Was Bolsonaro's 2018 electoral victory an institutional fluke?

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Abstract

Presidential Elections are critical moments for polyarchical systems, particularly in contexts of high social tension. In this regard, the 2018 presidential election in Brazil, which used a two-round system, is a significant case study. Intriguingly, the most divisive candidates went to the second round. Was this an institutional accident? Pairwise and positional voting procedures embody different generalizations of a majoritarian credo that underpins such elections. The paper mobilizes both perspectives and, using representative survey data, reconstructs the top four preferences of the Brazilian electorate a week before the election. The analysis reveals that the electoral winner, Jair Messias Bolsonaro, was a Condorcet Winner, but may not have been the Borda Winner. Conversely, the second-round loser, Fernando Haddad, was a Condorcet Loser. Furthermore, the paper explores possible alternative scenarios under different feasible sets of candidates through simulations, contributing to a deeper understanding of the role of decision procedures in critical junctures.

1 Introduction

The escalating polarization, the looming threat of democratic backsliding, and a broader concern for the longevity of democratic polities have sparked a resurgence of interest in the role of institutions in either mitigating or exacerbating destabilizing dynamics and in the adaptability of the political system in the face of both internal and external stressors (Bednar 2021; Chiopris, Nalepa, and Vanberg 2021; V. Ostrom 1997). Given their pivotal role in the input-output relationship between society and the state, electoral institutions are naturally among the institutions under scrutiny (Wang et al. 2021). Consequently, a segment of the literature on collective choice has speculated whether the recent electoral victories of divisive candidates are a consequence of informationally deficient decision procedures (Potthoff and Munger 2021; Kurrild-Klitgaard 2018; Woon et al. 2020).

Jair Messias Bolsonaro is a hallmark example of a divisive candidate, and large margins in his favor marked his rise to power in the 2018 Brazilian presidential election. Was his victory simply a byproduct of the decision procedure? Despite the various historical contingencies that could explain his triumph, it is natural to question the role of the electoral system in his success. After all, it is widely recognized that the outcome of collective choices is fundamentally dependent on the voting procedure (Riker 1982). Like Brazil's two-round system, most electoral systems primarily use the first index of the voters' preferences (Grofman and Feld 2004). How would Bolsonaro have performed under informationally richer voting procedures, such as methods based on pairwise comparisons and positional voting procedures? Could he, and arguably other democratically elected destabilizing candidates, be products of decision procedures that favor divisive candidates over more inclusive ones (Igersheim et al. 2022)? What criteria should be used to argue that a candidate's victory was an institutional artifact? Would the result have changed with a different set of candidates?

The findings of this paper reveal that even though Bolsonaro was the Condorcet winner, he would not have been victorious under all reasonable alternative voting methods.

Notably, he was not the Borda winner. His opponent in the second round, Fernando Haddad, was a Condorcet loser among the top four candidates and would not have defeated Bolsonaro under any positional voting method. The third-place candidate, Ciro Gomes, had a slight positional advantage over Bolsonaro, with the potential to beat him in 53% of the positional voting methods. Ciro's advantage was especially pronounced in methods that emphasized rejection. This case highlights a partial conflict between typical evaluative positions harking back to the historic Borda-Condorcet debate. In order to arrive at these results, I first revisit the perspectives of Borda and Condorcet on voting procedures. Subsequently, I utilize a pre-electoral representative survey to reconstruct the full 4-top preferences - complete binary relations - of the Brazilian population. This augmented data is then used to simulate electoral outcomes under alternative methods for the top 4 and 3 candidate sets. Finally, I discuss the significance of the results and conclude by delineating the limitations of this endeavor.

2 Theory

The recognition that collective decision outcomes are intrinsically tied to the voting procedures implemented and that these procedures vary in their adherence to evaluative criteria naturally sparks curiosity about the alternative routes that could have been pursued during pivotal electoral moments(Tabarrok and Spector 1999; Kamiński 1999; E. Ostrom 1986). At the same time, this understanding presents a quandary: What criteria should guide the counterfactual analysis among all the potential ones? Does this task not seem inherently arbitrary, considering one could presumably retrofit a suitable choice of voting method to correspond with a pre-determined desired outcome(Riker 1982)?

A key point to anchor this discussion is the observation that political actors themselves contemplate these procedural properties, which become instrumental in legitimizing their position in the political sphere (McLean 2002; E. Ostrom 2009). Therefore, instead of adopting an imposing "philosopher-king" stance and forcing external values as if they were universally agreeable, we can observe the set of values that the agents themselves deploy (Binmore 2005).

Particularly prominent among polarizing or divisive candidates -those that have strong support at the top choices of voters, but also have a high share of the bottom choices among the electorate[1] - are claims of strength and legitimacy based on the notion of popular mandate (Tabarrok 2001), a congenial resource for politicians that, despite being elected, face widespread rejection or opposition.

Nevertheless, what exactly constitutes a mandate? At its most basic, a politician is understood to possess a mandate as long as they have emerged victorious under the established voting procedure. This perceived mandate is bolstered when there is a considerable margin between the vote share or score of the winning actor and the candidate who comes in second. These interpretations of the mandate are grounded in a fundamental majoritarian principle, a core component of the democratic ideal. This principle suggests that if all alternatives and voters are treated equally, then the option that garners the most support should be declared the winner (Dahl 1989). This "monotonic/majoritarian mindset" is inherent to both the "minimal" and the "marginal" mandates: the minimal mandate asserts that if a candidate is elected, they must have received more votes than the others. In contrast, the marginal mandate posits that greater support equals a stronger mandate. This majoritarian value fortifies the legitimacy claims of elected candidates. Generally, the wider the vote margin, the more a political leader can assert they have the backing of the populace (Grossman et al. 2022). Alternatively, discrediting an opposition's mandate

1. The associated concept of an inclusive candidate is defined by Igersheim et al. (2022, p.6) as those who "receive widespread support from the voters but with no strong feeling of rejection or attachment."

by alleging electoral fraud is a tactic that yet again hinges on this central tenet of democratic ethos. It exploits the majoritarian principle, suggesting that an unfair process has undermined the true "popular choice".

