

# Ideologically Motivated Biases in a Multiple Issues Opinion Model

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## Abstract

It has been observed people tend to have opinions that are far more internally consistent than it would be reasonable to expect. Here, we study how that observation might emerge from changing how agents trust the opinions of their peers in a model for opinion dynamics with multiple issues. A previous Bayesian inspired opinion model for continuous opinions is extended to include multiple issues. In the original model, agents tended to trust less opinions that were too different from their own. We investigate the properties of the extended model in its natural form. And we also introduce the possibility the trust of the agent might depend not only on the specific issue but on the average opinions over the many issues. By adopting such a ideological point of view, we observe an important decrease in the spread of individual opinions.

## 1 Introduction

Here, we will present an opinion dynamics model [12, 29, 27, 28, 72, 19, 50] where the opinions exist over an one-dimensional axis and each agent has a best estimate on more than one issue over that axis. Describing opinions over a spatial landscape is an usual way to describe policy alternatives and agents' preferences. The geometrical properties of the space are usually defined by mapping from similarity to proximity of the political agents [22, 45, 77]. Such a description can capture the common notion of parties or policies being more "to the left" or "right". If they're similar then they're closer [79, 60]. The spatial metaphor is not restricted to politics it is commonly used in the literature about opinion dynamics and actually based upon a long research tradition in cognitive science [2, 8, 30]. Major opinions, including political ones, tend to be formed not from only one issue but from how each person feels about a number of them. Locating someone in a left versus right or liberal versus conservative axis, therefore, requires inspecting the opinions of that person in not only one but several issues that constitute the agent ideological positioning [11].

Proximity of opinions as basis for how much an agent would trust others is a basic principle in Bounded Confidence models [19, 33, 6, 25], besides being clear common sense. It is also a fundamental consequence of what recent experiments show about our cognition. We tend to evaluate our own knowledge with excessive overconfidence [63, 78]. Even when people might gain or lose money, we tend to trust our own estimates far more than we should [3, 65, 13]. Not only that, when looking for information, we actively search for those who agree with us [61]. The picture that has emerged recently from decades of experiments in cognition is that our reasoning skills might have evolved primarily for the purpose of argumentation [57, 59]. That is, fitting with our group, by convincing the group or getting convinced by it might have been much more important from an evolutive point of view than being correct. When our groups are defined by the ideas they defend, as they often are, and even more so, in political problems, that means a desire to conform and trust those who already agree with us and distrust those who do not [53]. Especially when dealing with identity-defining ideas, our reason seems to trick us to defend our own preferences [40, 42] over looking for truth. Even in scientific studies, that kind of selective trust seems to have led us to not achieve the best conclusions [52, 51]. Trust becomes something we reserve mostly for those we already agree with [58].

For many problems, it makes sense to consider different issues as having components in more than one single dimension [80]. However, it is often the case that describing the problem as one-dimensional can be justified. We can certainly see this as a first approximation along the most relevant dimension. In that case, the one dimensional case is just a projection along a direction where variation seems especially important of higher-dimensional problems. From an application point of view, it is usual to find discussions to be simplified over only one main disagreement.

One traditional way to model this type of scenario is to use continuous opinions over a fixed interval, as it is done in the Bounded Confidence (BC) models [19, 33]. While discrete models [27, 28, 72] can be very useful at describing choices, they are not easiest way to represent strength of opinion unless a continuous variable is associated to the choice [50]. Discrete models also tend to lack a scale where we can compare opinions and decide which one is more conservative or more liberal.

On the other hand, continuous models are not particularly well suited for problems involving discrete decisions. As we will not deal with those kinds of problems here, they are a natural choice. Indeed, continuous opinions models have been proposed for several different problems on how opinions spread on a society [18, 81], from questions about the spread of extremism [6, 31, 5, 4, 25] to other issues such as how different networks [44, 1, 15, 35] or the uncertainty of each agent [17] might change how agents influence each other.

Here, we will use a continuous opinion model created by Bayesian-like reasoning [49], inspired by the Continuous Opinions and Discrete Actions (CODA) model [50, 48]. That model was shown previously [49] to provide the same qualitative results as BC models. While a little less simple, the Bayesian basis makes for a clearer interpretation of the meaning of the variables. That makes

extending the model and interpreting new results simple. That approach is also consistent with a bounded rationality variant interpretation of the spatial model of political decision making [36, 64].

Variations of how each agent estimate how trustworthy other agents are will also be introduced. Here, we use a function of trust  $p^*$  that plays a role similar to the threshold value in BC models. While  $p^*$  is not a simple discontinuous cut-off, it is a function of the distance between the opinions of the agent and the neighbor. If only one issue is debated, that distance is uniquely defined. However, if we have multiple issues represented on the same one-dimensional line, it is not clear which distance we should use. That happens because we have the opinion of agent  $i$  on a specific issue  $o_i$  and we can compare it to  $o_j$  to estimate how much  $i$  trusts  $j$ . However, as there are several issues we could also use the mean opinion of  $i$  over all its issues and the same goes for  $j$ . Therefore, we will study two additional cases: in the first alternative, we will change  $p^*$  to  $p^{**}$ , determined by the distance between the neighbor and the agent average opinions. In the second alternative, we will use a  $p^{***}$  calculated from the opinion of the neighbor and the average opinion of the agent. The idea here is to make the behavior of our agents closer to what experiments show about human reasoning. We have observed that our reasoning about political problems can be better described as ideologically motivated [39, 73, 14]. Indeed, our opinions tend to come in blocks even when the issues are logically independent [38]. Our reasoning abilities seem to exist more to defend our main point of views [57, 59] and our cultural identity [41] than to find the best answer. In that context, evaluating others by how they differ from us as a whole, instead of in each issue, is a model variation worth exploring.

## 2 The Model

Bayesian reasoning, from a descriptive point of view, is quite simple. It assumes an individual has an initial opinion that is updated as new data is learned. The update rules are obtained as a consequence of knowing how the data is generated. In general terms, it is basically a normative model for how opinions should change. Of course, actual people do not use data perfectly, but the structure of any bayesian analysis is already that of an opinion dynamics model.

The model is, then, grounded on a Bayesian foundation for opinion dynamics [48]. The reason for such a foundation is twofold. First, opinion dynamics models tend to be created on an ad hoc basis without any clear connection between them and how each may contribute to the accumulation of knowledge [23]. The Bayesian approach provides flexible first principles that allow us to circumvent this problem. It is a framework where the assumptions agents make about communication and the opinions of others are used to generate the model. Second, even though it is well documented that people don't faithfully follow the principle of Bayesian rationality<sup>1</sup>, many works also indicate that it is a good

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<sup>1</sup>For a formal definition of Bayesian rationality see [32, p.104].

*approximation* and that humans are imperfectly bayesian [76, 43, 74, 47, 32, 9]. Agents here are also not perfectly Bayesian, as their models are incomplete.

In this framework every agent has a prior probability distribution about the issue at stake that represents their knowledge. Agents communicate and the probability that the value stated by a neighbor  $j$ , what here we'll call the opinion ( $o$ ), is true is the likelihood that  $i$  uses, following an approximate Bayesian rule, to update its prior probability distribution such that **posterior opinion distribution  $\propto$  prior  $\times$  likelihood**<sup>2</sup>.

Here, the population will consist of  $N$  fully connected agents (an agent  $i$  can interact with any other agent  $j$ ). Each agent  $i$  will have an opinion  $0 \leq o_{ik} \leq 1$ , where  $k = 1, \dots, n$  is a specific issue. We assume each agent opinion about issue  $k$  can be represented as a value  $o_{ik}$  at the range of possible values for  $o$ s. Agents also have an uncertainty  $\sigma$  associated to their average estimate  $o$ . The uncertainty  $\sigma$  could be different for each agent and also updated during the interactions [49]. For the sake of simplicity, however, we will assume the uncertainty  $\sigma_i = \sigma$  is identical for (almost) all agents and it does not change. The set of opinions for each agent on all possible issues will define its ideological profile  $I_i = ((o_{i1}, \sigma), \dots, (o_{in}, \sigma))$ , where  $n$  is the number of issues,  $o$  is the opinion about the issue and  $\sigma$  is the global uncertainty [55]. The arithmetic mean  $x_i$  of agent  $i$  opinions in each issue will be called here the ideal point for each agent, that is, it defines the agent ideological position at the dimension of interest [7]. Obviously  $x_i = \frac{1}{n} \sum_{k=1}^n o_{ik}$ .

