Working Paper

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1 Introduction

Democracies might not elect democrats. In recent years, countries as diverse as Hungary, Brazil, or even the USA have witnessed the election of closet or wannabe authoritarians (Lührmann et al. 2018; Chiopris, Nalepa, and Vanberg 2021). After being elected, such politicians typically adopt a set of autocratization strategies that test the democratic robustness of the political system (Sørensen and Ansell 2021; Bermeo 2016). Their election is, thus, a critical juncture for a polity (Thelen 2009). The centrality of the electoral moment has lead a strand of the literature in collective choice to wonder whether the success of wannabe autocrats is an artifact of the current electoral systems (Potthoff and Munger 2021; Kurrild-Klitgaard 2018; Woon et al. 2020). This paper shows that this hypothesis is not true in the case of the Brazilian presidential election of 2018. Bolsonaro was the Condorcet Winner, Borda Winner, and would have won under most of the positional voting procedures. Therefore, his victory was based upon a solid mandate (Tabarrok 2001).

Though it is debatable whether we can characterize the aforementioned cases as instances of a broader autocratization wave (Cassani and Tomini 2020; Waldner and Lust 2018), Bolsonaro's autocratic credentials are unmistakable. Even a restricted democratic minimalist assessment criteria would single out, for instance, his accusations against the integrity of the electoral system, the attacks on the Supreme Court and the illegal electoral maneuvers between the presidential two-rounds in 2022 as manifest anti-democratic postures (Przeworski, Shapiro, and Hacker-Cordon 1999; Schmitter 1994).

Despite the multiple historical contingencies that might explain his victory, one might naturally wonder what role the electoral system has had in it. After all, it is well-known that the outcome of collective choices is fundamentally dependent on the voting procedure (Riker 1982). As Brazil's majority with run-off system, most electoral systems use merely the first index of the voters' preference rankings (Grofman and Feld 2004) . How would he have fared under informationally richer voting procedures, such as pairwise comparison methods and positional voting procedures? Was he, and arguably other democratically elected wannabe-autocrats, a product of decision procedures that favor divisive candidates over more inclusive ones (Igersheim et al. 2022)? Would the result have changed with a different set of candidates?

To answer those questions, I make use of a pre-electoral representative survey to reconstruct the full 4-top rankings of the Brazilian population, using a bayesian preference learning model (Sørensen et al. 2019). Subsequently, I use this augmented data to simulate the electoral results under both the Borda and Condorcet procedures, and simulate the result for all positional procedures in 3 candidate sets. Finally, I discuss the significance of the results and conclude by pointing out the limitations of the endeavor.

2 Theory

It is non-trivial to trace alternative, counterfactual, political scenarios, inasmuch the political system is a complex web of agents, situations, discourses and institutions (Bednar 2018; Simon 1991; Parsons 1949; V. Ostrom 1997). However, institutions stand out among

those components as a locus of attention. Not only do they matter, as North (1991) argued, but they can be designed (Coleman 1990). Varying part(s) of the institutional matrix gives us insight into what possible adjacent paths can or could have ensued (North 2018). Nevertheless, focusing primarily on the institutions still leaves us facing a system of rule combinations (E. Ostrom 1986). Sticking to "close" counterfactuals (Menzel 2021; King and Zeng 2006) is one way of reducing the analysis's complexity while improving our inferences' reasonableness.

Within the subset of polyarchical systems, elections are consistently enumerated as central to their functioning (Riker 1982). No wonder ever since the inception of the representative government, opening the black box of the input-output process performed by voting mechanisms has emerged as a recurrent preoccupation of politologists(Rothschild 2005; McLean 2014). Their centrality is composed of the realization that the result of collective decisions is inseparable from the decision procedure being used and that such procedures differ in terms of their consistency with normative evaluative criteria (D. Saari 2001; Sen 2012). Voting procedures have, thus, a privileged position within the institutional matrix of a polity, and my analysis will focus on their role in a the critical moment of the Brazilian polity (Kaminski, Lissowski, and Swistak 1998; Kamiński 1999; Tabarrok and Spector 1999).

If the target system of interest had bestowed complexity upon us, now the theory emerges as a new source of complexity. There are dozens, maybe hundreds, of different voting procedures with distinct properties (Pacuit 2019; Felsenthal and Nurmi 2018; Dan S Felsenthal and Machover 2012). Majority rule, however, is a centerpiece of the democratic credo which underlies contemporary polyarchical systems (Dahl 1989). Nevertheless, its shortcomings are well understood, at least since the founding of the formal study of voting procedures by Condorcet and Borda. Their responses to the question of how can one then extend majoritarianism to more than two alternatives constitute the foundation of the two main broad classes of decision procedures: pairwise voting methods and positional voting methods.

Condorcet extended the majority rule to pairwise majority rule: apply majority rule to all pairwise comparisons. One possible and, particularly strong, condition that generalizes majoritarianism is the Condorcet criterion: a a decision procedure is Condorcet consistent if it selects the candidate if there is any that wins in all pairwise majority contests, the Condorcet Winner (CW) (Dan S. Felsenthal and Machover 1992). Borda, on the other hand, devised a scoring scheme: if there are say 3 alternatives $\{A, B, C\}$ and an agent i has ranking $B > C > A$ then the Borda score in i's ranking for each alternative is $A : B : C = 0 : 2 : 1$[1]. The Borda score for the whole profile, all voters rankings, is the sum of each alternative score at each voter ranking. The Borda Winner (BW) is the alternative with highest score. Underlying this scoring system, there is also a latent generalization of majority rule: the Borda score is equivalent to summing over the pairwise votes the alternative gets in a pairwise tournament (Nurmi 2002).

