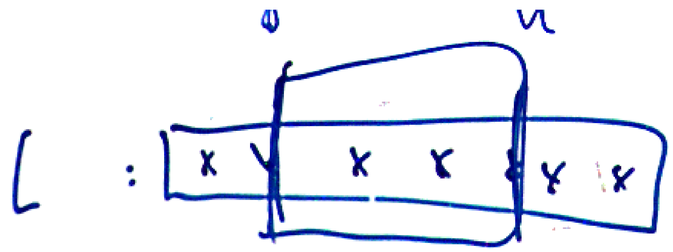


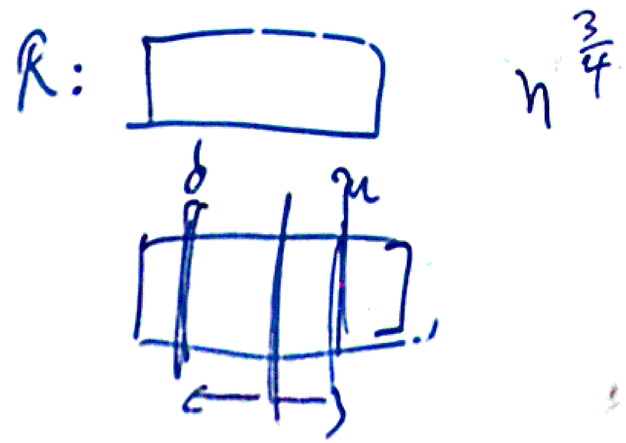
o u

o, u

11



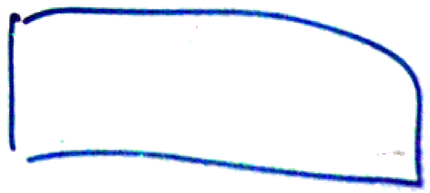


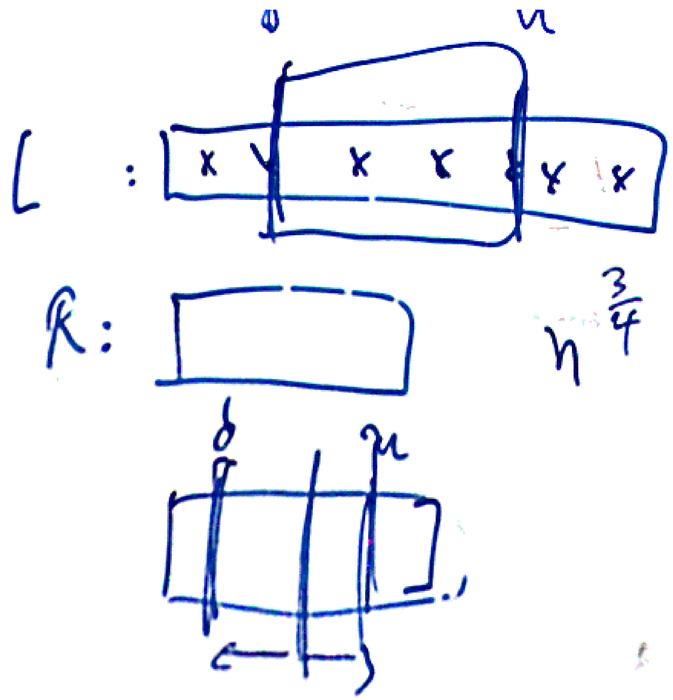


d, u

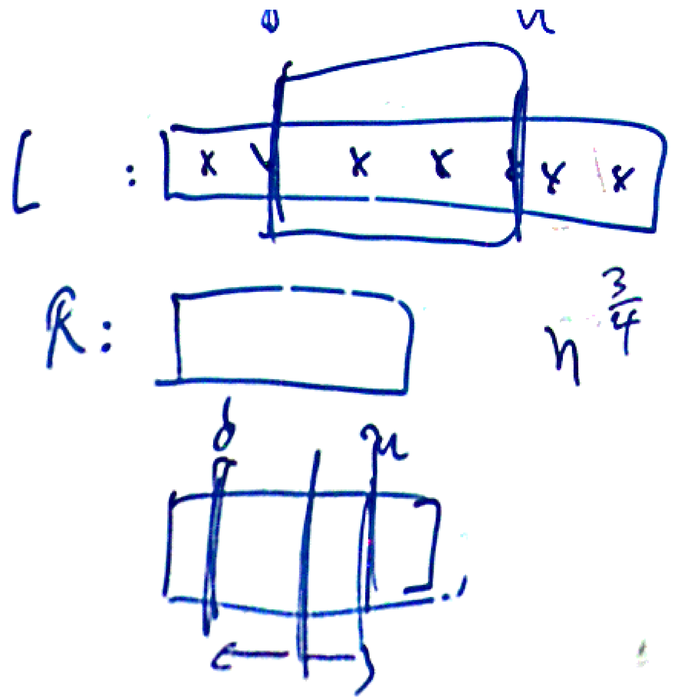
$d \quad u$

11

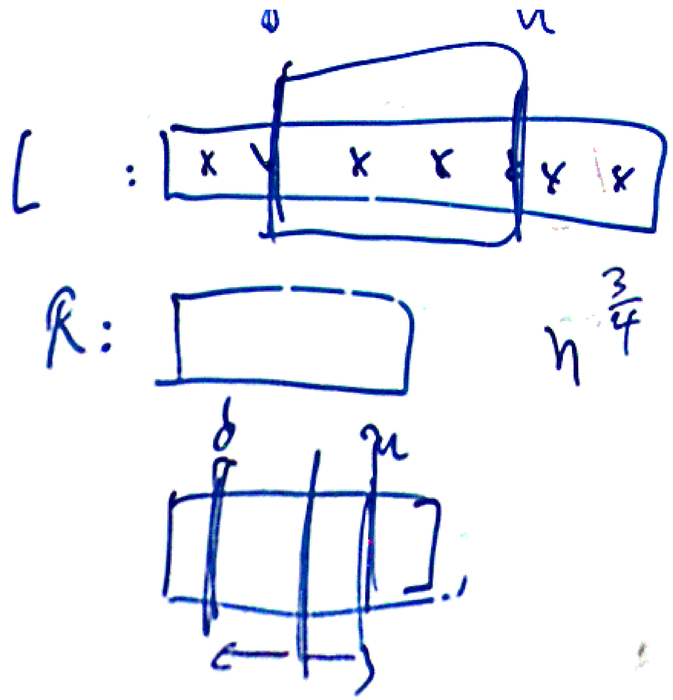




$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \,/\, R[i] \le m \right\} \right|$$

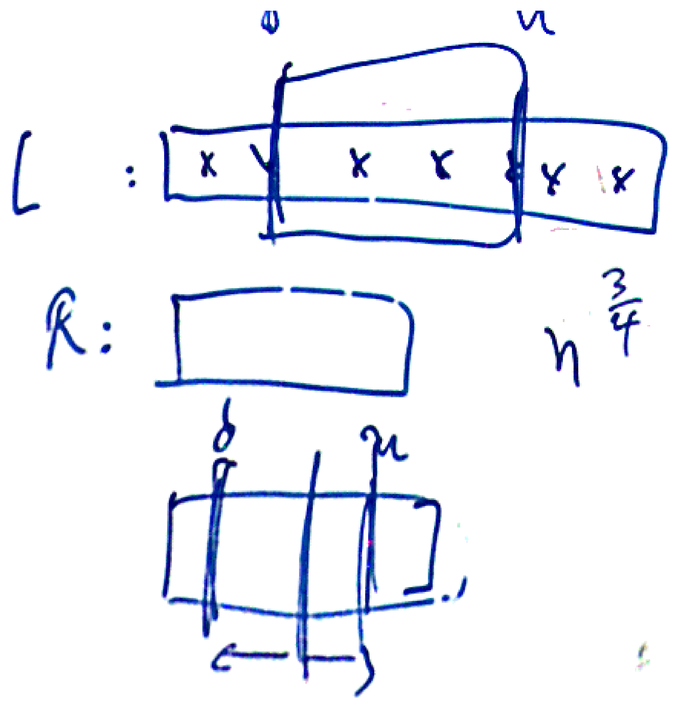


$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \,/\, R[i] \le m \right\} \right|$$



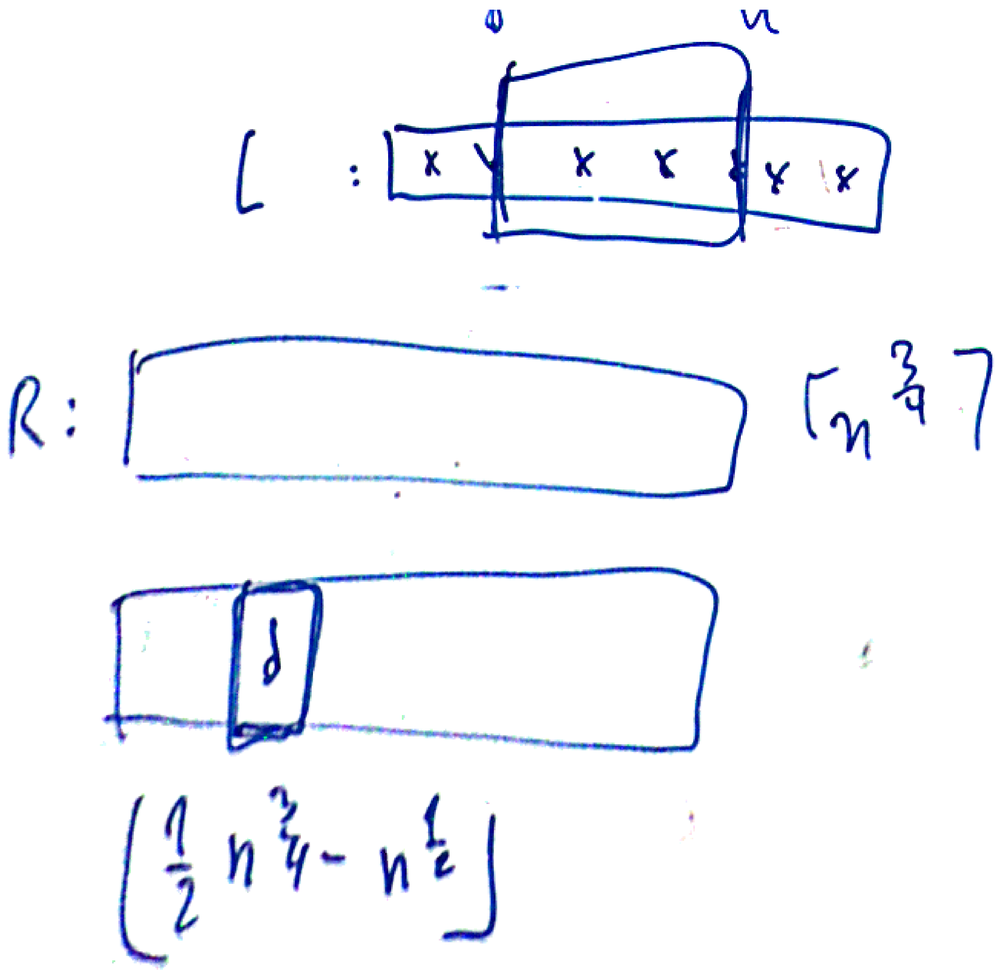
$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \;/\; R[i] \le m \right\} \right|$$

Y_1



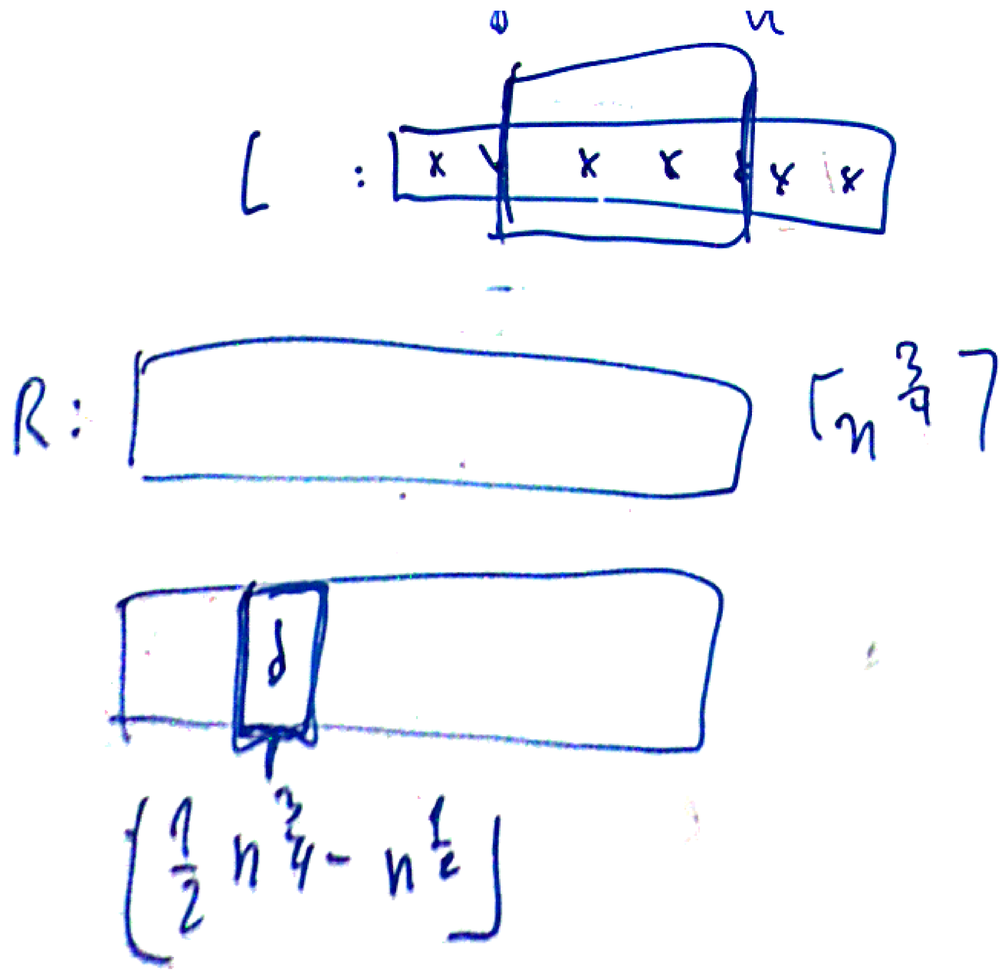
$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \;/\; R[i] \le m \right\} \right|$$

$$Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor$$



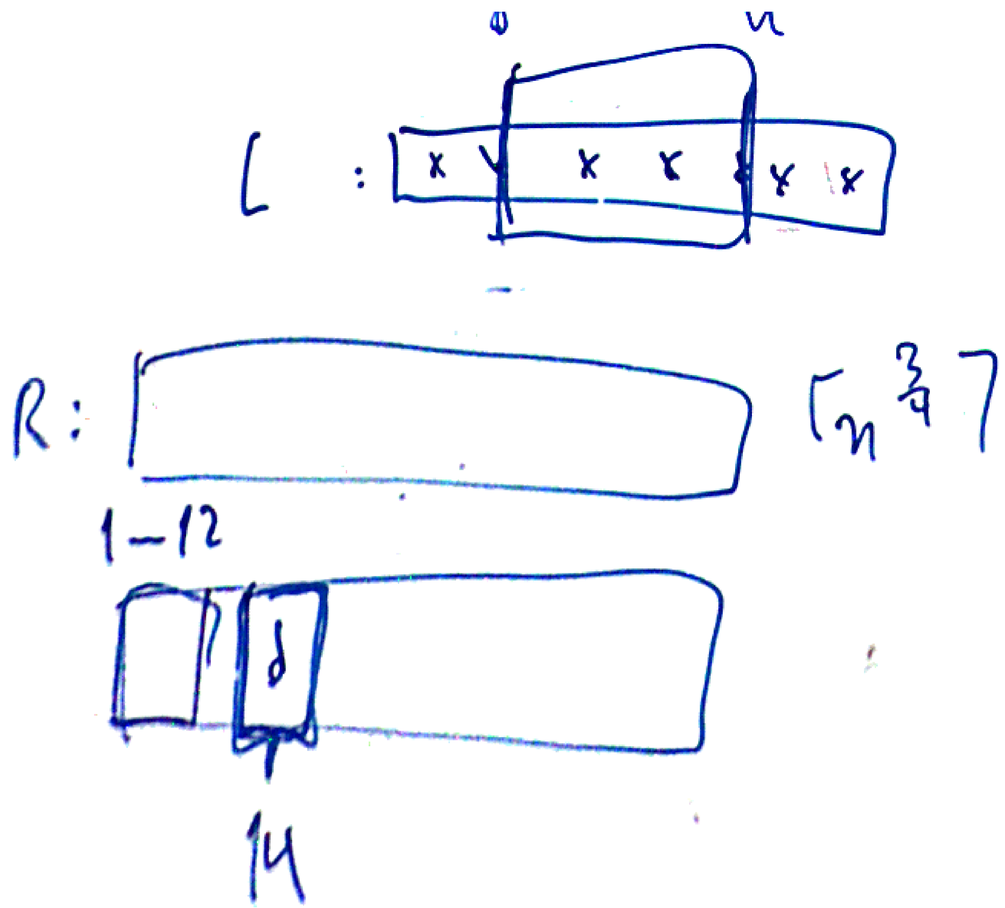
$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \;/\; R[i] \le m \right\} \right|$$

$$Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor$$



$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \;\middle|\; R[i] \le m \right\} \right|$$

