1. New

defined by f

Let:

- $f : \mathbb{R}^n \to \mathbb{R}^n$

Then, $(\mathbb{R}^n, \mathbb{N}, \phi)$ is defined by $f$if:

- $$\begin{array}{rcl} \phi : \mathbb{R}^n \times \mathbb{N} & \longrightarrow & \mathbb{R}^n \\ x & \longmapsto & f^n(x) \end{array}$$

We denote:

- $(\mathbb{R}^n, \mathbb{N}, f)$ n-D

Henon's application

Let:

- $$\begin{array}{rcl} f : \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ (x, y) & \longmapsto & (-x^2 + 0.4y, x) \end{array}$$

Study:

- Fixed points of f

Demonstration:

$(0, 0), (-0$

$6, -0$

$6)$ fixed points

Invariant curve

Let:

- γ differentiable curve
- $p \in \mathbb{R}^n$

Then, γ is invariant if:

- $\forall\ x \in \gamma* :$

$$o(x) \subset \gamma*$$

Then, γ is converges to $p$if:

- $\forall\ x \in \gamma* :$

$$o(x) \xrightarrow{n} p$$

Stable & Unstable

Let:

- $f : \mathbb{R}^n \to \mathbb{R}^n$
- $p \in \mathbb{R}^n$ m-periodic point

Then, p is stable if:

- $\forall\ \varepsilon \in \mathbb{R}^+ :$

  $\exists\ \delta \in \mathbb{R}^+ :$

  $\forall\ x \in B(p, \varepsilon) :$

  $\forall\ n \in \mathbb{N} :$

  $$f^{nm}(x) \in B(p, \varepsilon)$$

Then, p is unstable if:

- p not stable

Attractive & Repulsive

Let:

- $f : \mathbb{R}^n \to \mathbb{R}^n$
- $p \in \mathbb{R}^n$ m-periodic point

Then, p is attractive if:

- p stable
- $\exists\, \varepsilon \in \mathbb{R}^+ :$

  $\forall\, x \in B(p, \varepsilon) :$

  $$f^{nm}(x) \xrightarrow{n} p$$

Then, p is repulsive if:

- p attractive by f^{-1}

Invariance of stability over periods

Let:

- $(\mathbb{R}^n, \mathbb{N}, f)$ n-D dynamical system
- $p \in \mathbb{R}^n$ k-periodic point
- χ character of periodic points

Then, holds:

- $\exists\, \sigma \in \mathrm{Im}(\chi) :$

  $\forall\, x \in o(p) :$

$$\chi(x) = \sigma$$

Demonstration:

i will