



## 1. New

**Invariance of stability over orbits**

Let:

$\cdot (M, \mathbb{N}, f)$  functional dynamical system

$\cdot x \in M$

Then, holds:

$\cdot \forall x' \in o(x) :$

$$\chi(x') = \chi(x)$$

Demonstration:

Follow 2 steps

Step 1 : *falta* :

*rows*

Step 2 : *attractiveness* :

$$\chi(x) = -1$$

$\exists \varepsilon \in \mathbb{R}^+ :$

$$x \in B_\varepsilon(x) \rightarrow f^{2n}(x) \xrightarrow{n} x$$

$$f \in \mathcal{C}^0(M) \rightarrow \exists \varepsilon_1 \in \mathbb{R}^+ :$$

$$f(B_{\varepsilon_1}(x_1)) \subset B_\varepsilon(x)$$

$$x \in B_{\varepsilon_1}(x_1) \rightarrow f(x) \in B_\varepsilon(x) \rightarrow f^{2n-1}(f(x)) \overset{n}{\longrightarrow} x$$

falta

## Multiplier

Let:

$\cdot (M, \mathbb{N}, f)$  functional dynamical system

$\cdot x \in M$

We namemultiplier of  $x$  to:

$\cdot Df(p)$

## Linear system

Let:

·  $(M, \mathbb{N}, f)$  functional dynamical system

Then,  $(M, \mathbb{N}, f)$  is linear if:

$$\begin{aligned} \cdot \exists A \in \mathcal{M}_{n \times n}(\mathbb{R}) : \\ f : \mathbb{R}^n &\longrightarrow \mathbb{R}^n \\ x &\longmapsto Ax \end{aligned}$$

## Linear property

Let:

·  $(M, \mathbb{N}, f)$  linear dynamical system

Then, holds:

$$\cdot \forall a, b \in \mathbb{R} :$$

$$\forall x, y \in M :$$

$$f(ax + by) = af(x) + bf(y)$$

Demonstration:

matrius

### Fixed points of linear applications

Let:

$\cdot (M, \mathbb{N}, f)$  linear dynamical system

Then, holds:

$\cdot 0 \in \text{Fix}(f)$

Demonstration:

*demonstration*

## Eigenvalue

Let:

$$\cdot A \in \mathcal{M}_{n \times n}(K)$$

$$\cdot \lambda \in K$$

Then,  $\lambda$  is an eigenvalue if:

$$\cdot \exists v \in K \setminus \{0\} :$$

$$Av = \lambda v$$

We denote:

$$\cdot \{ \lambda \in K \mid \lambda \text{ eigenvalue} \} : \text{Spec}(A)$$

## Eigenvector

Let:

- $A \in \mathcal{M}_{n \times n}(K)$
- $\lambda \in K$  eigenvalue
- $v \in K$

Then,  $v$  is an eigenvector of eigenvalue  $\lambda$  if:

- $Av = \lambda v$

We denote:

- $\{v \in K \mid v \text{ eigenvector of eigenvalue } \lambda\} : Ker_{\lambda}(K)$

## Jordan form of 2-D real linear maps

Let:

- $A \in \mathcal{M}_{2 \times 2}(\mathbb{R})$
- $\chi_A(t)$  characteristic polynomial of  $A$

Then, holds:

- $\exists \beta$  base of  $K$  :
 
$$\begin{cases} A = \lambda, 0, 0, \mu & \#Z(\chi_A(t)) = 2 \\ A = \lambda, 1, 0, \lambda & \#Z(\chi_A(t)) = 1 \\ A = \alpha, \beta, -\beta, \alpha & \#Z(\chi_A(t)) = 0 \end{cases}$$

Demonstration:

*demonstration*



## Topology of 2-D real linear maps

Let:

- $A \in \mathcal{M}_{2 \times 2}(\mathbb{R})$
- $\lambda \neq \mu$  eigenvalues of  $A$

Then, holds:

- $|\lambda|, |\mu| < 1 \rightarrow (0, 0)$  attractive
- $|\lambda| > |\mu| \rightarrow$  tangent to  $y = 0$
- $|\mu| > |\lambda| \rightarrow$  tangent to  $x = 0$
- $|\mu| = |\lambda| \rightarrow$  only invariant lines
- 
- $|\lambda|, |\mu| > 1 \rightarrow (0, 0)$  repulsive
- equivalent to other case

Demonstration:

*demonstration*