Some title

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Abstract

1 Introduction

Here, we will present an opinion dynamics model [1, 2, 3, 4, 5, 6, 7] where the opinions exist over an one-dimensional axis and each agent has a best estimate on more than one issue over that axis. Describing opinions over a spatial landscape is an usual way to describe policy alternatives and agents' preferences. The geometrical properties of the space are usually defined by mapping from similarity and proximity of the political agents [8, 9]. Such a description can capture the common notion of parties or policies being more "to the left" or "right". If they're similar then they're closer [10, 11]. Major opinions, including political ones, tend to be formed not from only one issue but from how each person feels about a number of them. Locating someone in a left versus right or liberal versus conservative axis, therefore, requires inspecting the opinions of that person in not only one but several issues that constitute the agent ideological positioning [12].

For many problems, it makes sense to consider different issues as having components in more than one single dimension [13]. However, it is often the case that describing the problem as one-dimensional can be justified. We can certainly see this as a first approximation along the most relevant dimension. In that case, the one dimensional case is just a projection along a direction where variation seems especially important of higher-dimensional problems. From an application point of view, it is usual to find discussions to be simplified over only one main disagreement.

One traditional way to model this type of scenario is to use continuous opinions over a fixed interval, as it is done in the Bounded Confidence (BC) models [6, 14]. While discrete models [3, 4, 5] can be very useful at describing choices, they are not easiest way to represent strength of opinion unless a continuous variable is associated to the choice [7]. Discrete models also tend to lack a scale where we can compare opinions and decide which one is more conservative or more liberal.

On the other hand, continuous models are not particularly well suited for problems involving discrete decisions. As we will not deal with those kinds of problems here, they are a natural choice. Indeed, continuous opinions models have been proposed for several different problems on how opinions spread on a society [15, 16], from questions about the spread of extremism [17, 18, 19, 20, 21, 22] to other issues such as how different networks [23, 24, 25, 26] or the uncertainty of each agent [27] might change how agents influence each other.

Here, we will use a continuous opinion model created by Bayesian-like reasoning [28], inspired by the Continuous Opinions and Discrete Actions (CODA) model [7, 29]. That model was shown previously [28] to provide the same qualitative results as BC models. While a little less simple, the Bayesian basis makes for a clearer interpretation of the meaning of the variables. That makes extending the model and interpreting new results simple. That approach is also consistent with a bounded rationality variant interpretation of the spatial model of political decision making [30, 31].

Variations of how each agent estimate how trustworthy other agents are will also be introduced. Here, we use a function of trust p^* that plays a role similar to the threshold value in BC models. While p^* is not a simple discontinuous cut-off, it is a function of the distance between the opinions of the agent and the neighbor. If only one issue is debated, that distance is uniquely defined. However, if we have multiple issues represented on the same one-dimensional line, it is not clear which distance we should use. That happens because we have the opinion of agent i on a specific issue o_i and we can compare it to o_j to estimate how much i trusts j. However, as there are several issues we could also use the mean opinion of i over all its issues and the same goes for j. Therefore, we will study two additional cases: in the first alternative, we will change p^* to p^{**}, determined by the distance between the neighbor and the agent average opinions. In the second alternative, we will use a p^{***} calculated from the opinion of the neighbor and the average opinion of the agent. The idea here is to make the behavior of our agents closer to what experiments show about human reasoning. We have observed that our reasoning about political problems can be better described as ideologically motivated [32, 33, 34]. Indeed, our opinions tend to come in blocks even when the issues are logically independent [35]. Our reasoning abilities seem to exist more to defend our main point of views [36, 37] and our cultural identity [38] than to find the best answer. In that context, evaluating others by how they differ from us as a whole, instead of in each issue, is a model variation worth exploring.

