Some title

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Abstract

1 Introduction

Here, we will present an opinion dynamics model [1, 2, 3, 4, 5, 6, 7] where the opinions exist over an one-dimensional axis and each agent has a best estimate on more than one issue over that axis. Describing opinions over a spatial landscape is an usual way to describe policy alternatives and agents' preferences. The geometrical properties of the space are usually defined by mapping from similarity to proximity of the political agents [8, 9]. Such a description can capture the common notion of parties or policies being more "to the left" or "right". If they're similar then they're closer [10, 11]. Major opinions, including political ones, tend to be formed not from only one issue but from how each person feels about a number of them. Locating someone in a left versus right or liberal versus conservative axis, therefore, requires inspecting the opinions of that person in not only one but several issues that constitute the agent ideological positioning [12].

For many problems, it makes sense to consider different issues as having components in more than one single dimension [13]. However, it is often the case that describing the problem as one-dimensional can be justified. We can certainly see this as a first approximation along the most relevant dimension. In that case, the one dimensional case is just a projection along a direction where variation seems especially important of higher-dimensional problems. From an application point of view, it is usual to find discussions to be simplified over only one main disagreement.

One traditional way to model this type of scenario is to use continuous opinions over a fixed interval, as it is done in the Bounded Confidence (BC) models [6, 14]. While discrete models [3, 4, 5] can be very useful at describing choices, they are not easiest way to represent strength of opinion unless a continuous variable is associated to the choice [7]. Discrete models also tend to lack a scale where we can compare opinions and decide which one is more conservative or more liberal.

On the other hand, continuous models are not particularly well suited for problems involving discrete decisions. As we will not deal with those kinds of problems here, they are a natural choice. Indeed, continuous opinions models have been proposed for several different problems on how opinions spread on a society [15, 16], from questions about the spread of extremism [17, 18, 19, 20, 21, 22] to other issues such as how different networks [23, 24, 25, 26] or the uncertainty of each agent [27] might change how agents influence each other.

Here, we will use a continuous opinion model created by Bayesian-like reasoning [28], inspired by the Continuous Opinions and Discrete Actions (CODA) model [7, 29]. That model was shown previously [28] to provide the same qualitative results as BC models. While a little less simple, the Bayesian basis makes for a clearer interpretation of the meaning of the variables. That makes extending the model and interpreting new results simple. That approach is also consistent with a bounded rationality variant interpretation of the spatial model of political decision making [30, 31].

Variations of how each agent estimate how trustworthy other agents are will also be introduced. Here, we use a function of trust p^* that plays a role similar to the threshold value in BC models. While p^* is not a simple discontinuous cut-off, it is a function of the distance between the opinions of the agent and the neighbor. If only one issue is debated, that distance is uniquely defined. However, if we have multiple issues represented on the same one-dimensional line, it is not clear which distance we should use. That happens because we have the opinion of agent i on a specific issue o_i and we can compare it to o_j to estimate how much i trusts j. However, as there are several issues we could also use the mean opinion of i over all its issues and the same goes for j. Therefore, we will study two additional cases: in the first alternative, we will change p^* to p^{**}, determined by the distance between the neighbor and the agent average opinions. In the second alternative, we will use a p^{***} calculated from the opinion of the neighbor and the average opinion of the agent. The idea here is to make the behavior of our agents closer to what experiments show about human reasoning. We have observed that our reasoning about political problems can be better described as ideologically motivated [32, 33, 34]. Indeed, our opinions tend to come in blocks even when the issues are logically independent [35]. Our reasoning abilities seem to exist more to defend our main point of views [36, 37] and our cultural identity [38] than to find the best answer. In that context, evaluating others by how they differ from us as a whole, instead of in each issue, is a model variation worth exploring.

2 The Model

Here, the population will consist of N agents fully connected (an agent i can interact with any other agent j). Each agent i will have an opinion $0 \leq o_{i,k} \geq 1$, where $k = 1, \ldots, n$ is a specific issue. We assume each agent opinion about issue k can represented as a value $o_{i,k}$ at the range of possible values for os. Agents also have an uncertainty σ associated to their average estimate o. The uncertainty σ could be different for each agent and also updated during the interactions [28]. For the sake of simplicity, however, we will assume the uncertainty $\sigma_i = \sigma$ is identical for (almost) all agents and it does not change. The set of opinions for each agent on all possible issues will define its ideological profile $I_i = ((o_{i,1}, \sigma), \ldots, (o_{i,n}, \sigma))$, where n is the number of issues, o is the opinion about the issue and σ is the global uncertainty [29]. The arithmetic mean x_i of agent i opinions in each issue will be called here the ideal point for each agent, that is, it defines the agent ideological position at the dimension of interest [39]. Obviously $x_i = \frac{1}{n}\sum_{k=1}^{n} o_k$.

In order to have agents with initial ideal points well distributed over the possible range, the initial valued for each $o_{i,k}$ was randomly drawn using a Beta $Be(\alpha, \beta)$ with random parameters α_i and β_i. Those parameters were drawn for each agent i from the ranges $(\alpha \in [1.1, 100], \beta \in [1.1, 100])$ by dividing those ranges by N equally spaced values and then assigning them to each agent after mixing the values as in a Latin hypercube sampling [40]. That is done to allow agents with a diverse value of initial ideal points and, at the same time, to keep the initial $o_{i,k}$s of each agent i correlated.

