

# Some title

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## Abstract

## 1 Introduction

Here, we will present an opinion dynamics model [1, 2, 3, 4, 5, 6, 7] where the opinions exist over an one-dimensional axis and each agent has a best estimate on more than one issue over that axis. Describing opinions over a spatial landscape is an usual way to describe policy alternatives and agents' preferences. The geometrical properties of the space are usually defined by mapping from similarity and proximity of the political agents [8, 9]. Such a description can capture the common notion of parties or policies being more "to the left" or "right". If they're similar then they're closer [10, 11]. Major opinions, including political ones, tend to be formed not from only one issue but from how each person feels about a number of them. Locating someone in a left versus right or liberal versus conservative axis, therefore, requires inspecting the opinions of that person in not only one but several issues that constitute the agent ideological positioning [12].

For many problems, it makes sense to consider different issues as having components in more than one single dimension [13]. However, it is often the case that describing the problem as one-dimensional can be justified. We can certainly see this as a first approximation along the most relevant dimension. In that case, the one dimensional case is just a projection along a direction where variation seems especially important of higher-dimensional problems. From an application point of view, it is usual to find discussions to be simplified over only one main disagreement.

One traditional way to model this type of scenario is to use continuous opinions over a fixed interval, as it is done in the Bounded Confidence (BC) models [6, 14]. While discrete models [3, 4, 5] can be very useful at describing choices, they are not easiest way to represent strength of opinion unless a continuous variable is associated to the choice [7]. Discrete models also tend to lack a scale where we can compare opinions and decide which one is more conservative or more liberal.

On the other hand, continuous models are not particularly well suited for problems involving discrete decisions. As we will not deal with those kinds of problems here, they are a natural choice. Indeed, continuous opinions models have been proposed for several different problems on how opinions spread on a society [15, 16], from questions about the spread of extremism [17, 18, 19, 20, 21, 22] to other issues such as how different networks [23, 24, 25, 26] or the uncertainty of each agent [27] might change how agents influence each other.

Here, we will use a continuous opinion model created by Bayesian-like reasoning [28], inspired by the Continuous Opinions and Discrete Actions (CODA) model [7, 29]. That model was shown previously [28] to provide the same qualitative results as BC models. While a little less simple, the Bayesian basis makes for a clearer interpretation of the meaning of the variables. That makes extending the model and interpreting new results simple. That approach is also consistent with a bounded rationality variant interpretation of the spatial model of political decision making [30, 31].

Variations of how each agent estimate how trustworthy other agents are will also be introduced. Here, we use a function of trust  $p^*$  that plays a role similar to the threshold value in BC models. While  $p^*$  is not a simple discontinuous cut-off, it is a function of the distance between the opinions of the agent and the neighbor. If only one issue is debated, that distance is uniquely defined. However, if we have multiple issues represented on the same one-dimensional line, it is not clear which distance we should use. That happens because we have the opinion of agent  $i$  on a specific issue  $o_i$  and we can compare it to  $o_j$  to estimate how much  $i$  trusts  $j$ . However, as there are several issues we could also use the mean opinion of  $i$  over all its issues and the same goes for  $j$ . Therefore, we will study two additional cases: in the first alternative, we will change  $p^*$  to  $p^{**}$ , determined by the distance between the neighbor and the agent average opinions. In the second alternative, we will use a  $p^{***}$  calculated from the opinion of the neighbor and the average opinion of the agent. The idea here is to make the behavior of our agents closer to what experiments show about human reasoning. We have observed that our reasoning about political problems can be better described as ideologically motivated [32, 33, 34]. Indeed, our opinions tend to come in blocks even when the issues are logically independent [35]. Our reasoning abilities seem to exist more to defend our main point of views [36, 37] and our cultural identity [38] than to find the best answer. In that context, evaluating others by how they differ from us as a whole, instead of in each issue, is a model variation worth exploring.

