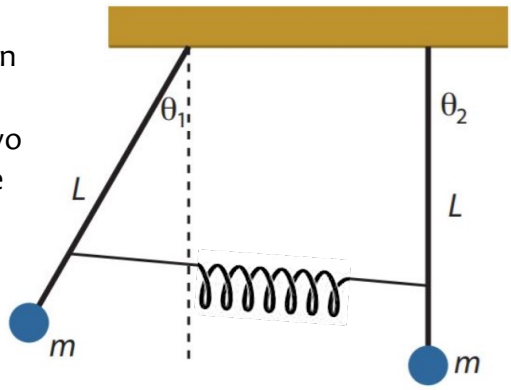
7c. Coupled Pendula

- **coupled_pendula.cpp**: Two identical pendula of length L = 1 are connected by a weak spring with constant k, as in the figure.
- If θ_1 and θ_2 are, respectively, the angles formed by the two pendula with the vertical, the equations of motion for the coupled system can be written as



$$L\ddot{\theta}_1 = -g\sin\theta_1 - kL\left(\sin\theta_1 - \sin\theta_2\right)$$
$$L\ddot{\theta}_2 = -g\sin\theta_2 + kL\left(\sin\theta_1 - \sin\theta_2\right)$$

- Choose *k = 0.8*, *g = 9.8* and use the initial condition

$$\theta_1(0) = -\frac{\pi}{4}, \quad \theta_2(0) = \omega_1(0) = \omega_2(0) = 0$$

where $\omega = d\theta/dt$.

- Solve the equations of motion using both RK4 and either Position-Verlet (PV) or Velocity-Verlet (VV). Use a single loop to advance both solutions in time with constant $\Delta t = 0.1$ and write the solution to disk using the standard multicolumn format.
- Break from the loop when the difference between the solutions generated by the two methods (on θ_1 or θ_2) exceeds *tol = 0.25*, i.e.:

$$\max\left(\epsilon_1, \epsilon_2\right) > tol, \quad \text{where} \quad \epsilon_j = \left|\theta_j^{\mathrm{RK4}} - \theta_j^{\mathrm{PV}}\right|/\pi \quad (j = 1, 2)$$

- Count the (approximate) number of turning points for the two pendula (for RK4).
- Upload your code with i) the output inserted in the comments at the beginning of the file, ii) the required library function at the end, e.g.

```
// Name: ..., Date: ...
//
// Code output:
// ****************************************************
// Loop break at nstep = ??; t = ??      # when you exit from the loop
//               eps1  = ??; eps2 = ??   # differences between the two solutions
//               tp1   = ??; tp2  = ??   # number of turning points
// ****************************************************
#include ...
...
int main()
{
  // code here
}

void RK4Step (...){

}

void  VerletStep(...){

}
```

7c. Coupled Pendula (cont)

- Also, upload a plot (**png** screen capture is fine) of the solution for **0 < t < 50**, showing *only* the angles θ_1 and θ_2 obtained with RK4 and the Verlet algorithm (4 plots in total) as a function of time.
- If you saved the output using a multi-column format, you may use the following Gnuplot script:

```
reset

# Set column indices
th1_RK4 = 2   # This is the column number for theta1 (RK4)
th2_RK4 = 3   # This is the column number for theta2 (RK4)


th1_PV = 6    # This is the column number for theta1 (Verlet)
th2_PV = 7    # This is the column number for theta2 (Verlet)

# Set plot speicifications
set grid
set key    font ",14"
set title  font ",14"
set tics   font ",14"
set xlabel "x"      font ",14"
set ylabel "theta"  font ",14"

# Plot
set xrange[0:50]
set yrange[-1:1]
  plot "coupled_pendula.dat" using 1:th1_RK4  title "th1 (RK4)"  w lines
replot "coupled_pendula.dat" using 1:th2_RK4  title "th2 (RK4)"  w lines

replot "coupled_pendula.dat" using 1:th1_PV  title "th2 (PV)"   w lines
replot "coupled_pendula.dat" using 1:th2_PV  title "th2 (PV)"   w lines
```