2 The Model

Here, the population will consist of N agents fully connected (an agent i can interact with any other agent j). Each agent i will have an opinion $0 \leq o_{i,k} \geq 1$, where $k = 1, \ldots, n$ is a specific issue. We assume each agent opinion about issue k can represented as a value $o_{i,k}$ at the range of possible values for os. Agents also have an uncertainty σ associated to their average estimate o. The uncertainty σ could be different for each agent and also updated during the interactions [28]. For the sake of simplicity, however, we will assume the uncertainty $\sigma_i = \sigma$ is identical for (almost) all agents and it does not change. The set of opinions for each agent on all possible issues will define its ideological profile $I_i = ((o_{i,1}, \sigma), \ldots, (o_{i,n}, \sigma))$, where n is the number of issues, o is the opinion about the issue and σ is the global uncertainty [29]. The arithmetic mean x_i of agent i opinions in each issue will be called here the ideal point for each agent, that is, it defines the agent ideological position at the dimension of interest [39]. Obviously $x_i = \frac{1}{n}\sum_{k=1}^{n} o_k$.

In order to have agents with initial ideal points well distributed over the possible range, the initial valued for each $o_{i,k}$ was randomly drawn using a Beta $Be(\alpha, \beta)$ with random parameters α_i and β_i. Those parameters were drawn for each agent i from the ranges $(\alpha \in [1.1, 100], \beta \in [1.1, 100])$ by dividing those ranges by N equally spaced values and then assigning them to each agent after mixing the values. That is, if we have 10 agents one element of the following list of possible pairs [(1.1, 56.0444), (23.0778, 34.0667), (45.0556, 100.0), (78.0222, 67.0333), (89.0111, 23.0778), (67.0333, 12.0889), (100.0, 78.0222), (56.0444, 1.1), (34.0667, 89.0111), (12.0889, 45.0556)] is assigned to each agent i. As a $Be(\alpha, \beta)$ has an expected value of $\frac{\alpha}{\alpha+\beta}$, that allows agents to start each simulation with quite diverse value of ideal points. That also means the initial $o_{i,k}$s of each agent i will tend to be correlated.

For its part, σ is a parameter of the model, not the agents. However, a proportion *p_intran* of the agents will have an unique $\sigma_{i,k} = 1e - 20$. This is done so that we can control for the impact of inflexible agents on the model dynamics [40, 41, 42].

How many agents are intransigent is also a parameter (coded as *p_intran*), and such σ is established at the initial condition by sampling the issue index from the I_i's length. This means that we've opted for an implementation in which agents are intransigents in a *single* issue.

Each iteration of the simulation is the application of two procedures: the opinion update through social influence and a random opinion update (noise). In the social influence procedure we draw two agents, i and j, from the population, where i will observe j opinion about a specific issue $k \in (1, \ldots, n)$. The issue k is randomly drawn over its possible its possible n values. That way, we have the corresponding pairs $(o_{i,k}, o_{j,k})$ and $(\sigma_{i,k}, \sigma_{j,k})$. Finally, the agent i updates its opinion $(o_{i,k})$ following a Bayesian rule: each agent has a Normal prior $f_i(\theta) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(\theta - o_i)^2}{2\sigma_i}}$; for the likelihood there is an initial chance p, updated to p^* that the agent j has information and a chance $1-p$ it knows nothing and its opinion is just a random non-informative draw which leads to $f(o_j|\theta) = pN(\theta, \sigma_j^2) + (1-p)U(0,1)$; from the expected value of the multiplication of the prior with the mixture likelihood we derive the update rule [29]:

$$o_{i,k}(t+1) = p^* \frac{o_{i,k}(t) + o_{j,k}(t)}{2} + (1 - p^*) o_{i,k}(t).$$

Wherein

$$p^* = \frac{p\frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{(\Delta_{ij})^2}{2\sigma_i^2}}}{p\frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{(\Delta_{ij})^2}{2\sigma_i^2}} + (1-p)}.$$

The Δ_{ij} term is equal to $o_{i,k}(t) - o_{j,k}(t)$. As mentioned, we also test cases in which it's equal to $x_i(t) - x_j(t)$ (the p^{**} case) and to $x_i(t) - o_{j,k}(t)$ (the p^{***}). p, for its part, is a global parameter used to model the likelihood of the other agent's (j) opinion being true [29].