## 2 The Model

Here, the population will consist of  $N$  agents fully connected (an agent  $i$  can interact with any other agent  $j$ ). Each agent  $i$  will have an opinion  $0 \leq o_{i,k} \leq 1$ , where  $k = 1, \dots, n$  is a specific issue. We assume each agent opinion about issue  $k$  can be represented as a value  $o_{i,k}$  at the range of possible values for  $o$ s. Agents also have an uncertainty  $\sigma$  associated to their average

estimate  $o$ . The uncertainty  $\sigma$  could be different for each agent and also updated during the interactions [28]. For the sake of simplicity, however, we will assume the uncertainty  $\sigma_i = \sigma$  is identical for (almost) all agents and it does not change. The set of opinions for each agent on all possible issues will define its ideological profile  $I_i = ((o_{i,1}, \sigma), \dots, (o_{i,n}, \sigma))$ , where  $n$  is the number of issues,  $o$  is the opinion about the issue and  $\sigma$  is the global uncertainty [29]. The arithmetic mean  $x_i$  of agent  $i$  opinions in each issue will be called here the ideal point for each agent, that is, it defines the agent ideological position at the dimension of interest [39]. Obviously  $x_i = \frac{1}{n} \sum_{k=1}^n o_k$ .

In order to have agents with initial ideal points well distributed over the possible range, the initial valued for each  $o_{i,k}$  was randomly drawn using a Beta  $Be(\alpha, \beta)$  with random parameters  $\alpha_i$  and  $\beta_i$ . Those parameters were drawn for each agent  $i$  from the ranges ( $\alpha \in [1.1, 100]$ ,  $\beta \in [1.1, 100]$ ) by dividing those ranges by  $N$  equally spaced values and then assigning them to each agent after mixing the values as in a Latin hypercube sampling [40]. That is done to allow agents with a diverse value of initial ideal points and, at the same time, to keep the initial  $o_{i,k}$ s of each agent  $i$  correlated.

While we could have  $\sigma$  as a measurement of each agent uncertainty and have it evolve with time, here we will keep it as a fixed parameter of the model. However, a proportion  $p_{intran}$  of agents will be stubborn about one single issue  $k$ , so that  $\sigma_{i,k} = 1e-20$ . That behavior is kept constant as the simulation unfolds, that is  $\sigma_{i,k}$  is also not updated by the model. A proportion of agents who are stubborn is introduced here so we can check if inflexible [41, 42, 43] have a significant impact on the outcomes of the model.

In each iteration of the simulation, two procedures are applied: the opinion update through social influence, and a random opinion update (noise). In the social influence procedure we draw two agents,  $i$  and  $j$ , from the population, where  $i$  will observe  $j$  opinion about a randomly drawn issue  $k \in (1, \dots, n)$ . Agent  $i$  updates its opinion  $(o_{i,k})$  following an approximate Bayesian rule. That rule is obtained by assuming each agent has a Normal prior  $f_i(\theta) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(\theta-o_i)^2}{2\sigma_i^2}}$  compatible with its parameters  $o_i$  and  $\sigma_i$ , where the index related to the issue  $k$  was omitted for the sake of simplifying the equation. The agents also assume a mixture likelihood where there is an initial chance  $p$ , updated to  $p^*$  that the agent  $j$  has information and a chance  $1 - p$  it knows nothing and its opinion is just a random non-informative draw. That is, the likelihood is given by  $f(o_j|\theta) = pN(\theta, \sigma_j^2) + (1 - p)U(0, 1)$ . While a full Bayesian treatment would produce a posterior distribution that is a mixture of two normals, here we will assume each agent only updates its expected value and it does not carry the full posterior information to the next iteration. That leads to the following update rule for the expected value  $o_{i,k}$  [29]:

$$o_{i,k}(t+1) = p^* \frac{o_{i,k}(t) + o_{j,k}(t)}{2} + (1 - p^*)o_{i,k}(t), \quad (1)$$

where  $p^*$  is given by

$$p^* = \frac{p \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(\Delta_{ij})^2}{2\sigma_i^2}}}{p \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(\Delta_{ij})^2}{2\sigma_i^2}} + (1-p)}. \quad (2)$$

Here  $\Delta_{ij} = o_{i,k}(t) - o_{j,k}(t)$  is the distance between the opinions on the  $k$  issue.  $\Delta_{ij}$  plays a similar role to the threshold parameter in the Bounded Confidence models by making distant opinions less influential. As  $\Delta_{ij}$  increases, it is easy to see that Equation 2 causes  $p^*$  to tend to zero. And, as that happens, the weights in average update Equation 1 change so that the previous value  $o_{i,k}(t)$  remains almost unchanged.