In order to have agents with initial ideal points well distributed over the possible range, the initial value for each  $o_{ik}$  was randomly drawn using a Beta  $Be(\alpha, \beta)$  with random parameters  $\alpha_i$  and  $\beta_i$ . Those parameters were drawn for each agent  $i$  from the ranges ( $\alpha \in [1.1, 100]$ ,  $\beta \in [1.1, 100]$ ) by dividing those ranges by  $N$  equally spaced values and then assigning them to each agent after mixing the values as in a Latin hypercube sampling [56]. That is done to allow agents with a diverse value of initial ideal points and, at the same time, to keep the initial  $o_{iks}$  of each agent  $i$  correlated.

While we could have  $\sigma$  as a measurement of each agent uncertainty and have it evolve with time, here we will keep it as a fixed parameter of the model. However, a proportion  $p\_intran$  of agents will be stubborn about one single issue  $k$ , so that  $\sigma_{ik} \approx 0$  (implemented as 1e-20). That behavior is kept constant as the simulation unfolds, that is  $\sigma_{ik}$  is also not updated by the model. A proportion of agents who are stubborn is introduced here so we can check if inflexible [26, 20, 54] have a significant impact on the outcomes of the model.

In each iteration of the simulation, two procedures are applied: the opinion update through social influence, and a random opinion update (noise). In the social influence procedure we draw two agents,  $i$  and  $j$ , from the population, where  $i$  will observe  $j$  opinion about a randomly drawn issue  $k \in (1, \dots, n)$ . Agent  $i$  updates its opinion ( $o_{ik}$ ) following an approximate Bayesian rule. That rule is obtained by assuming each agent has a Normal prior  $f_i(\theta) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(\theta-o_i)^2}{2\sigma_i^2}}$  compatible with its parameters  $o_i$  and  $\sigma_i$ , where the index related to the is-

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<sup>2</sup>For a more extensive definition and derivations see [48] and [55] respectively.

sue  $k$  was omitted for the sake of simplifying the equation. The agents also assume a mixture likelihood where there is an initial chance  $p$ , updated to  $p^*$  that the agent  $j$  has information and a chance  $1 - p$  it knows nothing and its opinion is just a random non-informative draw. That is, the likelihood is given by  $f(o_j|\theta) = pN(\theta, \sigma_j^2) + (1 - p)U(0, 1)$ . While a full Bayesian treatment would produce a posterior distribution that is a mixture of two normals, here we will assume each agent only updates its expected value and it does not carry the full posterior information to the next iteration. That leads to the following update rule for the expected value  $o_{i,k}$  [55]:

$$o_{ik}(t+1) = p^* \frac{o_{ik}(t) + o_{jk}(t)}{2} + (1 - p^*)o_{ik}(t), \quad (1)$$

where  $p^*$  is given by

$$p^* = \frac{p \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(\Delta_{ij})^2}{2\sigma_i^2}}}{p \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(\Delta_{ij})^2}{2\sigma_i^2}} + (1 - p)}. \quad (2)$$

Here  $\Delta_{ij} = o_{ik}(t) - o_{jk}(t)$  is the distance between the opinions on the  $k$  issue. From Equation 1, we can see  $p^*$  plays the role of the probability agent  $i$  assigns to  $j$  being correct. While that role was initially played by  $p$ , as  $i$  learns about  $j$  opinion,  $i$  also updates its opinion on the chances that  $j$  might know something. That is why we need an updated value when the posteriori average is calculated. As  $\Delta_{ij}$  increases, it is easy to see that Equation 2 causes  $p^*$  to tend to zero. Given this relationship,  $\Delta_{ij}$  plays the obvious role of distance between the opinions, while  $\sigma_i^2$  plays the role of the threshold parameter in the Bounded Confidence models. As  $\Delta_{ij}$  increases as larger multiples of  $\sigma_i^2$ , that makes distant opinions less influential. As that happens, the weights in average update Equation 1 change so that the previous value  $o_{i,k}(t)$  remains almost unchanged. Trust depends on the ratio

The role of  $\Delta_{ij}$  on the trust between the agents suggests we might change its definition to check how it might better reflect the actual behavior of humans. In particular, people tend to trust better those who have a similar ideology. That means trust might not depend on the specific issue alone, but on the average over all issues. While that is not the model we obtain from applying Bayes rule, such a change makes sense as an attempt to have a better model. In order to check that possibility we will also implement two other cases by changing the way  $\Delta_{ij}$  is calculated. In the first variation,  $p^*$  will be substituted by  $p^{**}$ , where  $p^{**}$  means  $\Delta_{ij} = x_i(t) - x_j(t)$ , that is, agent  $i$  will observe the average ideological position of  $j$  in order to estimate its trust. That assumed  $i$  has more information than only the value  $o_{jk}(t)$ . To check what happens when only  $o_{jk}(t)$  can be observed by agent  $i$ , we will introduce a second variant case where  $i$  compares  $o_{jk}(t)$  to its own average. That is,  $\Delta_{ij} = x_i(t) - o_{jk}(t)$  and we represent that case by  $p^{***}$ .

In order to better understand the model dynamics we also introduce noise as the second procedure. At each iteration, another agent  $i$  is randomly chosen and

its opinion changed due to random noise. That is,  $i$  opinion becomes  $o_{ik}(t+1) = o_{ik}(t) + r$  where  $r$  is drawn from a Normal distribution with mean 0 and standard deviation  $\rho$ . If agent  $i$  is intransigent in issue  $k$  it won't its  $o_{ik}$  opinion when chosen by the noise algorithm. Moreover, if  $o_{ik}(t) + r > 1$  then  $o_{ik}(t+1) = 1$ . Likewise, if  $o_{ik}(t) + r < 0$  then  $o_{ik}(t+1) = 0$ . Noise is introduced here as a way of accounting for the effect of factors not related to the modeled social influence [23] and it is interesting to verify if small noises can lead to drastic changes in systemic properties [46].

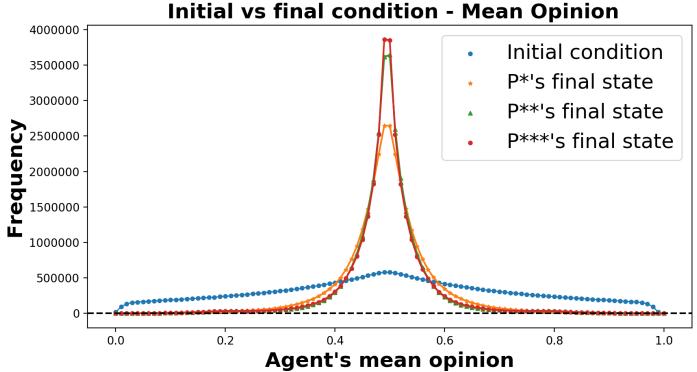
### 3 Model Results

To better understand the model behavior, we ran simulations using as range for the parameters the values:

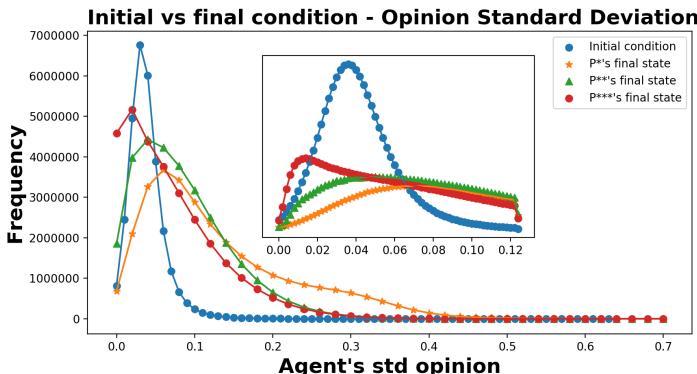
$\sigma$	$n$	$p$	$p\_intran$	$N$	$\rho$
[0.01, 0.5]	[1, 10]	[0.1, 0.99]	[0.0, 0.1]	[500, 5000]	(0.0, 0.1)

Table 1: Parameters' Bounds

The parameter space was explored by two sweeps of its parameters: one sampling of 70,000 quasi-random low-discrepancy sequence draws on all parameters [69, 68], that generate evenly spaced points, and another of 60,000 keeping  $N = 500$  so that we can compare different realizations. Each of those draws correspond to a complete run of the model. An interesting question in any opinion dynamics model is if agents can reach consensus, if they diverge, or something between those two states. That can be observed by comparing the initial mean and standard deviation of each agent's opinions. Figure 1 shows histograms for those variables at the initial condition and also for the final opinions corresponding to the  $(p^*, p^{**}, p^{***})$  cases. The upper graphic shows the distribution of the mean opinions  $x_i$ , and the lower one the distribution of the standard deviation of the opinions of each agent  $s_i^2 = \frac{1}{n} \sum_{k=1}^n (o_{ik} - x_i)^2$ . That is, for each case we sampled 60,000 parameterizations guaranteeing that we're filling evenly the space of parameters' possible values. Then we collected the mean and standard deviation of the opinions of each agent for each parameterization both at the initial and final conditions. Figure 1 then gives a general view of the model behavior as we are plotting many different combinations of parameters' values.



