1. Holomorphic functions

Conjugation

Let:

$$\cdot \quad \begin{array}{rccc} \bar{a} & : \mathbb{C} & \longrightarrow & \mathbb{C} \\ & z & \longmapsto & \bar{z} \end{array}$$

Then, $\bar{a}$ is not holomorphic :

$$u_x = 1$$

$$u_y = 0$$

$$v_x = 0$$

$$v_y = -1$$

$\forall\ z \in \mathbb{C}$:

$$-1 \neq 1 \to f \text{ not holomorphic in } z$$

Quadratic norm

Let:

- $\begin{aligned} f : \mathbb{C} &\longrightarrow \mathbb{C} \\ z &\longmapsto |z|^2 \end{aligned}$
- $f_{\mathbb{R}^2}$ component decomposition of f

Then, f is holomorphic in 0:

f differentiable in $\mathbb{R}^2$ polinomial

$\forall\ z \in \mathbb{C}$:

$u_x(x,y) = 2x$

$u_y(x,y) = 2y$

$v_x(x,y) = 0$

$v_y(x,y) = 0$

$u_x = v_y \leftrightarrow x = 0$

$u_y = -v_x \leftrightarrow y = 0$

f holomorphic function in $z \leftrightarrow z = 0$

Non preserving angles function

Let:

$\cdot\ f(z) = z^2$

Then, f is conform in $\mathbb{R} \setminus \{0\}$:

$$f(\{(x,0) \in \mathbb{C} \mid x > 0\}) = \{(x,0) \in \mathbb{C} \mid x > 0\}$$

$$f(\{(x,0) \in \mathbb{C} \mid x < 0\}) = \{(x,0) \in \mathbb{C} \mid x > 0\}$$

$$ang(A,B) = \pi \neq 0 = ang(f(A), f(B))$$

Exponential

Let:

$\cdot\ a\ :\ 0$

$\cdot\ c_n\ :\ \frac{1}{n}$

Then, $\sum\limits_{n\geq 0} c_n(z-a)^n$ is convergent in D1 :

$$\lim_n \frac{|c_n|}{|c_{n+1}|} = \lim_n \frac{n+1}{n} = 1 \to R = 1$$

CH$\to$ $D(0,1)$ convergent

$\mathbb{C} \setminus D(0,1)$ divergent

$f' = f$

Geometric series

Let:

$\cdot\, a \,:\, 0$

$\cdot\, c_n : 0$

Then, $\sum z^n n[0]$ is convergent in $\mathbb{D}$:

$R = \frac{c_n}{c_{n+1}} = 1$ Then, holds:

$\cdot\ \sum z^n n[0] = \frac{1}{1-z}$

$\cdot\ \sum n z^{n-1} n[0] = \frac{1}{(1-z)^2}$

$\cdot\ \sum \frac{z^{n+1}}{n+1} n[0] = -\log(1-z)$

Demonstration:

$\forall\ z \in \mathbb{D}$:

$\sum z^n n[0]$ geometric series

$\sum z^n n[0] = \frac{1}{1-z}$

II differentiating

III integrating

Series not centered in 0

Let:

- a : i
- c_n : $\frac{n+1}{5^{n+1}}$

Then, *item* is a/an entity :

$\sum \frac{n(z-i)^{n-1}}{5^n} n[1]$

$= \frac{1}{5} \sum n \frac{z-i}{5}^{n-1} n[1] = \frac{1}{5} \sum n u^{n-1} n[1]$

$S(u) = \tilde{S}'(u)$

$\tilde{S}(u) = \frac{1}{5} \sum u^n n[1] = \frac{u}{5(1-u)}$

$S(u) = \frac{1}{5(1-u)^2}$

$S(z) = \frac{5}{(5+i-z)^2}$ over $D(i, 5)$

Radius of convergence without quotient test

Let:

- $\sum \frac{(-1)^n}{n(n+1)} (z-2)^{n(n+1)} n[1]$

Then, R is a/an entity :

$\lim_{c_{n+1}} c_n \nexists$

$\lim_n \frac{1}{n(n+1)}^{\frac{1}{n(n+1)}} = 1$

ignore zeros