The threshold role of  $\Delta_{ij}$  suggests we might change its definition to check how it might better reflect the actual behavior of humans. In particular, people tend to trust better those who have a similar ideology. That means trust might not depend on the specific issue alone, but on the average over all issues. While that is not the model we obtain from applying Bayes rule, such a change makes sense as an attempt to have a better model. In order to check that possibility we will also implement two other cases by changing the way  $\Delta_{ij}$  is calculated. In the first variation,  $p^*$  will be substituted by  $p^{**}$ , where  $p^{**}$  means  $\Delta_{ij} = x_i(t) - x_j(t)$ , that is, agent  $i$  will observe the average ideological position of  $j$  in order to estimate its trust. That assumed  $i$  has more information than only the value  $o_{j,k}(t)$ . To check what happens when indeed only  $o_{j,k}(t)$  indeed what??, we will introduce a second variant case where  $i$  compares  $o_{j,k}(t)$  to its own average. That is,  $\Delta_{ij} = x_i(t) - o_{j,k}(t)$  and we represent that case by  $p^{***}$ .

In order to better understand the model dynamics we also introduce noise as the second procedure. At each iteration, another agent  $i$  is randomly chosen and its opinion changed due to random noise. That is,  $i$  opinion becomes  $o_{i,k}(t+1) = o_{i,k}(t) + r$  where  $r$  is drawn from a Normal distribution with mean 0 and standard deviation  $\rho$ . If agent  $i$  is intransigent in issue  $k$  it won't its  $o_{i,k}$  opinion when chosen by the noise algorithm. Moreover, if  $o_{i,k}(t) + r > 1$  then  $o_{i,k}(t+1) = 1$ . Likewise, if  $o_{i,k}(t) + r < 0$  then  $o_{i,k}(t+1) = 0$ . Noise is introduced here as a way of accounting for the effect of factors not related to the modeled social influence [44] and it is interesting to verify if small noises can lead to drastic changes in systemic properties [45].

### 3 Model Results

To better understand the model behavior, we run simulations using as range for the parameters the values:

$\sigma$	$n$	$p$	$p\_intran$	$N$	$\rho$
[0.01, 0.5]	[1, 10]	[0.1, 0.99]	[0.0, 0.1]	[500, 5000]	(0.0, 0.1)

Table 1: Parameters' Bounds

The parameter space was explored by sampling the parameter space 70,000 times using quasi-random low-discrepancy sequences [46], that generate evenly spaced points. An important question in any opinion dynamics model is if agents can reach consensus. That can be observed by measured the standard deviation of the population mean opinions ( $Y_{std}$ ) for the a system final state. Figure 1 shows histograms for the values of ( $Y_{std}$ ) for initial conditions and also for the sets of simulations corresponding to the  $(p^*, p^{**}, p^{***})$  cases.