(a) Initial x Final state:  
Distribution of mean opinions



(b) Initial x Final state:  
Distribution of the standard deviations of opinions

Figure 1: Difference between initial and final opinion distributions for 60.000 parameterizations. In all cases,  $N = 500$ .

The histograms show that, in general, opinions have a tendency to move towards the middle value. This seems to suggest most parameter values lead to consensus or near it. However, consensus is not always achieved, as shown by the tails of the mean distribution at Figure 1, even if the final distribution shows that extreme opinions become less common. And we can also observe in the distribution of standard deviations  $s_i$  that the opinions of each agent on each issue tend to become more diverse, as, for most cases,  $s_i$  seems to increase from its initial values. The only exception is the  $p^{***}$  scenario. There, we still see an increase in the frequency of large values of  $s_i$ . But there are also many cases where there is a strong tendency for  $s_i$  to become smaller. Or, in other words, in several scenarios, the agents' opinions on the issues tended to gather

much closer to their own mean opinions  $x_i$  as a consequence of the dynamics.

The cases where the average opinions remain a little spread, while rare, correspond to scenarios where bi-partisanship survives, with agents opinions surviving at both extreme positions. This suggests that, in general, the model can be described as one with similarity biased influence [23]. This tendency to consensus seems to be a little weaker at the  $p^*$  case when compared to the two other cases,  $p^{**}$  and  $p^{***}$ .

Figure 1 (b) also shows that in all cases  $(p^*, p^{**}, p^{***})$  the standard deviation of the opinions distribution becomes more spread. While there are situations where  $s_i$  tends to become smaller, signaling the agent opinions become closer to its mean, the reverse also happens. The tails for larger values of  $s_i$  show that the intercation with other agents quite often led to a more diverse set of opinions over the different issues. As a matter of fact, except for the  $p^{***}$  scenarios, a larger internal spread than in the initial conditions seem to be the usual result. For  $p^{***}$ , while we also observe that strengthenig of the spread for many parameter values, we also see that an increase in cases with small  $s_i$ . That is, the dynamics can lead to stronger internal consistency.

The distributions for the mean and standard deviation might look contradictory at a first glance. But they are information about different quantities. A tendency of the  $x_i$  towards central value while  $s_i$  increases is simple to understand. It just shows that, while opinions on different issues might be spreading more, including towards more extreme values, the mean of every issue opinion of the agents show a clear central tendency. That the mean would have a central tendency should not be seen as a surprise. But that does not necessarily correspond to what happens to the agents opinions on specific issues  $o_{ik}$ .

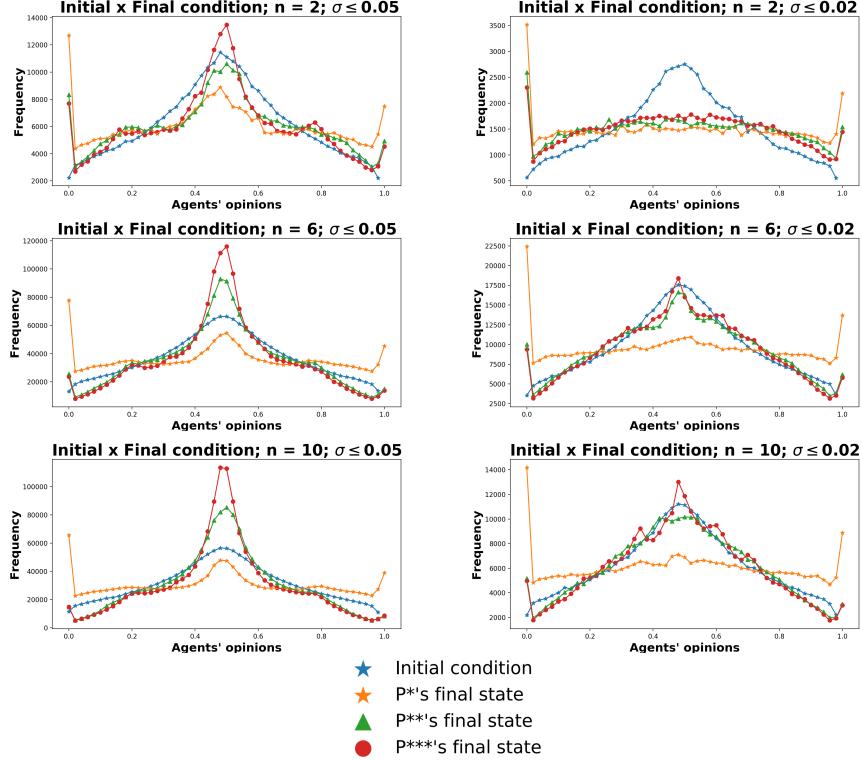


Figure 2: Histogram for the final distribution of values of  $o_{ik}$ . Initial distribution is also shown for comparison.

The behavior of the opinions about specific issues  $o_{ik}$  values when we have small values of  $\sigma$  can be observed in the Figure 2. The graphics show separate cases for different number  $n$  of issues and distinct small values of  $\sigma$ . The stronger central tendency. While the most important peak is still around 0.5, the distribution of values is now much more spread. That reflects the fact that, as  $\sigma$  decreases, consensus becomes much harder and a fraction of the agents find agreement at other values. Indeed, in every graphic, for the  $p^*$  scenario, central opinions become less common than in initial conditions, while more extreme values become predominant, showing a clear tendency away from consensus. The same is not true for the more centralizing versions,  $p^{**}$  and  $p^{***}$ , that seems to show a more varied behavior. For  $\sigma \leq 0.05$ , both cases show some tendency to consensus. As for  $\sigma \leq 0.02$ , we see very small changes in the initial distribution, when  $n = 6$  or  $n = 10$ , and a clear drive to disagreement when  $n = 2$ . In all scenarios, however, it is clear that the tendency to disagreement is weaker when update is done following the  $p^{**}$  and  $p^{***}$  rules than what we observe for  $p^*$ .

The histograms, however, don't show the whole story of which parameters influence the system behavior. With that in mind we performed a Sobol sensitivity analysis [68] using as outcome both the observed standard deviation of

agent's mean opinions  $Y std^2 = \frac{1}{n} \sum_{k=1}^n (x_i - \bar{x}_i)^2$  and the standard deviation  $S_s$  of the collection of  $N$  internal standard deviations  $s_i$ . The Sobol indexes decompose the impact of parameters on the variance of the output. The higher the value of index the bigger the impact of the parameter on the output. First order Sobol indexes include linear and non-linear contributions of the parameters, while total Sobol indexes also include all the interaction effects between parameters [75]. If there are only three parameters, the total effect  $S_{T1}$  of the first parameter  $X_1$  is given by the equation  $S_{T1} = S_1 + S_{12} + S_{13} + S_{123}$  where  $S_i = \frac{V[E(Y|X_i)]}{V(Y)}$ ;  $S_{12}$  is the impact at the variance of the output  $Y$  of the interaction of  $X_1$  and  $X_2$ ; that is, their combined effect minus their first order effects:  $S_{ij} = \frac{V[E(Y|X_i, X_j)] - V[E(Y|X_i)] - V[E(Y|X_j)]}{V(Y)}$  [69]. For our simulations, we obtain the estimates:

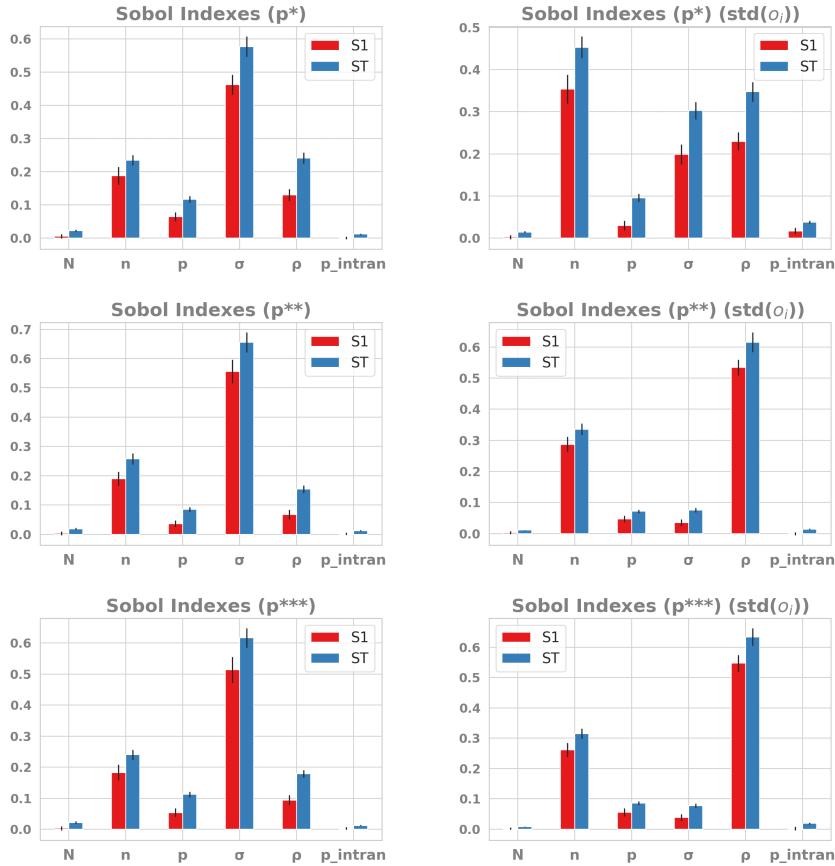


Figure 3: Sobol Indexes for the three cases ( $p^*$ ,  $p^{**}$ ,  $p^{***}$ ). First column shows the analysis for  $Y std$  and the second column, for  $S_s$ .

The sensitivity analysis in Figure 3 shows that  $\sigma$ ,  $n$ , and  $\rho$  have the most

influence on the final values of  $Y_{std}$  and  $S_s$ .  $p$  seems to still have some smaller influence and both  $N$  and  $p_{intran}$  seem to make no difference for the range of values we used. It is also interesting to notice that the same general behavior appears in the three scenarios for  $Y_{std}$ .  $\sigma$  being the parameter that explains most of the variance was expected and it is consistent with martins12b. It is interesting to see that two of the new parameters, the number of issues  $n$  and the noise  $\rho$ , also play an important role in explaining the total variance of  $Y_{std}$ .

When we look at the standard deviation of the agent's opinions,  $S_s$ , however, we see that  $\sigma$  is the most relevant parameter only for the  $p^*$  case. For  $p^{**}$  and  $p^{***}$ , however,  $\sigma$  seems to matter very little, basically as much as  $p$ . And the noise  $\rho$  seems to have most of the impact on  $S_s$ . This change is not so hard to understand. As we are speaking of how much change we observe in the standard deviations of each agent opinions  $s_i$ , there is a fundamental difference between the  $p^*$  scenario and the  $p^{**}$  and  $p^{***}$  ones. On the first one, the opinion of  $i$  in each issue does not depend on its own opinion on other issues. However, as in both  $p^{**}$  and  $p^{***}$  agent  $i$  uses its own mean to decide own much to trust other agents, it is natural  $i$  opinions will tend to a mean value. So, while for  $p^*$ ,  $s_i$  is driven by how much the opinions get spread in general and, therefore, by  $\sigma$ ,  $p^{**}$  and  $p^{***}$  scenarios suggest that the tendency to keep consistent opinions predominates and the variation in the final results is driven mostly by the noise. This is compatible with what we observed Figure 1 for the distribution of  $s_i$ . There, we had that the  $p^*$  showed it was common to observe agents with very little internal consistency, equivalent to large amounts of  $s_i$ . The tail for high  $s_i$ , however, was significantly smaller for  $p^{**}$  and  $p^{***}$ , with  $p^{***}$  even showing cases where very small  $s_i$  became more frequent than in the initial distribution.

A sensitivity analysis, however, does not show the direction of the impact. Figure 4 shows scatter plots for  $Y_{std}$  as a function of the main parameters. Each point corresponds to the outcome of one implementation of the model for the value of the parameter at the  $x$ -axis and the observed value of  $Y_{std}$ . Similar scatter plots for  $S_s$  showed similar behaviors and, therefore, are not shown here.

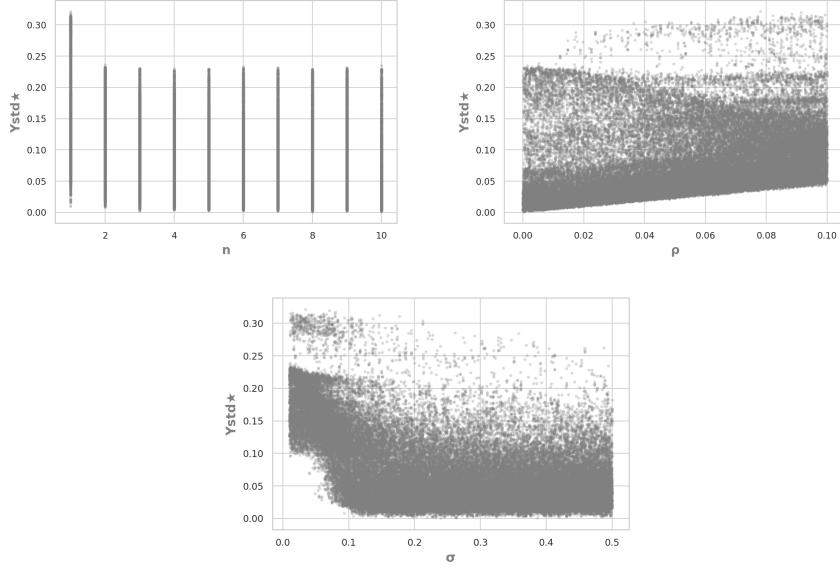


Figure 4: Scatterplots for the parameters with highest impact. Each dot corresponds to the result obtained in a single run.

The negative impact of  $\sigma$  in the population' opinion dispersion is expected: a higher  $\sigma$  means agents are easier to influence. As they are connected to all the other agents, the more uncertain they are, the more centralized the agents' mean opinions will tend to become. The plots also show that the exact value of  $\sigma$  seems to matter little above approximately 0.1. Therefore we restrict our following analysis to  $\sigma$  in the  $(0.0, 0.1]$  range. The effect of  $\rho$  is also expected: the bigger the noise, the more dispersed the final state of the system is. The number of issues  $n$  seems to have little influence unless it is  $n = 1$ . When there is only one issue, we see scenarios where the opinions clearly move away to stronger polarizations, with ( $Y_{std}$ ) around 0.3. Those cases are no longer observed as we have at least two issues. That is probably an artifact of initial conditions. As initial conditions were randomly drawn, extreme values in all issues become more and more rare with increasing number of issues  $n$  and, therefore, there are very few agents to push others to the more extreme regions.

Now we have a general picture, looking at typical behaviors for specific parameter values helps us understand the model better. We ran cases where we kept  $\rho = 0.05$ ;  $N = 500$ ;  $p\_intran = 0.0$  fixed, for 500.000 iterations, and test combinations of  $\sigma = (0.02, 0.04, 0.1)$  and  $n = (1, 5, 10)$ . The time series at Figure 5 show the time evolution of the opinions of all 500 agents for a typical run of the the  $p**$  case. Only results for  $p**$  are shown here because the time series for  $p^*$  and  $p**$  are visually almost identical to the ones in the figures. We see that larger values  $\sigma$  seem to lead opinions towards the center: the bigger the  $\sigma$  is, the more agents will tend to move closer to the mean.

