Introdução á Análise de Estabilidade

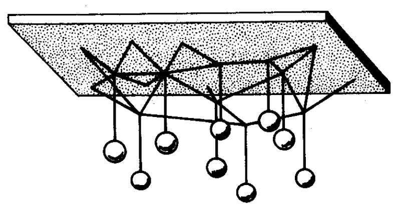
performed with a power-flow computer program (Chapter 7), ensure that phase angles across transmission lines are not too large, that bus voltages are close to nominal values, and that generators, transmission lines, transformers, and other equipment are not overloaded.

Transient stability, the main focus of this chapter, involves major disturbances such as loss of generation, line-switching operations, faults, and sudden load changes. Following a disturbance, synchronous machine frequencies undergo transient deviations from synchronous frequency (60 Hz), and machine power angles change. The objective of a transient stability study is to determine whether or not the machines will return to synchronous frequency with new steady-state power angles. Changes in power flows and bus voltages are also of concern.

Elgerd [2] gives an interesting mechanical analogy to the power-system transient stability program. As shown in Figure 13.1, a number of masses representing synchronous machines are interconnected by a network of elastic strings representing transmission lines. Assume that this network is initially at rest in steady-state, with the net force on each string below its break point, when one of the strings is cut, representing the loss of a transmission line. As a result, the masses undergo transient oscillations and the forces on the strings fluctuate. The system will then either settle down to a new steady-state operating point with a new set of string forces, or additional strings will break, resulting in an even weaker network and eventual system collapse. That is, for a given disturbance, the system is either transiently stable or unstable.

Figure 13.1

Mechanical analog of power-system transient stability [2]

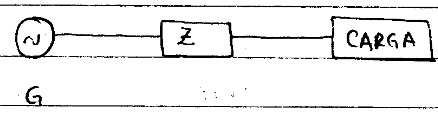


In today's large-scale power systems with many synchronous machines interconnected by complicated transmission networks, transient stability studies are best performed with a digital computer program. For a specified disturbance, the program alternately solves, step by step, algebraic power-flow equations representing a network and nonlinear differential equations representing synchronous machines. Both predisturbance, disturbance, and postdisturbance computations are performed. The program output includes power angles and frequencies of synchronous machines, bus voltages, and power flows versus time.

In many cases, transient stability is determined during the first swing of

4.1. Introdução:

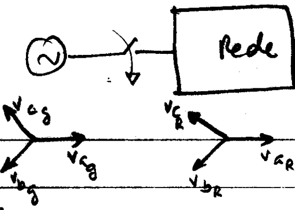
Consideremos o seguinte sistema:



onde G é um gerador síncrono
Z é impedância
a carga pode ser:
- impedância
- Motor de indução
- Motor síncrono
- Combinação dos anteriores
- barra infinita.

A estabilidade do gerador síncrono é a capacidade para se manter em sincronismo* com o sistema ao qual está conectado.

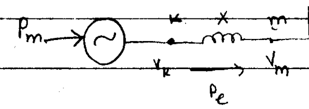
* operar em paralelo na mesma velocidade.



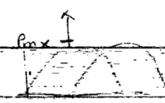
$$\Delta \bar{V}_r = \bar{V}_{a_g} - \bar{V}_{a_R} \approx 0$$

se a 'carga' for uma ou várias máquinas síncronas pode acontecer o caso em que o limite de potência ativa (fornecido e recebido) P_{max},

$$P_{km} = -P_{mk} = \frac{V_k V_m}{X} \operatorname{sen} \delta$$



seja **excedido**, o que acarretará perda de sincronismo. (p.ex.: mais potência entrando que saindo)



Interpretação Física das conseqüências da perda de sincronismo:

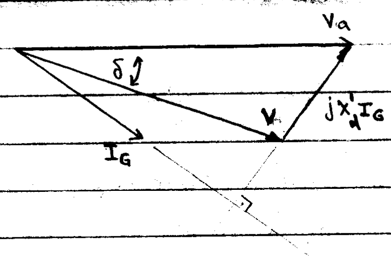
limite de Potência ativa transferido excedido. → Máquina Síncrona perdendo sincronismo

→ Rotação de maior ou menor velocidade p/ gerar tensão à freqüência do sistema → 'escorregamento' entre tensão gerada e conjunto de tensões do sistema.

→ grandes sobre correntes → Disparo dos dispositivos de proteção. → isolamento da Maquina.

4.3 Modelo da máquina síncrona:

Assumindo o modelo da máquina síncrona (simplificado) temos fasorialmente,



onde,

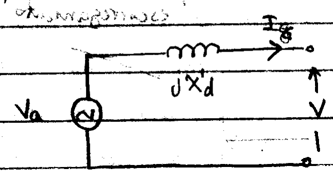
V é a tensão nos terminais do gerador

V_a é a tensão interna do gerador

I_G é a corrente entregue pelo ~~no~~ gerador

X'_d é a reactância transitória

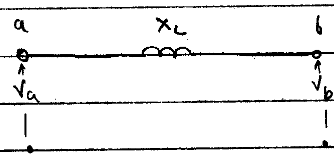
O circuito equivalente é,



$$P_G = \frac{V_a V \operatorname{sen}\delta}{X'_d}$$

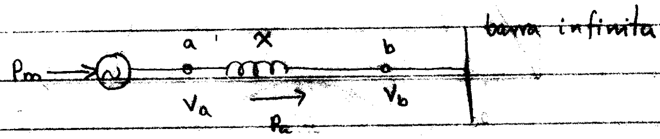
onde $\delta = \angle V_a - \angle V$

por outro lado, na linha de transmissão ligando duas barras



$P_{ab} = \frac{V_a V_b}{X_L} \operatorname{sen} \delta$, onde $\delta = \angle V_a - \angle V_b$

vamos considerar o seguinte modelo para nossos estudos:



onde,

P_m é a potência mecânica de entrada à máquina

V_a é a tensão ~~atrás~~ da impedância síncrona.

(tensão interna do gerador - circuito equivalente)

X é a impedância da máquina e da linha de transmissão (reativas)

$X = x'_d + x_L$

Para um sistema sem perdas e em estado estacionário, temos:

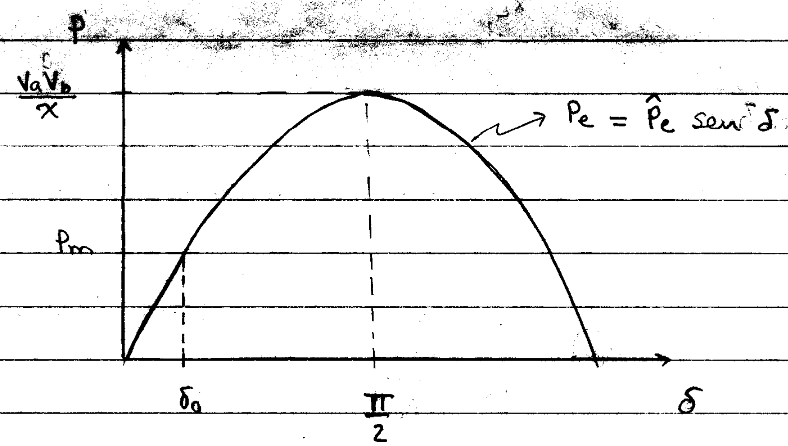
$$P_m = P_e = \frac{V_a V_b}{X} \operatorname{sen} \delta \qquad (4.1)$$

onde, δ é o ângulo de carga , $\delta = \angle V_a - \angle V_b$

P_m é a potência mecânica aplicada à máquina

P_e é a potência elétrica saindo da máquina.

graficamente, temos:



com δ_0 o ângulo de carga do sistema

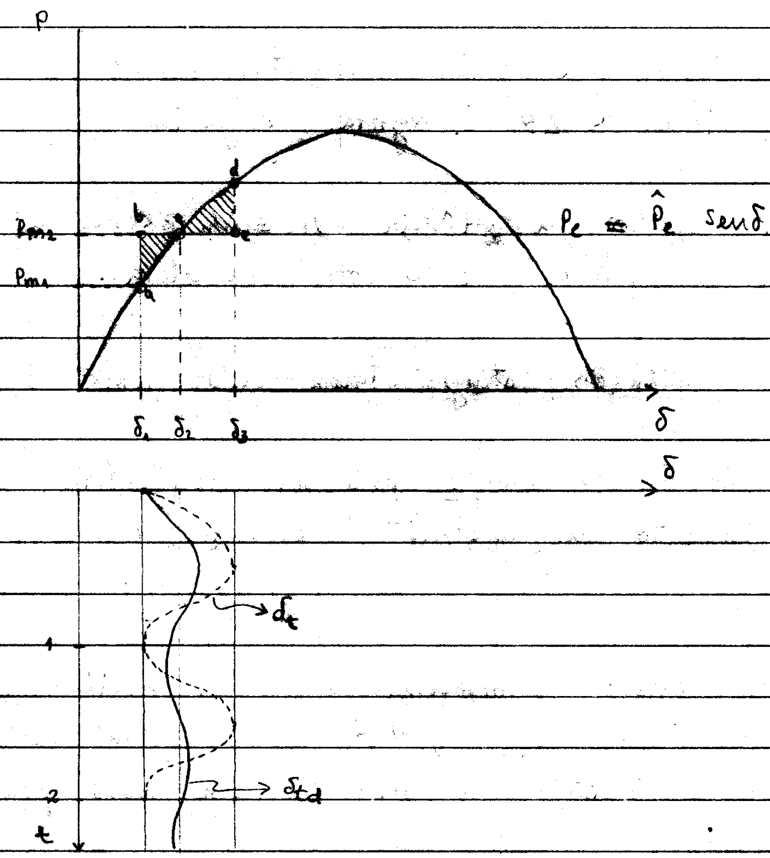
- Para incrementos graduais da potência mecânica de entrada junto com a excitação da máquina (para preservar a tensão V_a constante) o valor máximo de P_e é atingido quando $\delta = \pi/2$

- se $P_m > \frac{V_a V_b}{X}$, então há mais potência entrando que saindo. Como conseqüência o rotor da máquina acelera e existe escorregamento entre os polos, acarretando perda de sincronismo.

Dizemos então que o <u>limite de estabilidade em estado estacionário</u> tem sido excedido.

Vamos ~~supor~~ agora que o sistema em regime permanente sofra uma perturbação ~~severa~~. brusca

Graficamente o sistema se comporta da seguinte maneira:



· A potência mecânica P_m foi bruscamente incrementada de P_{m1} a P_{m2} (de ⓐ para ⓑ, na figura).

· O excesso de potência de entrada com relação à potência de saída é, $\Delta P = P_{m2} - P_e \operatorname{sen}\delta_1$.

- O exesso acarreta uma aceleração do rotor até o novo ponto de equilíbrio (c) e a diferença entre P_m e P_e representa a potência de aceleração.
- Devido à inércia o rotor não se move instantaneamente de δ_1 para δ_2, mas seguindo a curva P_e de (a) até (c)

- O rotor não consegue parar instantaneamente e ultrapassa δ_2. A diferença ΔP é agora negativa.

- com ΔP negativo a potência de saída exede a potência de entrada e o rotor desacelera.

- a sobre oscilação atinge algum valor máximo δ_3

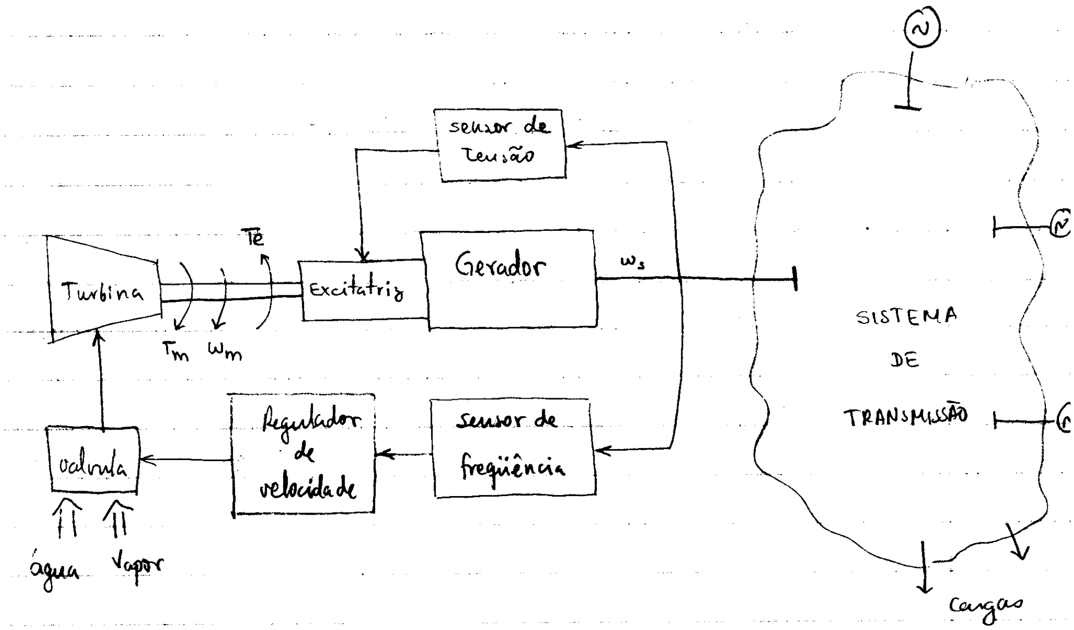
- A variação δ no tempo pode ser oscilatória sem amortecimento em ausência de perdas (curva δ_t da figura)

Na prática existe amortecimento e as oscilações diminuem (curva δ_{td}), Neste caso a estabilidade é mantida e δ oscila em torno de um novo ponto de equilíbrio

- O estudo de estabilidade consiste em determinar se o sincronismo é recuperado depois do sistema sofrer uma perturbação.

4.2 Equação de Oscilação:

Esquema geral de um gerador síncrono (e seus controles), conectado ao sistema de transmissão:



Torque eletromagnético $\longrightarrow$ interação

- Campo Magnético Rotor $\}\rightarrow$ excitatriz (controle reg. de Ten.
- Campo Magnético estator $\}$ correntes armadur.

$\downarrow$

acoplamento c/ a rede. (Controle frequência)

Torque eletromagnético reflete o acoplamento entre cada gerador e todos os outros elementos do sistema.

comportamento dinâmico:

Torque Líquido = variação do momento angular

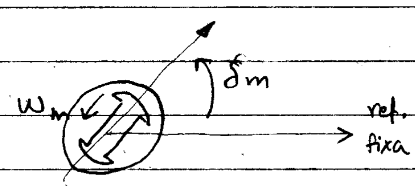
$$T_a = T_m - T_e = \frac{d}{dt}(I\,\omega_m) \qquad (4.2)$$

onde:

$T_m \rightarrow$ Torque mecânico (N.m)
$T_e \rightarrow$ Torque eletromagnético (N.m)
$T_a \rightarrow$ Torque de aceleração (N.m)
$I \rightarrow$ Momento de inércia ($Kg.m^2$)
$\omega_m \rightarrow$ velocidade angular mecânica (rd/s)

sendo:
$$\omega_m = \frac{d}{dt}\delta_m \qquad (4.3)$$

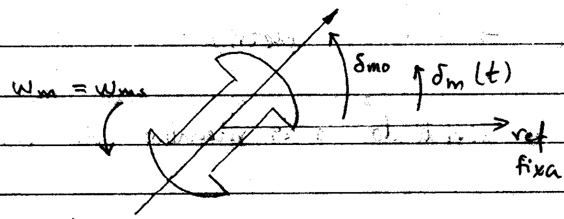
onde $\delta_m \rightarrow$ Posição angular do rotor (ângulo mecânico em radianos) em relação a uma referência fixa.



temos:

$$T_m - T_e = I \frac{d^2\delta_m}{dt^2} \qquad (4.4.)$$

supondo a máquina girando na velocidade síncrona $\omega_m = \omega_{ms}$



$$\delta_m(t) = \delta_{mo} + \omega_{ms} t$$

a posição angular do rotor em radianos elétricos pode ser obtida levando-se em conta que:

$$\delta_{(elétrico)} = \frac{P}{2} \delta_m \qquad (\text{rd elétricos})$$

$$\omega_s = \frac{P}{2} \omega_{ms} = 2\pi f^0 \qquad \left(\frac{\text{rd elétricos}}{s}\right)$$

onde:

$P \rightarrow$ número de polos do gerador

$f^0 \rightarrow$ freqüência nominal síncrona (60 Hz).

a equação 4.4. fica,

$$T_m - T_e = \frac{I}{P/2} \frac{d^2\delta}{dt^2} \qquad (4.5)$$

multiplicando por ω_{ms},

$$P_m - P_e = \frac{\omega_{ms}\, I}{P/2} \frac{d^2\delta}{dt^2} \quad , \text{MW} \qquad (4.6.)$$

onde,

$P_m \rightarrow$ Potência mecânica

$P_e \rightarrow$ " elétrica

multiplicando e dividindo o 2º membro de (4.6.) por ω_{ms},

$$P_m - P_e = \left(\frac{I\,\omega_{ms}^2}{2}\right) \frac{1}{\pi f^0} \frac{d^2\delta}{dt^2} \quad \text{MW} \qquad (4.7.)$$

ou

$$P_m - P_e = \frac{W}{\pi f^0} \frac{d^2\delta}{dt^2} \quad \text{MW} \qquad (4.8)$$

onde:

$$W \triangleq \frac{I\,\omega_{ms}^2}{2}$$

dividindo (4.8) pelos MVA base, temos em p.u.:

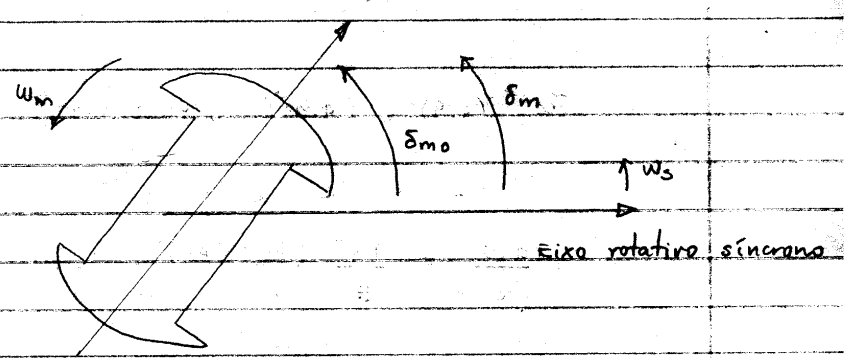
$$\boxed{P_m - P_e = \frac{H}{\pi f^0} \frac{d^2\delta}{dt^2}} \qquad (4.9)$$

onde: $H = \frac{W}{\text{MVA}}$ (s) constante de inercia do gerador

H → pode ~~ser~~ interpretado como sendo o tempo em segundos durante o qual a energia cinética armazenada poderia ser convertida na potencia nominal da máquina, sem a alimentação da turbina.

a análise da estabilidade transitória se interessa pelo movimento relativo dos rotores.