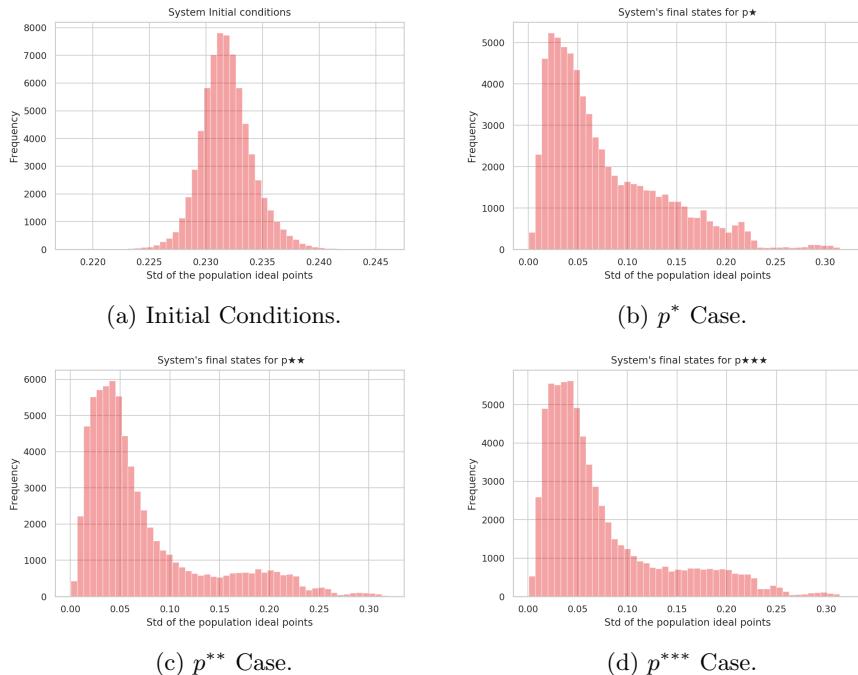


Figure 1: Standard deviations of the population mean opinions ( $Y_{std}$ ) for initial conditions and final conditions after 1,000,000 iterations.

The histograms show that, in most cases, ( $Y_{std}$ ) is significantly smaller than the initial random situation. The peak at small values of ( $Y_{std}$ ) seem to suggest a tendency to consensus in many cases. However, consensus is not always achieved, as we can see for the solid number of cases where ( $Y_{std}$ ) stays close to the initial values. Although rare, we can also observe a few cases where ( $Y_{std}$ ) becomes larger. That can be explained by cases where bi-partisanship becomes

stronger, with agents tending to both extreme positions. That suggest that, in general, the model can be described as one with similarity biased influence [44].

The histograms, however, don't show which values of the parameters cause those changes. They also say nothing about which parameters are relevant and which ones have little influence on the final outcome. With that in mind we perform a Sobol sensitivity analysis [47]. The Sobol indexes decompose the impact of parameters on the variance of the output. The higher the value of index the bigger the impact of the parameter on the output. First order Sobol indexes include linear and non-linear contributions of the parameters, while total Sobol indexes also include all the interaction effects between parameters [48]. Suppose we have a model with three parameters. Then the total effect  $S_{T1}$  of the first parameter  $X_1$  is given by the equation  $S_{T1} = S_1 + S_{12} + S_{13} + S_{123}$  where  $S_i = \frac{V[E(Y|X_i)]}{V(Y)}$ ;  $S_{12}$  is the impact at the variance of the output  $Y$  of the interaction of  $X_1$  and  $X_2$ ; that is, their combined effect minus their first order effects:  $S_{ij} = \frac{V[E(Y|X_i, X_j)] - V[E(Y|X_i)] - V[E(Y|X_j)]}{V(Y)}$  [46]. We then find the following indexes:

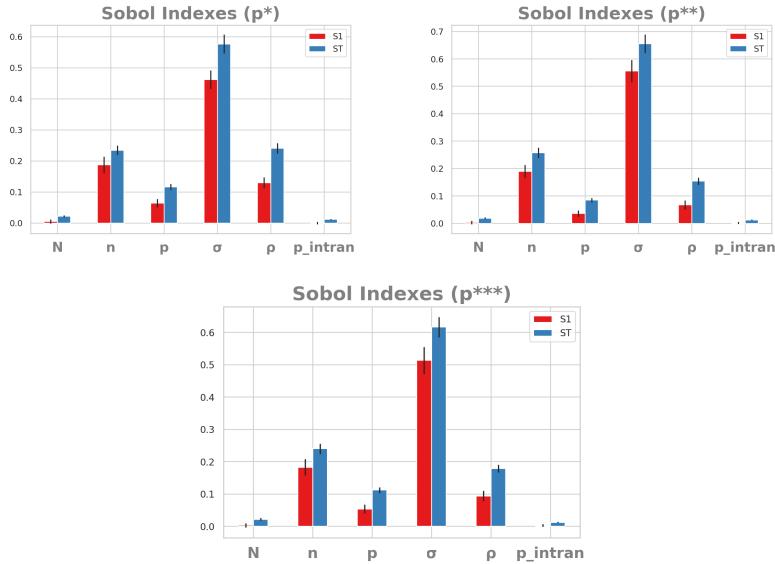


Figure 2: Sobol Indexes for the three cases ( $p^*, p^{**}, p^{***}$ )

