Majoritarian principles and critical junctures: an analysis of Brazil's 2018 presidential election

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Abstract

Presidential Elections are critical moments for polyarchical systems, particularly in contexts of high social tension. The 2018 presidential election in Brazil used a two-round system, yet the most divisive candidates went to the second round. Pairwise and positional voting procedures embody different generalizations of a majoritarian credo that underpins such elections. The paper mobilizes both perspectives and, using representative survey data, reconstructs the top four preferences of the Brazilian electorate a week before the election. It shows that the electoral winner, Jair Messias Bolsonaro, was a Condorcet Winner, but may have not been the Borda Winner, while the second-round loser, Fernando Haddad, was a Condorcet Loser. Furthermore, possible alternative scenarios under different feasible sets of candidates are simulated, contributing to understanding the role of decision procedures in critical junctures.

1 Introduction

The rise of polarization, the specter of democratic backsliding, and more generally an apprehension with the survival prospects of democratic polities[1] have led to a renewed interest in what institutions mitigate vs. instigate destabilizing dynamics, or that increase the adaptability of the political system vis-à-vis both internal and external stressors (Bednar 2021; Chiopris, Nalepa, and Vanberg 2021; Aligica and Tarko 2014; V. Ostrom 1997). Given their centrality to the input-output relation between society and the state, electoral institutions naturally figure among the set of such institutions under scrutiny (Wang et al. 2021), and a branch of the literature on collective choice has wondered whether the current electoral victories of divisive candidates have not been an effect of informationally poor decision procedures (Potthoff and Munger 2021; Kurrild-Klitgaard 2018; Woon et al. 2020). This paper shows, however, that in 2018 presidential election in Brazil the election of a polarizing candidate, Jair Messias Bolsonaro, was not simply an artifact of the decision procedure. Despite that, neither would he have won under any reasonable voting method. I demonstrate that, in this case, there is a partial conflict between typical evaluative positions, harking back to the Borda-Condorcet debate.

Despite the multiple historical contingencies that might explain his victory, one might naturally wonder what role the electoral system has had in it. After all, it is well-known that the outcome of collective choices is fundamentally dependent on the voting procedure (Riker 1982). As Brazil's two-round system, most electoral systems merely use the first index of the voters' preference rankings (Grofman and Feld 2004). How would he have fared under informationally richer voting procedures, such as methods based on pairwise comparisons

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1. For a critical perspective on the current academic discourse on democratic backsliding see Little and Meng (2023).

and positional voting procedures? Was he, and arguably other democratically elected destabilizing candidates, a product of decision procedures that favor divisive candidates over more inclusive ones (Igersheim et al. 2022)? What criteria should be used to argue that a candidate's victory was an institutional artifact? Would the result have changed with a different set of candidates?

To answer those questions, I, first, revisit the perspectives of Borda and Condorcet on voting procedures. After that, I use a pre-electoral representative survey to reconstruct the full 4-top rankings of the Brazilian population and use this augmented data to simulate electoral outcomes, under alternative methods, for the top 4 and 3 candidate sets. Finally, I discuss the significance of the results and conclude by pointing out the limitations of this endeavor.

2 Theory

The realization that the result of collective decisions is inseparable from the voting procedures being used and that such procedures differ in terms of their consistency with evaluative criteria naturally leads us to wonder what possible adjacent paths could have been taken in key electoral moments (Tabarrok and Spector 1999; Kamiński 1999; E. Ostrom 1986). Concurrently, this realization puts us in a conundrum: what, among all possible criteria, should be used to motivate the counterfactual analysis? Isn't this endeavor inherently arbitrary, given that one can always retrofit a choice of voting method that matches an a-priori desired outcome (Riker 1982)? The anchoring point is to note that political actors themselves reflect on those procedural properties, which end up being levers for their legitimacy claims within the political game (McLean 2002; E. Ostrom 2009). Thus, rather than assuming a philosopher-king stance and imposing external values as if they were universally agreeable, we can look at what set of values the agents themselves mobilize (Binmore 2005). Particularly prominent among polarizing or divisive candidates -those that have strong support at the top choices of voters, but also have a high share of the bottom choices among the electorate[2]- are claims of strength and legitimacy based on the notion of popular mandate, a congenial resource for politicians that, despite being elected, face widespread rejection or opposition.

Nevertheless, what is a mandate? At a minimum, a politician has a mandate as long as he has won under the voting procedure. An actor has more mandate the more significant the difference between its vote share or score vs. the second most well-voted candidate. Such an agent has more marginal mandate rather than just having the minimal mandate bestowed by winning. Note that both notions of the mandate are related to a fundamental majoritarian credo which is part of the democratic ideal: that if both alternatives and voters are deemed equals, then the alternative that receives more support should be the winner (Dahl 1989). This "monotonic/majoritarian mindset" underlies the minimal mandate, since if a candidate was elected, it received more votes than the others, and the stronger marginal mandate, since more support means more mandate. That majoritarian value is what sustains claims of legitimacy of elected candidates. Roughly, the higher the marginal mandate, the more backing of popular support an Executive leader can claim to have (Grossman et al. 2022). Alternatively, to deny the opposition has achieved a mandate by claiming the electoral process is fraudulent is a maneuver that again mobilizes this focal point of the democratic ethos.

