Documento - Análisis Factorial - Marcelo Chávez

April 6, 2023

1 Construcción de Índices con Análisis Multivariante

1.1 Descripción

El presente documento sirve para la aprobación del curso sobre Construcción de Índices con Análisis Multivariante.

1.2 Contenido

- Librerías a emplearse
- Lectura de la base de datos
- Descripción de la base de datos
- Perfilamiento de variables
- KMO
- Gráfico de Factores
- Matriz de Componentes Rotados
- Comunalidades
- Análisis de la Varianza
- Selección de Factores
- Resultados del AF

1.3 Fecha de elaboración

- 26 de marzo de 2023

1.4 Lenguaje de programación

- Python 3.10

1.5 Desarrollado por:

- Ing. Marcelo Chávez

1. Librerías a utilizar:

[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from factor_analyzer import FactorAnalyzer
import warnings
```

```
warnings.filterwarnings('ignore')
```

2. Lectura de la base de datos fuente: Para el presente ejercicio se ha utilizado el data set de Kaggle: **COVID-19 effect on Pollution**

[2]:
```
db_pollution = pd.read_csv(r"C:
↪\Users\marcelochavez\Documents\INDICES_INDICADORES\db\station_day.csv",
                          dtype={"Outcome":str})
db_pollution
```

[2]:
```
       StationId        Date  PM2.5    PM10    NO    NO2    NOx    NH3    CO  \
0          AP001  2017-11-24  71.36  115.75  1.75  20.65  12.40  12.19  0.10
1          AP001  2017-11-25  81.40  124.50  1.44  20.50  12.08  10.72  0.12
2          AP001  2017-11-26  78.32  129.06  1.26  26.00  14.85  10.28  0.14
3          AP001  2017-11-27  88.76  135.32  6.60  30.85  21.77  12.91  0.11
4          AP001  2017-11-28  64.18  104.09  2.56  28.07  17.01  11.42  0.09
…          …           …     …       …     …     …      …      …     …
99716      WB013  2020-04-27  11.51   31.71  1.73  10.21  11.93  21.52  0.54
99717      WB013  2020-04-28   9.38   33.70  1.77  12.39  14.15  22.44  0.64
99718      WB013  2020-04-29  16.86   32.17  2.00  14.26  16.27  24.19  0.57
99719      WB013  2020-04-30  15.17   32.10  2.02  12.40  14.41  25.58  0.61
99720      WB013  2020-05-01  17.05   29.15  1.70  10.54  12.22  24.21  0.65

         SO2      O3  Benzene  Toluene  Xylene    AQI    AQI_Bucket
0      10.76  109.26     0.17     5.92    0.10    NaN           NaN
1      15.24  127.09     0.20     6.50    0.06  184.0      Moderate
2      26.96  117.44     0.22     7.95    0.08  197.0      Moderate
3      33.59  111.81     0.29     7.63    0.12  198.0      Moderate
4      19.00  138.18     0.17     5.02    0.07  188.0      Moderate
…        …       …        …        …     …      …             …
99716   6.92   28.90     1.33     8.96     NaN   38.0          Good
99717   6.09   26.34     1.35     9.47     NaN   39.0          Good
99718   7.27   29.62     1.66    10.45     NaN   44.0          Good
99719  11.53   33.93     0.96     8.92     NaN   46.0          Good
99720   6.51   23.48     0.77     7.91     NaN   57.0  Satisfactory

[99721 rows x 16 columns]
```

[3]:
```
print("El nro de registros del data set es:", db_pollution.shape[0])
print("El nro de variables del data set es:", db_pollution.shape[1])
```

