


# Homework 4

③ A matching  $M$  of graph  $G$  is maximal if no other edges of  $G$  can be added to  $M$

• A matching  $M$  is perfect if every vertex of  $G$  is incident to exactly one edge of the matching  $M$ . Also the degree of every vertex in the subgraph should have a degree 1.

• Maximal matching can be computed greedily

• Not every  $G$  has perfect matching. Ex: 

④ Fractional matching assigns to each edge in  $E$  a fraction  $[0,1]$  s. that for every vertex  $\sum_{e \in E(v)} f(e) \leq 1$ .

Then matching is a special case of fractional matching in which if edge  $e$  is in  $M$  then  $f(e)=1$  else  $f(e)=0$ . It follows

that for whichever  $M$   $\sum_{e \in E(v)} f(e) \leq 1$  for every  $v \in V$ . Also if we have perfect match the equality holds.

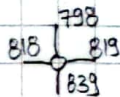
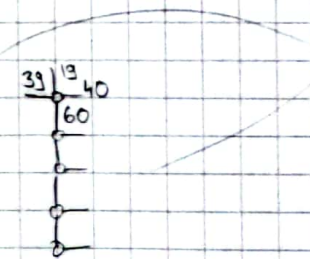
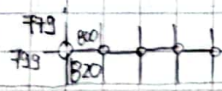
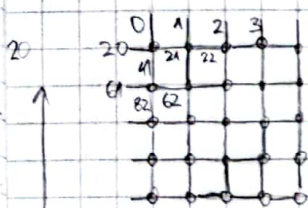
• Let graph  $G$  have  $n$  - edges. Let  $M_i$  be a matching that contains only one edge  $i$ . Then  $\bigcup M_i = G$ . Let  $M_i$  have the edge  $i$  weighted as 1. Then every weighted matching is:

$$\alpha_1 M_1 + \alpha_2 M_2 + \dots + \alpha_n M_n$$

s. that the sum of matches corresponding to the same vertex

$$\text{is } \sum \alpha_i \leq 1$$

⑤



added external edges which will have weight = 0



⑥ Even though we relaxed the matching problem we still as a result got a matching.

⑦ As a sensible jump move I removed the worst 3, 5 or 10 edges which had the lowest weight.

⑧ How close to  $x^*$  we can get pretty much depends on the number of steps that we take and the 1, 2 or 3-adjacency. Using 3-adjacency, jump moves and 10000 step the difference of  $f(x^*)$  and  $f(x_{\min})$  was only 38.



## Exercise 6: $x^*$ solution output from terminal

```
Weight: 0.0 soution: 4.724506678311034e-10
Weight: 1.7072386343133985 soution: 0.999999996703919
Weight: 1.1525390429142615 soution: 1.0158528885136048e-09
Weight: 1.5762883601668132 soution: 1.3640613908186706e-08
Weight: 1.606715046382856 soution: 0.9999999854372826
Weight: 1.424130671302386 soution: 1.5462297144799156e-08
Weight: 1.736444235624723 soution: 2.268246924228738e-09
Weight: 1.9343670147690148 soution: 6.45155614096176e-09
Weight: 1.9255685129067763 soution: 0.9999999928585426
Weight: 1.450839371404132 soution: 8.203825941623025e-09
Weight: 1.1132380458407551 soution: 1.243057907946173e-09
Weight: 1.9848411989623345 soution: 0.9999999511378935
Weight: 1.838898086445934 soution: 5.000747661826529e-08
Weight: 1.124662681203267 soution: 3.1381895722888826e-09
Weight: 1.9208418826173723 soution: 0.9999999769440047
Weight: 1.8698963620621283 soution: 2.368162668891683e-08
Weight: 1.518838057126072 soution: 0.9999999755432981
Weight: 1.5912754357449292 soution: 7.638052332967752e-09
Weight: 1.3990027038701303 soution: 1.9702673671986546e-09
Weight: 1.0547616388220313 soution: 1.3611582751066796e-09
Weight: 0.0 soution: 2.06820919930639e-09
Weight: 0.0 soution: 4.724506678311034e-10
Weight: 0.0 soution: 4.6110469642492254e-10
Weight: 0.0 soution: 5.509908124152164e-10
Weight: 0.0 soution: 4.321371464722306e-10
Weight: 0.0 soution: 5.55500585980154e-10
Weight: 0.0 soution: 5.194891615558129e-10
Weight: 0.0 soution: 3.5720541713942195e-10
Weight: 0.0 soution: 4.790406003578889e-10
Weight: 0.0 soution: 3.80502074275783e-10
Weight: 0.0 soution: 9.251045817632189e-10
Weight: 0.0 soution: 3.8348795752932375e-10
Weight: 0.0 soution: 3.8840831880108816e-10
Weight: 0.0 soution: 4.419571254768186e-10
Weight: 0.0 soution: 1.0476443211904967e-09
Weight: 0.0 soution: 2.6469743946183236e-10
Weight: 0.0 soution: 1.1525544778114362e-09
Weight: 0.0 soution: 3.492473394418922e-10
Weight: 0.0 soution: 8.595199801504681e-10
Weight: 0.0 soution: 3.1484157010931774e-10
Weight: 0.0 soution: 2.06820919930639e-09
This fractional matching contains 200.00000041706016 egdes
Cost which we were maximizing is: 338.20277289933085
```

### Exercise 7: solution for MaxWeightMatching and comparison with $x^*$

```
Weight: 1.9255685129067763 soution: 1.0  
Weight: 1.450839371404132 soution: 0.0  
Weight: 1.1132380458407551 soution: 0.0  
Weight: 1.9848411989623345 soution: 0.0  
Weight: 1.838898086445934 soution: 1.0  
Weight: 1.124662681203267 soution: 0.0  
Weight: 1.9208418826173723 soution: 1.0  
Weight: 1.8698963620621283 soution: 0.0  
Weight: 1.518838057126072 soution: 0.0  
Weight: 1.5912754357449292 soution: 1.0  
Weight: 1.3990027038701303 soution: 0.0  
Weight: 1.0547616388220313 soution: 1.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
Weight: 0.0 soution: 0.0  
This matching contains 185.0 egdes  
Cost which we were maximizing is: 300.8976888623149
```