However, with more than 2 alternatives, the majoritarian mindset is not as clear-cut as a cyclical profile of voters' rankings, such as [xyz, yzx, zxy], reminds us. Nonetheless, it remains a centerpiece of the democratic paradigm. How can, then, one extend majoritarianism to more than two alternatives? Borda and Condorcet grappled with this problem and gave different answers. Condorcet extended the majority rule to pairwise majority rule: apply majority rule to all pairwise comparisons. One possible condition that generalizes majoritarianism is the Condorcet criterion: a decision procedure is Condorcet consistent if it selects the candidate, if there is any, that wins in all pairwise majority contests. This alternative is called a Condorcet winner (CW). Borda, on the other hand, devised a scoring scheme: if there are, say, 3 alternatives $\{A, B, C\}$ and an agent i has ranking $B \succ C \succ A$ then the Borda score in i's ranking for each alternative is $A : B : C = 0 : 2 : 1$. Alternatively, it can also be coded as $1 : 3 : 2$. The Borda score for the entire profile is the sum of each alternative score at each voter ranking, and the candidate with the highest score, the Borda winner (BW), wins. It is equivalent to adding the number of votes an alternative got in each pairwise comparison against the other alternatives (Nurmi 1999). As such, it is another way of generalizing the "majoritarian/monotonic" perspective to more than two alternatives.

Being Condorcet consistent is arguably the primary normative benchmark for a voting method in single-candidate elections, while the Borda perspective could be considered the leading contender (Regenwetter et al. 2006; Felsenthal 2011; Nurmi 2002). While being plausible generalizations of the majoritarian credo, they also offer more robust and informationally more demanding views of mandate. If the candidate is a CW, it would have won under all possible majority pairwise comparisons against the other candidates. A CW is a candidate with a pairwise mandate. The Copeland scores could be a more general measure of pairwise mandate since the CW may select an empty set. However, this generalization is unnecessary in the context of this paper. The BW lends itself to a similar interpretation, but the notion of mandate can be strengthened here. The Borda count can be seen as one method within a family of methods that assign weights to positions in the ballot. The higher the proportion of positional voting systems that the candidate would have won had the election used it, what Tabarrok (2001) has called positional stability, the higher the positional mandate of the candidate.

Suppose a candidate wins under a voting procedure that only uses the top choice of the electorate but is neither a BW nor a CW. In that case, in this generalized majoritarian perspective, it has a weaker mandate than if it were both - which would signal a comprehensive social base. Thus, a candidate who wins under the current voting procedure but is neither a BW nor a CW could be considered an artifact of the procedure. In the latter case, the procedure would be just "tracking" a broader support pattern for the alternative. The concepts of pairwise and positional mandates will serve as the main analytical tools in this paper for comprehending both the popular support a candidate garners and the role of the decision procedure in their election.

Even though the pairwise and positional perspectives of popular support/mandate generalize a widely held democratic principle, they are not captured by electoral processes that only have voters' first choice as input. As such, they are not typically mobilized by politicians. Nonetheless, this information, which has been repeatedly rediscovered in acts of political reflexivity (McLean 2014), can be queried to tell a more refined story about the backing a candidate has among the electorate. Such a broader informational backdrop underlies current research on the case of the United States and Donald Trump's electoral victory. Regardless of the specificities of each paper, all presuppose that the informational

paucity of only focusing on top choices blinds the States' socio-technical translation of popular support into political input (the choice of a candidate). For instance, Potthoff and Munger (2021), Woon et al. (2020), and Kurrild-Klitgaard (2018) debate whether Donald Trump was a CW in the primaries, with recommendations of voting procedures that better track what the CW is, after all. Igersheim et al. (2022) goes a step further: they argue that not only was Trump neither, but Sanders was the actual Borda and Condorcet Winner, and generally the "best" candidate, if by best one understand to be a candidate being the most inclusive and winning under the most alternative decision procedures. I doubt this support distribution for Sanders would hold had he been a viable candidate. Moreover, changing the voting procedure implies changing the candidates' campaign strategies. Nevertheless, this is an intriguing result. An analogous line of reasoning would make us wonder whether a similar conclusion could be drawn about Bolsonaro: he would not have either pairwise or positional mandate. We will see, however, that an unambiguous conclusion cannot be drawn in the Brazilian case.

3 Case/Data

Jair Messias Bolsonaro was elected as Brazil's president in 2018, following more than 20 years as a congressman, during which he primarily served as a low-clergy politician defending the interests of the military and local police forces in Rio de Janeiro. The 2018 electoral landscape in Brazil was marked by widespread rejection of the traditional political elite, especially the Labor Party (Partido dos Trabalhadores - PT). This sentiment was fueled by recent corruption scandals and the impeachment of the previous president, Dilma Rousseff, a member of the Labor Party.

Among the 13 contestants in the 2018 Brazilian presidential election, the leading candidates were Jair Bolsonaro, a rightist; Fernando Haddad, a leftist from the Labor Party (PT); Geraldo Alckmin, a center-right candidate; and Ciro Gomes, a center-left candidate. The election followed a two-round system. In the first round, 8.79% of the votes were White/Null, meaning they were not counted, and there was a 20% abstention rate. The result of the valid votes was as follows: Bolsonaro received 46.3%, Haddad 29.28%, Ciro 12.47%, Alckmin 4.76%, and Others 7.19%. Among the 9 other candidates, João Amoêdo had the highest vote share at 2.5%, with all others receiving less than 1%.

In the second round, the result was Bolsonaro 55.12% and Haddad 44.78%. White/Null votes constituted 9.57%, and the abstention rate was 21.3%. Bolsonaro's victory, with more than a 10% margin over his second-round opponent, was decisive. However, he contested the result, claiming that he would have won in the first round with over 50% of the valid votes if the elections had not been rigged, though no evidence of fraud was found. Bolsonaro continued to cast doubt on the electoral institutions leading up to the 2022 election, which he lost to Lula by a margin of 1%.

It is important to highlight two events that significantly marked the 2018 election. First, the leading leftist candidate, Lula, was prevented from running due to his arrest at the beginning of the electoral campaigns. This process was later deemed suspicious in 2021, as the judge[2] was found to be cooperating with the prosecutor, and Lula won in 2022 in an electoral process marred by irregularities favoring Bolsonaro. The support distribution for Lula was markedly different from that for Haddad, who was merely his replacement. PT's campaign hinged on the possibility of Lula being released, and Haddad primarily positioned himself as Lula's candidate. However, his popularity was nowhere near Lula's, and he inherited the high rejection of his party at the time. Second, on September 6, 2018, a month before the first round, Bolsonaro was the target of an assassination attempt. This

2. Bolsonaro nominated this judge as Minister of Justice.

knife attack likely altered his pattern of support.

The dataset used for the analysis comes from a representative street survey conducted on October 2, 2018, by DataFolha, an independent research institute highly esteemed and trusted by Brazilian experts[3] . This date was less than a week before the election's first round on October 7, 2018. The survey focused on one question in particular, which serves as the sole variable in our analysis: pairwise comparisons between the four top candidates. These comparisons included: Alckmin × Bolsonaro, Alckmin × Ciro, Alckmin × Haddad, Bolsonaro × Ciro, Bolsonaro × Haddad, and Haddad × Ciro, with no option for indifference.