$$Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor$$

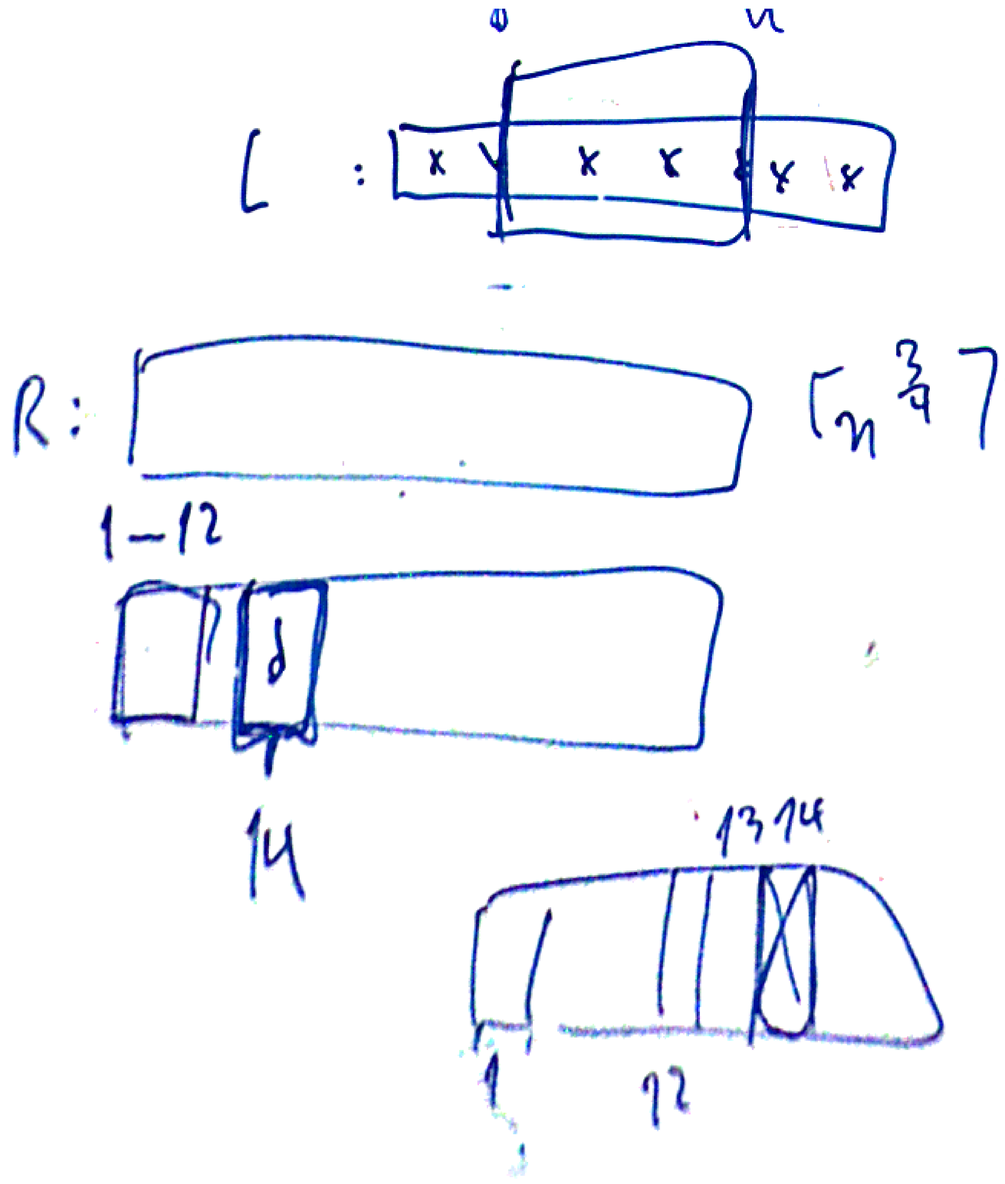


$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \;/\; R[i] \le m \right\} \right|$$

$$Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor$$

$$\parallel$$

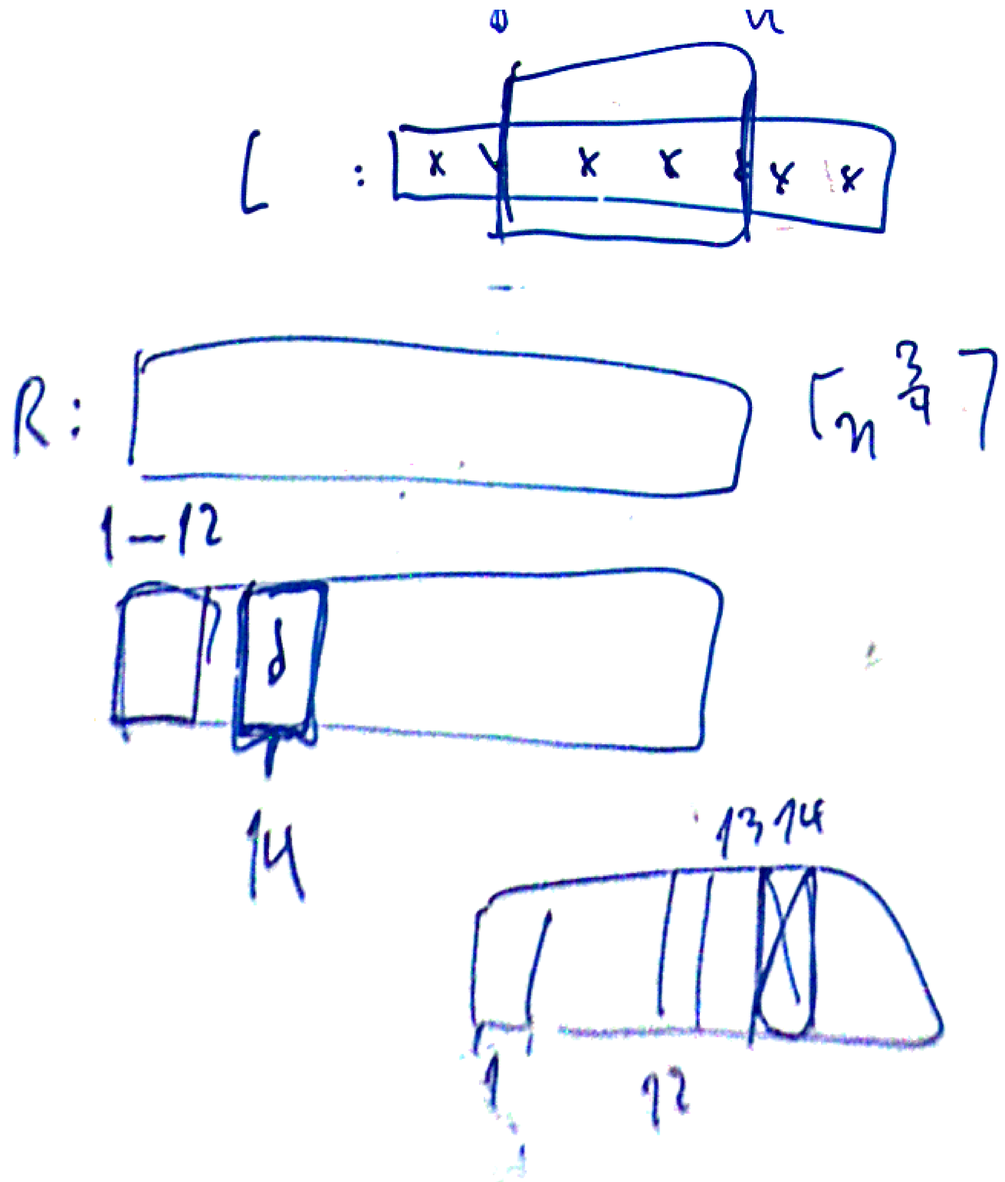
$$Y_1 < 14$$



$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \right. \right.$$
$$\left. R[i] \le m \right\} \Big|$$

$$Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor$$
$$\parallel$$
$$14$$

$$Y_1 < 14$$
$$\parallel$$
$$12$$



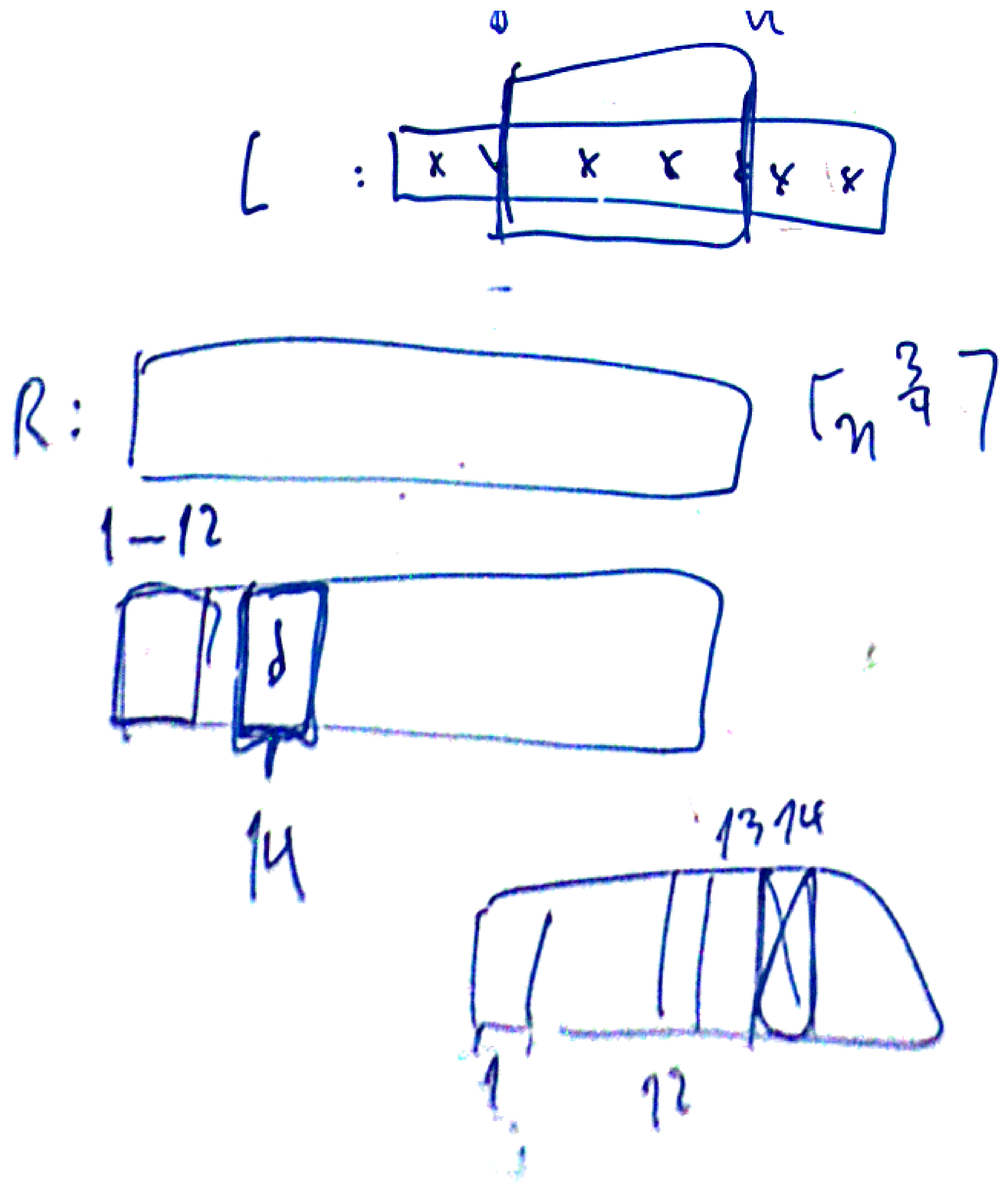
$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \right\} \right|$$

$$Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor$$

$$\parallel$$

$$14$$

$$Y_1 < 12$$

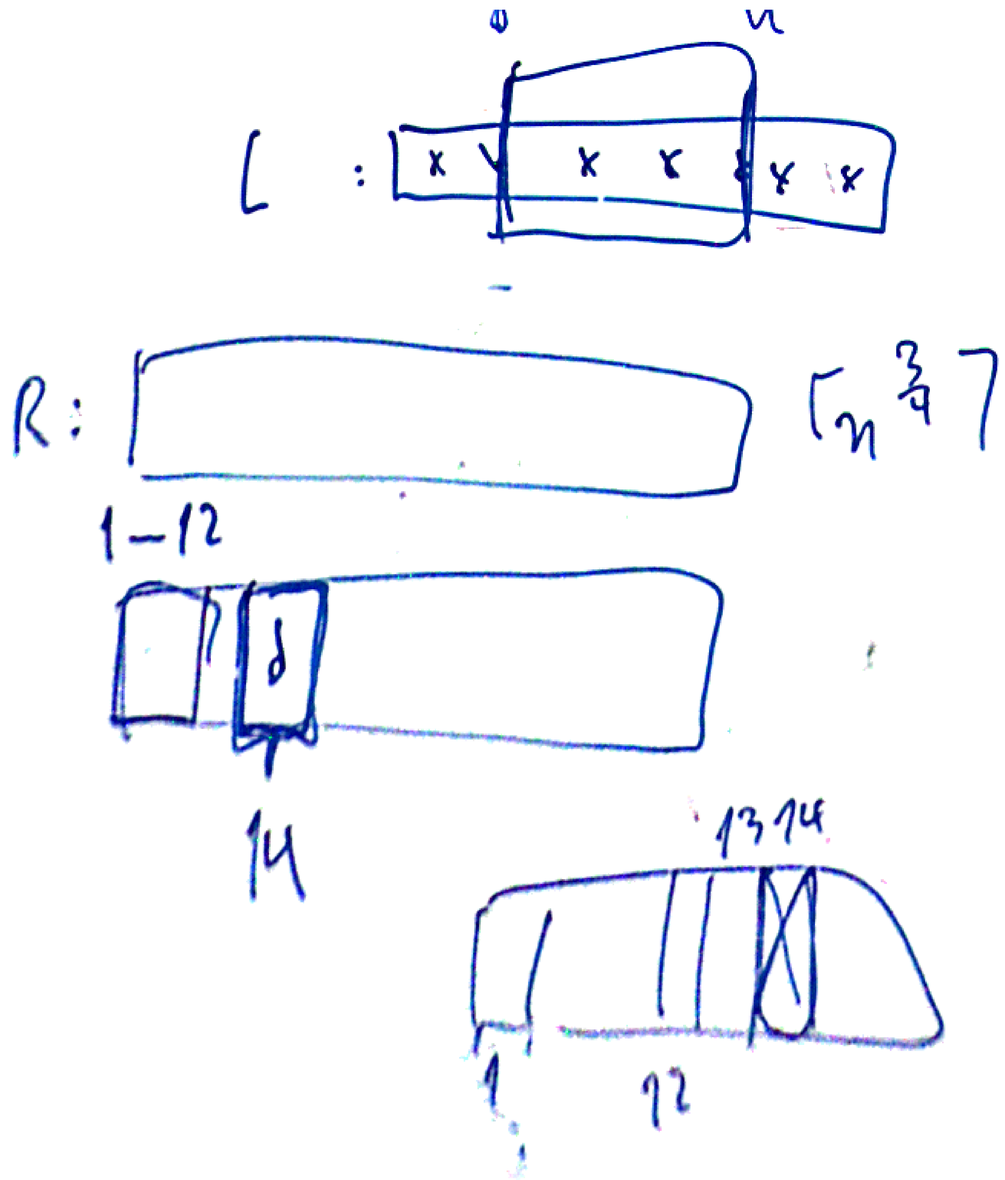


$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \right. \right.$$
$$\left. R[i] \le m \right\} \Big|$$

$$Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor$$

$$\overset{\shortparallel}{14}$$

$$\underset{\underset{12}{\shortparallel}}{Y_1} <$$

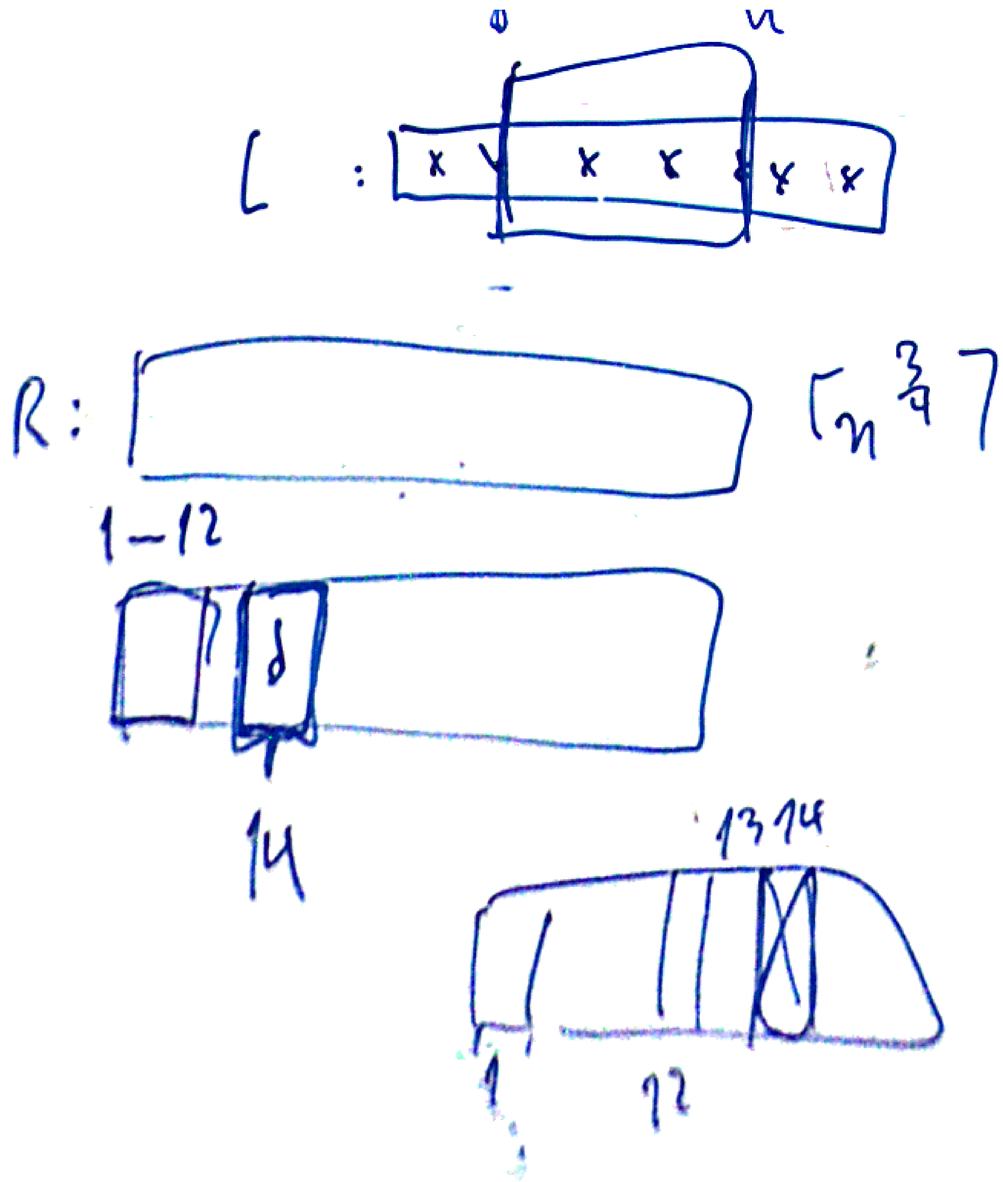


$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \;/\; R[i] \le m \right\} \right|$$

$$Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor$$

$$\overset{\shortparallel}{14}$$

$$\underset{\underset{12}{\shortparallel}}{Y_1} <$$



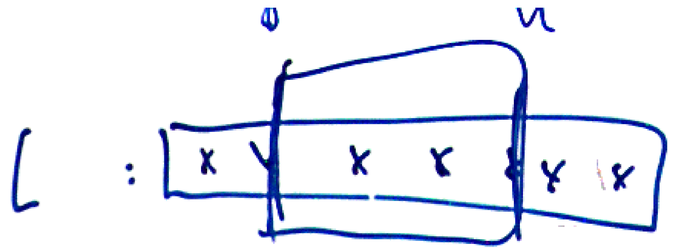
$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \right\} \right|$$

