Better Understanding Triple Differences Estimators

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Field Paper Presentation

Motivation

Triple Differences

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- Triple Differences (DDD) extend to cases where the parallel trends (PT) assumption in DiD *may not hold*.
  - **Ex:** PT can be violated due to the presence of a *time-varying confounder* that *changes differently across states*.
- When the PT assumption is questionable, researchers often augment the design by adding another *placebo* comparison group to recover the treatment effect of interest.
- DDD designs address this issue by finding a *within-state comparison group* that **is not exposed** to the treatment but is **affected** by the time-varying confounder.

Running Example: Muralidharan and Prakash (2017)

- The researchers examine how giving bicycles to *girls* influences their *enrollment in secondary schools*.
- **Distance effect:** Female enrollment drops significantly with increased distance to school (Panel B).
- **Pre-program gap:** The attrition for female in secondary school enrollment is higher than male enrollment.
- The Cycle program in *Bihar, India* aimed to reduce the gender gap in secondary school enrollment.

Figures/panel_b.png

Running Example: Muralidharan and Prakash (2017)

- PT assumption is questionable because the bicycle program coincided with *rapid growth* and *increased education spending* in Bihar, unlike the control state, Jharkhand.
- Changes in girls' secondary school enrollment cannot be attributed to the *Cycle Program* directly.

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- Changes in girls' secondary school enrollment cannot be attributed to the *Cycle Program* directly.
- DDD to the rescue!
- Boys in Bihar **are not** exposed to the policy but are **affected** by the expansion in education spending.

What's the appeal of DDD compared to DiD?

- DDD allow us to take into account:

  1. Location-specific trends: Those related to the difference in education spending in Bihar vs. Jharkhand.

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What's the appeal of DDD compared to DiD?

- DDD allow us to take into account:

  1. Location-specific trends: Those related to the difference in education spending in Bihar vs. Jharkhand.

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  2. Partition-specific trends: Those related to the inherent disparities in girls' access to education in India.

     This would be ruled out if we were to do DiD by dropping observations of Jharkhand.

What’s the appeal of DDD compared to DiD?

Putting everything together, in DDD we allow to use all the information to control for location-specific trends and partition-specific trends, which otherwise would arise questionable results using DiD.

	No Eligible	Eligible
Treated	(Bihar, boys)	(Bihar, girls)
Control	(Jharkhand, boys)	(Jharkhand, girls)

Contribution

- Although it is also widely used in empirical work, DDD hasn't received as much attention as DiD.
- The key question in this paper is: How can we leverage our DiD knowledge to approach DDD?
  - We study identification, estimation, and inference procedures for DDD designs.
  - We derive the semiparametric efficiency bound for DDD designs and demonstrate that DDD estimators using a doubly robust representation reach this bound.
  - We extend our framework to staggered DDD designs.

Framework

Some notation

We have access to a sample of n units available, $i = 1, 2, \cdots, n$

- T time periods: $t = 1, 2, \ldots, T$.
- Different groups adopt a policy in different time periods g. Let $G \in \mathcal{G} \subset \{2, ..., T\} \cup \{\infty\}$ denote the time when group g is first-adopt the policy, with the notion that if a group is *"never-treated"*, $G = \infty$.
- Within each set of groups, we have two partitions (defined by some well-known criterion), $\ell \in P \equiv \{0, 1\}$. This determines *eligibility status*.
- Let $D_i \in \mathcal{D} \subset \{2, ..., T\} \cup \{\infty\}$ denote the time unit i is first-treated, with the notion that if a unit is *"never-treated"*, $D_i = \infty$.
- Note that $\mathcal{D} = \mathcal{G}$ such that:

$$D = \begin{cases} d & \text{if } G = g \equiv d \wedge P = 1, \\ \infty & \text{if } (G = g \wedge P = 0) \text{ or } (G = g' \wedge P \in \{0, 1\}, \text{ with } g' > g) \end{cases}$$

Ex: Units in $G = 2$ with $P = 1$ are treated at time $D = 2$, otherwise the unit remains untreated ($D = \infty$).

