I. Logarithm determinations

Logarithm determinations

Let:

- $\begin{array}{rccc} log: & \mathbb{C} \setminus (-\infty, 0] & \longrightarrow & \mathbb{C} \\ & 1 & \longmapsto & 4\pi i \end{array}$ logarithm determination
- $\begin{array}{rccc} f: & \mathbb{C} \setminus [1, \infty) & \longrightarrow & \mathbb{C} \\ & z & \longmapsto & -log(2-2z) \end{array}$

Then, holds:

- $f \in \mathcal{H}(\mathbb{C} \setminus [1, \infty))$

Demonstration:

$$\begin{array}{rccc} g: & \mathbb{C} \setminus [1, \infty) & \longrightarrow & \mathbb{C} \setminus (-\infty, 0] \\ & z & \longmapsto & 2-2z \end{array}$$

g well defined :

$\forall\, z \in \mathbb{C} \setminus [1, \infty)$ „ $Im(g(z)) = 0$:

$Im(2-2z) = 0 \to -2Im(z) = 0 \to Im(z) = 0 \to Re(z) < 1$

$Re(g(z)) = Re(2-2z) = 2 - 2Re(z) > 0$

$g \in \text{Pol}\mathbb{C} \setminus [1, \infty)) \to g \in \mathcal{H}(\mathbb{C} \setminus [1, \infty))$

$log \in \mathcal{H}(\mathbb{C} \setminus (-\infty, 0])$

$f = -log \circ g \in \mathcal{H}(\mathbb{C} \setminus [1, \infty))$