Working Paper

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1 Introduction

The rise of polarization, the specter of democratic backsliding, and more generally an apprehension with the survival prospects of democratic polities have lead to a renewed interest on what institutions mitigate vs instigate destabilizing dynamics, or that increase the adaptability of the political system vis-á-vis both internal and external stressors (Bednar 2021; Przeworski 2019; Chiopris, Nalepa, and Vanberg 2021; Aligica 2014). Given their centrality to the input-output between society and the state, electoral institutions naturally figure among the set of such institutions under scrutiny (Wang et al. 2021), and a strand of the literature in collective choice has wondered whether the current electoral victories of divisive candidates have not been an effect of informationally poor decision procedures (Potthoff and Munger 2021; Kurrild-Klitgaard 2018; Woon et al. 2020). This paper shows, however, that in the 2018 presidential election in Brazil, a highly polarized one, the election of a polarizing candidate, Jair Messias Bolsonaro, was not an artifact of the decision procedure.

Despite the multiple historical contingencies that might explain his victory, one might naturally wonder what role the electoral system has had in it. After all, it is well-known that the outcome of collective choices is fundamentally dependent on the voting procedure (Riker 1982). As Brazil's majority with run-off system, most electoral systems use merely the first index of the voters' preference rankings (Grofman and Feld 2004). How would he have fared under informationally richer voting procedures, such as pairwise comparison methods and positional voting procedures? Was he, and arguably other democratically elected destabilizing candidate, a product of decision procedures that favor divisive candidates over more inclusive ones (Igersheim et al. 2022)? Would the result have changed with a different set of candidates?

To answer those questions, I make use of a pre-electoral representative survey to reconstruct the full 4-top rankings of the Brazilian population, using a preference learning model (Sørensen et al. 2019). Subsequently, I use this augmented data to simulate the electoral results under both the Borda and Condorcet procedures, and simulate the result for all positional procedures in the top 4 and 3 candidate sets. Finally, I discuss the significance of the results and conclude by pointing out the limitations of the endeavor.

2 Theory

There are multiple ways in which electoral institutions might affect the polity. Even if we restrict our attention to a single rule, the aggregation procedure that transforms ballots into a choice of a candidate (Goodin and List 2006), two perspectives of analysis ensue: a static and a dynamic. From a static point of view the decision procedures respect differing sets of criteria such as monotonicity, neutrality, anonimity, manipulability and so on (Nurmi 1999). From a dynamic point of view, both politicians and voters continuously adapt to the rule in a system of feedbacks between the state and society (Wang et al. 2021). In this article, I'll focus on two ways a decision procedure might reinforce destabilizing dynamics: by electing polarizing candidates and by electing candidates with weak mandate.

A polarizing or divisive candidate has strong support in the top choice of voters, but also has a high share of bottom choices among the electorate (Igersheim et al. 2022). The notion of a mandate of a candidate lets us situate both methods vis-à-vis the one-choice informational environment typical of large-scale majoritarian elections. At a minimum, a candidate has a mandate as long as it has won under the voting procedure. A candidate has more mandate, however, the more significant the difference between its vote share or score vs. the second most well-voted candidate.

The destabilizing effects of electing divisive candidates or candidates with weak mandate are well-documented in the literature (Kaminski 2015; Lührmann et al. 2018; Baldassarri and Page 2021; Bednar 2021). The focus of the paper is on the effect of decision procedures on that phenomenon. But why would, after all, voting procedures have such effect? For both a weak mandate and a polarizing candidate in the same example, suppose there are 11 voters with preference A $>$ B $>$ C $>$ D, 10 voters with B $>$ C $>$ D $>$ A, 10 voters with C $>$ B $>$ D $>$ A, 10 voters with D $>$ B $>$ C $>$ A. A is naturally the plurality winner, even though it was only the top choice of 0.27% of the electorate and was the bottom choice of the rest. It would lose in pairwise comparisons to all other alternatives. In this example, the alternative that would win in all such pairwise comparisons would be B, a Condorcet Winner(BW). If we assigned weights $(3, 2, 1, 0)$ to positions in the ranking and summed the scores B would again the be winner, in this case the Borda Winner (BW).

Note that being a CW is an even more robust notion of having a mandate: if the candidate is a CW, then it would have won under all possible majority pairwise comparisons against the other candidates. The BW lends itself to a similar interpretation. Suppose a candidate wins under a voting procedure that only uses the top choice of the electorate, but is neither a BW nor a CW. In that case, it has less mandate, in this generalized majoritarian perspective, than if it is both, which signals a comprehensive social basis. In the former case, a candidate who wins under the current voting procedure, but is neither a BW nor a CW could be considered an artifact of the procedure. In the latter case, on the other hand, the procedure would be just "tracking" a broader pattern of support for the alternative.

This notion of mandate can be strengthened in the case of the Borda Count. The Borda count can be seen as one method within a family of methods that assign weights to positions in the ballot. In one extreme, the plurality voting method assigns score 1 to the top choice and 0 to all others. On the other extreme is the antiplurality voting method, which assigns 1 to all positions besides the last one. Between the two extremes are all possible ways of assigning a score to the ballots of the electorate. The higher the proportion of positional voting systems that the candidate would have won had the election used it, what Tabarrok (2001) has called positional stability, the higher the mandate of the candidate.

