Block I

Propositions

1. Discrete dynamical systems

introduction

Fixed points theorem

Let:

- $I \subset \mathbb{R}$ open
- $f : I \to I$ differentiable
- $x \in I$

Then, holds:

- $|f'(x)| < 1 \to x$ attractive
- $|f'(x)| > 1 \to x$ repulsive

Demonstration:

demonstration

Attractiveness of periodic points does not involve the chosen point

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$ n-periodic point
- $\{x_i\}_{i=1}^{r}$ orbit of x

Then, holds:

- x attractive $\leftrightarrow \forall\ x' \in o(x)$:

  x' attractive

Demonstration:

$\forall\ x' \in o(x)$:

$$f^{n'}(x') = \prod_{i=1}^{r} f'(x_i) = f^{n'}(x)$$

Partition of attraction set

Let:

$\cdot\ (M, \mathbb{N}, f)$ functional dynamical system

$\cdot\ x$ n-periodic point

$\cdot\ o(x)$ orbit of x

Then, holds:

$\cdot\ \forall\ x' \in o(x)$:

$\exists\, \mathcal{U} \subset M$ open :

$\forall\ y \in \mathcal{U}$:

$$f^n(y) \xrightarrow{n} x'$$

Demonstration:

demonstration

Homeomorphisms are monotonous

Let:

- $f : \mathbb{R} \to \mathbb{R}$ homeomorphism

Then, holds:

- f monotonous

Demonstration:

no demonstration

Homeomorphisms and n-periodic points

Let:

$\cdot$ $f\ :\ \mathbb{R} \to \mathbb{R}$ homeomorphism (M, T, ϕ) dynamical system defined

by f

Then, holds:

$\cdot$ $\forall\ n \in \mathbb{N}$:

$\nexists\ x \in M$ „ x n-periodic point

Demonstration:

graphically

Sarkovskii's theorem

Let:

- $f : I \to I$

- $(M, \mathbb{N}, f)$ dynamical system

Then, holds:

- $\exists\, x \in M$:

    $o(x)$ k-period

- $\to \forall\, l \in \mathbb{N}$ „ $l > k$:

    $\exists\, x' \in M$:

        x' l-period

Invariance of stability over periods

Let:

· $(\mathbb{R}^n, \mathbb{N}, f)$ n-D dynamical system

· $p \in \mathbb{R}^n$ k-periodic point

· χ character of periodic points

Then, holds:

· $\exists\ \sigma \in \mathrm{Im}(\chi)$:

$\forall\ x \in o(p)$:

$\chi(x) = \sigma$

Demonstration:

i will

2. 2-D linear dynamical systems

Invariance of stability over orbits

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$

Then, holds:

- $\forall\ x' \in o(x)$:

  $\chi(x') = \chi(x)$

Demonstration:

Follow 2 steps

Step 1 : $falta$:

$rows$

Step 2 : $attractiveness$:

$\chi(x) = -1$

$\exists\ \varepsilon \in \mathbb{R}^{+}$:

$x \in B_{\varepsilon}(x) \to f^{2n}(x) \xrightarrow{n} x$

$f \in \mathcal{C}^0(M) \to \exists\, \varepsilon_1 \in \mathbb{R}^+ :$

$$f(B_{\varepsilon_1}(x_1)) \subset B_\varepsilon(x)$$

$x \in B_{\varepsilon_1}(x_1) \to f(x) \in B_\varepsilon(x) \to f^{2n-1}(f(x)) \xrightarrow{n} x$

falta

Linear property

Let:

· $(M, \mathbb{N}, f)$ linear dynamical system

Then, holds:

· $\forall\ a, b \in \mathbb{R}$:

$\forall\ x, y \in M$:

$$f(ax + by) = af(x) + bf(y)$$

Demonstration:

matrius

Fixed points of linear applications

Let:

$\cdot$ $(M, \mathbb{N}, f)$ linear dynamical system

Then, holds:

$\cdot$ $0 \in \mathrm{Fix}(f)$

Demonstration:

demonstration

Jordan form of 2-D real linear maps

Let:

· $A \in \mathcal{M}_{2\times 2}(\mathbb{R})$

· $\chi_A(t)$ characteristic polynomial of A

Then, holds:

· $\exists\ \beta$ base of K :

$$\begin{cases} A = \lambda, 0, 0, \mu & \#Z(\chi_A(t)) = 2 \\ A = \lambda, 1, 0, \lambda & \#Z(\chi_A(t)) = 1 \\ A = \alpha, \beta, -\beta, \alpha & \#Z(\chi_A(t)) = 0 \end{cases}$$

Demonstration:

demonstration

Topology of 2-D real linear maps

Let:

· $A \in \mathcal{M}_{2\times 2}(\mathbb{R})$

· $\lambda \neq \mu$ eigenvalues of A

Then, holds:

· $|\lambda|, |\mu| < 1 \rightarrow (0,0)$ attractive

· $|\lambda| > |\mu| \rightarrow$ tangent to y = 0

· $|\mu| > |\lambda| \rightarrow$ tangent to x = 0

· $|\mu| = |\lambda| \rightarrow$ only invariant lines

·

· $|\lambda|, |\mu| > 1 \rightarrow (0,0)$ repulsive

· equivalent to other case

Demonstration:

demonstration