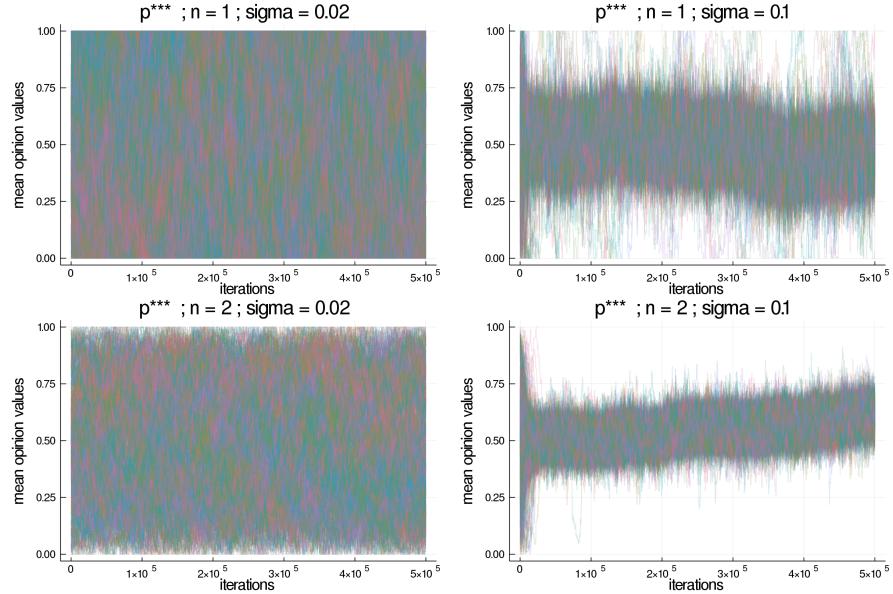


Figure 5: Time series for the parameterization:  $\rho = 0.05, N = 500, p\_intran = 0.0, n = 1$  or  $2$

Figure 6 shows a similar time evolution series for  $n = 5$  issues. Here, even for small values of  $\sigma$  we observe the opinions tend to avoid the more extreme values. As commented before, that seems to be an artifact of initial random draws. While it is easy to draw the most extreme values with only one draw, for  $n = 5$ , as we are close to the end of the range, no values outside that range are possible. So, the most extreme values require that, at the start, agents should have all their five issues drawn as extreme. As that is rare, those few that do start there tend to be attracted to still extreme positions, but a little less so. However, there actually seems to be a stronger tendency towards more central opinions. That can be observed as the increase in  $\sigma$  leads to a position closer to consensus than we observed in Figure 5 for the  $n = 1$  case.

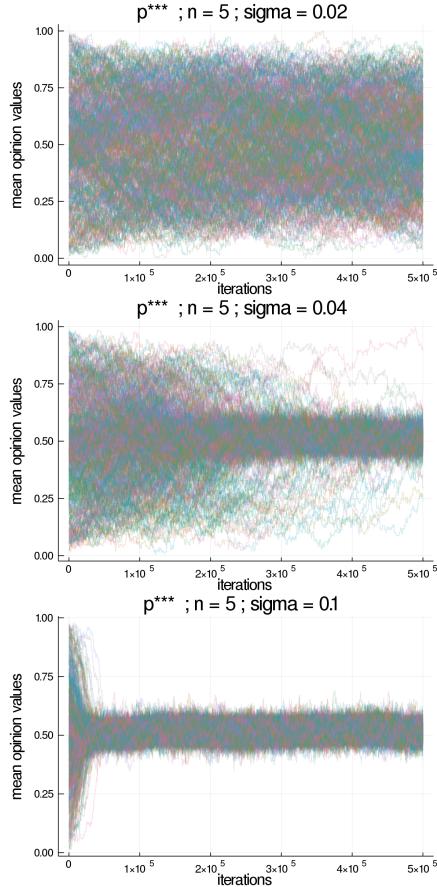


Figure 6: Time series for the parameterization:  $\rho = 0.05, N = 500, p\_intran = 0.0$

A possible reason we need to check is the fact we are measuring the mean opinion values  $x_i$ . As  $\rho$  changes a single  $o_i$  at each iteration, that means a higher  $n$  should imply a lesser impact of  $\rho$  on the mean opinion of the agent. That could explain why, for a larger  $n$ , consensus might come easier. To test if that was what was actually happening, we ran the same scenarios but increasing  $\rho$  with  $n$ . As the noises are independent, their combined effect would mean they would have their variances  $\rho^2$  added. To check that we made runs where  $\rho_2 = \sqrt{n} * \rho_1$ . The effects of increasing  $\rho$  that way in the case of Figure 6 can be seen at Figure 7. A comparison with Figure 5 shows a larger spread around the consensus, suggesting there might be other effects here other than random fluctuations.

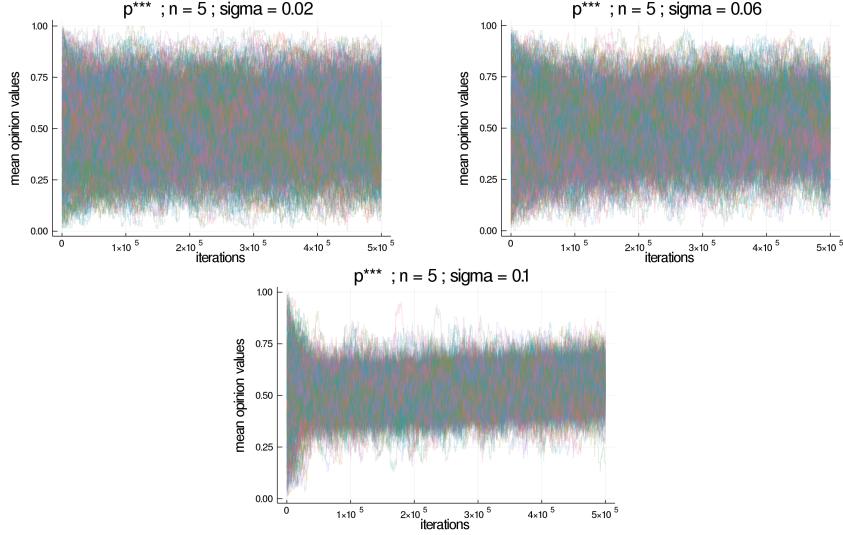


Figure 7: Time series for the parameterization:  $\rho \approx 0.12, N = 500, p\_intran = 0.0$

Heretofore we've tested parameterizations with noise. To verify the influence of the existence of noise, we also ran cases with  $\rho$  close to zero, that is  $\rho = 10^{-5}$ . The most obvious difference we can see is that the population mean opinion values converge to well defined values. In parameter combinations in which  $\sigma = 0.1$  the tendency is convergence to values close to 0.5. An interesting distinction between the cases in this parameterization is that  $p^{**}$  and  $p^{***}$  always converge to 0.5, independently of the number of issues. Alternatively, in the  $p^*$  case this happens when  $n = 1$ , but when we have  $n = 5$  or 10 there are other values of convergence, more as we increase  $n$ , even though the centralizing tendency remains.

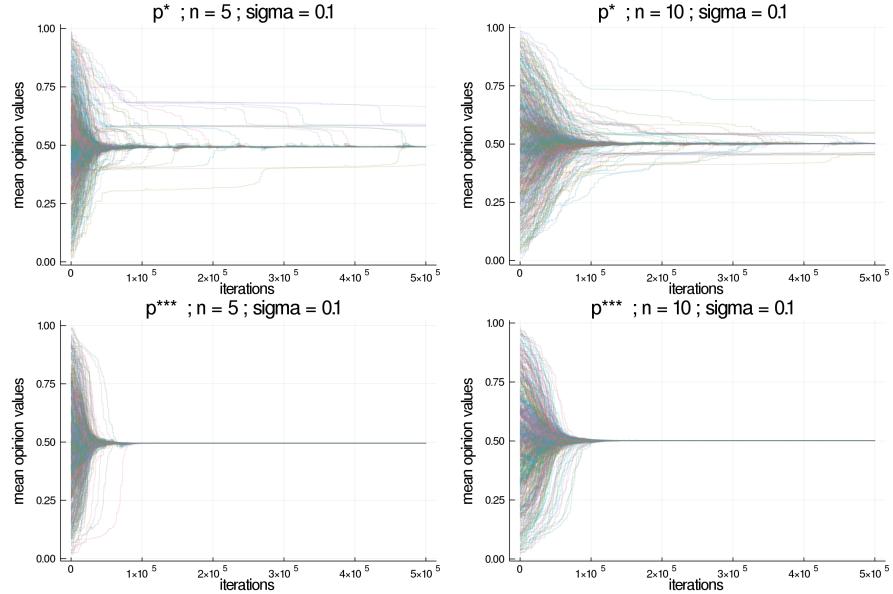


Figure 8: Time series for the parameterization:  $\rho = 1e-5$ ,  $N = 500$ ,  $p\_intran = 0.0$

When  $\sigma$  is around 0.02 or 0.04, we observe another difference between cases:  $p^*$  has more convergence values than  $p^{**}$  and  $p^{***}$ . Figure 9 illustrates this system behavior for  $\sigma = 0.02$ . We can also see that the standard deviations  $s_i$  are very different for the  $p^*$  and  $p^{***}$  cases. While for  $p^*$ , the values of  $s_i$  range from small values around 0.02 to much larger values, up to 0.2, in the  $p^{***}$  scenario we see that  $s_i$  tends to diminish with time, the majority of the observed values getting close to zero.

