1. Non-linear maps

Let:

$$\cdot \quad \begin{array}{rccc} f: & \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ & (x,y) & \longmapsto & (2y+x^2-2xy, 2-2x+2y+xy-y^2) \end{array}$$

Study:

· Stability of fixed points of f

Demonstration:

Fixed points:

$$f(x,y)=(x,y) \leftrightarrow \begin{cases} 2y+x^2-2xy = x \\ 2-2x+y+xy-y^2=0 \end{cases}$$

Case $x \neq 1$

$$\leftrightarrow \begin{cases} y = \frac{x(x-1)}{2(x-1)} \\ 2-2x+y+xy-y^2=0 \end{cases} \quad \leftrightarrow \begin{cases} y = \frac{x}{2} \\ 2-2x+x/2+x^2/2-x^2/4=0 \end{cases}$$

$$\leftrightarrow \begin{cases} y = \frac{x}{2} \\ x^2-6x+8=0 \end{cases} \quad \leftrightarrow \begin{cases} y = \frac{x}{2} \\ x \in \{2,4\} \end{cases}$$

Case $x = 1$

$$2-2+y+y-y^2=0 \leftrightarrow y^2-2y=0 \leftrightarrow y \in \{0,2\}$$

$$\mathrm{Fix}(f) = \{(1,0),(1,2),(2,1),(4,2)\}$$

Stability:

$$Df(x,y) = \begin{pmatrix} 2x-2y & 2-2x \\ y-2 & -2y+x+2 \end{pmatrix}$$

$$Df(1,0) = \begin{pmatrix} 2 & 0 \\ -2 & 3 \end{pmatrix}$$

$$Df(1,2) = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}$$

$Df(2,1) = \begin{pmatrix} 4 & -2 \\ 0 & 2 \end{pmatrix}$

$Df(4,2) = \begin{pmatrix} 2 & -2 \\ -1 & 2 \end{pmatrix}$

$\chi_{Df(1,0)}(t) = t^2 - 5t + 6$

$\chi_{Df(1,2)}(t) = t^2 + 3t + 2$

$\chi_{Df(2,1)}(t) = t^2 - 6t + 8$

$\chi_{Df(4,2)}(t) = t^2 - 4t + 2$

$\sigma(Df(1,0)) = \{2,3\} \to (1,0)$ repulsive node

$\sigma(Df(1,2)) = \{-2,-1\}$ degenerated case

$\sigma(Df(2,1)) = \{2,4\} \to (2,1)$ repulsive node

$\sigma(Df(4,2)) = \{2 \pm \sqrt{2}\} \to (4,2)$ saddle point

$x = 1$is an invariant curve:

$\forall \ (x,y) \in \mathbb{R}^2 \quad " \quad x = 1$:

$f_1(1,y) = 2y + 1^2 - 2y = 1$

$f(1,y) \in \{(x,y) \in \mathbb{R}^2 \mid x = 1\}$

Stable & Unstable invariant subspaces

$\lambda_1 \ := \ 2 + \sqrt{2}$

$\lambda_2 \ := \ 2 - \sqrt{2}$

$E_{\lambda_1}(A) = Ker\begin{pmatrix} -\sqrt{2} & -2 \\ -1 & -\sqrt{2} \end{pmatrix} = \{(x,y) \in \mathbb{R}^2 \mid y = -\frac{\sqrt{2}}{2}x\}$

$E_{\lambda_2}(A) = Ker\begin{pmatrix} \sqrt{2} & -2 \\ -1 & \sqrt{2} \end{pmatrix} = \{(x,y) \in \mathbb{R}^2 \mid y = \frac{\sqrt{2}}{2}x\}$

$E^s =< (1, -\frac{\sqrt{2}}{2} >$

$E^u = < (1, \frac{\sqrt{2}}{2} >$

f has an invariant attractive variety tangent to $(4, 2) + E^s$ and an invariant repulsive variety tangent to $(4, 2) + E^u$