É conveniente então medir a posição ângular do rotor em relação a um eixo de referência que gire na velocidade síncrona (eixo rotativo síncrono).



$$\omega = \frac{p}{2}\,\omega_m$$

$$\delta = \frac{p}{2}\,\delta_m$$

$$\delta_0 = \frac{p}{2}\,\delta_{m0}$$

Pode-se escrever: $\delta = \delta_0 + (\omega - \omega_s)\,t$ (4.10)

δ_0 → posição angular inicial

Derivando (4.10) em relação ao tempo,

$$\frac{d\delta}{dt} = \omega - \omega_s \qquad (4.11)$$

em situação de regime,

$$\frac{d\delta}{dt} = 0 \Rightarrow \omega = \omega_s \Rightarrow \delta = \delta_0$$

em transitório:

$$\frac{d\delta}{dt} \neq 0 \Rightarrow \omega \neq \omega_s \Rightarrow \delta \text{ varia}$$

as equações (4.9) e (4.11), reescritas,

$$\boxed{\begin{aligned} \frac{d^2\delta}{dt^2} &= \frac{d\omega}{dt} = \frac{\pi f^0}{H}(P_m - P_e) = \frac{\omega_s}{2H}(P_m - P_e) \\ \frac{d\delta}{dt} &= \omega - \omega_s \end{aligned}} \qquad (4.12)$$

são as equações de oscilação (SWING), do gerador síncrono. Elas descrevem o seu comportamento dinâmico.

Para estudar a estabilidade transitória, é necessário estudar as funções $\delta(t)$ dos geradores, funções estas, obtidas pela resolução das equações (4.12).

As curvas $\delta \times t$ são chamadas curvas de oscilação dos geradores.

- Uma solução analítica da equação (4.12) não é possível, já que $P_e = \hat{P}_e \operatorname{sen} \delta$ implica numa não linearidade. É necessário utilizar técnicas numéricas para determinar $\delta(t)$.

- A equação de oscilação (4.13) incluindo amortecimento é,

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} + k_d \frac{d\delta}{dt} = P_m - P_e \qquad (4.13)$$

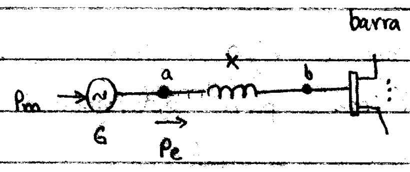
onde k_d é o coeficiente de amortecimento.

- As equações (4.12) e (4.13) são válidas sempre que se assuma como hipóteses:

  - P_m é mantida constante durante o período do transitório, ou seja, não existe ação do controle Pf
  - P_e é mantida constante durante o período do transitório, ou seja, não existe ação de regulação automática de tensão.
  - O gerador síncrono representado pelo seu modelo equivalente simplificado.

Exemplo 4.1.:

seja o seguinte sistema,



$V_b = 1.$ pu

$V_a = 1.12$ p.u.

$x = .4$ reactância da linha e da máquina

$H = 2$ s

$P_m = .75$ pu

inicialmente em regime permanente temos,

$$P_m^0 = .75 \text{ pu} = P_e^0 = \frac{V_a V_b}{x} \operatorname{sen} \delta^0 =$$

$$= \frac{(1.12)(1.)}{.4} \operatorname{sen} \delta^0 \Rightarrow \delta^0 = 15.5^\circ$$

suponha que acontecem os seguintes eventos:

- Repentinamente acontece um curto circuito entre a rede e o gerador G.

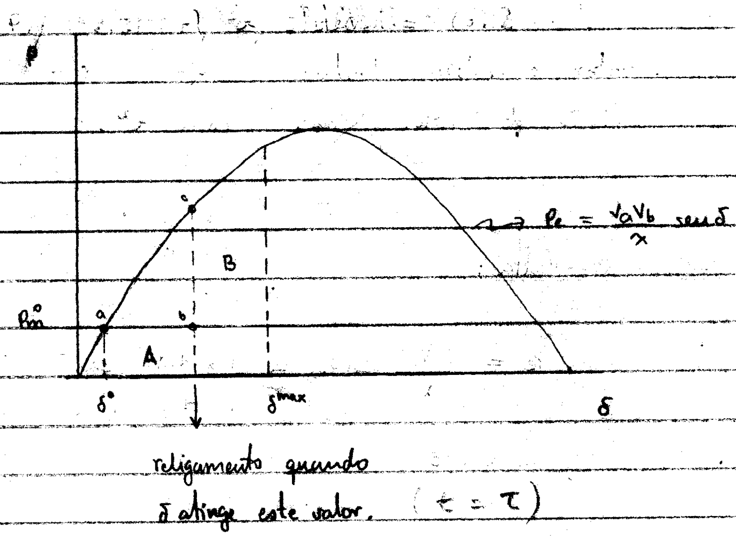
- Os relés protetores ordenam a desconeção instantanea da linha.

- Uma fração de segundo depois a linha é religada, suponda que a falta tenha sido retirada, voltando à operação normal.

- No período em que acontece a falta a saída P_e do gerador será igual a zero.

Determinar as equações de oscilação que descrevem os eventos.

Graficamente:



Período em falta:

$$\left.\begin{aligned} P_e &= 0 \\ P_m &= .75 \end{aligned}\right\} \Rightarrow$$ Potência mecânica de entrada acelera o rotor (área A).

a equação de oscilação (4.12) neste período é,

$$\frac{d^2\delta}{dt^2} = \frac{\pi f^0}{H} P_m = \frac{\pi\, 60}{2} * .75 = 70.68 \text{ rd/s}^2 = 4045\ °/s^2$$

Integrando esta equação diferencial e sabendo que

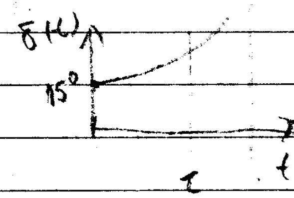
$$\left.\frac{d\delta}{dt}\right|_{t=0} = 0 \qquad e \qquad \delta(0) = 15.5^\circ$$

temos,

$$\delta(t) = 2022.5\, t^2 + c_1 t + c_2$$

$$\delta(t) = 2022.5\, t^2 + 15.5^\circ \quad \text{graus elétricos.}$$

a posição do rotor cresce com t^2.



Período Pós-falta:

$$P_e = \frac{V_a V_b}{x} \operatorname{sen}\delta = \frac{1 \times 1.12}{.4} \operatorname{sen}\delta$$

$$P_m = .75$$

desaceleração do rotor (área B da figura)

da equação de oscilação:

$$\frac{d^2\delta}{dt^2} = \frac{\pi f^\circ}{H}(P_m - P_e) = \frac{\pi f^\circ}{H}(.75 - 2.8 \operatorname{sen}\delta)$$

ou,

$$0.011 \frac{d^2\delta}{dt^2} = 0.75 - 2.8 \operatorname{sen}\delta$$

Cond. iniciais: $\left.\frac{d\delta}{d}\right|_{t=\tau}$, $\delta(\tau)$

Esta equação deve ser resolvida por métodos numéricos.

para verificar a estabilidade do sistema a equação $\delta(t)$ durante a falta e a equação $\delta(t)$ pós falta, devem representar a continuidade temporal dos eventos. Isto é, a primeira equação fornece as condições $\delta(T)$ e $\dot{\delta}(T)$ que são condições iniciais para a solução da segunda equação.

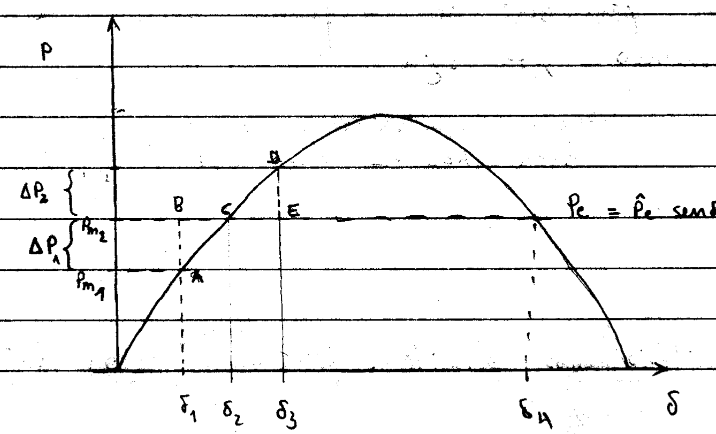
As condições iniciais para esta última equação diferencial supondo que o religamento é feito no tempo τ_r, são:

$$\delta(\tau_r) = 2022.5\ \tau_r^2 + 15.5^\circ$$

$$\left.\frac{d\delta}{dt}\right|_{t=\tau_r} = 4045\ \tau_r$$

4.4. Critério de Áreas iguais.

Podemos determinar o máximo valor de δ, para um sistema simples, a partir do diagrama $P-\delta$.



Em regime, $P_{m1} = P_e$, acontecendo uma perturbação P_{m2},

$$\Delta P_1 = T\,\omega_s = P_{m2} - P_{m1}$$

o torque de aceleração é,

$$T = \frac{P_{m2} - P_e}{\omega_s}$$

A energia ganha pelo rotor durante a mudança angular de δ_1 para δ_2 (aceleração) é,

energia ganha $(P_{m2} > P_e)^*$ $= \int_{\delta_1}^{\delta_2} T\,d\delta = \frac{1}{\omega_s}\int_{\delta_1}^{\delta_2} (P_{m2} - P_e)\,d\delta \approx$ área ABC

4.12

energia perdida pelo rotor $(P_e > P_{m2})^*$ $= \int_{\delta_2}^{\delta_3} T\,d\delta = \frac{1}{\omega_s}\int_{\delta_2}^{\delta_3} (P_e - P_{m2})\,d\delta \approx$ área CDE

- Ignorando a existência de (amortecimento) perdas durante as oscilações, a energia ganha deve ser igual à energia perdida para que a maquina permaneça em sincronismo, ou seja

área ABC = área CDE (4.14)

Este critério nos permite determinar o máximo valor que pode atingir δ para preservar a estabilidade, sem usar a equação de oscilação.

temos então, neste caso,

$$(\delta_2-\delta_1)\,P_{m_2} - \int_{\delta_1}^{\delta_2} \hat{P}_e \operatorname{sen}\delta \, d\delta = \int_{\delta_2}^{\delta_3} \hat{P}_e \operatorname{sen}\delta \, d\delta - P_{m_2}(\delta_3-\delta_2)$$

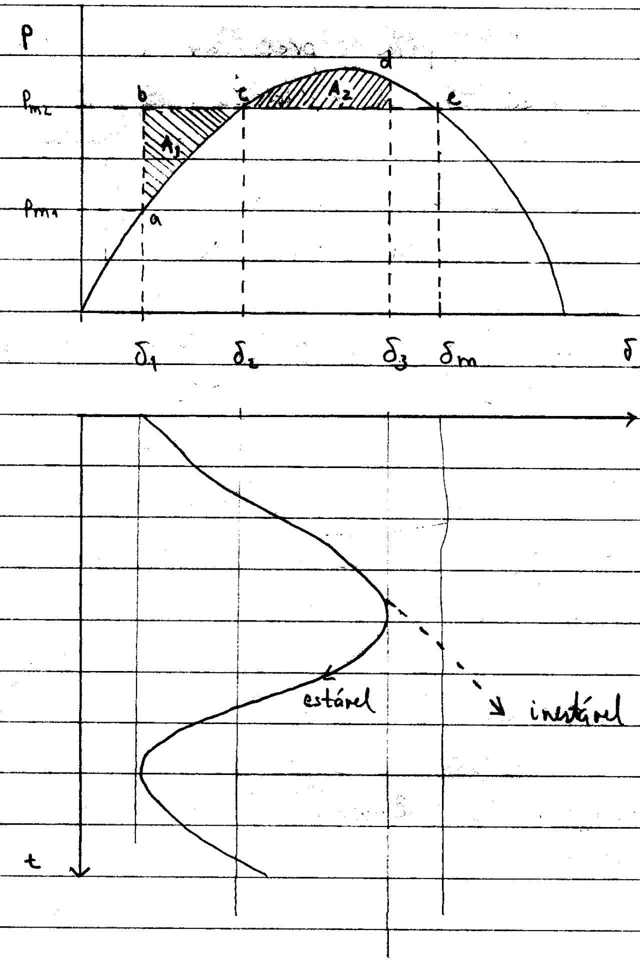
os valores δ_1 e δ_2 com estado estacionário podem ser calculados apartir de

$$P_{m_1} = \hat{P}_e \operatorname{sen}\delta_1$$

$$P_{m_2} = \hat{P}_e \operatorname{sen}\delta_2$$

logo, δ_3 pode ser calculada.

utilizemos este critério para a determinação do <u>máximo incremento</u> em Pm (de forma a assegurar) a estabilidade possível mantendo ainda

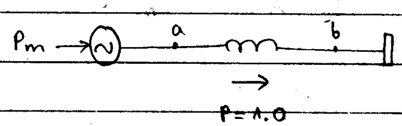


- O sistema será estável se existe uma área A_2 no mínimo igual a A_1 que possa ser colocada acima de P_{m2}.

- Se $A_1 >$ área cde $\Rightarrow$ $\delta_3 > \delta_m$, neste caso $P_{m_2} > P_e$ e o rotor continua a acelerar perdendo à estabilidade.

· O limite P_{m2} para o qual A_1 = área cde pode ser calculado gráficamente ou analiticamente apartir deste critério. P_{m2} é o limite de estabilidade transitório para as condições de operação.

Exemplo 4.2 :



$V_a = 1.3$

$V_b = 1.0$

$$x = x_L + x'_d = .2 + .3 = .5 \quad pu.$$

Acontece uma falta trifásica elétricamente perto dos terminais do gerador e instantaneamente a linha de transmissão é desligada do sistema.

Determinar o ângulo de carga antes do qual a falta deve ser eliminada por uma proteção para preservar a estabilidade.

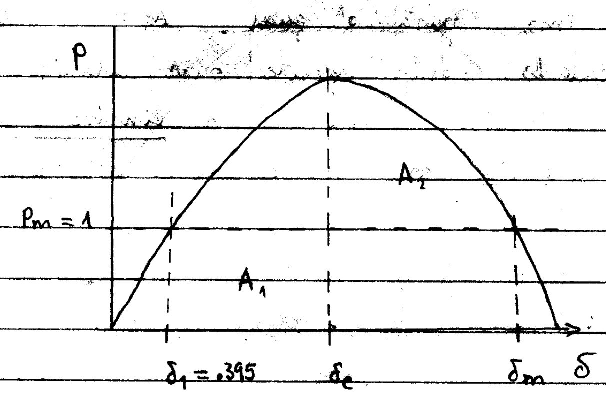
Qual é o tempo crítico no qual é atingido o ângulo se a máquina tem uma constante de inercia $H = 2$ s?

Em regime permanente,

$$P_m = P_e = \frac{V_a V_b}{x} \operatorname{sen}\delta_1 = \frac{1.3 \times 1}{.3 + .2} \operatorname{sen}\delta_1 = 1$$

$$\operatorname{sen}\delta_1 = \frac{1}{2.6} = .385 \Rightarrow \delta_1 = .395 \text{ rd}$$

graficamente,



No período em falta a potência elétrica de saída do gerador é considerada nula, já que não esta enviando potência à rede.

$P_e \to 0$ e P_m entrando é transformada em energia de aceleração do rotor.

