# 1. Holomorphic functions

## Incremental quotient

Let:

- $\mathcal{U} \subset \mathbb{C}$ open
- $f : \mathcal{U} \to \mathbb{C}$
- $p \in \mathcal{U}$

We name incremental quotient of $f$ in $p$ to:

- $\lim_{z \to p} \frac{f(z) - f(p)}{z - p}$

We denote:

- $f'(p)$

## Holomorphic function

Let:

- $\mathcal{U} \subset \mathbb{C}$ open
- $f : \mathcal{U} \to \mathbb{C}$
- $p \in \mathcal{U}$

Then, $f$ is holomorphic over $p$ if:

- $\exists f'(p)$

Then, $f$ is holomorphic over $U$ if:

- $\forall\ p \in \mathcal{U}$:

  $\exists f'(p)$

We denote:

- $\{f : \mathcal{U} \to \mathbb{C} \mid f \text{ holomorphic over } \mathcal{U}\} : \mathcal{H}(\mathcal{U})$
- $f \in \mathcal{H}(\mathbb{C}) : f$ entire

### Real components

Let:

- $f \in \mathcal{H}(\mathcal{U})$

We name first real component of $f$ to:

- $$\begin{array}{rcl} f_{Re} : \mathcal{U} & \longrightarrow & \mathbb{R} \\ z & \longmapsto & Re(f(z)) \end{array}$$

We name second real component of $f$ to:

- $$\begin{array}{rcl} f_{Im} : \mathcal{U} & \longrightarrow & \mathbb{R} \\ z & \longmapsto & Im(f(z)) \end{array}$$

### Real dual

Let:

- $f \in \mathcal{H}(\mathcal{U})$

- $f_{Re}, f_{Im}$ real components of $f$

We name real dual of $f$ to:

- $$\begin{array}{rcl} f_{\mathbb{R}^2} : \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ (x,y) & \longmapsto & (f_{Re}(x+yi), f_{Im}(x+yi)) \end{array}$$

We denote:

- $property \ : \ notation$

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