The notion of a mandate of a candidate lets us situate both methods vis-à-vis the one-choice informational environment typical of large-scale majoritarian elections. At a minimum, a candidate has a "mandate" as long as it has won under the voting procedure. A candidate has more mandate, however, the more significant the difference between its vote share or score vs. the second most well-voted candidate. Note that being a CW is an even more robust notion of having a mandate: if the candidate is a CW, then it would have won under all possible majority pairwise comparisons against the other candidates. The pairwise implementation of the Borda Count lends itself to a similar interpretation. Suppose a candidate wins under a voting procedure that only uses the top choice of the

1. Alternatively, it can be coded as $1 : 3 : 2$, or any scoring in which the difference between pair of subsequent scores is constant throughout the scoring vector (D. G. Saari 2012) .

electorate, but is neither a BW nor a CW. In that case, it has less mandate, in this generalized majoritarian perspective, than if it is both, which signals a comprehensive social basis. In the former case, a candidate who wins under the current voting procedure, but is neither a BW nor a CW could be considered an artifact of the procedure. In the latter case, on the other hand, the procedure would be just "tracking" a broader pattern of support for the alternative.

This notion of mandate can be strengthened in the case of the Borda Count. The Borda count can be seen as one method within a family of methods that assign weights to positions in the ballot. In one extreme, the plurality voting method assigns score 1 to the top choice and 0 to all others. On the other extreme is the antiplurality voting method, which assigns 1 to all positions besides the last one. Between the two extremes are all possible ways of assigning a score to the ballots of the electorate. The higher the proportion of positional voting systems that the candidate would have won had the election used it, what Tabarrok (2001) has called positional stability, the higher the mandate of the candidate.

Focusing on the Condorcet Criterion and the positional voting methods family also provide information lost by relying only on the top choice on how divided the population is between the candidates. Igersheim et al. (2022) argue that voting procedures that rely only on the top choice bias the voting system in favor of what they call divisive candidates: they receive a high proportion of first choices, but are also highly rejected, receiving a high proportion of bottom positions in the citizens' preferences. In contrast, inclusive candidates would be the ones that receive wide support, but not necessarily high top nor bottom ranking positions in the electorate. However, if the voting procedure only uses the top choice, then it is throwing away this distribution of support. In a context of rising social/political polarization this amounts to the political system positively reinforcing the ongoing conflict, and potentially destabilizing, dynamics (Svolik 2018; Shi et al. 2017; Baldassarri and Page 2021; Axelrod, Daymude, and Forrest 2021).

This broader informational backdrop underlies current research on the case of the United States and Donald Trump's electoral victory. Potthoff and Munger (2021), Woon et al. (2020), and Kurrild-Klitgaard (2018) debate whether Donald Trump was a Condorcet Winner in the primaries. Igersheim et al. (2022) goes a step further: not only was he not, they argue that Sanders was the actual Borda and Condorcet Winner, and generally the "best" candidate, if by best one understand a candidate being the most inclusive and winning under the most alternative decision procedures. We will see, however, that no such conclusion can be drawn in the Brazilian case.

3 Case/Data

Jair Messias Bolsonaro was elected the president of Brazil in 2018. For more than 20 years as a congressman, he defended the interests of the military and police forces primarily. The 2018 electoral scenario in Brazil was one of high rejection of the traditional political elite, particularly of the Labour Party (Partido dos Trabalhadores - PT), after corruption scandals and an impeachment process of the previous president, Dilma Roussef, a Labor politician. The main contestants, among 13, were him, a rightist candidate; Fernando Haddad, a leftist candidate from PT; Geraldo Alckmin, a center-right candidate; and Ciro Gomes, a center-left candidate. The presidential election in Brazil follows a two-round system. In the first round, 8.79% of the votes were White/Null, which means the voting procedure does not count them. The result of the valid votes was the following: Bolsonaro:Haddad:Ciro:Alckmin:Others = $46.3 : 29.28 : 12.47 : 4.76 : 7.19$. Among the 9 other candidates, the highest vote share was João Amoêdo's with 2.5%. All others got less than 1%. Moreover, There was a 20% abstention. In the second round the result

Number of Pairwise Comparisons	Frequency
1	15
2	42
3	462
4	118
5	503
6	1797

Table 1: Frequency of pairwise comparisons in the dataset.

was: Bolsonaro:Haddad = 55.12% : 44.78%. White/Null votes were 9.57% of the total electorate. The abstention in this round was 21.3%.

The dataset used for the analysis is a representative street survey done on 10/02/2018, less than a week before the first round (10/07/2018). DataFolha did this survey, an independent research institute highly esteemed and trusted by brazilian experts[2]. One question, in particular, is the only variable in our analysis: pairwise comparisons between the 4 top candidates. With it is possible to reconstruct the full 4-top ranking of the voter. Preliminary pre-processing has led me to drop 171 observations where all pairwise comparisons were missing and 132 in which they were cyclic. This leaves us with 2937 out of 3240 observations. As Table 1 shows only 1797 observations compared all 4 candidates. As such, we have to augment the data with transitive closures for 1140 observations, by methods discussed in the next section.

4 Methods

I use a Bayesian Mallows model to infer the missing rankings (Sørensen et al. 2019). The model has the following mathematical form:

- PDF for $\mathbf{r} \in \mathcal{P}_n$: $P(\mathbf{r} \mid \alpha, \rho) = \frac{1}{Z_n(\alpha)} \exp\left[-\frac{\alpha}{n} d(\mathbf{r}, \boldsymbol{\rho})\right] 1_{\mathcal{P}_n}(\mathbf{r})$
- Likelihood : $P\left(\mathbf{R}_1, \ldots, \mathbf{R}_N \mid \alpha, \boldsymbol{\rho}\right) = \frac{1}{Z_n(\alpha)^N} \exp\left[-\frac{\alpha}{n} \sum_{j=1}^{N} d\left(\mathbf{R}_j, \boldsymbol{\rho}\right)\right] \prod_{j=1}^{N} 1_{\mathcal{P}_n}\left(\mathbf{R}_j\right)$
- Prior for α: $\pi(\alpha \mid \lambda) = \frac{\lambda \exp(-\lambda\alpha) 1_{[0,\alpha_{\max}]}(\alpha)}{1-\exp(-\lambda\alpha_{\max})}$
- Uniform prior for ρ
- Posterior:

$$P\left(\alpha, \rho \mid \mathbf{R}_1, \ldots, \mathbf{R}_N\right) \propto \frac{1_{\mathcal{P}_n}(\boldsymbol{\rho})}{Z_n(\alpha)^N} \exp\left[-\frac{\alpha}{n} \sum_{j=1}^{N} d\left(\mathbf{R}_j, \rho\right) - \lambda\alpha\right] 1_{[0,\alpha_{\max}]}(\alpha).$$

