Financial Mathematics Cheat Sheet

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Capitalization and discount

	Capitalization		Discount
Simple	$C_n = C_0 \cdot (1 + i \cdot n)$	$i_m = \dfrac{i}{m}$	$C_0 = C_n(1 - d \cdot n)$ $\quad d_m = \dfrac{d}{m}$ **Rational** $\quad C_0 = \dfrac{C_n}{1 + i \cdot n}$
Compound	$C_n = C_0 \cdot (1 + i)^n$	$i_m = (1 + i)^{1/m} - 1$ $i = (i_m + 1)^m - 1$	$C_0 = C_n \cdot (1 - d)^n$

Notes: C_n capital in $t = n$, C_0 capital in $t = 0$, n periods, m subperiods, i interest rate, d discount rate. There is also the so-called **fractional capitalization**, $i_m = \frac{j(m)}{m}$, where $j(m)$ is the nominal interest rate payable per m.

Clearing interest formulas

Simple interest formulas	Compound interest formulas
$C_n = C_0 \cdot (1 + i \cdot n)$	$C_n = C_0 \cdot (1 + i)^n$
$C_0 = \dfrac{C_n}{1 + i \cdot n}$	$C_0 = \dfrac{C_n}{(1 + i)^n}$
$i = \dfrac{\frac{C_n}{C_0} - 1}{n}$	$i = \left(\dfrac{C_n}{C_0}\right)^{1/n} - 1$
$n = \dfrac{\frac{C_n}{C_0} - 1}{i}$	$n = \dfrac{\log\left(\frac{C_n}{C_0}\right)}{\log(1 + i)}$

Annuities

		Unitary	Variable in geometric progression
Temporal	**Postpayable**	$a_{n\rceil i} = \dfrac{1 - (1 + i)^{-n}}{i}$ $S_{n\rceil i} = a_{n\rceil i} \cdot (1 + i)^n$	$A(C;q)_{n\rceil i} = \begin{cases} C \cdot \dfrac{1 - \left(\dfrac{q}{1+i}\right)^n}{1 + i - q} & \text{si} \quad q \neq 1 + i \\ C \cdot \dfrac{n}{1 + i} & \text{si} \quad q = 1 + i \end{cases}$ $S(C;q)_{n\rceil i} = A(C;q)_{n\rceil i} \cdot (1 + i)^n$
	Prepayable	$\ddot{a}_{n\rceil i} = (1 + i) \cdot a_{n\rceil i}$ $\ddot{S}_{n\rceil i} = (1 + i) \cdot S_{n\rceil i}$	$\ddot{A}(C;q)_{n\rceil i} = (1 + i) \cdot A(C;q)_{n\rceil i}$ $\ddot{S}(C;q)_{n\rceil i} = (1 + i) \cdot S(C;q)_{n\rceil i}$
Perpetual	**Postpayable**	$a_{\infty\rceil i} = \dfrac{1}{i}$	$A(C;q)_{\infty\rceil i} = \begin{cases} C \cdot \dfrac{1}{1 + i - q} & \text{si} \quad q < 1 + i \\ \text{Infinito} & \text{si} \quad q \geq 1 + i \end{cases}$
	Prepayable	$\ddot{a}_{\infty\rceil i} = (1 + i) \cdot a_{\infty\rceil i}$	$\ddot{A}(C;q)_{\infty\rceil i} = (1 + i) \cdot A(C;q)_{\infty\rceil i}$

Notes:

q = factor.

Present discounted value, example, $V_0 = C \cdot a_{n\rceil i}$

Final capitalized value, example, $V_n = C \cdot S_{n\rceil i}$

Amortization table

Period	Interest rate	Amortization term	Interest payment	Amortization payment	Outstanding capital	Amortized capital
0	-	-	-	-	C_0	-
t_1	i_1	a_1	$I_1 = C_0 \cdot i_1$	$A_1 = a_1 - I_1$	$C_1 = C_0 - A_1$	$M_1 = C_0 - C_1$
t_2	i_2	a_2	$I_2 = C_1 \cdot i_2$	$A_2 = a_2 - I_2$	$C_2 = C_1 - A_2$	$M_2 = C_1 - C_2$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
t_s	i_s	a_s	$I_s = C_{s-1} \cdot i_s$	$A_s = a_s - I_s$	$C_s = C_{s-1} - A_s$	$M_s = C_{s-1} - C_s$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
t_n	i_n	a_n	$I_n = C_{n-1} \cdot i_n$	$A_n = a_n - I_n$	$C_n = C_{n-1} - A_n = 0$	$M_n = C_0 - C_n = M_{n-1} + A_n = C_0$

