1 Euclides' proof of there being infinitely many prime numbers

1.1 Statement

Let P be prime numbers. There are infinitely many of them.

1.2 Statement

Equivalently, let P be prime numbers. There is no $p \in P$ such that $\forall p' \in P : p' \leq p$.

1.2.1 Proof

Assume there is p as defined above. then $(p \cdot p - 1) - 1$ is not divisible by any p', and is greater than p, which gives us condradiction, which could only be because the assumption is false, thus, there is no maximum element in P.

2 Mersenne's proof that for composite n $M_n = 2^n - 1$ is composite as well

2.1 proof

consider $n = a \cdot b$ and $2^n - 1 = ((2^a)^{b-1} \cdot 2^a) - 1$. Then ...