Block I

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1. 1st laboratory

Orbit analysis

Let:

$$. \begin{array}{rccc} f & : \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \longmapsto & x^3 + \frac{1}{4}x \end{array}$$

Demonstration:

Formalization :

Consider (M, T, ϕ) where:

$M = \mathbb{R}$

$T = \mathbb{N}$

$$\begin{array}{rccc} \phi & : \mathbb{R} \times \mathbb{N} & \longrightarrow & \mathbb{R} \\ & (x, n) & \longmapsto & f^n(x) \end{array}$$

We will denote $\phi(x, n) = f^n(x)$ as x_n

Fixed points :

$\forall\ x \in \mathbb{R}$:

$$f(x) = x \leftrightarrow x^3 + \frac{1}{4}x - x = 0 \leftrightarrow x^3 - \frac{3}{4}x = 0$$

$$\leftrightarrow x = 0\ \vee\ x^2 - \frac{3}{4} = 0 \leftrightarrow x \in \{0, \pm\frac{\sqrt{3}}{2}\}$$

The fixed points set is $\{0, \pm\frac{\sqrt{3}}{2}\}$

Graphic analysis :

Parity: f is odd

$\forall\ x \in \mathbb{R}$:

$$f(-x) = (-x)^3 + \frac{(-x)}{4} = -(x^3 + \frac{x}{4}) = -f(x)$$

Monotonicity: f is increasing

$\forall\ x \in \mathbb{R}$:

$$f'(x) = 3x^2 + \frac{1}{4} > 0$$

Convexity: f is concave over $\mathbb{R}^-$ and convex over $\mathbb{R}^+$

$\forall\ x \in \mathbb{R}^-$:

$$f''(x) = 6x \leq 0$$

$\forall\ x \in \mathbb{R}^+$:

$$f''(x) = 6x \geq 0$$