However, with more than 2 alternatives, the majoritarian mindset is not as clear-cut as a profile, voters' rankings, such as [xyz, yzx, zxy] reminds us. Nonetheless, it remains a centerpiece of the democratic paradigm. How can, then, one extend majoritarianism

2. The correspondent concept of an inclusive candidate is defined by Igersheim et al. (2022, p.6) as those that "get widespread support from the voters but with no strong feeling of rejection or attachment".

to more than two alternatives? Borda and Condorcet grappled with this problem and gave different answers. Condorcet extended the majority rule to pairwise majority rule: apply majority rule to all pairwise comparisons. One possible condition that generalizes majoritarianism is what is known as the Condorcet criterion: a decision procedure is Condorcet consistent if it selects the candidate, if there is any, that wins in all pairwise majority contests. This alternative is called a Condorcet winner (CW) (Felsenthal 2011). Borda, on the other hand, devised a scoring scheme: if there are say 3 alternatives $\{A, B, C\}$ and an agent i has ranking $B > C > A$ then the Borda score in i's ranking for each alternative is $A : B : C = 0 : 2 : 1$[3]. The Borda score for the full profile is the sum of each alternative score at each voter ranking, and the candidate with the highest score, the Borda winner (BW), wins. It is equivalent to adding the number of votes an alternative got in each pairwise comparison against the other alternatives (Nurmi 1999). As such, it is another way of generalizing the "majoritarian/monotonic" perspective to more than two alternatives.

Being Condorcet consistent is arguably the primary normative benchmark for a voting method in single-candidate elections, while the Borda perspective could be considered the leading contender (Regenwetter et al. 2006; Felsenthal 2011; Nurmi 2002). While being plausible generalizations of the majoritarian credo, they also offer stronger and informationally more demanding views of mandate. If the candidate is a CW, it would have won under all possible majority pairwise comparisons against the other candidates. I will say a CW is a candidate with pairwise mandate[4]. The BW lends itself to a similar interpretation, but the notion of mandate can be strengthened here. The Borda count can be seen as one method within a family of methods that assign weights to positions in the ballot. In one extreme, the plurality voting method assigns a score of 1 to the top choice and 0 to all others. On the other extreme is the antiplurality voting method, which assigns 1 to all positions besides the last one. Between the two extremes are all possible ways of assigning a score to the ballots of the electorate. The higher the proportion of positional voting systems that the candidate would have won had the election used it, what Tabarrok (2001) has called positional stability, the higher the positional mandate of the candidate.

Suppose a candidate wins under a voting procedure that only uses the top choice of the electorate but is neither a BW nor a CW. In that case, it has less mandate, in this generalized majoritarian perspective, than if it were both - which would signal a comprehensive social basis. Thus, a candidate who wins under the current voting procedure but is neither a BW nor a CW could be considered an artifact of the procedure. In the latter case, the procedure would be just "tracking" a broader support pattern for the alternative. Altogether, the notions of pairwise and positional mandates will be the primary lenses in this paper to understand both the popular support a candidate receives and the role of the decision procedure in its election.

Even though the pairwise and positional perspectives of popular support/mandate generalize a widely held democratic principle, they are not captured by electoral processes that only have as input voters' first choice. As such, they are not typically mobilized by politicians. Nonetheless, this information, which has been repeatedly rediscovered in acts of political reflexivity (McLean 2014), can be queried to tell a more refined story about the backing a candidate has among the electorate. Such a broader informational backdrop underlies current research on the case of the United States and Donald Trump's electoral victory. Regardless of the specificities of each paper, all presuppose that the informational paucity of only focusing on top choices blinds the States' socio-technical translation of popular support into political input (the choice of a candidate). For instance, Potthoff and

3. Alternatively, it can also be coded as $1 : 3 : 2$.

4. The Copeland scores could be a more general measure of pairwise mandate or even the Kemeny scores for each permutation, but this generalization is unnecessary in the context of this paper.

Munger (2021), Wo[illegible]et al. (2020), and Kurrild-Klitgaard (2018) debate whether Donald Trump was a CW in the primaries, with recommendations of voting procedures that better track what the CW is, after all. Igersheim et al. (2022) goes a step further: they argue that not only was Trump neither, but Sanders was the actual Borda and Condorcet Winner, and generally the "best" candidate, if by best one understand a candidate being the most inclusive and winning under the most alternative decision procedures[5]. An analogous line of reasoning would lead us to hypothesize that a similar conclusion could be drawn about Bolsonaro: he would not have either pairwise or positional mandate. We will see, however, that an unambiguous conclusion cannot be drawn in the Brazilian case.

3 Case/Data

Jair Messias Bolsonaro was elected the president of Brazil in 2018. For more than 20 years as a congressman, he was primarily a low clergy politician defending the interests of the military and local police forces of the state of Rio de Janeiro. The 2018 electoral scenario in Brazil was one of high rejection of the traditional political elite, particularly of the Labor Party (Partido dos Trabalhadores - PT), after corruption scandals and an impeachment process of the previous president, Dilma Roussef, a Labor politician.

The main contestants, among 13, were him, a rightist candidate; Fernando Haddad, a leftist candidate from PT; Geraldo Alckmin, a center-right candidate; and Ciro Gomes, a center-left candidate. The presidential election in Brazil follows a two-round system. In the first round, 8.79% of the votes were White/Null, which means the voting procedure does not count them. Moreover, there was a 20% abstention. The result of the valid votes was, in percentages, the following: Bolsonaro:Haddad:Ciro:Alckmin:Others = 46.3 : 29.28 : 12.47 : 4.76 : 7.19. Among the 9 other candidates, the highest vote share was João Amoêdo's with 2.5%. All others got less than 1%. In the second round, the result was: Bolsonaro:Haddad = 55.12 : 44.78. White/Null votes were 9.57% of the total electorate. The abstention in this round was 21.3%. With more than 10% more than his second-round opponent, Bolsonaro rose to power backed by a solid marginal mandate. Nevertheless, he still contested the result and argued that he would have won in the first round had the elections not been frauded[6]. He continued putting the electoral institutions under suspicion with a view to the 2022 election, which he lost to Lula with a margin of 1%.