Note that both notions, CW and BW, require a broader informational input from voters (their full ranking), while outputting a result, in principle, more robust to divisiveness (since rejection is taken into account) and more refined in terms of mandate (since we've moved from simply winning to winning either in all pairwise comparisons or even in most or all possible positional voting procedures). This broader informational backdrop underlies current research on the case of the United States and Donald Trump's electoral victory. Regardless of the specifities of each paper, all pressupose that the informational paucity of only focusing on top choices blinds the States' socio-technical translation of popular support into political input (the choice of candidate). For instance, Potthoff and Munger (2021), Woon et al. (2020), and Kurrild-Klitgaard (2018) debate whether Donald Trump was a Condorcet Winner in the primaries, with recommendations of voting procedures that better track what is the CW after all. Igersheim et al. (2022) goes a step further: they argue that not only Trump was neither, but that Sanders was the actual Borda and Condorcet

Number of Pairwise Comparisons	Frequency
1	15
2	42
3	462
4	118
5	503
6	1797

Table 1: Frequency of pairwise comparisons in the dataset.

Winner, and generally the "best" candidate, if by best one understand a candidate being the most inclusive and winning under the most alternative decision procedures. We will see, however, that no such conclusion can be drawn in the Brazilian case.

3 Case/Data

Jair Messias Bolsonaro was elected the president of Brazil in 2018. For more than 20 years as a congressman, he was primarily low clergy politician defending the interests of the military and local police forces of the state of Rio de Janeiro. The 2018 electoral scenario in Brazil was one of high rejection of the traditional political elite, particularly of the Labour Party (Partido dos Trabalhadores - PT), after corruption scandals and an impeachment process of the previous president, Dilma Roussef, a Labor politician. The main contestants, among 13, were him, a rightist candidate; Fernando Haddad, a leftist candidate from PT; Geraldo Alckmin, a center-right candidate; and Ciro Gomes, a center-left candidate. The presidential election in Brazil follows a two-round system. In the first round, 8.79% of the votes were White/Null, which means the voting procedure does not count them. The result of the valid votes was the following: Bolsonaro:Haddad:Ciro:Alckmin:Others = 46.3 : 29.28 : 12.47 : 4.76 : 7.19. Among the 9 other candidates, the highest vote share was João Amoêdo's with 2.5%. All others got less than 1%. Moreover, There was a 20% abstention. In the second round the result was: Bolsonaro:Haddad = 55.12% : 44.78%. White/Null votes were 9.57% of the total electorate. The abstention in this round was 21.3%.

The dataset used for the analysis is a representative street survey done on 10/02/2018, less than a week before the first round (10/07/2018). DataFolha did this survey, an independent research institute highly esteemed and trusted by brazilian experts[1]. One question, in particular, is the only variable in our analysis: pairwise comparisons between the 4 top candidates. With it is possible to reconstruct the full 4-top ranking of the voter. Preliminary pre-processing has led me to drop 171 observations where all pairwise comparisons were missing and 132 in which they were cyclic. This leaves us with 2937 out of 3240 observations. As Table 1 shows only 1797 observations compared all 4 candidates. As such, we have to augment the data with transitive closures for 1140 observations, by methods discussed in the next section.

4 Methods

I use a preference learning model to infer the incomplete rankings. A preference learning model is a statistical model tailored for the estimation of a "consensus" ranking from the data, the imputation of incomplete rankings, and for the clustering of the data intro groups for predictive purposes (Liu et al. 2019). Particularly, I use the Bayesian Mallows model as

1. I had access to the survey data, code-book and questionnaire by creating an account and requesting access to them, available for educational/research purposes, at https://www.cesop.unicamp.br.

implemented[2] in the BayesMallows R package (Sørensen et al. 2019; Lu and Boutilier 2011). The idea is to use a statistical model with distributions over the space of permutations P_n. The likelihood, $P(\mathbf{r} \mid \alpha, \rho)$, of a voter profile R, is a pdf conditional on what is the consensus ranking in the population,ρ, and what is the concentration around the consensus rankig α. The model assumes a exponential decay for the probability of a ranking happening as the distance between it and the consensus increases. Thus, for a prior for α an exponential distribution is assumed, while for ρ an uniform prior is assumed. A joint posterior is then numerically calculated with a MCMC algorithm (Sørensen et al. 2019). Finally, the distribution is used to infer latent rankings for the incomplete rankings seen in the sample[3] (Sørensen et al. 2019). I ran the chain for 5000 iterations, a standard in bayesian inference.

TODO: appendix with diagnostics

After the rankings are inferred, I compute the Borda Count and Condorcet Procedure for the top 4 candidates, calculate the victory scenarios for all positional voting methods, and visualize the results for 3 candidates using Saari's representation triangles[4] (D. G. Saari 2012).

Saari's representation triangles provide a way of visualizing all possible positional voting results of an election[5]. Consider Figure 1. The closer to a vertex, the better the vertex's position in the social ranking. Region 1 corresponds to the social ranking of $A > B > C$, while Region 4 corresponds to the social ranking of $C > B > A$. The lines separating the regions represent indifference. The point in which all lines meet corresponds to $A \sim B \sim C$, while the line separating Region 1 and 2 would correspond to $A > B \sim C$. The three dots are the results for the antiplurality, the borda and the plurality voting methods. The line connecting the antiplurality and plurality results, the extremes, denotes all possible positional results, including the result for the Borda Count. The Borda Count point is always closer to the antiplurality result. In this example, most positional voting methods would have agreed with the plurality procedure outcome of B as the winner.