```
El nro de registros del data set es: 99721
El nro de variables del data set es: 16
```

3. Descripción de la base de datos:

About this Dataset

Context Air is what keeps humans alive. Monitoring it and understanding its quality is of immense importance to our well-being.

Content The dataset contains air quality data and AQI (Air Quality Index) at hourly and daily level of various stations across multiple cities in India.

A tutorial of how AQI is calculated is available here: https://www.kaggle.com/rohanrao/calculating-aqi-air-quality-index

Cities Ahmedabad Aizawl Amaravati Amritsar Bengaluru Bhopal Brajrajnagar Chandigarh Chennai Delhi Ernakulam Gurugram Guwahati Hyderabad Jaipur Jorapokhar Kochi Kolkata Lucknow Mumbai Patna Shillong Talcher Thiruvananthapuram Acknowledgements

The data has been made publicly available by the Central Pollution Control Board: https://cpcb.nic.in/ which is the official portal of

Fuente URL: https://www.kaggle.com/code/parulpandey/breathe-india-covid-19-effect-on-pollution/notebook

3. Perfilamiento de variables:

[4]:
```python
def perfilamiento(df):
    # Catálogo de variables:
    catalogo_variables = pd.DataFrame(df.dtypes).rename(columns={0:
 ↪'tipo_variable'})
    catalogo_variables['len_max'] = [columnData.str.len().max() if columnData.
 ↪dtype == object else columnData.max() for columnName, columnData in df.
 ↪items()]
    catalogo_variables['len_min'] = [columnData.str.len().min() if columnData.
 ↪dtype == object else columnData.min() for columnName, columnData in df.
 ↪items()]
    catalogo_variables = pd.concat([catalogo_variables, df.isnull().sum()],␣
 ↪axis=1).rename(columns={0:'absoluto_missing'})
    catalogo_variables = catalogo_variables.loc[catalogo_variables.index !=␣
 ↪"Unnamed: 0"]
    porcentaje_missing = pd.DataFrame(((df.isnull().sum() / len(df))*100).
 ↪round(2))
    porcentaje_missing = porcentaje_missing.loc[porcentaje_missing.index !=␣
 ↪"Unnamed: 0"].rename(columns={0:"porcentaje_missing"})
    # selección de las columnas numéricas
    num_cols = df.select_dtypes(include=['float64', 'int64']).columns.tolist()
    # resumen estadístico descriptivo de las columnas numéricas
    resumen_estadistico = df[num_cols].describe().round(2)
    # cálculo de la correlación de las columnas numéricas
    corr_matrix = df[num_cols].corr()
    catalogo_variables = pd.concat([catalogo_variables, porcentaje_missing],␣
 ↪ignore_index=False, axis=1)
    return  catalogo_variables, resumen_estadistico, corr_matrix
```

```
catalogo_variables, resumen_estadistico, corr_matrix =␣
 ↪perfilamiento(db_pollution)
```

Tipo de variable y missings:

[5]:
```
catalogo_variables
```

[5]:
```
           tipo_variable  len_max  len_min  absoluto_missing
StationId         object     5.00     5.00                 0  \
Date              object    10.00    10.00                 0
PM2.5            float64  1000.00     0.02             20737
PM10             float64  1000.00     0.01             41021
NO               float64   470.00     0.01             16277
NO2              float64   448.05     0.01             15764
NOx              float64   467.63     0.00             14915
NH3              float64   418.90     0.01             45811
CO               float64   175.81     0.00             12491
SO2              float64   195.65     0.01             23977
O3               float64   963.00     0.01             24508
Benzene          float64   455.03     0.00             29670
Toluene          float64   454.85     0.00             36265
Xylene           float64   170.37     0.00             79749
AQI              float64  2049.00     8.00             20128
AQI_Bucket        object    12.00     4.00             20128

            porcentaje_missing
StationId                 0.00
Date                      0.00
PM2.5                    20.80
PM10                     41.14
NO                       16.32
NO2                      15.81
NOx                      14.96
NH3                      45.94
CO                       12.53
SO2                      24.04
O3                       24.58
Benzene                  29.75
Toluene                  36.37
Xylene                   79.97
AQI                      20.18
AQI_Bucket               20.18
```

ANÁLISIS DE MISSING VALUES: Todas las variables que miden la contaminación del aire presentan valores perdidos, para lo cual se proceder a eliminar todos los registros que tienen missing values. En la siguiente línea se encuentra dicho proceso:

[6]:
```
db_pollution_new = db_pollution.copy().dropna()
print("El nro de registros del data set es:", db_pollution_new.shape[0])
print("El nro de variables del data set es:", db_pollution_new.shape[1])
```