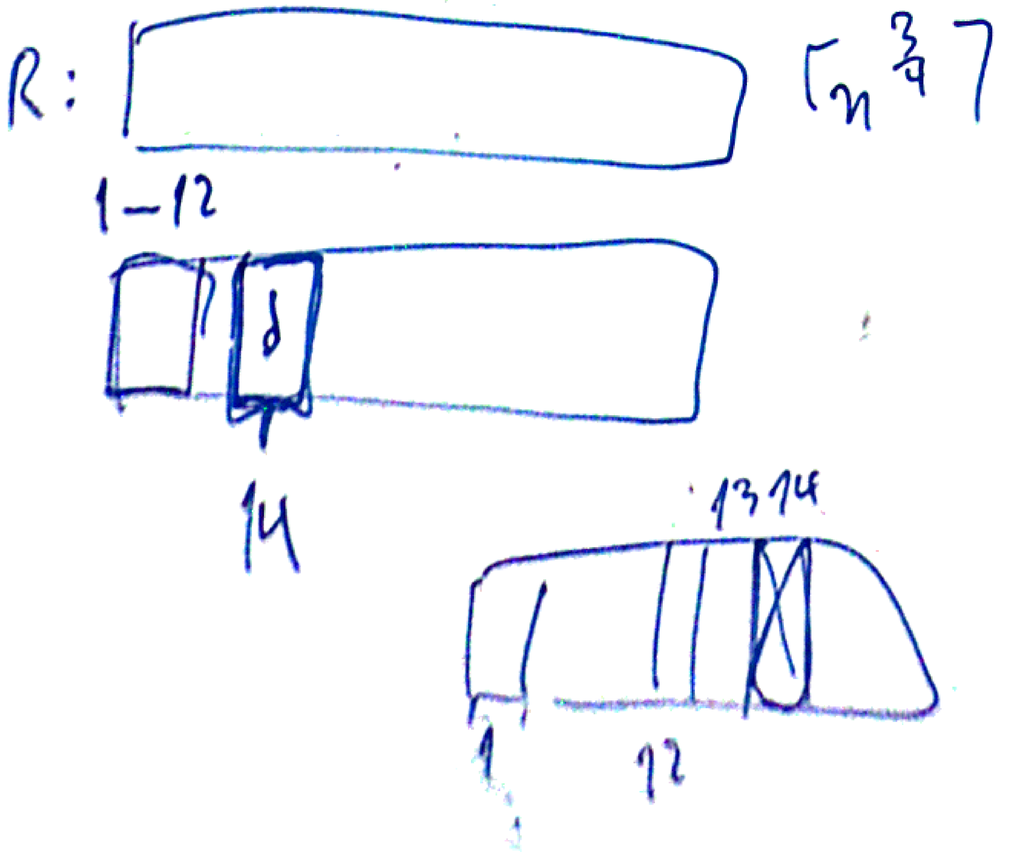
$$Pr\left(Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor\right) = p = \frac{1}{100}$$

$$Pr(\quad) = p$$

$$Pr(\quad) = p$$

$$Pr(A \vee B \vee C) = 3p = \frac{1}{100}$$





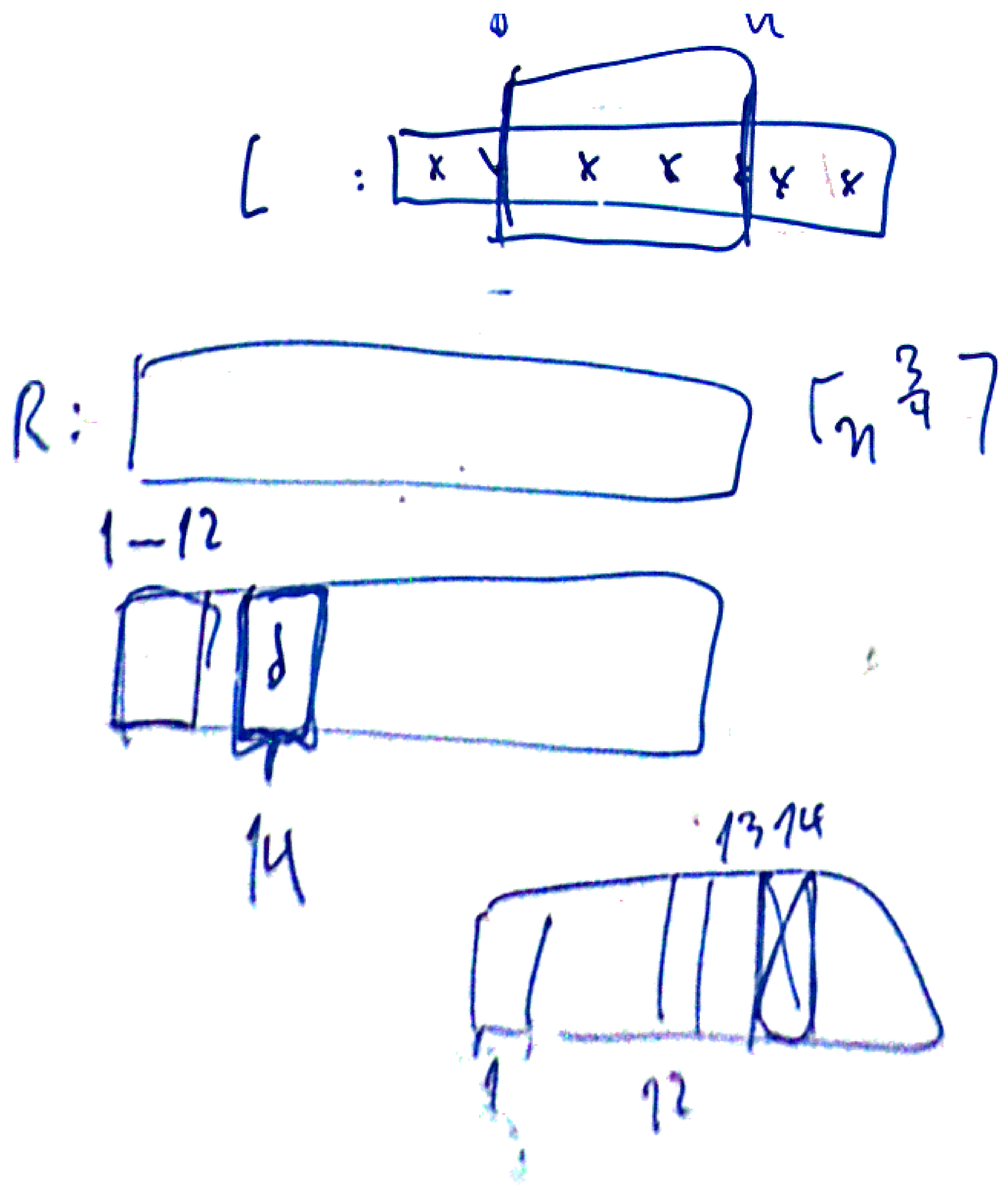
$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \right\} \right|$$

$$Pr\left(Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor\right) = p = \frac{1}{100}$$

$$Pr(\quad) = p$$

$$Pr(\quad) = p$$

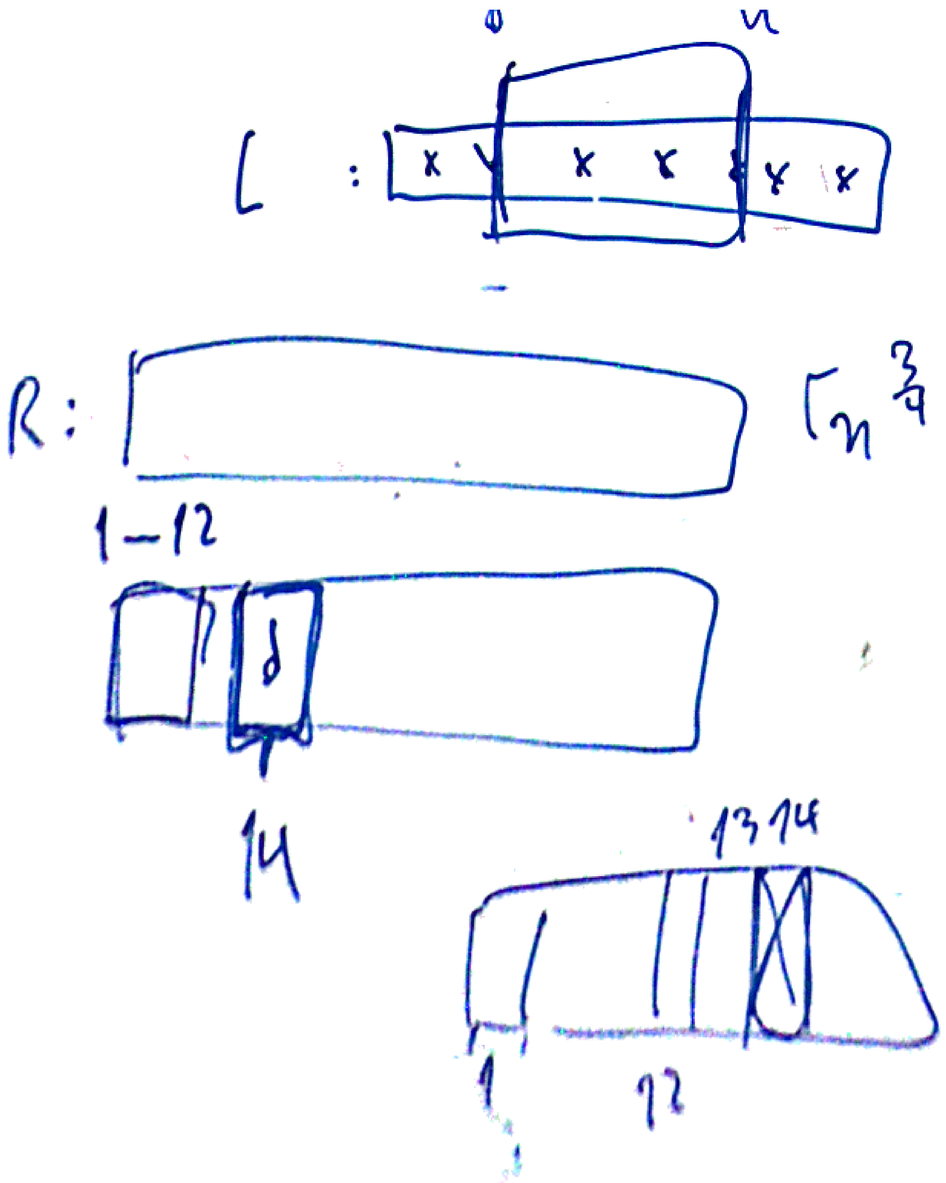
$$Pr(A \vee B \vee C) = 3p = \frac{1}{100}$$



$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil \} |$

$Pr(\text{retorna resultado}) = p$

$X = i$

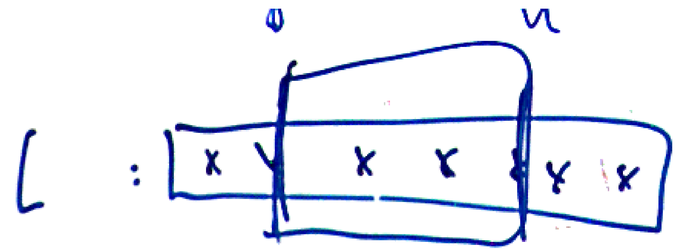


$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \right\} \right|$$

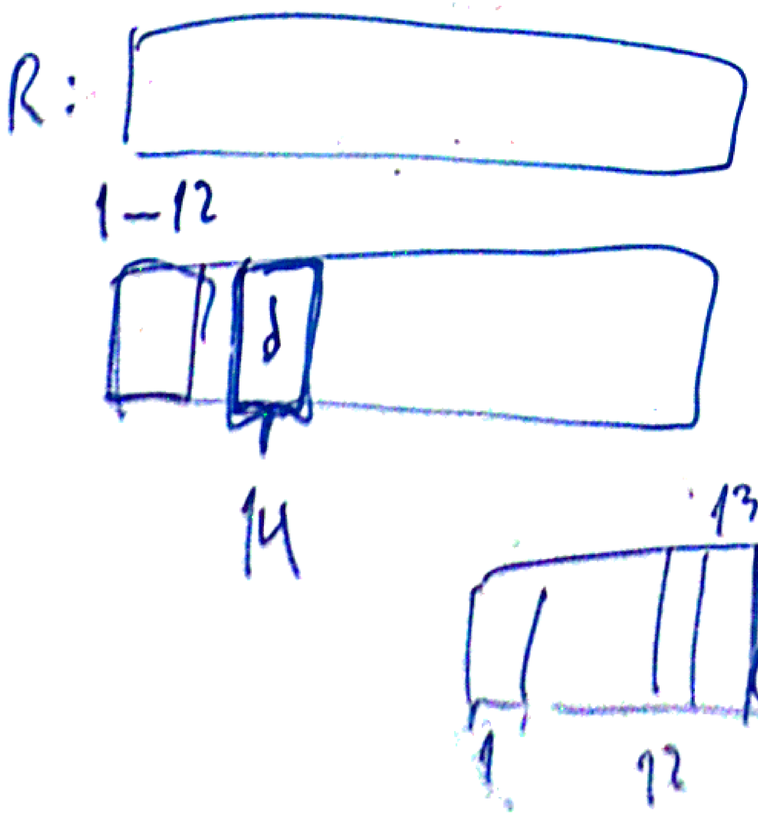
$Pr(\text{retorna resultado}) = p$

$Pr(X = i)$

$i \geq 1$



$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil \}|$

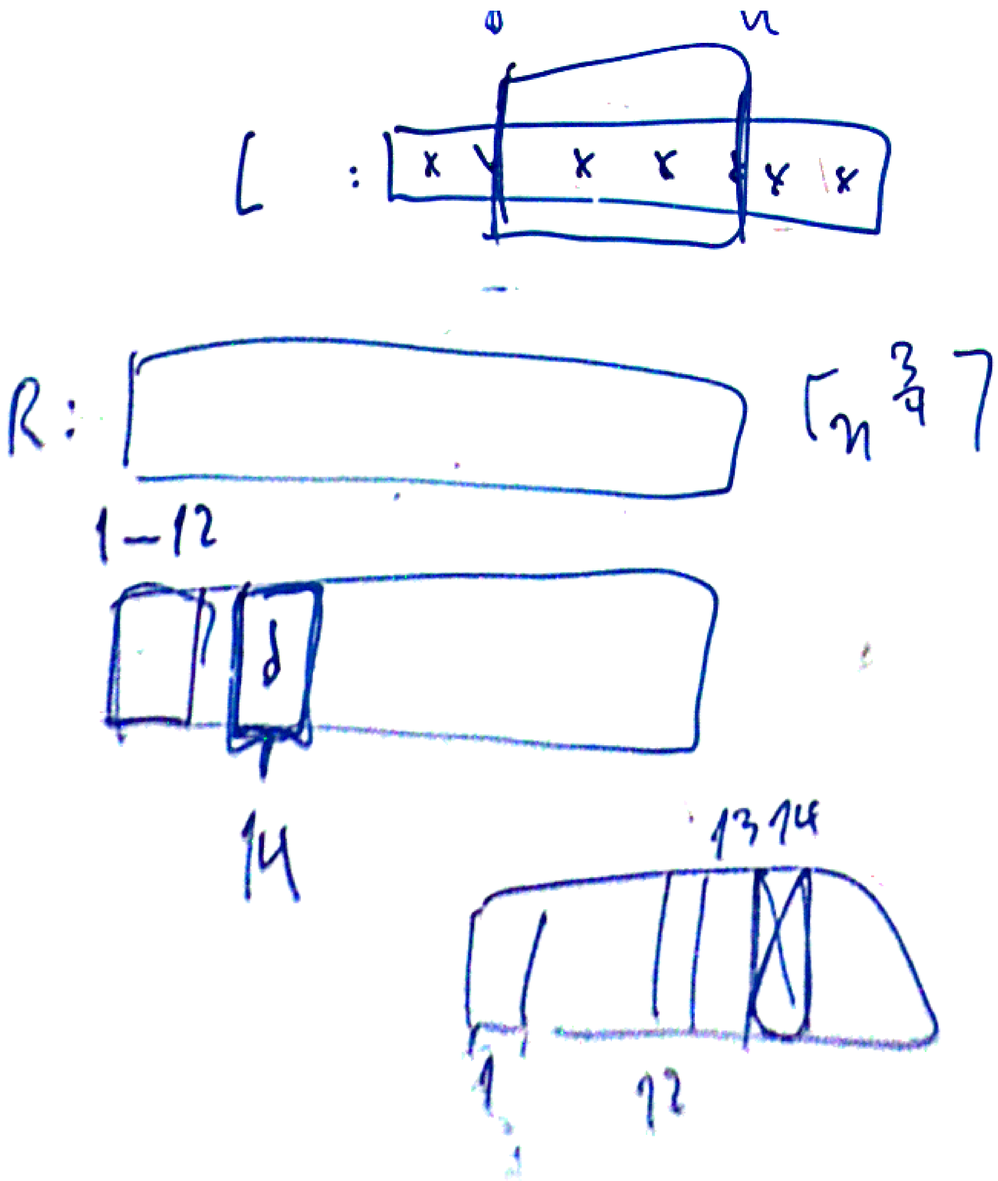


$Pr(\text{retorna resultado}) = p$

$Pr(X = i) = (1-p)^{i-1} \cdot p$

$i \geq 1$

$E(X) =$



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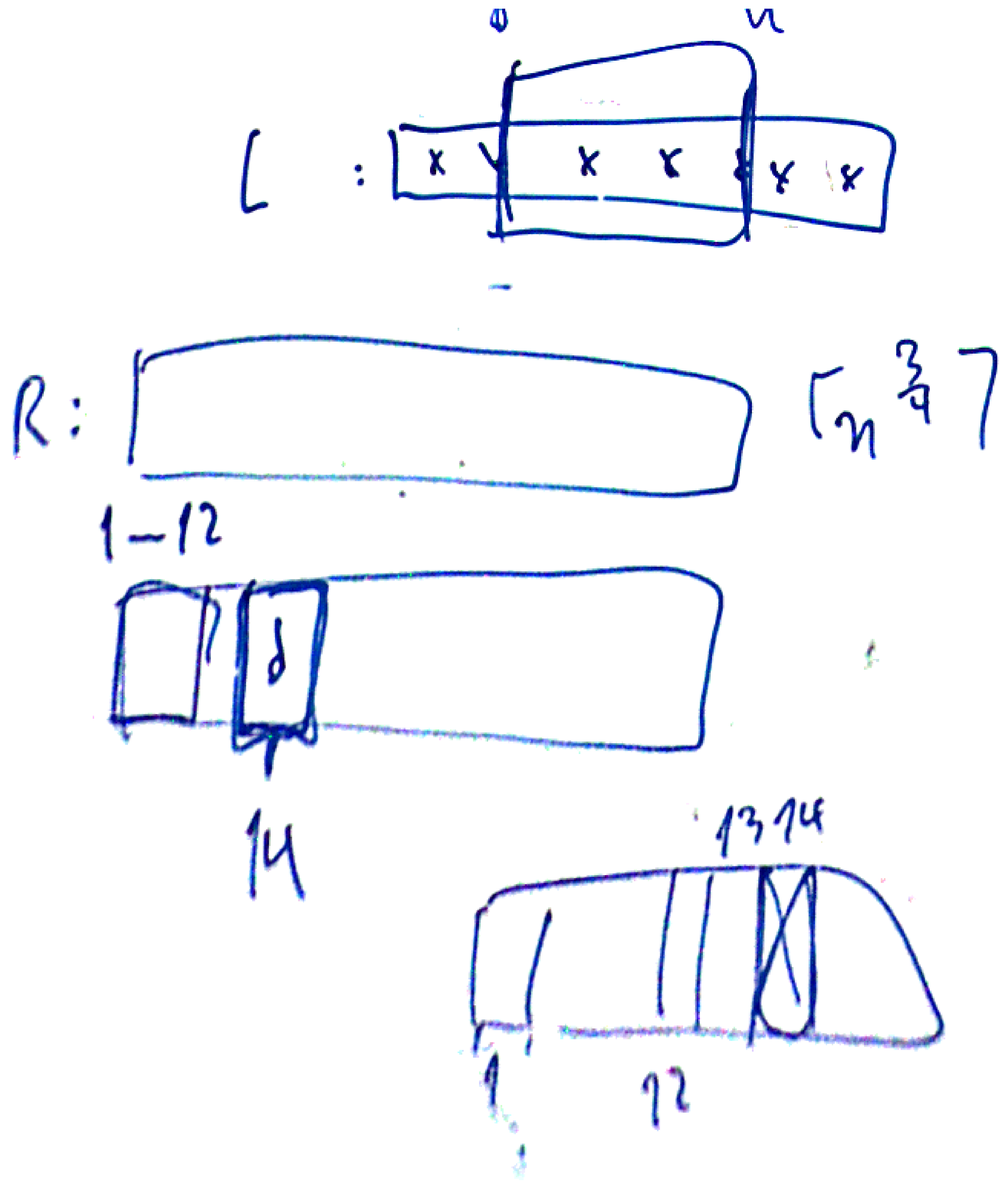


