1. New

logarithm is holomorphic

Let:

$\cdot \, log \; : \; \mathbb{C} \setminus e^{i\alpha}(-\infty, 0] \to B_\alpha$

Then, holds:

$\cdot \, log \in \mathcal{H}$

$\cdot \, log' = \frac{1}{z}$

Demonstration:

log continuous

$\exp(\log(z)) = z$

$\omega = \log(z)$

$\omega_0 = \log(z_0)$

$\exp(\omega) = z$

$\exp(\omega_0) = z_0$

log continuous $\to w \xrightarrow{n} w_0$ if $z = z_0$

$\frac{log(z) - log(z_0)}{z - z_0} = \frac{w - w_0}{e^w - e^{w_0}}$

$$= \lim_{w \to w_0} \frac{1}{\frac{e^w - e^{w_0}}{w - w_0}} = \frac{1}{e^{w_0}} = \frac{1}{z_0}$$

Complex Powers

Let:

$$\cdot\ z^{w} = e^{wlogz} = e^{w(log|z|+iarg(z)}$$

Then, holds:

$$\cdot\ z^{w} = \exp(n\log(z))$$

Demonstration:

$$z = \exp(\log(z))$$

$$z^{w} = \exp(\log(z))^{n} = \prod_{i=1}^{n} \exp(\log(z)) = \exp(n\log(z))$$

i powers

Let:

- $i \in \mathbb{C}$

Then, i^i is an infinite value set :

$\forall\ k \in \mathbb{Z}$:

$$i^i = \exp(i \log(i)) = \exp(i^2(\frac{\pi}{2} + 2k\pi)) = \exp(-(\frac{\pi}{2} + 2k\pi))$$

Different values of powers

Let:

- $z, w \in \mathbb{C}$

Then, holds:

- $w \in \mathbb{Z} \to z^w$ unique
- $w = \frac{p}{q} \in \mathbb{Q} \to z^w$ has q values
- $w \in \mathbb{R} \setminus \mathbb{Q} \to z^w$ has infinite values

Different logarithm definitions

Let:

- $\Omega = \mathbb{C} \setminus \{(x, y) \in \mathbb{C} \mid x < 0, y = 0\}$
- $\Omega_2 = \mathbb{C} \setminus \{(x, y) \in \mathbb{C} \mid x > 0, y = x\}$
- $\Omega_3 = \mathbb{C} \setminus \{(x, y) \in \mathbb{C} \mid x \in (-1, 0), y = 0\} \cup \{(x, y) \in \mathbb{C} \mid x = -1, y \in$

$(0, \pi)\} \cup \{(x, y = \in \mathbb{C} \mid x > -1, y = \frac{\pi}{2}\}$

Study:

- $arg(i), arg(2i)$ in function of $arg(1)$

Start:

$arg(1) = 0 \to arg(i) = \frac{\pi}{2}$

$arg(1) = -2\pi \to arg(i) = \frac{-3\pi}{2}$

$arg(i) = \frac{-3\pi}{2} \to arg(1) = -2\pi$

$arg(1) = 0 \to arg(i) = \frac{-3\pi}{2}$

$arg(1) = -2\pi \to arg(\pi) = \frac{-7\pi}{2}$

$arg(1) = 0 \to arg(i) = \frac{\pi}{2}$

$arg(1) = 0 \to arg(2i) = \frac{-3\pi}{2}$

$arg(i) = \frac{-3\pi}{2} \to arg(2i) = \frac{-7\pi}{2}$

Simple fraction form

Let:

- $f \in \mathrm{Rat}\mathbb{C})$

We name simple fraction form of f to:

- $\sum_{i=1}^{r} \frac{a_i}{(x - a_i)} = f$