The sensitivity analysis in Figure 2 shows that  $\sigma$ ,  $n$ , and  $\rho$  have the most influence on the final values of ( $Y std$ ).  $p$  seems to still have some smaller influence and both  $N$  and  $p_{intran}$  seem to make no difference. It is also interesting to notice that the same behavior appears in the three.  $\sigma$  being the parameter that explains most of the variance was expected and it is consistent with [29]. It is interesting to see that two of the new parameters, the number of issues  $n$  and the noise  $\rho$ , also play an important role in explaining the total variance.

The sensitivity analysis, however, does not show the direction of the impact, which we investigate through scatter plots shown in Figure 3. Each value of ( $Y_{std}$ ) in those graphics correspond to the outcome of one implementation of the model for the value of the parameter at the  $x$ -axis.

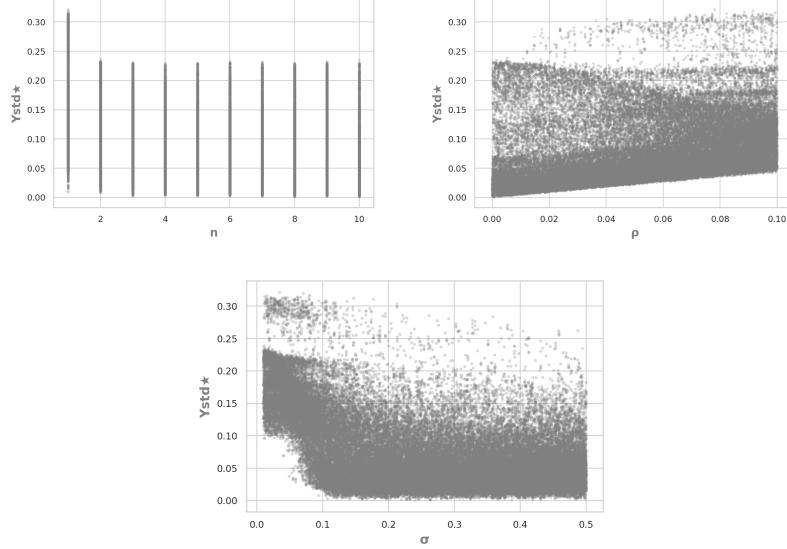


Figure 3: Scatterplots for the parameters with highest impact. Each dot corresponds to the result obtained in a single run.

The negative impact of  $\sigma$  in the population' opinion dispersion is to be expected given the update rule: a higher  $\sigma$  means that agents are easier to influence. As they are connected to all the other agents, the more uncertain they are, the more centralized the agents' mean opinions will tend to become. The plots also show that the exact value of  $\sigma$  seems to matter little above approximately 0.1. Therefore we restrict our following analysis to the  $(0.0, 0.1]$  range. The effect of  $\rho$  is also expected: the bigger the noise, the more dispersed the final state of the system is. The number of issues  $n$  seems to have little influence unless it is  $n = 1$ . When there is only one issue, we see results where the opinions clearly move away to stronger polarizations, with ( $Y_{std}$ ) around 0.3. Those cases are no longer observed as we have at least two issues.

After a general investigation we turn to the analysis of specific parametrizations. For that let's start by fixing the following parameters as constants:  $\rho = 0.05$  ;  $N = 500$  ;  $p\_intran = 0.0$ , run the simulation for 500.000 iterations, and test combinations of  $\sigma = (0.02, 0.04, 0.1)$  and  $n = (1, 5)$ . The following time series exhibit the typical behavior of the system for the parameterizations. As shown by Figure 4,  $\sigma$  has the effect of leading the dynamics of the simulation to the center as it increases: the bigger the  $\sigma$  the closest to the mean the pop-

ulation is . However, when the number of issues also increases that tendency is not clear, since the noise disperses them, even though  $\sigma$  has a bigger impact:

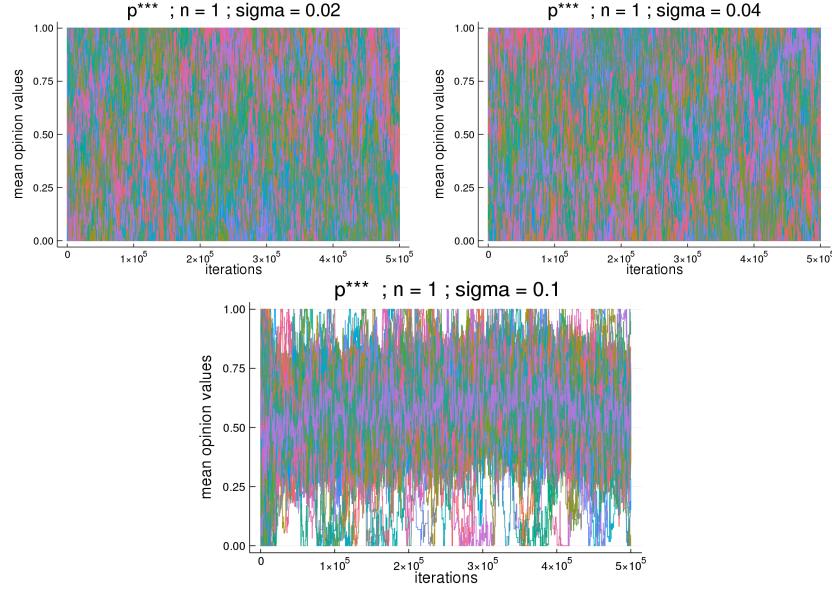


Figure 4: Time series for the parameterization:  $\rho = 0.05, N = 500, p\_intran = 0.0, n = 1$

On the other hand, when we also increase the number of issues the centralizing effect of  $\sigma$  becomes stronger:

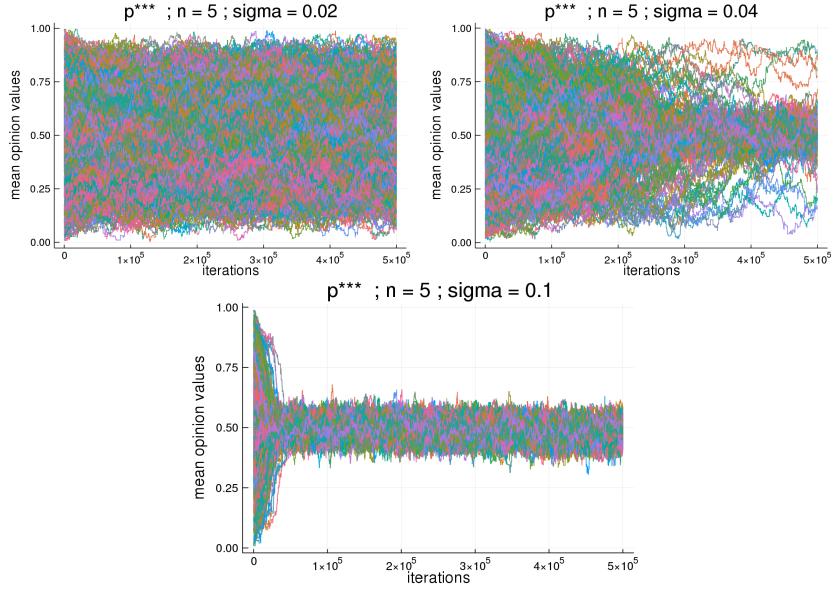


Figure 5: Time series for the parameterization:  $\rho = 0.05, N = 500, p\_intran = 0.0$

The reason for that is : we're measuring the mean opinion values ( $x_i$ ) and  $\rho$  changes a single  $o_i$  at each iteration which means that a higher  $n$  implies a lesser impact of  $\rho$  on the mean opinion of the agent, since she will have  $n - 1$  other opinions stabilizing her mean opinion at a point in the opinion spectrum. As  $\sigma$  is the parameter that dominates the model update rule it interacts with  $n$ , which enforces  $\sigma$  effect whenever we test higher  $ns$ . This effect holds even when we raise  $\rho$  together with  $n$ . Let's use the same parameterization as the last plot but with a new  $\rho$  such that  $\rho_2 = \sqrt{n} * \rho_1 = \sqrt{10} * 0.05$ . The interaction between  $n$  and  $\sigma$  still happens, with bigger  $n$  stabilizing the noise effect and contributing to the centralizing effect of  $\sigma$ , even though a bigger  $\rho$  leads to more noise around the mean.

