Understanding the Central Limit Theorem through the exponential distribution

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Synopsis

In this report, we examine the central limit theorem by doing many simulations of random samplings of the exponential distribution. We measure the distribution of the means of these observations and show that it is approximately normally distributed as $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$.

Load some libraries

```
library(ggplot2)
```

Setup

For the rest of this project, we set the rate parameter, lambda, to 0.2, the number of simulations, N, to 1000, and the number of drawings for each simulation, n, to 40.

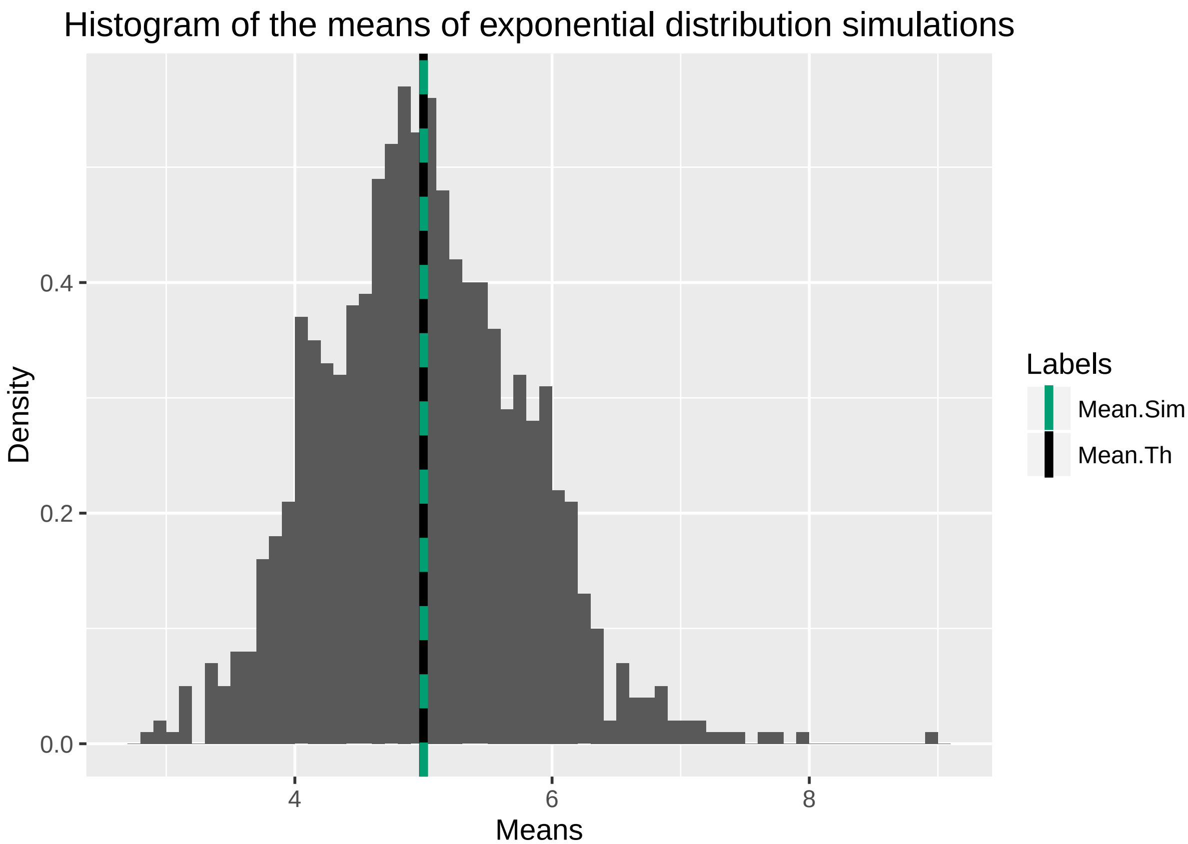
```
lambda <- 0.2
N <- 1000
n <- 40
mean_theory <- 1/lambda
var_theory <- 1/(lambda*lambda)
var_theory_means <- var_theory/n
```

Let's run 1000 simulations of the exponential distribution. Each time we do a simulation, we save the mean to a vector of means. We also compute the mean of the sample and the variance. We also set the seed of each drawing so we can be sure that we can exactly reproduce the results.

```
sim_means <- NULL
for (i in 1 : N){
    set.seed(i)
    sim_means <- c(sim_means, mean(rexp(n,lambda)))
}
mean_sim_means <- mean(sim_means)
var_sim_means <- var(sim_means)
```

