# 1. Information & Decision

## Regularity

Let:

- $(\Omega, \mathcal{A}, \{P_\theta\}_{\theta \in \Theta})$ 1-D **real** statistical model
- $L$ likelihood function of $(\Omega, \mathcal{A}, \{P_\theta\}_{\theta \in \Theta})$

Then, $(\Omega, \mathcal{A}, \{P_\theta\}_{\theta \in \Theta})$ is regular if:

- $\Theta$ open
- $\forall\ \theta_1, \theta_2 \in \Theta$ :

$$\{x \in \Omega \mid L(x, \theta_1) = 0\} = \{x \in \Omega \mid L(x, \theta_2) = 0\}$$

- $\forall\ \theta \in \Theta$ :

$\exists\ f\ :\ \Omega \to \mathbb{R}^+$ :

$\exists\ \mathcal{E}_\theta \subset \Theta$ :

$\mathcal{E}_\theta$ neighborhood of $\theta$

$\forall\ \theta' \in \mathcal{E}_\theta$ :

$$\mid \partial_\theta \log(L(x,\theta)) \mid \vee \mid \partial_{\theta^2} \log(L(x,\theta))\theta \mid\ \leq$$

$f(x)$

- $\forall\ \theta \in \Theta$ :

$$E_x \Big(\ \mid \partial_\theta \log(L(x,\theta)) \mid^2\ \Big) \text{ finite}$$

## Fisher's information

Let:

$\cdot\ (\Omega, \mathcal{A}, \{P_\theta\}_{\theta\in\Theta})$ 1-D regular statistical model

$\cdot\ L$ likelihood function of $(\Omega, \mathcal{A}, \{P_\theta\}_{\theta\in\Theta})$

We name Fisher's information of $(\Omega, \mathcal{A}, \{P_\theta\}_{\theta\in\Theta})$ to:

$$\cdot \quad \begin{array}{rccc} f : & \Theta & \longrightarrow & \mathbb{R} \\ & \theta & \longmapsto & E_x\Big( \mid \log(L(x,\theta)) \mid^2 \Big) \end{array}$$

We denote:

$\cdot\ f(\theta)\ :\ I_F(\theta)$

## Kullback's information

Let:

$\cdot\ (\Omega, \mathcal{A}, \{P_\theta\}_{\theta\in\Theta})$ d-D statistical model

We name Kullback's information of $(\Omega, \mathcal{A}, \{P_\theta\}_{\theta\in\Theta})$ to:

$$\cdot \quad \begin{array}{rccc} f : & \Theta^2 & \longrightarrow & \mathbb{R} \\ & (\theta_1, \theta_2) & \longmapsto & E_{\theta_2}\Big( \log(\frac{L(x,\theta_1)}{L(x,\theta_2)}) \Big) \end{array}$$

We denote:

$\cdot\ f((\theta_1, \theta_2))\ :\ I_K(\theta_1 \mid \theta_2)$

### Decision

Let:

· $(\Omega, \mathcal{A}, \{P_\theta\}_{\theta \in \Theta})$ d-D **measurable** statistical model

· $(D, \mathcal{D})$ measurable space

· $f \,:\, \Omega \to D$

Then, $f$ is a decision if:

· $f$ measurable

We denote:

· $\{f \,:\, \Omega \to D \mid f$ measurable $\} \,:\, \Xi$

### Decision order

Let:

· $(\Omega, \mathcal{A}, \{P_\theta\}_{\theta \in \Theta})$ d-D **measurable** statistical model

· $(D, \mathcal{D})$ measurable space

· $f_1, f_2 \,:\, \Omega \to D$ decisions

Then, $f_1$ is better than $f_2$ if:

· *conditions*

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**Loss function**

Let:

- $(\Omega, \mathcal{A}, \{P_\theta\}_{\theta \in \Theta})$ d-D statistical model

- $(D, \mathcal{D})$ measurable space

- $W : D \times \Theta \to \mathbb{R}^+$

Then, $W$ is a loss function if:

- $W$ measurable

- $\forall\, d \in D$ " $d$ correct :

  $W(d, \theta) = 0$

- $\forall\, d_1, d_2 \in D$ " $d_1$ better than $d_2$ :

  $W(d_1, \theta) \leq W(d_2, \theta)$

**Risk function**

Let:

- $(\Omega, \mathcal{A}, \{P_\theta\}_{\theta \in \Theta})$ d-D statistical model

- $(D, \mathcal{D})$ measurable space

- $W : D \times \Theta \to \mathbb{R}^+$ loss function

We name risk function of $W$ to:

$$\cdot \quad \begin{array}{rccc} R : & \Xi \times \Theta & \longrightarrow & \mathbb{R} \\ & (\chi, \theta) & \longmapsto & E_x\left( \left(W(\chi(x), \theta)\right) \right) \end{array}$$