# Some title

Marcelo V. Maciel
André C. R. Martins
NISC - EACH, Universidade de São Paulo
Av. Arlindo Bétio, 1000, São Paulo, 03828-080, Brazil

**Abstract**

## 1 Introduction

In this paper, we will propose a opinion dynamics model [1, 2, 3, 4, 5, 6, 7] to explore the consequences of the existence of issues that can be interpreted as opinions over an one-dimensional axis. It's usual to think about policy alternatives and agents' preferences spatially (geometrically), that is, through a mapping from similarity to proximity [8, 9]. The model then captures the daily notion of parties or policies being more "to the left" or "right" than others, that is, if they're similar then they're closer [10, 11]. Major opinions, including political ones, tend to be formed from how each person feels about a number of issues. Locating someone in a left versus right or liberal versus conservative axis, therefore, requires inspecting the opinions of that person in not only one but a number of different issues that constitute the ideological positioning [12].

While it would make sense to consider different issues as having components in more than one single dimension [13], looking at the problem as one-dimensional can be justified in several ways. We can certainly see this as a first approximation along the most relevant dimension. In this case, we are simply investigating the projection of higher-dimensional problems along a direction where variation seems especially important. And, from the point of view of applications, it is usual to find discussions to be simplified over a main disagreement. Even though there are many variants of this modeling strategy, for our work what matters is that this naturally leads to the use of continuous opinion models such as the Bounded Confidence (BC) models [6, 14]. While discrete models [3, 4, 5] can be very useful at describing choices, they are not easiest way to represent strength of opinion. Discrete models also do not naturally provide a scale where we can compare opinions and decide which one is more to the right or more liberal.

On the other hand, continuous models are not particularly well suited for problems involving discrete decisions. As we will not deal with those kinds of problems here, they are a natural choice. Indeed, continuous opinions models have been proposed for several different problems on how opinions spread on a society [15, 16], from questions about the spread of extremism [17, 18, 19, 20, 21, 22] to other issues such as how different networks [23, 24, 25, 26] or the uncertainty of each agent [27] might change how agents influence each other.

Here, we will use a continuous opinion model created by Bayesian-like reasoning [28], inspired by the Continuous Opinions and Discrete Actions (CODA) model [7, 29]. The model was shown previously [28] to provide the same qualitative results as BC models. While a little less simple, the Bayesian basis make for a more clear interpretation of the meaning of the variables, as we extend the model and need to interpret the new results, and is consistent with a boundedly rational variant interpretation of the spatial model of political decision making [30, 31].

We will also study variations of our model where the function of trust $p^*$ will not be influenced by the distance between the opinions of the agent and the neighbor on the specific issue they are debating. Instead, we will test two other cases: in the first alternative, $p^*$ will be determined by the distance between the neighbor and the agent average opinions (the $p^{**}$ case); in the second alternative, $p^*$ is derived from the opinion of the neighbor and the average opinion of the agent ( the $p^{***}$ case). The idea here is to make the behavior of our agents closer to what experiments show about human reasoning. We have observed that our reasoning about political problems can be better described as ideologically motivated [32, 33, 34]. Indeed, our opinions tend to come in blocks even when the issues are logically independent [35]. Our reasoning abilities seem to exist more to defend our main point of views [36, 37] and our cultural identity [38] than to find the best answer. In that context, evaluating other by how they differ from us as a whole, instead of in each issue, is a model variation worth exploring.

## 2 The Model

The model is an agent-based social simulation [39]. At the initial condition of the simulation we have a population of $N$ agents which have an ideological profile $I_i = ((o_{i,1}, \sigma), \ldots, (o_{i,n}, \sigma))$ , where $n$ is the number of issues, $o$ is the opinion about the issue and $\sigma$ is a global variable which can be interpreted as the uncertainty about the issue [29]. Another attribute is the agent's ideological position at the dimension of interest, or ideal point [40], which we treat as the arithmetic mean of its opinions in each issue $x_i = \frac{1}{n}\sum_{k=1}^{n} o_k$.

The initial $o_i$s for each issue are sampled from $\text{Beta}(\alpha, \beta)$ distributions where each agent is associated with its own pair $(\alpha \in [1.1, 100], \beta \in [1.1, 100])$ . The reason for this is that if we sample the $o$s from an Uniform distribution as we increase the number of issues ($n$) the closer to the center of the dimension the agents' ideological position ($x$) would be. Using a Beta distribution prevents this, lets the initial $o$s of each agent to be correlated, since they're drawn from the agent's own Beta, and lets us have an initial population of agents with positions distributed along the ideological spectrum, instead of clustered around the center.