Building block of the analysis

- Let $Y_{i,t}(d)$ be the potential outcome for unit i, at time t, if this unit is first treated at time period d.
- A parameter that is interesting and has clear economic interpretation is the $ATT(g,t)$ (Callaway and Sant'Anna, 2021).

Definition (Parameter of interest: ATT(g,t))

Average Treatment Effect at time t of starting treatment at time g, among the units that indeed started treatment at time g.

$$ATT(g,t) := \mathbb{E}\left[Y_t(d) - Y_t(\infty) | G = g, P = 1\right], \; for \; t \geq g.$$

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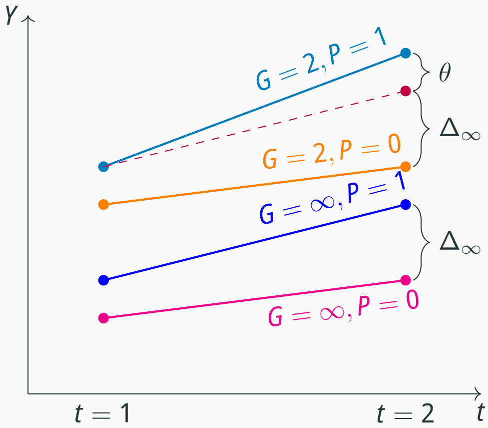
$$ATT(g,t) := \mathbb{E}\left[Y_t(d) - Y_t(\infty) | G = g, P = 1\right], \; for \; t \geq g.$$

- Then, our identification problem comes from the fact that we never observe $\mathbb{E}\left[Y_t\left(\infty\right)|G = g, P = 1\right]$ in $t \geq g$.

Canonical DDD Design

2x2x2 DDD without covariates

For simplicity, let's start with the canonical 2x2x2 DDD. Thus, $g \in G = \{2, \infty\}$ and $\ell \in P = \{0, 1\}$.



$$\theta = \Big[\Big(\mathbb{E}[Y_{t=2}|G=2,P=1] - \mathbb{E}[Y_{t=1}|G=2,P=1]\Big) - \Big(\mathbb{E}[Y_{t=2}|G=2,P=0] - \mathbb{E}[Y_{t=1}|G=2,P=0]\Big)\Big] - \Big[\Big(\mathbb{E}[Y_{t=2}|G=\infty,P=1] - \mathbb{E}[Y_{t=1}|G=\infty,P=1]\Big) - \Big(\mathbb{E}[Y_{t=2}|G=\infty,P=0] - \mathbb{E}[Y_{t=1}|G=\infty,P=0]\Big)\Big]$$

- Note that this is the difference of two DiD's: one among $G = 2$ across eligible groups, and one among $G = \infty$ across eligible groups.

Recovering the ATT using 3WFE Regression

- When there are only 2 time periods and *no covariates*, the following three-way fixed-effects (3WFE) regression specification can be used to recover the ATT:

$$
\begin{aligned}
Y_{i,t} \quad = \quad & \alpha_0 + \gamma_{0,1} 1_{\{G_i=2\}} + \gamma_{0,2} 1_{\{P_i=1\}} + \gamma_{0,3} 1_{\{T_i=2\}} \\
& + \gamma_{0,4} 1_{\{G_i=2\}} 1_{\{P_i=1\}} + \gamma_{0,5} 1_{\{G_i=2\}} 1_{\{T_i=2\}} + \gamma_{0,6} 1_{\{P_i=1\}} 1_{\{T_i=2\}} \\
& + \beta_0^{3wfe} 1_{\{G_i=2\}} 1_{\{P_i=1\}} 1_{\{T_i=2\}} + \varepsilon_{i,t},
\end{aligned}
$$

- We can show that $\beta_0^{3wfe} = \theta$ (Olden and Møen, 2022).

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These results suggest that we can estimate ATT in the canonical DDD either (i) by a difference between two DiDs or (2) by a saturated 3WFE regression.

What happens when covariates play an important role?

Assumptions

- Adding covariates in the above 3WFE specification would imply additional restrictions to the DGP:
  - Homogeneous treatment effects in covariates.
  - Rule out covariate-specific trends in both the treated and comparison groups.