Where $\rho \in \mathcal{P}_n$ is a location parameter representing the consensus ranking, $\alpha \geq 0$ is a scale parameter , $Z_n(\alpha)$ is a normalizing function , $d(\cdot, \cdot)$ is a right invariant distance among rankings , $1_{\mathcal{P}_n}(\mathbf{r})$ is an indicator function for the set $\mathcal{P}_n$ which equals one when $\mathbf{r} \in \mathcal{P}_n$ and zero otherwise; and $\mathbf{R}_j = (R_{1j}, \ldots, R_{nj})$ is the ranking for voter $j, j = 1, \ldots, N$ (Sørensen et al. 2019). The inferred rankings are sampled from the numerically computed posterior distribution. After the rankings are inferred, I compute the Borda Count and Condorcet Procedure for the top 4 candidates and visualize the results for 3 candidates using Saari's representation triangles[3] (D. G. Saari 2012).

2. I had access to the survey data, code-book and questionnaire by creating an account and requesting access to them, available for educational/research purposes, at https://www.cesop.unicamp.br.

3. It is also possible to represent the result for 4 candidates in a 2d plot by opening the tetrahedron of possible results(D. Saari 2001). However, this is a work in progress.

Saari's representation triangles provide a way of visualizing all possible positional voting results of an election[4]. Consider Figure **??**. The closer to a vertex, the better the vertex's position in the social ranking. Region 1 corresponds to the social ranking of $A_1 > A_2 > A_3$, while Region 4 corresponds to the social ranking of $A_3 > A_2 > A_1$. The lines separating the regions represent indifference. The point in which all lines meet corresponds to $A_1 \sim A_2 \sim A_3$, while the line separating Region 1 and 2 would correspond to $A_1 > A_2 \sim A_3$. The three dots are the results for the antiplurality, the borda and the plurality voting methods. The line connecting the antiplurality and plurality results, the extremes, denotes all possible positional results, including the result for the Borda Count. The Borda Count point is always closer to the antiplurality result. In this example, most positional voting methods would have agreed with the plurality procedure outcome of A_2 as the winner.

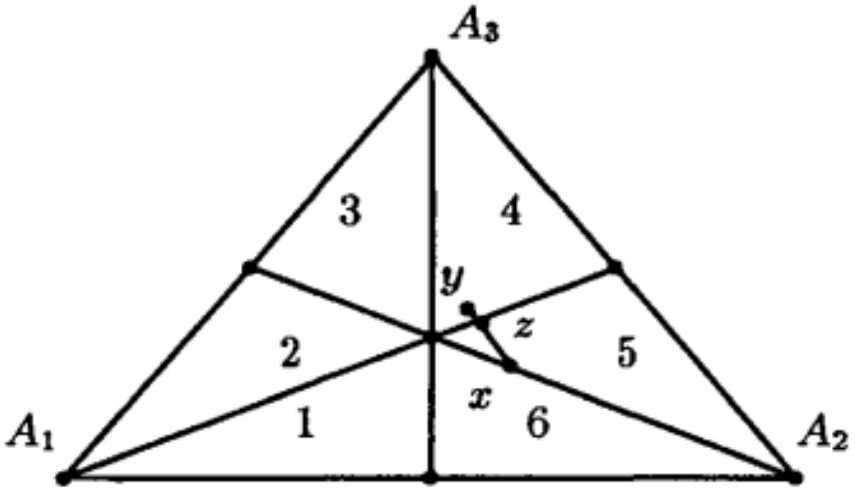

Figure 1: Source: Nurmi (2002).

A further complication is a mismatch between the survey's plurality result and the actual result of the first round. This is typical in surveys, and might be due to strategic voting, social desirability bias (not wanting to be seen as "polarized"), or systematic refusal of part of the electorate to answer the survey. Any imputation technique will reproduce this top choice discrepancy, thus I'll show directly the mismatch between the imputation and the actual vote[5]. I ran the imputation algorithm on the top 4 candidates, which means that in the sample the 7.19% that voted for others is kept constant. The result after applying the algorithm is Bolsonaro:Haddad:Ciro:Alckmin:Others = 36.7.7% : 17.3% : 14.1% : 7.19%. Thus, Bolsonaro and Haddad are undervoted in the sample, while Ciro and Alckmin are overvoted[6]. If we were simply transferring the top choices from over-voted to under-voted we could simply, say, transfer Alckmin $\rightarrow$ Bolsonaro and Ciro $\rightarrow$ Hadddad. However, there are 24 permutations of the top 4 candidates, with 6 for each candidate in which they are the top alternative. This gives leeway to many possible transfers. I have designed an algorithm, described in Appendix **??**, that, while respecting who can transfer and who needs votes, respects the Kemeny's Distance between the rankings and picked the transferred table that minimizes the euclidean distance between the inferred

4. For a complete exposition of this method see D. G. Saari (1995) or Nurmi (2002).

5. In a further version of this working paper I'll show this directly, then contrast with the imputed data. They, thankfully, match,I'm just time-constrained.

6. Remember the actual result was Bolsonaro:Haddad:Ciro:Alckmin:Others = 46.3 : 29.28 : 12.47 : 4.76 : 7.19.

table and the actual election. Two such inferred tables lead to the following proportion: Bolsonaro:Haddad:Ciro:Alckmin:Others = 46.19 : 29.32 : 12.51 : 4.77 : 7.19. Since the application of electoral procedures to those tables leads to qualitatively similar results, I'll just present the results for one of them. The results for the other is in Appendix **??**

5 Results

The inferred rankings[7] are summarized in Figure **??**. Indeed, the candidates that went to the second round were the most divisive ones. Ciro, however, could be considered more inclusive than Alckmin, since the proportion of last choices within Alckmin's subset of voters is higher than Ciro's. However, there is also a relevant distinction among the divisive candidates: Haddad's rejection was higher than his support, while the opposite held for Bolsonaro.