With these strict comparisons for all survey respondents, it is possible to reconstruct their full ranking of the top four candidates. Preliminary pre-processing led me to drop 171 observations where all pairwise comparisons were missing and 132 where they were cyclic, leaving 2937 out of 3240 observations. Table 1 shows that only 1797 observations compared all four candidates. Therefore, I had to augment the data with transitive closures for 1140 observations, using methods discussed in the next section.

Number of Pairwise Comparisons	Frequency
1	15
2	42
3	462
4	118
5	503
6	1797

Table 1: Frequency of pairwise comparisons in the dataset.

4 Methods

I impute the missing data using the **R** package mice (multiple imputation by chained equations), one of the standard packages for this task. It fills the missing values in a row using the values of the other columns by an iterative series of predictive models (Buuren 2018). Under the hood, it offers a menu of possible predictive models, such as Bayesian linear regression, predictive mean matching, logistic regression, polytomous regression, classification trees, and random forests. Among the classes of methods that could be applied to the missing voting data, given its categorical nature, the polytomous regression was the only one that did not introduce cyclic rankings or repeated alternatives in the ranking, and as such, was the one I used[4].

A further complication is a mismatch between the survey's plurality result and the actual result of the first round. This is typical in surveys and might be due to strategic voting, social desirability bias (not wanting to be seen as "extreme"), or systematic refusal of part of the electorate to answer the survey (Nishimura, Wagner, and Elliott 2016). Any imputation technique will reproduce this top-choice discrepancy since it inherits this problem from the survey. In the survey, the distribution of vote shares among the candidates was as follows: Bolsonaro garnered 36.81%, Haddad 24.96%, Ciro 17.06%, Alckmin 13.97%, and Others 7.2%. Thus, Bolsonaro and Haddad are undervoted in the sample,

3. I had access to the survey data, code-book, and questionnaire by creating an account and requesting access to them, available for educational/research purposes, at https://www.cesop.unicamp.br.

4. Besides the polytomous regression, I tested predictive mean matching, classification trees, and random forests. All introduced cyclic rankings, sometimes in large amounts (as in the case of random forests).

while Ciro and Alckmin are overvoted[5]. However, transferring is complicated because we are working with the full rankings, which gives leeway to many possible transfer methods. To solve this problem, I developed a systematic transfer method, as discussed in the following subsection.

4.1 The Transfer Method

To get a hold of the problem, consider Table 2, which shows some of the rankings for Alckmin and Bolsonaro after the imputation. If we are to transfer from Alckmin to Bolsonaro, we are led to the problem of first picking which ranking at the source should be chosen and then which ranking at the target should receive votes while respecting how much the source has in excess and how much the target needs. Which row from the set $\{1, 2, 3\}$ should transfer votes to which row of the set $\{4, 5, 6\}$?

	1	2	3	4	frequency	proportion
1	Alckmin	Bolsonaro	Ciro	Haddad	93	0.03
2	Alckmin	Ciro	Bolsonaro	Haddad	63	0.02
3	Alckmin	Haddad	Bolsonaro	Ciro	14	0.00
4	Bolsonaro	Alckmin	Ciro	Haddad	556	0.18
5	Bolsonaro	Ciro	Alckmin	Haddad	366	0.12
6	Bolsonaro	Alckmin	Haddad	Ciro	59	0.02

Table 2: Some pre-transfer proportions of Alckmin/Bolsonaro's rankings

The process of sorting and transferring rankings can be done systematically. We begin by organizing the rankings based on Bolsonaro's position. The rankings are sorted into three sets: those where he is in the second position (e.g., (Alckmin, Bolsonaro, Ciro, Haddad), (Alckmin, Bolsonaro, Haddad, Ciro)), third position (e.g., (Alckmin, Ciro, Bolsonaro, Haddad), (Alckmin, Haddad, Bolsonaro, Ciro)), and last position (e.g., (Alckmin, Ciro, Haddad, Bolsonaro), (Alckmin, Haddad, Ciro, Bolsonaro)).

Suppose we select a source ranking from the first sorted set (second position). We then need to determine the target ranking among those where Bolsonaro is the first choice. The target is chosen based on the minimal Kemeny's distance to the source ranking (Nurmi 2002). The Kemeny distance measures the number of transpositions (switching of pairs) needed to transform one permutation into another. The transfer from the source ranking is then the **min**(number of votes the source has, total votes the under-voted needs, total votes the over-voted can give)[6]. The frequencies of the source and target rankings are then updated, along with the total votes needed by the under-voted and the total votes the over-voted can give. If the under-voted does not need other votes, the algorithm breaks the loop and goes to another over-voted $\rightarrow$ under-voted transfer. If not, it checks if the over-voted can still transfer votes to the current target under-voted. If yes, it picks another source ranking in the sorted rankings sets and repeats until the source has run out of votes it can give, or the target has received enough votes. It then goes to another over-voted $\rightarrow$ under-voted transfer candidate pair.

Ultimately, this leads to 24 possible orders for transfer sequences from over-voted to under-voted. For example, one sequence might be Alckmin $\rightarrow$ Bolsonaro, then Alckmin $\rightarrow$ Haddad, then Ciro $\rightarrow$ Haddad, then Ciro $\rightarrow$ Bolsonaro. The entire transference process results in six transfers that minimize the Euclidean distance between the inferred plurality

5. Remember the actual result was Bolsonaro:Haddad:Ciro:Alckmin:Others = 46.3 : 29.28 : 12.47 : 4.76 : 7.19.

6. This **min** ensures that the transfer does not lead to negative numbers, nor gives more or less than needed.

result and the actual result of the first round. Since the results are invariant between them, only one analysis is reported. After the adjustments, the new distribution of vote shares is as follows: Bolsonaro received 46.32%, Haddad 29.26%, Ciro 12.45%, Alckmin 4.77%, and Others 7.19%.

Having imputed the missing rankings and transferred them to match the first-round results, I identify the Borda Winner (BW) and Condorcet Winner (CW) among the top four candidates. Then, I calculate and plot all counterfactual victory scenarios for positional voting methods. Finally, I visualize the positional outcomes for alternative sets of three candidates using Saari's outcome simplex, a concept I will elaborate on in the following subsection.