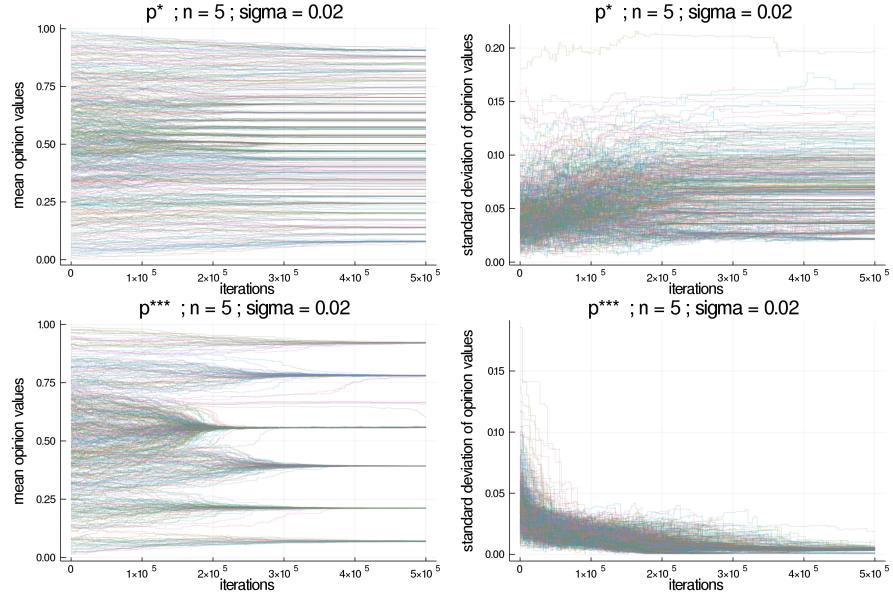


Figure 9: Time series for the parameterization:  $\rho = 1e-5$ ,  $N = 500$ ,  $p\_intran = 0.0$

The reason for that lies in how  $\Delta$  (that controls how much an agent will trust others) is calculated in each case: in the  $p^{**}$  and  $p^{***}$  cases the update rules make use of mean opinion values. That introduces a tendency for distinct issues opinions to move towards each other. The  $p^*$  update rule, however, works with single issue opinions. As the issues are uncoupled, that allows the agents to keep an ideologically more diverse profile.

Another impact of the number of issues, as shown in Figure 10, is that a higher  $n$  leads to a longer time for the convergence. The reason is that we're only changing one opinion by iteration, so naturally a higher  $n$  means the agents will take longer to be influenced. The relationship here is roughly linear such that the plot the region at  $5 \times 10^5$  iterations when  $n = 10$  is very similar to the corresponding region at  $0.5 \times 10^5$  when  $n = 1$ .

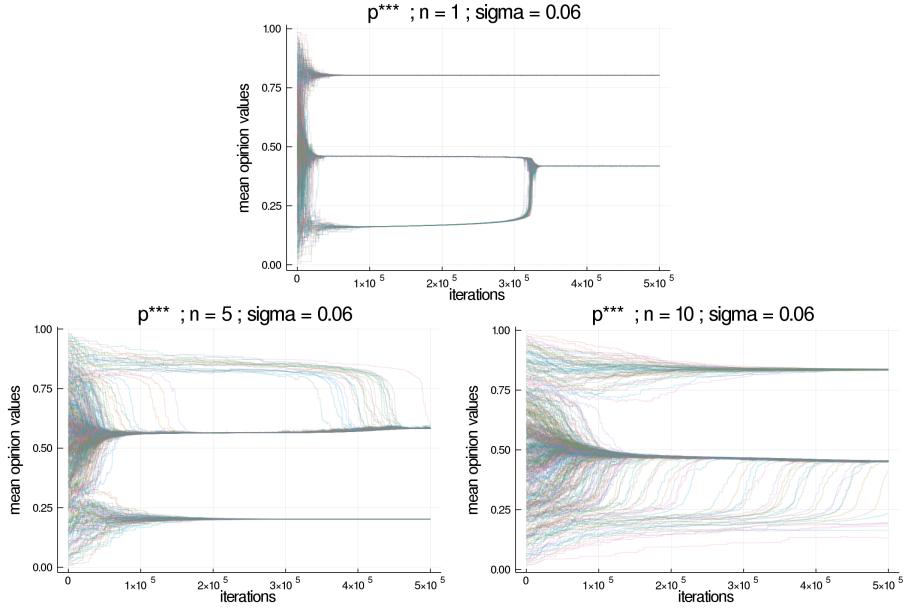


Figure 10: Time series for the parameterization:  $\rho = 1e-5$ ,  $N = 500$ ,  $p\_intran = 0.0$ .

## 4 Conclusions

Here, we have extended a previous model for continuous opinions [49] to include the case where agents have opinions on  $n$  issues. We explored the impact of the main parameters on the final results. The  $\sigma$  parameter, that determines how certain each agent is of his opinion, was observed to be a major influence in the outcome of the simulations and, therefore, plays the same qualitative role in the system dynamics that threshold values plays in Bounded Confidence models. Differently from Bounded Confidence models, the global sensitivity analysis shows that in our model its opinion certainty, and not trust, that is the main driver of the dynamics<sup>3</sup>. The number of issues  $n$  as well as the amount of noise  $\rho$  were also shown by our sensitivity analysis to have important effects on the outcomes, while the initial general amount of trust  $p$  as well as the number of agents  $N$  seemed to matter quantitatively little.

As the model is based on an update rule where opinions always move towards agreement (even if by a negligible amount), the tendency to central opinions is expected. That is especially true for the average values  $x_i$ . Since they are an average of several opinions, a strong tendency to central values is expected from simple statistical considerations. In order to see evidence of cases where the final opinions show a splitting between several final values, we do have to look at the individual opinions on each issue,  $o_{ik}$ . It is interesting to see that, at

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<sup>3</sup>However, the trust functions also matter as recapitulated below.

least for the regular trust function  $p^*$ , the final distributions for  $o_{ik}$  show that specific opinions might become quite spread over the range of possible values.

Testing different forms of trust functions also provided interesting results. A Bayesian analysis of the problem suggests the rational choice would be just comparing the opinions of the agents on the specific issue. That is, how much each agent  $i$  trusts an agent  $j$  should be a function of the distance between their opinions on the subject  $k$  they are debating, that is,  $o_{ik} - o_{jk}$ . However, as humans do show a lot of ideologically motivated reasoning [57, 59, 41], it makes sense to change how trust is calculated to a situation that is more compatible with experiments [37, 62, 66, 24]. Two possibilities were considered here. In the first one,  $p^{**}$ , the trust was dependent on the distance between the agent average estimates over all issues, that is, the trust function was a function of  $x_i - x_j$ . The second possibility,  $p^{***}$ , considered that agent  $i$  was observing  $j$  opinion only on issue  $k$  and, therefore, could only compare that value to its own average, that is the trust function was a function of  $o_{ik} - x_j$ .

Compared to the non-ideological case, both ideological trust functions showed a tendency that each agent would have its own opinions show less diversity over the set of issues. This could be easily observed by the larger proportion of smaller values of  $s_i$ . That effect was particularly stronger for  $p^{***}$ . As the issues were always treated as independent, that tendency does correspond to observing some irrational consistency [38]. The tendency to consistency was not perfect, though, as large values of  $s_i$  were also observed as much more common than those in the initial distributions. That suggests that, while the mechanism of ideological trust might play an important role in the existence of the irrational consistency effect, it probably can not account for the whole of it. Other effects, such as confirmation biases, are also probably important to describe the whole effect.