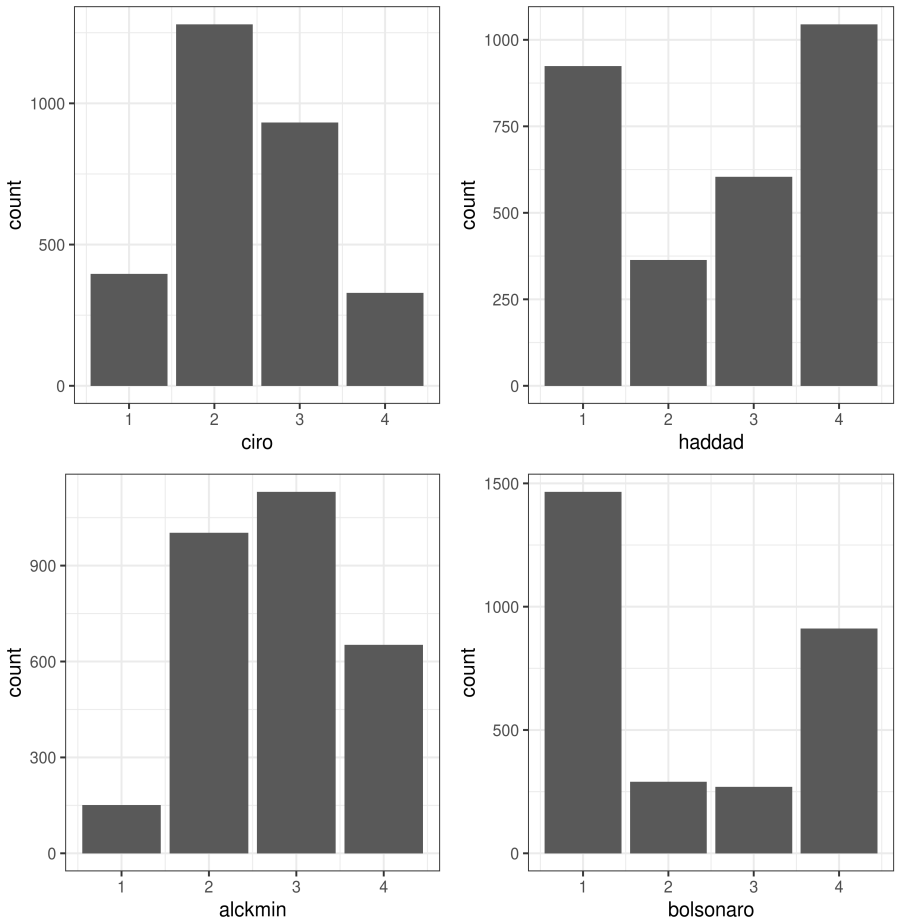

Figure 2: Frequency candidates appear at each position

But what does this support distribution mean from the point of view of the Borda and Condorcet procedures? Table **??** shows what we can infer from the imputed data. Despite being a divisive candidate, Bolsonaro would have won in all pairwise majority comparisons against any of the other top candidates. Haddad, however, would have lost against Ciro, who would only have lost against Bolsonaro. Unlike what was widely believed at the time, and was the motto of his campaign, Ciro most likely would not have won

7. The whole table is shown in Appendix **??**.

against Bolsonaro in a second round. He was not the "anti-Bolsonaro", but merely an "anti-Haddad", together with Bolsonaro. Alckmin, the candidate with the longest television time and the broadest supporting coalition would have lost against any other top candidates. He was the Condorcet Loser in the top set. The same pattern is reflected in the Borda Scores: Bolsonaro > Ciro > Haddad > Alckmin.

	alckmin	bolsonaro	ciro	haddad
alckmin	0	-1	-1	-1
bolsonaro	1	0	1	1
ciro	1	-1	0	1
haddad	1	-1	-1	0

(a) Pairwise Majority Comparisons

	borda score
alckmin	6527
bolsonaro	8185
ciro	7617
haddad	7041

(b) Borda scores

Table 2: Borda and Condorcet results for final inferred ranking 1

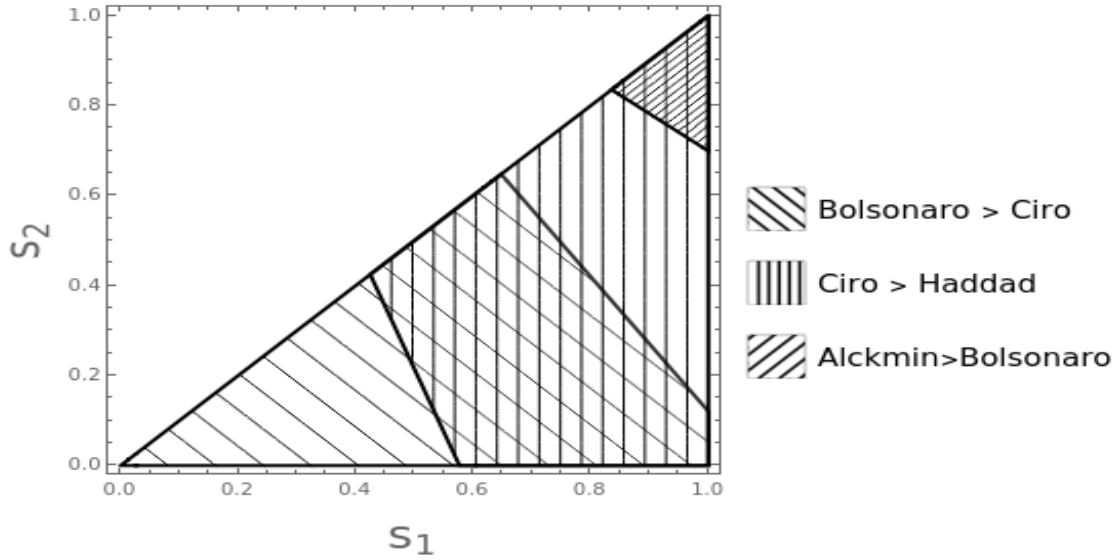

Figure 3: Source: Nurmi (2002).