For its part, $\sigma$ is a global variable, that is, a parameter of the model. A certain proportion of the agents will have an unique $\sigma_{i,k} = 1e - 20$, so that we can control for the impact of *intransigent* agents on the model dynamics [41]. How many agents are intransigent is also a parameter (coded as *p_intran* ), and such $\sigma$ is established at the initial condition by sampling the issue index from the $I_i$'s length.

An iteration of the simulation is the application of two procedures: the opinion update through social influence and a random opinion update (noise). In the social influence procedure we draw a single agent $i$ from the population. We then draw another agent $j$ from the population. Afterwards, we draw one of the issues $k \in (1, \ldots, n)$ so that we have the corresponding pairs $(o_{i,k}, o_{j,k})$ and $(\sigma_{i,k}, \sigma_{j,k})$. Finally, the agent $i$ updates its opinion $(o_{i,k})$ following the equation

$$o_{i,k}(t+1) = p^* \frac{o_{i,k}(t) + o_{j,k}(t)}{2} + (1 - p^*) o_{i,k}(t).$$

Wherein

$$p^* = \frac{p \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(\Delta_{ij})^2}{2\sigma_i^2}}}{p \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(\Delta_{ij})^2}{2\sigma_i^2}} + (1-p)}.$$

The $\Delta_{ij}$ term is equal to $o_{i,k}(t) - o_{j,k}(t)$. As mentioned, we also test cases in which it's equal to $x_i(t) - x_j(t)$ ( the $p^{**}$ case ) and to $x_i(t) - o_{j,k}(t)$ ( the $p^{***}$). $p$, for its part, is a global parameter used to model the likelihood of the other agent's ($j$) opinion being true [29].

Furthermore, there is the noise: we draw another agent $i$ whose opinion $o_{i,k}(t+1)$ is equal to $o_{i,k}(t) + r$ where $r$ is taken from a Normal distribution of mean 0 and standard deviation $\rho$. $\rho$ is then a global parameter of the simulation. From a theoretical point of view the noise is justified as a way of accounting for the effect of factors not related to social influence that make the agents change their opinion about issues [42]. A further methodological justification is that small perturbations in the local behavior of agents may lead to drastic changes in systemic properties [43]. If an agent $i$ is intransigent in an issue $k$ it won't randomly change its $o_{i,k}$ opinion if its chosen by the noise algorithm. Moreover, if $o_{i,k}(t) + r > 1$ then $o_{i,k}(t+1) = 1$. Likewise, if $o_{i,k}(t) + r < 0$ then $o_{i,k}(t+1) = 0$.

# 3 Model Results

To have a general understanding about the model behavior we first established the bounds of the parameters:

| $\sigma$ | $n$ | $p$ | $p\_intran$ | $N$ | $\rho$ |
|---|---|---|---|---|---|
| $[0.01, 0.5]$ | $[1, 10]$ | $[0.1, 0.99]$ | $[0.0, 0.1]$ | $[500, 5000]$ | $(0.0, 0.1]$ |

Table 1: Parameters' Bounds

To sweep the parameter space we sample 70.000 parameterizations taken from quasi-random low-discrepancy sequences [44], that generate evenly spaced points. After running the simulation for 1.000.000 iterations we take the standard deviation of the population mean opinions ($Y std$) as a system final state measure. Histograms of the initial condition viz-a-viz the three cases ($p^{*}, p^{**}, p^{***}$) final state lets us understand the general tendency impinged by the update rules:

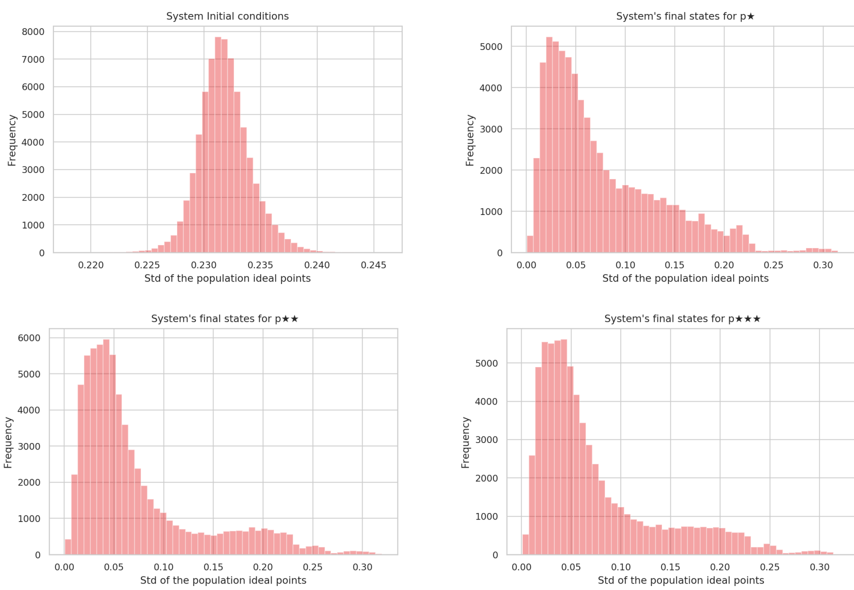


Figure 1: System initial condition x final state

The general tendency of the model is one of biased assimilation [42]. The histograms, however, don't show which parameter is the most important to explain this trend. With that in mind we perform a Sobol sensitivity analysis [45] which generate the following indexes:

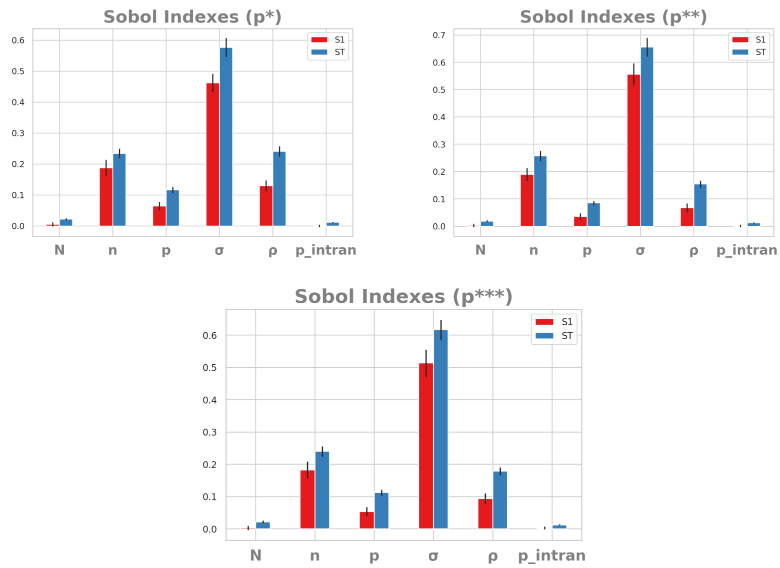


Figure 2: Sobol Indexes for the three cases $(p^*, p^{**}, p^{***})$

The sensitivity analysis shows that the most important parameters are : $\sigma, n, \rho$, and that the three cases have the same qualitative behavior. $\sigma$ being the parameter that explains the most the variance of the system measure is consistent with [29], while the relevance of the number of issues and the noise is a new result. The sensitivity analysis, however, does not show the direction of the impact, which we investigate through scatter plots:

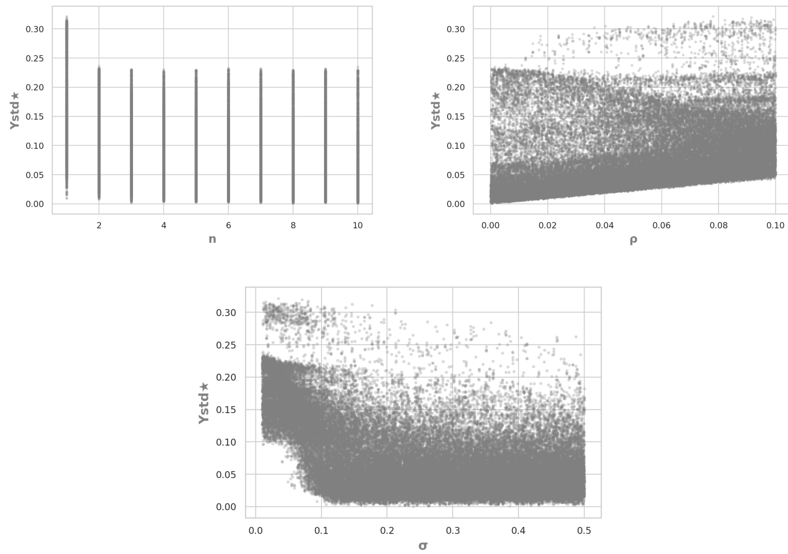


Figure 3: Scatterplots for the parameters with highest impact

The negative impact of $\sigma$ in the population' opinion dispersion is to be expected given the update rule: a higher $\sigma$ means that agents are easier to influence and as they're connected to all the others the more uncertain they the more centralized the agents' mean opinions after they interact with their neighbors. The plots also show that $\sigma$ has the same impact on the system whenever its bigger than approximately 0.1.Therefore we restrict our following analysis to the $(0.0, 0.1]$ range. The effect of $\rho$ is also expected: the bigger the noise more dispersed is the final state of the system.

