

1. New

Homeomorphisms are monotonous

Let:

$\cdot f : \mathbb{R} \rightarrow \mathbb{R}$ homeomorphism

Then, holds:

$\cdot f$ monotonous

Demonstration:

no demonstration

Homeomorphisms and n-periodic points

Let:

· $f : \mathbb{R} \rightarrow \mathbb{R}$ homeomorphism (M, T, ϕ) dynamical system defined
by f

Then, holds:

· $\forall n \in \mathbb{N}$:
 $\exists x \in M$ „ x n-periodic point

Demonstration:

graphically

Decreasing function orbits

Let:

· *declarations*

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Show that:

· *statements*

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Demonstration:

f corta en un punto

f decreasing $\rightarrow f^2$ increasing

$f^{2n} \xrightarrow{n}$ fixed point of f

10.

Let:

$$\cdot f : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \in \mathbb{C}^\infty$$

$$\cdot f(0) = 0$$

$$\cdot p \in \mathbb{R}^+ \setminus \{0\} \quad \parallel f'(p) \geq 0$$

$$\cdot f' \text{ decreasing}$$

Show that:

$$\cdot \forall p \in \mathbb{R}^+ \setminus \{0\}:$$

$$f^n(x) \xrightarrow{n} p$$

Demonstration:

$$f' \text{ decreasing} \rightarrow f'' < 0 \rightarrow f \text{ concave}$$

$$f \text{ positive} \rightarrow f \text{ has no extrema} \rightarrow f' > 0 \rightarrow f \text{ increasing}$$

$$f \text{ has only one fixed point}$$

$$\text{Suppose 2 fixed points : } p, p'$$

$$IVT \rightarrow \exists c \in (0, p'):$$

$$f'(c) = 1$$

$$f'(p) < 1 \rightarrow p \text{ attractive } IVT \rightarrow \text{dont exist more fixed points}$$

$$\rightarrow f'(c') = 1 \not\leq 1$$

$$\forall x \in (0, p):$$

$$f(x) > x$$

$$\forall x \in \mathbb{R} \quad \parallel x > p:$$

$$f(x) < x$$

$$f \text{ increasing} \rightarrow f([0, p]) = [0, p]$$