1. New

Invariance of stability over orbits

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$

Then, holds:

- $\forall\ x' \in o(x)$:

  $\chi(x') = \chi(x)$

Demonstration:

Follow 2 steps

Step $1 : \quad falta :$

$rows$

Step $2 : \quad attractiveness :$

$\chi(x) = -1$

$\exists\ \varepsilon \in \mathbb{R}^+$:

$$x \in B_\varepsilon(x) \to f^{2n}(x) \xrightarrow{n} x$$

$$f \in \mathcal{C}^0(M) \to \exists\, \varepsilon_1 \in \mathbb{R}^+ :$$

$$f(B_{\varepsilon_1}(x_1)) \subset B_\varepsilon(x)$$

$$x \in B_{\varepsilon_1}(x_1) \to f(x) \in B_\varepsilon(x) \to f^{2n-1}(f(x)) \xrightarrow{n} x$$

falta

Multiplier

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$

We namemultiplier of x to:

- $Df(p)$

Linear system

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system

Then, $(M, \mathbb{N}, f)$ is linear if:

- $\exists\ A \in \mathcal{M}_{n\times n}(\mathbb{R})$:

$$\begin{array}{rccc} f & : \mathbb{R}^n & \longrightarrow & \mathbb{R}^n \\ & x & \longmapsto & Ax \end{array}$$

Linear property

Let:

- $(M, \mathbb{N}, f)$ linear dynamical system

Then, holds:

- $\forall\ a, b \in \mathbb{R}$:

  $\forall\ x, y \in M$:

$$f(ax + by) = af(x) + bf(y)$$

Demonstration:

matrius

Fixed points of linear applications

Let:

- $(M, \mathbb{N}, f)$ linear dynamical system

Then, holds:

- $0 \in \mathrm{Fix}(f)$

Demonstration:

demonstration

Eigenvalue

Let:

- $A \in \mathcal{M}_{n \times n}(K)$

- $\lambda \in K$

Then, λ is an eigenvalue if:

- $\exists \, v \in K \setminus \{0\}$:

$$Av = \lambda v$$

We denote:

- $\{\lambda \in K \mid \lambda \text{ eigenvalue } \} \; : \; Spec(A)$

Eigenvector

Let:

· $A \in \mathcal{M}_{n \times n}(K)$

· $\lambda \in K$ eigenvalue

· $v \in K$

Then, v is an eigenvector of eigenvalue λif:

· $Av = \lambda v$

We denote:

· $\{v \in K \mid v \text{ eigenvector of eigenvalue } \lambda\} \ : \ Ker_{\lambda}(K)$

Jordan form of 2-D real linear maps

Let:

· $A \in \mathcal{M}_{2 \times 2}(\mathbb{R})$

· $\chi_A(t)$ characteristic polynomial of A

Then, holds:

· $\exists \, \beta$ base of K :

$$\begin{cases} A = \lambda, 0, 0, \mu & \#Z(\chi_A(t)) = 2 \\ A = \lambda, 1, 0, \lambda & \#Z(\chi_A(t)) = 1 \\ A = \alpha, \beta, -\beta, \alpha & \#Z(\chi_A(t)) = 0 \end{cases}$$

Demonstration:

demonstration

Topology of 2-D real linear maps

Let:

- $A \in \mathcal{M}_{2\times 2}(\mathbb{R})$
- $\lambda \neq \mu$ eigenvalues of A

Then, holds:

- $|\lambda|, |\mu| < 1 \to (0,0)$ attractive
- $|\lambda| > |\mu| \to$ tangent to y $= 0$
- $|\mu| > |\lambda| \to$ tangent to x $= 0$
- $|\mu| = |\lambda| \to$ only invariant lines
-
- $|\lambda|, |\mu| > 1 \to (0,0)$ repulsive
- equivalent to other case

Demonstration:

demonstration