Furthermore, there is the noise: we draw another agent i whose opinion $o_{i,k}(t+1)$ is equal to $o_{i,k}(t) + r$ where r is taken from a Normal distribution of mean 0 and standard deviation ρ. ρ is then a global parameter of the simulation. From a theoretical point of view the noise is justified as a way of accounting for the effect of factors not related to social influence that make the agents change their opinion about issues [43]. A further methodological justification is that small perturbations in the local behavior of agents may lead to drastic changes in systemic properties [44]. If an agent i is intransigent in an issue k it won't randomly change its $o_{i,k}$ opinion if its chosen by the noise algorithm. Moreover, if $o_{i,k}(t) + r > 1$ then $o_{i,k}(t+1) = 1$. Likewise, if $o_{i,k}(t) + r < 0$ then $o_{i,k}(t+1) = 0$.

3 Model Results

To have a general understanding about the model behavior we first established the bounds of the parameters:

σ	n	p	p_intran	N	ρ
$[0.01, 0.5]$	$[1, 10]$	$[0.1, 0.99]$	$[0.0, 0.1]$	$[500, 5000]$	$(0.0, 0.1]$

Table 1: Parameters' Bounds

To sweep the parameter space we sample 70.000 parameterizations taken from quasi-random low-discrepancy sequences [45], that generate evenly spaced points. After running the simulation for 1.000.000 iterations we take the standard deviation of the population mean opinions ($Y std$) as a system final state measure. Histograms of the initial condition viz-a-viz the three cases (p^*, p^{**}, p^{***}) final state lets us understand the general tendency impinged by the update rules:

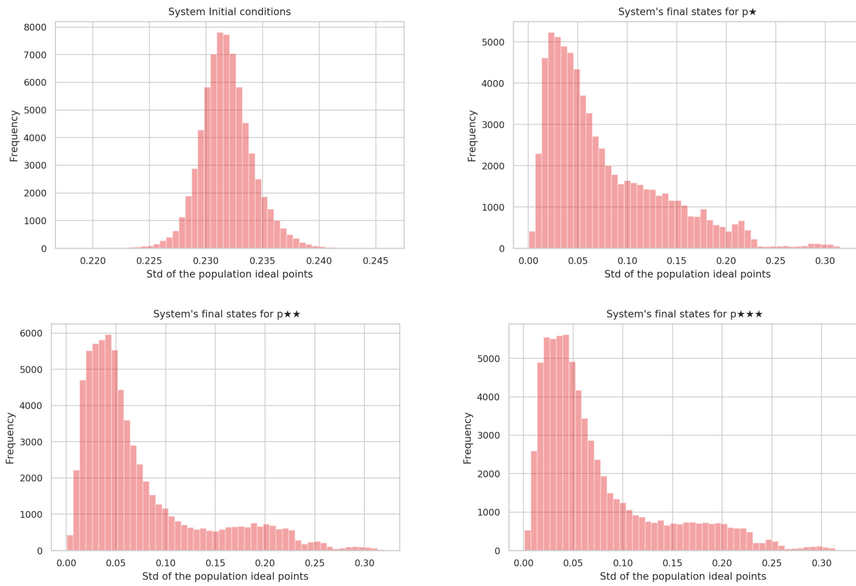

Figure 1: System initial condition x final state

The general tendency of the model is one of biased assimilation [43]. The histograms, however, don't show which parameter is the most important to explain this trend. With that in mind we perform a Sobol sensitivity analysis [46] which generate the following indexes:

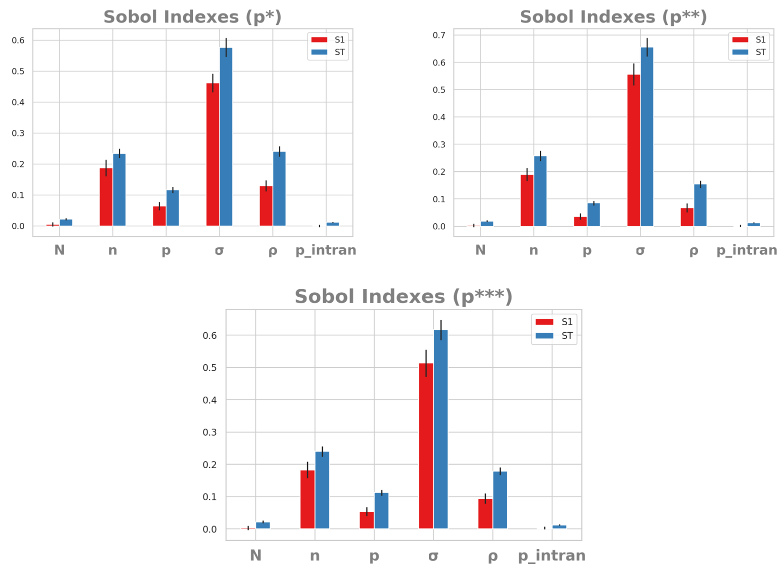

Figure 2: Sobol Indexes for the three cases (p^{*}, p^{**}, p^{***})

The sensitivity analysis shows that the most important parameters are : σ, n, ρ, and that the three cases have the same qualitative behavior. σ being the parameter that explains the most the variance of the system measure is consistent with [29], while the relevance of the number of issues and the noise is a new result. The sensitivity analysis, however, does not show the direction of the impact, which we investigate through scatter plots:

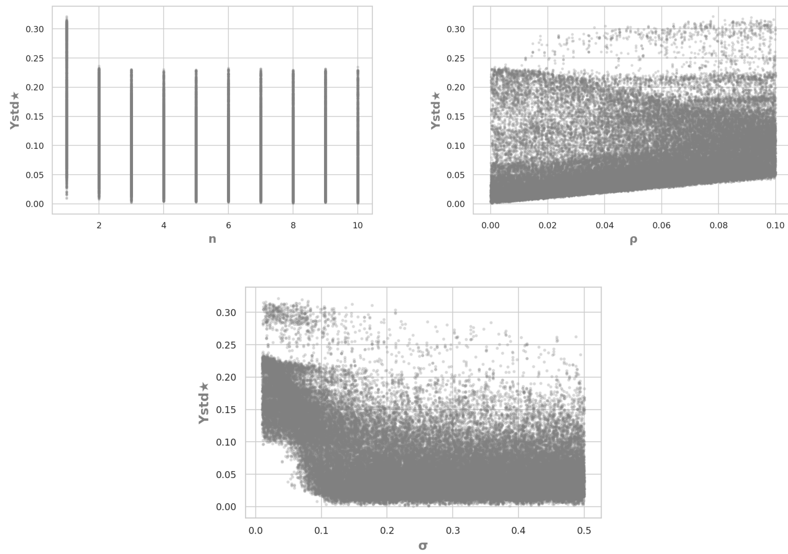

Figure 3: Scatterplots for the parameters with highest impact

The negative impact of σ in the population' opinion dispersion is to be expected given the update rule: a higher σ means that agents are easier to influence and as they're connected to all the others the more uncertain they the more centralized the agents' mean opinions after they interact with their neighbors. The plots also show that σ has the same impact on the system whenever its bigger than approximately 0.1.Therefore we restrict our following analysis to the $(0.0, 0.1]$ range. The effect of ρ is also expected: the bigger the noise more dispersed is the final state of the system.

After a general investigation we turn to the analysis of specific parametizations. For that let's start by fixing the following parameters as constants: $\rho = 0.05$; $N = 500$; $p\_intran = 0.0$, run the simulation for 500.000 iterations, and test combinations of $\sigma = (0.02, 0.04, 0.1)$ and $n = (1, 5)$. As show by 4, σ has the effect of leading the dynamics of the simulation to the center as it increases: the bigger the σ the closest to the mean the population is . However,when the number of issues also increases that tendency is not clear, since the noise disperses them, even though σ has a bigger impact:

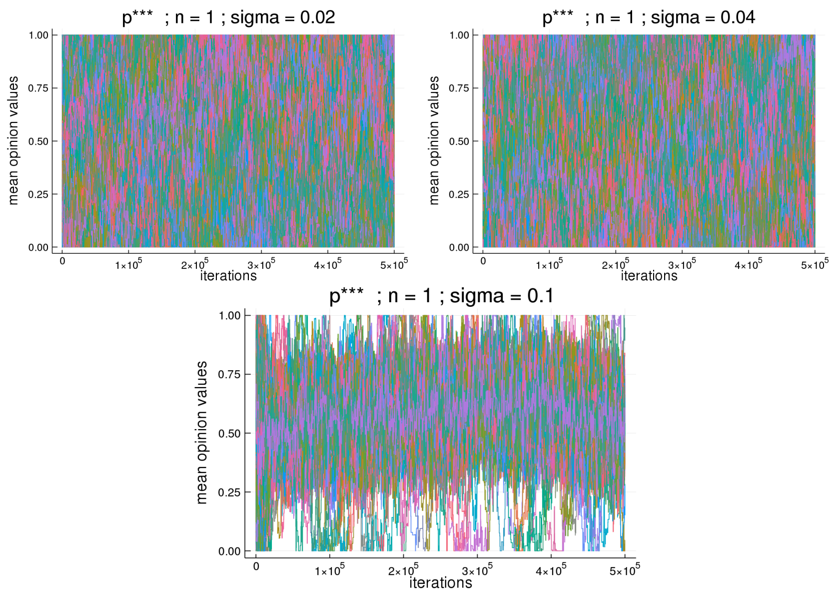

Figure 4: Time series for the parameterization: $\rho = 0.05, N = 500, p\_intran = 0.0, n = 1$

On the other hand, when we also increase the number of issues the centralizing effect of σ becomes stronger:

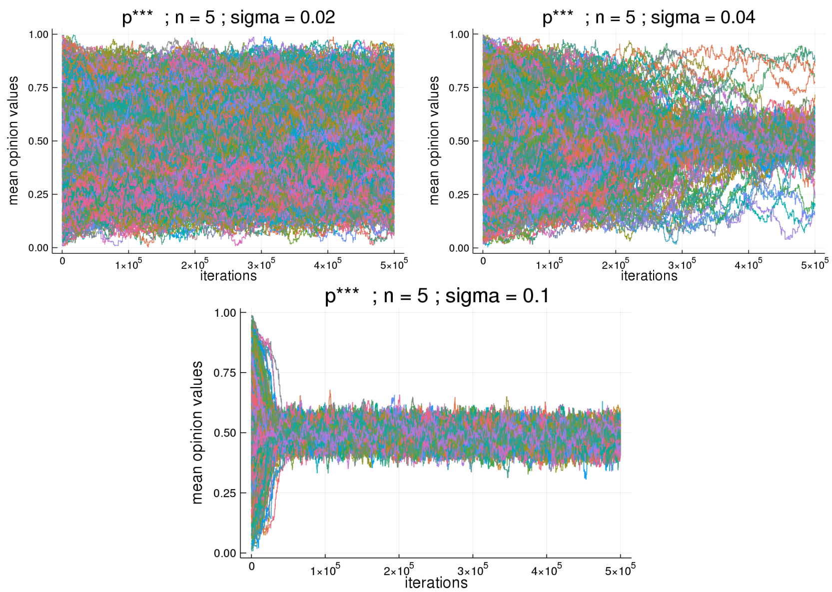

Figure 5: Time series for the parameterization: $\rho = 0.05, N = 500, p\_intran = 0.0$

The reason for that is : we're measuring the mean opinion values (x_i) and ρ changes a single o_i at each iteration which means that a higher n implies a lesser impact of ρ on the mean opinion of the agent, since she will have $n-1$ other opinions stabilizing her mean opinion at a point in the opinion spectrum. As σ is the parameter that dominates the model update rule it interacts with n, which enforces σ effect whenever we test higher ns. This effect holds even when we raise ρ together with n. Let's use the same parameterization as the last plot but with a new ρ such that $\rho_2 = \sqrt{n} * \rho_1 = \sqrt{10} * 0.05$. The interaction between n and σ still happens, with bigger n stabilizing the noise effect and contributing to the centralizing effect of σ, even though a bigger ρ leads to more noise around the mean.