While we could have σ as a measurement of each agent uncertainty and have it evolve with time, here we will keep it as a fixed parameter of the model. However, a proportion *p_intran* of agents will be stubborn about one single issue k, so that $\sigma_{i,k} = 1e-20$. That behavior is kept constant as the simulation unfolds, that is $\sigma_{i,k}$ is also not updated by the model. A proportion of agents who are stubborn is introduced here so we can check if inflexible [41, 42, 43] have a significant impact on the outcomes of the model.

In each iteration of the simulation, two procedures are applied: the opinion update through social influence, and a random opinion update (noise). In the social influence procedure we draw two agents, i and j, from the population, where i will observe j opinion about a randomly drawn issue $k \in (1, \ldots, n)$. Agent i updates its opinion $(o_{i,k})$ following an approximate Bayesian rule. That rule is obtained by assuming each agent has a Normal prior $f_i(\theta) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(\theta - o_i)^2}{2\sigma_i}}$ compatible with its parameters o_i and σ_i, where the index related to the issue k was omitted for the sake of simplifying the equation. The agents also assume a mixture likelihood where there is an initial chance p, updated to p^* that the agent j has information and a chance $1 - p$ it knows nothing and its opinion is just a random non-informative draw. That is, the likelihood is given by $f(o_j|\theta) = pN(\theta, \sigma_j^2) + (1 - p)U(0, 1)$. While a full Bayesian treatment would produce a posterior distribution that is a mixture of two normals, here we will assume each agent only updates its expected value and it does not carry the full posterior information to the next iteration. That leads to the following update rule for the expected value $o_{i,k}$ [29]:

$$o_{i,k}(t + 1) = p^* \frac{o_{i,k}(t) + o_{j,k}(t)}{2} + (1 - p^*)o_{i,k}(t), \tag{1}$$

where p^* is given by

$$p^* = \frac{p\frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{(\Delta_{ij})^2}{2\sigma_i^2}}}{p\frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{(\Delta_{ij})^2}{2\sigma_i^2}} + (1-p)}. \qquad (2)$$

Here $\Delta_{ij} = o_{i,k}(t) - o_{j,k}(t)$ is the distance between the opinions on the k issue. Δ_{ij} plays a similar role to the threshold parameter in the Bounded Confidence models by making distant opinions less influential. As Δ_{ij} increases, it is easy to see that Equation2 causes p^* to tend to zero. And, as that happens, the weights in average update Equation 1 change so that the previous value $o_{i,k}(t)$ remains almost unchanged.

The threshold role of Δ_{ij} suggests we might change its definition to check how it might better reflect the actual behavior of humans. In particular, people tend to trust better those who have a similar ideology. That means trust might not depend on the specific issue alone, but on the average over all issues. While that is not the model we obtain from applying Bayes rule, such a change makes sense as an attempt to have a better model. In order to check that possibility we will also implement two other cases by changing the way Δ_{ij} is calculated. In the first variation, p^* will be substituted by p^{**}, where p^{**} means $\Delta_{ij} = x_i(t) - x_j(t)$, that is, agent i will observe the average ideological position of j in order to estimate its trust. That assumed i has more information than only the value $o_{j,k}(t)$. To check what happens when indeed only $o_{j,k}(t)$ indeed what??, we will introduce a second variant case where i compares $o_{j,k}(t)$ to its own average. That is, $\Delta_{ij} = x_i(t) - o_{j,k}(t)$ and we represent that case by p^{***}.

take a look at this sentence

In order to better understand the model dynamics we also introduce noise as the second procedure. At each iteration, another agent i is randomly chosen and its opinion changed due to random noise. That is, i opinion becomes $o_{i,k}(t+1) = o_{i,k}(t) + r$ where r is drawn from a Normal distribution with mean 0 and standard deviation ρ. If agent i is intransigent in issue k it won't its $o_{i,k}$ opinion when chosen by the noise algorithm. Moreover, if $o_{i,k}(t) + r > 1$ then $o_{i,k}(t+1) = 1$. Likewise, if $o_{i,k}(t) + r < 0$ then $o_{i,k}(t+1) = 0$. Noise is introduced here as a way of accounting for the effect of factors not related to the modeled social influence [44] and it is interesting to verify if small noises can lead to drastic changes in systemic properties [45].

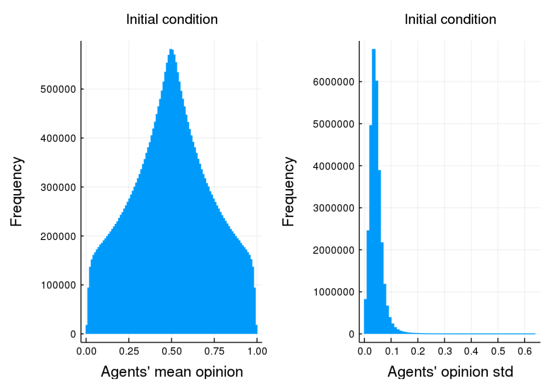
3 Model Results

To better understand the model behavior, we run simulations using as range for the parameters the values:

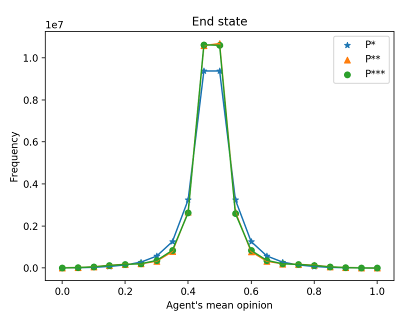
σ	n	p	p_intran	N	ρ
$[0.01, 0.5]$	$[1, 10]$	$[0.1, 0.99]$	$[0.0, 0.1]$	$[500, 5000]$	$(0.0, 0.1]$