```
El nro de registros del data set es: 8430
El nro de variables del data set es: 16
```

[7]:
```
catalogo_variables, resumen_estadistico, corr_matrix =␣
↪perfilamiento(db_pollution_new)
```

[8]:
```
catalogo_variables
```

[8]:
```
           tipo_variable  len_max  len_min  absoluto_missing
StationId         object     5.00     5.00                 0  \
Date              object    10.00    10.00                 0
PM2.5            float64   734.56     1.31                 0
PM10             float64   830.10     5.77                 0
NO               float64   262.00     0.10                 0
NO2              float64   254.78     0.10                 0
NOx              float64   331.50     0.02                 0
NH3              float64   269.93     0.10                 0
CO               float64     4.74     0.00                 0
SO2              float64    67.26     0.10                 0
O3               float64   138.18     0.03                 0
Benzene          float64   165.41     0.00                 0
Toluene          float64   259.03     0.00                 0
Xylene           float64   133.60     0.00                 0
AQI              float64   692.00    19.00                 0
AQI_Bucket        object    12.00     4.00                 0

            porcentaje_missing
StationId                  0.0
Date                       0.0
PM2.5                      0.0
PM10                       0.0
NO                         0.0
NO2                        0.0
NOx                        0.0
NH3                        0.0
CO                         0.0
SO2                        0.0
O3                         0.0
Benzene                    0.0
Toluene                    0.0
Xylene                     0.0
AQI                        0.0
AQI_Bucket                 0.0
```

Análisis Exploratorio de variables:

[9]:
```
resumen_estadistico
```

[9]:
```
         PM2.5     PM10       NO      NO2      NOx      NH3       CO      SO2  \
count  8430.00  8430.00  8430.00  8430.00  8430.00  8430.00  8430.00  8430.00
mean     55.31   111.85    12.57    34.19    31.25    18.15     0.70     9.53
std      45.50    70.06    20.25    23.75    30.02    13.04     0.45     7.84
min       1.31     5.77     0.10     0.10     0.02     0.10     0.00     0.10
25%      27.45    64.83     2.59    15.47    12.97     9.94     0.42     4.00
50%      46.26   101.63     6.12    28.99    22.75    15.19     0.62     7.47
75%      68.41   141.58    12.99    48.41    38.24    23.43     0.89    12.39
max     734.56   830.10   262.00   254.78   331.50   269.93     4.74    67.26