It is relevant to notice that two events marked the 2018 election. First, the leading leftist candidate, Lula, was prevented from running. He was arrested at the beginning of the electoral campaigns, the process was deemed suspicious in 2021 since the judge[7] was cooperating with the prosecutor, and he won in 2022, in an electoral process marked by irregularities in favor of Bolsonaro. The distribution of support for Lula was markedly different from Haddad, who was merely his replacement. PT's campaign was predicated on the possibility of Lula being released, and Haddad posed primarily as Lula's candidate. Nonetheless, his popularity was nowhere near Lula's, and he inherited the high rejection of his party at the time. Second, on 09/06/2018, a month before the first round, there was an assassination attempt against Bolsonaro. The knife attack most likely changed his pattern of support.

The dataset used for the analysis is a representative street survey done on 10/02/2018, less than a week before the first round (10/07/2018). DataFolha, an independent research institute highly esteemed and trusted by Brazilian experts[8], did this survey. One question,

5. I doubt this distribution of support for Sanders would hold had he been a viable candidate.
6. No evidence has been found of any rigging of the elections in Brazil, despite his claims.
7. Bolsonaro nominated this judge Minister of Justice.
8. I had access to the survey data, code-book, and questionnaire by creating an account and requesting access to them, available for educational/research purposes, at https://www.cesop.unicamp.br.

Number of Pairwise Comparisons	Frequency
1	15
2	42
3	462
4	118
5	503
6	1797

Table 1: Frequency of pairwise comparisons in the dataset.

in particular, is the only variable in our analysis: pairwise comparisons between the 4 top candidates. With it is possible to reconstruct the full 4-top ranking of the voter. Preliminary pre-processing has led me to drop 171 observations where all pairwise comparisons were missing and 132 in which they were cyclic. This leaves us with 2937 out of 3240 observations. As Table 1 shows only 1797 observations compared all 4 candidates. As such, we have to augment the data with transitive closures for 1140 observations by methods discussed in the next section.

4 Methods

I impute the missing data using the **R** package mice (multiple imputation by chained equations), one of the standard packages for this task. It fills the missing values in a row by using the values of the other columns, by an iterative series of predictive models (Buuren 2018). Under the hood, it offers a menu of possible predictive models, such as bayesian linear regression, predictive mean matching, logistic regression, polytomous regression, classification trees and random forests. Among the classes of methods that could be applied to the missing voting data, given its categorical nature, the polytomous regression was the only one that did not introduce cyclic rankings, or repeated alternatives in the ranking, and as such, was the one I used[9].

A further complication is a mismatch between the survey's plurality result and the actual result of the first round. This is typical in surveys and might be due to strategic voting, social desirability bias (not wanting to be seen as "extreme"), or systematic refusal of part of the electorate to answer the survey (Nishimura, Wagner, and Elliott 2016). Any imputation technique will reproduce this top-choice discrepancy since it inherits this problem from the survey. The share in the survey is Bolsonaro:Haddad:Ciro:Alckmin:Others $= 36.81 : 24.96 : 17.06 : 13.97 : 7.2$. Thus, Bolsonaro and Haddad are undervoted in the sample, while Ciro and Alckmin are overvoted[10]. However, transferring is complicated by the fact that we are working with the full rankings, which gives leeway to many possible ways of transferring. For instance, consider Table 2, which shows some of the rankings for Alckmin and Bolsonaro after the imputation. If we are to transfer from Alckmin to Bolsonaro, we are led to the problem of first picking which ranking at the source should be chosen and then which ranking at the target should receive votes while respecting how much the source has in excess and how much the target needs. Which row from the set $\{1, 2, 3\}$ should transfer votes to which row of the set $\{4, 5, 6\}$?

A natural sorting of which ranking should be the source is the position Bolsonaro is in the ranking. We start with rankings in which he is in the second position ((Alckmin, Bolsonaro, Ciro, Haddad), (Alckmin, Bolsonaro, Haddad, Ciro)), then third position ((Al-

9. Besides the polytomous regression, I tested predictive mean matching, classification trees, and random forests. All introduced cyclic rankings, sometimes in large amounts (as in the case of random forests).

10. Remember the actual result was Bolsonaro:Haddad:Ciro:Alckmin:Others $= 46.3 : 29.28 : 12.47 : 4.76 : 7.19$.

	1	2	3	4	frequency	proportion
1	Alckmin	Bolsonaro	Ciro	Haddad	93	0.03
2	Alckmin	Ciro	Bolsonaro	Haddad	63	0.02
3	Alckmin	Haddad	Bolsonaro	Ciro	14	0.00
4	Bolsonaro	Alckmin	Ciro	Haddad	556	0.18
5	Bolsonaro	Ciro	Alckmin	Haddad	366	0.12
6	Bolsonaro	Alckmin	Haddad	Ciro	59	0.02

Table 2: Some pre-transfer proportions of Alckmin/Bolsonaro's rankings

ckmin, Ciro, Bolsonaro, Haddad), (Alckmin, Haddad, Bolsonaro, Ciro)), then last position ((Alckmin, Ciro, Haddad, Bolsonaro), ((Alckmin, Haddad, Ciro, Bolsonaro))).