After a general investigation we turn to the analysis of specific parametizations. For that let's start by fixing the following parameters as constants: $\rho = 0.05$ ; $N = 500$ ; $p\_intran = 0.0$, run the simulation for 500.000 iterations, and test combinations of $\sigma = (0.02, 0.04, 0.1)$ and $n = (1, 5)$. As show by 4, $\sigma$ has the effect of leading the dynamics of the simulation to the center as it increases: the bigger the $\sigma$ the closest to the mean the population is . However,when the number of issues also increases that tendency is not clear, since the noise disperses them, even though $\sigma$ has a bigger impact:

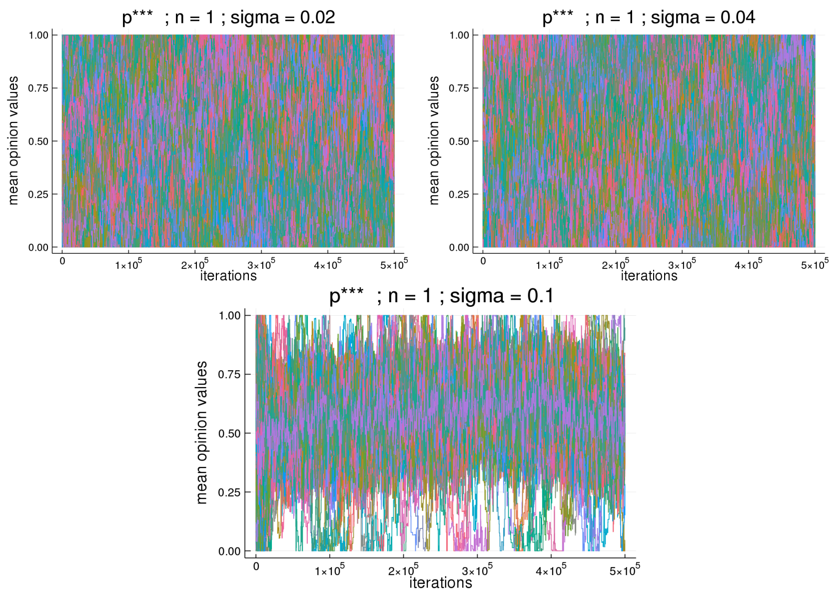


Figure 4: Time series for the parameterization: $\rho = 0.05, N = 500, p\_intran = 0.0, n = 1$

On the other hand, when we also increase the number of issues the centralizing effect of $\sigma$ becomes stronger:

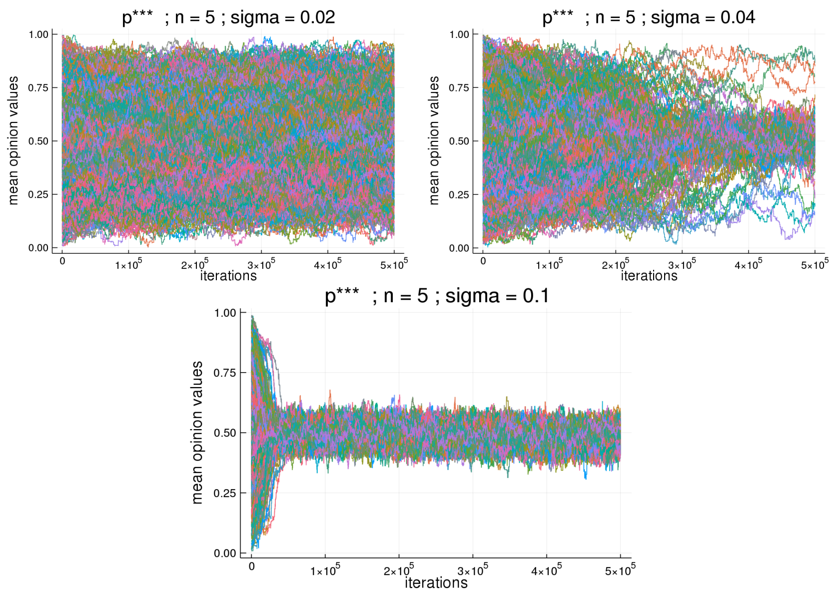


Figure 5: Time series for the parameterization: $\rho = 0.05, N = 500, p\_intran = 0.0$