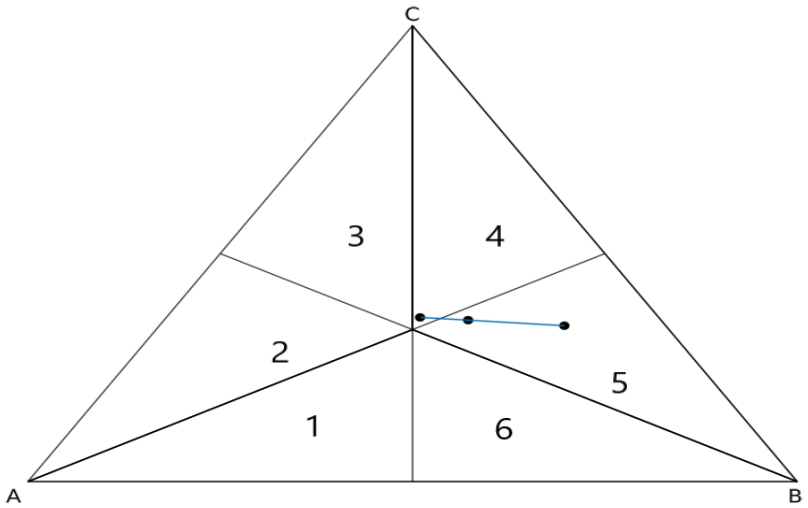

Figure 1: Saari representation triangle

To calculate all positional voting victories I use two facts proved by Donald Saari

2. See https://github.com/ocbe-uio/BayesMallows.

3. The data is augmented while the markov chain is running, and is continuously updated throughout the algorithm loop. I just recover the last iteration of the chain.

4. It is also possible to represent the result for 4 candidates in a 2d plot by opening the tetrahedron of possible results(D. Saari 2001). However, this is a work in progress.

5. For a complete exposition of this method see D. G. Saari (1995) or Nurmi (2002).

(D. G. Saari 2012; D. Saari 2001): first, any positional voting method for 4 candidates can be seen as assigning weights to ranking positions in a standardized manner $(1, s_1, s_2, 0)$, where $0 \leq s_2 \leq s_1 \leq 1$; second, all such procedures will lie in the convex hull of the plurality, antiplurality and vote for two procedures, with respective weights of $(1,0,0,0), (1,1,1,0), (1,1,0,0)$. Calculating scenarios amounts, thus, to just vary the values of s_1 and s_2.

A further complication is a mismatch between the survey's plurality result and the actual result of the first round. This is typical in surveys, and might be due to strategic voting, social desirability bias (not wanting to be seen as "polarized"), or systematic refusal of part of the electorate to answer the survey. Any imputation technique will reproduce this top choice discrepancy, thus I'll show directly the mismatch between the imputation and the actual vote[6]. I ran the imputation algorithm on the top 4 candidates, which means that in the sample the 7.19% that voted for others is kept constant. The result after applying the algorithm is Bolsonaro:Haddad:Ciro:Alckmin:Others = 36.7.7% : 17.3% : 14.1% : 7.19%. Thus, Bolsonaro and Haddad are undervoted in the sample, while Ciro and Alckmin are overvoted[7]. If we were simply transferring the top choices from over-voted to under-voted we could simply, say, transfer Alckmin → Bolsonaro and Ciro → Hadddad. However, there are 24 permutations of the top 4 candidates, with 6 for each candidate in which they are the top alternative. This gives leeway to many possible transfers. I have designed an algorithm, described in Appendix A, that, while respecting who can transfer and who needs votes, respects the Kemeny's Distance between the rankings and picked the transferred table that minimizes the euclidean distance between the inferred table and the actual election. Two such inferred tables lead to the following proportion: Bolsonaro:Haddad:Ciro:Alckmin:Others = 46.19 : 29.32 : 12.51 : 4.77 : 7.19. Since the application of electoral procedures to those tables leads to qualitatively similar results, I'll just present the results for one of them. The results for the other is in Appendix C

5 Results

The inferred rankings[8] are summarized in Figure 2. Indeed, the candidates that went to the second round were the most divisive ones. Ciro, however, could be considered more inclusive than Alckmin, since the proportion of last choices within Alckmin's subset of voters is higher than Ciro's. There is also a relevant distinction among the divisive candidates: Haddad's rejection was higher than his top-choice support, while the opposite held for Bolsonaro.

6. In a further version of this working paper I'll show this directly, then contrast with the imputed data. They, thankfully, match,I'm just time-constrained.