However, neither the CW nor the family of positional voting methods are, in general, Independent of the Alternative Set(Kaminski 2015). That is if we drop or add candidates, the "social" ranking might change without respecting the ordering of the baseline set of alternatives. Consider the Borda-induced social ranking in this case: Bolsonaro > Ciro > Haddad > Alckmin. If by dropping Alckmin, the ranking changes to Ciro > Bolsonaro > Haddad, then the Borda Count, in this case, would be inducing a "paradoxical." result. In Figure **??** we consider alternative scenarios by dropping one of the top 4 candidates. Not only can we determine if there is any "alternative set stability", but also take into account the positional stability of the result.

candidates	w_s **tallies**
Alckmin	0.3415*s_1 + 0.385*s_2 + 0.0514
Bolsonaro	0.0988*s_1 + 0.0919*s_2 + 0.499
Ciro	0.4358*s_1 + 0.3173*s_2 + 0.1348
Haddad	0.1238*s_1 + 0.2057*s_2 + 0.3147

Table 3: w_s vector of each top 4 candidate

candidates	antiplurality	vote for two	plurality
Alckmin	0.7779	0.3929	0.0514
Bolsonaro	0.6897	0.5978	0.499
Ciro	0.8879	0.5706	0.1348
Haddad	0.6442	0.4385	0.3147

Table 4: Tallies for boundary positional methods (dropping "Others")

candidates	alckmin	bolsonaro	ciro	haddad
alckmin	-	0.05	0.0	0.25
bolsonaro	0.95	-	0.69	1.0
ciro	1.0	0.31	-	0.76
haddad	0.75	0.0	0.24	-

Table 5: Percentage of positional victories of row against column

In terms of alternative set stability, the positional voting procedures are eminently well-behaved in this case. The only reversal seems to be the antiplurality ranking if Haddad had dropped. In this case, Figure **??** (d), the antiplurality ranking was[8] Ciro > Alckmin > Bolsonaro > Haddad, but now the ranking is Ciro > Bolsonaro > Alckmin. The reason is that Haddad's votes were most likely transferred more to Ciro than to Alckmin. The antiplurality point, however, is very close to a tie between Bolsonaro and Alckmin, which means this can also be an artifact of sampling uncertainty.

What about positional stability? Notice that in all scenarios where Bolsonaro is still in the alternative set he would have been both the plurality and the Borda winner, and most positional voting methods would have elected him. The only counterfactual positional results in which he would not have been elected is if a voting procedure that emphasized rejection had been used, and Ciro would then have been elected. Those scenarios, however, are in these cases, Figure **??** (a,c,d), always a minority. His victory, thus, was not a fluke, or an artifact of institutional technology. Not only he would have won under both Borda and Condorcet methods, but his victory was also stable under both alternative sets of alternatives and alternative positional methods. This amounts to a hoop test falsification of the idea that divisive closet authoritarians are an artifact of the current decision procedures (Mahoney and Goertz 2006). It may well be true in the USA, but does not hold in general. Even though electoral systems based on just the first positions of citizens' preferences do ignore, by definition, the distribution of support the candidates have throughout the whole rankings, and we could expect that divisive candidates would fare worse under informationally richer decision procedures, a divisive candidate can still be a CW with high positional stability. Therefore, highly polarized scenarios can lead to the election of a divisive candidate regardless of which decision procedure is being used.

However, figure **??** (c) presents an interesting scenario. Though, as expected, Haddad would have been the plurality winner[9], now his victory would not have been positionally stable. In Table **??** it was shown that Ciro would have beaten him with a majority pairwise comparison, which gives credence to affirming that Ciro would have won under a majority with run-off system. In this scenario the most inclusive candidate would have been elected. Within Latin America, Brazil sticks out for its lack of transitional justice (Nalepa 2022). The remnants of the old regime were never punished, and Bolsonaro is an instance of such

8. have to confirm that. I even have the code for it written somewhere.

9. As shown in Figure **??** in Appendix **??**, this is a case in which the vote transfers differ. In the alternative vote transfer, in this scenario, Ciro would have tied with Haddad in the plurality case, but would have won under all other positional voting methods.

group. It is no coincidence after being elected he assigned 70% more military to public administration positions traditionally held by civilians. Or that in his term the army re-emerged in the country as a prominent political actor, with generals feeling comfortable to periodically remind the civil society that the military forces function as a "moderating power"[10] in the political system. If wolves in sheep clothes are at the door, as characterized by Chiopris, Nalepa, and Vanberg (2021), then maybe citizens should not leave it open for them. This leads to an empirical implication for the subset of transitional democracies (those that are still in the process of democratization (Svolik 2008)): the election of closet authoritarians is more probable in countries that have a military dictatorship past, but didn't go through a transitional justice process (Geddes 1999). Testing it is a subject of future studies.