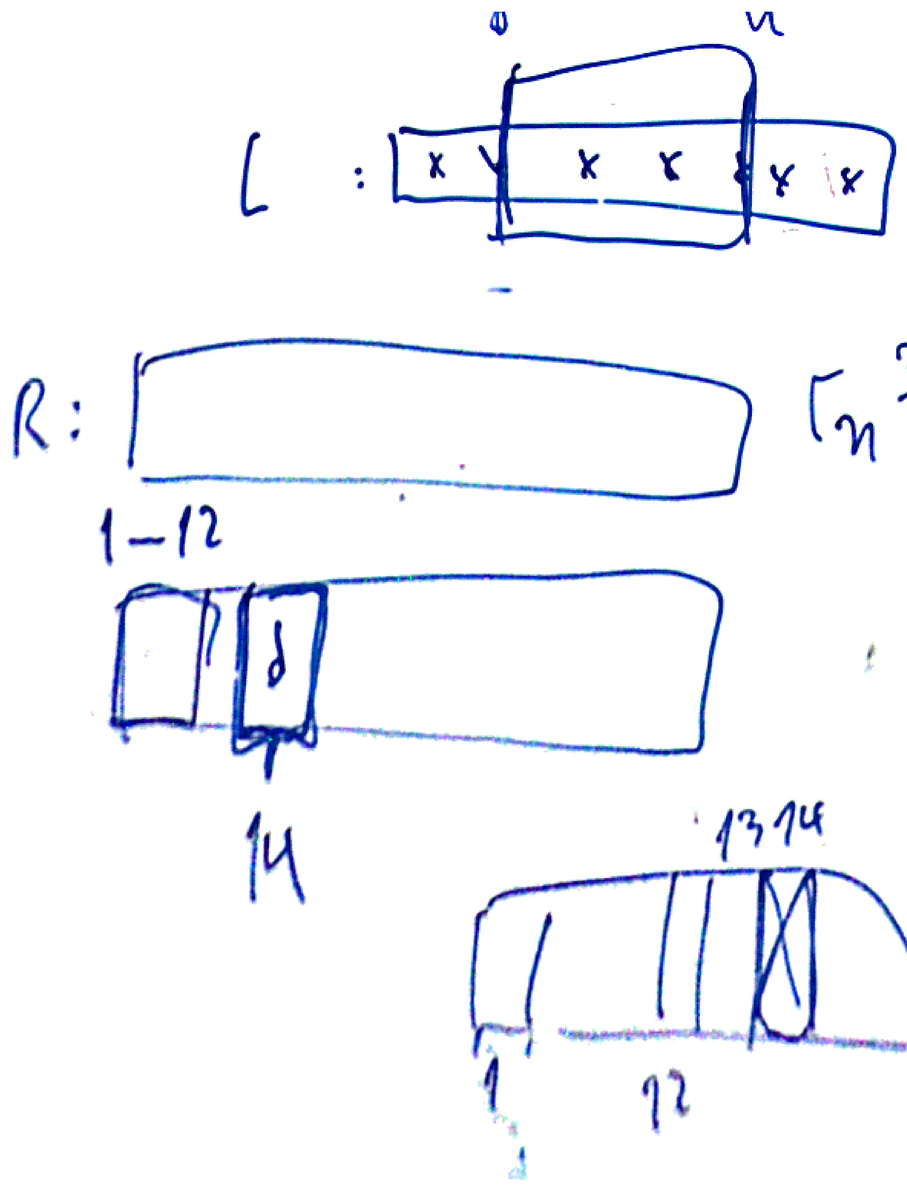
$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \right\} \right|$$

$$Pr(\text{retorna resultado}) = p$$

$$Pr(X = i) = (1-p)^{i-1} \cdot p$$

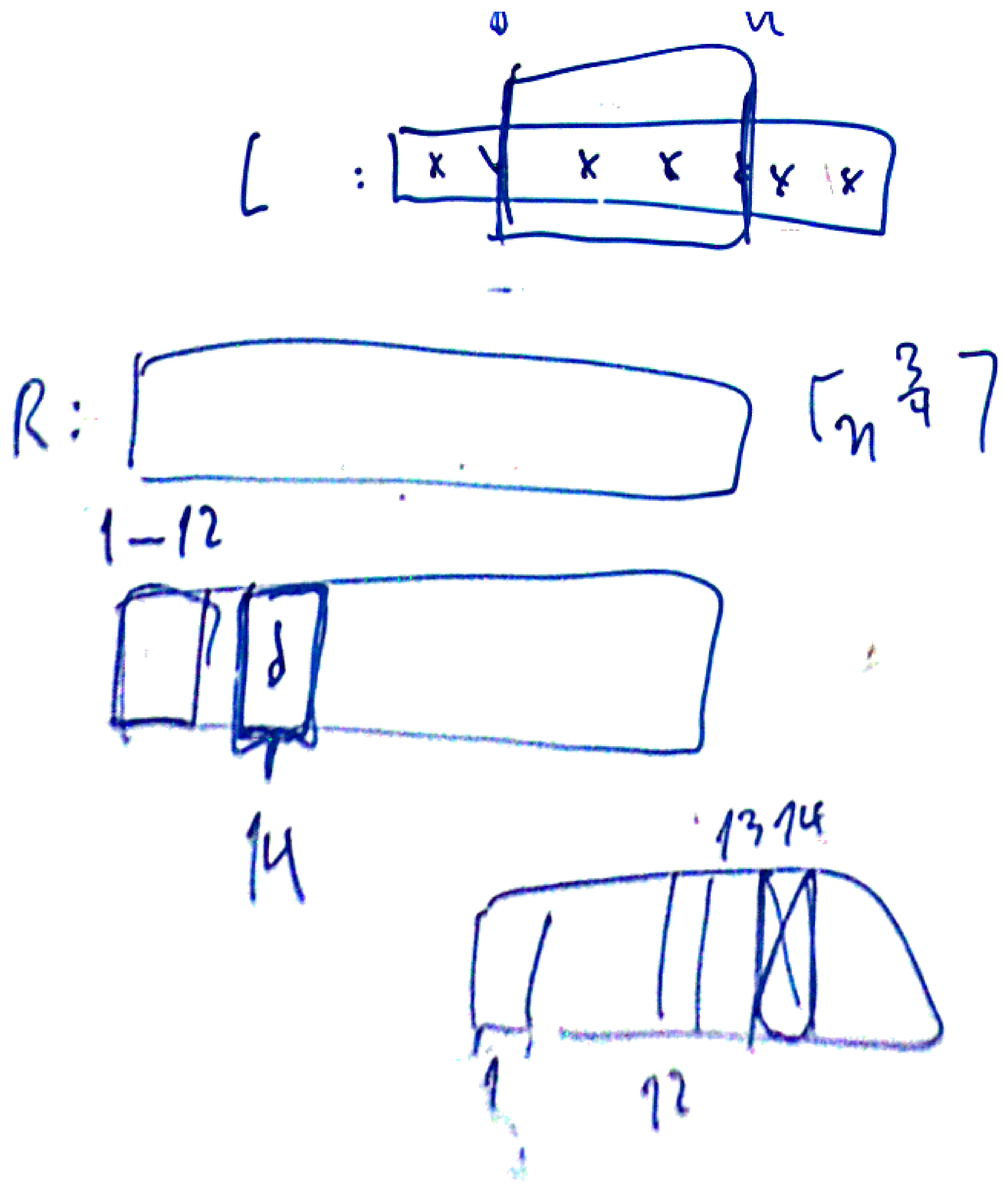
$$i \geq 1$$

$$E(X) = \frac{1}{p}$$



$$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \}|$$

$$\Pr\left(Y_1 < \left\lceil \tfrac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rceil \right) \le ?$$



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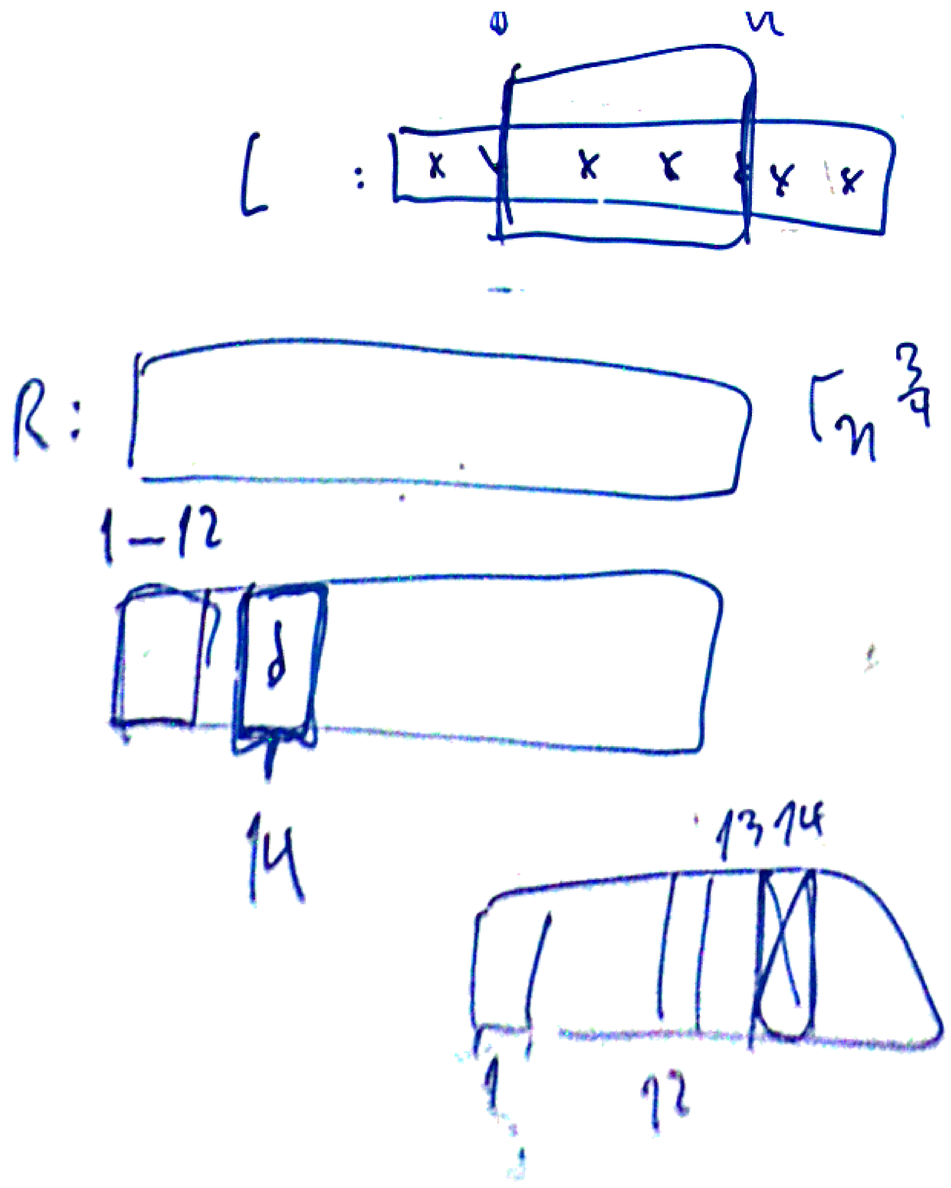

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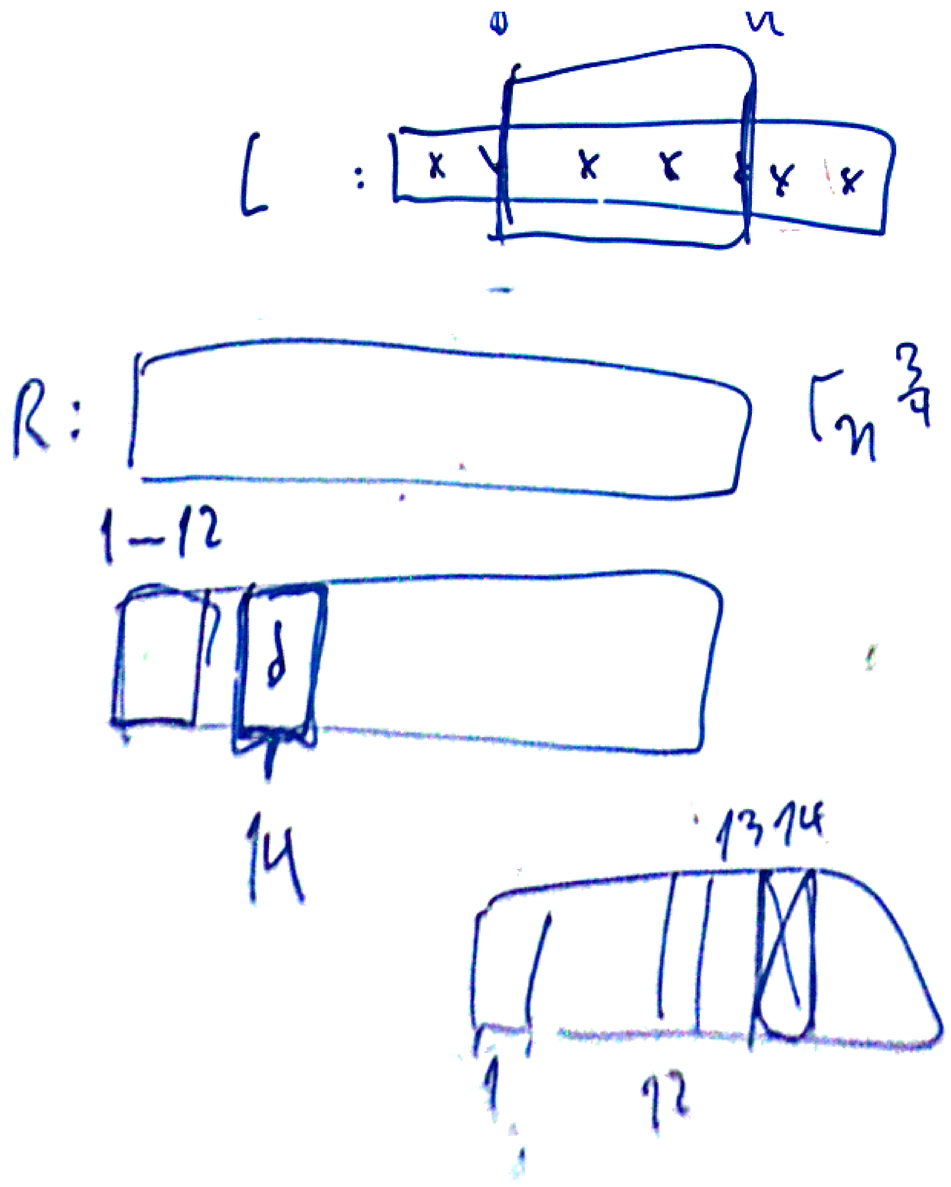

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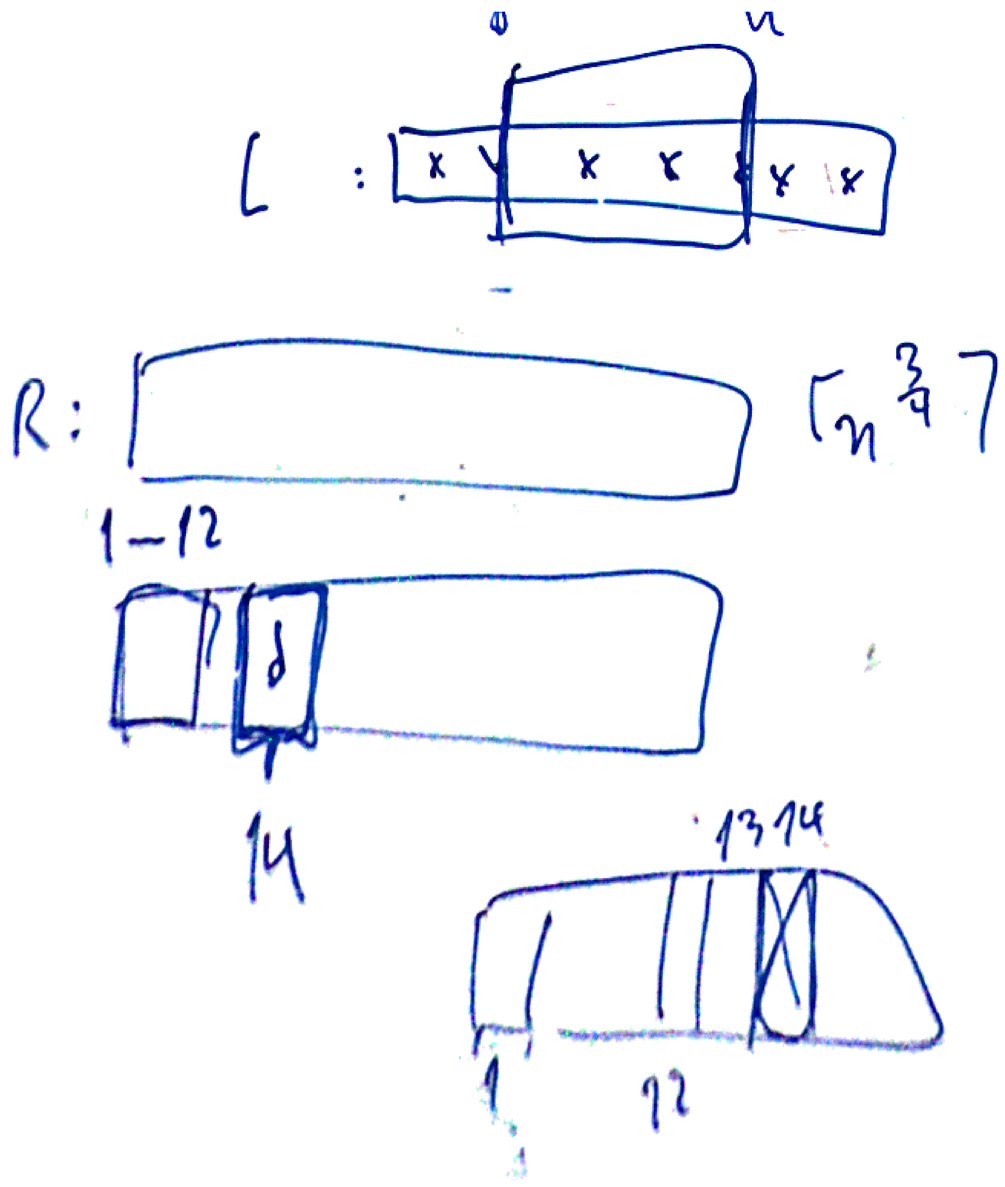