Table 1: Parameters' Bounds

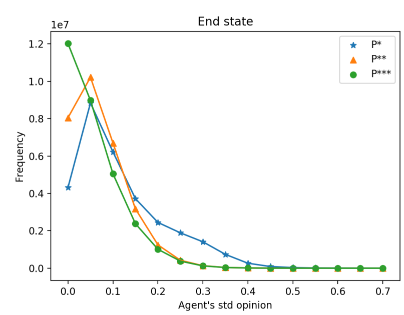
The parameter space was explored by two sweeps of its parameters: one sampling of 70,000 times using quasi-random low-discrepancy sequences [46], that generate evenly spaced points, and another of 60,000 times keeping $N = 500$ so that we can compare different runs. An important question in any opinion dynamics model is if agents can reach consensus. That can be observed by comparing the initial mean ($\frac{1}{n}\sum_{k=1}^{n} o_k$) and standard deviation ($\sqrt{\frac{1}{n}\sum_{k=1}^{n}(o_k - x_i)^2}$) of each agent's opinions. Figure 1 shows histograms for these variables at the initial condition and also for the sets of simulation's end states corresponding to the (p^*, p^{**}, p^{***}) cases.



(a) Initial Conditions:
mean and standard deviation of agents' opinions.



(b) Final state:
agents' mean opinions.



(c) Final state:
standard deviations of opinions

Figure 1: Initial and final states for 60.000
parameterizations. $N = 500$ for comparability.

The histograms show that, in general, we have a greater central tendency than the initial random situation. The peak at the value 0.5 is now (Figure 1 (b)) $\approx 10^7$ while at the initial condition ((Figure 1 (a)) was $\approx 5 * 10^5$ while the histograms become more centered. This seems to suggest a tendency to consensus in many cases. However, consensus is not always achieved, as shown by the tails of Figure 1 (b) and since there is also a change at the standard deviation distributions (Figure 1 (c)). That can be explained by cases where bipartisanship becomes stronger, with agents tending to both extreme positions. That suggest that, in general, the model can be described as one with similarity biased influence [44].

Ver se consigo descrever os histogramas melhor, agora não ta saindo.

The histograms, however, don't show which values of the parameters cause those changes. They also say nothing about which parameters are relevant and which ones have little influence on the final outcome. With that in mind we perform a Sobol sensitivity analysis [47]. The Sobol indexes decompose the impact of parameters on the variance of the output. The higher the value of index the bigger the impact of the parameter on the output. First order Sobol indexes include linear and non-linear contributions of the parameters, while total Sobol indexes also include all the interaction effects between parameters [48]. If there are only three parameters, the total effect S_{T1} of the first parameter X_1 is given by the equation $S_{T1} = S_1 + S_{12} + S_{13} + S_{123}$ where $S_i = \frac{V[E(Y|X_i)]}{V(Y)}$; S_{12} is the impact at the variance of the output Y of the interaction of X_1 and X_2; that is, their combined effect minus their first order effects: $S_{ij} = \frac{V[E(Y|X_i,X_,)] - V[E(Y|X_i)] - V[E(Y|X_j)]}{V(Y)}$ [46]. For our simulations, we obtain the estimates:

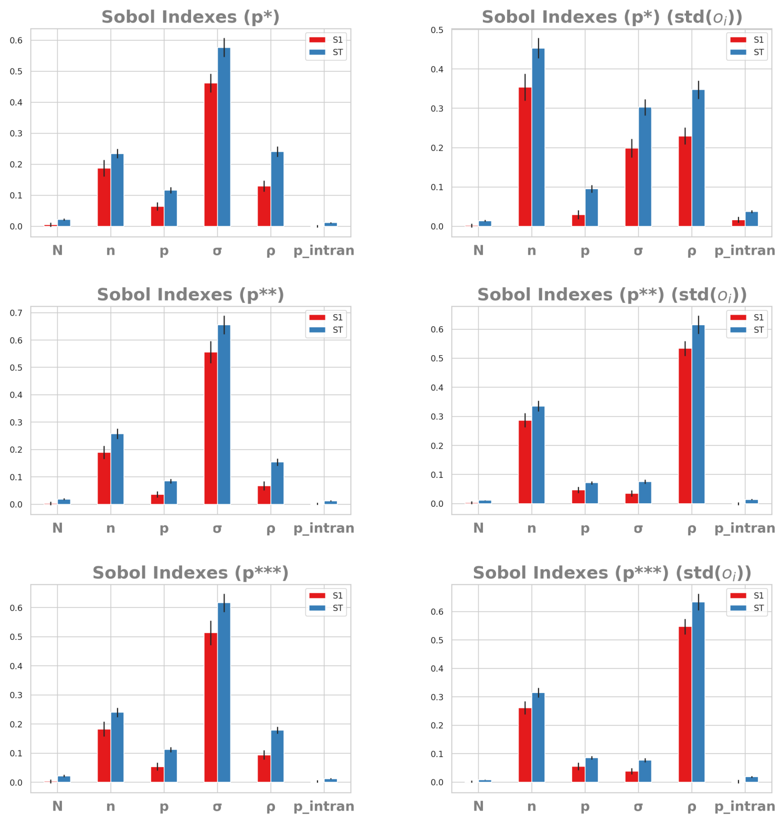


Figure 2: Sobol Indexes for the three cases (p^*, p^{**}, p^{***})

The sensitivity analysis in Figure2 shows that σ, n, and ρ have the most influence on the final values of $(Y\,std)$. p seems to still have some smaller influence and both N and p_{intran} seem to make no difference. It is also interesting to notice that the same behavior appears in the three. σ being the parameter that explains most of the variance was expected and it is consistent with [29]. It is interesting to see that two of the new parameters, the number of issues n and the noise ρ, also play an important role in explaining the total variance.