The reason for that is : we're measuring the mean opinion values ($x_i$) and $\rho$ changes a single $o_i$ at each iteration which means that a higher $n$ implies a lesser impact of $\rho$ on the mean opinion of the agent, since she will have $n-1$ other opinions stabilizing her mean opinion at a point in the opinion spectrum. As $\sigma$ is the parameter that dominates the model update rule it interacts with $n$, which enforces $\sigma$ effect whenever we test higher $n$s. This effect holds even when we raise $\rho$ together with $n$. Let's use the same parameterization as the last plot but with a new $\rho$ such that $\rho_2 = \sqrt{n} * \rho_1 = \sqrt{10} * 0.05$. The interaction between $n$ and $\sigma$ still happens, with bigger $n$ stabilizing the noise effect and contributing to the centralizing effect of $\sigma$, even though a bigger $\rho$ leads to more noise around the mean.

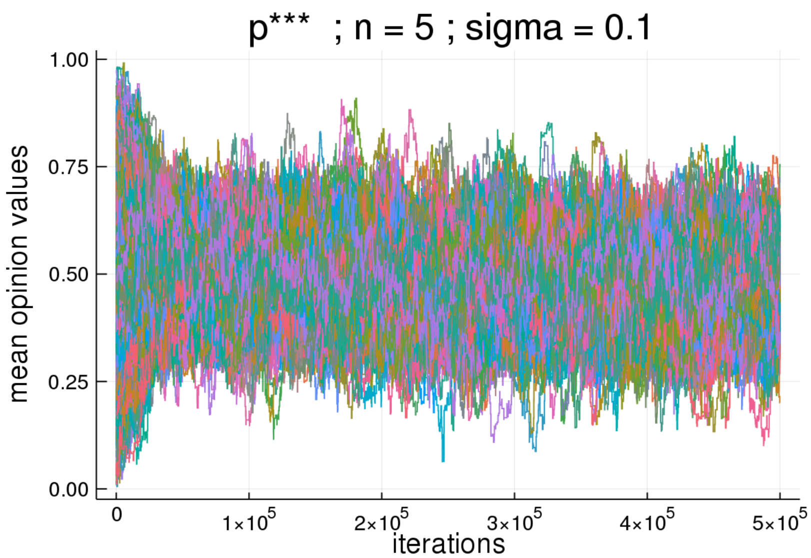


Figure 6: Time series for the parameterization: $\rho \approx 0.12, N = 500, p\_intran = 0.0$

Heretofore we've tested parameterizations with noise, but what happens if we lower $\rho$ to a value close to zero, such as 1e-5? The first difference is that the population mean opinion values converge to certain values. In parameter combinations in which $\sigma = 0.1$ the tendency is convergence to values close to 0.5. An interesting distinction between the cases in this parameterization is that $p^{**}$ and $p^{***}$ always converge to 0.5, independently of the number of issues. Alternatively, in the $p^{*}$ case this happens when $n = 1$, but when we have $n = 5$ or 10 there are other values of convergence, more as we increase $n$, even though the centralizing tendency remains.

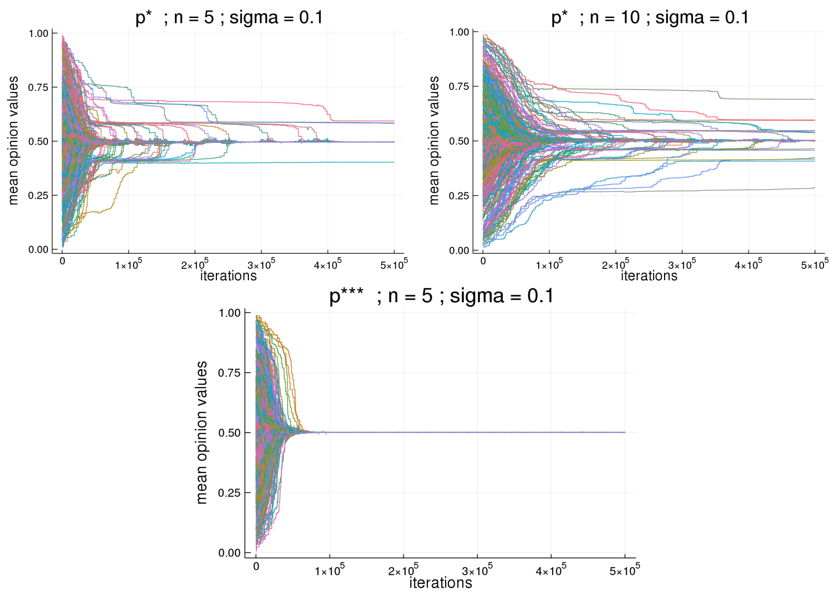


Figure 7: Time series for the parameterization: $\rho = 1e-5, N = 500, p\_intran = 0.0$