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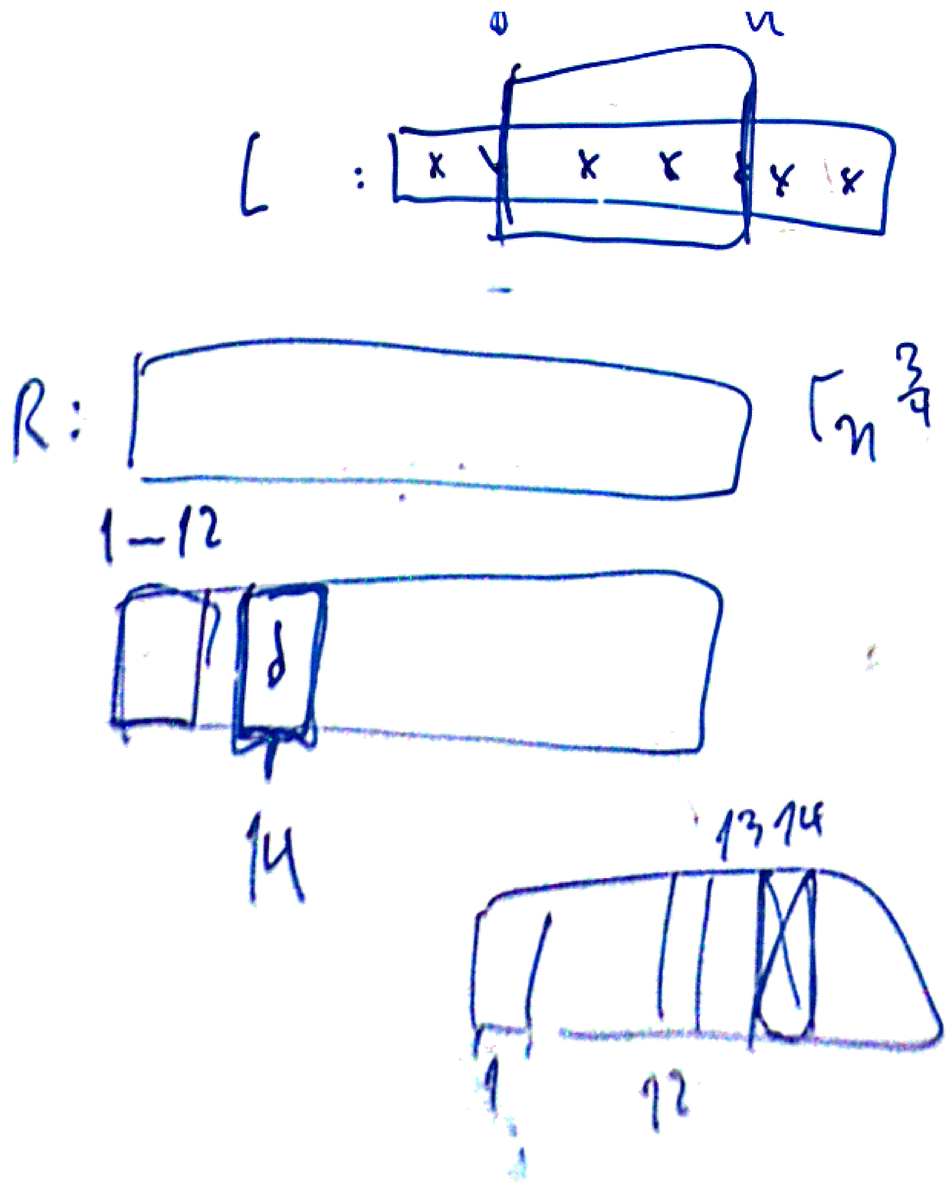

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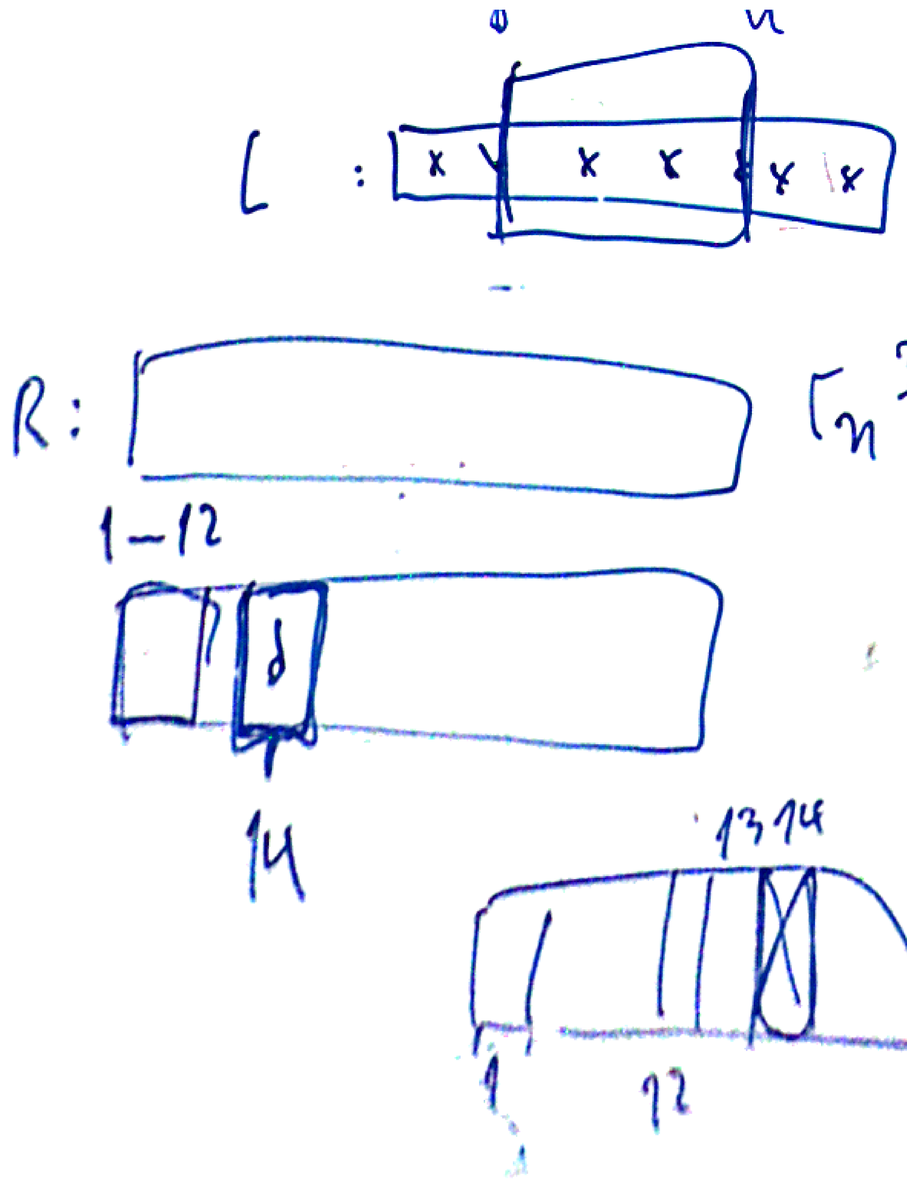
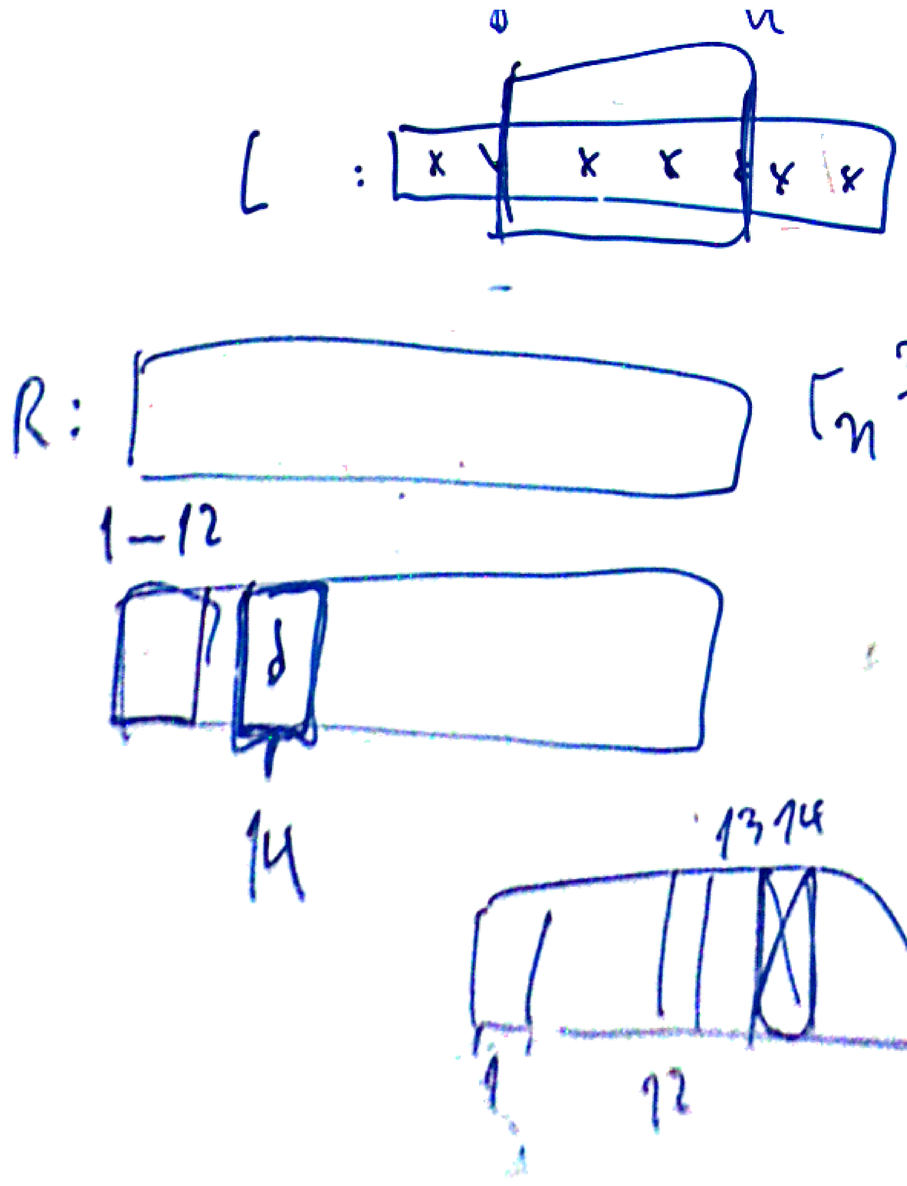


$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \}|$

$Pr(Y_1 < \lceil \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \rceil)$?



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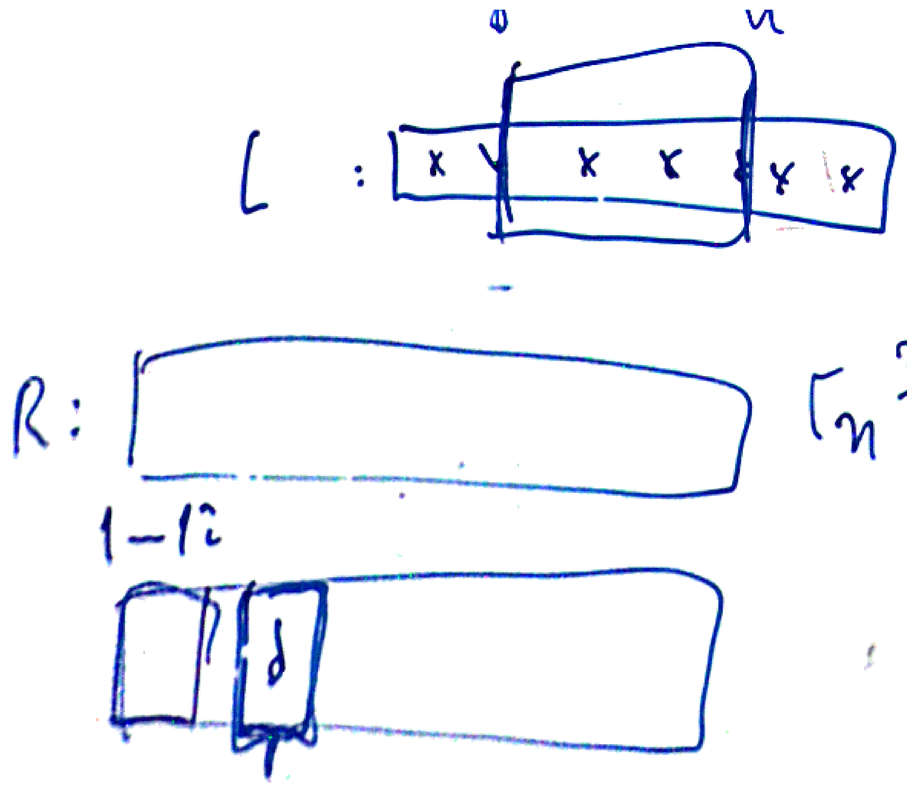

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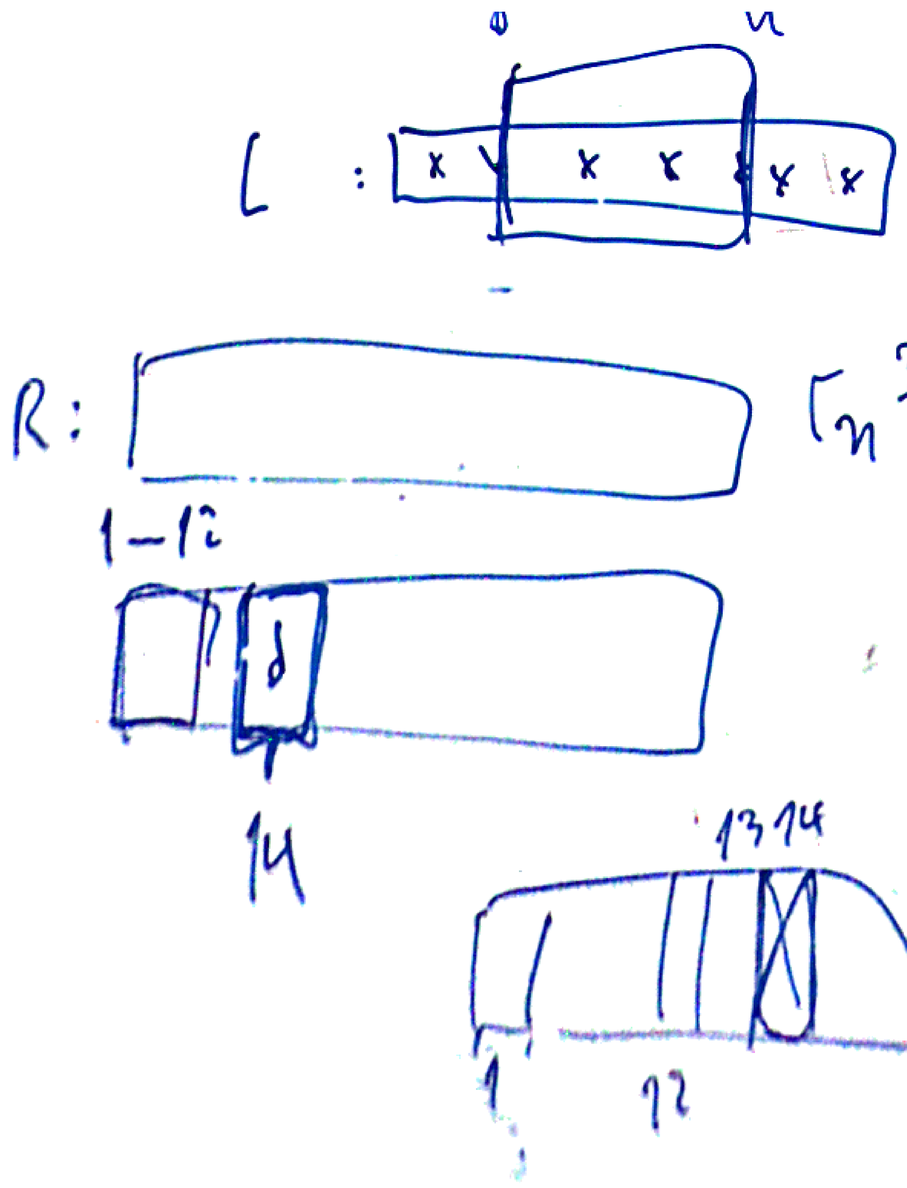