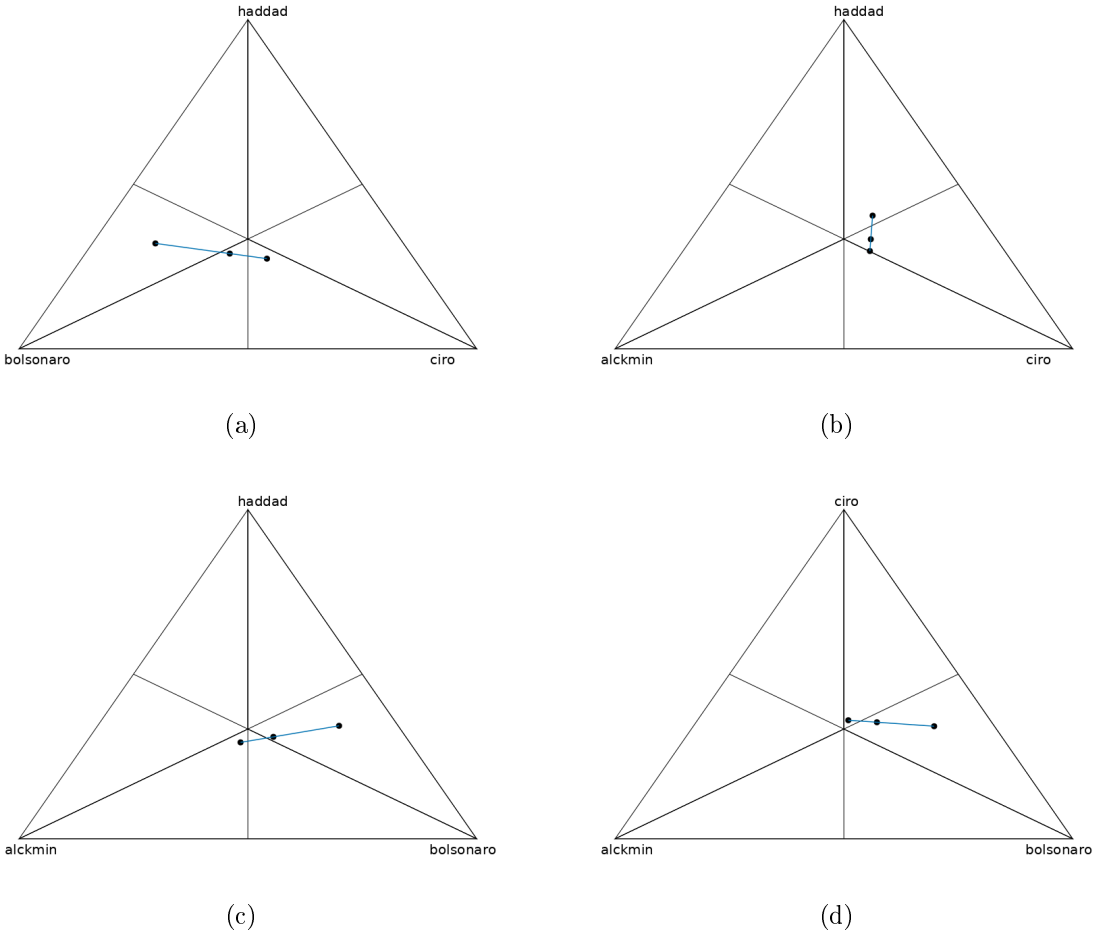

Figure 4: Positional results after dropping one candidate

I still intend to :

- Elaborate on the statistical model, and show diagnostics for it;
- Implement the opened tetrahedron with its procedure hull. It will be the first empirical paper to do that;
- Implement a decomposition of the votes for 4 candidates. Again, it would be the first empirical paper to do that (Nurmi did that for 3 candidates);
- Do highschool geometry on the procedure line and give a measure to assertions such as : "most positional methods would elect Bolsonaro in such scenario"

10. As the Emperor used to do before the Independence.

6 Conclusion

The paper contributes to the analysis of the institutional robustness of polyarchical systems, by considering credible alternative voting procedures' outcomes and properties at a critical juncture in Brazil's political history. We have demonstrated empirically that, contrary to established theoretical expectations, Bolsonaro's victory was not caused by the decision procedure.

The most glaring limitation of the paper is that agents adapt to new institutional environments. By assuming a direct translation between preferences and behavior we are ignoring strategic voting. The percentage of strategic voting in a large scale election, however, is an open empirical problem (Straeten et al. 2010; Kawai and Watanabe 2013). Nevertheless, a combination of game-theoretic models with a simulation parameterized by the inferred ranking distribution is a route of research that I intend to pursue.

The research used only one variable from the dataset, the pairwise comparisons, to simulate alternative scenarios. However, socio-demographic variables from the dataset could have been used to strengthen the data imputation procedure. Speaking of imputation, simpler imputations, such as the Impartial Culture assumption (Regenwetter et al. 2006), could have been used as benchmarks to compare with the imputation through the bayesian mallows model. Moreover, it is necessary to properly analyze the pattern of missingness of pairwise comparisons. Roughly less than half of the dataset is constituted of incomplete pairwise comparisons, and there might be valuable information on the agent's preferences contained in patterns of missingness (McElreath 2020).

Other voting procedures could have been used. Particularly, truncated positional voting methods could have been directly applied to the raw data (Terzopoulou and Endriss 2021). Moreover, we have been disregarding indifference in the agent's preferences. Again, this is valuable information, and it is known that forcing strict preferences when indifference exists leads to artificial inflation of the profile inconsistency (Gehrlein 2010). Finally, though we have analyzed the four top candidates, there is no direct relationship between the result of an election with positional procedures when we have a subset of the alternatives vs when we have the whole set of candidates (D. Saari 2001). It is well known, for instance, that the Borda Count violates WARP precisely because it is not contraction-expansion consistent (Schwartz 2018). Nonetheless, we expect that the results will not reverse, given that the bottom candidates could be deemed irrelevant to most of the population (and as such, be tied in the bottom of the rankings), and the patterns seen when dropping only one candidate. This is a subject for a more thorough analysis, in any case. Finally, even though we have analyzed scenarios in which candidates were removed, and alternative voting procedures could have been used, it would be more realistic to simulate the formation of coalitions and how voters would have reacted to those. The assumption of a pure additive transfer of votes, implicit when we removed candidates, is not necessarily true with coalitions, insofar voters of a center-left candidate, for instance, could actually, vote for the center-right candidate if they are alienated by an alliance with the Left, which in the case of the election under scrutiny, was highly rejected, as shown in our analysis.

