1. One-dimensional discrete dynamical systems

Dynamical system

Let:

- M manifold
- T monoid
- $\phi : M \times T \to M$

Then, (M, T, ϕ) is a dynamical system if:

- $\forall\ x \in X$:

  $\phi(x, 0) = 0$

  $\forall\ t_1, t_2 \in T$:

  $\phi(\phi(x, t_1), t_2) = \phi(x, t_1 + t_2)$

Dimension

Let:

- (M, T, ϕ) dynamical system

We name dimension of (M, T, ϕ) to:

- $\dim(M)$

We denote:

- $dim(M) = n\ :\ (M, T, \phi)$ n-D dynamical system

Discrete & Continuous

Let:

- (M, T, ϕ) dynamical system

Then, (M, T, ϕ) is discrete if:

- $T \overset{\subset}{\sim} \mathbb{N}$

Then, (M, T, ϕ) is continuous if:

- $T \subset \mathbb{R}$,, T open

Defined by a function

Let:

- (M, T, ϕ) dynamical system
- $f : M \to M$

Then, (M, T, ϕ) is a dynamical system defined by f if:

- $T = \mathbb{N}$
- $$\begin{aligned} \phi : M \times \mathbb{N} &\longrightarrow M \\ (x, n) &\longmapsto f^n(x) \end{aligned}$$

We denote:

- (M, T, ϕ) dynamical system defined by f : $(M, \mathbb{N}, f)$

Orbit

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$

We name orbit of x to:

- $\{f^n(x)\}_{n \in \mathbb{N}}$

We denote:

- $o(x)$

n-periodic point

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$
- $n \in \mathbb{N}$

Then, x is a n-periodic point if:

- $f^n(x) = x$
- $\forall\, n' \in \mathbb{N}$ „ $n' < n$:

  $f^{n'}(x) \neq x$

We denote:

- $n = 1$: x fixed point

Attractive & Repulsive

Let:

- $(M, \mathbb{N}, f)$ metrical dynamical system
- $x \in M$ m-periodic point

Then, x is attractive if:

- $\exists\, \mathcal{U} \subset M$ open :

  $\forall\, y \in \mathcal{U}$:

  $\exists\, N \in \mathbb{N}$:

  $\forall\, n \in \mathbb{N}$ „ $n \geq N$:

  $f^{nm}(y) \in \mathcal{U}$

Then, x is repulsive if:

- $\forall\, \mathcal{U} \subset M$ „ $x \in \mathcal{U}$:

  $\forall\, y \in \mathcal{U}$:

  $\exists\, N \in \mathbb{N}$:

  $\forall\, n \in \mathbb{N}$ „ $n \geq N$:

  $f^{nm}(x) \notin \mathcal{U}$

Attraction set

Let:

- $(M, \mathbb{N}, f)$ dynamical system
- $x \in M$ attractive m-periodic point
- $o(x)$ orbit of x

We name attraction set of x to:

- $\{y \in M \mid \exists\, x' \in o(x) \;:\; f^{nm}(y) \xrightarrow{n} x'\}$

We denote:

- $A(x)$

Neutral point

Let:

- $(M, \mathbb{N}, f)$ differentiable dynamical system
- $x \in M$

Then, x is a neutral point if:

- $f'(x) \in \{-1, 1\}$

Feeble attractive & repulsive points

Let:

- $(M, \mathbb{N}, f)$ $\mathcal{C}^3$ dynamical system
- $x \in M$

Then, x is feeble attractive point if:

- $f'(x) = 1$
- $f''(x) = 0$
- $f'''(x) > 0$

Then, x is feeble repulsive point if:

- $f'(x) = 1$
- $f''(x) = 0$
- $f'''(x) < 0$

Multiplier

Let:

- $(M, \mathbb{N}, f)\mathcal{C}^1$ dynamical system
- $x \in M$

We name multiplier of x to:

- $f'(x)$