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| 1. The field of complex numbers |
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introduction

The field of complex numbers

Let:

$$\begin{aligned} \cdot \quad & + : \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}^2 \\ & ((a, b), (c, d)) \longmapsto (a + c, b + d) \\ \cdot \quad & \cdot : \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}^2 \\ & ((a, b), (c, d)) \longmapsto (ac - bd, ad + bd) \end{aligned}$$

We name the field of complex numbers to:

$$\cdot (\mathbb{R}^2, +, \cdot)$$

We denote:

$$\cdot (\mathbb{R}^2, +, \cdot) : \mathbb{C}$$

$$\cdot (0, 1) \in \mathbb{C} : i$$

$$\cdot (a, b) \in \mathbb{C} : a + bi$$

$$\cdot \pi_1(\mathbb{C}) : Re$$

$$\cdot \pi_2(\mathbb{C}) : Im$$

Conjugation

We name complex conjugation to:

$$\begin{aligned} \cdot \quad & f : \mathbb{C} \longrightarrow \mathbb{C} \\ & (a, b) \longmapsto (a, -b) \end{aligned}$$

We denote:

$$\cdot f((a, b)) : \overline{(a, b)}$$

Norm

We name `complex norm` to:

$$\begin{array}{lcl} \cdot & f : \mathbb{C} & \longrightarrow \mathbb{R} \\ & (a, b) & \longmapsto a^2 + b^2 \end{array}$$

We denote:

$$\cdot f((a, b)) : |(a, b)|$$

Polar form

We name `polar form` to:

$$\begin{array}{lcl} \cdot & f : \mathbb{C} & \longrightarrow \mathbb{R}^+ \times [0, 2\pi) \\ & (a, b) & \longmapsto (\sqrt{|(a, b)|}, \arctan(\frac{b}{a})) \end{array}$$

We denote:

$$\cdot f((a, b)) : \text{polar form of } (a, b)$$

Unit sphere projection

We name `unit sphere projection` to:

$$\begin{array}{lcl} \cdot & f : \mathbb{C} & \longrightarrow S^1 \\ & z & \longmapsto \frac{z}{|z|} \end{array}$$

We denote:

$$\cdot f(z) : \pi(z)$$

Root of unity

Let:

$$\cdot z \in \mathbb{C}$$

Then, z is a root of unity if:

$$\cdot \exists n \in \mathbb{N} \quad z^n = 1$$

We denote:

$$\cdot \{z \in \mathbb{C} \mid z \text{ root of unity} \} : S^1$$

Disk

Let:

$$\cdot p \in \mathbb{C}$$

$$\cdot r \in \mathbb{R}^+ \setminus \{0\}$$

We name Disk centered in p and radius r to:

$$\cdot \{z \in \mathbb{C} \mid |z - p| < r\}$$

We denote:

$$\cdot \{z \in \mathbb{C} \mid |z - p| < r\} : D^1$$