Obviously, there is still much more work to be done. The article does not intend to reproduce faithfully any specific concrete phenomenon since it's a “credible world” [71, 70] whose purpose is to investigate theoretically the relationship between parameters and system rules in an isolated artificial environment theoretically similar to the real world. We see this theoretical endeavor as part of a general research project which intends to understand concrete opinion dynamics occurring in human societies, through empirical and theoretical investigations. If we determine a specific concrete opinion system/phenomenon the model could be calibrated and validated with data and compared with other models. Furthermore, there are many possible extensions to the model. Salience, when one issue matters more than the others for an agent, is one these [34, 10]. Investigating the role of filter algorithms, bots, media, strategic behavior, in opinion dynamics are others examples [67, 21]. Adding any of those elements would contribute to the dimensionality and complexity of the model [16]. Consequently, those extensions are beyond the scope of the paper.

## 5 Acknowledgement

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## References

- [1] Daron Acemoglu and Asuman Ozdaglar. “Opinion Dynamics and Learning in Social Networks.” In: *Dynamic Games and Applications* 1.1 (Mar. 2011), pp. 3–49. DOI: 10.1007/s13235-010-0004-1. URL: <https://doi.org/10.1007/s13235-010-0004-1>.
- [2] Janet Aisbett and Greg Gibbon. “A general formulation of conceptual spaces as a meso level representation.” In: *Artificial Intelligence* 133.1-2 (2001), pp. 189–232.
- [3] Joseph W. Alba and J. Wesley Hutchinson. “Knowledge Calibration: What Consumers Know and What They Think They Know.” In: *Journal of Consumer Research* 27.2 (2000), pp. 123–156.
- [4] Giacomo Albi, Lorenzo Pareschi, and Mattia Zanella. *Opinion dynamics over complex networks: kinetic modeling and numerical methods*. 2016. URL: <https://arxiv.org/abs/1604.00421>.
- [5] Meysam Alizadeh et al. “Intergroup Conflict Escalations Lead to More Extremism.” In: *JASSS-The Journal Of Artificial Societies And Social Simulation* 14.4 (2014), p. 4.
- [6] F. Amblard and G. Deffuant. “The role of network topology on extremism propagation with the relative agreement opinion dynamics.” In: *Physica A* 343 (2004), pp. 725–738.
- [7] David A Armstrong et al. *Analyzing spatial models of choice and judgment with R*. CRC Press, 2014.
- [8] Fred Attneave. “Dimensions of similarity.” In: *The American journal of psychology* 63.4 (1950), pp. 516–556.
- [9] Chris L Baker et al. “Rational quantitative attribution of beliefs, desires and percepts in human mentalizing.” In: *Nature Human Behaviour* 1.4 (2017), p. 0064.
- [10] Chitralekha Basu. “Bridging Spatial and Saliency Theory: Party Size and Issue Selection in Campaigns.” In: *Political Science Research and Methods* (2019), pp. 1–15.
- [11] Kenneth Benoit, Michael Laver, et al. *Party policy in modern democracies*. Routledge, 2006.
- [12] C. Castellano, S. Fortunato, and V. Loreto. “Statistical Physics of social dynamics.” In: *Reviews of Modern Physics* 81 (2009), pp. 591–646.

- [13] Chloe Chaplin and Julia Shaw. “Confidently Wrong: Police Endorsement of Psycho-Legal Misconceptions.” In: *Journal of Police and Criminal Psychology* (2015), pp. 1–9.
- [14] Ryan L. Claassen and Michael J. Ensley. “Motivated Reasoning and Yard-Sign-Stealing Partisans: Mine is a Likable Rogue, Yours is a Degenerate Criminal.” In: *Political Behavior* (2015), pp. 1–19. doi: 10.1007/s11109-015-9313-9.
- [15] Abhimanyu Das, Sreenivas Gollapudi, and Kamesh Munagala. “Modeling Opinion Dynamics in Social Networks.” In: *Proceedings of the 7th ACM International Conference on Web Search and Data Mining*. WSDM ’14. New York, New York, USA: ACM, 2014, pp. 403–412. ISBN: 978-1-4503-2351-2. doi: 10.1145/2556195.2559896. URL: <http://doi.acm.org/10.1145/2556195.2559896>.
- [16] Scott De Marchi. *Computational and mathematical modeling in the social sciences*. Cambridge University Press, 2005.
- [17] G. Deffuant. “Comparing Extremism Propagation Patterns in Continuous Opinion Models.” In: *JASSS-The Journal Of Artificial Societies And Social Simulation* 9.3 (2006), p. 8.
- [18] G. Deffuant, F. Amblard, and T. Weisbuch G.and Faure. “How can extremism prevail? A study based on the relative agreement interaction model.” In: *JASSS-The Journal Of Artificial Societies And Social Simulation* 5.4 (2002), p. 1.
- [19] G. Deffuant et al. “Mixing beliefs among interacting agents.” In: *Adv. Compl. Sys.* 3 (2000), pp. 87–98.
- [20] Guillaume Deffuant et al. “How can extremism prevail? A study based on the relative agreement interaction model.” In: *Journal of artificial societies and social simulation* 5.4 (2002).
- [21] Fei Ding et al. “An evolutionary game theory model of binary opinion formation.” In: *Physica A: Statistical Mechanics and its Applications* 389.8 (2010), pp. 1745–1752.
- [22] Anthony Downs. “An economic theory of political action in a democracy.” In: *Journal of Political Economy* 65.2 (1957), pp. 135–150.
- [23] Andreas Flache et al. “Models of Social Influence: Towards the Next Frontiers.” In: *Journal of Artificial Societies and Social Simulation* 20.4 (2017), p. 2. ISSN: 1460-7425. doi: 10.18564/jasss.3521. URL: <http://jasss.soc.surrey.ac.uk/20/4/2.html>.
- [24] DJ Flynn, Brendan Nyhan, and Jason Reifler. “The nature and origins of misperceptions: Understanding false and unsupported beliefs about politics.” In: *Political Psychology* 38 (2017), pp. 127–150.
- [25] Daniel W. Franks et al. “Extremism Propagation in Social Networks with Hubs.” In: *Adaptive Behavior* 16.4 (2008), pp. 264–274.

- [26] S. Galam. “Local dynamics vs. social mechanisms: A unifying frame.” In: *Europhysics Letters* 70.6 (2005), pp. 705–711.
- [27] S. Galam, Y. Gefen, and Y. Shapir. “Sociophysics: A new approach of sociological collective behavior: Mean-behavior description of a strike.” In: *J. Math. Sociol.* 9 (1982), pp. 1–13.
- [28] S. Galam and S. Moscovici. “Towards a theory of collective phenomena: Consensus and attitude changes in groups.” In: *Eur. J. Soc. Psychol.* 21 (1991), pp. 49–74.
- [29] Serge Galam. *Sociophysics: A Physicist’s Modeling of Psycho-political Phenomena*. Springer, 2012.
- [30] Peter Gardenfors. “Conceptual spaces as a framework for knowledge representation.” In: *Mind and Matter* 2.2 (2004), pp. 9–27.
- [31] Floriana Gargiulo and Alberto Mazzoni. “Can Extremism Guarantee Pluralism?” In: *JASSS-The Journal Of Artificial Societies And Social Simulation* 11.4 (2008), p. 9.
- [32] Herbert Gintis. *Individuality and entanglement: the moral and material bases of social life*. Princeton University Press, 2016.
- [33] R. Hegselmann and U. Krause. “Opinion Dynamics and Bounded Confidence Models, Analysis and Simulation.” In: *Journal of Artificial Societies and Social Simulations* 5.3 (2002), p. 3.
- [34] Melvin J Hinich, Michael C Munger, et al. *Analytical politics*. Cambridge University Press, 1997.
- [35] Haibo Hu. “Competing opinion diffusion on social networks.” In: *Royal Society Open Science* 4.11 (2017). DOI: 10.1098/rsos.171160. eprint: <http://rsos.royalsocietypublishing.org/content/4/11/171160.full.pdf>. URL: <http://rsos.royalsocietypublishing.org/content/4/11/171160>.
- [36] Macartan Humphreys and Michael Laver. “Spatial models, cognitive metrics, and majority rule equilibria.” In: *British Journal of Political Science* 40.1 (2010), pp. 11–30.
- [37] Jennifer Jerit and Jason Barabas. “Partisan perceptual bias and the information environment.” In: *The Journal of Politics* 74.3 (2012), pp. 672–684.
- [38] Robert Jervis. *Perception and Misperception in International Politics*. Princeton University Press, 1976.
- [39] John T. Jost et al. “Political Conservatism as Motivated Social Cognition.” In: *Psychol. Bull.* 129.3 (2003), pp. 339–375.
- [40] Dan Kahan. “Fixing the communication failure.” In: *Nature* 463 (2010), pp. 296–297.
- [41] Dan M. Kahan, Hank Jenkins-Smith, and Donald Braman. “Cultural cognition of scientific consensus.” In: *Journal of Risk Research* 14 (2011), pp. 147–174.