- A condição para que seja mantida a estabilidade é que $A_1 = A_2$, logo,

$$\delta_m = \pi - .395 = 2.747 \text{ rd}$$

$$A_1 = 1 \times (\delta_c - \delta_1) = \delta_c - .395$$

$$A_2 = \int_{\delta_c}^{\delta_m} 2.6 \operatorname{sen}\delta \, d\delta - 1 \times (2.747 - \delta_c)$$

$$= 2.6(-\cos\delta_m + \cos\delta_c) - 2.747 + \delta_c$$

com $A_1 = A_2$,

$$\delta_c - .395 = 2.4 + 2.6 \cos\delta_c - 2.747 + \delta_c$$

$$\cos\delta_c = -\frac{.048}{2.6} \Rightarrow \delta_c = \cancel{89^\circ}\ 91^\circ \equiv 1.59 \text{ rd}$$

Para a preservação da estabilidade, a linha deve ser religada antes que o ângulo de carga atinja δ_c que é chamado de 'ângulo de eliminação crítico'.

Cont. pag 140

$$\frac{d^2\delta}{dt^2} = \frac{\pi f^0}{H} \cdot P_m = \frac{\pi\,(60)}{2} \cdot (1) = 94.25 \text{ rd/s}^2$$

$$\frac{d\delta}{dt} = 94.25\, t + c_1$$

$$\delta(t) = \frac{94.25}{2} t^2 + c_1 t + c_2$$

$$\left.\frac{d\delta}{dt}\right|_{t=0} = 0 \Rightarrow c_1 = 0$$

$$\delta(0) = 0.395 \text{ rd} = c_2$$

$$\delta(t) = \frac{94.25}{2} t^2 + 0.395$$

$$\delta_c = \delta(t = \tau_c) = 1.59 = \frac{94.25}{2} \tau_c^2 + 0.395$$

Exemplo 4.3.

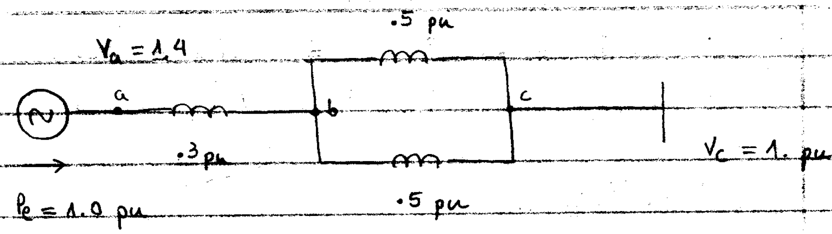


fig (a).

. um dos circuitos da linha de transmissão sofre uma falta bifásica à terra. Durante a ocorrência desta falta as reactâncias do sistema são;

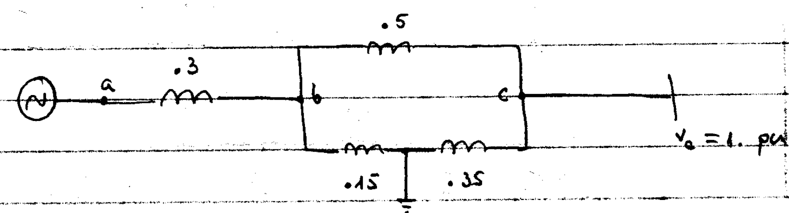


fig (b)

Determinar o ângulo crítico de eliminação antes do qual os dispositivos protetores da linha em falta devem atuar para preservar a estabilidade.

Antes da falta, (fig a)

$$P_e = \frac{1.4 \times 1}{X} \operatorname{sen} \delta$$

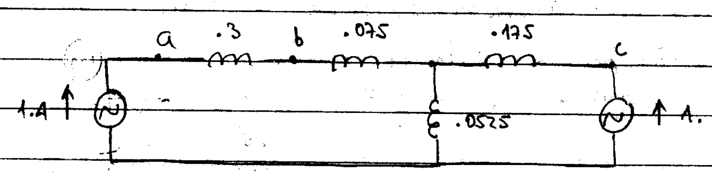
$X \rightarrow$ reactância efetiva entre a e c

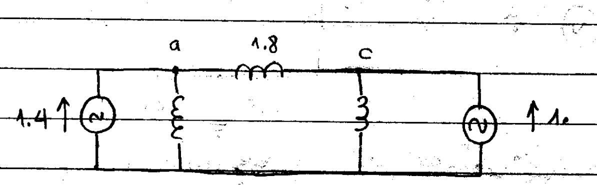
$$1 = \frac{1.4 \times 1}{.25 + .3} \operatorname{sen} \delta_1 = 2.545 \operatorname{sen} \delta_1 = P_e$$

$$\operatorname{sen} \delta_1 = .3928 \Rightarrow \delta_1 = 23.13°$$

Durante a falta,

da (fig. b), através de sucessivas transformações delta-estrela, estrela-delta, podemos determinar a reatância equivalente, assim





somente a reatância entre ⓐ e ⓒ precisa ser calculada, uma vez que as reatâncias shunt não contribuem à transferência de potência entre ⓐ e ⓒ.

com isto temos então,

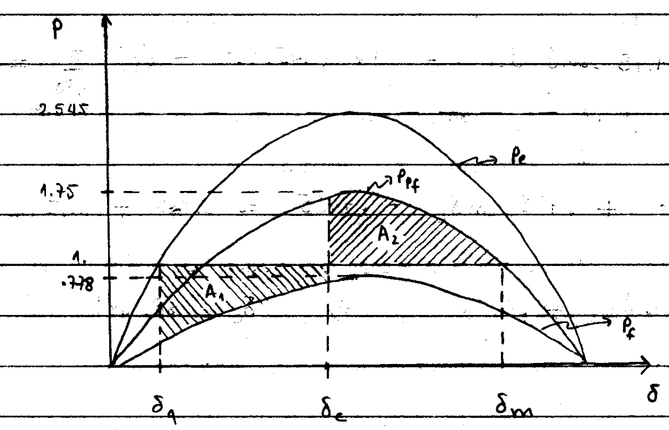
$$P_f = \frac{1.4 \times 1.}{1.8} \operatorname{sen}\delta = .778 \operatorname{sen}\delta$$

após a ocorrência da falta: da fig (a),

só temos um circuito em funcionamento,

$$P_{pf} = \frac{1.4 \times 1.}{.8} \operatorname{sen}\delta = 1.75 \operatorname{sen}\delta$$

graficamente, temos:



persistindo a falta $\rightarrow$ o torque de aceleração é produzido pela diferença entre P_m e P_f,

$$\Delta P = P_m - P_f > 0 \quad \therefore \quad \frac{d^2\delta}{dt^2} > 0$$

eliminando a falta $\rightarrow$ o torque de desaceleração é produzido pela diferença entre P_{pf} e P_m,

$$\Delta P = P_m - P_{pf} < 0 \quad \therefore \quad \frac{d^2\delta}{dt^2} < 0$$

determinação do ângulo crítico:

o ângulo máximo de oscilação, δ_m,

$$1 = 1.75 \operatorname{sen} \delta_m \Rightarrow \delta_m = 145.15°$$

para assegurar a estabilidade, $A_1 = A_2$,

valores em p.u.

$$1 \times (\delta_c - 23.13°)\frac{\pi}{180} - \int_{\delta_1}^{\delta_c} .778 \operatorname{sen}\delta \, d\delta =$$

$$= \int_{\delta_c}^{\delta_m} 1.75 \operatorname{sen}\delta \, d\delta - 1 \times (145.15° - \delta_c)\frac{\pi}{180} =$$

$$= -.4037 + .778 \cos\delta_c - .7155 = 1.436 + 1.75 \cos\delta_c - 2.533$$

$$.977 \cos\delta_c = .0222 \Rightarrow \delta_c = 88.69°$$

4.5. considerações na estabilidade dinâmica

- critério de áreas iguais determina-se o ângulo crítico para a eliminação da falta.

- Este critério não permite a obtenção do tempo crítico para a eliminação da falta

- Este tempo crítico é necessário para a especificação do desempenho dos dispositivos de proteção.

- o tempo crítico pode ser calculado pela integração numérica da equação de oscilação.

- uma equação de oscilação mais geral e precisa é,

$$M \frac{d^2\delta}{dt^2} + K_d \frac{d\delta}{dt} = P_m(t) - P_e(\delta, t) \qquad (4.15)$$

com $M = \frac{H}{\pi f_0}$

$P_m(t)$ → Potência mecânica de entrada assumindo a ação de controle Pf.

$P_e(\delta, t)$ → Potência elétrica de saída como função do tempo devido à ação do controle de regulação de tensão.

- considerando à ação dos controladores, observamos que :

  + Devido à constante de tempo associada ao controle mecânico, no primeiro intervalo (1 a 1.5 sg) depois de acontecida a falta não se experimenta nenhuma resposta deste controle.

  + O controle elétrico (regulação de tensão automática) responde instantaneamente mantendo a tensão nos terminais do gerador no seu nível nominal.

  + a característica ângulo potência, na realidade, não é senoidal: A forma da curva depende da característica de exitação da máquina.

  + O período transitório é seguido por um período dinâmico durante o qual o efeito do controle mecânico entra em jogo e interatua com o controle de regulação de tensão automático.

  + Um sistema pode ser estável na primeira oscilação (transitório) e ser posteriormente inestável dinamicamente na segunda ou subseqüêntes oscilações.

Os seguintes fatores devem ser levados em conta para a análise de estabilidade do sistema :

  - Topologia do sistema
  - impedâncias nas linhas de transmissão

- fluxo de carga do sistema antes da falta
- Tipo de falta e impedância de falta
- localização da falta
- velocidade de eliminação da falta
- Inercia de todas as máquinas girando
- sistema de exitação e características da regulação automática de tensão
- características do controle mecânico
- reatâncias das máquinas e constantes de amortecimento.

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12.5 ELGERD.

4.6. Solução computacional da equação da oscilação.

$$M \frac{d^2\delta}{dt^2} + k_d \frac{d\delta}{dt} = P_m - \hat{P}_e \operatorname{sen}\delta \qquad (4.16)$$

com $M = H/\pi f^0$

variáveis de estado: $x_1 \triangleq \delta$ angulo de carga rd

$x_2 \triangleq \dot{\delta}$ velocidade angular do rotor rd/s

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \triangleq \begin{bmatrix} \delta \\ \dot{\delta} \end{bmatrix}$$

a equação (4.16) fica,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{M}\left(P_m - P_e - k_d\, x_2\right)$$

em geral,

$$\dot{x}_1 = f_1(x_1, x_2)$$
$$\dot{x}_2 = f_2(x_1, x_2)$$

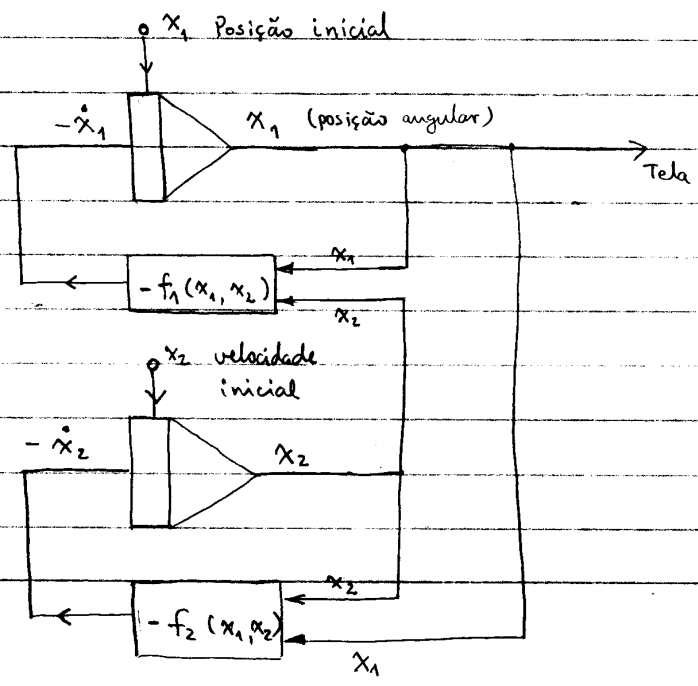
onde

f_1 é linear em x_2 e independente de x_1
f_2 é linear em x_2 e não linear em x_1

vetorialmente,

$$\underline{\dot{x}} = \underline{f}(\underline{x})$$

analogicamente para cada gerador temos a seguinte configuração:



A simulação analógica para o exemplo 4.1. é,

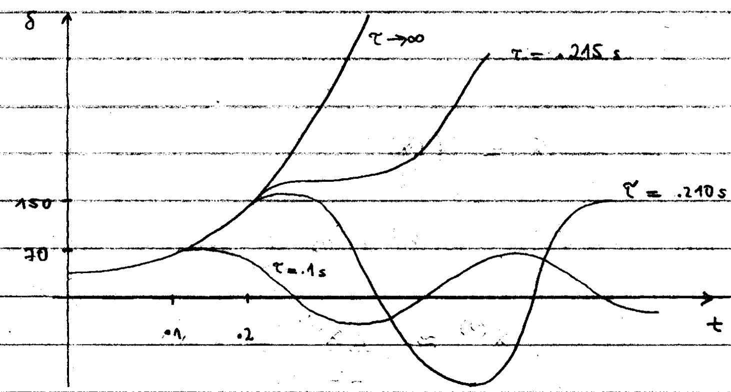
No período em falta: $\delta(t) = 2022.5\, t^2 + 15.5°$

Supondo que a falta é eliminada em $t = \tau$, as condições iniciais para o período pós falta são:

$x_1(\tau) = 2022.5\, \tau^2 + 15.5°$ posição inicial (pós falta)

$x_2(\tau) = 4050\, \tau$ velocidade inicial (pós falta).

Dependendo do valor de τ podem-se obter vários resultados:



Simulação Digital:

- a variável independente t é discretizada nos intervalos de tempo, $t^{(0)}$, $t^{(1)}$, ..., $t^{(\nu)}$, ..., não necessàriamente equidistantes.

• ideia :

Partindo de um estado inicial conhecido $\underline{x}^{(0)}$, calcular novos estados $\underline{x}^{(1)}$, $\underline{x}^{(2)}$, ..., $x^{(\nu)}$, ...

• Existem vários métodos de integração numérica : Euler, Euler Modificado, Runge-kutta, Trapezoidal.

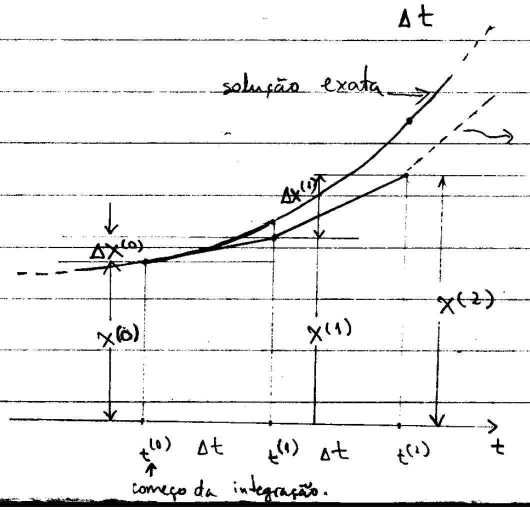
Método de Euler :

caso escalar :

$$\dot{x} = f(x)$$

para $t = t^{(\nu)}$, estabelecemos a seguinte precisão :

$$\dot{x}^{(\nu)} \approx \frac{\Delta x^{(\nu)}}{\Delta t} = f(x)$$



idéia do processo:

PASSO 0 $\quad t = t^{(0)}$ e $x = x^{(0)}$

calcular: $\Delta x^{(0)} = f(x^{(0)}) \cdot \Delta t$

PASSO 1 $\quad t^{(1)} = t^{(0)} + \Delta t$

$$x^{(1)} = x^{(0)} + f(x^{(0)})\,\Delta t$$

PASSO 2 $\quad t^{(2)} = t^{(1)} + \Delta t$

$$x^{(2)} = x^{(1)} + f(x^{(1)})\,\Delta t$$

$\vdots$

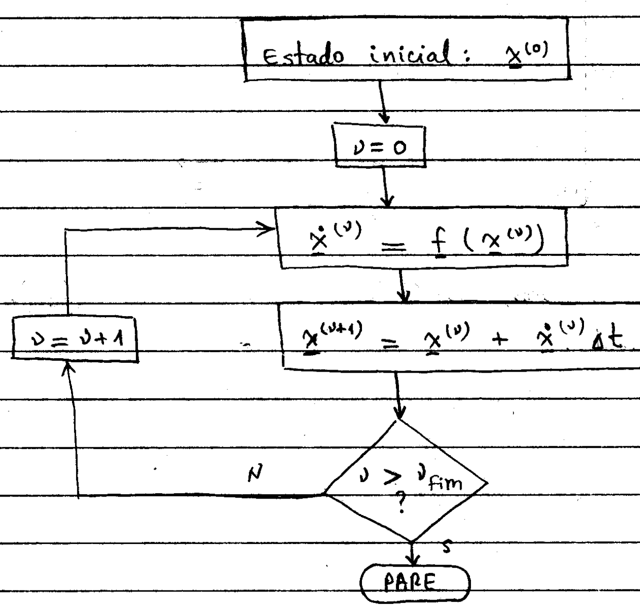
etc.