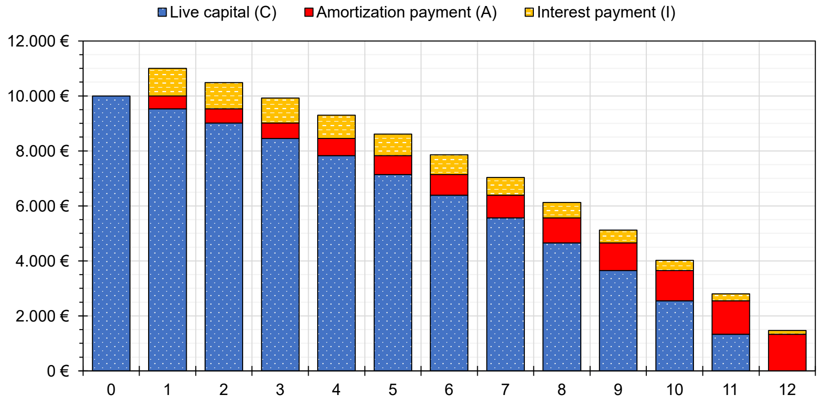
Loans

	French	American	Italian	Geometric progression terms
i	Constante	-	-	-
a	Constante; $a = \frac{C_0}{a_{n\rceil i}}$	$a_s = I_s = C_0 \cdot i_s \quad \text{si} \quad s \neq n$ $a_n = I_n + C_0 = C_0 \cdot i_s + C_0 \quad \text{si} \quad s = n$	$a_{s+1} = a_s - i \cdot A$	$a = \frac{C_0}{A(1;q)_{n\rceil i}}$
I	-	-	$I_{s+1} = I_s - i \cdot A$	-
A	$A_s = A_1 \cdot (1+i)^{s-1}$	$A_s = 0 \quad \text{si} \quad s \neq n$ $A_n = C_0 \quad \text{si} \quad s = n$	Constante; $A = \frac{C_0}{n}$	-
C	$C_0 = A_1 \cdot S_{n\rceil i}$	$C_s = C_0 \quad \text{si} \quad s \neq n$ $C_n = 0 \quad \text{si} \quad s = n$	-	-
Recursive method	$C_s = C_{s-1} \cdot (1+i) - a$	-	$C_s = C_{s-1} - A$	$C_s = C_{s-1} \cdot (1+i) - a \cdot q^{s-1}$
Prospective method	$C_s = a \cdot a_{n-s\rceil i}$	-	$C_s = C_{n-s} \cdot A$	$C_s = A(a \cdot q^s; q)_{n-s\rceil i}$
Retrospective method	$C_s = C_0 \cdot (1+i)^s - a \cdot S_{s\rceil i}$	-	$C_s = C_0 - s \cdot A$	$C_s = C_0 \cdot (1+i)^s - S(a;q)_{s\rceil i}$

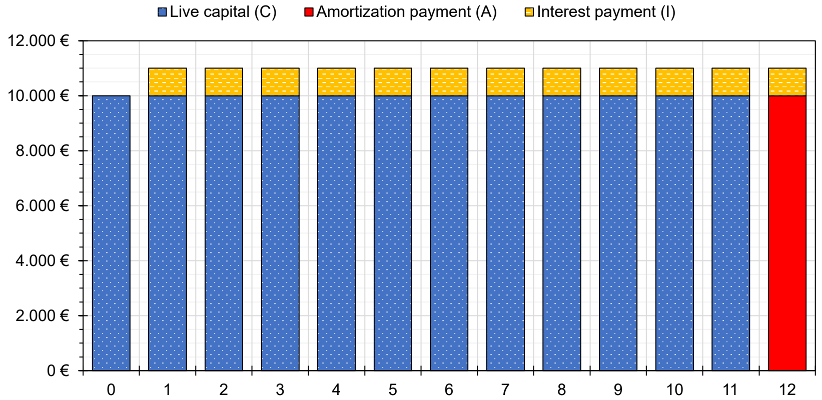
Graphical analysis of loans

Suppose a loan with principal $C_0 = 10.000$ at an interest rate $i = 10\%$ ending at $n = 12$ in which an amount is paid in each period that will depend on the type of loan.

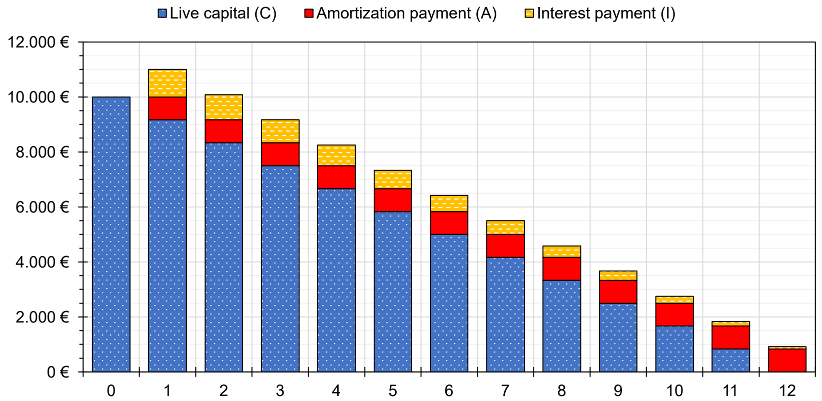
French loan: constant interest rate, all amortization terms are constant, amortization payments vary in geometric progression of factor $(1 + i)$:



American loan: the debtor only pays interest at the end of each period except in the last, in which he also pays the nominal amount of the loan; the payments are only interest, the live capital does not vary until the last period.



Italian loan: constant amortization payments, amortization terms and interest payments decrease in arithmetic progression $(-i \cdot A)$.



Loan with geometric progression terms: amortization terms vary in geometric progression of factor q (in this case, $q = 1.05$).