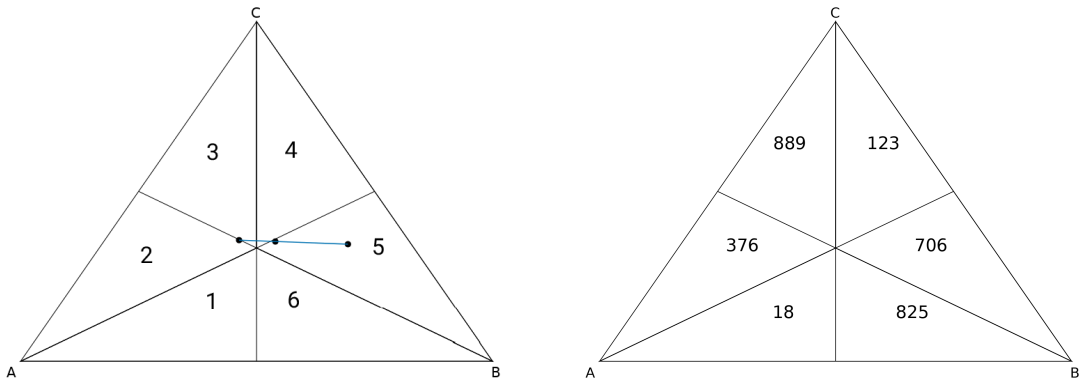
4.2 Saari's Outcome Simplex

Positional voting methods are a class of voting systems that assign scores to positions in voters' preferences. These methods are characterized by the principle that the higher an alternative is ranked in a voter's preference, the higher the score it receives. A better-ranked alternative should thus receive a score at least as large as the next best-ranked alternative.

Donald Saari has proved several facts that help understand and calculate positional voting victories. Any positional voting method that respects the above constraint can be defined by a vector of weights $W_j = (w_1, \ldots, w_n)$. For example, in a three-candidate election, plurality voting can be represented by the vector $(1, 0, 0)$, while the Borda count is represented by $(3, 2, 1)$. Note that plurality could also be coded as $(2, 0, 0)$ or even $(2, 1, 1)$, and the result would not change. Similarly, the Borda count could be associated with the vector $(2, 1, 0)$, with no change in the outcome.

This flexibility in representation comes from an equivalence relation: any two weighting vectors W_1 and W_2 that satisfy $W_1 = aW_2 + b(1, \ldots, 1)$ for $a, b > 0$ will have the same result in terms of outcome ordering (Saari 1995). This relation can be exploited to standardize positional voting methods.

For four candidates, the weighting vectors can be expressed as $(1, s_1, s_2, 0)$, where $0 \leq s_2 \leq s_1 \leq 1$. In plurality, $s_1 = s_2 = 0$, while in the Borda count, $s_1 = \frac{2}{3}$ and $s_2 = \frac{1}{3}$. All such positional voting methods for three candidates will lie on the line connecting plurality with antiplurality $(1, 1, 0)$. For four candidates, all such procedures will lie in the convex hull of plurality, antiplurality, and vote-for-two procedures, with respective weights of $(1, 0, 0, 0), (1, 1, 1, 0), (1, 1, 0, 0)$. Calculating scenarios, thus, amounts to varying the values of s_1 and s_2 (Saari 1995, 2001).



(a) Point-share triangle

(b) Profile triangle

Figure 1: Saari's triangles

Saari's outcome simplex, or point-share triangle (Eggers 2020), makes use of the structure of positional voting methods to provide a way of visualizing all possible positional voting results of an election[7]. Consider Figure 1a. The closer to a vertex, the better the vertex's position in the social ranking. Region 1 corresponds to the social ranking of $A \succ B \succ C$, while Region 4 corresponds to the social ranking of $C \succ B \succ A$. The lines separating the regions represent indifference. The point at which all lines meet corresponds to $A \sim B \sim C$, while the line separating Region 1 and 2 would correspond to $A \succ B \sim C$. The three dots are the results of the antiplurality, the Borda, and the plurality voting methods. The Borda Count point is always closer to the antiplurality result. In this example, most positional voting methods would have agreed with the plurality procedure outcome of B as the winner. A related triangle, the representation triangle, or profile triangle (Eggers 2020), will represent a profile compactly. Figure 1b shows the equivalent profile triangle. In each ranking region, we plot the frequency of votes that match that region's ranking. For 4 candidates, we can use analog representations by "opening" the 3-simplex/tetrahedron and plotting onto its polyhedral net - the arrangement of polyhedrons in the plane that, when folded, become the faces of the simplex.

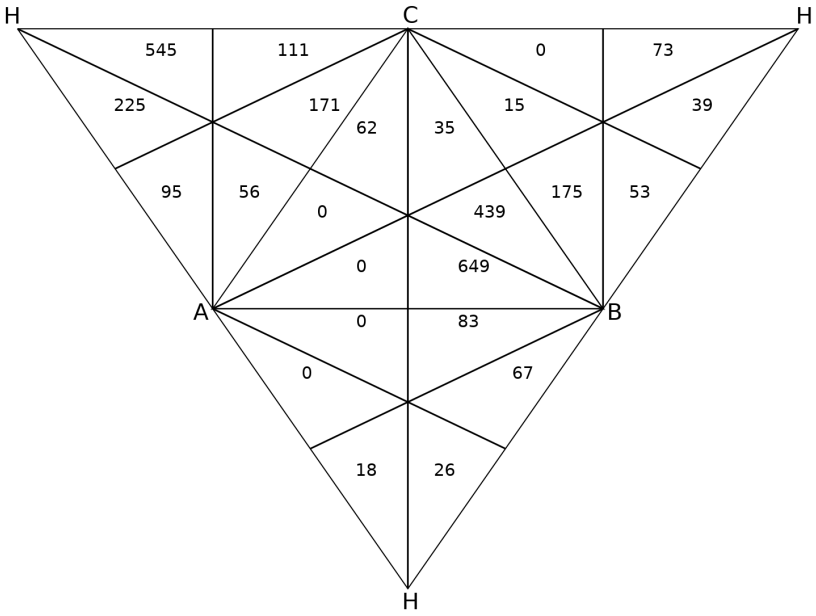
5 Results

The inferred rankings are shown in Figure 2a and summarized in Figure 2b. Among all ways of transferring from over-voted to under-voted while respecting the Kemeny distance, the transference that best matched the rankings in the survey with the actual first-round result led all rankings in which Alckmin appears as the first choice to be of type Alckmin $\succ$ Ciro $\succ$ Haddad $\succ$ Bolsonaro or Alckmin $\succ$ Haddad $\succ$ Ciro $\succ$ Bolsonaro. Notably, no ranking of type Alckmin $\succ$ Bolsonaro $\succ$ _ $\succ$ _ remained, a substantively interesting outcome given that the algorithm only considered the ordinal distance but still transferred mostly from voters that ranked Center-Right $\succ$ Right.