7. Remember the actual result was Bolsonaro:Haddad:Ciro:Alckmin:Others = 46.3 : 29.28 : 12.47 : 4.76 : 7.19.

8. The whole table is shown in Appendix B.

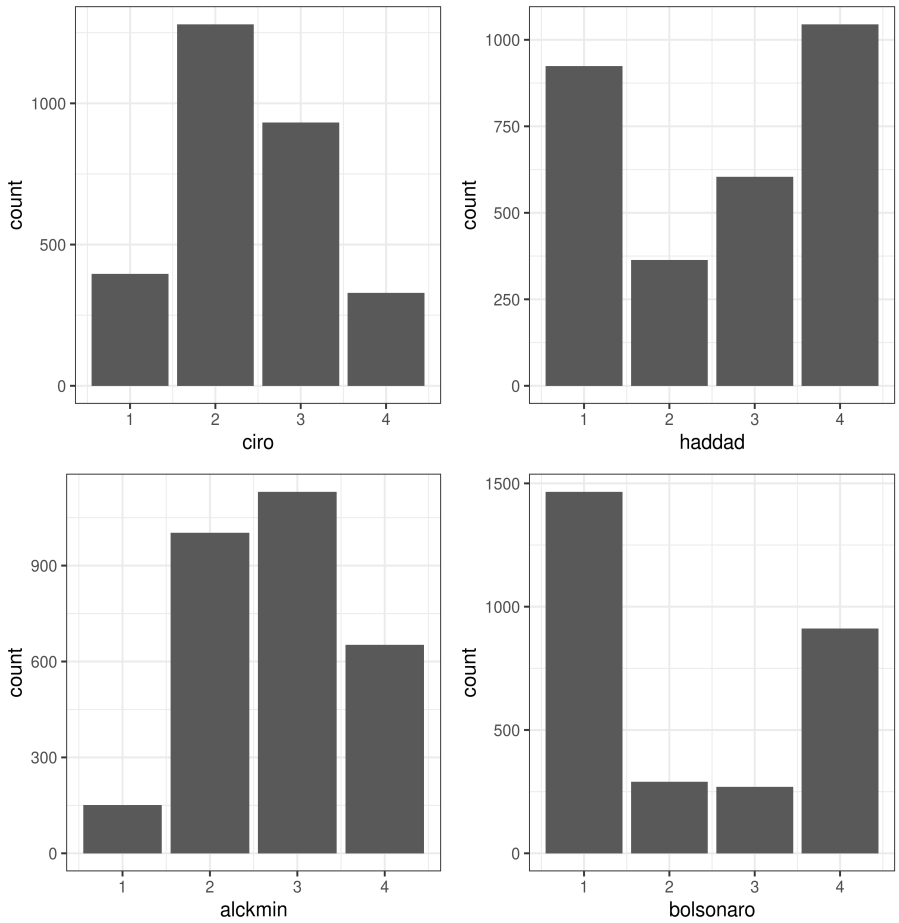

Figure 2: Frequency candidates appear at each position

But what does this support distribution mean from the point of view of the Borda and Condorcet procedures? Table 2 shows what we can infer from the imputed data. Despite being a divisive candidate, Bolsonaro would have won in all pairwise majority comparisons against any of the other top candidates. Haddad, however, would have lost against Ciro, who would only have lost against Bolsonaro. Unlike what was widely believed at the time, and was the motto of his campaign, Ciro most likely would not have won against Bolsonaro in a second round. He was not the "anti-Bolsonaro", but merely an "anti-Haddad", together with Bolsonaro. Alckmin, the candidate with the longest television time and the broadest supporting coalition would have lost against any other top candidates. He was the Condorcet Loser in the top set. The same pattern is reflected in the Borda Scores: Bolsonaro > Ciro > Haddad > Alckmin.

	alckmin	bolsonaro	ciro	haddad
alckmin	0	-1	-1	-1
bolsonaro	1	0	1	1
ciro	1	-1	0	1
haddad	1	-1	-1	0

(a) Pairwise Majority Comparisons

	borda score
alckmin	6527
bolsonaro	8185
ciro	7617
haddad	7041

(b) Borda scores

Table 2: Borda and Condorcet results for final inferred ranking 1

The counterfactual analysis can be deeper in the case of positional voting methods. As discussed in the methods section, ith 4 candidates, all results will lie in the convex hull of

candidates	w_s **tallies**
Alckmin	$0.3415s_1 + 0.385s_2 + 0.0514$
Bolsonaro	$0.0988s_1 + 0.0919s_2 + 0.499$
Ciro	$0.4358s_1 + 0.3173s_2 + 0.1348$
Haddad	$0.1238s_1 + 0.2057s_2 + 0.3147$

Table 3: w_s vector of each top 4 candidate

candidates	**alckmin**	**bolsonaro**	**ciro**	**haddad**
alckmin	-	0.05	0.0	0.25
bolsonaro	0.95	-	0.69	1.0
ciro	1.0	0.31	-	0.76
haddad	0.75	0.0	0.24	-

Table 4: Percentage of positional victories of row against column

three positional voting procedures: plurality, antiplurality and vote for two. Note that the score of a candidate will be of the form $a + bs_1 + cs_2$, where a is the share it received of votes in the first position, b in the second, and d in the third position of voters rankings. Therefore, the scores of each candidate in the inferred ranking for the 2018 election can be found by assigning values to the equations of Table 3. For instance, if we set $s_1 = s_2 = 0$ we recover the plurality score, after ignoring "Other" candidates.

Algebraic manipulation allows us to, then, answer the following question: Under what scenarios would candidate A have beaten candidate B, in the universe of positional voting methods? Table 4 gives the percentage of secenarios in which this would have happened. It shows that even though Bolsonaro was both the Borda and Condorcet Winner, there were scenarios in which he would have lost to the more inclusive candidates, Ciro and Alckmin. In Alckmin's case this would only have happened in 5% of the cases. However, Ciro could have beaten him in $\approx 30\%$ of the positional voting methods. Surprisingly, Haddad, who went to the second round with Bolsonaro would never have beaten him. The explanation for that is the following: as shown in Figure 2, Haddad and Bolsonaro were both divisive candidates. However, Bolsonaro had more support than Haddad. They were not equally supported/rejected. Given that they were both divisive, most of their support was in the top choice, they would have fared equally well or badly under the same positional voting methods, but since Bolsonaro had more first votes and was less in the bottom of the rankings than Haddad he actually "positionally dominated" Haddad. The same logic applies to a other surprising result: Alckmin would never have beaten Ciro.

Naturally, percentages don't show what scenarios, say, Ciro would have beaten Bolsonaro. Intuitively, we expect that voting procedures that emphasize rejection or more of the middle region of the rankings should give advantage to inclusive candidates. Since the positional voting methods with four candidates are determined by their s_1 and s_2 weights, we can actually visualize all scenarios by varying them, as in Figure 3. It shows the scenarios Bolsonaro × Ciro, Ciro × Haddad, and Alckmin × Bolsonaro. Note that as expected the only way Alckmin could have beaten Bolsonaro would be if s_1 and s_2 were close to 0.8. Remember that when both are 1 the voting procedure is equivalent to saying which candidate you don't like. However, this universe of cases was dominated by Ciro, who would have beaten Bolsonaro in any combination of s_1 and s_2 higher than the line connecting the points $(0.64, 0.64)$ and $(1, 0.12)$. The plot also shows what combinations of weights lead to 76% of Ciro > Haddad: any combination of weights higher than the line segment between $(0.42, 0.42)$ and $(0.57, 0.0)$.