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A The vote transfer algorithm

If we are to transfer from Alckmin to Bolsonaro we are lead to the problem of first picking which ranking at the source should be chosen then which ranking at the target should receive votes, while respecting how much the source has in excess and how much the target needs[11]. A natural sorting of which ranking should be the source is the position of Bolsonaro in the ranking. We start with rankings in which he is in the second position ((Alckmin, Bolsonaro, Ciro, Haddad), (Alckmin, Bolsonaro, Haddad, Ciro)), then third position ((Alckmin, Ciro, Bolsonaro, Haddad), (Alckmin, Haddad, Bolsonaro, Ciro)), then last position ((Alckmin, Ciro, Haddad, Bolsonaro), ((Alckmin, Haddad, Ciro, Bolsonaro))). Let's call those sets the sorted rankings sets. Suppose we picked a source ranking from the first sorted rankings set. What should be the target ranking, among the rankings which have Bolsonaro as the first choice? We transfer to the ranking that has minimal Kemeny's distance to the source ranking (Nurmi 2002). The Kemeny distance counts the number of transpositions (switching of pairs) needed to go from one permutation to another permutation. Thus, we transfer from the source ranking the min(number of votes the source ranking has, the total number of votes the under-voted needs, the total number of votes the over-voted can give). We update the source ranking frequency, the target ranking frequency, the total number of votes the under-voted needs, and the total number of votes the over-voted can give. If the under-voted doesn't need any other votes we stop and go to another over-voted $\rightarrow$ under-voted transfer. If not we check if the over-voted can still transfer votes to the current target under-voted. If yes we pick another source ranking in the sorted rankings sets and repeat until either the source has run out of votes it can given or the target has received enough votes. If not we go to another over-voted $\rightarrow$ under-voted transfer. In the end, this leads to 24 possible transfer sequences from over-voted to under-voted. One possible sequence is Alckmin $\rightarrow$ Bolsonaro, then Alckmin $\rightarrow$ Haddad, then Ciro $\rightarrow$ Haddad, then Ciro $\rightarrow$ Bolsonaro. Another possible sequence is Alckmin $\rightarrow$ Bolsonaro, then Alckmin $\rightarrow$ Haddad, then Ciro $\rightarrow$ Bolsonaro, then Ciro $\rightarrow$ Haddad. This leads to 6 transfers that minimize the euclidean distance between the inferred plurality result and actual result of the first round. However, there are actually two equivalence classes among those 6: 3 have the same proportion for each ranking, while the other 3 have the same proportion for each ranking. The new inferred proportion for both classes is: Bolsonaro:Haddad:Ciro:Alckmin:Others $= 46.19 : 29.32 : 12.51 : 4.77 : 7.19$.

11. In a further version of this paper I'll describe this with the help of a pseudo-code of the algorithm.

B Inferred Ranking Table

ranking vectors	percentage
haddad >ciro >alckmin >bolsonaro	17.19
bolsonaro >alckmin >ciro >haddad	13.99
bolsonaro >ciro >alckmin >haddad	13.72
bolsonaro >ciro >haddad >alckmin	8.85
haddad >alckmin >ciro >bolsonaro	7.12
bolsonaro >haddad >ciro >alckmin	6.26
ciro >alckmin >haddad >bolsonaro	5.04
bolsonaro >alckmin >haddad >ciro	4.36
haddad >ciro >bolsonaro >alckmin	3.51
alckmin >bolsonaro >ciro >haddad	2.79
bolsonaro >haddad >alckmin >ciro	2.72
ciro >alckmin >bolsonaro >haddad	2.72
ciro >bolsonaro >alckmin >haddad	2.04
ciro >haddad >alckmin >bolsonaro	1.67
haddad >bolsonaro >ciro >alckmin	1.57
alckmin >bolsonaro >haddad >ciro	1.5
ciro >haddad >bolsonaro >alckmin	1.19
haddad >bolsonaro >alckmin >ciro	1.16
haddad >alckmin >bolsonaro >ciro	0.92
ciro >bolsonaro >haddad >alckmin	0.82
alckmin >haddad >bolsonaro >ciro	0.54
alckmin >ciro >bolsonaro >haddad	0.31

Table 6: Inferred rankings

C The results for the other inferred table

ranking vectors	percentage
bolsonaro >alckmin >ciro >haddad	13.99
bolsonaro >ciro >haddad >alckmin	13.69
haddad >ciro >alckmin >bolsonaro	12.36
bolsonaro >ciro >alckmin >haddad	11.1
haddad >alckmin >ciro >bolsonaro	9.33
haddad >ciro >bolsonaro >alckmin	6.13
ciro >alckmin >haddad >bolsonaro	5.04
bolsonaro >alckmin >haddad >ciro	4.36
bolsonaro >haddad >ciro >alckmin	4.05
alckmin >bolsonaro >ciro >haddad	2.79
bolsonaro >haddad >alckmin >ciro	2.72
ciro >alckmin >bolsonaro >haddad	2.72
ciro >bolsonaro >alckmin >haddad	2.04
ciro >haddad >alckmin >bolsonaro	1.67
haddad >bolsonaro >ciro >alckmin	1.57
alckmin >bolsonaro >haddad >ciro	1.5
ciro >haddad >bolsonaro >alckmin	1.19
haddad >bolsonaro >alckmin >ciro	1.16
haddad >alckmin >bolsonaro >ciro	0.92
ciro >bolsonaro >haddad >alckmin	0.82
alckmin >haddad >bolsonaro >ciro	0.54
alckmin >ciro >bolsonaro >haddad	0.31

Table 7: Second minimizing inferred rankings - one ranking got 0 votes after the transfer