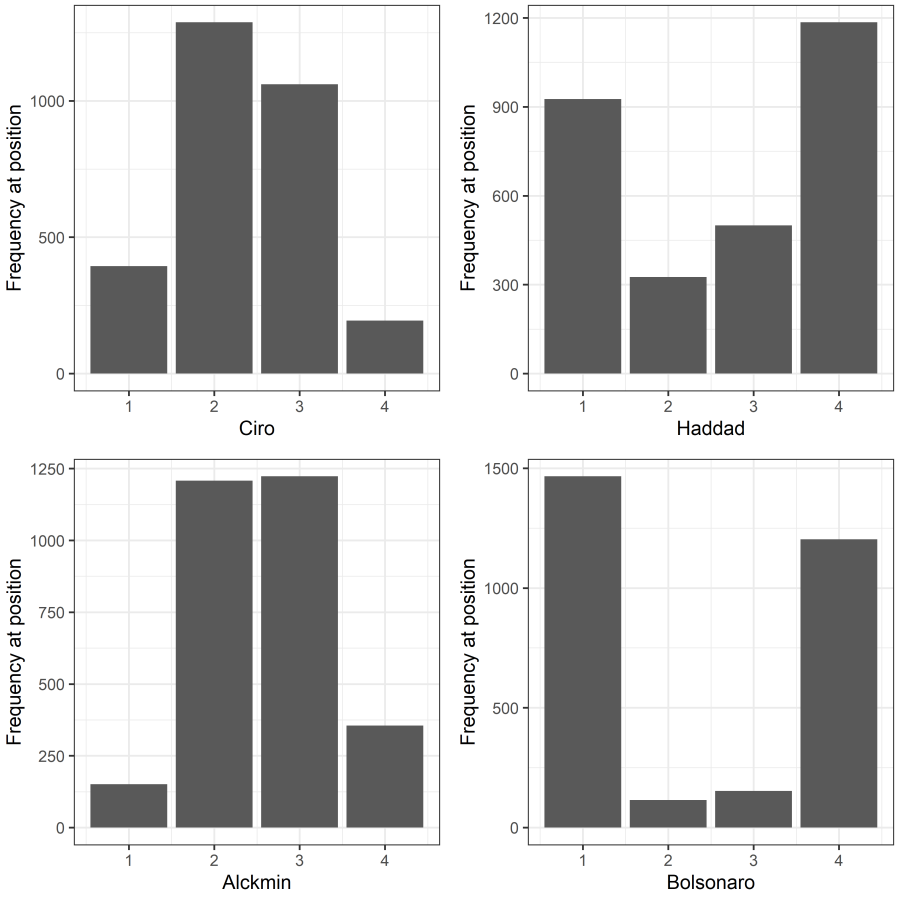
The most blatant pattern in Figure 2b is that the candidates who went to the second round were the most divisive, as expected. Among the more inclusive candidates, Ciro has more second than third choices, while Alckmin's support is roughly equally balanced between those two positions in the voters' preferences. Moreover, Ciro has more first choices and fewer last choices than Alckmin. There is also a difference among the divisive candidates: Haddad's rejection was higher than his top-choice support, while the opposite held for Bolsonaro. The differences within-group, inclusive vs. divisive, will be relevant to understanding how each candidate fares against others.

What does this support distribution mean from the point of view of the BW and CW? Table 3 shows what we can infer from the imputed data. Recall that we are using data from a survey before the first round and will extrapolate to consider what could have happened in general. Despite being divisive, Bolsonaro would have won in all pairwise majority comparisons against other top candidates. Haddad, however, would have lost against all others. He was a Condorcet Loser despite going to the second round. Ciro would only have lost against Bolsonaro, while Alckmin could only have won against Haddad. Unlike what was widely believed at the time and was the motto of his campaign, it is uncertain whether Ciro would have won against Bolsonaro in the second round. From a pairwise perspective, he was not the "anti-Bolsonaro", but merely an "anti-Haddad", even more than Bolsonaro. Alckmin, the candidate with the longest television time and the broadest supporting coalition, would have lost against Haddad, who was merely a substitute for Lula. However, the pattern is not reflected in the Borda Scores, which implies the ranking: Ciro $\succ$ Bolsonaro $\succ$ Alckmin $\succ$ Haddad.

7. For a complete exposition of this method see Saari (1995) or Nurmi (2002).



(a) Opened representation tetrahedron



(b) Frequencies at each position in the rankings

Figure 2: Profile after imputation and rankings transference

Nevertheless, the raw Borda scores of Ciro and Bolsonaro are very similar. If we standardize them, we see that the candidates are practically tied. Suppose we consider the

sampling error, imputation, and transfer degree of freedom. In that case, the most likely conclusion is that the Borda Ranking was Ciro $\succeq$ Bolsonaro $\succeq$ Alckmin $\succeq$ Haddad. Note that if we take a positional perspective, then yes, Ciro was indeed the main contestant against Bolsonaro. Nevertheless, this obviously could not be captured by majority with run-off, which excluded him from the second round.

	Alckmin	Bolsonaro	Ciro	Haddad
Alckmin	0.0%	-12.63%	-16.99%	8.27%
Bolsonaro	12.63%	0.0%	5.48%	7.46%
Ciro	16.99%	-5.48%	0.0%	16.65%
Haddad	-8.27%	-7.46%	-16.65%	0.0%

(a) Pairwise Margins

	Borda Score	Standardized Borda Score
Alckmin	7029	0.464
Bolsonaro	7718	0.543
Ciro	7756	0.547
Haddad	6867	0.446

(b) Borda Count Outcome

Table 3: Condorcet and Borda Outcomes

Now, what about the positional mandate? As discussed in the methods section, with 4 candidates, all results will lie in the convex hull of three positional voting procedures: plurality, antiplurality, and vote for two. The normalized score of a candidate will be of the form $q_{s_i} = a_i + b_i s_1 + c_i s_2$, where a_i is the share i received of votes in the first position, b_i in the second, and c_i in the third position of voters rankings. Therefore, the scores of each candidate in the inferred ranking for the 2018 election can be found by assigning values to the equations of Table 4. For instance, if we set $s_1 = s_2 = 0$ we recover the plurality score, after ignoring "Other" candidates.

candidates	q_s tallies
Alckmin	$0.4113s_1 + 0.4165s_2 + 0.0514$
Bolsonaro	$0.0392s_1 + 0.0521s_2 + 0.4992$
Ciro	$0.4387s_1 + 0.3612s_2 + 0.1341$
Haddad	$0.1109s_1 + 0.1703s_2 + 0.3154$

Table 4: q_s for each candidate

We can, then, represent the results by an opened outcome tetrahedron, as roughly depicted in Figure 3. As depicted in the legend, the black upside triangle in the triangle close to the B vertex is the plurality result, the black downside triangle in the triangle close to the C vertex is antiplurality, the black dot that is close both to the C and B vertex is the vote for two results, and the diamond that touches the line segment from A to H is the Borda count. To ease interpretation, Figure 3a shows the areas in which each candidate would have been top-ranked, while Figure 3b shows the areas in which each candidate would appear in the second position in the social ranking. As expected, the decision procedures emphasizing the top choice awarded Bolsonaro and Haddad to the extent that the Condorcet loser went to the second round. Note, however, that Haddad would be second placed only in a small region of the hull.