- [42] Dan M. Kahan et al. “The polarizing impact of science literacy and numeracy on perceived climate change risks.” In: *Nature Climate Change* 2 (2012), pp. 732–735.
- [43] C. Kemp et al. “A probabilistic model of theory formation.” In: *Cognition* 114.2 (2010), pp. 165–196.
- [44] Evgenii Kurmyshev, Héctor A. Juárez, and Ricardo A. González-Silva. “Dynamics of bounded confidence opinion in heterogeneous social networks: Concord against partial antagonism.” In: *Physica A: Statistical Mechanics and its Applications* 390.16 (2011), pp. 2945–2955. ISSN: 0378-4371. DOI: <https://doi.org/10.1016/j.physa.2011.03.037>. URL: <http://www.sciencedirect.com/science/article/pii/S0378437111002706>.
- [45] Michael Laver. “Measuring policy positions in political space.” In: *Annual Review of Political Science* 17 (2014), pp. 207–223.
- [46] Michael Macy and Milena Tsvetkova. “The signal importance of noise.” In: *Sociological Methods & Research* 44.2 (2015), pp. 306–328.
- [47] André C. R. Martins. “Adaptive Probability Theory: Human Biases as an Adaptation.” Cogprint preprint at <http://cogprints.org/4377/>. 2005.
- [48] André C. R. Martins. “Bayesian updating as basis for opinion dynamics models.” In: *AIP Conf. Proc.* 1490 (2012), pp. 212–221.
- [49] André C. R. Martins. “Bayesian Updating Rules in Continuous Opinion Dynamics Models.” In: *Journal of Statistical Mechanics: Theory and Experiment* 2009.02 (2009). arXiv:0807.4972v1, P02017.
- [50] André C. R. Martins. “Continuous Opinions and Discrete Actions in Opinion Dynamics Problems.” In: *Int. J. of Mod. Phys. C* 19.4 (2008), pp. 617–624.
- [51] André C. R. Martins. *Don't choose theories: Normative inductive reasoning and the status of physical theories*. arXiv:1701.00213. 2017. URL: <https://arxiv.org/abs/1701.00213>.
- [52] André C. R. Martins. *Stop the tests: Opinion bias and statistical tests*. Arxiv preprint: 1611.06545. 2016. URL: <https://arxiv.org/abs/1611.06545>.
- [53] André C. R. Martins. “Thou shalt not take sides: Cognition, Logic and the need for changing how we believe.” In: *Frontiers in Physics* 4.7 (2016). ISSN: 2296-424X. DOI: 10.3389/fphy.2016.00007. URL: [http://www.frontiersin.org/interdisciplinary\\_physics/10.3389/fphy.2016.00007/abstract](http://www.frontiersin.org/interdisciplinary_physics/10.3389/fphy.2016.00007/abstract).
- [54] André C. R. Martins and Serge Galam. “The building up of individual inflexibility in opinion dynamics.” In: *Phys. Rev. E* 87 (2013). arXiv:1208.3290, p. 042807.
- [55] André CR Martins. “Bayesian updating rules in continuous opinion dynamics models.” In: *Journal of Statistical Mechanics: Theory and Experiment* 2009.02 (2009), P02017.

- [56] Michael D McKay, Richard J Beckman, and William J Conover. “A comparison of three methods for selecting values of input variables in the analysis of output from a computer code.” In: *Technometrics* 42.1 (2000), pp. 55–61.
- [57] Hugo Mercier. “Reasoning Serves Argumentation in Children.” In: *Cognitive Development* 26.3 (2011), pp. 177–191.
- [58] Hugo Mercier and Dan Sperber. *The Enigma of Reason*. Harvard University Press, 2017.
- [59] Hugo Mercier and Dan Sperber. “Why do humans reason? Arguments for an argumentative theory.” In: *Behavioral and Brain Sciences* 34 (2011), pp. 57–111.
- [60] Nicholas R. Miller. “The spatial model of social choice and voting.” In: *Handbook of social choice and voting*. Ed. by Jack C. Heckelman and Nicholas R. Miller. Edward Elgar, Cheltenham, 2015, pp. 163–181.
- [61] Raymond S. Nickerson. “Confirmation Bias: A Ubiquitous Phenomenon in Many Guises.” In: *Review of General Psychology* 2.2 (1998), pp. 175–220.
- [62] Brendan Nyhan and Jason Reifler. “When corrections fail: The persistence of political misperceptions.” In: *Political Behavior* 32.2 (2010), pp. 303–330.
- [63] Stuart Oskamp. “Overconfidence in case-study judgments.” In: *Journal of Consulting Psychology* 29.3 (1965), pp. 261–265.
- [64] Elinor Ostrom. “A behavioral approach to the rational choice theory of collective action: Presidential address, American Political Science Association, 1997.” In: *American political science review* 92.1 (1998), pp. 1–22.
- [65] Anna-Katharina Praetorius et al. “Judgment Confidence and Judgment Accuracy of Teachers in Judging Self-Concepts of Students.” In: *The Journal of Educational Research* 106.1 (2013), pp. 64–76. DOI: 10 . 1080 / 00220671.2012.667010.
- [66] Justin Reedy, Chris Wells, and John Gastil. “How voters become misinformed: An investigation of the emergence and consequences of false factual beliefs.” In: *Social Science Quarterly* 95.5 (2014), pp. 1399–1418.
- [67] Camille Roth. “Algorithmic distortion of informational landscapes.” In: *arXiv preprint arXiv:1907.10401* (2019).
- [68] Andrea Saltelli, Karen Chan, E Marian Scott, et al. *Sensitivity analysis*. Vol. 1. Wiley New York, 2000.
- [69] Andrea Saltelli et al. *Global sensitivity analysis: the primer*. John Wiley & Sons, 2008.
- [70] Robert Sugden. “Credible worlds, capacities and mechanisms.” In: *Erkenntnis* 70.1 (2009), pp. 3–27.

- [71] Robert Sugden. “Credible worlds: the status of theoretical models in economics.” In: *Journal of economic methodology* 7.1 (2000), pp. 1–31.
- [72] K. Sznajd-Weron and J. Sznajd. “Opinion evolution in a closed community.” In: *Int. J. Mod. Phys. C* 11 (2000), p. 1157.
- [73] Charles S. Taber and Milton Lodge. “Motivated Skepticism of Political Beliefs.” In: *American Journal of Political Science* 50.3 (2006), pp. 755–769.
- [74] Ernö Téglás et al. “Pure Reasoning in 12-Month-Old Infants as Probabilistic Inference.” In: *Science* 332.6033 (2011), pp. 1054–1059.
- [75] Guus Ten Broeke, George Van Voorn, and Arend Ligtenberg. “Which sensitivity analysis method should I use for my agent-based model?” In: *Journal of Artificial Societies and Social Simulation* 19.1 (2016), p. 5.
- [76] J. B. Tenenbaum, C. Kemp, and P. Shafto. “Theory-based Bayesian models of inductive reasoning.” In: *Inductive reasoning*. Ed. by A. Feeney and E. Heit. Cambridge University Press., 2007.
- [77] Michael Tomz and Robert P Van Houweling. “Candidate positioning and voter choice.” In: *American Political Science Review* 102.3 (2008), pp. 303–318.
- [78] Claire I. Tsai, Joshua Klayman, and Reid Hastie. “Effects of amount of information on judgment accuracy and confidence.” In: *Organizational Behavior and Human Decision Processes* 107 (2008), 97–105. DOI: 10.1016/j.obhdp.2008.01.005.
- [79] Robert P Van Houweling and Paul M Sniderman. “The political logic of a Downsian space.” In: *Institute of Governmental Studies* (2005).
- [80] R. Vicente, André C. R. Martins, and N. Caticha. “Opinion Dynamics of Learning Agents: Does Seeking Consensus Lead to Disagreement?” In: *Journal of Statistical Mechanics: Theory and Experiment* 2009 (2009). arXiv:0811.2099, P03015.
- [81] G. Weisbuch, G. Deffuant, and F. Amblard. “Persuasion Dynamics.” In: *Physica A* 353 (2005), pp. 555–575.