Suppose we picked a source ranking from the first sorted rankings set. What should be the target ranking among the rankings which have Bolsonaro as the first choice? I transfer to the ranking that has minimal Kemeny's distance to the source ranking (Nurmi 2002). The Kemeny distance counts the number of transpositions (switching of pairs) needed to go from one permutation to another permutation. Thus, I transfer from the source ranking the **min**(number of votes the source ranking has, the total number of votes the under-voted needs, the total number of votes the over-voted can give)[11]. I update the source ranking frequency, the target ranking frequency, the total number of votes the under-voted needs, and the total number of votes the over-voted can give. If the under-voted does not need any other votes, the algorithm breaks the loop and goes to another over-voted → under-voted transfer. If not, it checks if the over-voted can still transfer votes to the current target under-voted. If yes, it picks another source ranking in the sorted rankings sets and repeats until the source has run out of votes it can give or the target has received enough votes. If not, it goes to another over-voted → under-voted transfer. In the end, this leads to 24 possible transfer sequences from over-voted to under-voted. One possible sequence is Alckmin → Bolsonaro, then Alckmin → Haddad, then Ciro → Haddad, then Ciro → Bolsonaro. Another possible sequence is Alckmin → Bolsonaro, then Alckmin → Haddad, then Ciro → Bolsonaro, then Ciro → Haddad. That transference process leads to 6 transfers that minimize the Euclidean distance between the inferred plurality result and the actual result of the first round. The results are invariant between them, so I only report the analysis for one of them. The new percentages are: Bolsonaro:Haddad:Ciro:Alckmin:Others = $46.32 : 29.26 : 12.45 : 4.77 : 7.19$.

After imputing the missing rankings and making the transfer of rankings to match the result for the first round, I identify the BW and CW among the top 4 candidates, calculate and plot all counterfactual victory scenarios for positional voting methods, and visualize the positional outcomes for alternative 3-candidates sets using Saari's outcome simplex (D. G. Saari 1995).

11. The **min** guarantees: we are not giving more than the source ranking has, which would lead to negative numbers; less or more than the undervoted needs; nor giving more than the over-voted should overall give (at some iteration in the loop, a ranking can have a higher frequency than both what the over-voted can give and the under-vote needs to receive).

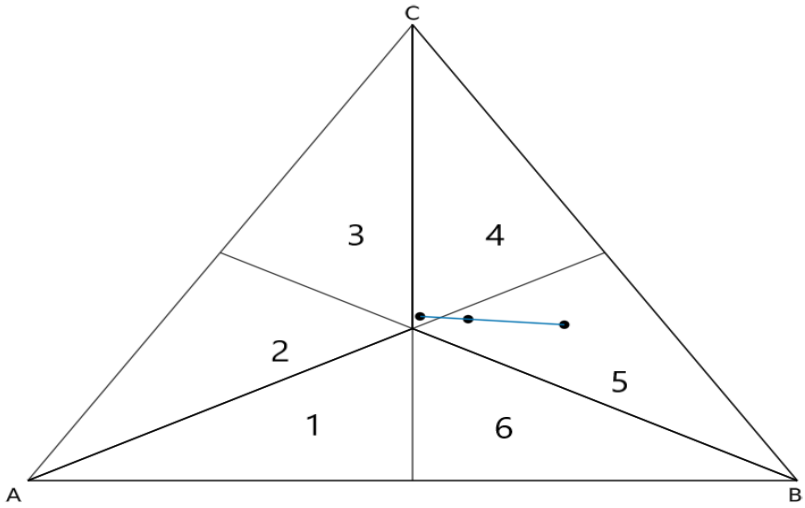


Figure 1: Saari's outcome simplex

Saari's outcome simplex, or point-share triangle (Eggers 2020), provides a way of visualizing all possible positional voting results of an election[12]. Consider Figure 1. The closer to a vertex, the better the vertex's position in the social ranking. Region 1 corresponds to the social ranking of $A > B > C$, while Region 4 corresponds to the social ranking of $C > B > A$. The lines separating the regions represent indifference. The point at which all lines meet corresponds to $A \sim B \sim C$, while the line separating Region 1 and 2 would correspond to $A > B \sim C$. The three dots are the results of the antiplurality, the Borda and the plurality voting methods. The line connecting the antiplurality and plurality results, the extremes, denotes all possible positional results, including the result for the Borda Count. It is called the procedure hull (D. Saari 2001). The Borda Count point is always closer to the antiplurality result. In this example, most positional voting methods would have agreed with the plurality procedure outcome of B as the winner. A related triangle, the representation triangle, or profile triangle (Eggers 2020), will be used to represent a profile compactly. In each ranking region, we plot the frequency of votes that match that region's ranking. For 4 candidates, we can use analog representations by "opening" the 3-simplex/tetrahedron and plotting onto its polyhedral net - the arrangement of polyhedrons in the plane that, when folded, become the faces of the simplex.