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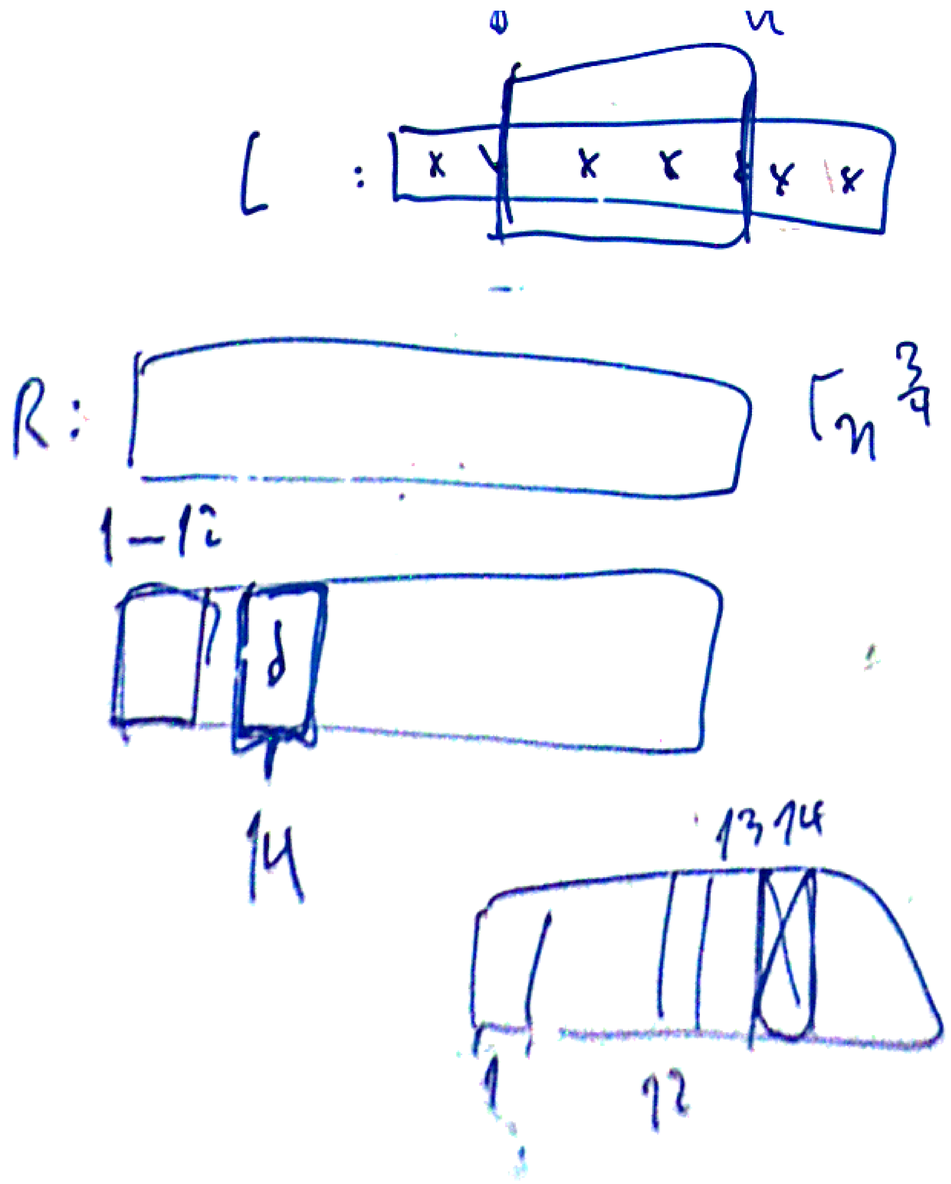

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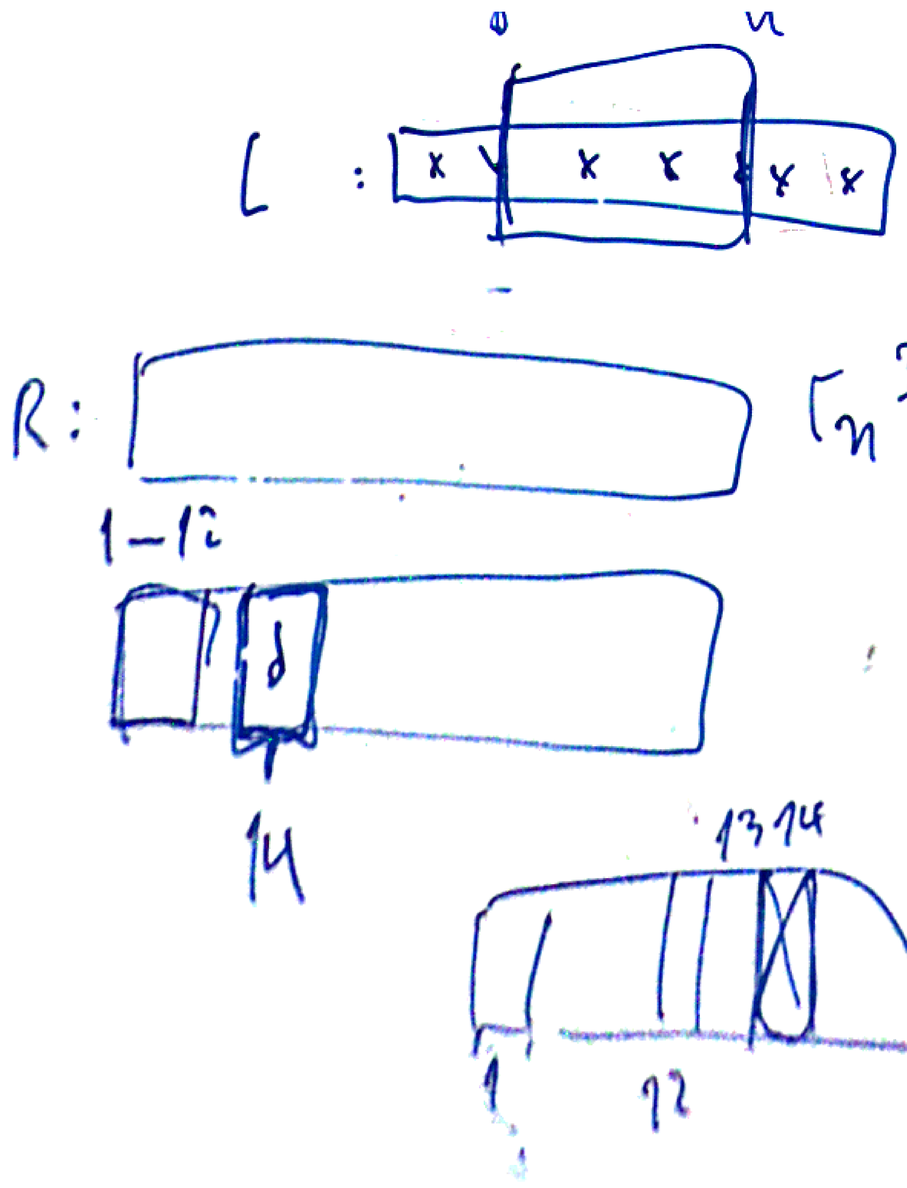

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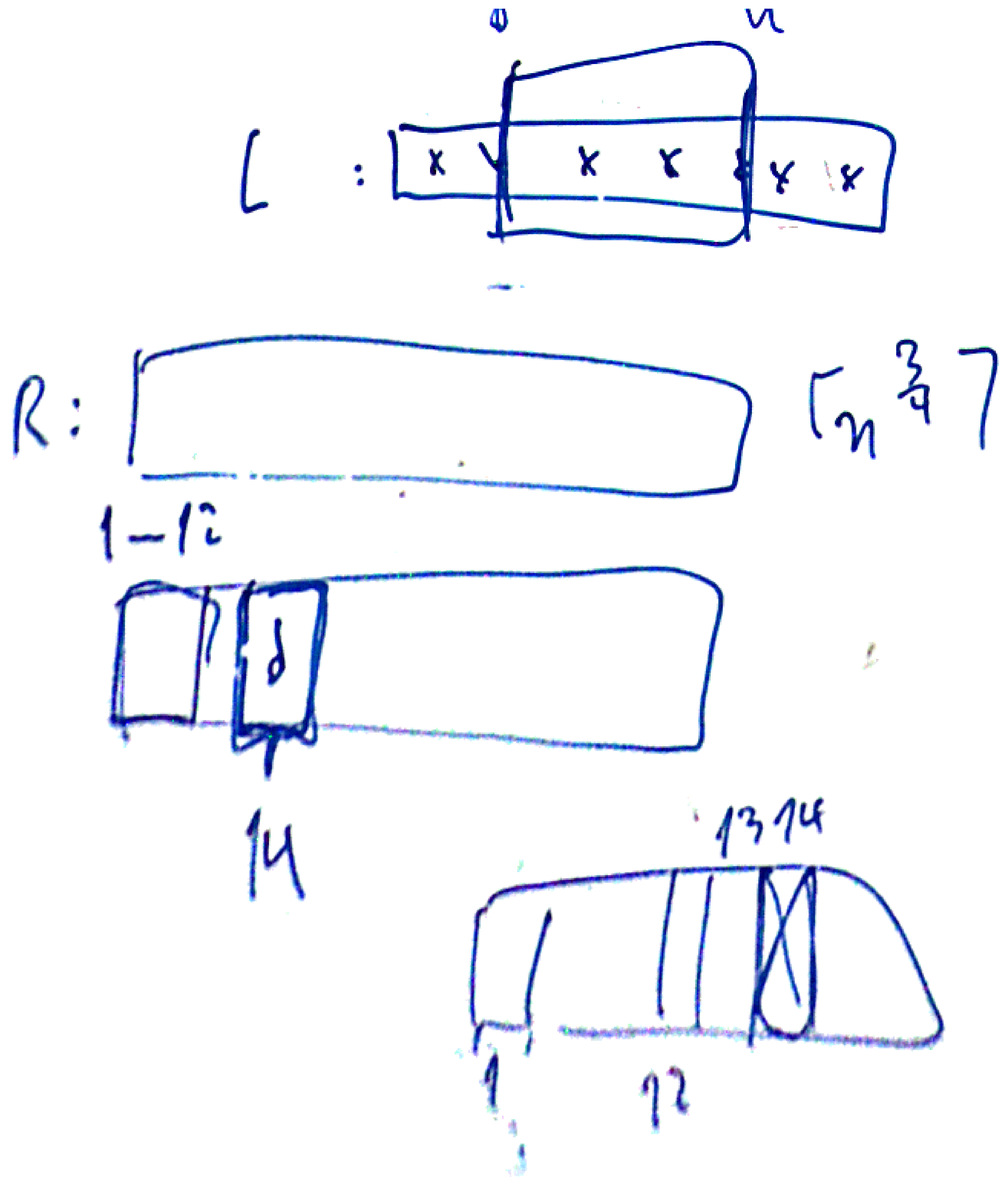
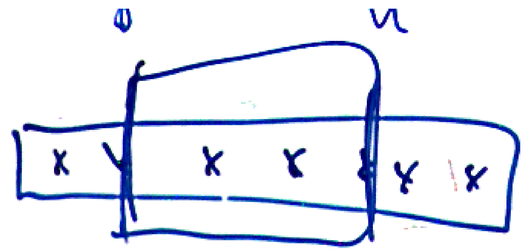


$$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \}|$$

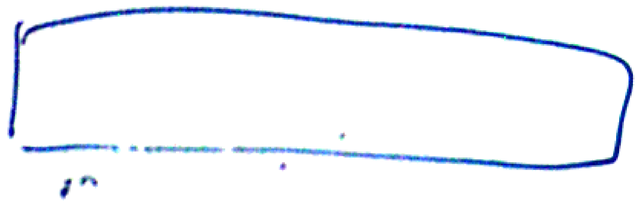
$$\Pr\left(Y_1 < \left\lceil \tfrac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rceil \right) \le ?$$

$$Y_1 = \sum$$

L :



R:



$\lceil n^{\frac{3}{4}} \rceil$

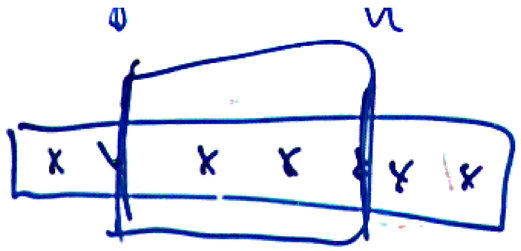
$$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \}|$$

$$\Pr\left(Y_1 < \left\lceil \tfrac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rceil \right) \le ?$$

$$X_i = \begin{cases} 0 & R[i] > m \\ 1 & R[i] \le m \end{cases}$$

$$Y_1 = \sum_{i=1}^{\lceil n^{\frac{3}{4}} \rceil} X_i$$

L :



R:

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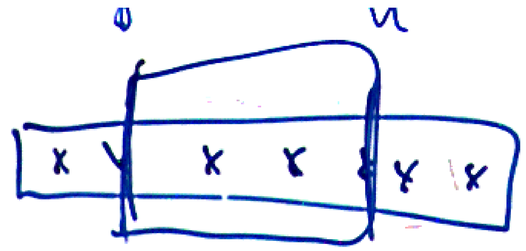
L :



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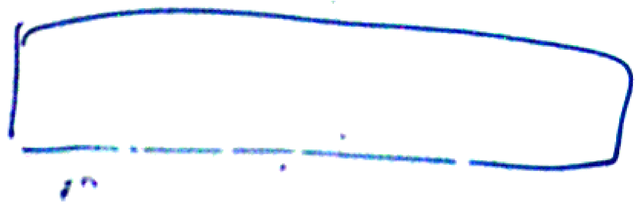
R :



$\lceil n^{\frac{3}{4}} \rceil$

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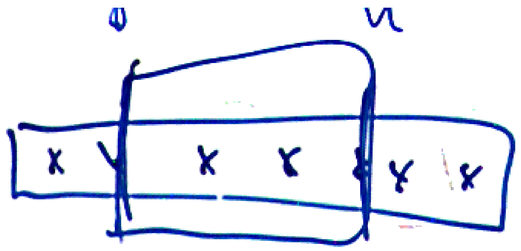
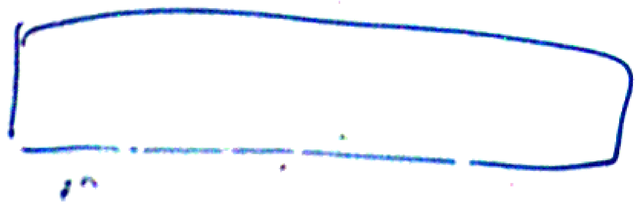


R:



$\lceil n^{\frac{3}{4}} \rceil$

$$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \}|$$

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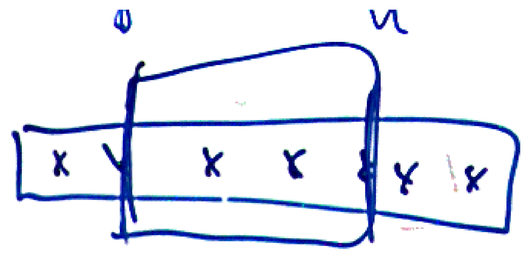
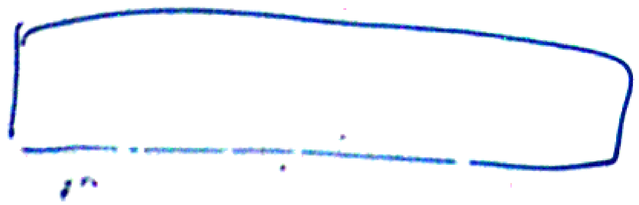
$$Y_1 = \sum_{i=1}^{\lceil n^{\frac{3}{4}} \rceil} X_i$$

L :



R:



$\lceil n^{\frac{3}{4}} \rceil$

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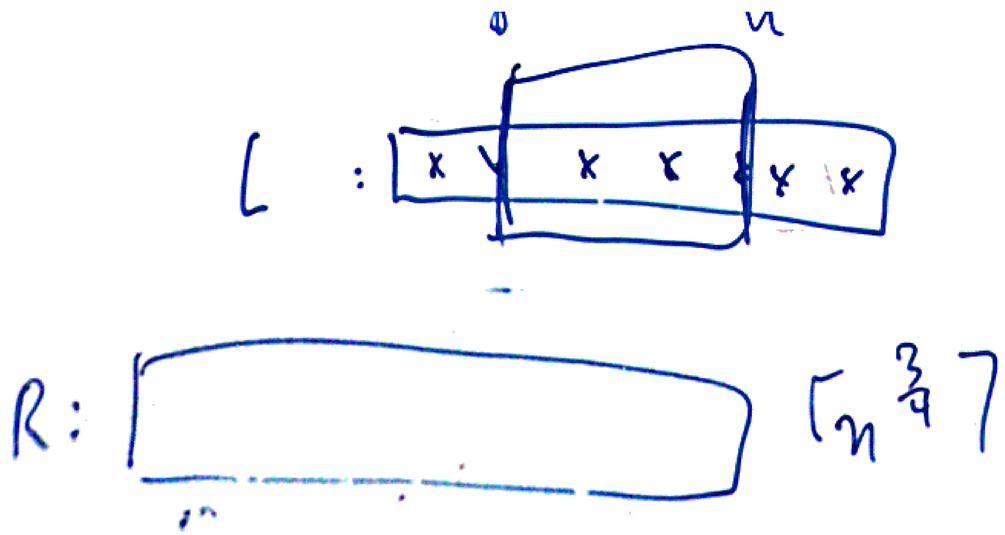

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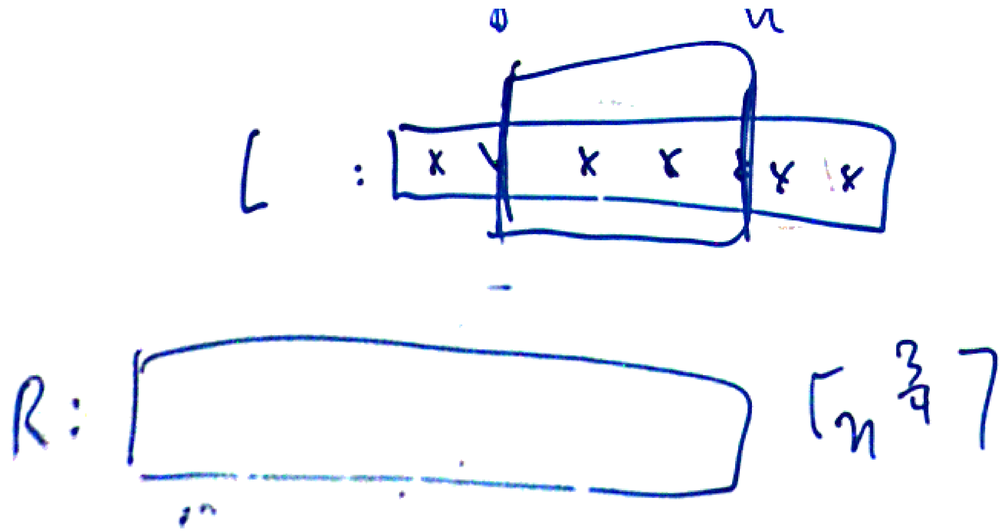