In the parameterization in which $\sigma$ is of intermediate value (for our range), such as 0.02 or 0.04, we observe another difference between cases: $p^*$ has more convergence values than $p^{**}$ and $p^{***}$. Figure 8 illustrates this system behavior:

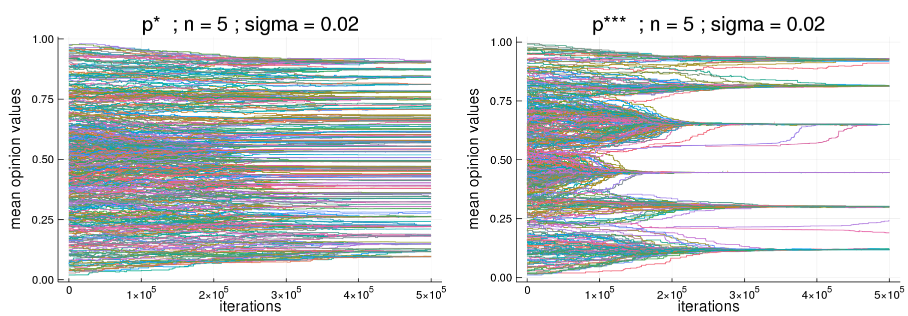


Figure 8: Time series for the parameterization: $\rho = 1e-5, N = 500, p\_intran = 0.0$

The reason for that lies in the $\Delta$ of each case: in the $p^{**}$ and $p^{***}$ cases the update rules make use of mean opinion values which facilitates the opinion convergence. The $p^*$ update rule works with single issue opinions which opens the possibility that there is little influence between agents given their ideological distance at the issue.

Another impact of the number of issues, as shown in Figure 9, is that a higher

$n$ leads to a longer time for the convergence to certain values. The reason is that we're only changing one opinion by iteration, so naturally a higher $n$ means the agents will take longer to be influenced. The relationship here is roughly linear such that the plot the region at $5 \times 10^5$ iterations when $n = 10$ is very similar to the corresponding region at $0.5 \times 10^5$ when $n = 1$.

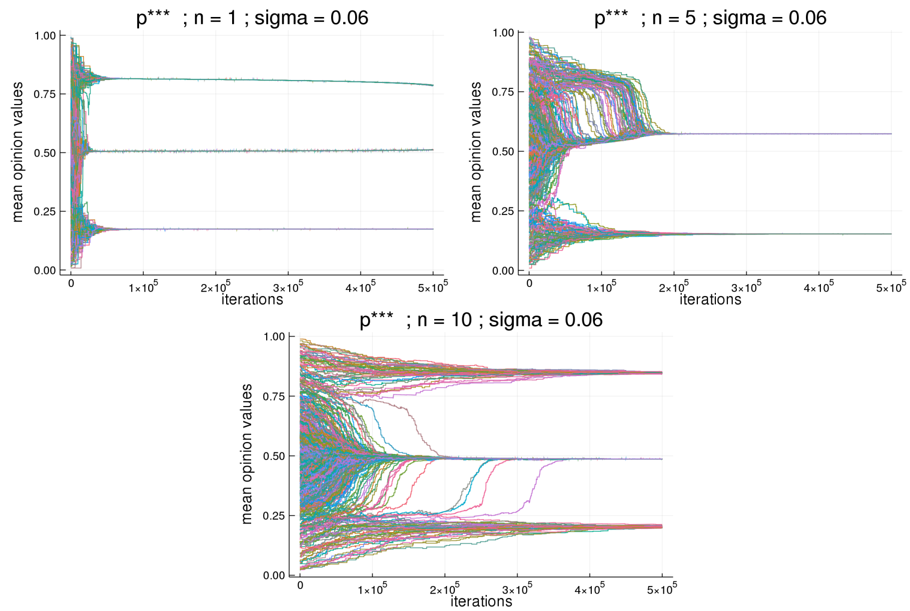


Figure 9: Time series for the parameterization: $\rho = 1e-5, N = 500, p\_intran = 0.0$.