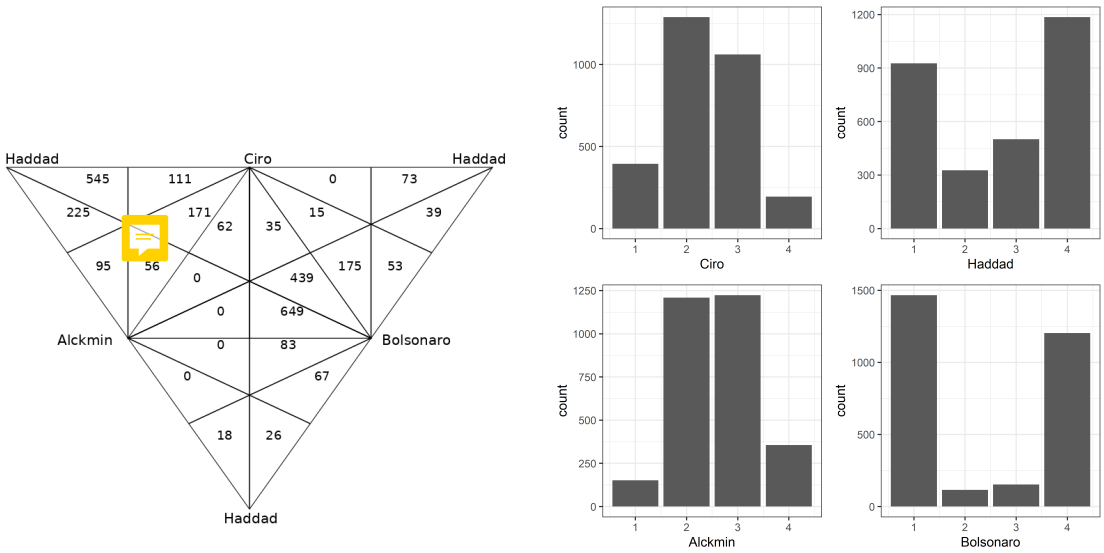
To calculate all positional voting victories, I use two facts proved by Donald Saari (D. G. Saari 1995; D. Saari 2001): first, any positional voting method for 4 candidates can be seen as assigning weights to rank positions in a standardized manner $(1, s_1, s_2, 0)$, where $0 \leq s_2 \leq s_1 \leq 1$; second, all such procedures will lie in the convex hull of the plurality, antiplurality and vote for two procedures, with respective weights of $(1, 0, 0, 0)$, $(1, 1, 1, 0)$, $(1, 1, 0, 0)$. Calculating scenarios amounts, thus, to vary the values of s_1 and s_2.

5 Results

The inferred rankings are shown in Figure 2a and summarized in Figure 2b. Among all ways of transferring from over-voted to under-voted, while respecting the Kemeny distance, the transference that best matched the rankings in the survey with the actual first-round result led all rankings in which Alckmin appears as the first choice to be of type Alckmin > Ciro > Haddad > Bolsonaro and Alckmin > Haddad > Ciro > Bolsonaro. Notably,

12. For a complete exposition of this method see D. G. Saari (1995) or Nurmi (2002).

no ranking of type Alckmin > Bolsonaro > _ > _ remained. The most blatant pattern in Figure 2b is that the candidates that went to the second round were, indeed, the most divisive ones. Among the more inclusive candidates, Ciro has more second choices than third choices, while Alckmin's support is equally balanced between those two positions in the voters' preferences. Moreover, Ciro has more first choices and fewer last choices than Alckmin. There is also a difference among the divisive candidates: Haddad's rejection was higher than his top-choice support, while the opposite held for Bolsonaro. That within-group, inclusive vs. divisive, differences will be relevant to understand how each candidate fares against the others.



(a) Opened representation tetrahedron

(b) Frequencies at each position

Figure 2: Profile after imputation and rankings transference

What does this support distribution mean from the point of view of the BW and CW? Table 3 shows what we can infer from the imputed data. Despite being divisive, Bolsonaro would have won in all pairwise majority comparisons against other top candidates. Haddad, however, would have lost against all others. He was a Condorcet Loser, despite going to the second round. Ciro would only have lost against Bolsonaro, while Alckmin could only have won against Haddad. Unlike what was widely believed at the time and was the motto of his campaign, it is uncertain whether Ciro would have won against Bolsonaro in the second round. From a pairwise perspective, he was not the "anti-Bolsonaro", but merely an "anti-Haddad", even more than Bolsonaro. Alckmin, the candidate with the longest television time and the broadest supporting coalition, would have lost against Haddad, who was merely a substitute for Lula. However, the pattern is not reflected in the Borda Scores, which implies the ranking: Ciro > Bolsonaro > Alckmin > Haddad. Nevertheless, the raw Borda scores of Ciro and Bolsonaro are very similar. If we standardize them, we see that the candidates are practically tied. If we take into account the sampling error, imputation, and transfer degree of freedom, then the most we should conclude is that the Borda Ranking was Ciro $\sim$ Bolsonaro > Alckmin > Haddad. Note that if we take a positional perspective, then yes, Ciro was indeed the main contestant against Bolsonaro. Nevertheless, this obviously could not be captured by the majority with run-off.

Now, what about the positional mandate? As discussed in the methods section, with 4 candidates, all results will lie in the convex hull of three positional voting procedures: plurality, antiplurality, and vote for two. The normalized score of a candidate will be of the form $q_{s_i} = a_i + b_i s_1 + c_i s_2$, where a_i is the share i received of votes in the first position, b_i

	Alckmin	Bolsonaro	Ciro	Haddad
Alckmin	0.0%	-12.63%	-16.99%	8.27%
Bolsonaro	12.63%	0.0%	5.48%	7.46%
Ciro	16.99%	-5.48%	0.0%	16.65%
Haddad	-8.27%	-7.46%	-16.65%	0.0%

(a) Pairwise Margins

	Borda Score	Standardized Borda Score
Alckmin	7029	0.464
Bolsonaro	7718	0.543
Ciro	7756	0.547
Haddad	6867	0.446

(b) Borda Count Outcome

Table 3: Condorcet and Borda Outcomes

in the second, and c_i in the third position of voters rankings. Therefore, the scores of each candidate in the inferred ranking for the 2018 election can be found by assigning values to the equations of Table 4. For instance, if we set $s_1 = s_2 = 0$ we recover the plurality score, after ignoring "Other" candidates.

candidates	q_s tallies
Alckmin	$0.4113s_1 + 0.4165s_2 + 0.0514$
Bolsonaro	$0.0392s_1 + 0.0521s_2 + 0.4992$
Ciro	$0.4387s_1 + 0.3612s_2 + 0.1341$
Haddad	$0.1109s_1 + 0.1703s_2 + 0.3154$

Table 4: q_s for each candidate

We can, then, represent the results by an opened outcome tetrahedron, as roughly depicted in Figure 3. The black upside triangle is the plurality result, the black downside triangle is antiplurality, the black dot is the vote for two results, and the diamond is the Borda count. As expected, the decision procedures that emphasize the top choice awarded Bolsonaro and Haddad to the extent that the Condorcet loser went to the second round. Note, however, that Haddad only does well in a small region of the hull. Moreover, we can see that there are decision procedures in which even Alckmin would have beaten both Bolsonaro and Haddad.

