Atividade 04 - Superfícies Parametrizadas e suas áreas
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Disciplina: Cálculo 3.

$x^2+y^2=4 \quad r=\sqrt{4}=2$

$$C: \begin{cases} x = r\cos(v) \\ y = r\operatorname{sen}(v) \\ z = u \end{cases}$$

$$C: \begin{cases} x = 2\cos(v) \\ y = 2\operatorname{sen}(v), & 0 \le v \le 2\pi \\ z = u & 0 \le u \le 3 \end{cases}$$

$\vec{r}(v,u) = (2\cos(v), 2\operatorname{sen}(v), u)$

⇒ calculando os vetores das tangentes parciais:

$$\left.\begin{aligned} \vec{r}_v = \frac{\partial \vec{r}}{\partial v} \Rightarrow \frac{\partial}{\partial v}(2\cos(v)) &= -2\operatorname{sen}(v) \\ \frac{\partial}{\partial v}(2\operatorname{sen}(v)) &= 2\cos(v) \\ \frac{\partial}{\partial v}(u) &= 0 \end{aligned}\right\} \vec{r}_v = (-2\operatorname{sen}(v), 2\cos(v), 0)$$

$$\left.\begin{aligned} \vec{r}_u = \frac{\partial \vec{r}}{\partial u} = \frac{\partial}{\partial u}(2\cos(v)) &= 0 \\ \frac{\partial}{\partial u}(2\operatorname{sen}(v)) &= 0 \\ \frac{\partial}{\partial u}(u) &= 1 \end{aligned}\right\} \vec{r}_u = (0,0,1)$$

• Produto vetorial $\vec{r}_v \times \vec{r}_u$

$$\vec{r}_v \times \vec{r}_u = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ -2\operatorname{sen}(v) & 2\cos(v) & 0 & -2\operatorname{sen}(v) & 2\cos(v) \\ 0 & 0 & 1 & 0 & 0 \end{array}\right.$$

$\vec{i}(2\cos(v) - 0) + \vec{j}(0 - (-2\operatorname{sen}(v))) + \vec{k}(0-0)$

$= \vec{i}(2\cos(v)) + \vec{j}(2\operatorname{sen}(v)) + \vec{k}(0)$

calculando a magnitude do produto vetorial:

$|\vec{r}_v \times \vec{r}_u| = \sqrt{a^2+b^2+c^2}$

$= \sqrt{(2\cos(v))^2 + (2\,\text{sen}(v))^2 + (0)^2}$

$= \sqrt{4\cos^2(v) + 4\,\text{sen}^2(v)} = \sqrt{4\underbrace{(\cos^2(v) + \text{sen}^2(v))}_{1}}$

$= \boxed{2}$

$\Rightarrow AL = \iint_D |\vec{r}_v \times \vec{r}_u|\, dA = \int_0^3 \underbrace{\int_0^{2\pi} 2\, dv}\, du$

$= 2v \Big|_{v=0}^{v=2\pi} = 4\pi - 0 = 4\pi$

$= \int_0^3 4\pi\, du = 4\pi \cdot u \Big|_{u=0}^{u=3} = 12\pi - 0 = \boxed{12\pi}$

$\boxed{AL = 12\pi}$