1. New

Numeric series

Numeric series

Let:

· $(c_k)_{k\in\mathbb{N}}$

Then, *item* is a/an entity if:

· *conditions*

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We denote:

· *property* : *notation*

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Convergence of complex series

Let:

· $\sum c_n n[0]$ complex series

Then, holds:

· $\sum c_n n[0]$ convergent $\leftrightarrow \sum Rec_n n[0]$ convergent $\wedge \sum Imc_n n[0]$

convergent

Demonstration:

demonstration

Absolutely convergent

Let:

· $\sum c_n n[0]$ series

Then, $\sum c_n n[0]$ is absolutely convergent if:

· $\sum |c_n| n[0]$ convergent

Absolutely convergent are convergent

Let:

· $\sum c_n n[0]$ absolutely convergent

Then, holds:

· $\sum c_n n[0]$ convergent

Demonstration:

$S_k \ := \ \sum c_n n[0][k]$

$\forall \ m \in \mathbb{N} \ \ " \ m < k$:

$|S_k - S_m| = |\sum c_n n[m+1][k]| \leq \sum |c_n| n[m+1][k]$

$\leq \sum |c_n| n[m+1] \xrightarrow{n} 0$

$|S_k - S_m| \xrightarrow{n} 0 \ \rightarrow \ (S_k)_k$ convergent $\rightarrow \ \sum c_n n[0]$ convergent

Series and norm

Let:

$\cdot \sum c_n n[0]$ convergent

Then, holds:

$\cdot |c_n| \xrightarrow{n} 0$

Demonstration:

$\sum c_n n[0]$ convergent $\leftrightarrow (S_n)_n$ convergent

$\rightarrow$ Cauchy $|S_n - S_m| \xrightarrow{n} 0$ por n y m $\rightarrow |S_n - S_{n-1}| \xrightarrow{n} 0$

$\rightarrow |c_n| \xrightarrow{n} 0$

Root test

Let:

$$\cdot \sum_{n\geq 0} c_n \text{ real series}$$
$$\cdot\, l \in \mathbb{R} \quad " \overline{\lim_{k}}\, |c_k|^{\frac{1}{k}} = l$$

Then, holds:

$$\cdot\, l > 1 \rightarrow \sum_{n\geq 0} c_n \notin \mathbb{R}$$
$$\cdot\, l < 1 \rightarrow \sum_{n\geq 0} c_n \in \mathbb{R}$$

Demonstration:

$$demonstration$$

Quotient test

Let:

- $\sum_{n \geq 0} c_n$ real series

Then, holds:

- $\exists\, l \in \mathbb{R}$:

$$\lim_{k} \frac{c_{k+1}}{c_k} = l$$

- $\overline{\lim_{c_k}} |c_k|^{\frac{1}{k}} k = l$