Should I talk about intransigents with $\sigma = 0.01$ ???

# 4 Conclusions

# 5 Acknowledgement

The author would like to thank Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), for the support to this work, under grant

# References

[1] C. Castellano, S. Fortunato, and V. Loreto. Statistical physics of social dynamics. *Reviews of Modern Physics*, 81:591–646, 2009.

[2] Serge Galam. *Sociophysics: A Physicist's Modeling of Psycho-political Phenomena*. Springer, 2012.

[3] S. Galam, Y. Gefen, and Y. Shapir. Sociophysics: A new approach of sociological collective behavior: Mean-behavior description of a strike. *J. Math. Sociol.*, 9:1–13, 1982.

[4] S. Galam and S. Moscovici. Towards a theory of collective phenomena: Consensus and attitude changes in groups. *Eur. J. Soc. Psychol.*, 21:49–74, 1991.

[5] K. Sznajd-Weron and J. Sznajd. Opinion evolution in a closed community. *Int. J. Mod. Phys. C*, 11:1157, 2000.

[6] G. Deffuant, D. Neau, F. Amblard, and G. Weisbuch. Mixing beliefs among interacting agents. *Adv. Compl. Sys.*, 3:87–98, 2000.

[7] André C. R. Martins. Continuous opinions and discrete actions in opinion dynamics problems. *Int. J. of Mod. Phys. C*, 19(4):617–624, 2008.

[8] Anthony Downs. An economic theory of political action in a democracy. *Journal of Political Economy*, 65(2):135–150, 1957.

[9] Michael Laver. Measuring policy positions in political space. *Annual Review of Political Science*, 17:207–223, 2014.

[10] Robert P Van Houweling and Paul M Sniderman. The political logic of a downsian space. *Institute of Governmental Studies*, 2005.

[11] Nicholas R. Miller. The spatial model of social choice and voting. In Jack C. Heckelman and Nicholas R. Miller, editors, *Handbook of social choice and voting*, pages 163–181. Edward Elagar, Cheltenham, 2015.

[12] Kenneth Benoit, Michael Laver, et al. *Party policy in modern democracies*. Routledge, 2006.

[13] R. Vicente, André C. R. Martins, and N. Caticha. Opinion dynamics of learning agents: Does seeking consensus lead to disagreement? *Journal of Statistical Mechanics: Theory and Experiment*, 2009:P03015, 2009. arXiv:0811.2099.

[14] R. Hegselmann and U. Krause. Opinion dynamics and bounded confidence models, analysis and simulation. *Journal of Artificial Societies and Social Simulations*, 5(3):3, 2002.

[15] G. Deffuant, F. Amblard, and T. Weisbuch, G.and Faure. How can extremism prevail? a study based on the relative agreement interaction model. *JASSS-The Journal Of Artificial Societies And Social Simulation*, 5(4):1, 2002.

[16] G. Weisbuch, G. Deffuant, and F. Amblard. Persuasion dynamics. *Physica A*, 353:555–575, 2005.

[17] F. Amblard and G. Deffuant. The role of network topology on extremism propagation with the relative agreement opinion dynamics. *Physica A*, 343:725–738, 2004.

[18] Floriana Gargiulo and Alberto Mazzoni. Can extremism guarantee pluralism? *JASSS-The Journal Of Artificial Societies And Social Simulation*, 11(4):9, 2008.

[19] Daniel W. Franks, Jason Noble, Peter Kaufmann, and Sigrid Stagl. Extremism propagation in social networks with hubs. *Adaptive Behavior*, 16(4):264–274, 2008.

[20] Meysam Alizadeh, Ali Coman, Michael Lewis, and Claudio Cioffi-Revilla. Integroup conflict escalations llead to more extremism. *JASSS-The Journal Of Artificial Societies And Social Simulation*, 14(4):4, 2014.

[21] Giacomo Albi, Lorenzo Pareschi, and Mattia Zanella. Opinion dynamics over complex networks: kinetic modeling and numerical methods. arXiv:1604.00421, 2016.

[22] Tung Mai, Ioannis Panageas, and Vijay V. Vazirani. Opinion dynamics in networks: Convergence, stability and lack of explosion. In Ioannis Chatzigiannakis, Piotr Indyk, Fabian Kuhn, , and Anca Muscholl, editors, *44th International Colloquium on Automata, Languages, and Programming*, number 140, pages 1–14, 2017.

