Wave and Quantum-like behavior at Macroscopic Scale (Analysis)

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A droplet can be kept bouncing on the surface of the same fluid on a bath that is vibrating vertically. This droplet is then free to move horizontally. This horizontal movement has proven to act like quantum and wave statistics in different situations. In this article the results of the series of experiments driven by Couder and Fort will be discussed.

When imagining a drops colliding with an interface of the same fluid it is acceptable to assume that the drop will merge with the fluid in order to minimize the surface area. While doing experiments with soap solutions under vertical vibration, J. Walker observed that the solution could keep a stable drop floating on the surface of the fluid.[6] A bath filled with a fluid can hold a droplet of the same fluid when put into vertical vibration of surface acceleration greater than g.[3]

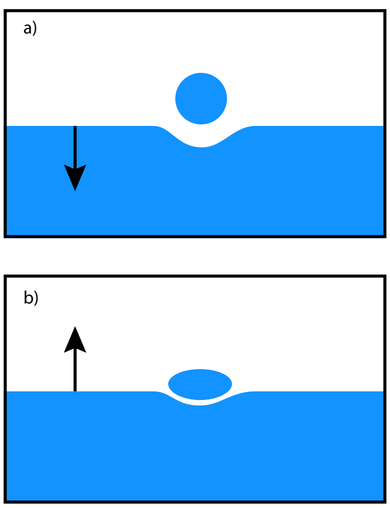

FIG. 1. Droplet bouncing on vibrating liquid surface. Arrows show bath moving a) down and b) up. Source: illustration by Marcel Soubkovsky based on [3]

The droplet is kept bouncing over the fluid and is then free to move horizontally due to vertical stabilization. At first, this horizontal movement seemed very random. Couder and Fort first noticed that the random nature of the horizontal movement of the droplet could be described by diffraction and interference waves.[2]. During a series of experiments, Couder and Fort placed a single slit on the vibrating bath. After successive repetitions, the final positions of the droplet described a diffraction wave.

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I. FARADAY WAVES

Faraday waves appear when the enclosure of a liquid is set into vibration. There is a critical value that when exceeded makes an otherwise flat static surface become unstable.[1]

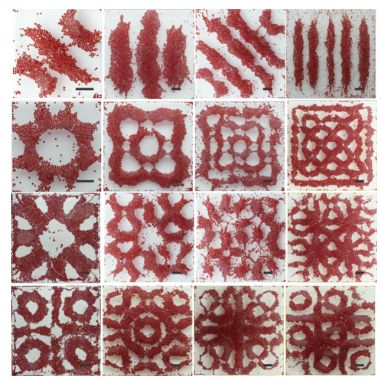

FIG. 2. Faraday waves used to assemble microscale polystyrene beads. Source: Creative Commons (Faraday Telsa)

As shown on figure 2, Faraday waves in a liquid can assemble particles on the noidal parts.

II. SIMILARITIES BETWEEN LIGHT DIFFRACTION PATTERNS AND THE DROPLET DIFFRACTION

A. Fraunhofer diffraction theory

The diffraction pattern curve provides an approximate fit to the histogram found by Couder and Fort's experiments. In Fraunhofer Wave Optics, the diffraction pattern can be observed when a beam of light passes through a narrow slit. If a screen is placed at a certain distance from the slit, a Fraunhofer diffraction pattern can be observed on the screen.[5] The pattern drawn by light is the one show in figure 3. The intensity can be calculated using the following formula

$$I = I_{max} \left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda} \right]^2$$

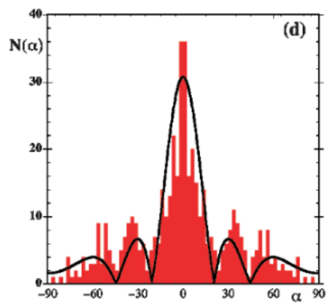

FIG. 3. Histogram obtained by Couder and Fort using a slit of width L = 14.8mm and a Faraday wavelength of λ_F = 6.95mm [2]

where I_{max} is the intensity at $\theta = 0$,a is the length of the slit,θ is the angle of deviation after the slit and λ is the wavelength of the beam. The expression can be simplified when considering only the position of the minima. The condition for intensity minima on a single slit problem states that

$$\frac{\pi a \sin\theta_{minima}}{\lambda} = m\pi$$

or

$$\sin\theta_{minima} = m\frac{\lambda}{a}$$

where $m = \pm 1, \pm 2, \pm 3, \ldots$

B. Couder and Fort's approximation

The histogram produced after several trials with the vibrating bath single slit setup could be compared to a diffraction wave produced by a Fraunhofer diffraction as shown in figure 3. Couder and Fort approximated the formula for the intensity of a Fraunhofer diffraction pattern to the pattern drawn by the final position of the droplet. The function for this specific problem in diffraction of a droplet can be represented as

$$f(\alpha) = A\left|\frac{\sin\left(\pi L \sin\alpha/\lambda_F\right)}{\pi L \sin\alpha/\lambda_F}\right|$$

where A is the maximum amplitude, L is the width of the slit and α the deviation angle of the droplet after the slit.