In what precise percentage of the cases would a candidate have beaten another? Note that by opening the tetrahedron, the information provided by the volume of the subregions of the procedure hull is lost[13]. Nevertheless, algebraic manipulation of the equations in Table 4 allows us to answer this question. Table 5 presents the percentage of scenarios in which this would have happened. It implies a more complex picture of what happened. Bolsonaro was the CW, was tied with Ciro as a BW and would have won against Ciro in roughly 47% of the positional voting methods. Nevertheless, it shows that there were scenarios in which he would have lost to the more inclusive candidates, Ciro and Alckmin. In Alckmin's case, this could have happened in surprising 30% of the cases. However, Ciro could have beaten him in most, $\approx 53\%$, of the positional voting methods. Surprisingly, Haddad, who went to the second round with Bolsonaro, would never have beaten him. The

13. Moreover, there can be some deformations in my implementation of Figure 3. The ensuing percentage computations, however, are exact.

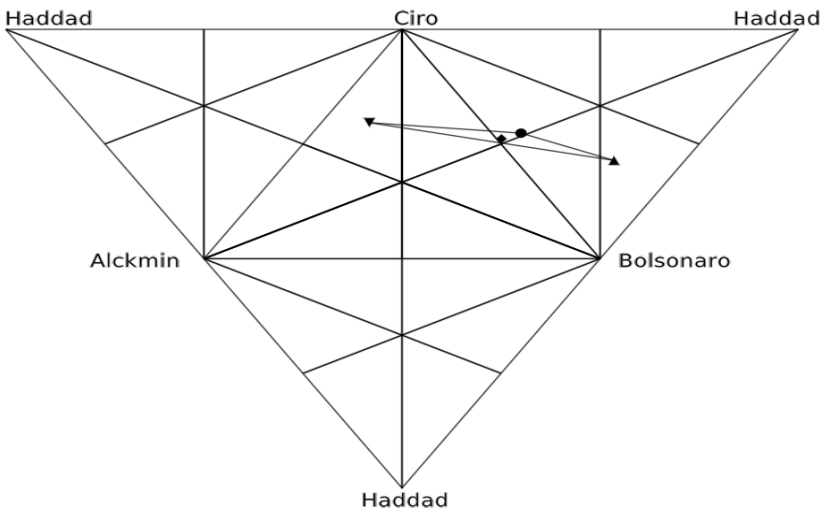


Figure 3: Saari's opened tetrahedron

explanation for that is the following: as shown in Figure 2b, Haddad and Bolsonaro were both divisive candidates, but Bolsonaro had more support than Haddad. They were not equally supported/rejected. Given that they were both divisive, most of their support was in the top choice, they would have fared equally well or badly under the same positional voting methods, but since Bolsonaro had more first votes and was less frequently in the bottom of the rankings than Haddad he actually "positionally dominated" Haddad. The same logic applies to another surprising result: Alckmin would never have beaten Ciro.

	Alckmin	Bolsonaro	Ciro	Haddad
Alckmin	0.0	0.31	0.0	0.58
Bolsonaro	0.69	0.0	0.47	1.0
Ciro	1.0	0.53	0.0	0.81
Haddad	0.42	0.0	0.19	0.0

Table 5: Proportion of victories in the positional voting procedure set

Naturally, proportions do not show what were the decision procedures in which, for instance, Ciro would have beaten Bolsonaro. Intuitively, voting procedures that emphasize rejection or more of the middle region of the rankings should give an advantage to inclusive candidates, which is qualitatively confirmed by Figure 3. Since the positional voting methods with four candidates are determined by their s_1 and s_2 weights, we can visualize all scenarios by varying them, as in Figure 4. It shows the scenarios Bolsonaro $\times$ Ciro, Ciro $\times$ Haddad, and Alckmin $\times$ Bolsonaro. Note that, as expected, the only way Alckmin could have beaten Bolsonaro would be if s_1 and s_2 were above 0.6. Remember that when both are 1, the voting procedure is antiplurality, a method equivalent to saying which candidate voters dislike. However, this universe of cases was dominated by Ciro, who would have beaten Bolsonaro in any combination of s_1 and s_2 higher than the line connecting the points $(0.51, 0.51)$ and $(0.9, 0)$. The plot also shows what combinations of weights lead to 81% of Ciro > Haddad: any combination to the right of the line segment connecting $(0.35, 0.35)$ and $(0.55, 0.0)$.

