1. The field of complex numbers

introduction

The field of complex numbers

Let:

- $$\begin{aligned} + : \mathbb{R}^2 \times \mathbb{R}^2 &\longrightarrow \mathbb{R}^2 \\ ((a,b),(c,d)) &\longmapsto (a+c, b+d) \end{aligned}$$
- $$\begin{aligned} \cdot : \mathbb{R}^2 \times \mathbb{R}^2 &\longrightarrow \mathbb{R}^2 \\ ((a,b),(c,d)) &\longmapsto (ac-bd, ad+bd) \end{aligned}$$

We name the field of complex numbers to:

- $(\mathbb{R}^2, +, \cdot)$

We denote:

- $(\mathbb{R}^2, +, \cdot) \ : \ \mathbb{C}$
- $(0,1) \in \mathbb{C} \ : \ i$
- $(a,b) \in \mathbb{C} \ : \ a+bi$
- $\pi_1(\mathbb{C}) \ : \ Re$
- $\pi_2(\mathbb{C}) \ : \ Im$

Conjugation

We name complex conjugation to:

- $$\begin{aligned} f : \mathbb{C} &\longrightarrow \mathbb{C} \\ (a,b) &\longmapsto (a,-b) \end{aligned}$$

We denote:

- $f((a,b)) \ : \ \overline{(a,b)}$

Norm

We name complex norm to:

$$\cdot \begin{array}{rcl} f\ :\mathbb{C} & \longrightarrow & \mathbb{R} \\ (a,b) & \longmapsto & \sqrt{a^2+b^2} \end{array}$$

We denote:

$$\cdot\, f((a,b))\ :\ |(a,b)|$$

Polar transformation

We name polar transformation to:

$$\cdot \begin{array}{rcl} f\ :\mathbb{C} & \longrightarrow & \mathbb{R}^{+} \times [0,2\pi) \\ (a,b) & \longmapsto & (\sqrt{|(a,b)|}, \arctan(\frac{b}{a})) \end{array}$$

We denote:

$$\cdot\, f((a,b))\ :\ (r,\theta)$$

Unit sphere projection

We name unit sphere projection to:

$$\cdot \begin{array}{rcl} f\ :\mathbb{C} & \longrightarrow & S^1 \\ z & \longmapsto & \frac{z}{|z|} \end{array}$$

We denote:

$$\cdot\, f(z)\ :\ \pi(z)$$

Roots of unity

Let:

- $z \in \mathbb{C}$

Then, z is a root of unity if:

- $\exists\ n \in \mathbb{N} \;\; " \; z^n = 1$

We denote:

- $\{z \in \mathbb{C} \mid z \text{ root of unity } \}\ :\ S^1$

Disk

Let:

- $p \in \mathbb{C}$
- $r \in \mathbb{R}^+ \setminus \{0\}$

We name Disk centered in p and radius r to:

- $\{z \in \mathbb{C} \mid |z - p| < r\}$

We denote:

- $\{z \in \mathbb{C} \mid |z - p| < r\}\ :\ D^1$

Components

Let:

· $f : \mathbb{C} \to \mathbb{C}$

We name real component of f to:

· $\begin{array}{rcl} f_{Re} : \mathbb{C} & \longrightarrow & \mathbb{R} \\ z & \longmapsto & Re(f(z)) \end{array}$

We name imaginary component of f to:

· $\begin{array}{rcl} f_{Im} : \mathbb{C} & \longrightarrow & \mathbb{R} \\ z & \longmapsto & Im(f(z)) \end{array}$

We name component decomposition of f to:

· $\begin{array}{rcl} f_{\mathbb{R}^2} : \mathbb{C} & \longrightarrow & \mathbb{R}^2 \\ (x,y) & \longmapsto & (f_{Re}(x+yi), f_{Im}(x+yi) \end{array}$

Component decomposition

Let:

· $f : \mathbb{C} \to \mathbb{C}$

· f_{Re}, f_{Im} real components of f

We name real dual of f to:

· $\begin{array}{rcl} f_{\mathbb{R}^2} : \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ (x,y) & \longmapsto & (f_{Re}(x+yi), f_{Im}(x+yi)) \end{array}$