Moreover, we can see that there are decision procedures in which even Alckmin would have beaten both Bolsonaro and Haddad, precisely in which Ciro would be top-ranked.

In contrast, Alckmin would have been second-ranked in the social ranking. Figure 3a also shows that the procedure hull is wholly contained in areas where Ciro and Bolsonaro would be top-ranked. If we look simultaneously at both Figure 3a and Figure 3b we can have a more thorough understanding of the positional scenario in the 2018 election. There is a small region in which Bolsonaro is top-ranked and Haddad is second-ranked, then a large area in which Bolsonaro is still top-ranked and Ciro is second-ranked, then in some procedures Ciro is top-ranked and Bolsonaro is second-ranked, and finally a small area where Ciro would be top-ranked while Alckmin would be placed second in the social ranking.

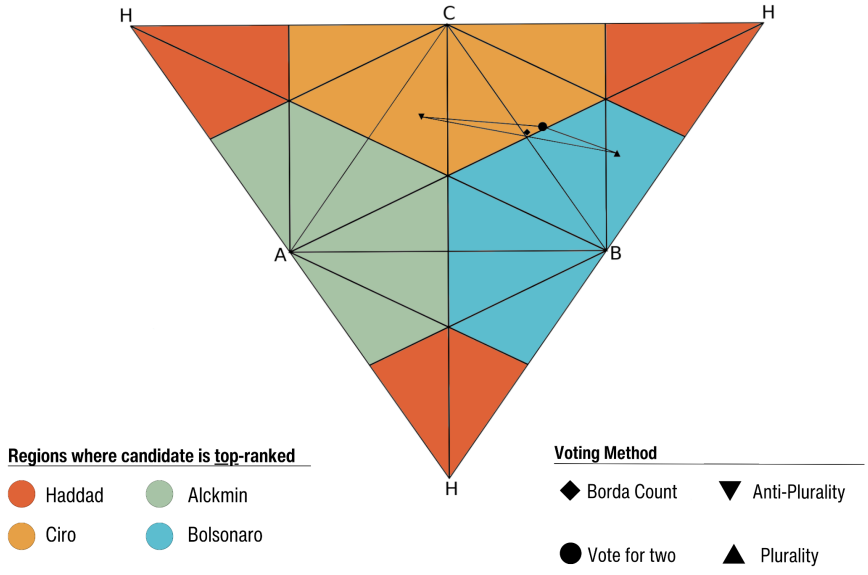
In what precise percentage of the cases would a candidate have beaten another? Note that by opening the tetrahedron, the information provided by the volume of the subregions of the procedure hull is lost. Nevertheless, algebraic manipulation of the equations in Table 4 allows us to answer this question. For instance, when is $0.4113s_1 + 0.4165s_2 + 0.0514 > 0.0392s_1 + 0.0521s_2 + 0.4992$ given the constraints as mentioned earlier on the values of the parameters? Solving this inequality gives us the intervals of the values of the parameters in which Alckmin would have beaten Bolsonaro in the set of positional voting procedures. The area of those intervals over the area of all the combinations gives us the percentage of scenarios in which a candidate could beat another, as shown in Table 5 while the actual Figure whose area is being used is shown in Figure 4.

Table 5 implies a more complex picture of what happened. Bolsonaro, the CW, was tied or lost to Ciro as a BW and would have won against Ciro in roughly 47% of the positional voting methods. Nevertheless, it shows that there were scenarios in which he would have lost to the more inclusive candidates, Ciro and Alckmin. In Alckmin's case, this could have happened in surprising 30% of the cases. However, Ciro could have beaten him in most, $\approx 53\%$ of the positional voting methods. Surprisingly, Haddad, who went to the second round with Bolsonaro, would never have beaten him. The explanation for that is that as shown in Figure 2b, Haddad and Bolsonaro were both divisive candidates, but Bolsonaro had more support than Haddad. They were not equally supported/rejected. Given that they were both divisive, most of their support was in the top choice, they would have fared equally well or badly under the same positional voting methods, but since Bolsonaro had more first votes and was less frequently in the bottom of the rankings than Haddad he actually "positionally dominated" Haddad. The same logic applies to another surprising result: Alckmin would never have beaten Ciro.

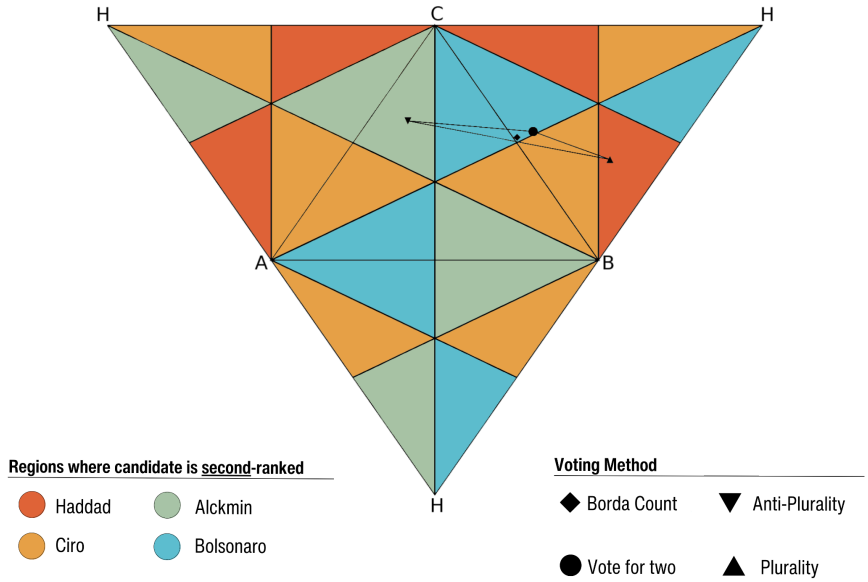
	Alckmin	Bolsonaro	Ciro	Haddad
Alckmin	0.0	0.31	0.0	0.58
Bolsonaro	0.69	0.0	0.47	1.0
Ciro	1.0	0.53	0.0	0.81
Haddad	0.42	0.0	0.19	0.0