[23] Evguenii Kurmyshev, Héctor A. Juárez, and Ricardo A. González-Silva. Dynamics of bounded confidence opinion in heterogeneous social networks: Concord against partial antagonism. *Physica A: Statistical Mechanics and its Applications*, 390(16):2945 – 2955, 2011.

[24] Daron Acemoglu and Asuman Ozdaglar. Opinion dynamics and learning in social networks. *Dynamic Games and Applications*, 1(1):3–49, Mar 2011.

[25] Abhimanyu Das, Sreenivas Gollapudi, and Kamesh Munagala. Modeling opinion dynamics in social networks. In *Proceedings of the 7th ACM International Conference on Web Search and Data Mining*, WSDM '14, pages 403–412, New York, NY, USA, 2014. ACM.

[26] Haibo Hu. Competing opinion diffusion on social networks. *Royal Society Open Science*, 4(11), 2017.

[27] G. Deffuant. Comparing extremism propagation patterns in continuous opinion models. *JASSS-The Journal Of Artificial Societies And Social Simulation*, 9(3):8, 2006.

[28] André C. R. Martins. Bayesian updating rules in continuous opinion dynamics models. *Journal of Statistical Mechanics: Theory and Experiment*, 2009(02):P02017, 2009. arXiv:0807.4972v1.

[29] André C. R. Martins. Bayesian updating as basis for opinion dynamics models. *AIP Conf. Proc.*, 1490:212–221, 2012.

[30] Macartan Humphreys and Michael Laver. Spatial models, cognitive metrics, and majority rule equilibria. *British Journal of Political Science*, 40(1):11–30, 2010.

[31] Elinor Ostrom. A behavioral approach to the rational choice theory of collective action: Presidential address, american political science association, 1997. *American political science review*, 92(1):1–22, 1998.

[32] John T. Jost, Jack Glaser, Arie W. Kruglanski, and Frank J. Sulloway. Political conservatism as motivated social cognition. *Psychol. Bull.*, 129(3):339–375, 2003.

[33] Charles S. Taber and Milton Lodge. Motivated skepticism of political beliefs. *American Journal of Political Science*, 50(3):755–769, 2006.

[34] Ryan L. Claassen and Michael J. Ensley. Motivated reasoning and yard-sign-stealing partisans: Mine is a likable rogue, yours is a degenerate criminal. *Political Behavior*, pages 1–19, 2015.

[35] Robert Jervis. *Perception and Misperception in International Politics*. Princeton University Press, 1976.

[36] Hugo Mercier. Reasoning serves argumentation in children. *Cognitive Development*, 26(3):177–191, 2011.

[37] Hugo Mercier and Dan Sperber. Why do humans reason? arguments for an argumentative theory. *Behavioral and Brain Sciences*, 34:57–111, 2011.

[38] Dan M. Kahan, Hank Jenkins-Smith, and Donald Braman. Cultural cognition of scientific consensus. *Journal of Risk Research*, 14:147–174, 2011.

[39] Scott De Marchi and Scott E Page. Agent-based models. *Annual Review of Political Science*, 17:1–20, 2014.

[40] David A Armstrong, Ryan Bakker, Royce Carroll, Christopher Hare, Keith T Poole, Howard Rosenthal, et al. *Analyzing spatial models of choice and judgment with R*. CRC Press, 2014.

[41] Guillaume Deffuant, Frédéric Amblard, Gérard Weisbuch, and Thierry Faure. How can extremism prevail? a study based on the relative agreement interaction model. *Journal of artificial societies and social simulation*, 5(4), 2002.

[42] Andreas Flache, Michael Mäs, Thomas Feliciani, Edmund Chattoe-Brown, Guillaume Deffuant, Sylvie Huet, and Jan Lorenz. Models of social influence: Towards the next frontiers. *Journal of Artificial Societies and Social Simulation*, 20(4):2, 2017.

[43] Michael Macy and Milena Tsvetkova. The signal importance of noise. *Sociological Methods & Research*, 44(2):306–328, 2015.

[44] Andrea Saltelli, Marco Ratto, Terry Andres, Francesca Campolongo, Jessica Cariboni, Debora Gatelli, Michaela Saisana, and Stefano Tarantola. *Global sensitivity analysis: the primer*. John Wiley & Sons, 2008.

[45] Andrea Saltelli, Karen Chan, E Marian Scott, et al. *Sensitivity analysis*, volume 1. Wiley New York, 2000.