1. Holomorphic functions

3. Cauchy-Riemman

Let:

$\cdot\ f \in \mathcal{H}(\mathbb{C})$ „ $Ref + Imf = c_a$

Show that:

$\cdot\ \exists\, a' \in \mathbb{C}:$

$f = c_{a'}$

Demonstration:

u, v real components of f

$u(x,y) + v(x,y) = a$

differentiate respect x and y

$u_x + v_x = 0$

$u_y + v_y = 0$

f holomorphic $\rightarrow f$ CR

$u_x - u_y = 0$

$u_y + u_x = 0$

$u_x, u_y, v_x, v_y = 0$

$\exists\, a_1 \in \mathbb{R}:$

$u = c_{a_1}$

$\exists\, a_2 \in \mathbb{R}:$

$v = c_{a_2}$

$f = c_{(a_1,a_2)}$

B.2 a)

Let:

$$\cdot \quad \begin{array}{rcl} u : \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ (x,y) & \longmapsto & \exp(y)\cos(x) \end{array}$$

Show that:

- $\exists\ f \in \mathcal{H}(\mathbb{C})$:

  u real component of f

Demonstration:

lab 1→ $u_{xx} + v_{yy} = 0$

$u_x = \exp(x)\cos(y)$

$u_{xx} = \exp(x)\cos(y)$

$u_y = -\exp(x)\sin(y)$

$u_{yy} = -\exp(x)\cos(y)$

ok

Calculate v using CR

$v_y = u_x = \exp(x)\cos(y)$

$v(x,y) = \int_{\mathbb{C}} \exp(x)\sin(y)dy = \exp(x)\sin(y) + \phi(x)$

$v_x = \partial_v x = \exp(x)\sin(y) + \phi'(x)$

$-u_y = \exp(x)\sin(y) + \phi'(x)$

CR→ $\phi'(x) = 0$

$\forall\ c \in \mathbb{R}$:

$\phi(x) = c$ ok

$v(x,y) = \exp(x)\sin(y)$

Preservation of angles

Let:

- γ_1, γ_2 plane arcs „ $\gamma_1(0) = \gamma_2(0)$

Then, holds:

- angle of $\gamma_1'(0)$ and $\gamma_2'(0)$ = angle $\sigma_1'(0), \sigma_2'(0)$

Demonstration:

rotations and homotecies let angles invariant