Nonetheless, the family of positional voting methods is not, in general, Independent of the Alternative Set (Kaminski 2015). If we drop or add candidates, the "social" ranking might change without respecting the ordering of the baseline set of alternatives. Consider

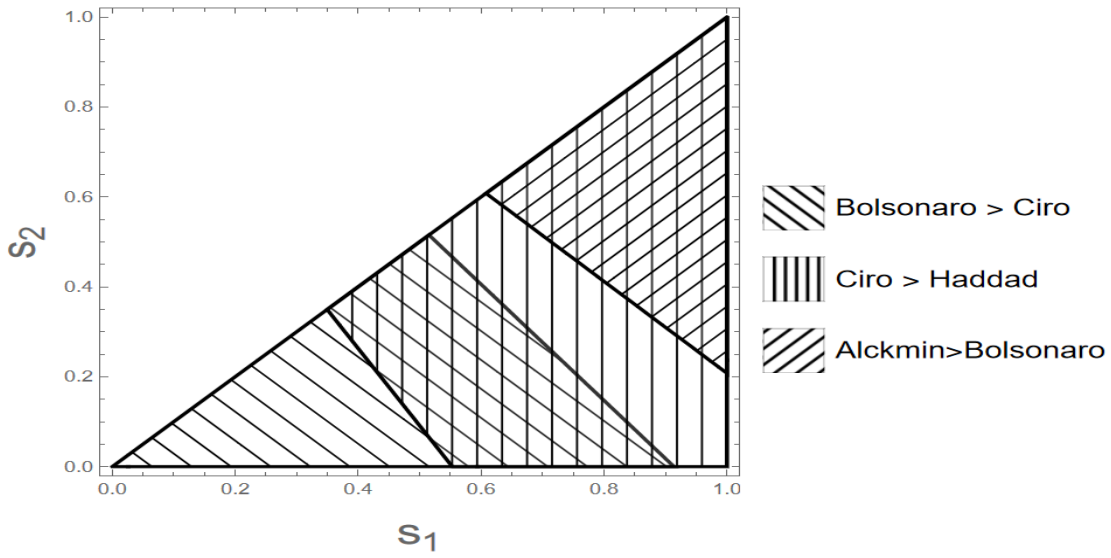


Figure 4: Victory in terms of values of s_1 and s_2

the Borda-induced social ranking in this case: Ciro $\sim$ Bolsonaro $>$ Haddad $>$ Alckmin. If by dropping Alckmin, the ranking changes to Bolsonaro $>$ Haddad $>$ Ciro $>$ Alckmin, then the Borda Count, in this case, would be inducing a "paradoxical" result. In Figure 5, I consider alternative scenarios by dropping one of the top 4 candidates.

The positional voting procedures are eminently well-behaved when dropping candidates from this dataset. There is a minor tilt toward Bolsonaro winning with the Borda Count in Figure 5d, but as I have previously argued, this seems like a tie, given the underlying uncertainty. Notice that in all scenarios where Bolsonaro is still in the alternative set, he would have been the plurality winner. However, he would have tied with Ciro under Borda and lost against him with decision procedures that put more weight on rejection, as in the 4 candidates analysis. In the scenario Ciro was not in the set, Bolsonaro would again only have lost against Alckmin, but in a minority of cases.

We have seen that besides having a high marginal mandate, Bolsonaro was also a CW. His victory, thus, was not a fluke or an artifact of institutional technology. Neither was he the candidate with the most positional mandate. This result revisits the Borda $\times$ Condorcet controversy. On the one hand, he was the CW, the primary normative benchmark for a voting procedure. On the other hand, in the Brazilian case, the Borda count would have been a more substantial barrier against a divisive candidate. Even though we could expect divisive candidates to have fared worse under informationally richer decision procedures, a divisive candidate can still be a CW with 47% positional stability/mandate. Yes, Ciro would have won against him in 53% of the positional methods, and at least would have tied with him in the Borda count, but most of the methods within this 53% emphasize rejection, and to give more weight to rejection vis-à-vis approval seems unreasonable under any set of normative expectations demanded of a decision procedure for large-scale democratic elections. Due to its symmetry, the Borda Count lies at a threshold: its constant decrease in assigning weights to the positions in the rankings guarantees that approval matters more than rejection, but without throwing away the rejection information. Therefore, highly polarized scenarios can lead to the election of a divisive candidate, which puts in dispute two reasonable metrics of support: being a CW vs. being a BW. This means the paper only gives partial support to the hypothesis that informationally richer decision procedures would be enough to contain divisive candidates, and two reasonable generalizations of the majoritarian credo end up in conflict.

However, Figure 5b presents an interesting scenario. Here, there is no conflict between

