1. Holomorphic functions

Conjugation

Let:

$$\bar{a} : \mathbb{C} \longrightarrow \mathbb{C}, \quad z \longmapsto \bar{z}$$

Then, $\bar{a}$ is not holomorphic :

$u_x = 1$

$u_y = 0$

$v_x = 0$

$v_y = -1$

$\forall\ z \in \mathbb{C}$:

$-1 \neq 1 \rightarrow f$ not holomorphic in z

Quadratic norm

Let:

- $$\begin{aligned} f\ :\mathbb{C} &\longrightarrow \mathbb{C} \\ z &\longmapsto |z|^2 \end{aligned}$$

- $f_{\mathbb{R}^2}$ component decomposition of f

Then, f is holomorphic in 0:

f differentiable in $\mathbb{R}^2$ polinomial

$\forall\ z \in \mathbb{C}$:

$$u_x(x,y) = 2x$$

$$u_y(x,y) = 2y$$

$$v_x(x,y) = 0$$

$$v_y(x,y) = 0$$

$$u_x = v_y \leftrightarrow x = 0$$

$$u_y = -v_x \leftrightarrow y = 0$$

f holomorphic function in $z \leftrightarrow z = 0$

Non preserving angles function

Let:

$\cdot\, f(z) = z^2$

Then, f is conform in $\mathbb{R} \setminus \{0\}$:

$f(\{(x,0) \in \mathbb{C} \mid x > 0\}) = \{(x,0) \in \mathbb{C} \mid x > 0\}$

$f(\{(x,0) \in \mathbb{C} \mid x < 0\}) = \{(x,0) \in \mathbb{C} \mid x > 0\}$

$ang(A,B) = \pi \neq 0 = ang(f(A), f(B))$

Exponential

Let:

$\cdot\, a \,:\, 0$

$\cdot\, c_n \,:\, \frac{1}{n}$

Then, $\sum_{n\geq 0} c_n(z-a)^n$ is convergent in D1 :

$\lim_n \frac{|c_n|}{|c_{n+1}|} = \lim_n \frac{n+1}{n} = 1 \to R = 1$

CH→ $D(0,1)$ convergent

$\mathbb{C} \setminus D(0,1)$ divergent

$f' = f$