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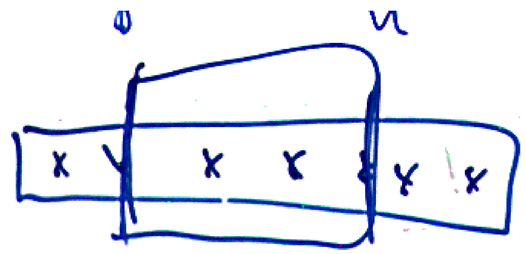
$$\Pr\left(Y_1 < \left\lceil \tfrac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rceil \right) \le ?$$

$$X_i = \begin{cases} 0 & R[i] > m \\ 1 & R[i] \le m \end{cases}$$

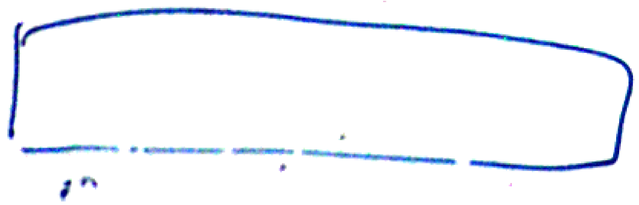
$$P(X_i = 1) =$$

L :



$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \}|$

R:



$\lceil n^{\frac{3}{4}} \rceil$

$\Pr\left(Y_1 < \left\lceil \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rceil\right) \le ?$

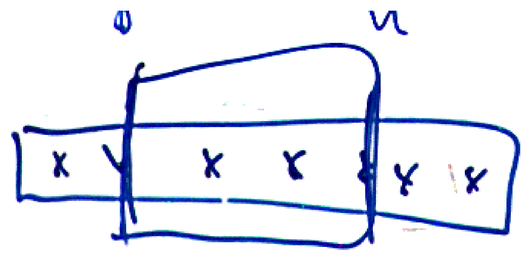
$$X_i = \begin{cases} 0 & R[i] > m \\ 1 & R[i] \le m \end{cases}$$

$P(X_i = 1) = \frac{\quad}{n}$

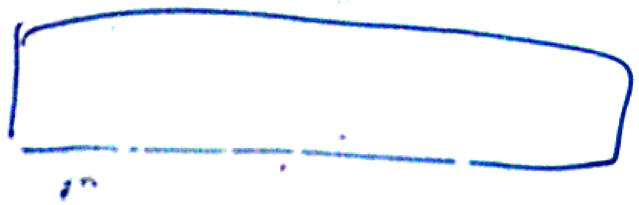
$n = 97$

$\frac{\quad}{97}$

L :



R:



$\lceil n^{\frac{3}{4}} \rceil$

$$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \}|$$

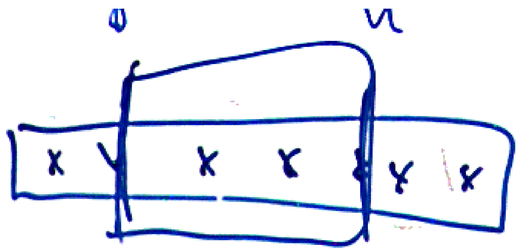
$$\Pr\left(Y_1 < \left\lceil \tfrac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rceil \right) \le ?$$

$$X_i = \begin{cases} 0 & R[i] > m \\ 1 & R[i] \le m \end{cases}$$

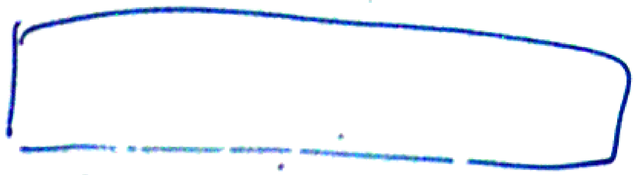
$$P(X_i = 1) = \frac{\lceil \frac{n}{2} \rceil}{n}$$

$n = 11$ $\quad \frac{6}{11}$

L :



R:



$\lceil n^{\frac{3}{4}} \rceil$

$$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \}|$$

$$\Pr\left(Y_1 < \left\lceil \tfrac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rceil \right) \le ?$$

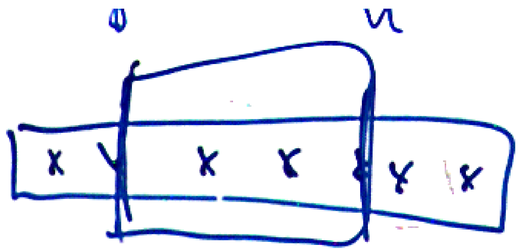
$$X_i = \begin{cases} 0 & R[i] > m \\ 1 & R[i] \le m \end{cases}$$

$\to p$

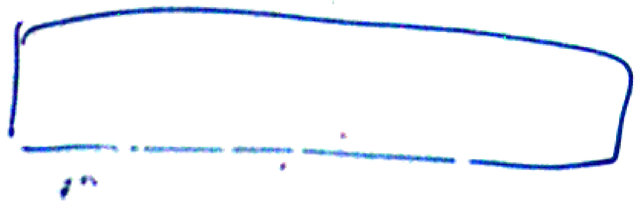
$$P(X_i = 1) = \frac{\lceil \frac{n}{2} \rceil}{n} = p$$

$$E(X_i) = p \quad Var(X_i)$$

L :



R :



$\lceil n^{\frac{3}{4}} \rceil$

$$X_i = \begin{cases} 0 & R[i] > m \\ 1 & R[i] \leq m \end{cases}$$

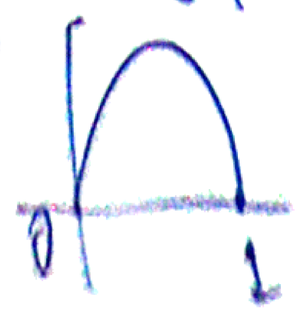
$\to p$

$$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \leq m \}|$$

$$\Pr\left(Y_1 < \left\lceil \tfrac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rceil\right) \leq ?$$

$$P(X_i = 1) = \frac{\lceil \frac{n}{2} \rceil}{n} = p$$

$$E(X_i) = p \qquad Var(X_i) = p(1-p)$$



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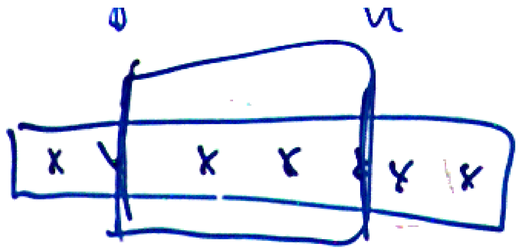
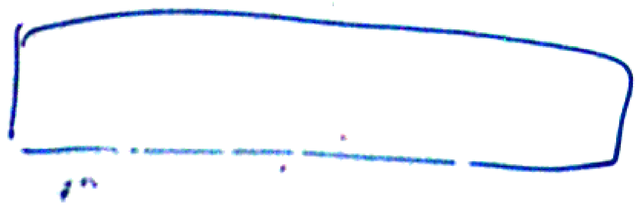


R:



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$$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \}|$$

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$$X_i = \begin{cases} 0 & R[i] > m \\ 1 & R[i] \le m \end{cases}$$

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$$P(X_i = 1) = \frac{\lceil \frac{n}{2} \rceil}{n} = p$$

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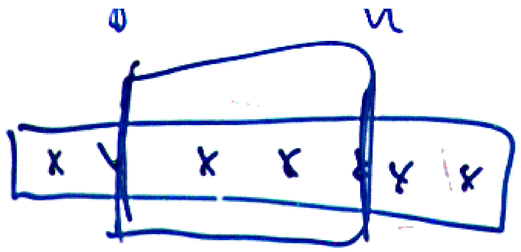
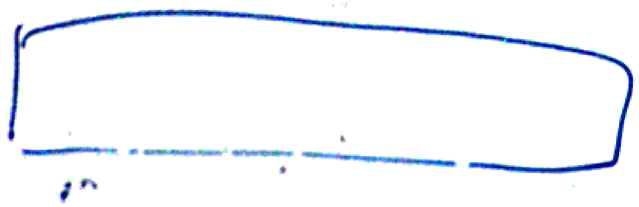
L :



$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \}|$

R:



$\lceil n^{\frac{3}{4}} \rceil$

$\Pr\left(Y_1 < \left\lceil \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rceil\right) \le ?$

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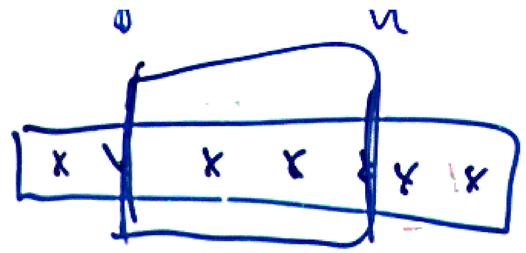
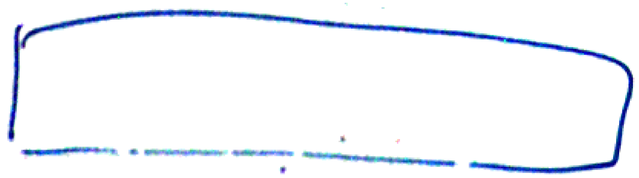
$E(X_i) = p \quad Var(X_i) = p(1-p)$

L :



R :



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$$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \}|$$

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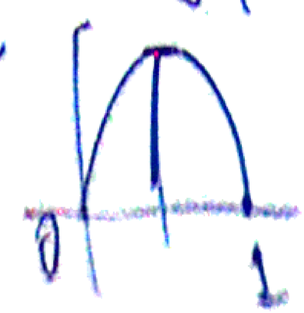
$$E(Y_1) = \lceil n^{\frac{3}{4}} \rceil \left(\frac{1}{2} + \frac{1}{2n} \right)$$

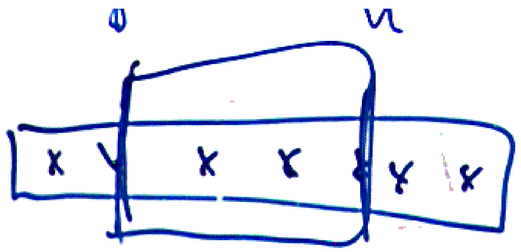
$$P(X_i = 1) = \frac{\lceil \frac{n}{2} \rceil}{n} = p$$

$$\mathrm{Var}(Y_1) \le \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil = m$$

$$E(X_i) = p \qquad \mathrm{Var}(X_i) = p(1-p)$$



L :



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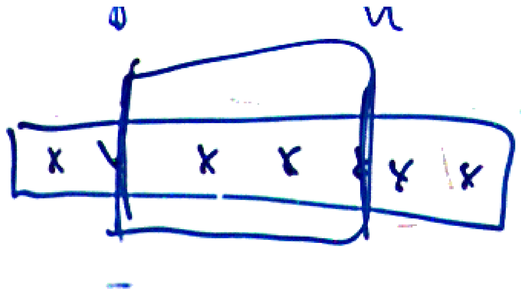
$$E(Y_1) = \lceil n^{\frac{3}{4}} \rceil \left(\frac{1}{2} + \frac{1}{2n} \right)$$

$$\Pr(|Y_1 - E(Y_1)| \ge a) \le \frac{Var(Y_1)}{a^2}$$

$$Var(Y_1) \le \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil = m$$

L :



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$$Var(Y_1) \le \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil = m$$

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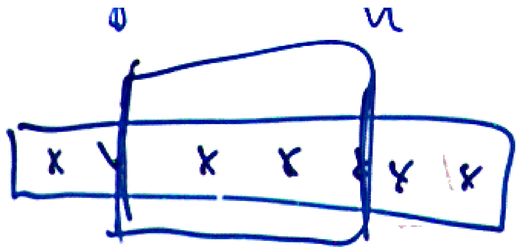

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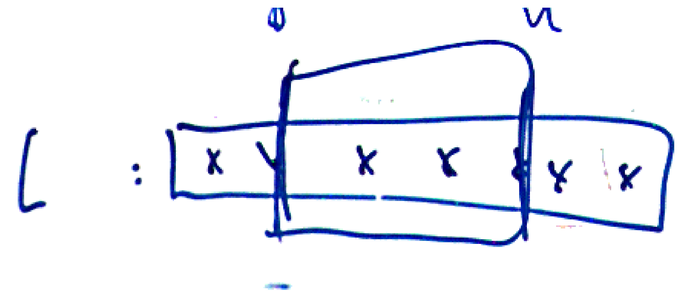
$$Var(Y_1) \le \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil = m$$



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$$Var(Y_1) \leq \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil = m$$

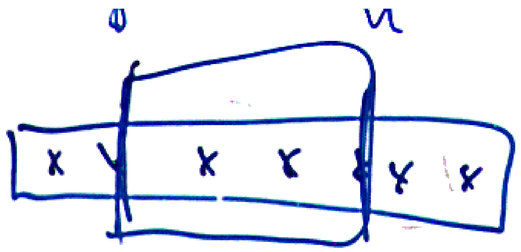
$$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \leq m \}|$$

$$Pr\left(Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor \right) \leq ?$$

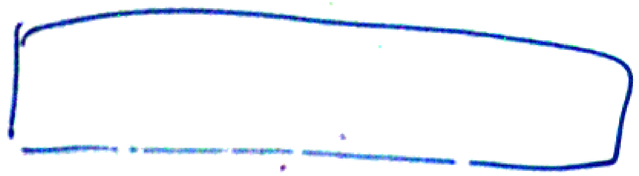
$$Pr\left(n^{\frac{1}{2}} < E(Y_1) - Y_1 \right) \leq$$

$$Pr\left(|E(Y_1) - Y_1| > n^{\frac{1}{2}} \right)$$

L :



R :



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$$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \}|$$

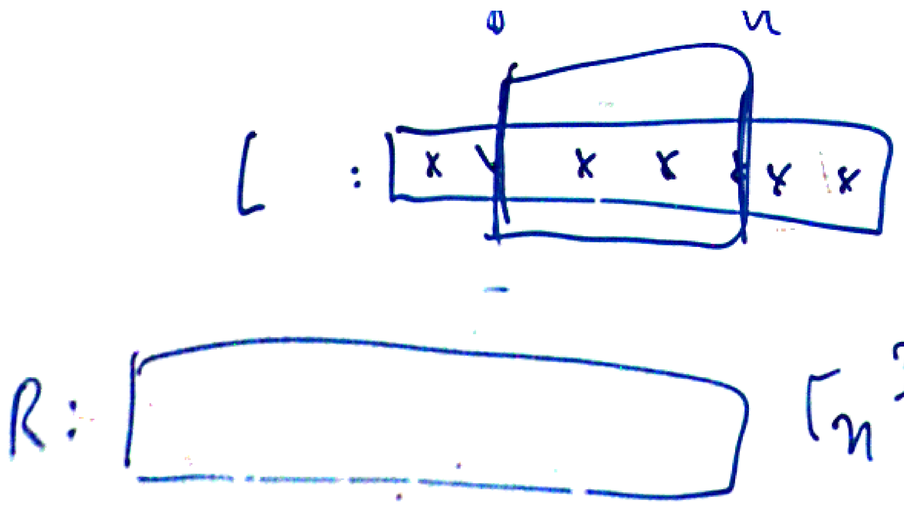
$$\Pr\left(Y_1 < \left\lfloor \tfrac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor\right) \le ?$$

$$\Pr\left(n^{\frac{1}{2}} < E(Y_1) - Y_1\right) \le$$

$$\Pr\left(|E(Y_1) - Y_1| \ge n^{\frac{1}{2}}\right)$$

$$E(Y_1) = \lceil n^{\frac{3}{4}} \rceil \left(\frac{1}{2} + \frac{1}{2n}\right)$$

$$Var(Y_1) \le \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil = m$$



$$Y_1 = \left| \{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \} \right|$$

$$E(Y_1) = \lceil n^{\frac{3}{4}} \rceil \left(\frac{1}{2} + \frac{1}{2n} \right)$$

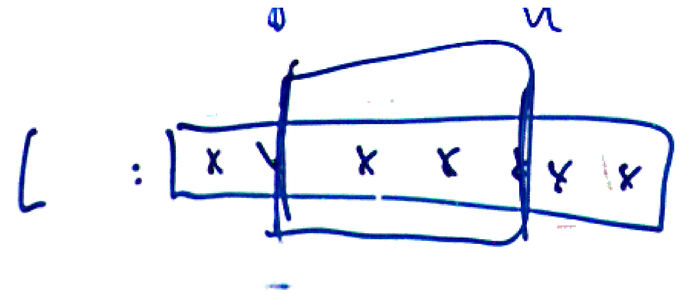
$$\mathrm{Var}(Y_1) \le \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil \quad = m$$

$$\Pr\left(Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor \right) \le ?$$

$$\Pr\left(n^{\frac{1}{2}} < E(Y_1) - Y_1 \right) \le$$

$$\Pr\left(|E(Y_1) - Y_1| \ge n^{\frac{1}{2}} \right)$$

$$\le \underline{\mathrm{Var}(Y_1)}$$



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$$Var(Y_1) \leq \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil = m$$

$$Y_1 = |\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \leq m \}|$$

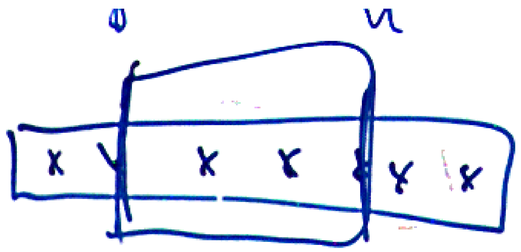
$$Pr\left(Y_1 < \left\lfloor \frac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor \right) \leq ?$$

$$Pr\left(n^{\frac{1}{2}} < E(Y_1) - Y_1 \right) \leq$$

$$Pr\left(|E(Y_1) - Y_1| \geq n^{\frac{1}{2}} \right)$$

$$\leq \frac{Var(Y_1)}{(n^{\frac{1}{2}})^2} \leq \frac{\frac{1}{4} \lceil n^{\frac{3}{4}} \rceil}{n}$$

L :



R :

$\lceil n^{\frac{3}{4}} \rceil$

$$Y_1 = \left| \left\{ i \in \{1, \dots, \lceil n^{\frac{3}{4}} \rceil\} \mid R[i] \le m \right\} \right|$$

$$\Pr\left(Y_1 < \left\lfloor \tfrac{1}{2} n^{\frac{3}{4}} - n^{\frac{1}{2}} \right\rfloor \right) \le ?$$

$$E(Y_1) = \lceil n^{\frac{3}{4}} \rceil \left(\frac{1}{2} + \frac{1}{2n} \right)$$

$$\Pr\left(n^{\frac{1}{2}} < E(Y_1) - Y_1 \right) \le$$

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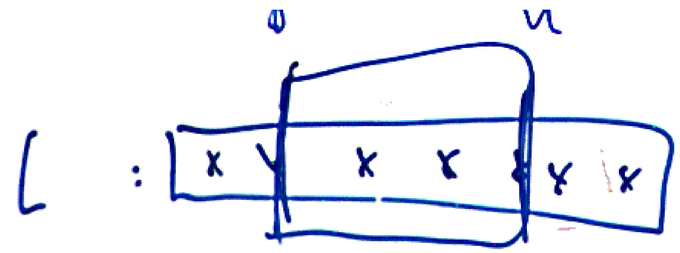
$$\le \frac{\mathrm{Var}(Y_1)}{(n^{\frac{1}{2}})^2} \le \frac{\frac{1}{4} \lceil n^{\frac{3}{4}} \rceil}{n}$$

$$\mathrm{Var}(Y_1) \le \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil = m$$