The sensitivity analysis, however, does not show the direction of the impact, which we investigate through scatter plots shown in Figure 3. Each value of $(Y\,std)$ in those graphics correspond to the outcome of one implementation of the model for the value of the parameter at the x-axis.

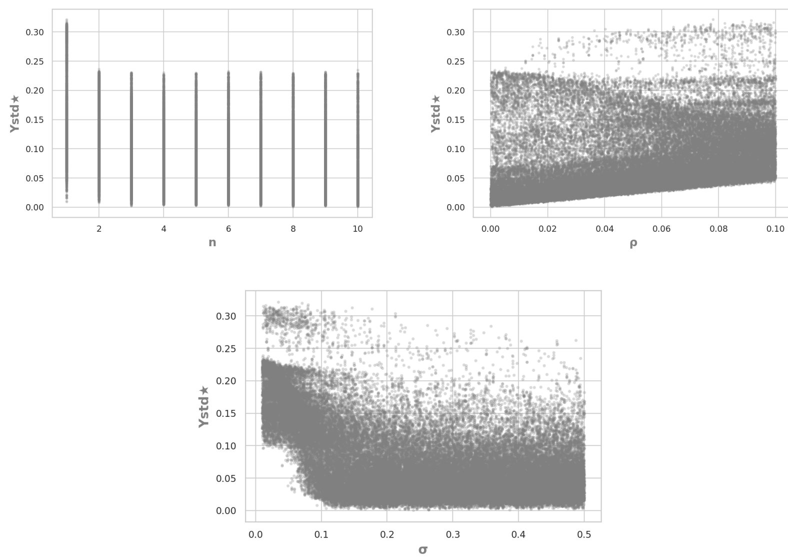


Figure 3: Scatterplots for the parameters with highest impact. Each dot corresponds to the result obtained in a single run.

The negative impact of σ in the population' opinion dispersion is expected: a higher σ means agents are easier to influence. As they are connected to all the other agents, the more uncertain they are, the more centralized the agents' mean opinions will tend to become. The plots also show that the exact value of σ seems to matter little above approximately 0.1. Therefore we restrict our following analysis to the $(0.0, 0.1]$ range. The effect of ρ is also expected: the bigger the noise, the more dispersed the final state of the system is. The number of issues n seems to have little influence unless it is $n = 1$. When there is only one issue, we see results where the opinions clearly move away to stronger polarizations, with $(Ystd)$ around 0.3. Those cases are no longer observed as we have at least two issues.

To better understand how parameters influence the results, we can look at typical behaviors for specific parameter values. We ran cases where we kept $\rho = 0.05$; $N = 500$; $p\_intran = 0.0$ fixed, for 500.000 iterations, and test combinations of $\sigma = (0.02, 0.04, 0.1)$ and $n = (1, 5)$. The time series at Figure 4 show the time evolution of the opinions of all 500 agents. The graphics exhibit the typical behavior of the system for the case where agent i estimates how j is trustworthy by comparing j opinion to its own average ($o_{i,k}$, that is the $p***$ case. Only results for $p***$ are show here because the time series for p^* and $p**$ are visually almost identical to the ones in the figuress. We see that larger values σ lead the opinions towards the center: the bigger the σ is, the more agents will tend to move closer to the mean.

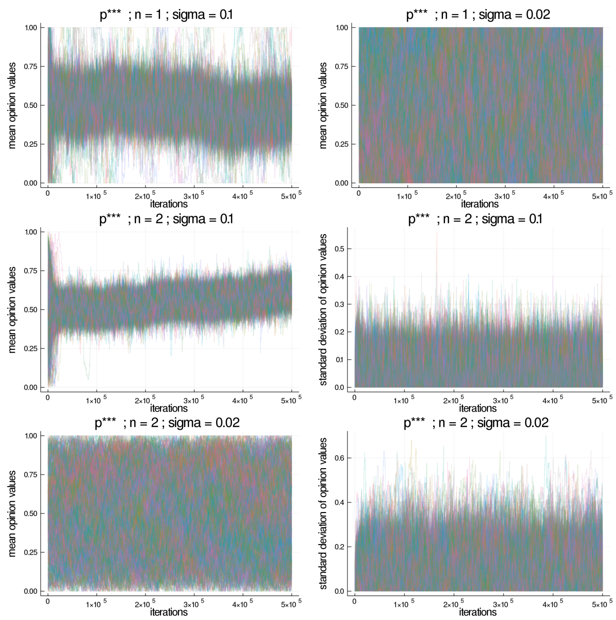


Figure 4: Time series for the parameterization: $\rho = 0.05, N = 500, p\_intran = 0.0, n = 1$ or 2

Figure 5 shows similar time evolution series for $n = 5$ issues. Here, even for small values of σ we observe the opinions tend to avoid the more extreme values. That seems to be an artifact of initial random draws. While it is easy to draw the most extreme values with only one draw, for $n = 5$, as we are close to the end of the range, no values outside that range are possible. So, the most extreme values require that, at the start, agents should have all their five issues drawn as extreme. As that is rare, those few that do start there tend to be attracted to still extreme positions, but a little less so. However, there also seems to be an actual tendency towards more central opinions. That can be oberved as σ increases and the tendency to consensus around a central position becomes even stronger than in the $n = 1$ case.

