

# 1 Vectors

## 1.1 Array-based implementation

THEOREM: Additive reallocation takes  $O(n)$  time, where  $n$  is number of elements added.

$\langle 1 \rangle 1$ . We copy each element of the array for each element added

PROOF: On each INSERT operation we grow our vector, so we copy the whole array, element by element.  $\square$

$\langle 1 \rangle 2$ . On average, the number of elements we copy is  $\frac{n+1}{2} + k$ , where  $k$  is length the vector had before we had our way with it.

PROOF: For each  $i \leftarrow [1..n]$ , we copy  $k+i$  elements on  $i$ -1st INSERT operation. If we pair them up as  $(i, n-i)$ , we get  $n+1$  elements copied per each of  $n$  pairs.  $\square$

$\langle 1 \rangle 3$ . Q.E.D.

$\langle 0 \rangle 1$ . Q.E.D.