$\lceil n^{\frac{3}{4}} \rceil$

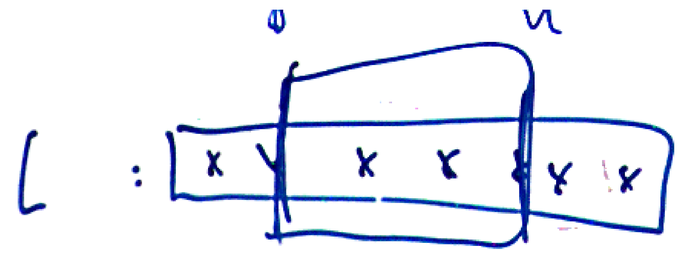
$$E(Y_1) = \lceil n^{\frac{3}{4}} \rceil \left(\frac{1}{2} + \frac{1}{2n} \right)$$

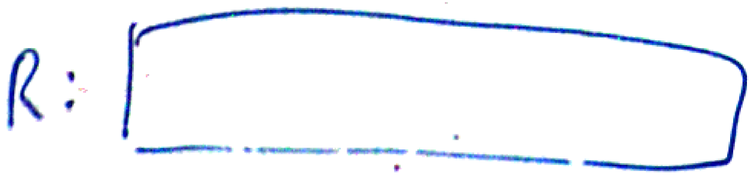
$$Var(Y_1) \leq \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil = m$$

$$Pr(A) \leq n^{-\frac{1}{4}}$$

$$Pr(B) \leq n^{-\frac{1}{4}}$$

$$Pr(C) \leq 4 \cdot n^{+\frac{1}{4}}$$





$\lceil n^{\frac{3}{4}} \rceil$

$$E(Y_1) = \lceil n^{\frac{3}{4}} \rceil \left(\frac{1}{2} + \frac{1}{2n} \right)$$

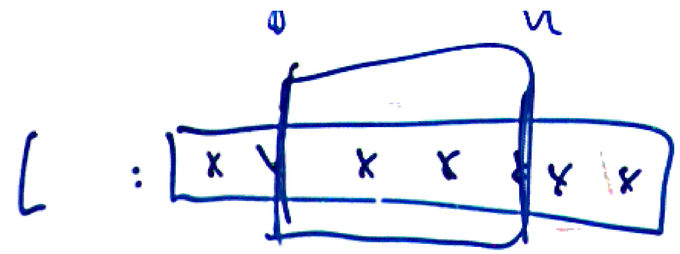
$$Var(Y_1) \leq \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil \quad = m$$

$$Pr(A) \leq n^{-\frac{1}{4}}$$

$$Pr(B) \leq n^{-\frac{1}{4}}$$

$$Pr(C) \leq 4 \cdot n^{+\frac{1}{4}}$$

$$Pr(A \vee B \vee C) \leq 6 \cdot n^{-\frac{1}{4}}$$



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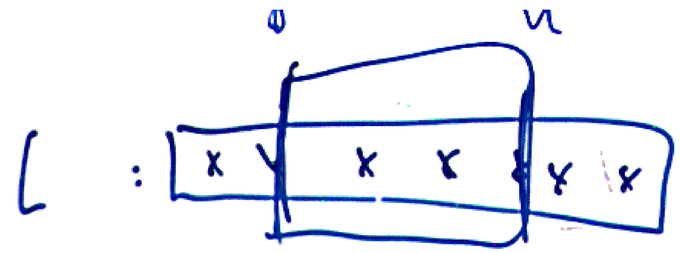

$\lceil n^{\frac{3}{4}} \rceil$

$$E(Y_1) = \lceil n^{\frac{3}{4}} \rceil \left(\frac{1}{2} + \frac{1}{2n} \right)$$

$$Var(Y_1) \leq \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil$$

$$Pr(A) \leq n^{-\frac{1}{4}}$$

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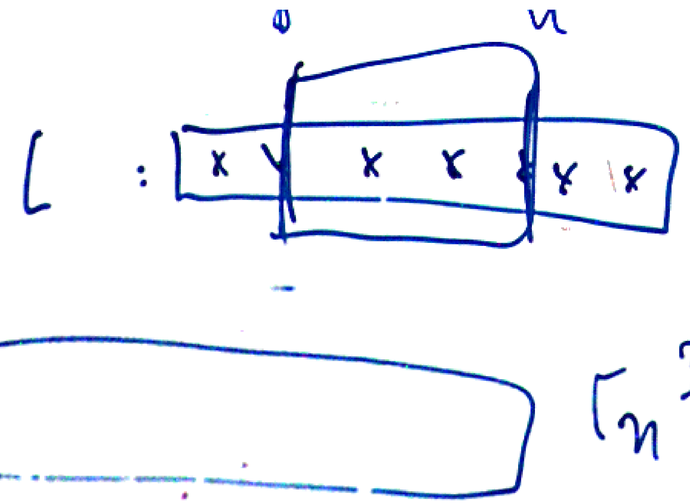
$$Pr(C) \leq 4 \cdot n^{+\frac{1}{4}}$$

$$Pr(A \vee B \vee C) \leq 6 \cdot n^{-\frac{1}{4}}$$

$$Pr(\text{no retorna}) \leq \frac{1}{n}$$



$$E(Y_1) = \lceil n^{\frac{3}{4}} \rceil \left(\frac{1}{2} + \frac{1}{2n} \right)$$

$$Var(Y_1) \leq \frac{1}{4} \lceil n^{\frac{3}{4}} \rceil \quad = m$$

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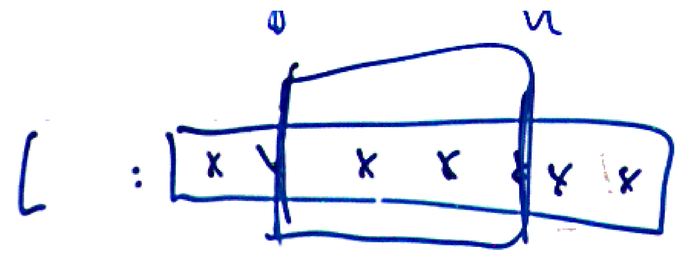
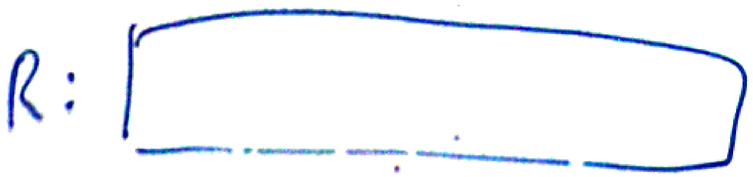
$$Pr(C) \leq 4 \cdot n^{+\frac{1}{4}}$$

$$Pr(A \vee B \vee C) \leq 6 \cdot n^{-\frac{1}{4}}$$

$$Pr(\text{no retorna}) \leq \frac{1}{10}$$

$$Pr(\text{retorna}) \geq \frac{1}{10}$$





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$$Pr(A \vee B \vee C) \leq 6 \cdot n^{-\frac{1}{4}}$$

$$Pr(\text{no retorna}) \leq \frac{9}{10}$$

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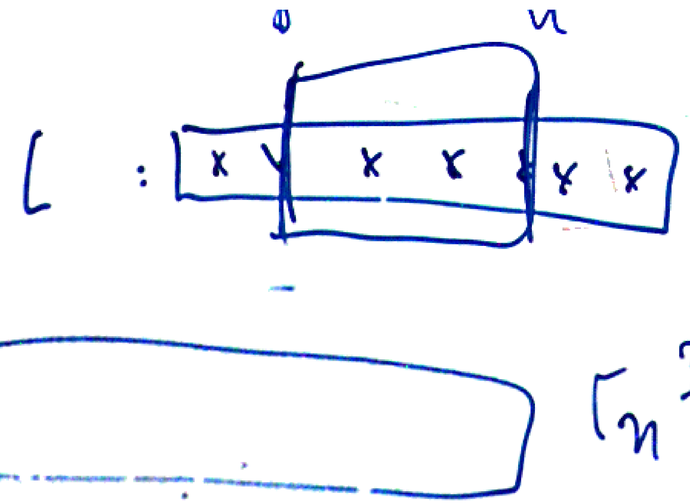

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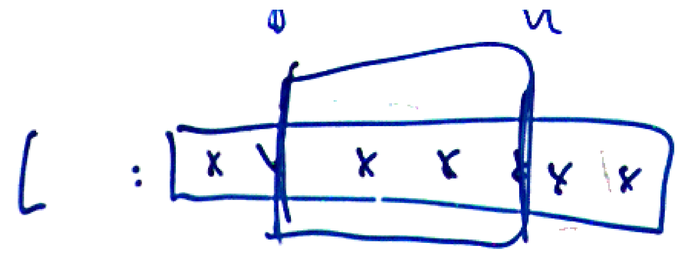

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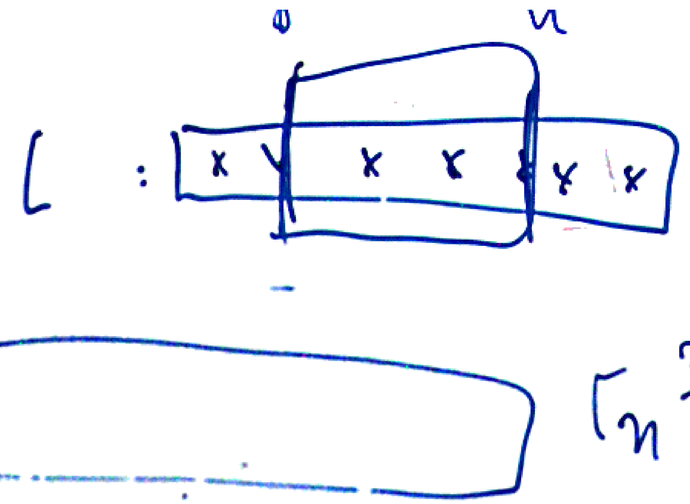

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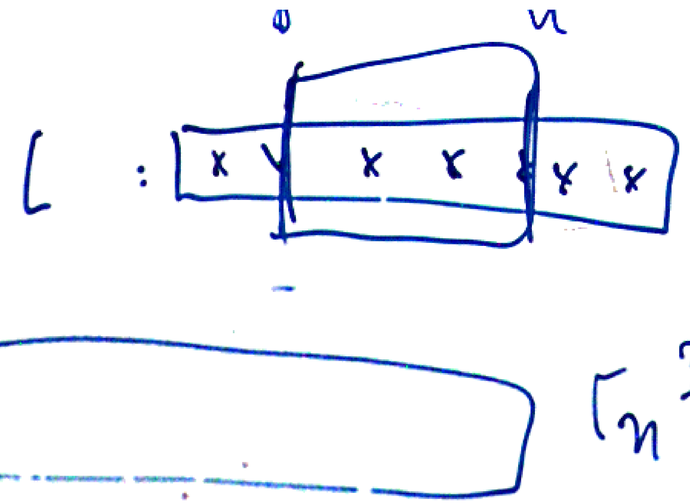

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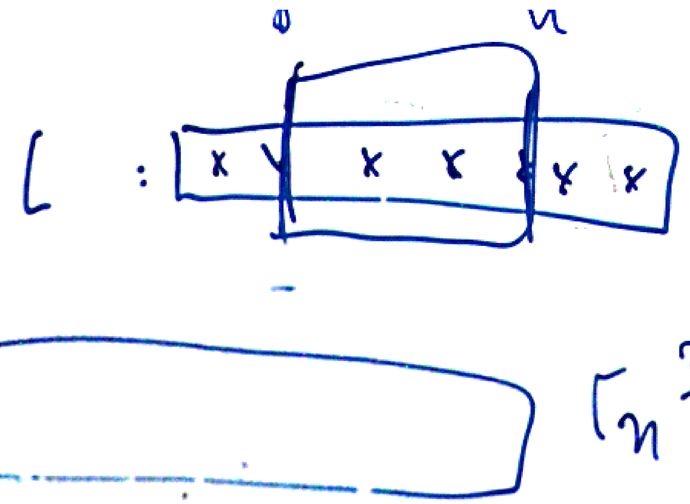

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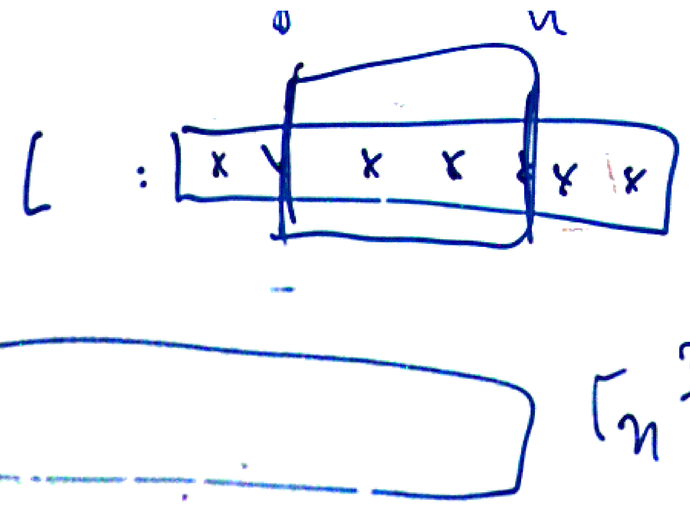

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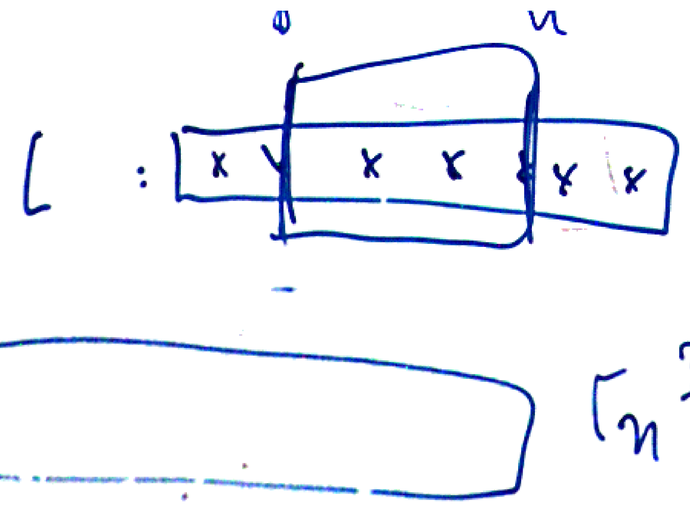

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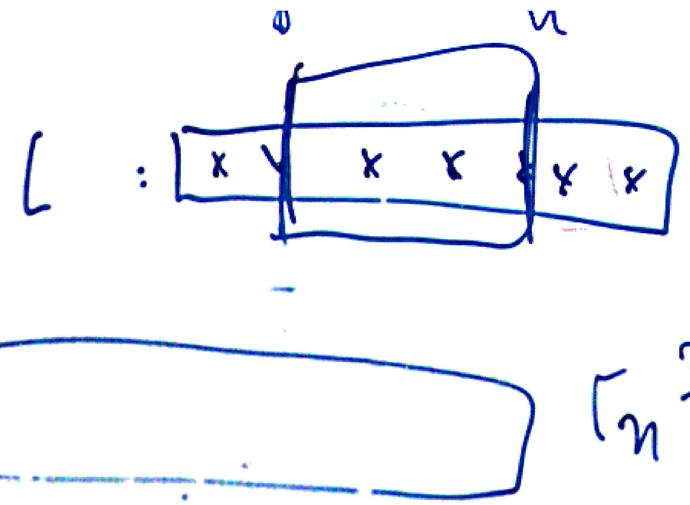

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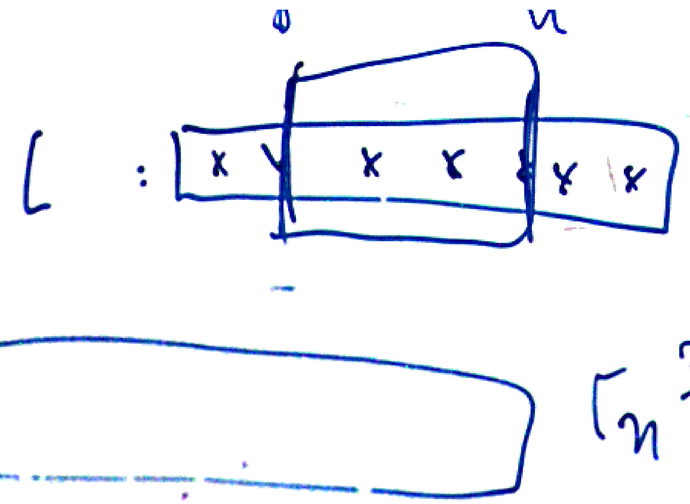

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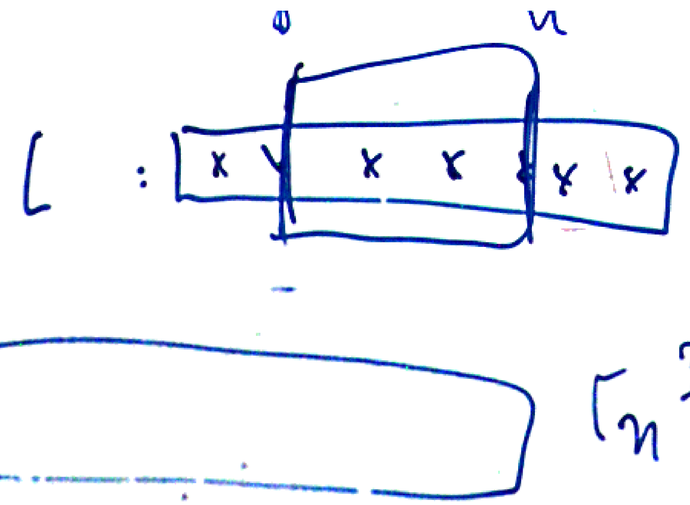

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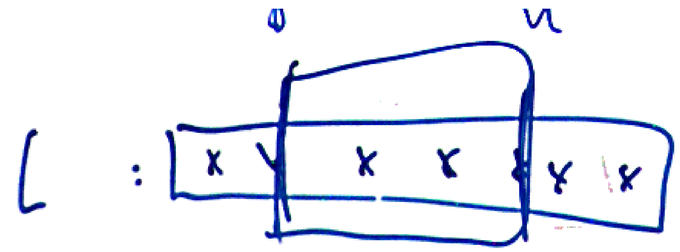

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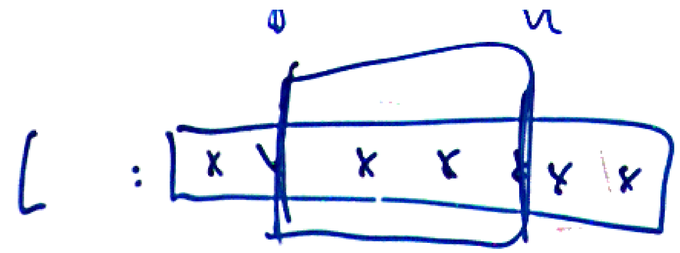

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