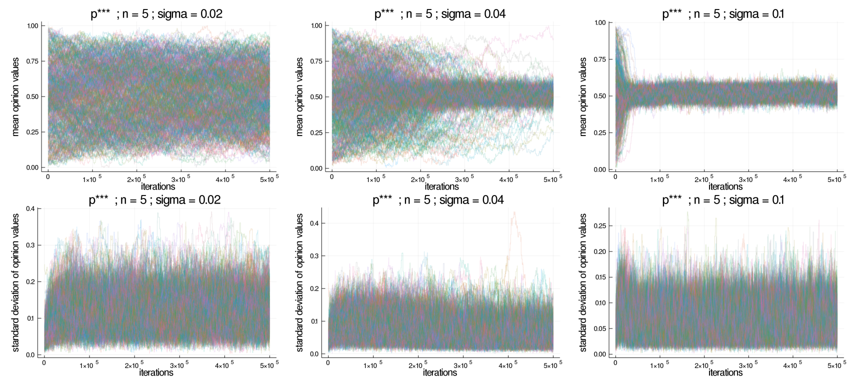


Figure 5: Time series for the parameterization: $\rho = 0.05, N = 500, p\_intran = 0.0$

The reason for that is : we're measuring the mean opinion values (x_i) and ρ changes a single o_i at each iteration which means that a higher n implies a lesser impact of ρ on the mean opinion of the agent, since she will have $n-1$ other opinions stabilizing her mean opinion at a point in the opinion spectrum. As σ is the parameter that dominates the model update rule it interacts with n, which enforces σ effect whenever we test higher ns. This effect holds even when we raise ρ together with n. Let's use the same parameterization as the last plot but with a new ρ such that $\rho_2 = \sqrt{n} * \rho_1 = \sqrt{10} * 0.05$. The interaction between n and σ still happens, with bigger n stabilizing the noise effect and contributing to the centralizing effect of σ, even though a bigger ρ leads to more noise around the mean.

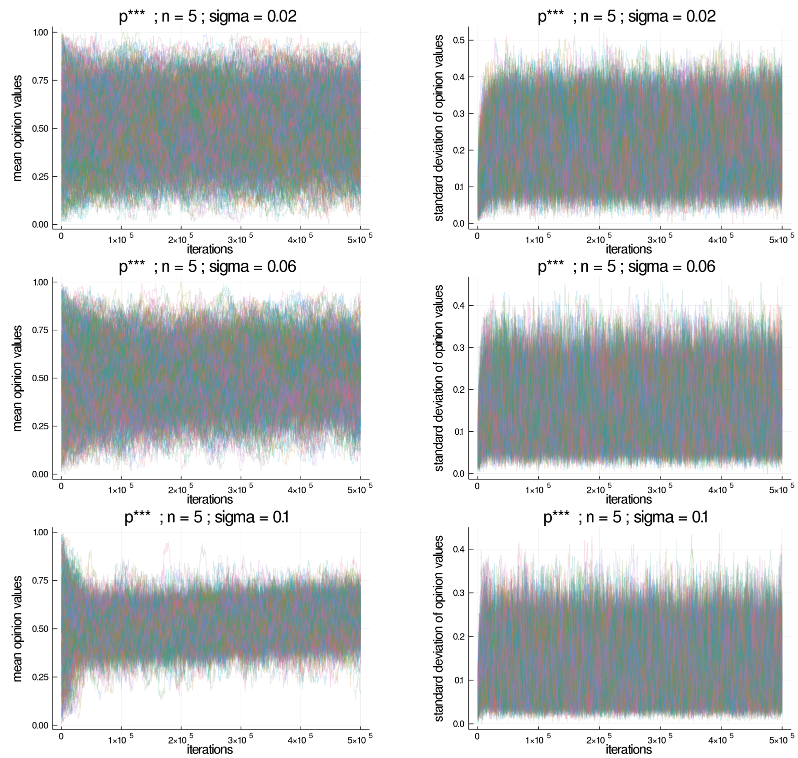


Figure 6: Time series for the parameterization: $\rho \approx 0.12, N = 500, p\_intran = 0.0$

Heretofore we've tested parameterizations with noise, but what happens if we lower ρ to a value close to zero, such as 1e-5? The first difference is that the population mean opinion values converge to certain values. In parameter combinations in which $\sigma = 0.1$ the tendency is convergence to values close to 0.5. An interesting distinction between the cases in this parameterization is that p^{**} and p^{***} always converge to 0.5, independently of the number of issues. Alternatively, in the p^{*} case this happens when $n = 1$, but when we have $n = 5$ or 10 there are other values of convergence, more as we increase n, even though the centralizing tendency remains.