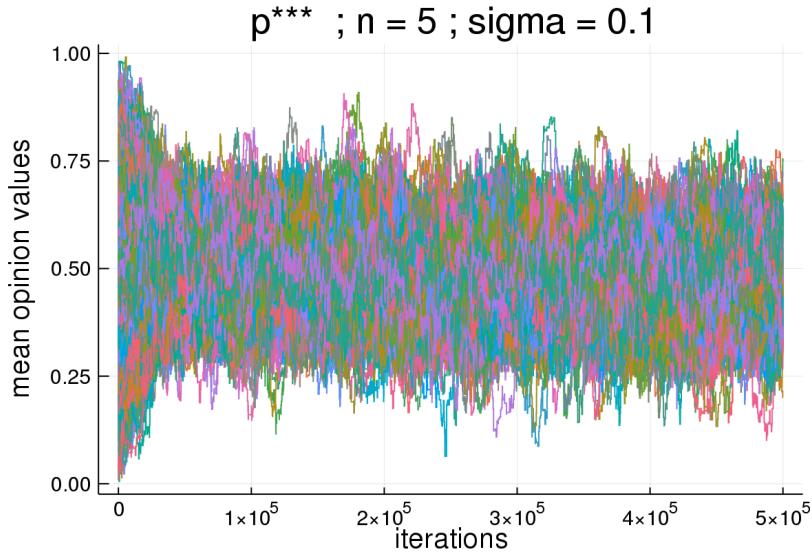


Figure 6: Time series for the parameterization:  $\rho \approx 0.12$ ,  $N = 500$ ,  $p\_intran = 0.0$

Heretofore we've tested parameterizations with noise, but what happens if we lower  $\rho$  to a value close to zero, such as 1e-5? The first difference is that the population mean opinion values converge to certain values. In parameter combinations in which  $\sigma = 0.1$  the tendency is convergence to values close to 0.5. An interesting distinction between the cases in this parameterization is that  $p^{**}$  and  $p^{***}$  always converge to 0.5, independently of the number of issues. Alternatively, in the  $p^*$  case this happens when  $n = 1$ , but when we have  $n = 5$  or 10 there are other values of convergence, more as we increase  $n$ , even though the centralizing tendency remains.

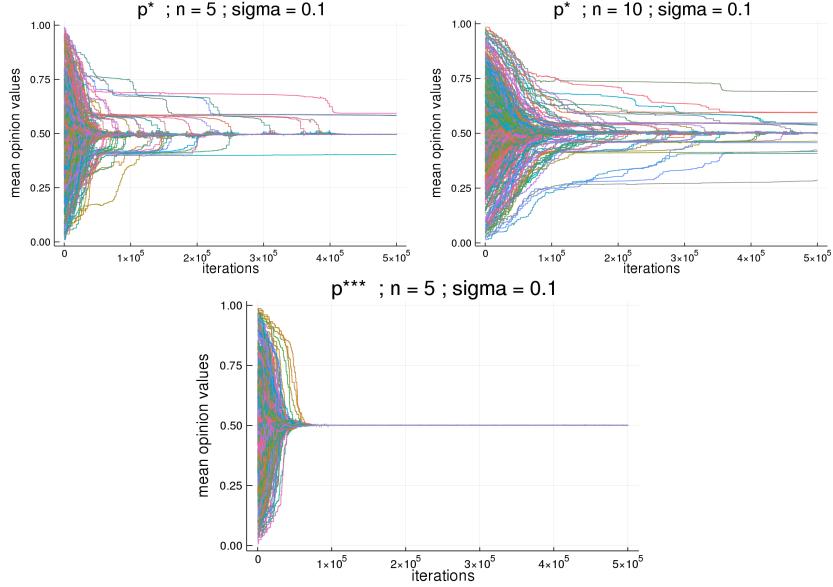


Figure 7: Time series for the parameterization:  $\rho = 1e-5, N = 500, p\_intran = 0.0$