Algoritmo:

$$x^{(\nu+1)} = x^{(\nu)} + f(x^{(\nu)}) \cdot \Delta t \;, \quad \nu = 0, 1, \ldots$$

vetorialmente,

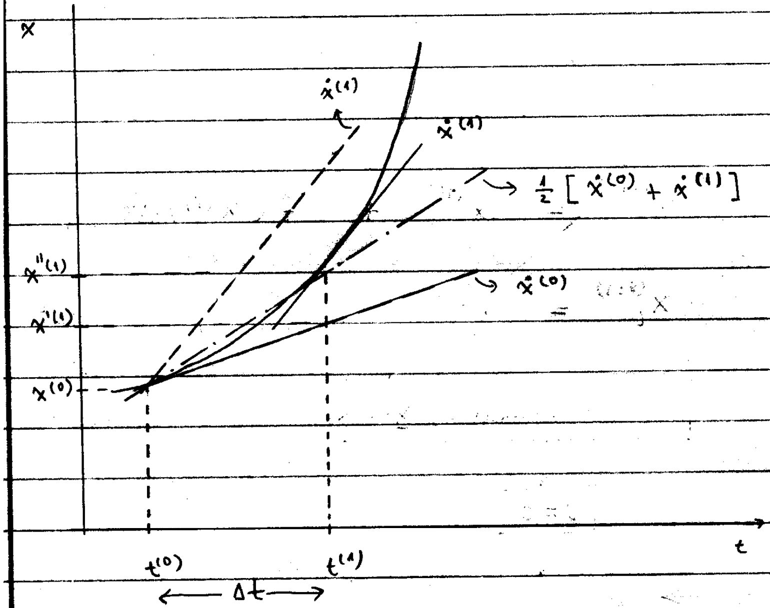
$$x_i^{(\nu+1)} = x_i^{(\nu)} + f_i(x_1^{(\nu)}, x_2^{(\nu)}, \ldots, x_n^{(\nu)}) \cdot \Delta t \;, \quad i = 1, \ldots, n$$



Desvantagens:

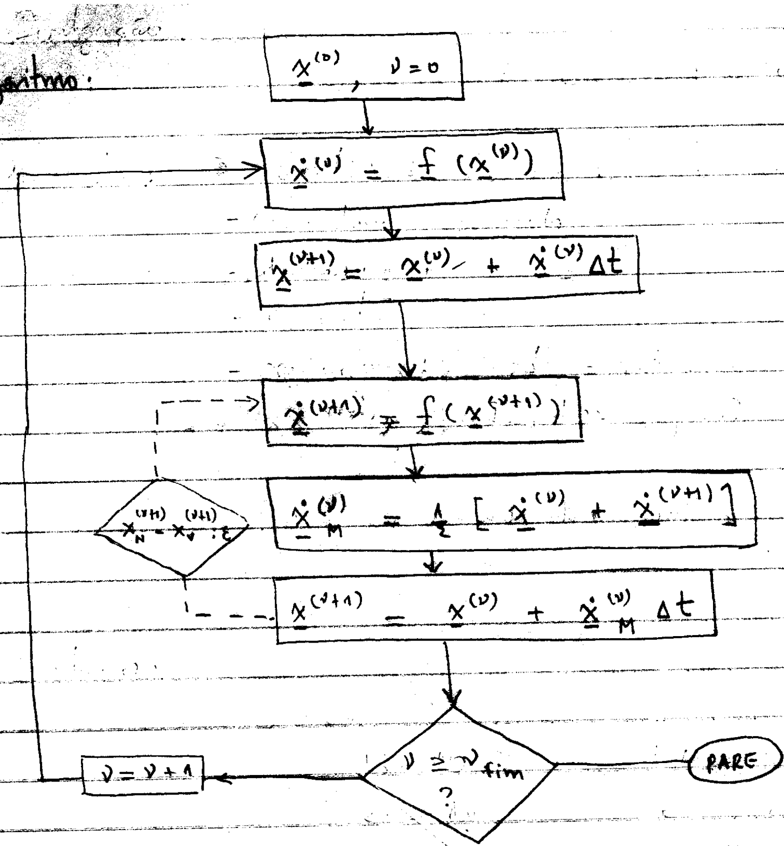
· imprecissão, pois as variáveis de estado no final de um intervalo $t + \Delta$ são calculadas a partir da derivada no começo do intervalo. Será introduzido um erro que aumentará na medida em que a derivada mude rapidamente dentro do intervalo Δt.

Método de Euler Modificado:



ideia: utilizar o valor médio das derivadas nos extremos do intervalo.

Algoritmo:



- O laço interno se refere a que o processo pode ser repetido até que nenhum melhoramento maior do que ε seja detectado.

Método de Runge-Kutta

Formula generica:

$$x^{(\nu+1)} = x^{(\nu)} + \sum_{\ell=1}^{m} a_\ell k_\ell$$

onde,

m - ordem do método Runge-Kutta

$a_\ell \Rightarrow$ constantes

$k_\ell \Rightarrow$ produtos de $\Delta t \cdot f(x)$ calculada em vários pontos.

SECTION 13.1

THE SWING EQUATION

Consider a generating unit consisting of a three-phase synchronous generator and its prime mover. The rotor motion is determined by Newton's second law, given by

$$J\alpha_m(t) = T_m(t) - T_e(t) = T_a(t) \tag{13.1.1}$$

where J = total moment of inertia of the rotating masses, kgm^2

α_m = rotor angular acceleration, rad/s^2

T_m = mechanical torque supplied by the prime mover minus the retarding torque due to mechanical losses, Nm

T_e = electrical torque that accounts for the total three-phase electrical power output of the generator, plus electrical losses, Nm

T_a = net accelerating torque, Nm

Also, the rotor angular acceleration is given by

$$\alpha_m(t) = \frac{d\omega_m(t)}{dt} = \frac{d^2\theta_m(t)}{dt^2} \tag{13.1.2}$$

$$\omega_m(t) = \frac{d\theta_m(t)}{dt} \tag{13.1.3}$$

where ω_m = rotor angular velocity, rad/s

θ_m = rotor angular position with respect to a stationary axis, rad

T_m and T_e are positive for generator operation. In steady-state T_m equals T_e, the accelerating torque T_a is zero, and, from (13.1.1), the rotor acceleration α_m is zero, resulting in a constant rotor velocity called *synchronous speed.* When T_m is greater than T_e, T_a is positive and α_m is therefore positive, resulting in increasing rotor speed. Similarly, when T_m is less than T_e, the rotor speed is decreasing.

It is convenient to measure the rotor angular position with respect to a synchronously rotating reference axis instead of a stationary axis. Accordingly, we define

$$\theta_m(t) = \omega_{msyn}t + \delta_m(t) \tag{13.1.4}$$

where ω_{msyn} = synchronous angular velocity of the rotor, rad/s

δ_m = rotor angular position with respect to a synchronously rotating reference, rad

Using (13.1.2) and (13.1.4), (13.1.1) becomes

$$J\frac{d^2\theta_m(t)}{dt^2} = J\frac{d^2\delta_m(t)}{dt^2} = T_m(t) - T_e(t) = T_a(t) \tag{13.1.5}$$

It is also convenient to work with power rather than torque, and to work in per-unit rather than in actual units. Accordingly, we multiply (13.1.5) by $\omega_m(t)$ and divide by S_{rated}, the three-phase voltampere rating of the generator:

$$\frac{J\omega_m(t)}{S_{rated}}\frac{d^2\delta_m(t)}{dt^2} = \frac{\omega_m(t)T_m(t) - \omega_m(t)T_e(t)}{S_{rated}}$$

$$= \frac{p_m(t) - p_e(t)}{S_{rated}} = p_{mp.u.}(t) - p_{ep.u.}(t) = p_{ap.u.}(t) \tag{13.1.6}$$

where $p_{mp.u.}$ = mechanical power supplied by the prime mover minus mechanical losses, per unit

$p_{ep.u.}$ = electrical power output of the generator plus electrical losses, per unit

Finally, it is convenient to work with a normalized inertia constant, called the H constant, which is defined as

$$H = \frac{\text{stored kinetic energy at synchronous speed}}{\text{generator voltampere rating}}$$

$$= \frac{\frac{1}{2}J\omega_{msyn}^2}{S_{rated}} \quad \text{joules/VA or per unit-seconds} \tag{13.1.7}$$

The H constant has the advantage that it falls within a fairly narrow range, normally between 1 and 10 p.u.-s, whereas J varies widely, depending on generating unit size and type. Solving (13.1.7) for J and using in (13.1.6),

$$2H \frac{\omega_m(t)}{\omega_{msyn}^2} \frac{d^2\delta_m(t)}{dt^2} = p_{mp.u.}(t) - p_{ep.u.}(t) = p_{ap.u.}(t) \tag{13.1.8}$$

Defining per-unit rotor angular velocity,

$$\omega_{p.u.}(t) = \frac{\omega_m(t)}{\omega_{msyn}} \tag{13.1.9}$$

Equation (13.1.8) becomes

$$\frac{2H}{\omega_{msyn}} \omega_{p.u.}(t) \frac{d^2\delta_m(t)}{dt^2} = p_{mp.u.}(t) - p_{ep.u.}(t) = p_{ap.u.}(t) \tag{13.1.10}$$

For a synchronous generator with P poles, the electrical angular acceleration α, electrical radian frequency ω, and power angle δ are

$$\alpha(t) = \frac{P}{2} \alpha_m(t) \tag{13.1.11}$$

$$\omega(t) = \frac{P}{2} \omega_m(t) \tag{13.1.12}$$

$$\delta(t) = \frac{P}{2} \delta_m(t) \tag{13.1.13}$$

Similarly, the synchronous electrical radian frequency is

$$\omega_{syn} = \frac{P}{2} \omega_{msyn} \tag{13.1.14}$$

The per-unit electrical frequency is

$$\omega_{p.u.}(t) = \frac{\omega(t)}{\omega_{syn}} = \frac{\frac{2}{P}\omega(t)}{\frac{2}{P}\omega_{syn}} = \frac{\omega_m(t)}{\omega_{msyn}} \tag{13.1.15}$$

Therefore, using (13.1.13–13.1.15), (13.1.10) can be written as

$$\frac{2H}{\omega_{syn}} \omega_{p.u.}(t) \frac{d^2\delta(t)}{dt^2} = p_{mp.u.}(t) - p_{ep.u.}(t) = p_{ap.u.}(t) \tag{13.1.16}$$

Equation (13.1.16), called the per-unit *swing equation*, is the fundamental equation that determines rotor dynamics in transient stability studies. Note that it is nonlinear due to $p_{ep.u.}(t)$, which is shown in Section 13.2 to be a nonlinear function of δ. Equation (13.1.16) is also nonlinear due to the $\omega_{p.u.}(t)$ term. However, in practice the rotor speed does not vary significantly from synchronous speed during transients. That is, $\omega_{p.u.}(t) \simeq 1.0$, which is often assumed in (13.1.16) for hand calculations.

Equation (13.1.16) is a second-order differential equation that can be rewritten as two first-order differential equations. Differentiating (13.1.4), and then using (13.1.3) and (13.1.12)–(13.1.14), we obtain

$$\frac{d\delta(t)}{dt} = \omega(t) - \omega_{syn} \tag{13.1.17}$$

Using (13.1.17) in (13.1.16),

$$\frac{2\text{H}}{\omega_{syn}}\,\omega_{p.u.}(t)\,\frac{d\omega(t)}{dt} = p_{mp.u.}(t) - p_{ep.u.}(t) = p_{ap.u.}(t) \tag{13.1.18}$$

Equations (13.1.17) and (13.1.18) are two first-order differential equations.

EXAMPLE 13.1

Generator per-unit swing equation and power angle during a short circuit

A three-phase, 60-Hz, 500-MVA, 15-kV, 32-pole hydroelectric generating unit has an H constant of 2.0 p.u.-s. (a) Determine ω_{syn} and ω_{msyn}. (b) Give the per-unit swing equation for this unit. (c) The unit is initially operating at $p_{mp.u.} = p_{ep.u.} = 1.0$, $\omega = \omega_{syn}$, and $\delta = 10°$ when a three-phase-to-ground bolted short circuit at the generator terminals causes $p_{ep.u.}$ to drop to zero for $t \geqslant 0$. Determine the power angle 3 cycles after the short circuit commences. Assume $p_{mp.u.}$ remains constant at 1.0 per unit. Also assume $\omega_{p.u.}(t) = 1.0$ in the swing equation.

Solution

a. For a 60-Hz generator,

$$\omega_{syn} = 2\pi 60 = 377 \quad \text{rad/s}$$

and, from (13.1.14), with P = 32 poles,

$$\omega_{msyn} = \frac{2}{\text{P}}\,\omega_{syn} = \left(\frac{2}{32}\right) 377 = 23.56 \quad \text{rad/s}$$

b. From (13.1.16), with H = 2.0 p.u.-s,

$$\frac{4}{2\pi 60}\,\omega_{p.u.}(t)\,\frac{d^2\delta(t)}{dt^2} = p_{mp.u.}(t) - p_{ep.u.}(t)$$

c. The initial power angle is

$$\delta(0) = 10° = 0.1745 \quad \text{radian}$$

Also, from (13.1.17), at $t = 0$,

$$\frac{d\delta(0)}{dt} = 0$$

Using $p_{mp.u.}(t) = 1.0$, $p_{ep.u.} = 0$, and $\omega_{p.u.}(t) = 1.0$, the swing equation from (b) is

$$\left(\frac{4}{2\pi 60}\right)\frac{d^2\delta(t)}{dt^2} = 1.0 \quad t \geqslant 0$$

Integrating twice and using the above initial conditions,

$$\frac{d\delta(t)}{dt} = \left(\frac{2\pi 60}{4}\right) t + 0$$

$$\delta(t) = \left(\frac{2\pi 60}{8}\right) t^2 + 0.1745$$

At $t = 3$ cycles $= \dfrac{3 \text{ cycles}}{60 \text{ cycles/second}} = 0.05$ second,

$$\delta(0.05) = \left(\frac{2\pi 60}{8}\right)(0.05)^2 + 0.1745$$

$$= 0.2923 \text{ radian} = 16.75°$$

■

EXAMPLE 13.2 **Equivalent swing equation: two generating units**

A power plant has two three-phase, 60-Hz generating units with the following ratings:

Unit 1: 500 MVA, 15 kV, 0.85 power factor, 32 poles, $H_1 = 2.0$ p.u.-s

Unit 2: 300 MVA, 15 kV, 0.90 power factor, 16 poles, $H_2 = 2.5$ p.u.-s

(a) Give the per-unit swing equation of each unit on a 100-MVA system base. (b) If the units are assumed to "swing together," that is, $\delta_1(t) = \delta_2(t)$, combine the two swing equations into one equivalent swing equation.

Solution a. If the per-unit powers on the right-hand side of the swing equation are converted to the system base, then the H constant on the left-hand side must also be converted. That is,

$$H_{new} = H_{old}\frac{S_{old}}{S_{new}} \quad \text{per unit}$$

Converting H_1 from its 500-MVA rating to the 100-MVA system base,

$$H_{1new} = H_{1old}\frac{S_{old}}{S_{new}} = (2.0)\left(\frac{500}{100}\right) = 10 \quad \text{p.u.-s}$$

Similarly, converting H_2,

$$H_{2new} = (2.5)\left(\frac{300}{100}\right) = 7.5 \quad \text{p.u.-s}$$

The per-unit swing equations on the system base are then

$$\frac{2H_{1new}}{\omega_{syn}}\omega_{1p.u.}(t)\frac{d^2\delta_1(t)}{dt^2} = \frac{20.0}{2\pi 60}\omega_{1p.u.}(t)\frac{d^2\delta_1(t)}{dt^2} = p_{m1p.u.}(t) - p_{e1p.u.}(t)$$

$$\frac{2H_{2new}}{\omega_{syn}}\omega_{2p.u.}(t)\frac{d^2\delta_2(t)}{dt^2} = \frac{15.0}{2\pi 60}\omega_{2p.u.}(t)\frac{d^2\delta_2(t)}{dt^2} = p_{m2p.u.}(t) - p_{e2p.u.}(t)$$

b. Letting:

$$\delta(t) = \delta_1(t) = \delta_2(t)$$

$$\omega_{p.u.}(t) = \omega_{1p.u.}(t) = \omega_{2p.u.}(t)$$

$$p_{mp.u.}(t) = p_{m1p.u.}(t) + p_{m2p.u.}(t)$$

$$p_{ep.u.}(t) = p_{e1p.u.}(t) + p_{e2p.u.}(t)$$

and adding the above swing equations

$$\frac{2(H_{1new} + H_{2new})}{\omega_{syn}}\omega_{p.u.}(t)\frac{d^2\delta(t)}{dt^2} = \frac{35.0}{2\pi 60}\omega_{p.u.}(t)\frac{d^2\delta(t)}{dt^2} = p_{mp.u.}(t) - p_{ep.u.}(t)$$

When transient stability studies involving large-scale power systems with many generating units are performed with a digital computer, computation time can be reduced by combining the swing equations of those units that swing together. Such units, which are called *coherent machines*, usually are connected to the same bus or are electrically close, and they are usually remote from network disturbances under study. ■

SECTION 13.2

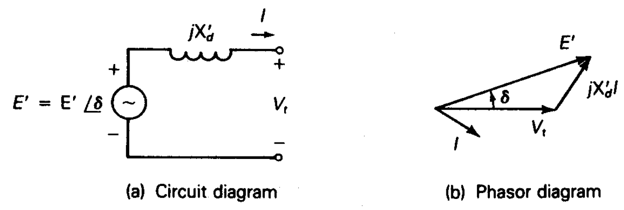
SIMPLIFIED SYNCHRONOUS MACHINE MODEL AND SYSTEM EQUIVALENTS

Figure 13.2 shows a simplified model of a synchronous machine, called the classical model, that can be used in transient stability studies. As shown, the synchronous machine is represented by a constant internal voltage E' behind its direct axis transient reactance X'_d. This model is based on the following assumptions:

1. The machine is operating under balanced three-phase positive-sequence conditions.

Figure 13.2

Simplified synchronous machine model for transient stability studies



2. Machine excitation is constant.
3. Machine losses, saturation, and saliency are neglected.

In transient stability programs, more detailed models can be used to represent exciters, losses, saturation, and saliency. However, the simplified model reduces model complexity while maintaining reasonable accuracy in stability calculations.