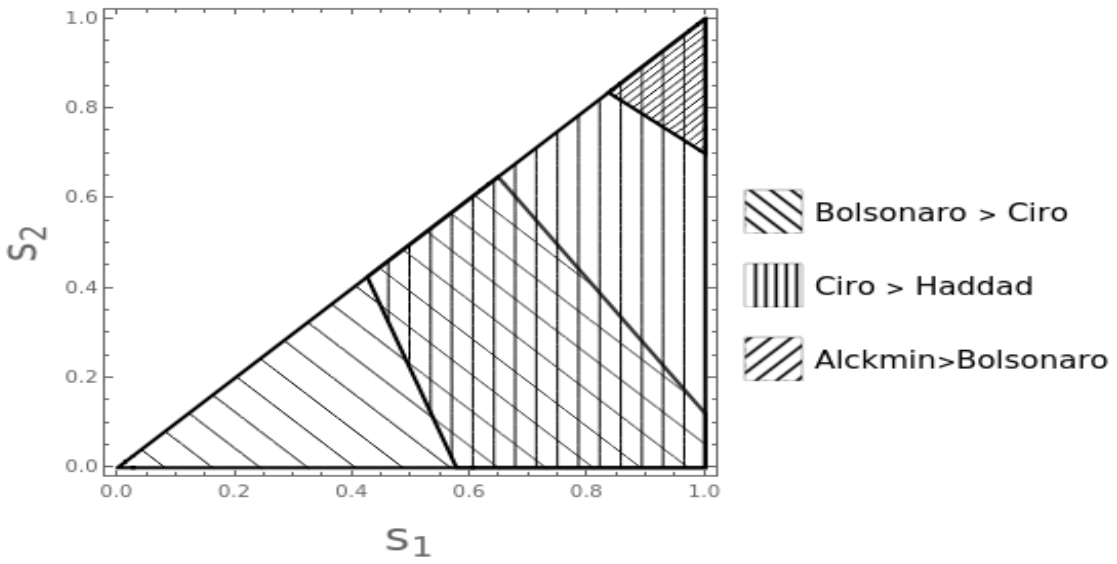

Figure 3: Victory in terms of values of s_1 and s_2

However, neither the CW nor the family of positional voting methods are, in general, Independent of the Alternative Set(Kaminski 2015). That is if we drop or add candidates, the "social" ranking might change without respecting the ordering of the baseline set of alternatives. Consider the Borda-induced social ranking in this case: Bolsonaro > Ciro > Haddad > Alckmin. If by dropping Alckmin, the ranking changes to Ciro > Bolsonaro > Haddad, then the Borda Count, in this case, would be inducing a "paradoxical." result. In Figure 4 we consider alternative scenarios by dropping one of the top 4 candidates. Not only can we determine if there is any "alternative set stability", but also take into account the positional stability of the result.

In terms of alternative set stability, the positional voting procedures are eminently well-behaved in this case. The only reversal is the antiplurality ranking if Haddad had dropped. In this case, Figure 4 (d), the antiplurality ranking was Ciro > Alckmin > Bolsonaro > Haddad, but now the ranking is Ciro > Bolsonaro > Alckmin. The reason is that Haddad's votes were most likely transferred more to Ciro than to Alckmin. The antiplurality point, however, is close to a tie between Bolsonaro and Alckmin, which means this can also be an artifact of sampling uncertainty.

What about positional stability across the alternative sets? Notice that in all scenarios where Bolsonaro is still in the alternative set he would have been both the plurality and the Borda winner, and most positional voting methods would have elected him. The only counterfactual positional results in which he would not have been elected is if a voting procedure that emphasized rejection had been used, and Ciro would then have been elected. Those scenarios, however, are in these cases, Figure 4 (a,c,d), always a minority. His victory, thus, was not a fluke, or an artifact of institutional technology. Not only he would have won under both Borda and Condorcet methods, but his victory was also stable under both alternative sets of alternatives and alternative positional methods. Even though electoral systems based on just the first positions of citizens' preferences do ignore, by definition, the distribution of support the candidates have throughout the whole rankings, and we could expect that divisive candidates would fare worse under informationally richer decision procedures, a divisive candidate can still be a CW with high positional stability. Therefore, highly polarized scenarios can lead to the election of a divisive candidate regardless of which reasonable decision procedure is used[9].

9. Emphasizing rejection rather than overall support across the ranking (as in, say, the Borda Count)

However, Figure 4 (c) presents an interesting scenario. Though, as expected, Haddad would have been the plurality winner[10], now his victory would not have been positionally stable. In Table 2a it was shown that Ciro would have beaten him with a majority pairwise comparison, which gives credence to affirming that Ciro would have won under a majority with run-off system. In this scenario the most inclusive candidate would have been elected. It seems the "only" way Bolsonaro would not have been elected in the 2018 election would have been if he never had actually been able to run. Given how Brazil sticks out for its lack of transitional justice and Bolsonaro's strong ties with the Old Regime this might be a lesson history has taught to the country.