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Assumption (Conditional Parallel Trends Assumption for DDD)

$$\begin{aligned} &\mathbb{E}\left[Y_{t=2}(\infty) - Y_{t=1}(\infty) | G = 2, P = 1, X\right] - \mathbb{E}\left[Y_{t=2}(\infty) - Y_{t=1}(\infty) | G = 2, P = 0, X\right] \\ &= \\ &\mathbb{E}\left[Y_{t=2}(\infty) - Y_{t=1}(\infty) | G = \infty, P = 1, X\right] - \mathbb{E}\left[Y_{t=2}(\infty) - Y_{t=1}(\infty) | G = \infty, P = 0, X\right] \ a.s. \end{aligned}$$

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$$=$$
$$\mathbb{E}\left[Y_{t=2}(\infty) - Y_{t=1}(\infty) | G = \infty, P = 1, X\right] - \mathbb{E}\left[Y_{t=2}(\infty) - Y_{t=1}(\infty) | G = \infty, P = 0, X\right] \ a.s.$$

Additional Assumptions: Strong Overlap & No Anticipation.

Doubly Robust and Semiparametric Efficiency

- Under the previous assumptions, the ATT can be identified via Regression Adjustments or IPW or any convex combination between them. RA & IPW

- However, this depends on the researcher's ability to accurately model outcome regression or propensity scores.
  - How would you choose this combination if the goal was to achieve **Doubly Robustness** (DR)?
  - How would you choose this combination if the goal was **efficiency**?

- We tackle these questions by deriving the *semiparametric efficiency bound* for the ATT in DDD setups.

- That usually leads to DR estimands, too.

Semiparametric Efficiency Bound

Proposition (Semiparametric Efficiency Bound for DDD)

Suppose that conditional PT, no-anticipation, and strong overlap assumptions are satisfied, and balanced panel data is available. Let $\theta(\Delta Y, X) := \Delta Y - m_{\Delta}^{G=2,P=0}(X) - m_{\Delta}^{G=\infty,P=1}(X) + m_{\Delta}^{G=\infty,P=0}(X)$, $S := (\Delta Y, G, P, X)$*. Then, the efficient influence function for the ATT is given by*

$$\begin{aligned}
\eta_{eff}(S) = {} & \omega_1^{G=2,P=1} \cdot \left(\theta(\Delta Y, X) - \theta \right) \\
& - \omega_0^{G=2,P=0}(X) \cdot \left(\Delta Y - m_{\Delta}^{G=2,P=0}(X) \right) \\
& - \omega_0^{G=\infty,P=1}(X) \cdot \left(\Delta Y - m_{\Delta}^{G=\infty,P=1}(X) \right) \\
& + \omega_0^{G=\infty,P=0}(X) \cdot \left(\Delta Y - m_{\Delta}^{G=\infty,P=0}(X) \right).
\end{aligned}$$

Furthermore, the semiparametric efficiency bound for the set of all regular, and asymptotic linear estimators of the ATT is $\mathbb{E}[\eta_{eff}(S)^2]$.

Weights Notation

DR DDD as a function of 3 DiDs

- We can take the expected value of $\eta_{\text{eff}}(W)$ and isolate θ given that any influence function has mean zero.
- Let's conveniently rewrite the propensity scores $\forall (g', \ell) \in \mathcal{S}_c$ as

$$p_{g',\ell}(X) = \mathbb{P}[G = 2, P = 1 | X, (G = 2, P = 1) \cup (G = g', P = \ell)],$$

- Finally, we get the following *DR-DDD* estimand for the ATT,

$$\begin{aligned}
\theta^{DR} &= \mathbb{E}\left[\left(\omega_1^{G=2,P=1} - \omega_0^{G=2,P=0}\left(p_{2,0}\left(X\right)\right)\right)\left(\Delta Y - m_{\Delta}^{G=2,P=0}\left(X\right)\right)\right] \Leftarrow DRDiD_{(2,1)}^{(2,0)} \\
&+ \mathbb{E}\left[\left(\omega_1^{G=2,P=1} - \omega_0^{G=\infty,P=1}\left(p_{\infty,1}(X)\right)\right)\left(\Delta Y - m_{\Delta}^{G=\infty,P=1}\left(X\right)\right)\right] \Leftarrow DRDiD_{(2,1)}^{(\infty,1)} \\
&- \mathbb{E}\left[\left(\omega_1^{G=2,P=1} - \omega_0^{G=\infty,P=0}\left(p_{\infty,0}(X)\right)\right)\left(\Delta Y - m_{\Delta}^{G=\infty,P=0}\left(X\right)\right)\right] \Leftarrow DRDiD_{(2,1)}^{(\infty,0)}
\end{aligned}$$