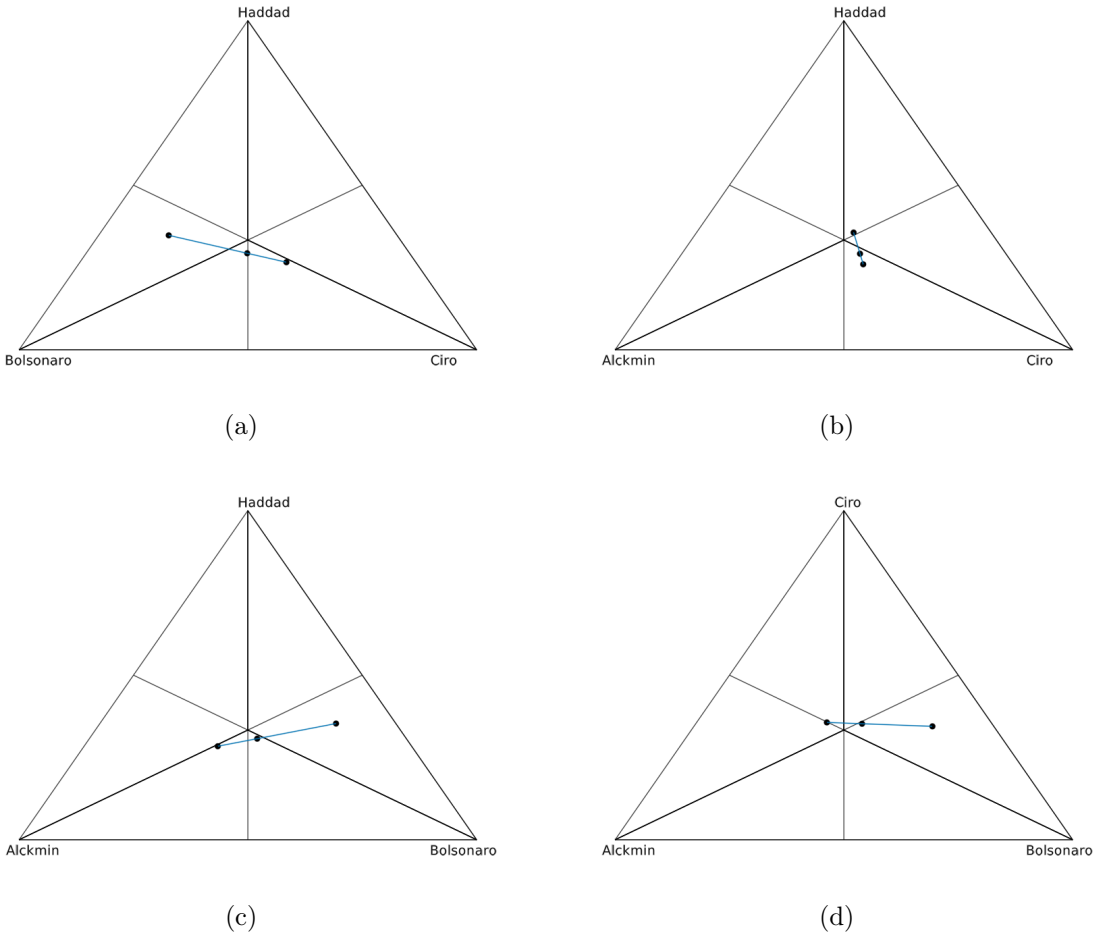


Figure 5: Positional results after dropping one candidate

the perspectives: under both positional and pairwise perspectives on a mandate, Haddad going to the second round was purely an institutional fluke. Even though it looks like Haddad would be a plurality winner if we dropped Bolsonaro, the plurality point is close to a tie with Ciro, and could be read as such, given the uncertainty. Ciro, thus, now would have almost 100% positional mandate. Moreover, in Table 3a it was shown that Ciro would have beaten him with a majority pairwise comparison, which gives credence to affirming that Ciro would have won under a majority with run-off system. In this scenario, the most inclusive candidate would have been elected. Ceteris paribus, it seems the only way an agreement between the Borda and Condorcet criteria could be guaranteed to exist in the Brazilian 2018 case would be if Bolsonaro had never been able to run. In this scenario, the Condorcet Loser would again be beaten, but now a candidate with a solid mandate, as endorsed both by the pairwise majority comparisons and the entire hull of positional methods would have been elected.

6 Conclusion

The paper contributes to the analysis of the institutional robustness of polyarchical systems by considering credible alternative voting procedures outcomes and properties at a critical juncture in Brazil's political history. First, I argued that the notions of pairwise and positional mandate can be derived from well-established axiological perspectives to evaluate whether an electoral result is solely an institutional fortuity. Then, I demonstrated that even though the aggregation procedure boosted Bolsonaro's victory, it was not merely

its effect, contrary to established theoretical expectations, but neither was he an undisputed winner under both aforementioned evaluative criteria for gauging the mandate of a candidate in democratic collective choice situations.

In terms of future research, contrasting Haddad with Lula and analyzing the effect of the knife attack should provide a more comprehensive picture of what happened in 2018. Moreover, the pipeline for the analysis is highly reproducible. Similar work can be done to surveys that contain pairwise comparisons between the top candidates, as many in Brazil do. As such, any other majoritarian election in Brazil could be analogously analyzed. Figure 5b could also be the starting point for an investigation of the selection of the pool of candidates allowed to compete for the Executive, particularly in transitional democracies, given Brazil's lax transitional justice, and Bolsonaro's intimate connection with the remnants of the Old Regime. First, from a positive point of view as a causal pathway for democratic backsliding (Svolik 2008; Nalepa 2022). Second, from a normative perspective. What, if anything, justifies restricting classes of actors from running for certain positions? What values would conflict here?

The most glaring limitation of that paper is that I used only one variable from the dataset, pairwise comparisons, to simulate alternative scenarios. However, socio-demographic variables from the dataset could have been used to strengthen the data imputation procedure. Moreover, roughly less than half of the dataset is constituted of incomplete pairwise comparisons, and there may be valuable information on the agent's preferences contained in patterns of missingness.

Another limitation is that agents adapt to new institutional environments. I am ignoring strategic voting by assuming a direct translation between preferences and behavior. However, the percentage of strategic voting in a large-scale election is an open empirical problem (Straeten et al. 2010; Kawai and Watanabe 2013). Nevertheless, a combination of game-theoretic models with a simulation parameterized by the inferred ranking distribution is a route of research that could be pursued.

Though I have analyzed the four top candidates, there can be discrepancies when we have a subset of the alternatives vs. when we have the whole set of candidates (D. Saari 2001; Kaminski 2015). It is well-known, for instance, that the Borda Count is susceptible to the winner-turns-loser paradox. Finally, even though I have analyzed scenarios in which candidates were removed, it would have been more realistic to simulate the formation of coalitions and how voters would have reacted to that (Kaminski, Lissowski, and Swistak 1998). The assumption of a pure additive transfer of votes, implicit when we removed candidates, is not necessarily valid with coalitions, insofar voters of a center-left candidate, for instance, could vote for the center-right candidate if they are alienated by an alliance with the Left, which, in the case of the election under scrutiny, was highly rejected.

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