1. New

Numeric series

Numeric series

Let:

- $(c_k)_{k\in\mathbb{N}}$

Then, *item* is a/an entity if:

- *conditions*
-

We denote:

- *property* : *notation*
-

Convergence of complex series

Let:

- $\sum_{n\geq 0} c_n$ complex series

Then, holds:

- $\sum_{n\geq 0} c_n$ convergent $\leftrightarrow \sum_{n\geq 0} Rec_n$ convergent $\wedge \sum Imc_n n[0]$ convergent

Demonstration:

demonstration

Absolutely convergent

Let:

- $\sum c_n n[0]$ series

Then, $\sum c_n n[0]$ is absolutely convergent if:

- $\sum |c_n| n[0]$ convergent

Absolutely convergent are convergent

Let:

- $\sum c_n n[0]$ absolutely convergent

Then, holds:

- $\sum c_n n[0]$ convergent

Demonstration:

$S_k := \sum c_n n[0][k]$

$\forall\ m \in \mathbb{N}$ „ $m < k$:

$|S_k - S_m| = |\sum c_n n[m+1][k]| \leq \sum |c_n| n[m+1][k]$

$\leq \sum |c_n| n[m+1] \xrightarrow{n} 0$

$|S_k - S_m| \xrightarrow{n} 0 \rightarrow (S_k)_k$ convergent $\rightarrow \sum c_n n[0]$ convergent

Series and norm

Let:

- $\sum c_n n[0]$ convergent

Then, holds:

- $|c_n| \xrightarrow{n} 0$

Demonstration:

$\sum c_n n[0]$ convergent $\leftrightarrow (S_n)_n$ convergent

$\rightarrow$ Cauchy $|S_n - S_m| \xrightarrow{n} 0$ por n y m $\rightarrow |S_n - S_{n-1}| \xrightarrow{n} 0$

$\rightarrow |c_n| \xrightarrow{n} 0$

Root test

Let:

- $\sum_{n \geq 0} c_n$ real series
- $l \in \mathbb{R}$,, $\overline{\lim_{k}} |c_k|^{\frac{1}{k}} = l$

Then, holds:

- $l > 1 \rightarrow \sum_{n \geq 0} c_n \notin \mathbb{R}$
- $l < 1 \rightarrow \sum_{n \geq 0} c_n \in \mathbb{R}$

Demonstration:

$demonstration$

Quotient test

Let:

- $\sum_{n\geq 0} c_n$ real series
- $l \in \mathbb{R}$ "

Then, holds:

- $\exists\, l \in \mathbb{R}$:

$$\lim_k \frac{c_{k+1}}{c_k} = l$$

- $\overline{\lim_{c_k}} |c_k|^{\frac{1}{k}} k = l$

Power series

Let:

- $\sum_{n \geq 0} a_n f_n$ complex valued sequence

Then, $\sum_{n \geq 0} a_n f_n$ is a power series if:

- $\forall\ n \in \mathbb{N}$:

$$\begin{aligned} f_n\ :\mathbb{C} &\longrightarrow \mathbb{C} \\ z &\longmapsto (z-a)^n \end{aligned}$$

Convergence radius

Let:

- *statements*
-

Then, *item* is a/an entity if:

- *conditions*
-

We denote:

- *property* : *notation*
-

Power series theorem

Let:

- $\sum_{n \geq 0} a_n c^n$ power series

Then, holds:

- $|z - a| < R \rightarrow$ absolutely convergent
- $|z - a| > R \rightarrow$ divergent
- convergent in $D(a, R)$
- $\begin{array}{rcl} f : \mathbb{C} & \longrightarrow & \mathbb{C} \\ z & \longmapsto & \sum_{n \geq 0} c_n (z - a)^n \end{array} \in \mathcal{H}(D(a, R))$
- $\forall\, z \in \mathbb{C}$:

$$f'(z) = \sum_{n\geq 0} nc_n(z-a)^{n-1} \text{ convergent}$$

· convergence radius of f' = convergence radius of f

Demonstration:

$\forall\ z \in \mathbb{C}$ " $|z-a| < R$:

Root test over $\sum_{n\geq 0} |c_n||z-a|^n \lim$

$(—c_n||z-a|^n)^{\frac{1}{n}})n = |z-a| \lim_n |c_n|^{\frac{1}{n}} = \dfrac{|z-a|}{R} < 1$ Root test → absolutely convergent $.z \in \mathbb{C}[|z-a| < R]$ $\forall\ \rho \in \mathbb{R}$ " $|z-a| < \rho < R$:

$\frac{1}{\rho} < \frac{1}{R}$

$\lim_n |c_n|^{1/n} = \dfrac{1}{R}$ → exists partial of $|c_n|^{1/n}$:

$|c_n||z-a|^n > \frac{|z-a|^n}{\rho^n}$ no $\xrightarrow{n} 0$

General term test → divergent

Exponential

Let:

- a : 0
- c_n : $\frac{1}{n}$

Then, $\sum_{n\geq 0} c_n(z-a)^n$ is convergent in D1 :

$$\lim_n \frac{|c_n|}{|c_{n+1}|} = \lim_n \frac{n+1}{n} = 1 \to R = 1$$

CH→ $D(0,1)$ convergent

$\mathbb{C} \setminus D(0,1)$ divergent

$f' = f$