Block I

Definitions

1. Statistic models

introduction

Statistical model

Let:

- Ω set
- $\mathcal{A}$ sigma-algebra over Ω
- $\mathcal{P} \subset \mathcal{F}(\Omega, \mathbb{R})$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is a statistical model if:

- $\forall\ f \in \mathcal{P}$:

  f probability distribution

Parametrized

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model
- $\Theta \subset \mathbb{R}^d$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is parametrized by Θif:

- $\mathcal{P} = \{P_\theta\}_{\theta \in \Theta}$

Likelihood

Let:

· $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ

We name likelihood function of $(\Omega, \mathcal{A}, \mathcal{P})$ to:

$$\cdot \quad \begin{array}{rcl} L\ :\Omega \times \Theta & \longrightarrow & \mathbb{R}^+ \\ (x, \theta) & \longmapsto & P_\theta(x) \end{array}$$

Exponential model

Let:

· $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ

· L likelihood function of $(\Omega, \mathcal{A}, \mathcal{P})$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is exponential if:

· $\exists\ \{f_i\}_{i=1}^r, f \in \mathcal{F}(\Omega, \mathbb{R})$ " $\forall\ i \in [1, r]_{\mathbb{N}}$:

f_i, f measurable

· $\exists\ \{\phi_i\}_{i=1}^r, \phi \in \mathcal{F}(\theta, \mathbb{R})$ " $\forall\ i \in [1, r]_{\mathbb{N}}$:

ϕ_i, ϕ measurable

· $\forall\ x \in \Omega$:

$\forall\ \theta \in \Theta$:

$$L(x, \theta) = \exp(\sum_{i=1}^r f_i(x)\phi_i(\theta) + f(x) + \phi(\theta))$$

2. Estimation

Statistic

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ m-D statistical model parametrized by Θ
- (M, Σ) measurable space
- $X : \Omega \to M$ random variable
- $f : M \to \mathbb{R}^m$

Then, f is a statistic if:

- f measurable

We denote:

- $f : T$

Sufficiency

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ m-D statistical model parametrized by Θ
- (M, Σ) measurable space
- $X : \Omega \to M$ random variable
- $T : M \to \mathbb{R}^m$ statistic

Then, T is sufficient if:

- $\forall\ \theta_1, \theta_2 \in \Theta$:

  $\forall\ x \in M$:

  $\forall\ t \in \mathbb{R}^m$:

$$P_{\theta_1}(X = x \mid T \circ X = t) = P_{\theta_2}(X = x \mid T \circ X = t)$$

Estimator

Let:

$\cdot$ $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ

$\cdot$ (M, Σ) measurable space

$\cdot$ $X : \Omega \to M$ random variable

$\cdot$ $T : M \to \mathbb{R}^k$ statistic

$\cdot$ $\theta \in \Theta$

Then, T is an estimator of θif:

$\cdot$ T approaches θ

Loss function

Let:

$\cdot$ $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ

$\cdot$ (M, Σ) measurable space

$\cdot$ $X : \Omega \to M$ random variable

$\cdot$ $T : M \to \mathbb{R}^k$ estimator

$\cdot$ $W : \mathbb{R}^k \times \Theta \to \mathbb{R}^+$

Then, W is a loss function if:

$\cdot$ $W(\theta, \theta) = 0$

Risk function

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ
- $x_1, \ldots, x_n$ observation of $X = X_1, \ldots, X_n) : \tilde{\Omega} \to \Omega$
- $T\ :\ \Omega \to \mathbb{R}^k$ estimator
- $W\ :\ \mathbb{R}^k \times \Theta \to \mathbb{R}^+$ loose function

We name risk function to:

- $\begin{array}{rcl} R_T\ : \Theta & \longrightarrow & \mathbb{R}^+ \\ \theta & \longmapsto & E_\theta(W(T, \theta)) \end{array}\ adsf$
- $asdfjkdaf$