            O3  Benzene  Toluene   Xylene      AQI
count  8430.00  8430.00  8430.00  8430.00  8430.00
mean     31.51     4.65    12.77     2.90   123.80
std      18.25    14.79    25.83     7.61    78.25
min       0.03     0.00     0.00     0.00    19.00
25%      18.83     0.18     1.20     0.10    74.00
50%      28.22     1.12     4.07     0.70   107.00
75%      40.77     3.63    12.92     2.68   142.00
max     138.18   165.41   259.03   133.60   692.00
```

Coeficiente de variación:

[10]:
```
def coef_variacion(df):
    # selecciona solo las columnas numéricas
    num_cols = df.select_dtypes(include=['float64', 'int64']).columns.tolist()
    des_std = df[num_cols].std()/df[num_cols].mean()
    return des_std
```

[11]:
```
coef_variacion(db_pollution_new)
```

[11]:
```
PM2.5      0.822619
PM10       0.626413
NO         1.610619
NO2        0.694618
NOx        0.960550
NH3        0.718350
CO         0.647255
SO2        0.822019
O3         0.579173
Benzene    3.181601
Toluene    2.022600
Xylene     2.621274
AQI        0.632106
dtype: float64
```

Análisis del CV: De la revisión del CV se observan variables que tienen mayor dispersión (de -1 a 1): NO, Benzene, Toluene, y Xylene, por lo cual serán excluidos del análisis

Matriz de Correlación:

[12]:
```
corr_matrix
```

[12]:

	PM2.5	PM10	NO	NO2	NOx	NH3	CO
PM2.5	1.000000	0.899029	0.477051	0.549542	0.553259	0.347301	0.612865 \
PM10	0.899029	1.000000	0.466039	0.560939	0.563885	0.342937	0.586988
NO	0.477051	0.466039	1.000000	0.432057	0.874927	0.236836	0.520822
NO2	0.549542	0.560939	0.432057	1.000000	0.704155	0.331459	0.464276
NOx	0.553259	0.563885	0.874927	0.704155	1.000000	0.301988	0.573869
NH3	0.347301	0.342937	0.236836	0.331459	0.301988	1.000000	0.275159
CO	0.612865	0.586988	0.520822	0.464276	0.573869	0.275159	1.000000
SO2	0.189603	0.224109	-0.006125	0.125957	0.037567	0.199884	0.102803
O3	-0.024981	-0.022058	-0.216404	-0.015600	-0.163291	-0.059098	-0.070788
Benzene	0.103686	0.115193	0.433082	0.103471	0.426687	0.090257	0.218287
Toluene	0.222077	0.237397	0.378883	0.346232	0.427640	0.054423	0.274064
Xylene	0.132979	0.157813	0.178278	0.251379	0.226008	0.028464	0.167609
AQI	0.919838	0.908759	0.490139	0.543730	0.586693	0.355752	0.614860

	SO2	O3	Benzene	Toluene	Xylene	AQI
PM2.5	0.189603	-0.024981	0.103686	0.222077	0.132979	0.919838
PM10	0.224109	-0.022058	0.115193	0.237397	0.157813	0.908759
NO	-0.006125	-0.216404	0.433082	0.378883	0.178278	0.490139
NO2	0.125957	-0.015600	0.103471	0.346232	0.251379	0.543730
NOx	0.037567	-0.163291	0.426687	0.427640	0.226008	0.586693
NH3	0.199884	-0.059098	0.090257	0.054423	0.028464	0.355752
CO	0.102803	-0.070788	0.218287	0.274064	0.167609	0.614860
SO2	1.000000	0.179632	-0.039887	0.093414	0.086383	0.188907
O3	0.179632	1.000000	-0.106756	-0.074450	-0.034643	0.013340
Benzene	-0.039887	-0.106756	1.000000	0.685674	0.154776	0.138371
Toluene	0.093414	-0.074450	0.685674	1.000000	0.513484	0.225213
Xylene	0.086383	-0.034643	0.154776	0.513484	1.000000	0.140339
AQI	0.188907	0.013340	0.138371	0.225213	0.140339	1.000000

4. Cálculo del KMO:

4.1 Selección para el cálculo del KMO previo al Análisis Factorial:

[13]:
```
db_pollution_new.columns
var_seleccion = ['PM2.5', 'PM10', 'NO2', 'NOx', 'NH3', 'CO', 'SO2', 'O3']
db_pollution_selec = db_pollution_new[var_seleccion]
db_pollution_selec
```

[13]:

	PM2.5	PM10	NO2	NOx	NH3	CO	SO2	O3
1	81.40	124.50	20.50	12.08	10.72	0.12	15.24	127.09

```
2         78.32  129.06  26.00   14.85  10.28  0.14  26.96  117.44
3         88.76  135.32  30.85   21.77  12.91  0.11  33.59  111.81
4         64.18  104.09  28.07   17.01  11.42  0.09  19.00  138.18
5         72.47  114.84  23.20   16.59  12.25  0.16  10.55  109.74
…         …      …       …       …      …      …     …      …
98073     80.93  174.84  56.74  139.70  29.32  1.18   9.36   15.96
98074    103.75  217.89  63.28  181.55  27.56  1.71  11.43   24.75
98075    115.40  225.76  70.41  213.77  35.25  2.11  10.49   22.11
98076     97.37  179.87  64.34  169.46  36.24  1.51   6.90   12.55
98077     89.13  179.89  56.37  134.84  37.98  1.30   8.13   23.21

