1. One-dimensional discrete dynamical system

introduction

Decreasing function orbits

Let:

- *declarations*
-

Show that:

- *statements*
-

Demonstration:

f corta en un punto

f decreasing $\rightarrow f^2$ increasing

$f^{2n} \xrightarrow{n}$ fixed point of f

9. Periodic points

Let:

$$
\cdot \quad \begin{array}{rcl} f \ : \mathbb{R} \times \mathbb{R}^{+} & \longrightarrow & \mathbb{R} \\ (x, r) & \longmapsto & r\frac{x}{1+x^2} \end{array}
$$

Study:

$\cdot$ Periodic points of f

Demonstration:

Graphical analysis :

f odd

f has 2 extrema in ± 1

$f \xrightarrow{n} 0$

Fixed points :

$f(x) = x \leftrightarrow x = \pm\sqrt{r-1}$

$f'(\pm\sqrt{r-1}) = \frac{2-r}{r}$

n-periodic points :

$f^n(x) = x$

10. Global orbit analysis

Let:

- $f : \mathbb{R}^+ \to \mathbb{R}^+ \in \mathbb{C}^\infty$
- $f(0) = 0$
- $p \in \mathbb{R}^+ \setminus \{0\}$ „ $f'(p) \geq 0$
- f' decreasing

Show that:

- $\forall\, x \in \mathbb{R}^+ \setminus \{0\}$:

$$f^n(x) \xrightarrow{n} p$$

Demonstration:

f' decreasing $\to f'' < 0 \to f$ concave

f positive $\to f$ has no extrema $\to f' > 0 \to f$ increasing

f has only one fixed point

Suppose 2 fixed points : p, p'

$IVT \to \exists\, c \in (0, p')$:

$$f'(c) = 1$$

$f'(p) < 1 \to p$ attractive $IVT \to$ dont exist more fixed points $\to f'(c') = 1 \not\leq 1$

$\forall\, x \in (0, p)$:

$$f(x) > x$$

$\forall\, x \in \mathbb{R}$ „ $x > p$:

$$f(x) < x$$

f increasing $\to f([0, p]) = [0, p]$