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Atividade 06 - Teorema de Gauss

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$$\vec{F}(x, y, z) = \underbrace{x^3 + y}_{P},\ \underbrace{y^3 + z}_{Q},\ \underbrace{z^3 + x}_{R}$$

Cubo de aresta 1

$0 \le x \le 1$

$0 \le y \le 1$

$0 \le z \le 1$

$$\operatorname{div}(\vec{F}) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 3x^2 + 3y^2 + 3z^2$$

$$\Rightarrow \Phi = \iint_S \vec{F} \cdot ds = \iiint_E \operatorname{div}(\vec{F})\, dV$$

$$\Rightarrow \Phi = \int_0^1 \int_0^1 \left[\int_0^1 (3x^2 + 3y^2 + 3z^2)\, dx \right] dy\, dz$$

$$= \frac{\cancel{3}x^3}{\cancel{3}} + 3y^2 \cdot x + 3z^2 x \Big|_{x=0}^{x=1} = 1 + 3y^2 + 3z^2 =$$

$$\int_0^1 \int_0^1 1 + 3y^2 + 3z^2\, dy\, dz$$

$$\Rightarrow 1 \cdot y + \frac{\cancel{3}y^3}{\cancel{3}} + 3z^2 \cdot y \Big|_{y=0}^{y=1} = 1 + 1 + 3z^2 = \int_0^1 1 + 1 + 3z^2\, dz$$

$$\Rightarrow 1 \cdot z + 1 \cdot z + \frac{\cancel{3}z^3}{\cancel{3}} \Big|_{z=0}^{z=1} = 1 + 1 + 1 \Rightarrow \boxed{\Phi = 3}$$