Table 5: Proportion of victories in the positional voting procedure set

Naturally, proportions do not show the decision procedures in which, for instance, Ciro would have beaten Bolsonaro. Intuitively, voting procedures that emphasize rejection or more of the middle region of the rankings should give an advantage to inclusive candidates, which is qualitatively confirmed by Figure 3. Since the positional voting methods with four candidates are determined by their s_1 and s_2 weights, we can visualize all scenarios by varying them, as in Figure 4. It shows the scenarios Bolsonaro $\times$ Ciro, Ciro $\times$ Haddad, and Alckmin $\times$ Bolsonaro. Note that, as expected, the only way Alckmin could have beaten Bolsonaro would be if s_1 and s_2 were above 0.6. Keep in mind that when both s_1 and s_2 are set to 1, the voting procedure becomes antiplurality. In this method, voters essentially indicate which candidate they dislike the most. However, this universe of cases



(a) Winning regions



(b) Regions candidate is second ranked

Figure 3: Saari's opened tetrahedron

was dominated by Ciro, who would have beaten Bolsonaro in any combination of s_1 and s_2 higher than the line connecting the points $(0.51, 0.51)$ and $(0.9, 0)$. The plot also shows what combinations of weights lead to 81% of Ciro $\succ$ Haddad: any combination to the right of the line segment connecting $(0.35, 0.35)$ and $(0.55, 0.0)$.

Nonetheless, the family of positional voting methods does not satisfy, in general, Independence of the Alternative Set (Kaminski 2015). If we drop or add candidates, the "social" ranking might change without respecting the ordering of the baseline set of alternatives. Consider the Borda-induced social ranking in this case: Ciro $\sim$ Bolsonaro $\succ$ Haddad $\succ$

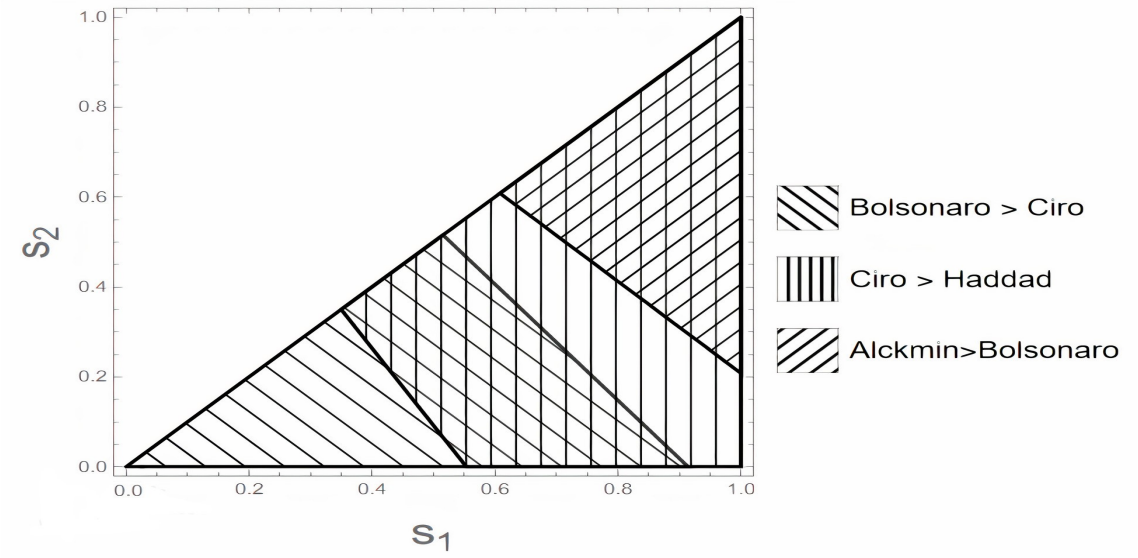


Figure 4: Pairwise victory for three selected pairs of candidates

Alckmin. If by dropping Alckmin, the ranking changes to Bolsonaro $\succ$ Haddad $\succ$ Ciro $\succ$ Alckmin, then the Borda Count, in this case, would be inducing a "paradoxical" result. In Figure 5, I consider alternative scenarios by dropping one of the top 4 candidates.

The positional voting procedures are eminently well-behaved when dropping candidates from this dataset. There is a minor tilt toward Bolsonaro winning with the Borda Count in Figure 5d, but as I have previously argued, this seems like a tie, given the underlying uncertainty. Notice that in all scenarios where Bolsonaro is still in the alternative set, he would have been the plurality winner. However, he would have lost or tied with Ciro under Borda or decision procedures that put more weight on rejection, as in the 4 candidates analysis. In the scenario where Ciro was not in the set, Bolsonaro would again only have lost against Alckmin in a minority of cases.

We have seen that besides having a high vote margin, Bolsonaro was also a CW. His victory, thus, was not a fluke or an artifact of institutional technology. However, he was probably not the candidate with the strongest positional mandate. This result revisits the Borda $\times$ Condorcet controversy. On the one hand, he was the CW, the primary normative benchmark for a voting procedure. On the other hand, in the Brazilian case, the Borda count would have been a more substantial barrier against a divisive candidate. Even though we could expect divisive candidates to have fared worse under informationally richer decision procedures, a divisive candidate can still be a CW with 47% positional stability/mandate. Yes, Ciro would have won against him in 53% of the positional methods and at least tied with him in the Borda count. However, most methods within this 53% emphasize rejection. Giving more weight to rejection vis-à-vis approval seems unreasonable under any normative expectations demanded of a decision procedure for large-scale democratic elections. Due to its symmetry, the Borda Count lies at a threshold: its constant decrease in assigning weights to the positions in the rankings guarantees that approval matters more than rejection, but without throwing away the rejection information. Therefore, highly polarized scenarios can lead to the election of a divisive candidate, which puts in dispute two reasonable metrics of support: being a CW vs. being a BW, thereby only giving partial support to the hypothesis that informationally richer decision procedures would be enough to contain divisive candidates, and two reasonable generalizations of the majoritarian credo end up in conflict.