Expanded Version MC Simulations

Conclusion

Highlights

- DDD is widely used in empirical research, but its properties have receive little attention.
- In its basic format, it is equivalent to running two separate DiD estimators and subtracting one from another.
  - This equivalence breaks down when covariates play an important role in the analysis.
- We derived semiparametric efficiency bound for DDD and proposed DR DDD estimands.
- We can leverage these results to tackle staggered treatment setups, too.

Next steps:

- Looking for an empirical application to illustrate the performance of the proposed estimator.
- A very fast package implementation in R is under construction.

Thanks!

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Some additional notation

- For $(g,\ell) \in \{2,\infty\} \times \{0,1\}$, let $\Delta Y = Y_{t=2} - Y_{t=1}$, and

$$
\begin{aligned}
m_{\Delta}^{G=g,P=\ell}(X) &:= \mathbb{E}\left[\Delta Y | G=g, P=\ell, X\right], \quad \text{(outcome regression)}. \\
p^{G=g,P=\ell}(X) &:= \mathbb{P}[G=g, P=\ell | X] \quad \text{(multi-valued propensity score)}.
\end{aligned}
$$

- For $(g,\ell) \in \mathcal{S}_c \equiv \{(\infty,0),(\infty,1),(2,0)\}$, let

$$
\omega_1^{G=2,P=1} := \frac{1_{\{G=2,P=1\}}}{\mathbb{E}[1_{\{G=2,P=1\}}]},
$$

$$
\omega_0^{G=g,P=\ell}(X) := \frac{\dfrac{1_{\{G=g,P=\ell\}} \cdot p^{G=2,P=1}(X)}{p^{G=g,P=\ell}(X)}}{\mathbb{E}\left[\dfrac{1_{\{G=g,P=\ell\}} \cdot p^{G=2,P=1}(X)}{p^{G=g,P=\ell}(X)}\right]}
$$

- Let $\theta(\Delta Y, X) := \Delta Y - m_{\Delta}^{G=2,P=0}(X) - m_{\Delta}^{G=\infty,P=1}(X) + m_{\Delta}^{G=\infty,P=0}(X)$, $S := (\Delta Y, G, P, X)$.

Go Back

Regression Adjustment and IPW identification

- We can show that if conditional PT, no-anticipation, and strong overlap assumptions are satisfied and balanced panel data is available, the ATT is identified via regression adjustments or IPW:
$$\theta = ATT^{RA} = ATT^{IPW},$$
where
$$ATT^{RA} := \mathbb{E}\left[\Delta Y | G = 2, P = 1\right] - \mathbb{E}\left[m_{\Delta}^{G=2,P=0}(X) + \left(m_{\Delta}^{G=\infty,P=1}(X) - m_{\Delta}^{G=\infty,P=0}(X) \right) \Big| G = 2, P = 1\right],$$
$$ATT^{IPW} := \mathbb{E}\left[\left(\left(w^{G=2,P=1}(G,P) - w^{G=2,P=0}(G,P,X)\right) - \left(w^{G=\infty,P=1}(G,P,X) - w^{G=\infty,P=0}(G,P,X)\right) \right) \Delta Y\right].$$

Go Back

DR DDD as a function of 3 DiDs

- To get a *DR-DDD* estimand for the ATT, isolate θ given that any influence function has mean zero. Go Back
- We can conveniently rewrite the propensity scores $\forall (g', \ell) \in \mathcal{S}_c$ as

$$p_{g',\ell}(X) = \mathbb{P}[G = 2, P = 1 | X, (G = 2, P = 1) \cup (G = g', P = \ell)],$$

