1. Statistic models

introduction

Statistical model

Let:

- Ω set
- $\mathcal{A}$ sigma-algebra over Ω
- $\mathcal{P} \subset \mathcal{F}(\Omega, \mathbb{R})$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is a statistical model if:

- $\forall\ f \in \mathcal{P}$:

    f probability distribution

Parametrized

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model
- $\Theta \subset \mathbb{R}^d$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is parametrized by Θif:

- $\mathcal{P} = \{P_\theta\}_{\theta \in \Theta}$

Likelihood

Let:

$\cdot$ $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ

We name likelihood function of $(\Omega, \mathcal{A}, \mathcal{P})$ to:

$$\cdot \quad \begin{array}{rcl} L\ :\Omega \times \Theta & \longrightarrow & \mathbb{R}^{+} \\ (x, \theta) & \longmapsto & P_{\theta}(x) \end{array}$$

Exponential model

Let:

$\cdot$ $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ

$\cdot$ L likelihood function of $(\Omega, \mathcal{A}, \mathcal{P})$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is exponential if:

$\cdot$ $\exists\ \{C_i\}_{i=1}^{r}, D \in \mathcal{F}(\theta, \mathbb{R})$ " $\forall\ i \in [1, r]_{\mathbb{N}}$:

C_i measurable

$\cdot$ $\exists\ \{\phi_i\}_{i=1}^{r}, \psi \in \mathcal{F}(\Omega, \mathbb{R})$ " $\forall\ i \in [1, r]_{\mathbb{N}}$:

ϕ_i measurable

$\cdot$ $\forall\ x \in \Omega$:

$\forall\ \theta \in \Theta$:

$$L(x, \theta) = \exp(\sum_{i=1}^{r} C_i(\theta)\phi_i(x) + D(\theta) + \psi(x))$$

entity

Let:

- $\cdot\, statements$

- $\cdot$

Then, *item* is a/an entity :

$proof$