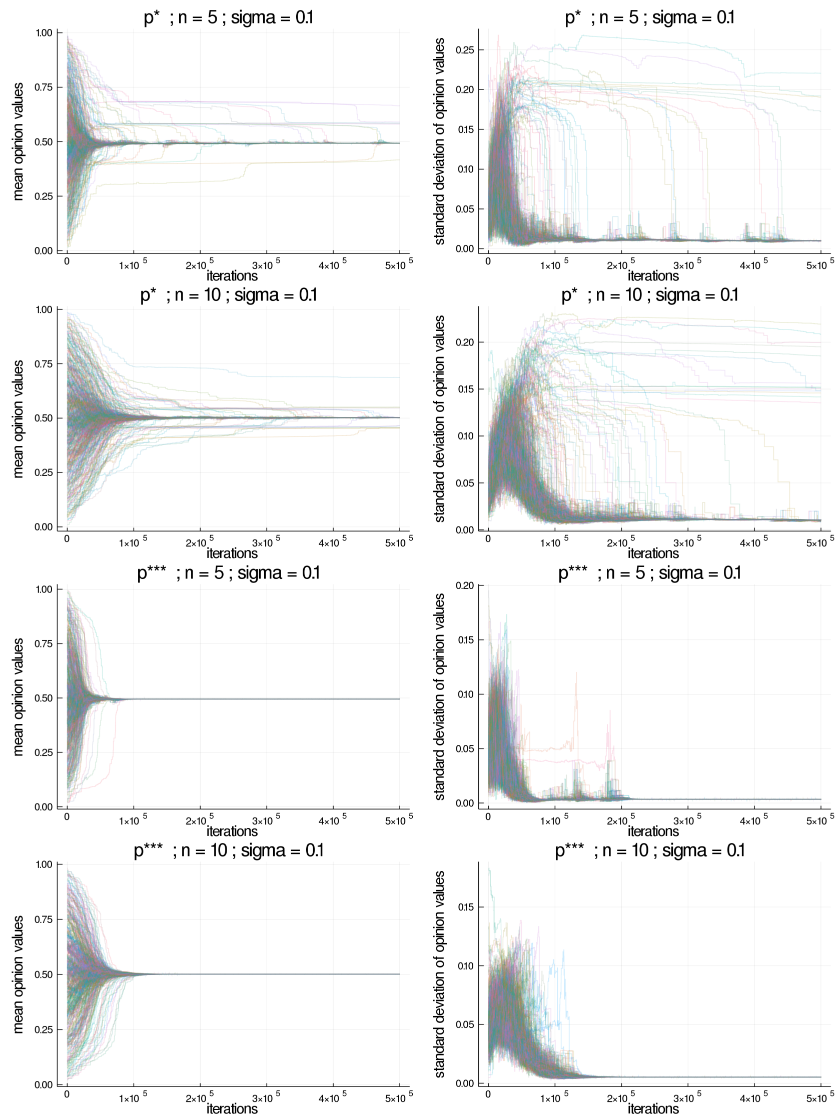


Figure 7: Time series for the parameterization: $\rho = 1e-5, N = 500, p\_intran = 0.0$

In the parameterization in which σ is of intermediate value (for our range), such as 0.02 or 0.04, we observe another difference between cases: p^* has more convergence values than p^{**} and p^{***}. Figure 8 illustrates this system behavior:

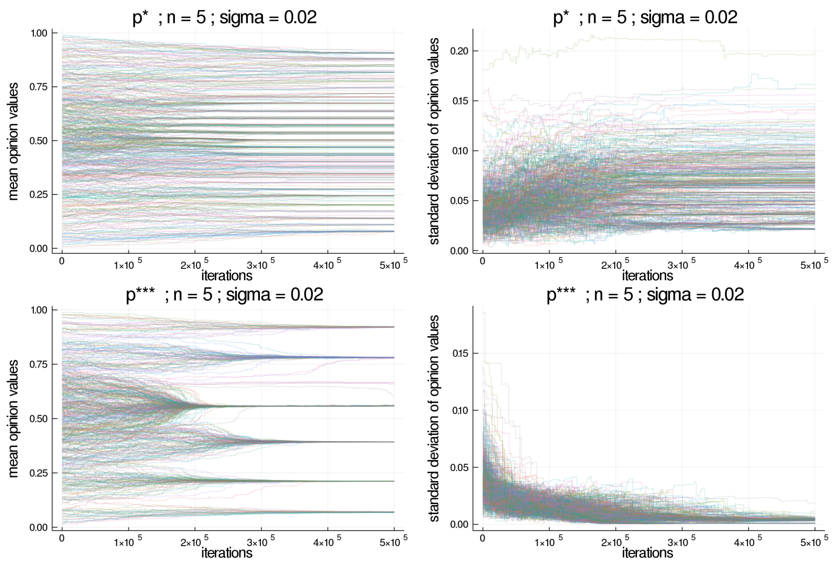


Figure 8: Time series for the parameterization: $\rho = 1e-5, N = 500, p\_intran = 0.0$

The reason for that lies in the Δ of each case: in the p^{**} and p^{***} cases the update rules make use of mean opinion values which facilitates the opinion convergence. The p^{*} update rule works with single issue opinions which opens the possibility that there is little influence between agents given their ideological distance at the issue.

Another impact of the number of issues, as shown in Figure 9, is that a higher n leads to a longer time for the convergence to certain values. The reason is that we're only changing one opinion by iteration, so naturally a higher n means the agents will take longer to be influenced. The relationship here is roughly linear such that the plot the region at 5×10^5 iterations when $n = 10$ is very similar to the corresponding region at 0.5×10^5 when $n = 1$.