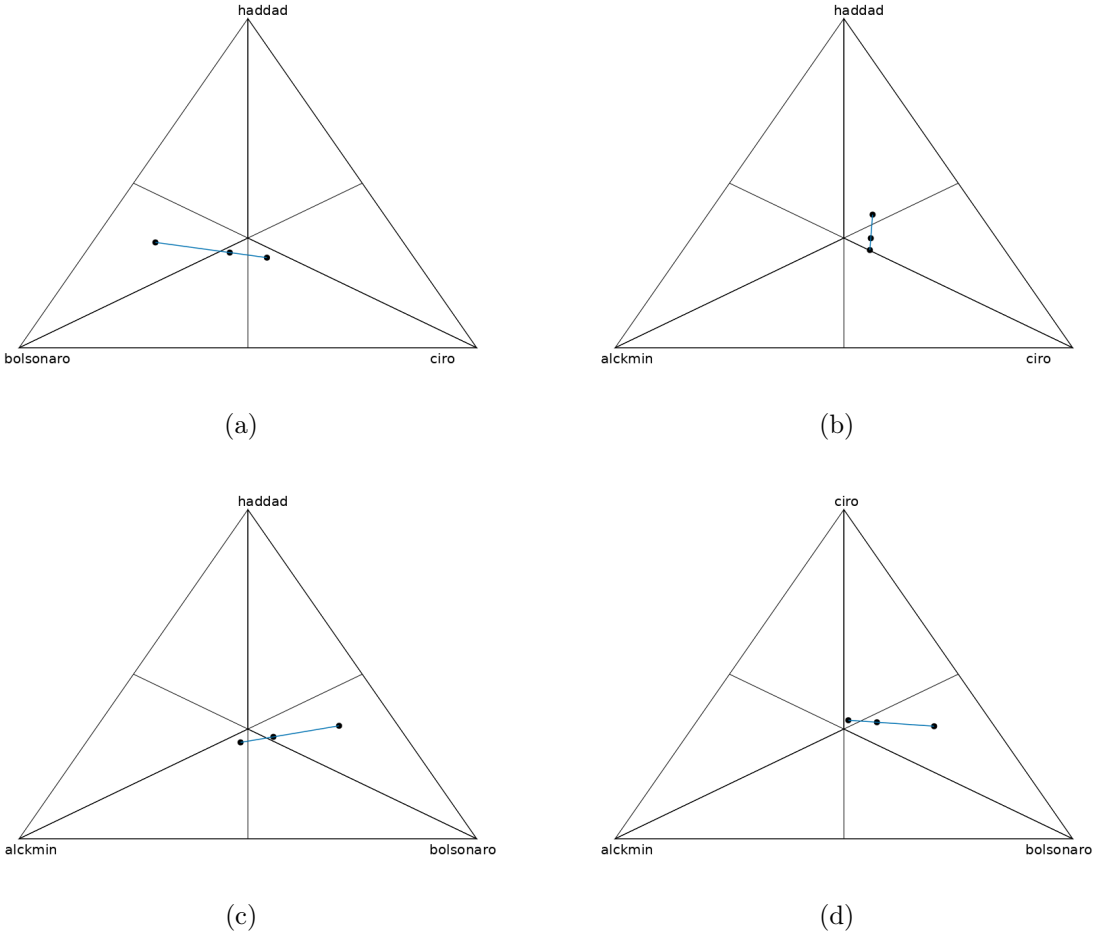

Figure 4: Positional results after dropping one candidate

6 Conclusion

The paper contributes to the analysis of the institutional robustness of polyarchical systems, by considering credible alternative voting procedures' outcomes and properties at a critical juncture in Brazil's political history. We have demonstrated empirically that, contrary to established theoretical expectations, Bolsonaro's victory was not caused by the decision procedure.

seems unreasonable under any set of normative expectations demanded of a decision procedure for large scale democratic elections.

10. As shown in Figure 6 in Appendix C, this is a case in which the vote transfers differ. In the alternative vote transfer, in this scenario, Ciro would have tied with Haddad in the plurality case, but would have won under all other positional voting methods.

The most glaring limitation of the paper is that agents adapt to new institutional environments. By assuming a direct translation between preferences and behavior we are ignoring strategic voting. The percentage of strategic voting in a large scale election, however, is an open empirical problem (Straeten et al. 2010; Kawai and Watanabe 2013). Nevertheless, a combination of game-theoretic models with a simulation parameterized by the inferred ranking distribution is a route of research that I intend to pursue.

The research used only one variable from the dataset, the pairwise comparisons, to simulate alternative scenarios. However, socio-demographic variables from the dataset could have been used to strengthen the data imputation procedure. Speaking of imputation, simpler imputations, such as the Impartial Culture assumption (Regenwetter et al. 2006), could have been used as benchmarks to compare with the imputation through the bayesian mallows model. Moreover, it is necessary to properly analyze the pattern of missingness of pairwise comparisons. Roughly less than half of the dataset is constituted of incomplete pairwise comparisons, and there might be valuable information on the agent's preferences contained in patterns of missingness (McElreath 2020).

Other voting procedures could have been used. Particularly, truncated positional voting methods could have been directly applied to the raw data (Terzopoulou and Endriss 2021). Moreover, we have been disregarding indifference in the agent's preferences. Again, this is valuable information, and it is known that forcing strict preferences when indifference exists leads to artificial inflation of the profile inconsistency (Gehrlein 2010). Finally, though we have analyzed the four top candidates, there is no direct relationship between the result of an election with positional procedures when we have a subset of the alternatives vs when we have the whole set of candidates (D. Saari 2001). It is well known, for instance, that the Borda Count violates WARP precisely because it is not contraction-expansion consistent (Schwartz 2018). Nonetheless, we expect that the results will not reverse, given that the bottom candidates could be deemed irrelevant to most of the population (and as such, be tied in the bottom of the rankings), and the patterns seen when dropping only one candidate. This is a subject for a more thorough analysis, in any case. Finally, even though we have analyzed scenarios in which candidates were removed, and alternative voting procedures could have been used, it would be more realistic to simulate the formation of coalitions and how voters would have reacted to those. The assumption of a pure additive transfer of votes, implicit when we removed candidates, is not necessarily true with coalitions, insofar voters of a center-left candidate, for instance, could actually, vote for the center-right candidate if they are alienated by an alliance with the Left, which in the case of the election under scrutiny, was highly rejected, as shown in our analysis.

