1. Holomorphic functions

Incremental quotient

Let:

- $\mathcal{U} \subset \mathbb{C}$ open
- $f : \mathcal{U} \to \mathbb{C}$
- $p \in \mathcal{U}$

We name incremental quotient of f in p to:

- $\lim_{z \to p} \dfrac{f(z) - f(p)}{z - p}$

We denote:

- $f'(p)$

Holomorphic function

Let:

- $\mathcal{U} \subset \mathbb{C}$ open
- $f : \mathcal{U} \to \mathbb{C}$
- $p \in \mathcal{U}$

Then, f is holomorphic over p if:

- $\exists f'(p)$

Then, f is holomorphic over U if:

- $\forall\, p \in \mathcal{U}$:

  $\exists f'(p)$

We denote:

- $\{f : \mathcal{U} \to \mathbb{C} \mid f$ holomorphic over $\mathcal{U}\} : \mathcal{H}(\mathcal{U})$
- $f \in \mathcal{H}(\mathbb{C}) : f$ entire

Cauchy-Riemman equations

Let:

- $u, v \,:\, \mathbb{R}^2 \to \mathbb{R}$
- $\begin{array}{rcl} f \,: \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ (x,y) & \longmapsto & (u((x,y)), v((x,y))) \end{array}$

Then, f is satisfies the Cauchy-Riemman equations if:

- $\exists\, u_x, u_y, v_x, v_y$
- $u_x = v_y$
- $u_y = -v_x$

We denote:

- $u_x + iv_x \,:\, f_x$
- $u_y + iv_y \,:\, f_y$

Conformal

Let:

- $\cdot\, \mathcal{U} \subset \mathbb{C}$ open
- $\cdot\, f\ : \mathcal{U} \to \mathbb{C}$
- $\cdot\, z \in \mathcal{U}$

Then, f is conformal in z if:

$\cdot\ \exists\ c \in \mathbb{C}\ \text{„}\ \ \forall\ I \subset \mathbb{R}\ \text{„}\ 0 \in I$:

$\forall\ \gamma\ :\ I \to \mathbb{R}^2\ \text{„}\ \gamma$ differentiable $\wedge\ \gamma(0) = z\ \wedge\ \gamma'(0) \neq 0$:

$$\frac{(f \circ \gamma)'(0)}{\gamma'(0)} = c$$

Then, f is conformal if:

$\cdot\ \ \forall\ z \in \mathcal{U}$:

f conformal in z