

1. One-dimensional discrete dynamical systems
Dynamical system

Let:

- M manifold
- T monoid
- $\phi : M \times T \rightarrow M$

Then, (M, T, ϕ) is a dynamical system if:

- $\forall x \in X:$

$$\phi(x, 0) = x$$

$$\forall t_1, t_2 \in T:$$

$$\phi(\phi(x, t_1), t_2) = \phi(x, t_1 + t_2)$$

Dimension

Let:

- (M, T, ϕ) dynamical system

We name dimension of (M, T, ϕ) to:

- $\dim(M)$

We denote:

- $\dim(M) = n : (M, T, \phi)$ n-D dynamical system

Discrete & Continuous

Let:

· (M, T, ϕ) dynamical system

Then, (M, T, ϕ) is discrete if:

· $T \simeq \mathbb{N}$

Then, (M, T, ϕ) is continuous if:

· $T \subset \mathbb{R}$ T open

Defined by a function

Let:

· (M, T, ϕ) dynamical system

· $f : M \rightarrow M$

Then, (M, T, ϕ) is a dynamical system defined by f if:

· $T = \mathbb{N}$

·
$$\begin{array}{ccc} \phi : M \times \mathbb{N} & \longrightarrow & M \\ (x, n) & \longmapsto & f^n(x) \end{array}$$

We denote:

· (M, T, ϕ) dynamical system defined by $f : (M, \mathbb{N}, f)$

Orbit

Let:

$\cdot (M, \mathbb{N}, f)$ functional dynamical system

$\cdot x \in M$

We name orbit of x to:

$\cdot \{f^n(x)\}_{n \in \mathbb{N}}$

We denote:

$\cdot o(x)$

n-periodic point

Let:

- (M, \mathbb{N}, f) functional dynamical system
- $x \in M$
- $n \in \mathbb{N}$

Then, x is a n-periodic point if:

- $f^n(x) = x$
- $\forall n' \in \mathbb{N} \quad n' < n:$
 $f^{n'}(x) \neq x$

We denote:

- $n = 1 : x$ fixed point

Attractive & Repulsive

Let:

- (M, \mathbb{N}, f) metrical dynamical system
- $x \in M$ m-periodic point

Then, x is attractive if:

- $\exists \mathcal{U} \subset M$ open :

$$\forall y \in \mathcal{U}:$$

$$\exists N \in \mathbb{N}:$$

$$\forall n \in \mathbb{N} \quad n \geq N:$$

$$f^{nm}(y) \in \mathcal{U}$$

Then, x is repulsive if:

- $\forall \mathcal{U} \subset M \quad x \in \mathcal{U}:$

$$\forall y \in \mathcal{U}:$$

$$\exists N \in \mathbb{N}:$$

$$\forall n \in \mathbb{N} \quad n \geq N:$$

$$f^{nm}(x) \notin \mathcal{U}$$

Attraction set

Let:

- (M, \mathbb{N}, f) dynamical system
- $x \in M$ attractive m-periodic point
- $o(x)$ orbit of x

We name attraction set of x to:

$$\cdot \{y \in M \mid \exists x' \in o(x) : f^{nm}(y) \xrightarrow{n} x'\}$$

We denote:

$$\cdot A(x)$$

Neutral point

Let:

- (M, \mathbb{N}, f) differentiable dynamical system
- $x \in M$

Then, x is a neutral point if:

$$\cdot f'(x) \in \{-1, 1\}$$

Feeble attractive & repulsive points

Let:

· $(M, \mathbb{N}, f) \mathcal{C}^3$ dynamical system

· $x \in M$

Then, x is feeble attractive point if:

· $f'(x) = 1$

· $f''(x) = 0$

· $f'''(x) > 0$

Then, x is feeble repulsive point if:

· $f'(x) = 1$

· $f''(x) = 0$

· $f'''(x) < 0$

Multiplier

Let:

· $(M, \mathbb{N}, f) \mathcal{C}^1$ dynamical system

· $x \in M$

We name multiplier of x to:

· $f'(x)$

Logistic

Let:

$$\cdot f : \mathbb{R} \rightarrow \mathbb{R}$$

$$\cdot (M, T, \phi) \text{ dynamical system defined by } f$$

Then, (M, T, ϕ) is a logistic dynamical system if:

$$\cdot \exists a \in \mathbb{R}:$$

$$\begin{array}{ccc} f : \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & ax(1-x) \end{array}$$

Chaos

Let:

$$\cdot f : \mathbb{R} \rightarrow \mathbb{R}$$

$$\cdot (M, T, \phi) \text{ dynamical system defined by } f$$

Then, (M, T, ϕ) is chaotic if:

$$\cdot \text{Fix}(f) \text{ dense in } \mathbb{R}$$

$$\cdot \exists x \in \mathbb{R}:$$

$$o(x) \text{ dense in } \mathbb{R}$$

$$\cdot f \text{ sensibility of } x_0$$

Sarkovskii's order

We name Sarkovskii's order to:

· *naming*

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We denote:

· *property : notation*

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