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A The vote transfer algorithm

If we are to transfer from Alckmin to Bolsonaro we are lead to the problem of first picking which ranking at the source should be chosen then which ranking at the target should receive votes, while respecting how much the source has in excess and how much the target needs[11]. A natural sorting of which ranking should be the source is the position of Bolsonaro in the ranking. We start with rankings in which he is in the second position ((Alckmin, Bolsonaro, Ciro, Haddad), (Alckmin, Bolsonaro, Haddad, Ciro)), then third position ((Alckmin, Ciro, Bolsonaro, Haddad), (Alckmin, Haddad, Bolsonaro, Ciro)), then last position ((Alckmin, Ciro, Haddad, Bolsonaro), ((Alckmin, Haddad, Ciro, Bolsonaro))). Let's call those sets the sorted rankings sets. Suppose we picked a source ranking from the first sorted rankings set. What should be the target ranking, among the rankings which have Bolsonaro as the first choice? We transfer to the ranking that has minimal Kemeny's distance to the source ranking (Nurmi 2002). The Kemeny distance counts the number of transpositions (switching of pairs) needed to go from one permutation to another permutation. Thus, we transfer from the source ranking the min(number of votes the source ranking has, the total number of votes the under-voted needs, the total number of votes the over-voted can give). We update the source ranking frequency, the target ranking frequency, the total number of votes the under-voted needs, and the total number of votes the over-voted can give. If the under-voted doesn't need any other votes we stop and go to another over-voted $\rightarrow$ under-voted transfer. If not we check if the over-voted can still transfer votes to the current target under-voted. If yes we pick another source ranking in the sorted rankings sets and repeat until either the source has run out of votes it can given or the target has received enough votes. If not we go to another over-voted $\rightarrow$ under-voted transfer. In the end, this leads to 24 possible transfer sequences from over-voted to under-voted. One possible sequence is Alckmin $\rightarrow$ Bolsonaro, then Alckmin $\rightarrow$ Haddad, then Ciro $\rightarrow$ Haddad, then Ciro $\rightarrow$ Bolsonaro. Another possible sequence is Alckmin $\rightarrow$ Bolsonaro, then Alckmin $\rightarrow$ Haddad, then Ciro $\rightarrow$ Bolsonaro, then Ciro $\rightarrow$ Haddad. This leads to 6 transfers that minimize the euclidean distance between the inferred plurality result and actual result of the first round. However, there are actually two equivalence classes among those 6: 3 have the same proportion for each ranking, while the other 3 have the same proportion for each ranking. The new inferred proportion for both classes is: Bolsonaro:Haddad:Ciro:Alckmin:Others $= 46.19 : 29.32 : 12.51 : 4.77 : 7.19$.

11. In a further version of this paper I'll describe this with the help of a pseudo-code of the algorithm. SHOULD I?

B Inferred Ranking Table

ranking vectors	percentage
haddad >ciro >alckmin >bolsonaro	17.19
bolsonaro >alckmin >ciro >haddad	13.99
bolsonaro >ciro >alckmin >haddad	13.72
bolsonaro >ciro >haddad >alckmin	8.85
haddad >alckmin >ciro >bolsonaro	7.12
bolsonaro >haddad >ciro >alckmin	6.26
ciro >alckmin >haddad >bolsonaro	5.04
bolsonaro >alckmin >haddad >ciro	4.36
haddad >ciro >bolsonaro >alckmin	3.51
alckmin >bolsonaro >ciro >haddad	2.79
bolsonaro >haddad >alckmin >ciro	2.72
ciro >alckmin >bolsonaro >haddad	2.72
ciro >bolsonaro >alckmin >haddad	2.04
ciro >haddad >alckmin >bolsonaro	1.67
haddad >bolsonaro >ciro >alckmin	1.57
alckmin >bolsonaro >haddad >ciro	1.5
ciro >haddad >bolsonaro >alckmin	1.19
haddad >bolsonaro >alckmin >ciro	1.16
haddad >alckmin >bolsonaro >ciro	0.92
ciro >bolsonaro >haddad >alckmin	0.82
alckmin >haddad >bolsonaro >ciro	0.54
alckmin >ciro >bolsonaro >haddad	0.31

Table 5: Inferred rankings

C The results for the other inferred table

ranking vectors	percentage
bolsonaro >alckmin >ciro >haddad	13.99
bolsonaro >ciro >haddad >alckmin	13.69
haddad >ciro >alckmin >bolsonaro	12.36
bolsonaro >ciro >alckmin >haddad	11.1
haddad >alckmin >ciro >bolsonaro	9.33
haddad >ciro >bolsonaro >alckmin	6.13
ciro >alckmin >haddad >bolsonaro	5.04
bolsonaro >alckmin >haddad >ciro	4.36
bolsonaro >haddad >ciro >alckmin	4.05
alckmin >bolsonaro >ciro >haddad	2.79
bolsonaro >haddad >alckmin >ciro	2.72
ciro >alckmin >bolsonaro >haddad	2.72
ciro >bolsonaro >alckmin >haddad	2.04
ciro >haddad >alckmin >bolsonaro	1.67
haddad >bolsonaro >ciro >alckmin	1.57
alckmin >bolsonaro >haddad >ciro	1.5
ciro >haddad >bolsonaro >alckmin	1.19
haddad >bolsonaro >alckmin >ciro	1.16
haddad >alckmin >bolsonaro >ciro	0.92
ciro >bolsonaro >haddad >alckmin	0.82
alckmin >haddad >bolsonaro >ciro	0.54
alckmin >ciro >bolsonaro >haddad	0.31

Table 6: Second minimizing inferred rankings - one ranking got 0 votes after the transfer