Each generator in the model is connected to a system consisting of transmission lines, transformers, loads, and other machines. To a first approximation the system can be represented by an "infinite bus" behind a system reactance. An infinite bus is an ideal voltage source that maintains constant voltage magnitude, constant phase, and constant frequency.

Figure 13.3 shows a synchronous generator connected to a system equivalent. The voltage magnitude V_{bus} and 0° phase of the infinite bus are constant. The phase angle δ of the internal machine voltage is the machine power angle with respect to the infinite bus.

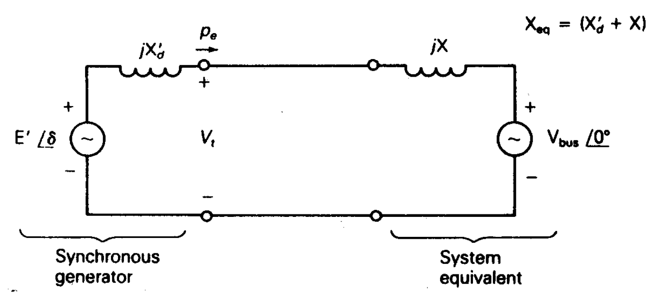
The equivalent reactance between the machine internal voltage and the infinite bus is $X_{eq} = (X'_d + X)$. From (7.10.3), the real power delivered by the synchronous generator to the infinite bus is

$$p_e = \frac{E'V_{bus}}{X_{eq}} \sin \delta \tag{13.2.1}$$

During transient disturbances both E′ and V_{bus} are considered constant in (13.2.1). Thus p_e is a sinusoidal function of the machine power angle δ.

Figure 13.3

Synchronous generator connected to a system equivalent



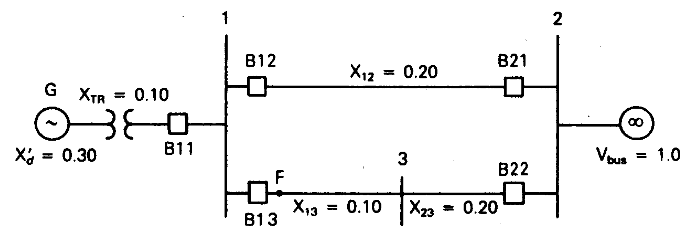
EXAMPLE 13.3

Generator internal voltage and real power output versus power angle

Figure 13.4 shows a single-line diagram of a three-phase, 60-Hz synchronous generator, connected through a transformer and parallel transmission lines to an infinite bus. All reactances are given in per-unit on a common system base. If the infinite bus receives 1.0 per unit real power at 0.95 p.f. lagging, determine (a) the internal voltage of the generator and (b) the equation for the electrical power delivered by the generator versus its power angle δ.

Figure 13.4

Single-line diagram for Example 13.3

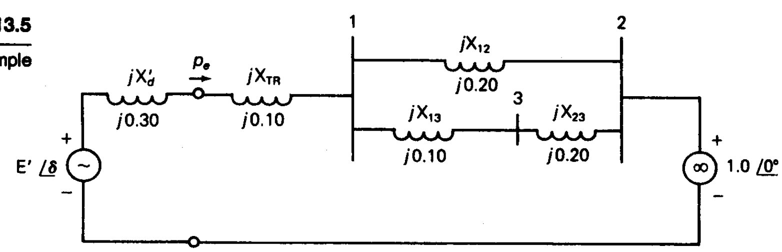


Solution a. The equivalent circuit is shown in Figure 13.5, from which the equivalent reactance between the machine internal voltage and infinite bus is

$$\begin{aligned} X_{eq} &= X_d' + X_{TR} + X_{12}\|(X_{13} + X_{23}) \\ &= 0.30 + 0.10 + 0.20\|(0.10 + 0.20) \\ &= 0.520 \quad \text{per unit} \end{aligned}$$

Figure 13.5

Equivalent circuit for Example 13.3



The current into the infinite bus is

$$I = \frac{P}{V_{bus}(\text{p.f.})} \underline{/-\cos^{-1}(\text{p.f.})} = \frac{(1.0)}{(1.0)(0.95)} \underline{/-\cos^{-1}0.95}$$

$$= 1.05263\underline{/-18.195°}$$

and the machine internal voltage is

$$E' = E'\underline{/\delta} = V_{bus} + jX_{eq}I$$
$$= 1.0\underline{/0°} + (j0.520)(1.05263\underline{/-18.195°})$$
$$= 1.0\underline{/0°} + 0.54737\underline{/71.805°}$$
$$= 1.1709 + j0.5200$$
$$= 1.2812\underline{/23.946°} \quad \text{per unit}$$

b. From (13.2.1),

$$p_e = \frac{(1.2812)(1.0)}{0.520} \sin\delta = 2.4638 \sin\delta \quad \text{per unit}$$

■

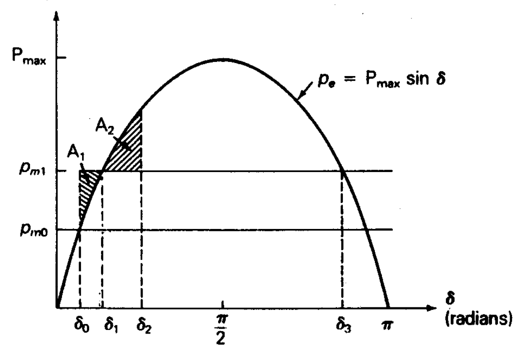
SECTION 13.3

THE EQUAL-AREA CRITERION

Consider a synchronous generating unit connected through a reactance to an infinite bus. Plots of electrical power p_e and mechanical power p_m versus power angle δ are shown in Figure 13.6. p_e is a sinusoidal function of δ, as given by (13.2.1).

Figure 13.6

p_e and p_m versus δ



Suppose the unit is initially operating in steady-state at $p_e = p_m = p_{m0}$ and $\delta = \delta_0$, when a step change in p_m from p_{m0} to p_{m1} occurs at $t = 0$. Due to rotor inertia, the rotor position cannot change instantaneously. That is, $\delta_m(0^+) = \delta_m(0^-)$; therefore, $\delta(0^+) = \delta(0^-) = \delta_0$ and $p_e(0^+) = p_e(0^-)$. Since $p_m(0^+) = p_{m1}$ is greater than $p_e(0^+)$, the acceleration power $p_a(0^+)$ is positive and, from (13.1.16), $(d^2\delta)/(dt^2)(0^+)$ is positive. The rotor accelerates and δ

increases. When δ reaches δ_1, $p_e = p_{m1}$ and $(d^2\delta)/(dt^2)$ becomes zero. However, $d\delta/dt$ is still positive and δ continues to increase, overshooting its final steady-state operating point. When δ is greater than δ_1, p_m is less than p_e, p_a is negative, and the rotor decelerates. Eventually, δ reaches a maximum value δ_2 and then swings back toward δ_1. Using (13.1.16), which has no damping, δ would continually oscillate around δ_1. However, damping due to mechanical and electrical losses causes δ to stabilize at its final steady-state operating point δ_1. Note that if the power angle exceeded δ_3, then p_m would exceed p_e and the rotor would accelerate again, causing a further increase in δ and loss of stability.

One method for determining stability and maximum power angle is to solve the nonlinear swing equation via numerical integration techniques using a digital computer. This method, which is applicable to multimachine systems, is described in Section 13.4. However, there is also a direct method for determining stability that does not involve solving the swing equation; this method is applicable for one machine connected to an infinite bus or for two machines. The method, called the *equal-area criterion*, is described in this section.

In Figure 13.6, p_m is greater than p_e during the interval $\delta_0 < \delta < \delta_1$, and the rotor is accelerating. The shaded area A_1 between the p_m and p_e curves is called the accelerating area. During the interval $\delta_1 < \delta < \delta_2$, p_m is less than p_e, the rotor is decelerating, and the shaded area A_2 is the decelerating area. At both the initial value $\delta = \delta_0$ and the maximum value $\delta = \delta_2$, $d\delta/dt = 0$. The equal-area criterion states that $A_1 = A_2$.

In order to derive the equal-area criterion for one machine connected to an infinite bus, assume $\omega_{\text{p.u.}}(t) = 1$ in (13.1.16), giving

$$\frac{2H}{\omega_{\text{syn}}}\frac{d^2\delta}{dt^2} = p_{m\text{p.u.}} - p_{e\text{p.u.}} \tag{13.3.1}$$

Multiplying by $d\delta/dt$ and using

$$\frac{d}{dt}\left[\frac{d\delta}{dt}\right]^2 = 2\left(\frac{d\delta}{dt}\right)\left(\frac{d^2\delta}{dt^2}\right)$$

(13.3.1) becomes

$$\frac{2H}{\omega_{\text{syn}}}\left(\frac{d^2\delta}{dt^2}\right)\left(\frac{d\delta}{dt}\right) = \frac{H}{\omega_{\text{syn}}}\frac{d}{dt}\left[\frac{d\delta}{dt}\right]^2 = (p_{m\text{p.u.}} - p_{e\text{p.u.}})\frac{d\delta}{dt} \tag{13.3.2}$$

Multiplying (13.3.2) by dt and integrating from δ_0 to δ,

$$\frac{H}{\omega_{\text{syn}}}\int_{\delta_0}^{\delta} d\left[\frac{d\delta}{dt}\right]^2 = \int_{\delta_0}^{\delta}(p_{m\text{p.u.}} - p_{e\text{p.u.}})\,d\delta$$

or

$$\frac{H}{\omega_{\text{syn}}}\left[\frac{d\delta}{dt}\right]^2\Bigg|_{\delta_0}^{\delta} = \int_{\delta_0}^{\delta}(p_{m\text{p.u.}} - p_{e\text{p.u.}})\,d\delta \tag{13.3.3}$$

The above integration begins at δ_0 where $d\delta/dt = 0$, and continues to an arbitrary δ. When δ reaches its maximum value, denoted δ_2, $d\delta/dt$ again equals zero. Therefore, the left-hand side of (13.3.3) equals zero for $\delta = \delta_2$ and

$$\int_{\delta_0}^{\delta_2} (p_{mp.u.} - p_{ep.u.})d\delta = 0 \tag{13.3.4}$$

Separating this integral into positive (accelerating) and negative (decelerating) areas, we arrive at the equal-area criterion

$$\int_{\delta_0}^{\delta_1} (p_{mp.u.} - p_{ep.u.})d\delta + \int_{\delta_1}^{\delta_2} (p_{mp.u.} - p_{ep.u.})d\delta = 0$$

or

$$\underbrace{\int_{\delta_0}^{\delta_1} (p_{mp.u.} - p_{ep.u.})d\delta}_{A_1} = \underbrace{\int_{\delta_1}^{\delta_2} (p_{ep.u.} - p_{mp.u.})d\delta}_{A_2} \tag{13.3.5}$$

In practice, sudden changes in mechanical power usually do not occur, since the time constants associated with prime mover dynamics are on the order of seconds. However, stability phenomena similar to that described above can also occur from sudden changes in electrical power, due to system faults and line switching. The following three examples are illustrative.

EXAMPLE 13.4

Equal-area criterion: transient stability during a three-phase fault

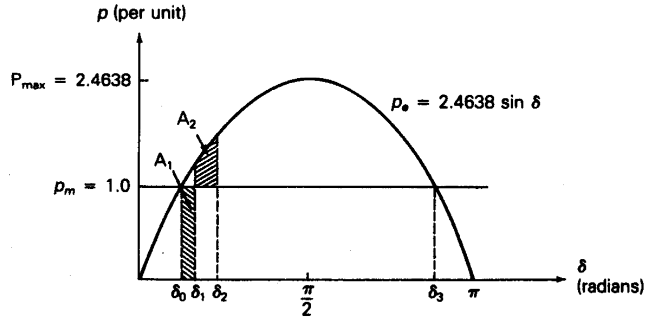
BA1

The synchronous generator shown in Figure 13.4 is initially operating in the steady-state condition given in Example 13.3, when a temporary three-phase-to-ground bolted short circuit occurs on line 1–3 at bus 1, shown as point F in Figure 13.4. Three cycles later the fault extinguishes by itself. Due to a relay misoperation, all circuit breakers remain closed. Determine whether stability is or is not maintained and determine the maximum power angle. The inertia constant of the generating unit is 3.0 per unit-seconds on the system base. Assume p_m remains constant throughout the disturbance. Also assume $\omega_{p.u.}(t) = 1.0$ in the swing equation.

Solution Plots of p_e and p_m versus δ are shown in Figure 13.7. From Example 13.3 the

Figure 13.7

p–δ plot for Example 13.4



initial operating point is $p_e(0^-)=p_m=1.0$ per unit and $\delta(0^+)=\delta(0^-)=\delta_0=23.95° = 0.4179$ radian. At $t=0$, when the short circuit occurs, p_e instantaneously drops to zero and remains at zero during the fault since power cannot be transferred past faulted bus 1. From (13.1.16), with $\omega_{\text{p.u.}}(t)=1.0$,

$$\frac{2H}{\omega_{\text{syn}}}\frac{d^2\delta(t)}{dt^2} = p_{m\text{p.u.}} \qquad 0 \leqslant t \leqslant 0.05 \quad \text{s}$$

Integrating twice with initial condition $\delta(0)=\delta_0$ and $\dfrac{d\delta(0)}{dt}=0$,

$$\frac{d\delta(t)}{dt} = \frac{\omega_{\text{syn}} p_{m\text{p.u.}}}{2H}\, t + 0$$

$$\delta(t) = \frac{\omega_{\text{syn}} p_{m\text{p.u.}}}{4H}\, t^2 + \delta_0$$

At $t = 3$ cycles $= 0.05$ second,

$$\delta_1 = \delta(0.05 \text{ s}) = \frac{2\pi 60}{12}(0.05)^2 + 0.4179$$

$$= 0.4964 \text{ radian} = 28.44°$$

The accelerating area A_1, shaded in Figure 13.7, is

$$A_1 = \int_{\delta_0}^{\delta_1} p_m d\delta = \int_{\delta_0}^{\delta_1} 1.0 d\delta = (\delta_1 - \delta_2) = 0.4964 - 0.4179 = 0.0785$$

At $t = 0.05$ s the fault extinguishes and p_e instantaneously increases from zero to the sinusoidal curve in Figure 13.7. δ continues to increase until the decelerating area A_2 equals A_1. That is,

$$A_2 = \int_{\delta_1}^{\delta_2} (p_{\max} \sin\delta - p_m)\, d\delta$$

$$= \int_{0.4964}^{\delta_2} (2.4638 \sin\delta - 1.0)\, d\delta = A_1 = 0.0785$$

Integrating,

$$2.4638[\cos(0.4964) - \cos\delta_2] - (\delta_2 - 0.4964) = 0.0785$$

$$2.4638 \cos\delta_2 + \delta_2 = 2.5843$$

The above nonlinear algebraic equation can be solved iteratively to obtain

$$\delta_2 = 0.7003 \text{ radian} = 40.12°$$

Since the maximum angle δ_2 does not exceed $\delta_3=(180° - \delta_0) = 156.05°$, stability is maintained. In steady-state, the generator returns to its initial operating point $p_{ess} = p_m = 1.0$ per unit and $\delta_{ss} = \delta_0 = 23.95°$.

Note that as the fault duration increases, the risk of instability also increases. The *critical clearing time*, denoted t_{cr}, is the longest fault duration allowable for stability. ■

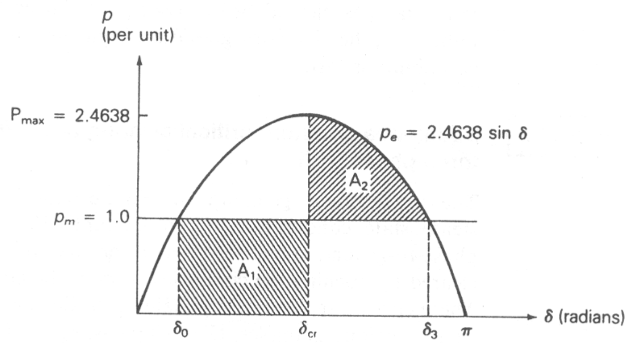
EXAMPLE 13.5

Equal-area criterion: critical clearing time for a temporary three-phase fault

Assuming the temporary short circuit in Example 13.4 lasts longer than 3 cycles, calculate the critical clearing time.