$$\begin{aligned}
\implies \theta^{DR} \quad &= \mathbb{E}\left[\left(\frac{1_{\{G=2,P=1\}}}{\mathbb{E}\left[1_{\{G=2,P=1\}}\right]} - \frac{\frac{p_{2,0}(X)\cdot 1_{\{G=2,P=0\}}}{1-p_{2,0}(X)}}{\mathbb{E}\left[\frac{p_{2,0}(X)\cdot 1_{\{G=2,P=0\}}}{1-p_{2,0}(X)}\right]}\right)\left(\Delta Y - m_{\Delta}^{G=2,P=0}(X)\right)\right] \\
&+\mathbb{E}\left[\left(\frac{1_{\{G=2,P=1\}}}{\mathbb{E}\left[1_{\{G=2,P=1\}}\right]} - \frac{\frac{p_{\infty,1}(X)\cdot 1_{\{G=\infty,P=1\}}}{1-p_{\infty,1}(X)}}{\mathbb{E}\left[\frac{p_{\infty,1}(X)\cdot 1_{\{G=\infty,P=1\}}}{1-p_{\infty,1}(X)}\right]}\right)\left(\Delta Y - m_{\Delta}^{G=\infty,P=1}(X)\right)\right] \\
&-\mathbb{E}\left[\left(\frac{1_{\{G=2,P=1\}}}{\mathbb{E}\left[1_{\{G=2,P=1\}}\right]} - \frac{\frac{p_{\infty,0}(X)\cdot 1_{\{G=\infty,P=0\}}}{1-p_{\infty,0}(X)}}{\mathbb{E}\left[\frac{p_{\infty,0}(X)\cdot 1_{\{G=\infty,P=0\}}}{1-p_{\infty,0}(X)}\right]}\right)\left(\Delta Y - m_{\Delta}^{G=\infty,P=0}(X)\right)\right]
\end{aligned}$$

Monte Carlo Simulations

Simulations for $T = 2$ with covariates

- For simplicity, we consider the scenario for panel data with $T = 2$ and we have access to generic data $W = (W_1, W_2, W_3, W_4)'$.
- WLOG, consider that the *eligibity of treatment* is given by binary well-know criterion $P = \{0, 1\}$ and let $(g, \ell) \in \mathcal{S}_c := \{(\infty, 0), (\infty, 1), (2, 0)\}$ and $\mathcal{S} := \mathcal{S}_c \cup \{(2, 1)\}$.
- Since we have 4 partitions in the data, we can model the selection into treatment as multinomial logistic link function.
- Outcome process can be modeled as linear regression onto W.
- We consider 4 DGPs:
  - **both** models are correctly specified;
  - Only **propensity score** is correctly specified;
  - Only **outcome model** is correctly specified;
  - **both** models are wrong.
- We compare our DR DDD estimator with:
  - 3WFE specification.
  - Difference between 2 Doubly Robust DiD (Sant'Anna & Zhao, 2020).

Results

	DGP 1: $\mathbb{E}\left[\eta_{\text{eff}}(W)^2\right] = 32.82$			DGP 2: $\mathbb{E}\left[\eta_{\text{eff}}(W)^2\right] = 32.52$			DGP 3: $\mathbb{E}\left[\eta_{\text{eff}}(W)^2\right] = 32.82$			DGP 4: $\mathbb{E}\left[\eta_{\text{eff}}(W)^2\right] = 32.52$		
	$\hat{\theta}_{ddd}$	$\hat{\theta}_{3wfe}$	$\hat{\theta}_{dr}$	$\hat{\theta}_{ddd}$	$\hat{\theta}_{3wfe}$	$\hat{\theta}_{dr}$	$\hat{\theta}_{ddd}$	$\hat{\theta}_{3wfe}$	$\hat{\theta}_{dr}$	$\hat{\theta}_{ddd}$	$\hat{\theta}_{3wfe}$	$\hat{\theta}_{dr}$
	$n = 1000$											
Bias	**0.0007**	-7.3298	-4.0199	**-0.0029**	-6.3178	-3.4484	**0.0255**	-3.8366	-1.9826	**0.0421**	-5.6247	-3.4447
RMSE	**0.1823**	8.4185	5.0331	**0.1799**	7.4545	4.5122	**1.4384**	5.1171	3.3235	**1.4498**	6.5893	4.3858
$\mathbb{E}[Var]$	43.5075	47341.2395	.	43.1213	47906.9263	.	2121.5766	45057.4548	.	2163.0339	45102.7697	.
Cov. 95	0.9650	0.9240	.	0.9730	0.9550	.	0.9600	0.9970	.	0.9530	0.9860	.
avg. length	**0.8110**	26.9471	.	**0.8071**	27.1033	.	**5.7012**	26.2999	.	**5.7553**	26.3134	.
	$n = 50000$											
Bias	**0.0008**	-7.3101	-4.0285	0.0007	-6.2891	-3.4389	-0.0039	-3.9647	-2.1148	0.1039	-5.4834	-3.3654
RMSE	**0.0257**	7.3348	4.0522	0.0257	6.3154	3.4636	0.2011	3.9929	2.1469	0.2345	5.5038	3.3857
$\mathbb{E}[Var]$	34.3777	47502.9417	.	33.9857	48240.7801	.	2059.7784	45339.0209	.	2113.8460	45453.0303	.
Cov. 95	0.9550	0.0000	.	0.9470	0.0000	.	0.9540	0.0000	.	0.9130	0.0000	.
avg. length	**0.1028**	3.8208	.	**0.1022**	3.8503	.	**0.7956**	3.7328	.	**0.8060**	3.7375	.

