1. New

Uniparametric dynamical system family

Let:

$\cdot\, I \subset \mathbb{R}\{f_\lambda : I \to I\}_{\lambda\in\Lambda}$

Then, $(M, T, f_\lambda)_\lambda$ is uniparametric dynamical system family if:

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Characterization of sella-node bifurcation points

Let:

$\cdot\, I \subset \mathbb{R}\{f_\lambda : I \to I\}_{\lambda\in\Lambda}$

$\cdot\, x \in I$

Then, holds:

$\cdot\, x$ SN bifurcation point $\leftrightarrow f(x) = x, f'(x) = 1$

$\cdot\, \frac{\partial}{\partial\lambda} f \neq 0$

$\cdot\, f''(x) \neq 0$

Demonstration:

demonstration

Quadratic function bifurcations

Let:

- $$\begin{array}{rcl} f : \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & a - x^2 \end{array}$$
- (M, T, f_c) dynamical system family

Then, f is bifurcates in -1/4 :

$f_{-\frac{1}{4}}(x) = x \leftrightarrow x = -\frac{1}{2}$

$f'_{-\frac{1}{4}}(x) = -2x$

$f'_{-\frac{1}{4}}(-\frac{1}{2}) = 1$

$\frac{\partial}{\partial a} f = 1 \neq 0$

$\frac{\partial}{\partial x^2} f = -2 \neq 0$

$sgn(1 * -2) = - \rightarrow -\frac{1}{2}$ SN

Pitchfork bifurcation

Let:

- (M, T, f_λ) dynamical system family
- $x \in M$

Then, x is pitchfork bifurcation if:

- x fixed point
- born of 2 fixed points
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Characterization of Pitchfork bifurcations

Let:

- (M, T, f_λ) dynamical system family
- $x \in M$

Then, holds:

- x Pitchfork $\leftrightarrow$
- $f(x) = x$
- $f'(x) = 1$
- $\frac{\partial}{\partial x^2} f = 0$
- $\frac{\partial}{\partial \lambda} f = 0$

Demonstration:

no demonstration

9. Fixed points and 2-periodic points

Let:

$$\cdot \begin{array}{rcl} f: \mathbb{R} \times \mathbb{R}^{+} & \longrightarrow & \mathbb{R} \\ (x, r) & \longmapsto & r\frac{x}{1+x^2} \end{array}$$

Study:

$\cdot$ Fixed points of f

$\cdot$ n-periodic points of f

Demonstration:

Graphical analysis f odd .f has 2 extrema in ± 1.$f \xrightarrow{n} 0$. Fixed points $f(x) = x \leftrightarrow x = \pm\sqrt{r-1}$.$f'(\pm\sqrt{r-1}) = \frac{2-r}{r}$ n-periodic points f