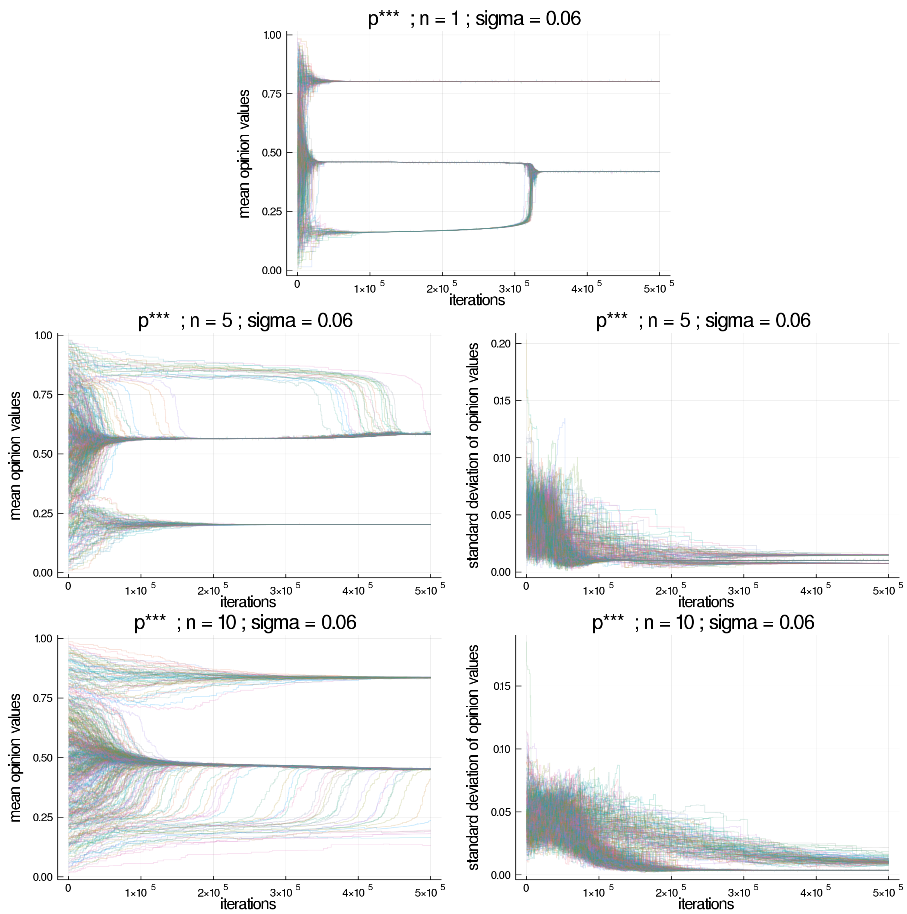


Figure 9: Time series for the parameterization: $\rho = 1e-5, N = 500, p\_intran = 0.0$.

Should I talk about intransigents with $\sigma = 0.01$???

4 Conclusions

5 Acknowledgement

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References

[1] C. Castellano, S. Fortunato, and V. Loreto. Statistical physics of social dynamics. *Reviews of Modern Physics*, 81:591–646, 2009.

[2] Serge Galam. *Sociophysics: A Physicist's Modeling of Psycho-political Phenomena*. Springer, 2012.

[3] S. Galam, Y. Gefen, and Y. Shapir. Sociophysics: A new approach of sociological collective behavior: Mean-behavior description of a strike. *J. Math. Sociol.*, 9:1–13, 1982.

[4] S. Galam and S. Moscovici. Towards a theory of collective phenomena: Consensus and attitude changes in groups. *Eur. J. Soc. Psychol.*, 21:49–74, 1991.

[5] K. Sznajd-Weron and J. Sznajd. Opinion evolution in a closed community. *Int. J. Mod. Phys. C*, 11:1157, 2000.

[6] G. Deffuant, D. Neau, F. Amblard, and G. Weisbuch. Mixing beliefs among interacting agents. *Adv. Compl. Sys.*, 3:87–98, 2000.

[7] André C. R. Martins. Continuous opinions and discrete actions in opinion dynamics problems. *Int. J. of Mod. Phys. C*, 19(4):617–624, 2008.

[8] Anthony Downs. An economic theory of political action in a democracy. *Journal of Political Economy*, 65(2):135–150, 1957.

[9] Michael Laver. Measuring policy positions in political space. *Annual Review of Political Science*, 17:207–223, 2014.

[10] Robert P Van Houweling and Paul M Sniderman. The political logic of a downsian space. *Institute of Governmental Studies*, 2005.

[11] Nicholas R. Miller. The spatial model of social choice and voting. In Jack C. Heckelman and Nicholas R. Miller, editors, *Handbook of social choice and voting*, pages 163–181. Edward Elagar, Cheltenham, 2015.

[12] Kenneth Benoit, Michael Laver, et al. *Party policy in modern democracies*. Routledge, 2006.

[13] R. Vicente, André C. R. Martins, and N. Caticha. Opinion dynamics of learning agents: Does seeking consensus lead to disagreement? *Journal of Statistical Mechanics: Theory and Experiment*, 2009:P03015, 2009. arXiv:0811.2099.

[14] R. Hegselmann and U. Krause. Opinion dynamics and bounded confidence models, analysis and simulation. *Journal of Artificial Societies and Social Simulations*, 5(3):3, 2002.

[15] G. Deffuant, F. Amblard, and T. Weisbuch, G.and Faure. How can extremism prevail? a study based on the relative agreement interaction model. *JASSS-The Journal Of Artificial Societies And Social Simulation*, 5(4):1, 2002.

[16] G. Weisbuch, G. Deffuant, and F. Amblard. Persuasion dynamics. *Physica A*, 353:555–575, 2005.

[17] F. Amblard and G. Deffuant. The role of network topology on extremism propagation with the relative agreement opinion dynamics. *Physica A*, 343:725–738, 2004.

[18] Floriana Gargiulo and Alberto Mazzoni. Can extremism guarantee pluralism? *JASSS-The Journal Of Artificial Societies And Social Simulation*, 11(4):9, 2008.

[19] Daniel W. Franks, Jason Noble, Peter Kaufmann, and Sigrid Stagl. Extremism propagation in social networks with hubs. *Adaptive Behavior*, 16(4):264–274, 2008.

[20] Meysam Alizadeh, Ali Coman, Michael Lewis, and Claudio Cioffi-Revilla. Integroup conflict escalations llead to more extremism. *JASSS-The Journal Of Artificial Societies And Social Simulation*, 14(4):4, 2014.