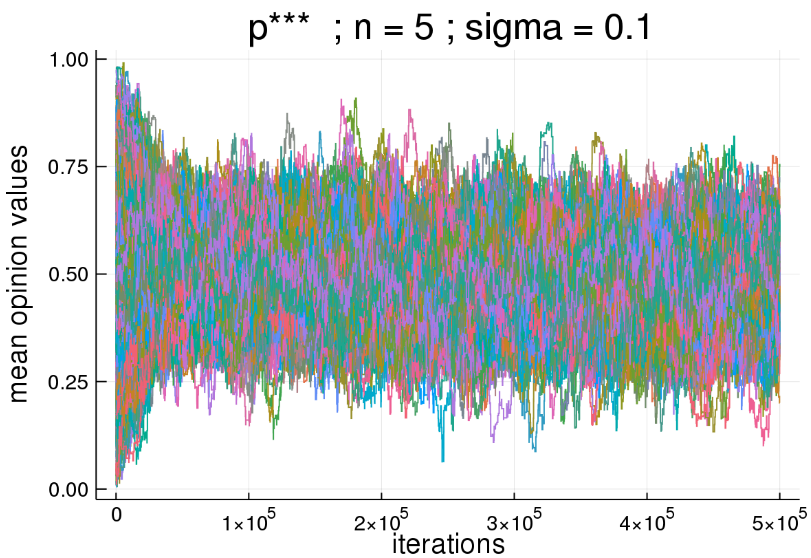

Figure 6: Time series for the parameterization: $\rho \approx 0.12, N = 500, p\_intran = 0.0$

Heretofore we've tested parameterizations with noise, but what happens if we lower ρ to a value close to zero, such as 1e-5? The first difference is that the population mean opinion values converge to certain values. In parameter combinations in which $\sigma = 0.1$ the tendency is convergence to values close to 0.5. An interesting distinction between the cases in this parameterization is that p^{**} and p^{***} always converge to 0.5, independently of the number of issues. Alternatively, in the p^{*} case this happens when $n = 1$, but when we have $n = 5$ or 10 there are other values of convergence, more as we increase n, even though the centralizing tendency remains.

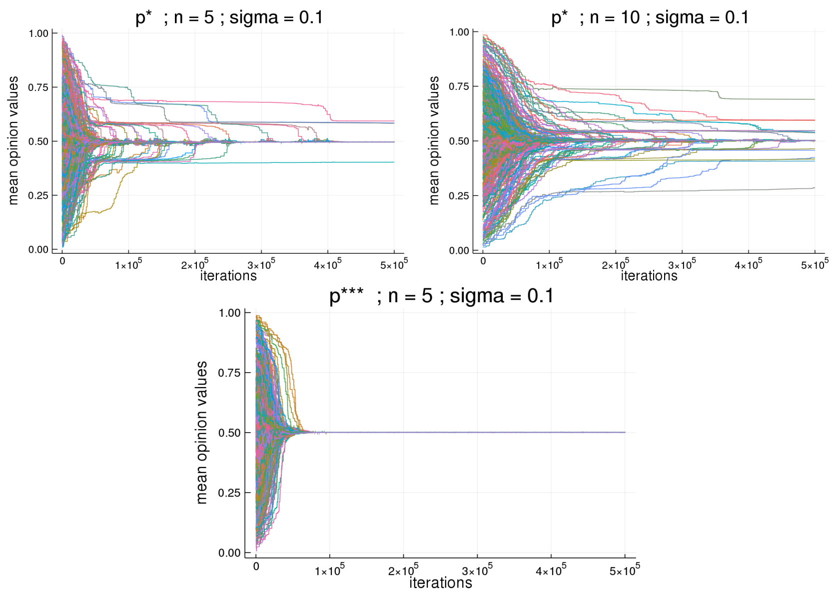

Figure 7: Time series for the parameterization: $\rho = 1e-5, N = 500, p\_intran = 0.0$

