1. New

defined by f

Let:

- $f\ :\ \mathbb{R}^n \to \mathbb{R}^n$

Then, $(\mathbb{R}^n, \mathbb{N}, \phi)$ is defined by $f$if:

- $\begin{array}{rcl} \phi\ : \mathbb{R}^n \times \mathbb{N} & \longrightarrow & \mathbb{R}^n \\ x & \longmapsto & f^n(x) \end{array}$

We denote:

- $(\mathbb{R}^n, \mathbb{N}, f)$ n-D

Henon's application

Let:

- $\begin{array}{rcl} f\ : \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ (x, y) & \longmapsto & (-x^2 + 0.4y, x) \end{array}$

Study:

- Fixed points of f

Demonstration:

$(0, 0), (-0$

$6, -0$

$6)$ fixed points

Invariant curve

Let:

- $\cdot\ \gamma$ differentiable curve

- $\cdot\ p \in \mathbb{R}^n$

Then, γ is invariant if:

- $\cdot\ \forall\ x \in \gamma* :$

  $$o(x) \subset \gamma*$$

Then, γ is converges to $p$if:

- $\cdot\ \forall\ x \in \gamma* :$

  $$o(x) \xrightarrow{n} p$$

Stable & Unstable

Let:

$\cdot\, f : \mathbb{R}^n \to \mathbb{R}^n$

$\cdot\, p \in \mathbb{R}^n$ m-periodic point

Then, p is stable if:

$\cdot\, \forall\ \varepsilon \in \mathbb{R}^+ :$

$\exists\ \delta \in \mathbb{R}^+ :$

$\forall\ x \in B(p, \varepsilon) :$

$\forall\ n \in \mathbb{N} :$

$f^{nm}(x) \in B(p, \varepsilon)$

Then, p is unstable if:

$\cdot\, p$ not stable

Attractive & Repulsive

Let:

- $f : \mathbb{R}^n \to \mathbb{R}^n$
- $p \in \mathbb{R}^n$ m-periodic point

Then, p is attractive if:

- p stable
- $\exists\, \varepsilon \in \mathbb{R}^+$:

  $\forall\, x \in B(p, \varepsilon)$:

  $$f^{nm}(x) \xrightarrow{n} p$$

Then, p is repulsive if:

- p attractive by f^{-1}

Invariance of stability over periods

Let:

- $(\mathbb{R}^n, \mathbb{N}, f)$ n-D dynamical system
- $p \in \mathbb{R}^n$ k-periodic point
- χ character of periodic points

Then, holds:

- $\exists\, \sigma \in \mathrm{Im}(\chi)$:

  $\forall\, x \in o(p)$:

$$\chi(x) = \sigma$$

Demonstration:

i will