[21] Giacomo Albi, Lorenzo Pareschi, and Mattia Zanella. Opinion dynamics over complex networks: kinetic modeling and numerical methods. arXiv:1604.00421, 2016.

[22] Tung Mai, Ioannis Panageas, and Vijay V. Vazirani. Opinion dynamics in networks: Convergence, stability and lack of explosion. In Ioannis Chatzigiannakis, Piotr Indyk, Fabian Kuhn, , and Anca Muscholl, editors, *44th International Colloquium on Automata, Languages, and Programming*, number 140, pages 1–14, 2017.

[23] Evguenii Kurmyshev, Héctor A. Juárez, and Ricardo A. González-Silva. Dynamics of bounded confidence opinion in heterogeneous social networks: Concord against partial antagonism. *Physica A: Statistical Mechanics and its Applications*, 390(16):2945 – 2955, 2011.

[24] Daron Acemoglu and Asuman Ozdaglar. Opinion dynamics and learning in social networks. *Dynamic Games and Applications*, 1(1):3–49, Mar 2011.

[25] Abhimanyu Das, Sreenivas Gollapudi, and Kamesh Munagala. Modeling opinion dynamics in social networks. In *Proceedings of the 7th ACM International Conference on Web Search and Data Mining*, WSDM '14, pages 403–412, New York, NY, USA, 2014. ACM.

[26] Haibo Hu. Competing opinion diffusion on social networks. *Royal Society Open Science*, 4(11), 2017.

[27] G. Deffuant. Comparing extremism propagation patterns in continuous opinion models. *JASSS-The Journal Of Artificial Societies And Social Simulation*, 9(3):8, 2006.

[28] André C. R. Martins. Bayesian updating rules in continuous opinion dynamics models. *Journal of Statistical Mechanics: Theory and Experiment*, 2009(02):P02017, 2009. arXiv:0807.4972v1.

[29] André C. R. Martins. Bayesian updating as basis for opinion dynamics models. *AIP Conf. Proc.*, 1490:212–221, 2012.

[30] Macartan Humphreys and Michael Laver. Spatial models, cognitive metrics, and majority rule equilibria. *British Journal of Political Science*, 40(1):11–30, 2010.

[31] Elinor Ostrom. A behavioral approach to the rational choice theory of collective action: Presidential address, american political science association, 1997. *American political science review*, 92(1):1–22, 1998.

[32] John T. Jost, Jack Glaser, Arie W. Kruglanski, and Frank J. Sulloway. Political conservatism as motivated social cognition. *Psychol. Bull.*, 129(3):339–375, 2003.

[33] Charles S. Taber and Milton Lodge. Motivated skepticism of political beliefs. *American Journal of Political Science*, 50(3):755–769, 2006.

[34] Ryan L. Claassen and Michael J. Ensley. Motivated reasoning and yard-sign-stealing partisans: Mine is a likable rogue, yours is a degenerate criminal. *Political Behavior*, pages 1–19, 2015.

[35] Robert Jervis. *Perception and Misperception in International Politics.* Princeton University Press, 1976.

[36] Hugo Mercier. Reasoning serves argumentation in children. *Cognitive Development*, 26(3):177–191, 2011.

[37] Hugo Mercier and Dan Sperber. Why do humans reason? arguments for an argumentative theory. *Behavioral and Brain Sciences*, 34:57–111, 2011.

[38] Dan M. Kahan, Hank Jenkins-Smith, and Donald Braman. Cultural cognition of scientific consensus. *Journal of Risk Research*, 14:147–174, 2011.

[39] David A Armstrong, Ryan Bakker, Royce Carroll, Christopher Hare, Keith T Poole, Howard Rosenthal, et al. *Analyzing spatial models of choice and judgment with R.* CRC Press, 2014.

[40] Michael D McKay, Richard J Beckman, and William J Conover. A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. *Technometrics*, 42(1):55–61, 2000.

[41] S. Galam. Local dynamics vs. social mechanisms: A unifying frame. *Europhysics Letters*, 70(6):705–711, 2005.

[42] Guillaume Deffuant, Frédéric Amblard, Gérard Weisbuch, and Thierry Faure. How can extremism prevail? a study based on the relative agreement interaction model. *Journal of artificial societies and social simulation*, 5(4), 2002.

[43] André C. R. Martins and Serge Galam. The building up of individual inflexibility in opinion dynamics. *Phys. Rev. E*, 87:042807, 2013. arXiv:1208.3290.

[44] Andreas Flache, Michael Mäs, Thomas Feliciani, Edmund Chattoe-Brown, Guillaume Deffuant, Sylvie Huet, and Jan Lorenz. Models of social influence: Towards the next frontiers. *Journal of Artificial Societies and Social Simulation*, 20(4):2, 2017.

[45] Michael Macy and Milena Tsvetkova. The signal importance of noise. *Sociological Methods & Research*, 44(2):306–328, 2015.

[46] Andrea Saltelli, Marco Ratto, Terry Andres, Francesca Campolongo, Jessica Cariboni, Debora Gatelli, Michaela Saisana, and Stefano Tarantola. *Global sensitivity analysis: the primer.* John Wiley & Sons, 2008.

[47] Andrea Saltelli, Karen Chan, E Marian Scott, et al. *Sensitivity analysis*, volume 1. Wiley New York, 2000.

[48] Guus Ten Broeke, George Van Voorn, and Arend Ligtenberg. Which sensitivity analysis method should i use for my agent-based model? *Journal of Artificial Societies and Social Simulation*, 19(1):5, 2016.