[8430 rows x 8 columns]
```

[14]:
```
# selección de las columnas numéricas
num_cols = db_pollution_new.select_dtypes(include=['float64', 'int64']).columns.
 ↪tolist()
db_diabetes_new = db_pollution[num_cols].copy()
```

5. Cálculo del KMO:

KMO

[15]:
```
from factor_analyzer.factor_analyzer import calculate_kmo
kmo_all, kmo_model = calculate_kmo(db_pollution_selec)
print(kmo_model.round(2))
```

```
0.78
```

Test de Barlett

[16]:
```
# Barlett's
from factor_analyzer.factor_analyzer import calculate_bartlett_sphericity
calculate_bartlett_sphericity(db_pollution_selec)
```

[16]:
```
(31773.710774973093, 0.0)
```

ANÁLISIS DEL KMO: Dado que el valor del KMO es estadísticamente significativo, esto nos indica que las variables seleccionadas del data frame son aptas para realizar un AF

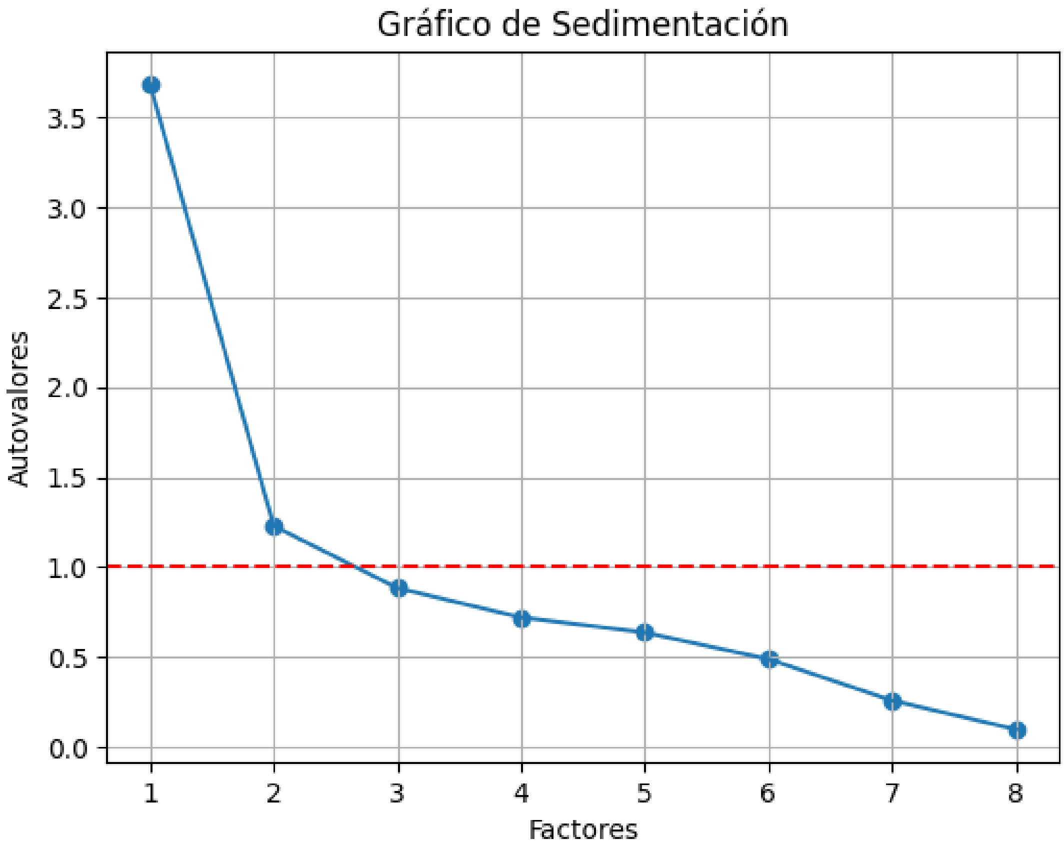
6. Análisis Gráfico de los Factores:

[17]:
```
fa = FactorAnalyzer(rotation=None, n_factors=len(db_pollution_selec.columns))
fa.fit(db_pollution_selec)