In the parameterization in which  $\sigma$  is of intermediate value (for our range), such as 0.02 or 0.04, we observe another difference between cases:  $p^*$  has more convergence values than  $p^{**}$  and  $p^{***}$ . Figure 8 illustrates this system behavior:

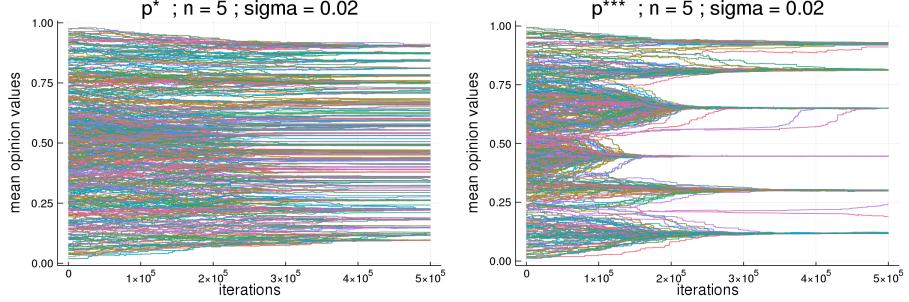


Figure 8: Time series for the parameterization:  $\rho = 1e-5, N = 500, p\_intran = 0.0$

The reason for that lies in the  $\Delta$  of each case: in the  $p^{**}$  and  $p^{***}$  cases the update rules make use of mean opinion values which facilitates the opinion convergence. The  $p^*$  update rule works with single issue opinions which opens the possibility that there is little influence between agents given their ideological distance at the issue.

Another impact of the number of issues, as shown in Figure 9, is that a higher

$n$  leads to a longer time for the convergence to certain values. The reason is that we're only changing one opinion by iteration, so naturally a higher  $n$  means the agents will take longer to be influenced. The relationship here is roughly linear such that the plot the region at  $5 \times 10^5$  iterations when  $n = 10$  is very similar to the corresponding region at  $0.5 \times 10^5$  when  $n = 1$ .

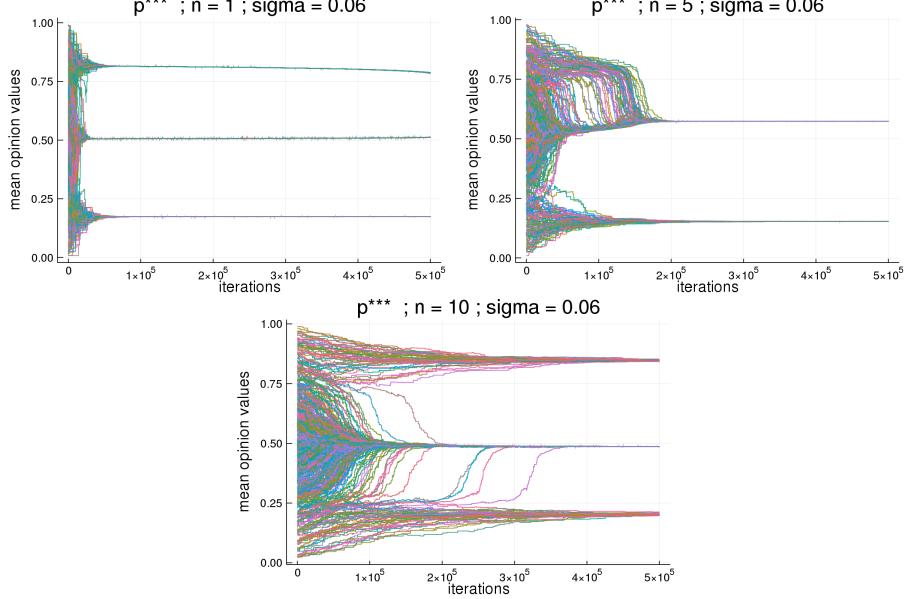


Figure 9: Time series for the parameterization:  $\rho = 1e-5$ ,  $N = 500$ ,  $p\_intran = 0.0$ .

Should I talk about intransigents with  $\sigma = 0.01$  ???

## 4 Conclusions

## 5 Acknowledgement

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