In the parameterization in which σ is of intermediate value (for our range), such as 0.02 or 0.04, we observe another difference between cases: p^* has more convergence values than p^{**} and p^{***}. Figure 8 illustrates this system behavior:

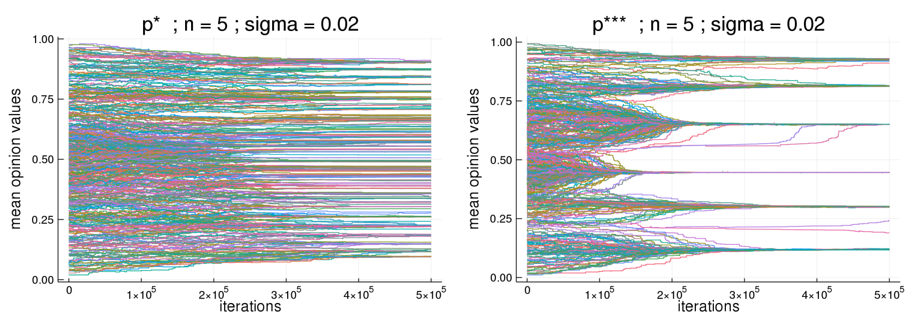

Figure 8: Time series for the parameterization: $\rho = 1e-5, N = 500, p\_intran = 0.0$

The reason for that lies in the Δ of each case: in the p^{**} and p^{***} cases the update rules make use of mean opinion values which facilitates the opinion convergence. The p^* update rule works with single issue opinions which opens the possibility that there is little influence between agents given their ideological distance at the issue.

Another impact of the number of issues, as shown in Figure 9, is that a higher

n leads to a longer time for the convergence to certain values. The reason is that we're only changing one opinion by iteration, so naturally a higher n means the agents will take longer to be influenced. The relationship here is roughly linear such that the plot the region at 5×10^5 iterations when $n = 10$ is very similar to the corresponding region at 0.5×10^5 when $n = 1$.

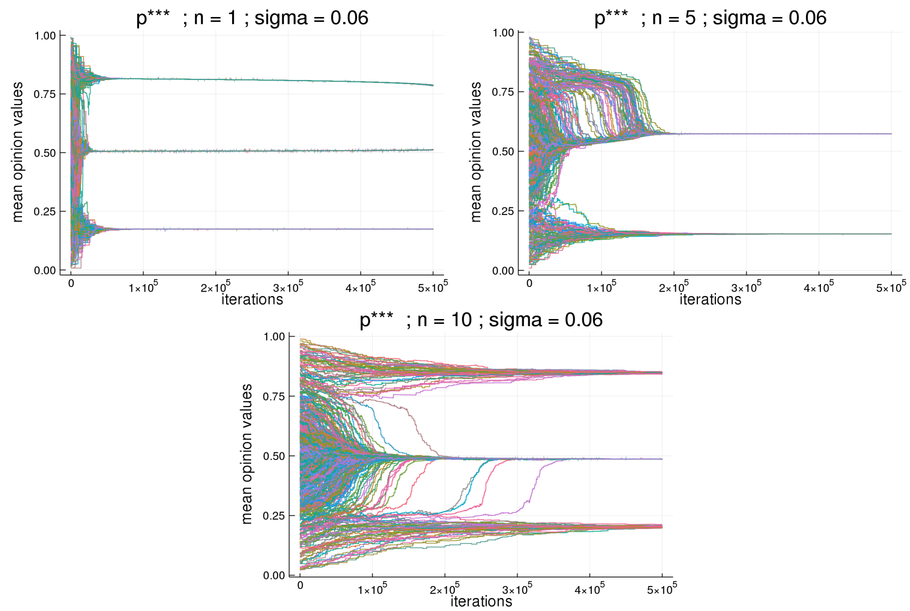

Figure 9: Time series for the parameterization: $\rho = 1e-5, N = 500, p\_intran = 0.0$.

Should I talk about intransigents with $\sigma = 0.01$???

4 Conclusions

5 Acknowledgement

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