However, Figure 5b presents an interesting scenario. There is no conflict between the perspectives: under both positional and pairwise perspectives on a mandate, Haddad

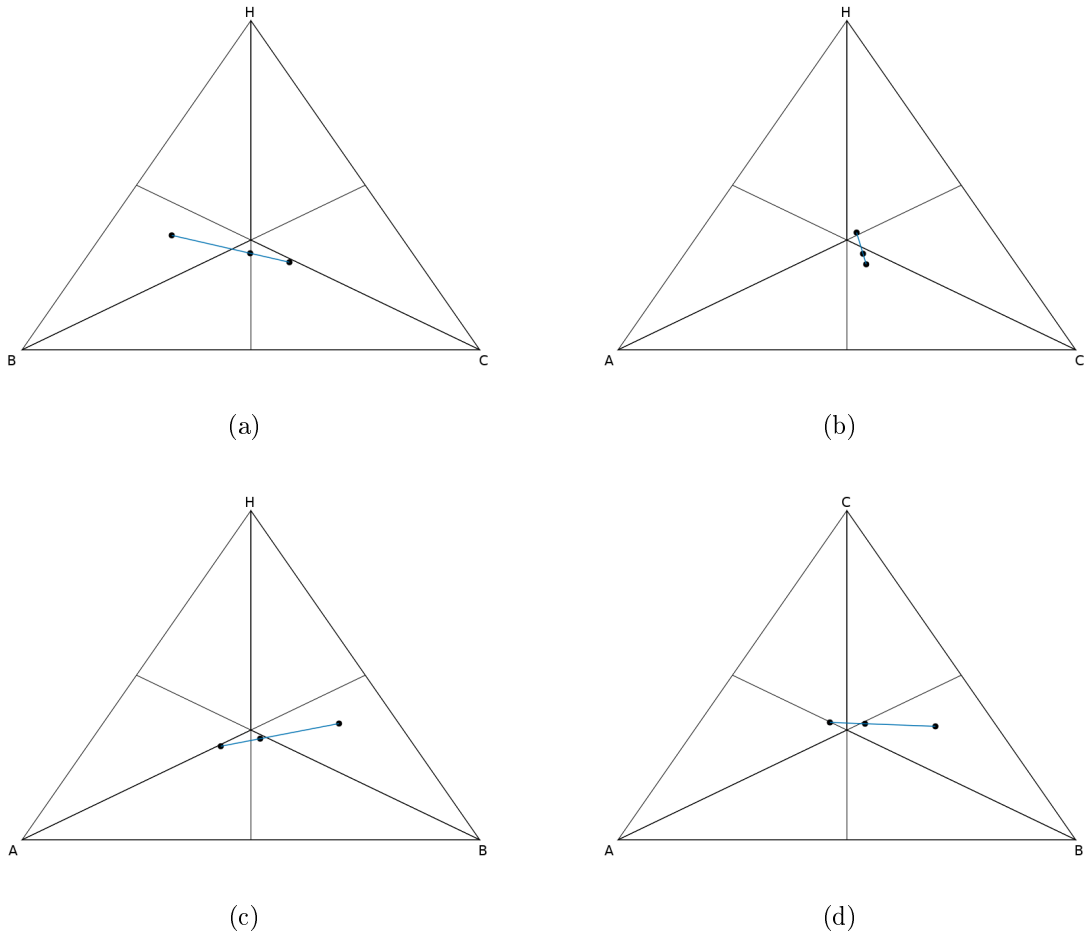


Figure 5: Positional results after dropping one candidate

going to the second round was purely an institutional fluke. Although Haddad would be a plurality winner if we dropped Bolsonaro, the plurality point is close to a tie with Ciro, and could be read as such, given the uncertainty. Ciro, thus, now would have almost 100% positional mandate. Moreover, in Table 3a, it was shown that Ciro would have beaten him with a majority pairwise comparison, which gives credence to affirming that Ciro would have won under a majority with run-off system. In this scenario, the most inclusive candidate would have been elected. Ceteris paribus, it seems the only way an agreement between the Borda and Condorcet criteria could be guaranteed to exist in the Brazilian 2018 case would be if Bolsonaro had never been able to run. In this scenario, the Condorcet Loser would again be beaten. However, now a candidate with a solid mandate, as endorsed by the pairwise majority comparisons and the entire hull of positional methods, would have been elected.

6 Conclusion

This paper investigates whether Bolsonaro's 2018 victory was a fluke or merely a result of the specific voting procedure used at the time. To answer this question, we first sought to understand what would constitute more than just an accidental outcome. I argued that the notions of pairwise and positional mandate can be derived from well-established axiological perspectives to evaluate whether an electoral result is solely an institutional fortuity. Then, I demonstrated that although the aggregation procedure boosted Bolsonaro's victory, it was not merely its effect, contrary to established theoretical expectations. However, neither

was he an undisputed winner under both aforementioned evaluative criteria for gauging the mandate of a candidate in democratic collective choice situations.

In terms of future research, contrasting Haddad with Lula and analyzing the effect of the knife attack should provide a more comprehensive picture of what happened in 2018. Moreover, the pipeline for the analysis is highly reproducible. Analogous analysis can be done for any case with majority with run-off presidential elections, or even for any survey that contains pairwise+ comparisons between the top candidates, as many in Brazil do. As such, any other majoritarian election in Brazil could be analogously analyzed. Figure 5b could+ also be the starting point for an investigation of the selection of the pool of+ candidates allowed to compete for the Executive, particularly in transitional+ democracies, given Brazil's lax transitional justice, and Bolsonaro's intimate+ connection with the remnants of the Dictatorship. First, from a positive point of view as a causal pathway for democratic backsliding+ (Svolik 2008; Nalepa 2022). Second, from a normative+ perspective. What, if anything, justifies restricting classes of actors from running for certain positions? What values would conflict here?

A limitation of that paper is that I used only one variable from the dataset, pairwise comparisons, to simulate alternative scenarios. However, socio-demographic variables from the dataset could have been used to strengthen the data imputation procedure. Moreover, roughly less than half of the dataset is constituted of incomplete pairwise comparisons, and there may be valuable information on the agent's preferences contained in patterns of missingness.

Another limitation is that agents adapt to new institutional environments. I am ignoring strategic voting by assuming a direct translation between preferences and behavior. However, the percentage of strategic voting in a large-scale election is an open empirical problem (Straeten et al. 2010; Kawai and Watanabe 2013). Nevertheless, a combination of game-theoretic models with a simulation parameterized by the inferred ranking distribution is a route of research that could be pursued.

Though I have analyzed the four top candidates, there can be discrepancies when we have a subset of the alternatives vs. when we have the whole set of candidates (Saari 2001; Kaminski 2015). It is well-known, for instance, that the Borda Count is susceptible to the winner-turns-loser paradox. Finally, even though I have analyzed scenarios in which candidates were removed, it would have been more realistic to simulate the formation of coalitions and how voters would have reacted to that (Kaminski, Lissowski, and Swistak 1998). The assumption of a pure additive transfer of votes, implicit when we removed candidates, is not necessarily valid with coalitions (Kaminski 2001), insofar voters of a center-left candidate, for instance, could vote for the center-right candidate if they are alienated by an alliance with the Left, which, in the case of the election under scrutiny, was highly rejected.

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