Figure 13.8

p–δ plot for Example 13.5



Solution The p–δ plot is shown in Figure 13.8. At the critical clearing angle, denoted δ_{cr}, the fault is extinguished. The power angle then increases to a maximum value $\delta_3 = 180° - \delta_0 = 156.05° = 2.7236$ radians, which gives the maximum decelerating area. Equating the accelerating and decelerating areas,

$$A_1 = \int_{\delta_0}^{\delta_{cr}} p_m d\delta = A_2 = \int_{\delta_{cr}}^{\delta_3} (P_{max}\sin\delta - p_m)\,d\delta$$

$$\int_{0.4179}^{\delta_{cr}} 1.0\,d\delta = \int_{\delta_{cr}}^{2.7236} (2.4638\sin\delta - 1.0)\,d\delta$$

Solving for δ_{cr},

$$(\delta_{cr} - 0.4179) = 2.4638[\cos\delta_{cr} - \cos(2.7236)] - (2.7236 - \delta_{cr})$$

$$2.4638\cos\delta_{cr} = +0.05402$$

$$\delta_{cr} = 1.5489 \text{ radians} = 88.74°$$

From the solution to the swing equation given in Example 13.4,

$$\delta(t) = \frac{\omega_{syn} p_{mp.u.}}{4H} t^2 + \delta_0$$

Solving

$$t = \sqrt{\frac{4H}{\omega_{syn} p_{mp.u.}} (\delta(t) - \delta_0)}$$

Using $\delta(t_{cr}) = \delta_{cr} = 1.5489$ and $\delta_0 = 0.4179$ radian,

$$t_{cr} = \sqrt{\frac{12}{(2\pi 60)(1.0)}(1.5489 - 0.4179)}$$
$$= 0.1897 \text{ s} = 11.38 \text{ cycles}$$

If the fault is cleared before $t = t_{cr} = 11.38$ cycles, stability is maintained. Otherwise, the generator goes out of synchronism with the infinite bus. That is, stability is lost. ■

EXAMPLE 13.6

Equal-area criterion: critical clearing angle for a cleared three-phase fault

The synchronous generator in Figure 13.4 is initially operating in the steady-state condition given in Example 13.3 when a permanent three-phase-to-ground bolted short circuit occurs on line 1–3 at bus 3. The fault is cleared by opening the circuit breakers at the ends of line 1–3 and line 2–3. These circuit breakers then remain open. Calculate the critical clearing angle. As in previous examples, H = 3.0 p.u.-s, $p_m = 1.0$ per unit and $\omega_{p.u.} = 1.0$ in the swing equation.

Solution From Example 13.3, the equation for the prefault electrical power, denoted p_{e1} here, is $p_{e1} = 2.4638 \sin \delta$ per unit. The faulted network is shown in Figure 13.9(a), and the Thévenin equivalent of the faulted network, as viewed from the generator internal voltage source, is shown in Figure 13.9(b). The Thévenin reactance is

$$X_{Th} = 0.40 + 0.20 \| 0.10 = 0.46666 \quad \text{per unit}$$

and the Thévenin voltage source is

$$V_{Th} = 1.0\underline{/0^\circ}\left[\frac{X_{13}}{X_{13} + X_{12}}\right] = 1.0\underline{/0^\circ}\frac{0.10}{0.30}$$
$$= 0.33333\underline{/0^\circ} \quad \text{per unit}$$

From Figure 13.9(b), the equation for the electrical power delivered by the generator to the infinite bus during the fault, denoted p_{e2}, is

$$p_{e2} = \frac{E'V_{Th}}{X_{Th}} \sin \delta = \frac{(1.2812)(0.3333)}{0.46666} \sin \delta = 0.9152 \sin \delta \quad \text{per unit}$$

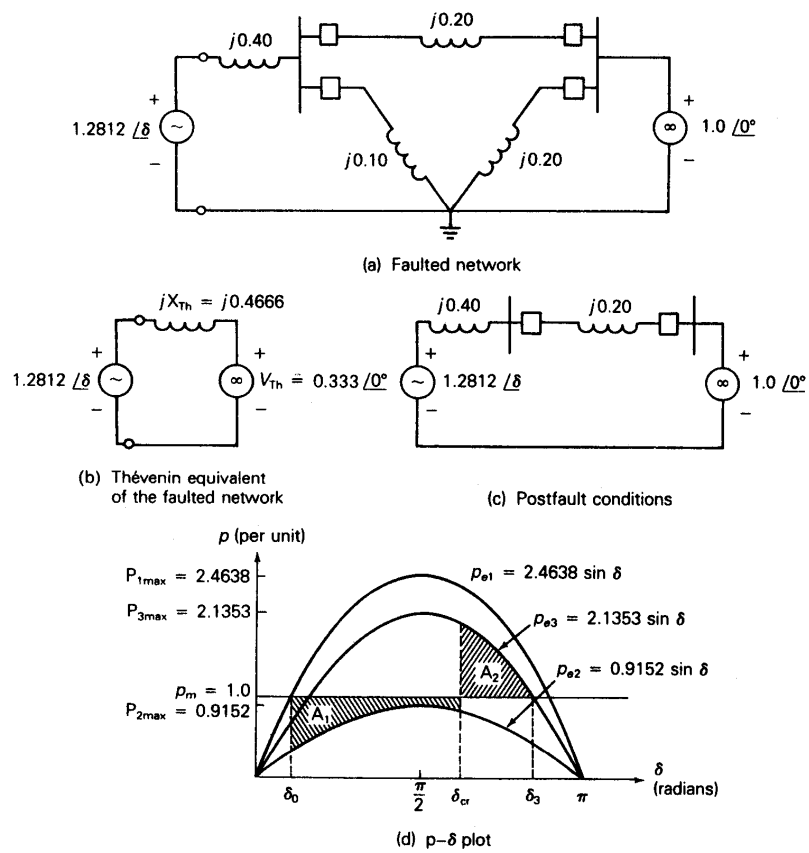
The postfault network is shown in Figure 13.9(c), where circuit breakers have opened and removed lines 1–3 and 2–3. From this figure, the postfault electrical power delivered, denoted p_{e3}, is

$$p_{e3} = \frac{(1.2812)(1.0)}{0.60} \sin \delta = 2.1353 \sin \delta \quad \text{per unit}$$

The p–δ curves as well as the accelerating area A_1 and decelerating area A_2 corresponding to critical clearing are shown in Figure 13.9(d). Equating A_1

Figure 13.9

Example 13.6



and A_2,

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (p_m - P_{2max} \sin \delta)\,d\delta = A_2 = \int_{\delta_{cr}}^{\delta_3} (P_{3max} \sin \delta - p_m)\,d\delta$$

$$\int_{0.4179}^{\delta_{cr}} (1.0 - 0.9152 \sin \delta)\,d\delta = \int_{\delta_{cr}}^{2.6542} (2.1353 \sin \delta - 1.0)\,d\delta$$

Solving for δ_{cr},

$$(\delta_{cr} - 0.4179) + 0.9152(\cos \delta_{cr} - \cos 0.4179)$$
$$= 2.1353(\cos \delta_{cr} - \cos 2.6542) - (2.6542 - \delta_{cr})$$
$$-1.2201 \cos \delta_{cr} = 0.4868$$
$$\delta_{cr} = 1.9812 \text{ radians} = 113.5°$$

If the fault is cleared before $\delta = \delta_{cr} = 113.5°$, stability is maintained. Otherwise, stability is lost. ■

SECTION 13.4

NUMERICAL INTEGRATION OF THE SWING EQUATION

The equal-area criterion is applicable to one machine and an infinite bus or to two machines. For multimachine stability problems, however, numerical integration techniques can be employed to solve the swing equation for each machine.

Given a first-order differential equation

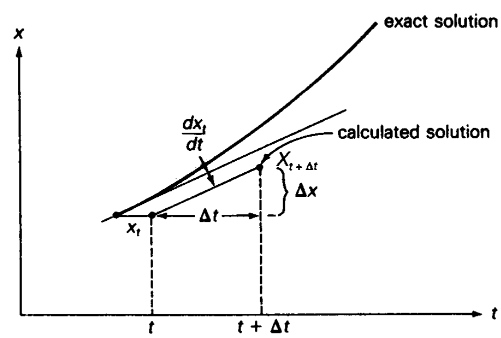
$$\frac{dx}{dt} = f(x) \tag{13.4.1}$$

one relatively simple integration technique is Euler's method [1], illustrated in Figure 13.10. The integration step size is denoted Δt. Calculating the slope at the beginning of the integration interval, from (13.4.1),

$$\frac{dx_t}{dt} = f(x_t) \tag{13.4.2}$$

Figure 13.10

Euler's method



The new value $x_{t+\Delta t}$ is calculated from the old valued x_t by adding the increment Δx,

$$x_{t+\Delta t} = x_t + \Delta x = x_t + \left(\frac{dx_t}{dt}\right)\Delta t \tag{13.4.3}$$

As shown in the figure, Euler's method assumes that the slope is constant over the entire interval Δt. An improvement can be obtained by calculating the slope at both the beginning and end of the interval, and then averaging these slopes. The modified Euler's method is illustrated in Figure 13.11. First, the slope at the beginning of the interval is calculated from

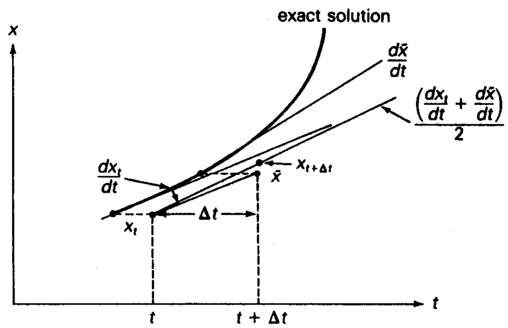


Figure 13.11

Modified Euler's method

(13.4.1) and used to calculate a preliminary value $\tilde{x}$ given by

$$\tilde{x} = x_t + \left(\frac{dx_t}{dt}\right)\Delta t \tag{13.4.4}$$

Next the slope at $\tilde{x}$ is calculated:

$$\frac{d\tilde{x}}{dt} = f(\tilde{x}) \tag{13.4.5}$$

Then the new value is calculated using the average slope:

$$x_{t+\Delta t} = x_t + \frac{\left(\frac{dx_t}{dt} + \frac{d\tilde{x}}{dt}\right)}{2}\Delta t \tag{13.4.6}$$

We now apply the modified Euler's method to calculate machine frequency ω and power angle δ. Letting x be either δ or ω, the old values at the beginning of the interval are denoted δ_t and ω_t. From (13.1.17) and (13.1.18), the slopes at the beginning of the interval are

$$\frac{d\delta_t}{dt} = \omega_t - \omega_{syn} \tag{13.4.7}$$

$$\frac{d\omega_t}{dt} = \frac{p_{ap.u.t}\,\omega_{syn}}{2H\omega_{p.u.t}} \tag{13.4.8}$$

where $p_{ap.u.t}$ is the per-unit accelerating power calculated at $\delta = \delta_t$, and $\omega_{p.u.t} = \omega_t/\omega_{syn}$. Applying (13.4.4), preliminary values are

$$\tilde{\delta} = \delta_t + \left(\frac{d\delta_t}{dt}\right)\Delta t \tag{13.4.9}$$

$$\tilde{\omega} = \omega_t + \left(\frac{d\omega_t}{dt}\right)\Delta t \tag{13.4.10}$$

Next, the slopes at $\tilde{\delta}$ and $\tilde{\omega}$ are calculated, again using (13.1.17) and (13.1.18):

$$\frac{d\tilde{\delta}}{dt} = \tilde{\omega} - \omega_{\text{syn}} \tag{13.4.11}$$

$$\frac{d\tilde{\omega}}{dt} = \frac{\tilde{p}_{ap.u.}\omega_{\text{syn}}}{2\text{H}\tilde{\omega}_{\text{p.u.}}} \tag{13.4.12}$$

where $\tilde{p}_{ap.u.}$ is the per-unit accelerating power calculated at $\delta = \tilde{\delta}$, and $\tilde{\omega}_{\text{p.u.}} = \tilde{\omega}/\omega_{\text{syn}}$. Applying (13.4.6), the new values at the end of the interval are

$$\delta_{t+\Delta t} = \delta_t + \frac{\left(\dfrac{d\delta_t}{dt} + \dfrac{d\tilde{\delta}}{dt}\right)}{2}\Delta t \tag{13.4.13}$$

$$\omega_{t+\Delta t} = \omega_t + \frac{\left(\dfrac{d\omega_t}{dt} + \dfrac{d\tilde{\omega}}{dt}\right)}{2}\Delta t \tag{13.4.14}$$

This procedure, given by (13.4.7)–(13.4.13), begins at $t = 0$ with specified initial values δ_0 and ω_0, and continues iteratively until $t = T$, a specified final time. Calculations are best performed using a digital computer.

EXAMPLE 13.7

Euler's method: computer solution to swing equation and critical clearing time

Verify the critical clearing angle determined in Example 13.6, and calculate the critical clearing time by applying the modified Euler's method to solve the swing equation for the following two cases:

Case 1 The fault is cleared at $\delta = 1.95$ radians $= 112°$ (which is less than δ_{cr})

Case 2 The fault is cleared at $\delta = 2.09$ radians $= 120°$ (which is greater than δ_{cr})

For calculations, use a step size $\Delta t = 0.01$ s, and solve the swing equation from $t = 0$ to $t = T = 0.85$ s.

Solution Equations (13.4.7)–(13.4.14) are solved by a digital computer program written in BASIC. From Example 13.6, the initial conditions at $t = 0$ are

$$\delta_0 = 0.4179 \quad \text{rad}$$

$$\omega_0 = \omega_{\text{syn}} = 2\pi 60 \quad \text{rad/s}$$

Also, the H constant is 3.0 p.u.-s, and the faulted accelerating power is

$$p_{ap.u.} = 1.0 - 0.9152 \sin\delta$$

The postfault accelerating power is

$$p_{ap.u.} = 1.0 - 2.1353 \sin\delta \quad \text{per unit}$$

The computer program and results at 0.02 s printout intervals are listed in Table 13.1. As shown, these results agree with Example 13.6, since the system

Table 13.1 Computer calculation of swing curves for Example 13.7

CASE 1 STABLE			CASE 2 UNSTABLE		
TIME s	DELTA rad	OMEGA rad/s	TIME s	DELTA rad	OMEGA rad/s
0.000	0.418	376.991	0.000	0.418	376.991
0.020	0.426	377.778	0.020	0.426	377.778
0.040	0.449	378.547	0.040	0.449	378.547
0.060	0.488	379.283	0.060	0.488	379.283
0.080	0.541	379.970	0.080	0.541	379.970
0.100	0.607	380.599	0.100	0.607	380.599
0.120	0.685	381.159	0.120	0.685	381.159
0.140	0.773	381.646	0.140	0.773	381.646
0.160	0.870	382.056	0.160	0.870	382.056
0.180	0.975	382.392	0.180	0.975	382.392
0.200	1.086	382.660	0.200	1.086	382.660
0.220	1.202	382.868	0.220	1.202	382.868
0.240	1.321	383.027	0.240	1.321	383.027
0.260	1.443	383.153	0.260	1.443	383.153
0.280	1.567	383.262	0.280	1.567	383.262
0.300	1.694	383.370	0.300	1.694	383.370
0.320	1.823	383.495	0.320	1.823	383.495
0.340	1.954	383.658	0.340	1.954	383.658
FAULT CLEARED			0.360	2.090	383.876
0.360	2.076	382.516	FAULT CLEARED		
0.380	2.176	381.510	0.380	2.217	382.915
0.400	2.257	380.638	0.400	2.327	382.138
0.420	2.322	379.886	0.420	2.424	381.546
0.440	2.373	379.237	0.440	2.511	381.135
0.460	2.413	378.674	0.460	2.591	380.902
0.480	2.441	378.176	0.480	2.668	380.844
0.500	2.460	377.726	0.500	2.746	380.969
0.520	2.471	377.307	0.520	2.828	381.288
0.540	2.473	376.900	0.540	2.919	381.824
0.560	2.467	376.488	0.560	3.022	382.609
0.580	2.453	376.056	0.580	3.145	383.686
0.600	2.429	375.583	0.600	3.292	385.111
0.620	2.396	375.053	0.620	3.472	386.949
0.640	2.351	374.446	0.640	3.693	389.265
0.660	2.294	373.740	0.660	3.965	392.099
0.680	2.221	372.917	0.680	4.300	395.426
0.700	2.130	371.960	0.700	4.704	399.079
0.720	2.019	370.855	0.720	5.183	402.689
0.740	1.884	369.604	0.740	5.729	405.683
0.760	1.723	368.226	0.760	6.325	407.477
0.780	1.533	366.773	0.780	6.941	407.812
0.800	1.314	365.341	0.800	7.551	406.981
0.820	1.068	364.070	0.820	8.139	405.711
0.840	0.799	363.143	0.840	8.702	404.819
0.860	0.516	362.750	0.860	9.257	404.934