III. SIMILARITIES BETWEEN LIGHT INTERFERENCE PATTERNS AND THE DROPLET HISTOGRAM

Couder and Fort proceeded then to study the possibility of producing an interference pattern with the vibrating bath system that was built for the study of the single slit diffraction. The single slit was replace by a double slit in order to reproduce Young's double slit experiment with a droplet on a vibrating bath. The droplet drew clear interference fringes that are well fitted by the following formula:

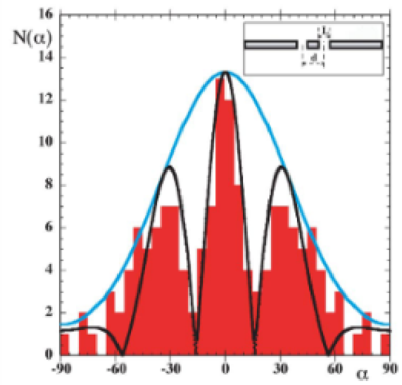

FIG. 4. Histogram for the deviation of 75 particles through two slits of width $L = 7.6$ mm, a distance $d = 14.3$ mm apart, the droplet's wavelength being $\lambda_F = 6.95$ mm ($L/\lambda_F = 1.1$ and $d/\lambda_F = 1.87$). The blue(light) line is the envelope due to the diffraction of a single slit while the curve in black is the optimum fit by the equation that is based on the interference pattern obtained for effective values $L/\lambda_F = 0.9$ and $d/\lambda_F = 1.7$. Source:[2]

$$f(\alpha) = A\left|\frac{\sin\left(\pi L \sin\alpha/\lambda_F\right)}{\pi L \sin\alpha/\lambda_F}\cos\left(\pi d \sin\alpha/\lambda_F\right)\right|$$

where d is the distance between the centers of the slits.

In the domain of wave optics, the interference produced by a double slit diffraction pattern is given by the expression:

$$I = I_{max}\cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$$

The approximation fits closely as shown in figure 4. This brought up many questions concerning the physical engine behind this phenomenon. The conclusions Couder and Fort brought are discussed further on.

IV. QUANTUM-LIKE BEHAVIOR

The vibrating bath with a droplet that was free to move horizontally was then studied on a circular corral by Harris, Moukhtar, Fort, Couder and Bush [4].

What was observed is that the droplet walking through a circular corral presents a behavior similar to quantum particles [4].

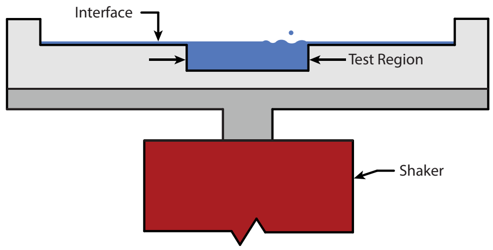

FIG. 5. Illustration showing the apparatus that was used by Harris, Moukhtar, Fort, Couder, and Bush in the study of a droplet confined to a circular corral. Source: [4]

A. Experimental Apparatus used by Harris, Moukhtar, Fort, Couder, and Bush

The apparatus consisted of the same engine as for the diffraction experiment[2], but now the bath is shaped as a circle. There is a deeper region on the bath called "Test region", and a shallow region that is kept only to maintain the interface as it is shown on figure 5.

B. Physical phenomenon

When put into action, the vibrating bath acceleration goes up to γg. When it overcomes the critical acceleration $\gamma_F g$, its free surface becomes unstable to Faraday waves with frequency $f/2$, as explained on section I.

The vibrating bath gives rise to a wave field the drives the droplet, which is put into movement towards a certain direction every time it ends up landing on a non-horizontal surface.

C. Results found by Harris, Moukhtar, Fort, Couder, and Bush

The variation of the droplet speed results in the graph seen on figure 6. It clearly draws a pattern on the shape of wavelike statistics. The authors discuss that the experiments driven by them show that the droplet confined in a geometry, in the long-memory limit, show a coherent statistical behavior just like quantum particles. This statistical behavior can be described by a linear wave theory. It is also discussed that the system used for the study of the droplet in a circular corral is strictly linked to the physical picture of quantum dynamics as seen by de Broglie[4].

[1] "Faraday wave," (2018), page Version ID: 871273450.
[2] Couder, Y. and Fort, E., Physical Review Letters **97** (2006), 10.1103/PhysRevLett.97.154101.
[3] Couder, Y., Fort, E., Gautier, C.-H., and Boudaoud, A., Physical Review Letters **94** (2005), 10.1103/PhysRevLett.94.177801.
[4] Harris, D. M., Moukhtar, J., Fort, E., Couder, Y., and Bush, J. W. M., APS (2013).
[5] Serway, R. and Jewett, J., *Physics for Scientists and Engineers with Modern Physics. 9th Edition* (Cengage Learning, Boston, 2014).
[6] Walker, J., Scientific American **The Amateur Scientist, June 1978**, 10.1038/scientificamerican0678-151.

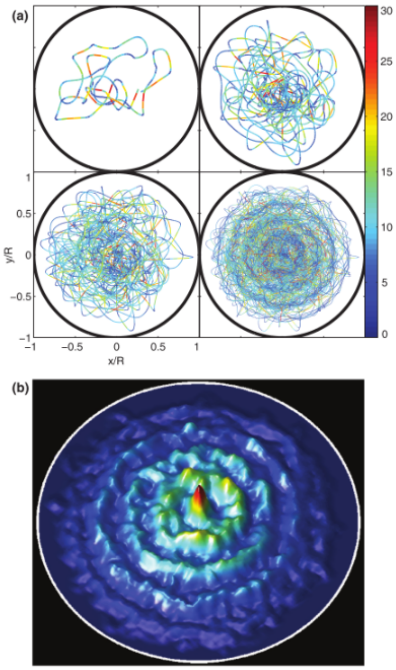

FIG. 6. (a) Trajectories of a droplet of diameter $D = 0.67mm$ walking in a circular corral with radius R = 14.3mm and depth $h_0 = 6.6$ mm, driven at $f = 70$ Hz, for which $\gamma_F = 3.7$. Trajectories of increasing length in the long-path-memory limit ($\Gamma = 0.011$) are color coded according to droplet speed (mm/s). (b) Probability distribution of the walking droplet's position. Source: [4]