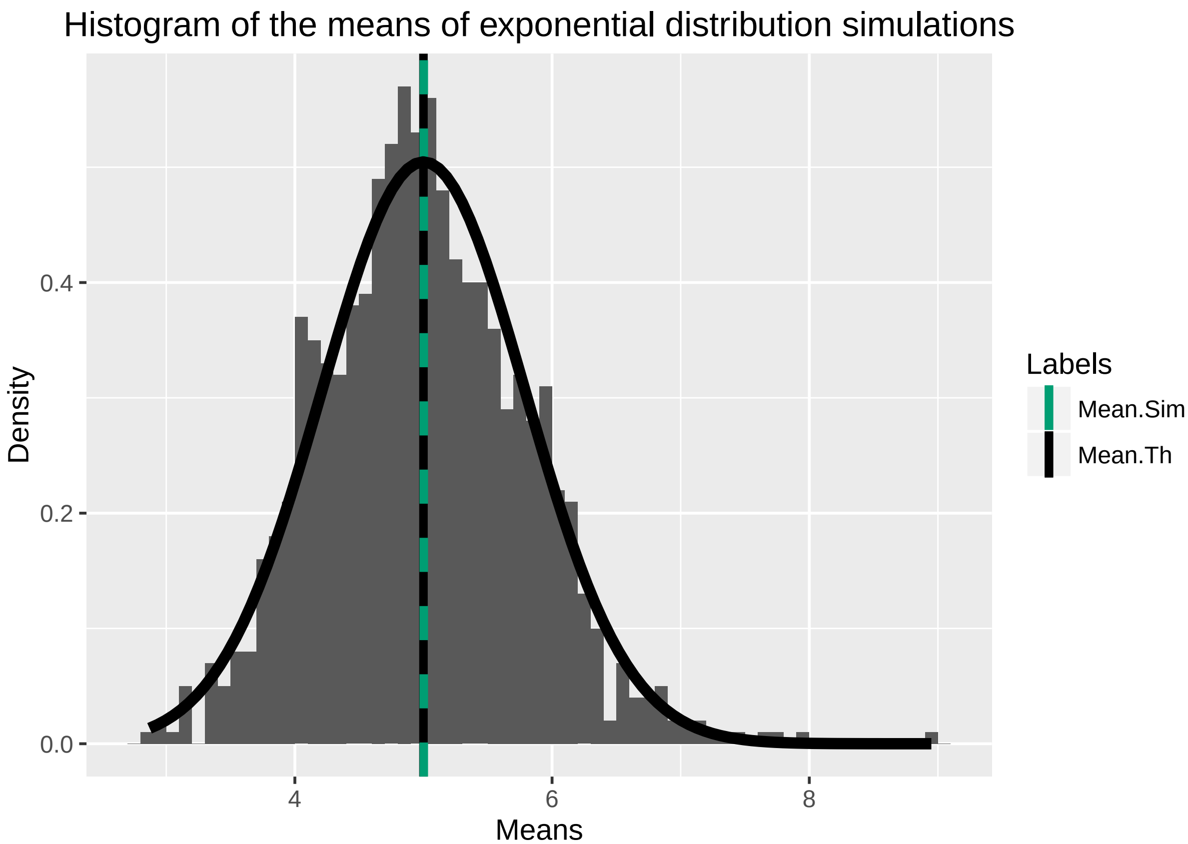
Comparing the sample mean to the theoretical mean

The sample mean is 5.002327 and the theoretical mean is 5. As you can see in this figure, the sample distribution is very closely centered to the theoretical distribution, as stated in the Central Limit Theorem.



Comparing the sample variance to the theoretical variance and examining the normal distribution

The sample variance is 0.6308244 and the theoretical variance is 0.625. As you can see in this figure, which is identical to the previous one with the expected normal distribution contour, the distribution of the means follows a normal distribution $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) = N(5, 0.625)$.



Histogram of the means of exponential distribution simulations
Density
0.0
0.2
0.4
4
6
8
Means
Labels
Mean.Sim
Mean.Th

Appendix

Code for the first histogram

```
ggplot() +
    aes(sim_means) +
    geom_histogram(aes(y=..density..),binwidth=0.1) +
    geom_vline(aes(xintercept=mean_theory,
                   colour="Mean.Th",
                   linetype = "Mean.Th"),
               size = 1.5) +
    geom_vline(aes(xintercept=mean_sim_means,
                   colour="Mean.Sim",
                   linetype = "Mean.Sim"),
               size = 1.5,
               show.legend=TRUE) +
    scale_colour_manual(name="Labels",
                        values=c(Mean.Sim="#009E73",Mean.Th="black")) +
    scale_linetype_manual(name="Labels",
                          values=c(Mean.Sim="dashed",Mean.Th="solid"),
                          guide=FALSE)+
    labs(x = "Means",
         y = "Density",
         title = "Histogram of the means of exponential distribution simulations")
```

Code for the second histogram

```
ggplot() +
    aes(sim_means) +
    geom_histogram(aes(y=..density..),binwidth=0.1) +
    geom_vline(aes(xintercept=mean_theory,
                   colour="Mean.Th",
                   linetype = "Mean.Th"),
               size = 1.5) +
    geom_vline(aes(xintercept=mean_sim_means,
                   colour="Mean.Sim",
                   linetype = "Mean.Sim"),
               size = 1.5,
               show.legend=TRUE) +
    scale_colour_manual(name="Labels",
                        values=c(Mean.Sim="#009E73",Mean.Th="black")) +
    scale_linetype_manual(name="Labels",
                          values=c(Mean.Sim="dashed",Mean.Th="solid"),
                          guide=FALSE)+
    stat_function(fun = dnorm,
                  size = 2,
                  args = list(mean = mean_theory, sd = sqrt(var_theory_means))) +
    labs(x = "Means",
         y = "Density",
         title = "Histogram of the means of exponential distribution simulations")
```