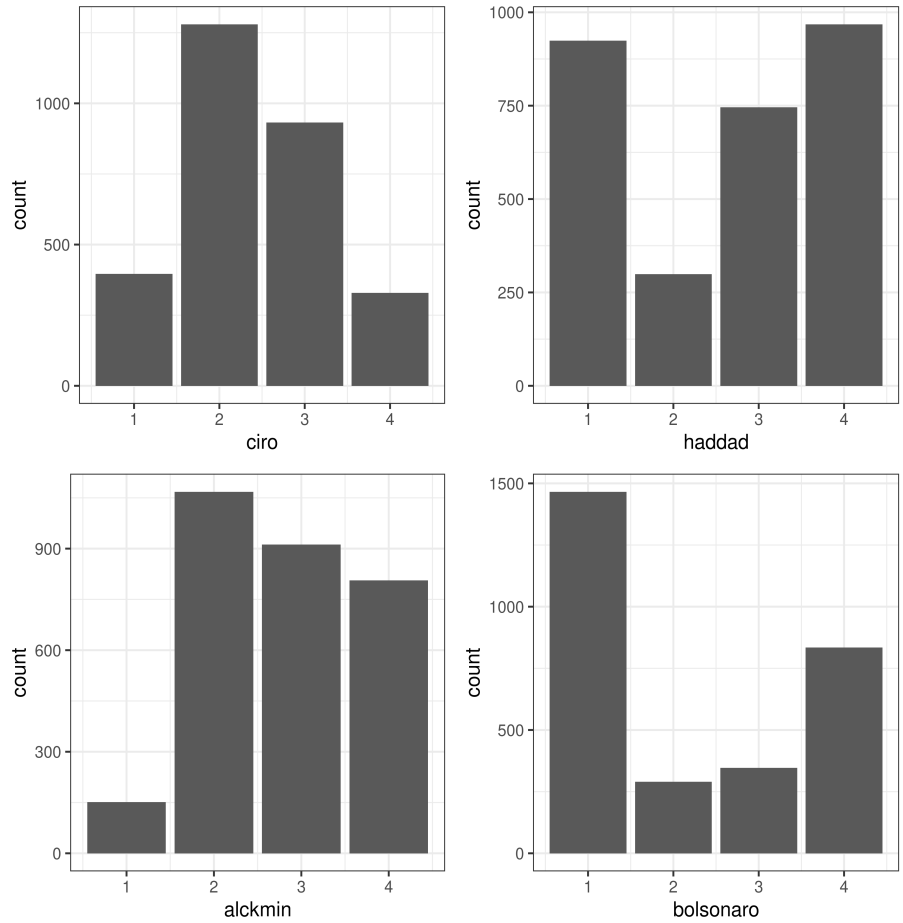


Figure 5: Number of times candidates appear at each position - final ranking 2

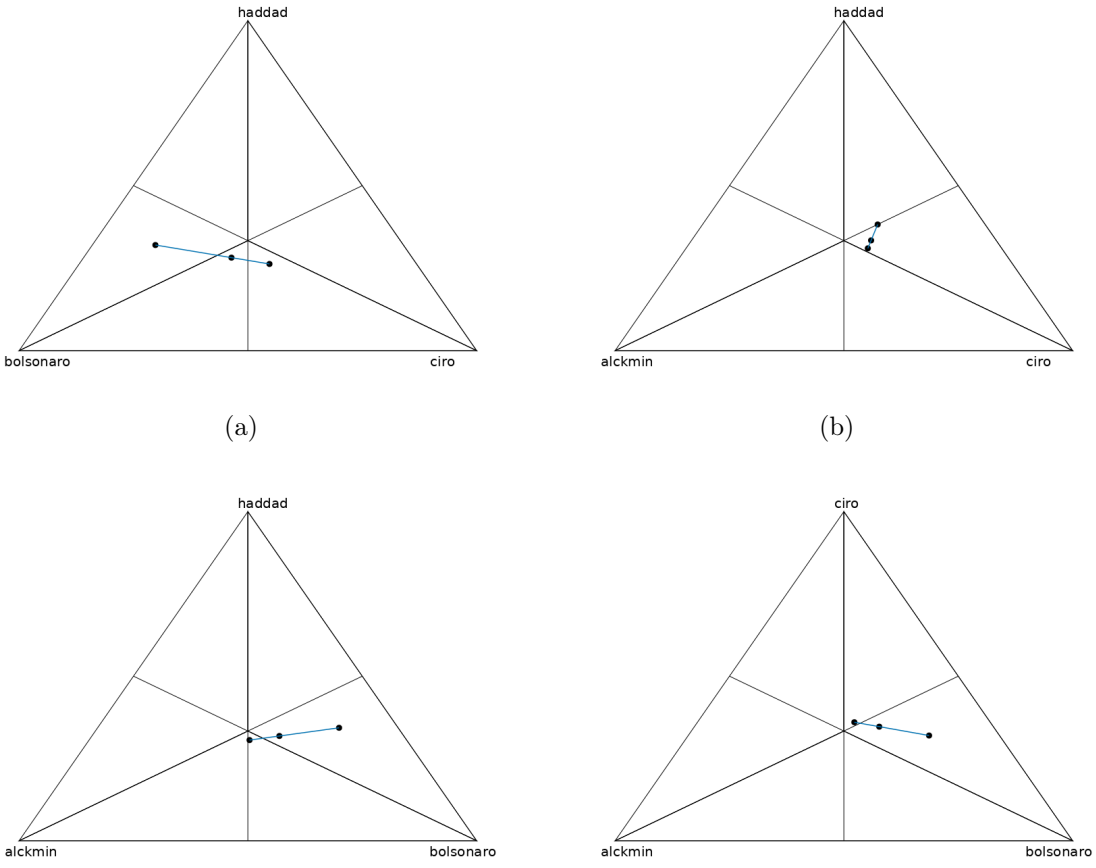
	alckmin	bolsonaro	ciro	haddad
alckmin	0	-1	-1	-1
bolsonaro	1	0	1	1
ciro	1	-1	0	1
haddad	1	-1	-1	0

(a) Pairwise Majority Comparisons

	borda2$other_info$count_max
alckmin	6438
bolsonaro	8262
ciro	7617
haddad	7053

(b) Borda scores

Table 7: Borda and Condorcet results for final inferred ranking 2



(a) (b) (c) (d)

Figure 6: Positional results after dropping one candidate - second possible vote transfer