

3d (C). Work by Gravitational Force

- [work_grav.cpp] A satellite is falling on Earth following a spiral-like orbit with equation:

$$r(\theta) = \frac{r_0}{\sqrt{1 + (\theta/b)^4}}$$

where $b=\pi$.

- Using its standard definition ($W = \int \mathbf{F} \cdot d\mathbf{s}$), compute the work done by the gravitational force on the satellite for $\theta \in [0, 4\pi]$ using Gaussian ($n_{\text{gauss}}=3$) and trapezoidal methods. In particular, determine the number of sub-intervals $n_{\text{sub}} = 4, 8, 16, \dots$ until the error

$$\varepsilon = |W_{n_{\text{sub}}} - W_{n_{\text{sub}}/2}|$$

(where $W_{n_{\text{sub}}}$ is the work obtained by integrating on n_{sub} intervals) falls below a given tolerance, i.e., $\rightarrow \varepsilon < \text{tol}$, with $\text{tol} = 1.e-6$. Assume $r_0 = GMm = 1$.

- Do you have an analytical solution for this problem? Use it as a comparison.
- Upload your code with i) the output inserted in the comments at the beginning of the file, ii) the required library function at the end, e.g.

```
// Name: Your name
// Date: 02 Nov 2023
//
// Code output:
// *****
// Trapezoidal:  n = ??; Wx = ??; Wy = ??; Wtot = Wx + Wy = ??
// Gaussian Int  n = ??; Wx = ??; Wy = ??; Wtot = Wx + Wy = ??
// (Exact = ??)
// *****
#include ...
...
int main()
{
    // code here
}

void Trapezoidal(...){

}

void Gauss(...){

}
```