1. Holomorphic functions

Incremental quotient

Let:

$\cdot\, \mathcal{U} \subset \mathbb{C}$ open

$\cdot\, f : \mathcal{U} \to \mathbb{C}$

$\cdot\, p \in \mathcal{U}$

We name incremental quotient of f in p to:

$$\cdot \lim_{z \to p} \frac{f(z) - f(p)}{z - p}$$

We denote:

$\cdot\, f'(p)$

Holomorphic function

Let:

- $\mathcal{U} \subset \mathbb{C}$ open
- $f : \mathcal{U} \to \mathbb{C}$
- $p \in \mathcal{U}$

Then, f is holomorphic over p if:

- $\exists f'(p)$

Then, f is holomorphic over U if:

- $\forall\ p \in \mathcal{U}$:

  $\exists f'(p)$

We denote:

- $\{f : \mathcal{U} \to \mathbb{C} \mid f \text{ holomorphic over } \mathcal{U}\} : \mathcal{H}(\mathcal{U})$
- $f \in \mathcal{H}(\mathbb{C}) : f$ entire

Cauchy-Riemman equations

Let:

- $u, v : \mathbb{R}^2 \to \mathbb{R}$
- $\begin{array}{rcl} f : \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ (x, y) & \longmapsto & (u((x, y)), v((x, y))) \end{array}$

Then, f is satisfies the Cauchy-Riemman equations if:

- $\exists\, u_x, u_y, v_x, v_y$
- $u_x = v_y$
- $u_y = -v_x$

We denote:

- $u_x + iv_x : f_x$
- $u_y + iv_y : f_y$

Conformal

Let:

- $\mathcal{U} \subset \mathbb{C}$ open
- $f : \mathcal{U} \to \mathbb{C}$
- $z \in \mathcal{U}$

Then, f is conformal in z if:

- $\exists\, c \in \mathbb{C}$:

  $\forall\, I \subset \mathbb{R}$ „ $0 \in I$:

  $\forall\, \gamma : I \to \mathbb{R}^2$ „ γ differentiable $\wedge\ \gamma(0) = z\ \wedge\ \gamma'(0) \neq$

0 :

$$\frac{(f\circ\gamma)'(0)}{\gamma'(0)} = c$$

Then, f is conformal if:

- $\forall\, z \in \mathcal{U}$:

  f conformal in z

Power series

Let:

- $\sum\limits_{n\geq 0} a_n f_n$ complex valued sequence

Then, $\sum\limits_{n\geq 0} a_n f_n$ is a power series if:

- $\forall\ n \in \mathbb{N}$:

$$\begin{array}{rccc} f_n & : \mathbb{C} & \longrightarrow & \mathbb{C} \\ & z & \longmapsto & (z-a)^n \end{array}$$

Convergence radius

Let:

- *statements*
-

Then, *item* is a/an entity if:

- *conditions*
-

We denote:

- *property* : *notation*
-

Absolutely convergent

Let:

· $\sum c_n n[0]$ series

Then, $\sum c_n n[0]$ is absolutely convergent if:

· $\sum |c_n| n[0]$ convergent

Numeric series

Let:

· $(c_k)_{k \in \mathbb{N}}$

Then, *item* is a/an entity if:

· *conditions*

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We denote:

· *property* : *notation*

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