Block I

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1. nth root determinations of a function

Cubic root determinations

Let:

- h_0, h_1, h_2 cubic root determinations over $\mathbb{C} \setminus (\mathbb{R}^- \times \{0\})$ with
- $h_0(1) = 1$
- $h_1(1) = \exp(\frac{2\pi i}{3})$
- $h_2(1) = \exp(\frac{4\pi i}{3})$

Study:

- $\mathrm{Im}(h_0)$,$\mathrm{Im}(h_1)$,$\mathrm{Im}(h_2)$
- Relationship with Log and Arg

Demonstration:

$\forall\ z \in \mathbb{C} \setminus (\mathbb{R}^- \times \{0\})$:

$\sqrt[3]{z} = \sqrt[3]{|\ z\ |}\exp(\frac{i(Arg(z)+2k\pi)}{3})$

$\forall\ k \in \dot{3}$:

$\sqrt[3]{z} = \sqrt[3]{|\ z\ |}\exp(\frac{iArg(z)}{3})$

$arg(z) = \frac{Arg(z)}{3}$

$Arg(z) \in (-\pi, \pi) \to arg(z) \in (\frac{-\pi}{3}, \frac{\pi}{3})$

$\Omega_0 \ := \ \{z \in \mathbb{C} \mid Arg(z) \in (\frac{-\pi}{3}, \frac{\pi}{3})\}$

$\forall\ k \in \dot{3} + 1$:

$\sqrt[3]{z} = \sqrt[3]{|\ z\ |}\exp(\frac{i(Arg(z)+2\pi)}{3})$

$arg(z) = \frac{Arg(z)}{3} + \frac{2\pi}{3}$

$Arg(z) \in (-\pi, \pi) \to arg(z) \in (\frac{\pi}{3}, \pi)$

$\Omega_1 \ := \ \{z \in \mathbb{C} \mid Arg(z) \in (\frac{\pi}{3}, \pi)\}$

$\forall\ k \in \dot{3} + 2$:

$\sqrt[3]{z} = \sqrt[3]{|\ z\ |}\exp(\frac{iArg(z)}{3})$

$arg(z) = \frac{Arg(z)}{3} + \frac{4\pi}{3}$

$Arg(z) \in (-\pi, \pi) \to arg(z) \in (\pi, \frac{5\pi}{3})\Omega_2 \ := \ \{z \in \mathbb{C} \mid Arg(z) \in (\pi, \frac{5\pi}{3})\}$

$h_0(1) = 1 \to \mathrm{Im}(h_0) = \Omega_0$

$h_1(1) = \exp(\frac{2\pi i}{3}) \rightarrow \mathrm{Im}(h_1) = \Omega_1$

$h_2(1) = \exp(\frac{4\pi i}{3}) \rightarrow \mathrm{Im}(h_2) = \Omega_2$

In particular:

$h_0(i) = \sqrt[3]{|\, i \,|}\exp(\frac{iArg(i)}{3}) = \exp(\frac{\pi}{6}i)$

$h_0(i) = \sqrt[3]{|\, i \,|}\exp(\frac{iArg(i)+2\pi}{3}) = \exp(\frac{5\pi}{6}i)$

$h_0(i) = \sqrt[3]{|\, i \,|}\exp(\frac{i(Arg(i)+4\pi)}{3}) = \exp(\frac{9\pi}{6}i)$