PROGRAM LISTING

```
10 REM EXAMPLE 13.7
20 REM SOLUTION TO SWING EQUATION
30 REM THE STEP SIZE IS DELTA
40 REM THE CLEARING ANGLE IS DLTCLR
50 DELTA + .01
60 DLTCLR = 1.95
70 J = 1
80 PMAX = .9152
90 PI = 3.1415927 #
100 T = 0
110 X1 = .4179
120 X2 = 2*PI*60
130 LPRINT " TIME   DELTA   OMEGA"
140 LPRINT "  s      rad      rad/s"
150 LPRINT USING "# # # # #.# # #"; T;X1;X2
160 FOR K = 1 TO 86
170 REM LINE 180 IS EQ(13.4.7)
180 X3 = X2 - (2*PI*60)
190 IF J = 2 THEN GOTO 240
200 IF X1 > DLTCLR OR X1 = DLTCLR THEN
      PMAX = 2.1353
210 IF X1 > DLTCLR OR X1 = DLTCLR THEN
      LPRINT" FAULT CLEARED"
220 IF X1 > DLTCLR OR X1 = DLTCLR THEN
      J = 2
230 REM LINES 240 AND 250 ARE EQ(13.4.8)
240 X4 = 1 - PMAX*SIN(X1)
250 X5 = X4*(2*PI*60)*(2*PI*60)/(6*X2)
260 REM LINE 270 IS EQ(13.4.9)
270 X6 = X1 + X3*DELTA
280 REM LINE 290 IS EQ(13.4.10)
290 X7 = X2 + X5*DELTA
300 REM LINE 310 IS EQ(13.4.11)
310 X8 = X7 - 2*PI*60
320 REM LINES 330 AND 340 ARE EQ(13.4.12)
330 X9 = 1 - PMAX*SIN(X6)
340 X10 = X9*(2*PI*60)*(2*PI*60)/(6*X7)
350 REM LINE 360 IS EQ(13.4.13)
360 X1 = X1 + (X3 + X8)*(DELTA/2)
370 REM LINE 380 IS EQ(13.4.14)
380 X2 = X2 + (X5 + X10)*(DELTA/2)
390 T = K*DELTA
400 Z = K/2
410 M = INT(Z)
420 IF M = Z THEN LPRINT USING
      "# # # # # . # # #";T;X1;X2
430 NEXT K
440 END
```

is stable for Case 1 and unstable for Case 2. Also from Table 13.1, the critical clearing time is between 0.34 and 0.36 s. ■

In addition to Euler's method, there are many other numerical integration techniques, such as Runge–Kutta, Picard's method, and Milne's predictor-corrector method [1]. Comparison of the methods shows a trade-off of accuracy versus computation complexity. The Euler method is a relatively simple method to compute, but requires a small step size Δt for accuracy. Some of the other methods can use a larger step size for comparable accuracy, but the computations are more complex.

SECTION 13.5

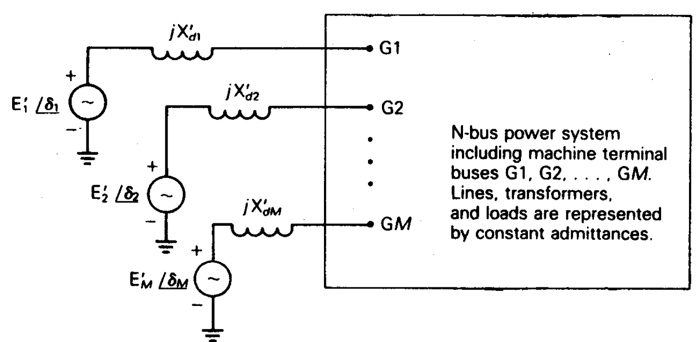
MULTIMACHINE STABILITY

The numerical integration methods discussed in Section 13.4 can be used to solve the swing equations for a multimachine stability problem. However, a method is required for computing machine output powers for a general network. Figure 13.12 shows a general N-bus power system with M synchronous machines. Each machine is the same as that represented by the simplified model of Figure 13.2, and the internal machine voltages are denoted $E'_1, E'_2, \ldots, E'_M$. The M machine terminals are connected to system buses denoted G1, G2, ..., GM in Figure 13.12. All loads are modeled here as constant admittances. Writing nodal equations for this network,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^{T} & Y_{22} \end{bmatrix} \begin{bmatrix} V \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{13.5.1}$$

Figure 13.12

N-bus power-system representation for transient stability studies



where

$$V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \quad \text{is the } N \text{ vector of bus voltages} \tag{13.5.2}$$

$$E = \begin{bmatrix} E'_1 \\ E'_2 \\ \vdots \\ E'_M \end{bmatrix} \quad \text{is the } M \text{ vector of machine voltages} \tag{13.5.3}$$

$$I = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_M \end{bmatrix} \quad \begin{array}{l} \text{is the } M \text{ vector of machine currents} \\ \text{(these are current sources)} \end{array} \tag{13.5.4}$$

$$\left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{12}^{\mathrm{T}} & Y_{22} \end{array}\right] \quad \text{is an } (N+M) \times (N+M) \text{ admittance matrix} \tag{13.5.5}$$

The admittance matrix in (13.5.5) is partitioned in accordance with the N system buses and M internal machine buses, as follows:

Y_{11} is $N \times N$

Y_{12} is $N \times M$

Y_{22} is $M \times M$

Y_{11} is similar to the bus admittance matrix used for power flows in Chapter 7, except that load admittances and inverted generator impedances are included. That is, if a load is connected to bus n, then that load admittance is added to the diagonal element Y_{11nn}. Also, $(1/jX'_{dn})$ is added to the diagonal element Y_{11GnGn}.

Y_{22} is a diagonal matrix of inverted generator impedances. That is,

$$Y_{22} = \begin{bmatrix} \dfrac{1}{jX'_{d1}} & & & 0 \\ & \dfrac{1}{jX'_{d2}} & & \\ & & \ddots & \\ 0 & & & \dfrac{1}{jX'_{dM}} \end{bmatrix} \tag{13.5.6}$$

Also, the kmth element of $\boldsymbol{Y}_{12}$ is

$$Y_{12km} = \begin{cases} \dfrac{-1}{jX'_{dn}} & \text{if } k = \text{G}n \text{ and } m = n \\ 0 & \text{otherwise} \end{cases} \tag{13.5.7}$$

Writing (13.5.1) as two separate equations,

$$\boldsymbol{Y}_{11}\boldsymbol{V} + \boldsymbol{Y}_{12}\boldsymbol{E} = \boldsymbol{0} \tag{13.5.8}$$

$$\boldsymbol{Y}_{12}^{\mathrm{T}}\boldsymbol{V} + \boldsymbol{Y}_{22}\boldsymbol{E} = \boldsymbol{I} \tag{13.5.9}$$

Assuming $\boldsymbol{E}$ is known, (13.5.8) is a linear equation in $\boldsymbol{V}$ that can be solved either iteratively or by Gauss elimination. Using the Gauss–Seidel iterative method given by (7.2.9), the kth component of $\boldsymbol{V}$ is

$$V_k(i+1) = \frac{1}{Y_{11kk}}\left[-\sum_{n=1}^{M} Y_{12kn}E_n - \sum_{n=1}^{k-1} Y_{11kn}V_n(i+1) - \sum_{n=k+1}^{N} Y_{11kn}V_n(i)\right] \tag{13.5.10}$$

After $\boldsymbol{V}$ is computed, the machine currents can be obtained from (13.5.9). That is,

$$\boldsymbol{I} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_M \end{bmatrix} = \boldsymbol{Y}_{12}^{\mathrm{T}}\boldsymbol{V} + \boldsymbol{Y}_{22}\boldsymbol{E} \tag{13.5.11}$$

The (real) electrical power output of machine n is then

$$p_{en} = \mathrm{Re}[E_n I_n^*] \qquad n = 1, 2, \ldots, M \tag{13.5.12}$$

We are now ready to outline a computation procedure for solving a transient stability problem. The procedure alternately solves the swing equations representing the machines and the above algebraic power-flow equations representing the network. We use the modified Euler method of Section 13.4 to solve the swing equations and the Gauss–Seidel iterative method to solve the power-flow equations. The procedure is now given in the following 11 steps.

Transient stability computation procedure

Step 1 Run a prefault power-flow program to compute initial bus voltages V_k, $k = 1, 2, \ldots, N$, initial machine currents I_n, and initial machine electrical power outputs p_{en}, $n = 1, 2, \ldots, M$. Set machine mechanical power outputs, $p_{mn} = p_{en}$. Set initial machine frequencies, $\omega_n = \omega_{\text{syn}}$. Compute the load admittances.

Step 2 Compute the internal machine voltages:

$$E_n = \mathrm{E}_n\underline{/\delta_n} = V_{\text{G}n} + (jX'_{dn})I_n \quad n = 1, 2, \ldots, M$$

where V_{Gn} and I_n are computed in Step 1. The magnitudes E_n will remain constant throughout the study. The angles δ_n are the initial power angles.

Step 3 Compute Y_{11}. Modify the $(N \times N)$ power-flow bus admittance matrix by including the load admittances and inverted generator impedances.

Step 4 Compute Y_{22} from (13.5.6) and Y_{12} from (13.5.7).

Step 5 Set time $t = 0$.

Step 6 Is there a switching operation, change in load, short circuit, or change in data? For a switching operation or change in load, modify the bus admittance matrix. For a short circuit, set the faulted bus voltage [in (13.5.10)] to zero.

Step 7 Using the internal machine voltages $E_n = \mathrm{E}_n\underline{/\delta_n}$, $n = 1, 2, \ldots, M$, with the values of δ_n at time t, compute the machine electrical powers p_{en} at time t from (13.5.10)–(13.5.12).

Step 8 Using p_{en} computed in Step 7 and the values of δ_n and ω_n at time t, compute the preliminary estimates of power angles $\tilde{\delta}_n$ and machine speeds $\tilde{\omega}_n$ at time $(t + \Delta t)$ from (13.4.7)–(13.4.10).

Step 9 Using $E_n = \mathrm{E}_n\underline{/\tilde{\delta}_n}$, $n = 1, 2, \ldots, M$, compute the preliminary estimates of the machine electrical powers $\tilde{p}_{en}$ at time $(t + \Delta t)$ from (13.5.10)–(13.5.12).

Step 10 Using $\tilde{p}_{en}$ computed in Step 9, as well as $\tilde{\delta}_n$ and $\tilde{\omega}_n$ computed in Step 8, compute the final estimates of power angles δ_n and machine speeds ω_n at time $(t + \Delta t)$ from (13.4.11)–(13.4.14).

Step 11 Set time $t = t + \Delta t$. Stop if $t \geq T$. Otherwise, return to Step 6.

EXAMPLE 13.8

Modifying power-flow Y_{bus} for application to multimachine stability

Consider a transient stability study for the power system given in Example 7.9, with the 200-Mvar shunt capacitor of Example 7.13 installed at bus 2. Machine transient reactances are $X'_{d1} = 0.20$ and $X'_{d2} = 0.10$ per unit on the system base. Determine the admittance matrices Y_{11}, Y_{22}, and Y_{12}.

Solution From Example 7.9, the power system has $N = 5$ buses and $M = 2$ machines. The second row of the 5×5 bus admittance matrix used for power flows is calculated in Example 7.9. Calculating the other rows in the same manner, we obtain

$$Y_{bus} = \begin{bmatrix} (0.932-j12.43) & 0 & 0 & 0 & (-0.932+j12.43) \\ 0 & (0.670-j7.115) & 0 & (-0.223+j2.480) & (-0.446+j4.96) \\ 0 & 0 & (1.865-j24.86) & (-1.865+j24.86) & 0 \\ 0 & (-0.223+j2.480) & (-1.865+j24.86) & (2.980-j37.0) & (-0.893+j9.92) \\ (-0.932+j12.43) & (-0.446+j4.960) & 0 & (-0.893+j9.920) & (2.271-j27.15) \end{bmatrix} \text{per unit}$$

To obtain Y_{11}, Y_{bus} is modified by including load admittances and inverted generator impedances. From Table 7.1, the load at bus 3 is $P_{L3} + jQ_{L3} = 0.2 + j0.1$ per unit and the voltage at bus 3 is $V_3 = 1.05$ per unit. Representing this load as a constant admittance,

$$Y_{load3} = \frac{P_{L3} - jQ_{L3}}{V_3^2} = \frac{0.2 - j0.1}{(1.05)^2} = 0.1814 - j0.0907 \quad \text{per unit}$$

Similarly, the load admittance at bus 2 is

$$Y_{load2} = \frac{P_{L2} - jQ_{L2}}{V_2^2} = \frac{2 - j0.7 + j0.5}{(0.968)^2} = 2.134 - j0.213$$

where V_2 is obtained from Example 7.13 and the 200-Mvar (0.5 per unit) shunt capacitor bank is included in the bus 2 load.

The inverted generator impedances are: for machine 1 connected to bus 1,

$$\frac{1}{jX'_{d1}} = \frac{1}{j0.20} = -j5.0 \text{ per unit}$$

and for machine 2 connected to bus 3,

$$\frac{1}{jX'_{d2}} = \frac{1}{j0.10} = -j10.0 \text{ per unit}$$

To obtain Y_{11}, add $(1/jX'_{d1})$ to the first diagonal element of Y_{bus}, add Y_{load2} to the second diagonal element, and add $Y_{load3} + (1/jX'_{d2})$ to the third diagonal element. The 5×5 matrix Y_{11} is then

$$Y_{11} = \begin{bmatrix} (0.932-j17.43) & 0 & 0 & 0 & (-0.932+j12.43) \\ 0 & (2.804-j7.328) & 0 & (-0.223+j2.480) & (-0.446+j4.96) \\ 0 & 0 & (2.0465-j34.951) & (-1.865+j24.86) & 0 \\ 0 & (-0.223+j2.480) & (-1.865+j24.86) & (2.980-j37.0) & (-0.893+j9.92) \\ (-0.932+j12.43) & (-0.446+j4.960) & 0 & (-0.893+j9.920) & (2.271-j27.15) \end{bmatrix} \text{per unit}$$

From (13.5.6), the 2×2 matrix Y_{22} is

$$Y_{22} = \begin{bmatrix} \frac{1}{jX'_{d1}} & 0 \\ 0 & \frac{1}{jX'_{d2}} \end{bmatrix} = \begin{bmatrix} -j5.0 & 0 \\ 0 & -j10.0 \end{bmatrix} \quad \text{per unit}$$

From Figure 7.2, generator 1 is connected to bus 1 (therefore, bus G1 = 1 and generator 2 is connected to bus 3 (therefore G2 = 3). From (13.5.7), the 5×2 matrix Y_{12} is

$$Y_{12} = \begin{bmatrix} j5.0 & 0 \\ 0 & 0 \\ 0 & j10.0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{per unit}$$

■

SECTION 13.6

PERSONAL COMPUTER PROGRAM: TRANSIENT STABILITY

The personal computer software package that accompanies this text includes the program entitled "TRANSIENT STABILITY," which computes machine power angles and frequencies in a balanced three-phase power system subjected to disturbances. This program also computes machine angular accelerations, machine electrical power outputs, and bus voltage magnitudes.

Input data for the program include: (1) the bus admittance matrix, initial bus voltages, initial machine currents, and initial machine electrical power outputs, all obtained from the program POWER FLOW described in Chapter 7; and (2) the per-unit inertia constant and direct axis transient reactance of each synchronous machine.

The program executes the transient stability computation procedure given in Section 13.5. The program user selects the type of each disturbance and the time at which each disturbance begins. Disturbance types include: switching operations (opening or closing circuit breakers selected by the program user), three-phase short circuits, changes in loads, and changes in input data. The program user also selects the integration step size Δt and the final time T.

Output data consist of the power angle, frequency, and electrical power output of each machine versus time, as well as bus voltages versus time. The progam user selects the outputs to be printed and the print time interval.

EXAMPLE 13.9

TRANSIENT STABILITY program: fault clearing with high-speed reclosure

Use the program TRANSIENT STABILITY to study a temporary three-phase short circuit at bus 5 of the power system given in Example 7.9. For prefault conditions, use the power-flow output given in Example 7.13, where a 200-Mvar shunt capacitor bank is installed at bus 2. Machine transient reactances are $X'_{d1} = 0.20$ and $X'_{d2} = 0.10$ per unit, as given in Example 13.8, and machine inertia constants are $H_1 = 5.0$ and $H_2 = 50$ p.u.-s (where machine 2 represents a large system). The short circuit is cleared by opening circuit breakers B1, B51, and B52 at $t = 0.05$ s (3 cycles), followed by reclosing these circuit breakers. Assume that the temporary fault has already self-extinguished when reclosure occurs.

Run the following two cases:

Case 1 Reclosure at $t = 0.27$ s (13 cycles after fault clearing)

Case 2 Reclosure at $t = 0.30$ s (15 cycles after fault clearing)

For computation purposes, select an integration step size $\Delta t = 0.01$ s and final time $T = 0.75$ s.