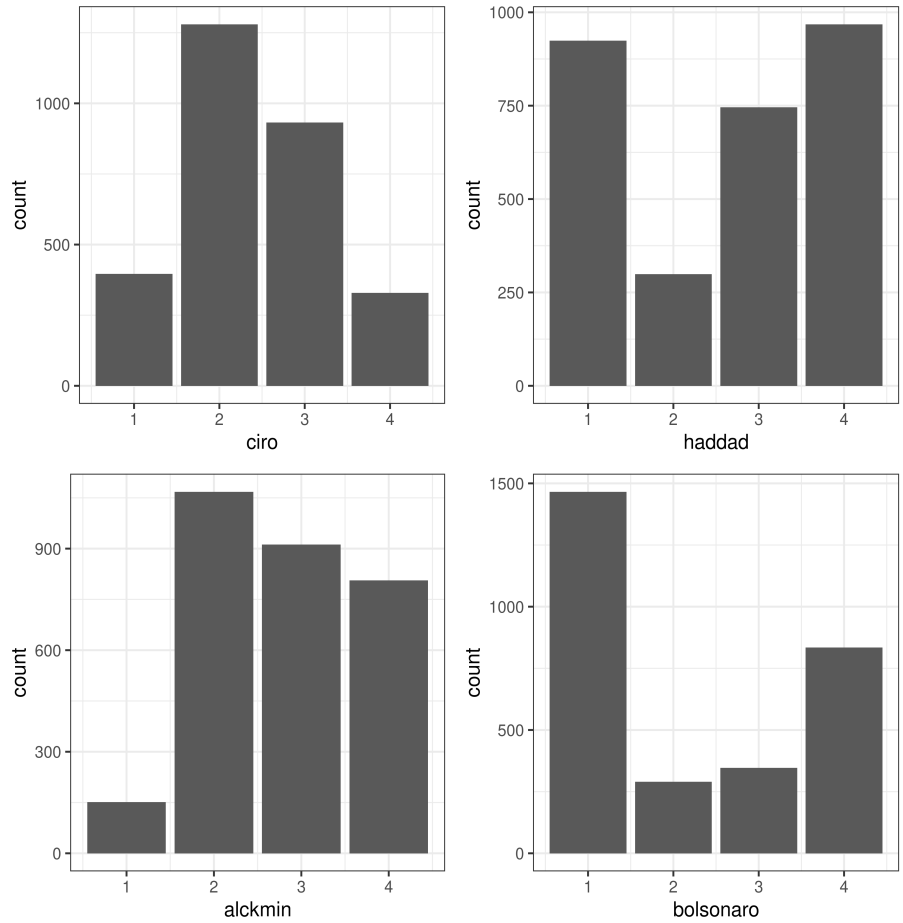


Figure 5: Number of times candidates appear at each position - final ranking 2

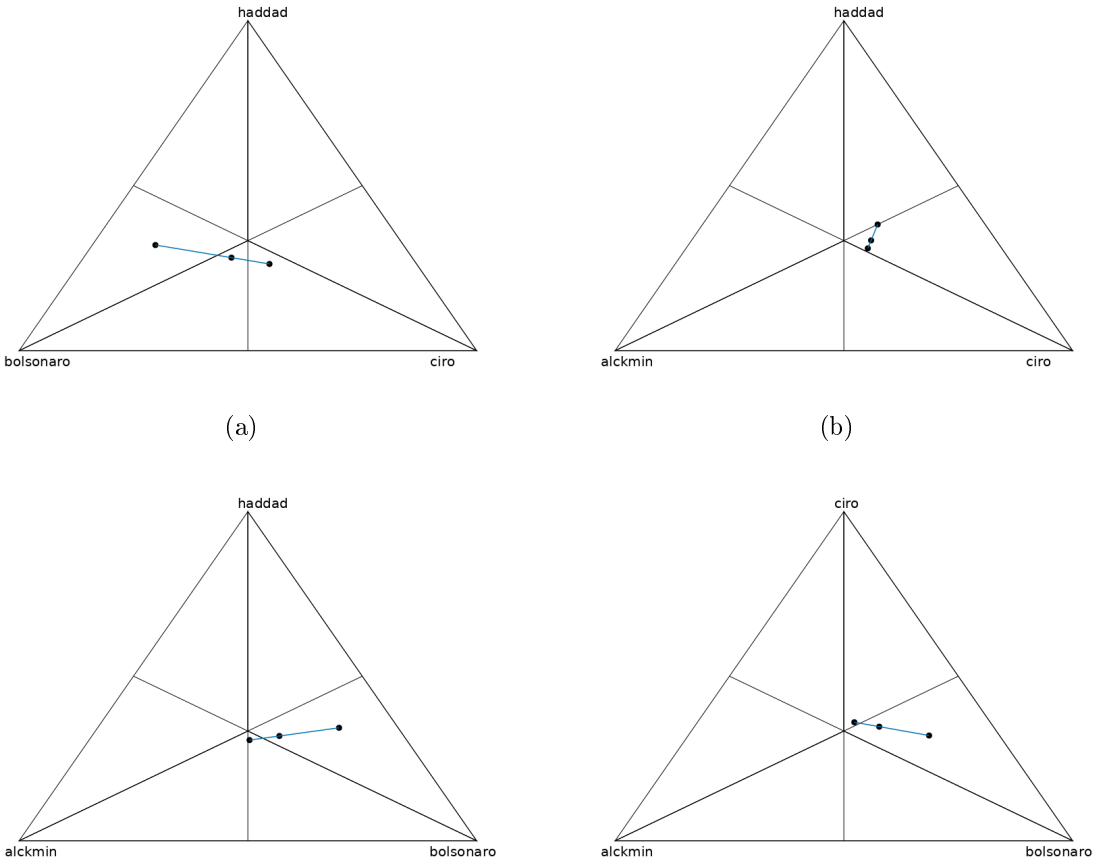
	alckmin	bolsonaro	ciro	haddad
alckmin	0	-1	-1	-1
bolsonaro	1	0	1	1
ciro	1	-1	0	1
haddad	1	-1	-1	0

(a) Pairwise Majority Comparisons

	borda2$other_info$count_max
alckmin	6438
bolsonaro	8262
ciro	7617
haddad	7053

(b) Borda scores

Table 8: Borda and Condorcet results for final inferred ranking 2



(a) (b)

(c) (d)

Figure 6: Positional results after dropping one candidate - second possible vote transfer