# 1. Orbit analysis

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Let:

$$\begin{aligned} f : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto x^3 + \tfrac{1}{4}x \end{aligned}.$$

Study:

· Orbit behavior of the real dynamical system defined by $f$

Demonstration:

Formalization :

Consider $(M, T, \phi)$ where:

$M = \mathbb{R}$

$T = \mathbb{N}$

$$\begin{aligned} \phi : \mathbb{R} \times \mathbb{N} &\longrightarrow \mathbb{R} \\ (x, n) &\longmapsto f^n(x) \end{aligned}$$

Study the orbits of $(\mathbb{R}, \mathbb{N}, \phi)$

We will denote $f^n(x)$ as $x_n$

Fixed points :

$\forall\ x \in \mathbb{R}$ :

$$f(x) = x \leftrightarrow x^3 + \tfrac{1}{4}x - x = 0 \leftrightarrow x^3 - \tfrac{3}{4}x = 0$$

$$\leftrightarrow x = 0 \ \vee \ x^2 - \tfrac{3}{4} = 0$$

$$x \text{ fixed point} \leftrightarrow x \in \{0, \pm\tfrac{\sqrt{3}}{2}\}$$

Graphic analysis :

Parity:

$\forall\ x \in \mathbb{R}$ :

$$f(-x) = (-x)^3 + \frac{(-x)}{4} = -(x^3 + \frac{x}{4}) = -f(x)$$

$f$ is odd

Monotonicity:

$\forall\ x \in \mathbb{R}$ :

$$f'(x) = 3x^2 + \frac{1}{4} > 0$$

$f$ is increasing over $\mathbb{R}$

Convexity:

$\forall\ x \in \mathbb{R}^-$ :

$$f''(x) = 6x \leq 0$$

$\forall\ x \in \mathbb{R}^+$ :

$$f''(x) = 6x \geq 0$$

$f$ is concave over $\mathbb{R}^-$ and convex over $\mathbb{R}^+$