# Check Eigenvalues
ev, v = fa.get_eigenvalues()
ev

plt.scatter(range(1,db_pollution_selec.shape[1]+1),ev)
plt.plot(range(1,db_pollution_selec.shape[1]+1),ev)
plt.title('Gráfico de Sedimentación')
```

```python
plt.xlabel('Factores')
plt.ylabel('Autovalores')
plt.axhline(y=1, color='r', linestyle='--')
plt.grid()
plt.show()
```



ANÁLISIS DEL GRÁFICO DE SEDIMENTACIÓN: En el gráfico se puede observar que máximo en dos dimensiones se puede resumir la varianza de las variables originales.

7. Matriz de Componentes Rotados con el Método Varimax

[18]:
```python
fa = FactorAnalyzer(rotation="varimax", n_factors=2)
fa.fit(db_pollution_selec)

# Check loadings
loadings = pd.DataFrame(fa.loadings_)
loadings.rename(columns = lambda x: 'Dimensión-' + str(x + 1), inplace=True)
loadings.index = db_pollution_selec.columns
loadings
```

[18]:

	Dimensión-1	Dimensión-2
PM2.5	0.926310	0.057106
PM10	0.933908	0.050690
NO2	0.591678	0.419971
NOx	0.559260	0.828554
NH3	0.394735	0.112495
CO	0.618897	0.276618
SO2	0.267926	-0.155719
O3	0.017911	-0.220189

ANÁLISIS DEL MÉTODO DE ROTACIÓN VARIMAX: Los loadings son una medida importante en el análisis factorial porque permiten identificar qué variables están más estrechamente relacionadas con cada factor.

8. Comunalidades

[19]:
```
communalities = fa.get_communalities()
communalities = pd.DataFrame(communalities, index=db_pollution_selec.T.index,␣
↪columns=['Comunalidades'])
communalities.sort_values(by=['Comunalidades'], ascending=True).head(8)
```

[19]:

	Comunalidades
O3	0.048804
SO2	0.096032
NH3	0.168471
CO	0.459551
NO2	0.526459
PM2.5	0.861312
PM10	0.874754
NOx	0.999272

ANÁLISIS DE LAS COMUNALIDADES: Las comunalidades representan la cantidad de información en cada variable que se comparte con los demás factores. En este sentido, las comunalidades de **PM2.5** (86%), **PM10** (87%), **NOx** (0.99%) indican la proporción de la varianza explicado por los factores extraídos en el AF.

9. Análisis de la Varianza en cada Factor

[20]:
```
factorVarianza = pd.DataFrame(fa.get_factor_variance()).round(2)
factorVarianza.rename(columns = lambda x: 'Factor-' + str(x + 1), inplace=True)
factorVarianza.index = ['SS Loadings',
                        'Proportion Variance',
                        'Cumulative Variance']
factorVarianza
```

[20]:

	Factor-1	Factor-2
SS Loadings	3.00	1.03
Proportion Variance	0.38	0.13
Cumulative Variance	0.38	0.50

10. Selección de Factores según el nivel de varianza

[21]:
```
def color_factor_importance(val):
    if np.abs(val) > 0.7:
        color = 'green'
    elif np.abs(val) > 0.5 and np.abs(val) < 0.7:
        color = 'blue'
    else:
        color = 'red'
    return 'color: %s' % color

loadings.style.applymap(color_factor_importance)
```

[21]:
```
<pandas.io.formats.style.Styler at 0x18d7a561810>
```

11. Resultados del AF en las variables de la base de contaminación atmosférica **CONSIDERACIÓN:** Tomando como valores significativos en cada dimensión a los que son mayores a **0.7**. Tenemos que el Índice de Contaminación Ambiental queda planteado de la siguiente forma:

$$ICA = 0.92 * PM2.5 + 0.93 * PM10 + 0.82 * NOx$$

Donde: **ICA** : **Í**ndice de **C**ontaminación **A**tmosférica