Monte Carlo Design

- Since we have 4 partitions in the data, we consider the following PS using a multinomial logistic link function:

$$p^{G=g,P=\ell}(W) = \begin{cases} \frac{\exp\left(f_{ps}^{g,\ell}(W)\right)}{1+\sum_{(g,\ell)\in\mathcal{S}_c}\exp\left(f_{ps}^{g,\ell}(W)\right)}, & \text{if } (g,\ell)\in\mathcal{S}_c \\ \frac{1}{1+\sum_{(g,\ell)\in\mathcal{S}_c}\exp\left(f_{ps}^{g,\ell}(W)\right)}, & \text{if } (g,\ell)=(2,1). \end{cases}$$

where, $f_{ps}^{g,\ell}(W) = \alpha_1^{g,\ell}W1 + \alpha_2^{g,\ell}W_2 + \alpha_3^{g,\ell}W_3 + \alpha_4^{g,\ell}W_4$

Monte Carlo Design

- Let $U \sim$ Uniform $[0, 1]$. The partition groups are assigned as follows

$$(g, \ell) := \begin{cases} (\infty, 0), & \text{if } U \leqslant p^{G=\infty, P=0}(W), \\ (\infty, 1), & \text{if } p^{G=\infty, P=0}(W) < U \leqslant p^{G=\infty, P=0}(W) + p^{G=\infty, P=1)}(W), \\ (2, 0), & \text{if } p^{G=\infty, P=0}(W) + p^{G=\infty, P=1}(W) < U \leqslant 1 - p^{G=2, P=1}(W), \\ (2, 1), & \text{if } 1 - p^{G=2, P=1}(W) < U. \end{cases}$$

- For the Outcome Regression process, define

$$f^{g,\ell}_{reg,G=2}(W) = \beta^{g,\ell}_{11} W1 + \beta^{g,\ell}_{21} W_2 + \beta^{g,\ell}_{31} W_3 + \beta^{g,\ell}_{41} W_4, \forall (g, \ell) \in \{(2, \ell)\}$$

$$f^{g,\ell}_{reg,G=\infty}(W) = \beta^{g,\ell}_{10} W1 + \beta^{g,\ell}_{20} W_2 + \beta^{g,\ell}_{30} W_3 + \beta^{g,\ell}_{40} W_4, \forall (g, \ell) \in \{(\infty, \ell)\}$$

- Let *time-invariant unobserved heterogeneity* be defined as $\nu(W, G, P) \sim N\left(f^{g,\ell}_{het}(W), 1\right), \forall (g, \ell) \in \mathcal{S}$ where,

$$f^{g,\ell}_{het}(W) = 1_{\{G=2\}} \cdot 1_{\{P=1\}} \cdot f^{g,\ell}_{reg,G=2}(W) + (1 - 1_{\{G=2\}}) \cdot 1_{\{P=1\}} \cdot f^{s,\ell}_{reg,G=\infty}(W)$$

Go Back