1. One-dimensional discrete dynamical systems

Dynamical system

Let:

- M manifold
- T monoid
- $\phi : M \times T \to M$

Then, (M, T, ϕ) is a dynamical system if:

- $\forall\, x \in X$:

  $\phi(x, 0) = 0$

  $\forall\, t_1, t_2 \in T$:

  $\phi(\phi(x, t_1), t_2) = \phi(x, t_1 + t_2)$

Dimension

Let:

- (M, T, ϕ) dynamical system

We name dimension of (M, T, ϕ) to:

- $\dim(M)$

We denote:

- $dim(M) = n \;:\; (M, T, \phi)$ n-D dynamical system

Discrete & Continuous

Let:

- (M, T, ϕ) dynamical system

Then, (M, T, ϕ) is discrete if:

- $T \underset{\sim}{\subset} \mathbb{N}$

Then, (M, T, ϕ) is continuous if:

- $T \subset \mathbb{R}$ „ T open

Defined by a function

Let:

- (M, T, ϕ) dynamical system
- $f : M \to M$

Then, (M, T, ϕ) is a dynamical system defined by f if:

- $T = \mathbb{N}$
- $\begin{array}{rcl} \phi : M \times \mathbb{N} & \longrightarrow & M \\ (x, n) & \longmapsto & f^n(x) \end{array}$

We denote:

- (M, T, ϕ) dynamical system defined by f : $(M, \mathbb{N}, f)$

Orbit

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$

We name orbit of x to:

- $\{f^n(x)\}_{n\in\mathbb{N}}$

We denote:

- $o(x)$

n-periodic point

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$
- $n \in \mathbb{N}$

Then, x is a n-periodic point if:

- $f^n(x) = x$
- $\forall\ n' \in \mathbb{N}$ „ $n' < n$:

  $f^{n'}(x) \neq x$

We denote:

- $n = 1\ :\ x$ fixed point

Attractive & Repulsive

Let:

- $(M, \mathbb{N}, f)$ metrical dynamical system
- $x \in M$ m-periodic point

Then, x is attractive if:

- $\exists\, \mathcal{U} \subset M$ open :

    $\forall\, y \in \mathcal{U}$:

    $\exists\, N \in \mathbb{N}$:

    $\forall\, n \in \mathbb{N}$ „ $n \geq N$:

    $f^{nm}(y) \in \mathcal{U}$

Then, x is repulsive if:

- $\forall\, \mathcal{U} \subset M$ „ $x \in \mathcal{U}$:

    $\forall\, y \in \mathcal{U}$:

    $\exists\, N \in \mathbb{N}$:

    $\forall\, n \in \mathbb{N}$ „ $n \geq N$:

    $f^{nm}(x) \notin \mathcal{U}$

Attraction set

Let:

- $(M, \mathbb{N}, f)$ dynamical system
- $x \in M$ attractive m-periodic point
- $o(x)$ orbit of x

We name attraction set of x to:

- $\{y \in M \mid \exists\, x' \in o(x) \;:\; f^{nm}(y) \xrightarrow{n} x'\}$

We denote:

- $A(x)$

Neutral point

Let:

- $(M, \mathbb{N}, f)$ differentiable dynamical system
- $x \in M$

Then, x is a neutral point if:

- $f'(x) \in \{-1, 1\}$

Feeble attractive & repulsive points

Let:

- $(M, \mathbb{N}, f)$ $\mathcal{C}^3$ dynamical system
- $x \in M$

Then, x is feeble attractive point if:

- $f'(x) = 1$
- $f''(x) = 0$
- $f'''(x) > 0$

Then, x is feeble repulsive point if:

- $f'(x) = 1$
- $f''(x) = 0$
- $f'''(x) < 0$

Multiplier

Let:

- $(M, \mathbb{N}, f)\mathcal{C}^1$ dynamical system
- $x \in M$

We name multiplier of x to:

- $f'(x)$

Logistic

Let:

- $f : \mathbb{R} \to \mathbb{R}$
- (M, T, ϕ) dynamical system defined by f

Then, (M, T, ϕ) is a logistic dynamical system if:

- $\exists\, a \in \mathbb{R}$:

$$\begin{aligned} f : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto ax(1-x) \end{aligned}$$

Chaos

Let:

- $f : \mathbb{R} \to \mathbb{R}$
- (M, T, ϕ) dynamical system defined by f

Then, (M, T, ϕ) is chaotic if:

- $Fix(f)$ dense in $\mathbb{R}$
- $\exists\, x \in \mathbb{R}$:

  $o(x)$ dense in $\mathbb{R}$

- f sensibility of x_0

Sarkovskii's order

We name Sarkovskii's order to:

- $naming$
-

We denote:

- $property$: $notation$
-

Saddle point

Let:

- $\mathcal{U} \subset \mathbb{R}^n$
- $f \in \mathcal{C}^1(\mathcal{U})$
- $x \in \mathcal{U}$

Then, x is a saddle point if:

- $f'(x) = 0$

Topologically equivalent

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system

- $(M, \mathbb{N}, f')$ functional dynamical system

Then, $(M, \mathbb{N}, f)$ is topologically equivalent to $(M, \mathbb{N}, f')$if:

- $Fix(f) = Fix(f')$

- $\forall\ x \in Fix(f)$:

$$character_f(x) = character_{f'}(x)$$

We denote:

- $(M, \mathbb{N}, f)\ (M, \mathbb{N}, f')$

Bifurcation point

Let:

- $(M, \mathbb{N}, f_\lambda)_{\lambda \in \Lambda}$ functional dynamical system family
- $\lambda' \in \Lambda$

Then, λ' is a bifurcation value if:

- $\forall\ \varepsilon \in \mathbb{R}^+$:

  $\exists\ \lambda'' \in (\lambda' - \varepsilon, \lambda' + \varepsilon)$:

  $(M, \mathbb{N}, f_{\lambda''})$ not topologically equivalent to $(M, \mathbb{N}, f_{\lambda'})$