Solution The power-flow output given in Example 7.13 is selected as the input to the TRANSIENT STABILITY program, along with the machine reactances and inertia constants given above. The first disturbance selected is a short circuit at bus 5 (that is, $V_5 = 0$) at time $t = 0$. The second disturbance selected is the opening of circuit breakers B1, B51, and B52 to clear the fault at $t = 0.05$ s. The third disturbance selected is the reclosure of these breakers at $t = 0.27$ s for Case 1 and at $t = 0.30$ s for Case 2. The condition $V_5 = 0$ is also removed.

Machine power angle outputs are listed in Table 13.2 and plotted in Figure 13.13 versus time for both cases. The printout interval is 0.03 s. As shown for Case 1, power angle δ_1 reaches a maximum value and then swings back. With damping included, δ_1 and δ_2 would settle down to new steady-state values.

Case 2 is unstable. The power angle δ_1 of machine 1 exceeds 180 degrees and diverges away from δ_2. That is, machine 1 pulls out of synchronism with machine 2 on the first swing.

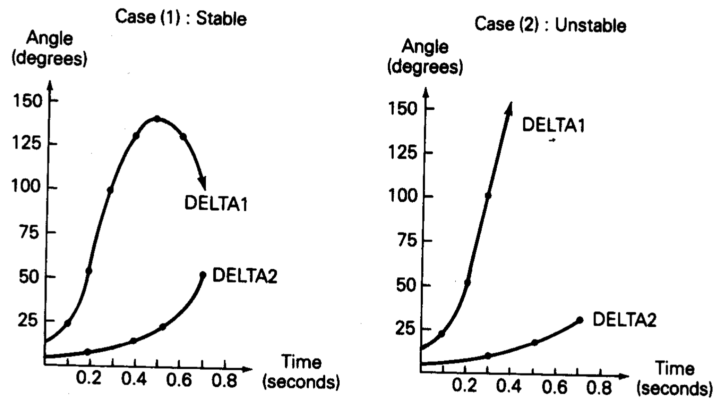
Table 13.2 Transient stability output for Example 13.9

CASE (1)				
TIME SECONDS	DELTA1 DEGREES	OMEGA1 RAD/SEC	DELTA2 DEGREES	OMEGA2 RAD/SEC
FAULT AT BUS 5				
0.00	11.16	376.99	6.10	376.99
0.03	12.02	377.99	6.19	377.10
FAULT CLEARED				
0.06	14.61	379.02	6.46	377.17
0.09	19.03	380.10	6.75	377.15
0.12	25.31	381.18	7.02	377.14
0.15	33.43	382.26	7.27	377.13
0.18	43.41	383.33	7.50	377.12
0.21	55.22	384.40	7.72	377.11
0.24	68.87	385.47	7.91	377.10
RECLOSURE				
0.27	84.36	386.53	8.09	377.09
0.30	99.39	384.95	8.47	377.34
0.33	111.78	383.48	9.29	377.60
0.36	121.78	382.17	10.56	377.86
0.39	129.66	381.01	12.27	378.11
0.42	135.66	379.98	14.41	378.36
0.45	139.97	379.02	16.99	378.61
0.48	142.66	378.09	19.98	378.86
0.51	143.72	377.11	23.39	379.10
0.54	143.02	376.04	27.24	379.35
0.57	140.37	374.82	31.52	379.61
0.60	135.47	373.43	36.24	379.87
0.63	128.04	371.87	41.41	380.13
0.66	117.82	370.22	47.00	380.37
0.69	104.82	368.67	52.99	380.57
0.72	89.39	367.48	59.27	380.71
0.75	72.47	366.98	65.73	380.76

CASE (2)				
TIME SECONDS	DELTA1 DEGREES	OMEGA1 RAD/SEC	DELTA2 DEGREES	OMEGA2 RAD/SEC
FAULT AT BUS 5				
0.00	11.16	376.99	6.10	376.99
0.03	12.02	377.99	6.19	377.10
FAULT CLEARED				
0.06	14.61	379.02	6.46	377.17
0.09	19.03	380.10	6.75	377.15
0.12	25.31	381.18	7.02	377.14
0.15	33.43	382.26	7.27	377.13
0.18	43.41	383.33	7.50	377.12
0.21	55.22	384.40	7.72	377.11
0.24	68.87	385.47	7.91	377.10
0.27	84.36	386.53	8.09	377.09
RECLOSURE				
0.30	101.66	387.59	8.25	377.08
0.33	118.65	386.21	8.63	377.34
0.36	133.51	385.13	9.44	377.59
0.39	146.83	384.43	10.68	377.83
0.42	159.27	384.11	12.30	378.03
0.45	171.49	384.17	14.25	378.21
0.48	184.17	384.64	16.48	378.36
0.51	198.01	385.54	18.93	378.46
0.54	213.78	386.90	21.51	378.52
0.57	232.34	388.78	24.15	378.52
0.60	254.56	391.15	26.73	378.45
0.63	281.23	393.92	29.14	378.31
0.66	312.80	396.75	31.25	378.12
0.69	348.90	399.05	33.02	377.93
0.72	388.05	400.20	34.52	377.82
0.75	427.95	399.94	35.95	377.85

Figure 13.13

Machine power angle swing curves for Example 13.9



SECTION 13.7

DESIGN METHODS FOR IMPROVING TRANSIENT STABILITY

Design methods for improving power-system transient stability include the following:

1. Improved steady-state stability
   - a. Higher system voltage levels
   - b. Additional transmission lines
   - c. Smaller transmission-line series reactances
   - d. Smaller transformer leakage reactances
   - e. Series capacitive transmission-line compensation
2. High-speed fault clearing
3. High-speed reclosure of circuit breakers
4. Single-pole switching
5. Larger machine inertia, lower transient rectance
6. Fast responding, high-gain exciters
7. Fast valving
8. Braking resistors

We discuss these design methods in the following paragraphs.

1. Increasing the maximum power transfer in steady-state can also improve transient stability, allowing for increased power transfer through the unfaulted portion of a network during disturbances. Upgrading voltage on

existing transmission or opting for higher voltages on new transmission increases line loadability (6.5.6). Additional parallel lines increase power-transfer capability. Reducing system reactances also increases power-transfer capability. Lines with bundled phase conductors have lower series reactances than lines that are not bundled. Oversized transformers with lower leakage reactances also help. Series capacitors reduce the total series reactances of a line by compensating for the series line inductance.

2. High-speed fault clearing is fundamental to transient stability. Standard practice for EHV systems is 1-cycle relaying and 2-cycle circuit breakers, allowing for fault clearing within 3 cycles (0.05 s). Ongoing research is presently aimed at reducing these to one-half cycle relaying and 1-cycle circuit breakers.
3. The majority of transmission-line short circuits are temporary, with the fault arc self-extinguishing within 5 to 40 cycles (depending on system voltage) after the line is deenergized. High-speed reclosure of circuit breakers can increase postfault transfer power, thereby improving transient stability. Conservative practice for EHV systems is to employ high-speed reclosure only if stability is maintained when reclosing into a permanent fault with subsequent reopening and lockout of breakers.
4. Since the majority of short circuits are single line-to-ground, relaying schemes and independent-pole circuit breakers can be used to clear a faulted phase while keeping the unfaulted phases of a line operating, thereby maintaining some power transfer across the faulted line. Studies have shown that single line-to-ground faults are self-clearing even when only the faulted phase is deenergized. Capacitive coupling between the energized unfaulted phases and the deenergized faulted phase is, in most cases, not strong enough to maintain an arcing short circuit [5].
5. Inspection of the swing equation, (13.1.16), shows that increasing the per-unit inertia constant H of a synchronous machine reduces angular acceleration, thereby slowing down angular swings and increasing critical clearing times. Stability is also improved by reducing machine transient reactances, which increases power-transfer capability during fault or postfault periods [*see* (13.2.1)]. Unfortunately, present-day generator manufacturing trends are toward lower H constants and higher machine reactances, which are a detriment to stability.
6. Modern machine excitation systems with fast thyristor controls and high amplifier gains (to overcome generator saturation) can rapidly increase generator field excitation after sensing low terminal voltage during faults. The effect is to rapidly increase internal machine voltages during faults, thereby increasing generator output power during fault and postfault periods. Critical clearing times are also increased [6].
7. Some steam turbines are equipped with fast valving to divert steam flows and rapidly reduce turbine mechanical power outputs. During faults near the generator, when electrical power output is reduced, fast valving action acts to balance mechanical and electrical power, providing reduced accel-

PROBLEMS

Section 13.1

13.1 A three-phase, 60-Hz, 400-MVA, 13.8-kV, 4-pole steam turbine-generating unit has an H constant of 5.0 p.u.-s. Determine: (a) ω_{syn} and ω_{msyn}; (b) the kinetic energy in joules stored in the rotating masses at synchronous speed; (c) the mechanical angular acceleration α_m and electrical angular acceleration α if the unit is operating at synchronous speed with an accelerating power of 400 MW.

13.2 Calculate J in kg m^2 for the generating unit given in Problem 13.1.

13.3 Generator manufacturers often use the term WR^2, which is the weight in pounds of all the rotating parts of a generating unit (including the prime mover) multiplied by the square of the radius of gyration in feet. WR^2/32.2 is then the total moment of inertia of the rotating parts in slug-ft^2. (a) Determine a formula for the stored kinetic energy in ft-lb of a generating unit in terms of WR^2 and rotor angular velocity ω_m. (b) Show that

$$H = \frac{2.31 \times 10^{-4}\mathrm{WR}^2(\mathrm{rpm})^2}{S_{\mathrm{rated}}} \quad \text{per unit-seconds}$$

where S_{rated} is the voltampere rating of the generator and rpm is the synchronous speed in r/min. Noted that 1 ft-lb = 746/550 = 1.356 joules. (c) Evaluate H for a three-phase generating unit rated 800 MVA, 3600 r/min, with WR^2 = 4,000,000 lb-ft^2.

13.4 The generating unit in Problem 13.1 is initially operating at $p_{mp.u.} = p_{ep.u.} = 0.7$ per unit, $\omega = \omega_{syn}$, and $\delta = 12°$ when a fault reduces the generator electrical power output by 70%. Determine the power angle δ five cycles after the fault commences. Assume that the accelerating power remains constant during the fault. Also assume that $\omega_{p.u.}(t) = 1.0$ in the swing equation.

13.5 Repeat Problem 13.4 for a bolted three-phase fault at the generator terminals that reduces the electrical power output to zero. Compare the power angle with that determined in Problem 13.4.

13.6 A third generating unit rated 400 MVA, 15 kV, 0.90 power factor, 16 poles, with $H_3 = 3.5$ p.u.-s is added to the power plant in Example 13.2. Assuming all three units swing together, determine an equivalent swing equation for the three units.

Section 13.2

13.7 The synchronous generator in Figure 13.4 delivers 0.9 per-unit real power at 1.08 per-unit terminal voltage. Determine: (a) the reactive power output of the generator; (b) the generator internal voltage; and (c) an equation for the electrical power delivered by the generator versus power angle δ.

13.8 The generator in Figure 13.4 is initially operating in the steady-state condition given in Problem 13.7 when a three-phase-to-ground bolted short circuit occurs at bus 3. Determine an equation for the electrical power delivered by the generator versus power angle δ during the fault.

Section 13.3

13.9 The generator in Figure 13.4 is initially operating in the steady-state condition given in Example 13.3 when circuit breaker B12 inadvertently opens. Use the equal-area criterion to calculate the maximum value of the generator power angle δ. Assume $\omega_{\text{p.u.}}(t) = 1.0$ in the swing equation.

13.10 The generator in Figure 13.4 is initially operating in the steady-state condition given in Example 13.3 when a temporary three-phase-to-ground short circuit occurs at point F. Three cycles later, circuit breakers B13 and B22 permanently open to clear the fault. Use the equal-area criterion to determine the maximum value of the power angle δ.

13.11 If breakers B13 and B22 in Problem 13.10 open later than 3 cycles after the fault commences, determine the critical clearing time.

13.12 Rework Problem 13.10 if circuit breakers B13 and B22 open after 3 cycles and then reclose when the power angle reaches 35°. Assume that the temporary fault has already self-extinguished when the breakers reclose.

13.13 The generator in Figure 13.4 is initially operating in the steady-state condition given in Problem 13.7 when circuit breaker B12 inadvertently opens. Use the equal-area criterion to calculate the maximum value of the generator power angle δ. Assume $\omega_{\text{p.u.}}(t) = 1.0$ in the swing equation.

13.14 The generator in Figure 13.4 is initially operating in the steady-state condition given in Problem 13.7 when a temporary three-phase-to-ground short circuit occurs at point F. Three cycles later, circuit breakers B13 and B22 permanently open to clear the fault. Use the equal-area criterion to calculate the maximum value of the generator power angle δ. Assume $\omega_{\text{p.u.}}(t) = 1.0$ in the swing equation.

13.15 If breakers B13 and B22 in Problem 13.14 open later than three cycles after the fault commences, determine the critical clearing time.

Section 13.4

13.16 Verify the maximum power angle determined in Problem 13.9 by applying the modified Euler's method to numerically integrate the swing equation. Write and run a computer program.

13.17 Investigate the effect of generating-unit damping torque on the maximum power angle in Problem 13.16. Damping, which is caused by friction and windage, can be represented by subtracting from the per-unit accelerating power $p_{a\text{p.u.}}(t)$ [used in (13.4.8) and (13.4.12)] the term $B\omega_{\text{p.u.}}(t)$, where B is a per-unit damping coefficient. Compare the maximum power angle using $B = 0.01$ per unit with that computed in Problem 13.16. Discuss the effect of generating-unit damping torques on stability.

13.18 Verify the critical clearing time determined in Problem 13.11 by applying the modified Euler's method. Write and run a computer program.

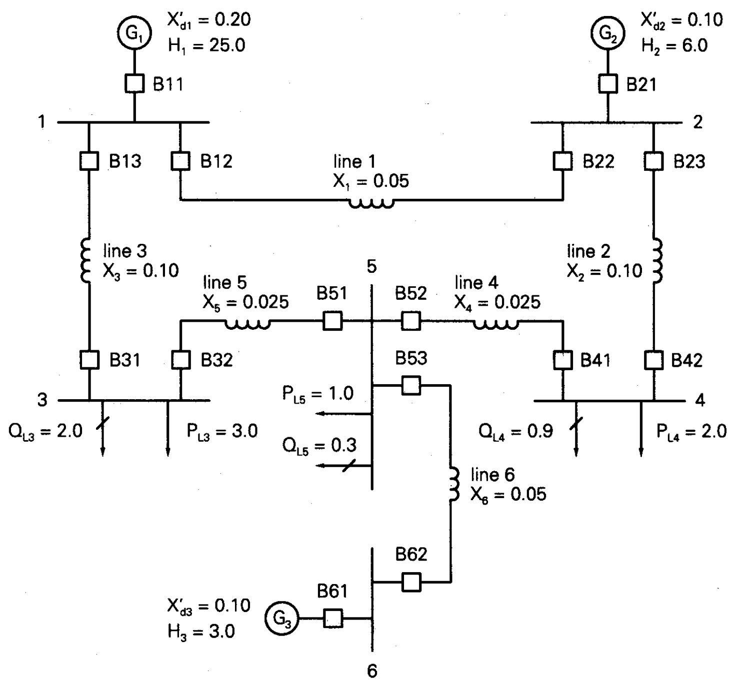
13.19 In Problem 13.12, assume that the circuit breakers open at $t = 3$ cycles and then reclose at $t = 24$ cycles (instead of when δ reaches 35°). Determine the maximum power angle by applying the modified Euler method. Write and run a computer program.

Section 13.5

13.20 Consider the six-bus power system shown in Figure 13.14, where all data is given in per-unit on a common system base. All resistances as well as transmission-line capacitances are neglected. (a) Determine the 6 × 6 per-unit bus admittance matrix Y_{bus} suitable for a power-flow computer program. (b) Determine the per-unit admittance matrices Y_{11}, Y_{12}, and Y_{22} given in (13.5.5), which are suitable for a transient stability study.

Figure 13.14

Single-line diagram of a six-bus power system (per-unit values are shown)



13.21 Modify the matrices Y_{11}, Y_{12}, and Y_{22} determined in Problem 13.20 for (a) the case when circuit breakers B12 and B22 open to remove line 1–2; and (b) the case when the load $P_{L4} + jQ_{L4}$ is removed.

Sections 13.6 and 13.7

13.22 Run the program TRANSIENT STABILITY for Example 13.9. Verify that Case 1 is stable and Case 2 is unstable.

13.23 Investigate the effect of varying the transient reactance X'_{d1} of machine 1 in Example 13.9. Run the program TRANSIENT STABILITY for (a) $X'_{d1} = 0.1$ and (b) $X'_{d1} = 0.4$ per unit. Discuss the effect of X'_{d1} on stability.

13.24 Investigate the effect of varying the inertia constant H in Example 13.9. Run the program TRANSIENT STABILITY for (a) $H_1 = 3.0$ and (b) $H_1 = 8.0$ p.u.-s. Discuss the effect of H_1 on stability.