# 1. Statistic models

*introduction*

**Exponential model**

Let:

$\cdot\, \Omega : \mathbb{R}^n$

$\cdot\, \mathcal{A} : \mathbb{B}(\mathbb{R}^n)$

$\cdot\, \theta : (\mu, \sigma^2)$

$\cdot\, \Theta : \mathbb{R} \times \mathbb{R}^+$

$\cdot\, \mathcal{P} : \{N(\theta)\}_{\theta \in \Theta}$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is an exponential model :

$\forall\ x \in \mathbb{R}^n$:

$$f_\theta(x) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{1}{2\sigma^2}(x_i - \mu)^2)$$
$$= (\tfrac{1}{\sqrt{2\pi\sigma^2}})^n \exp(-\tfrac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2)$$
$$L(x,\theta) = \exp(\tfrac{n}{2}\log(\tfrac{1}{2\pi\sigma^2}) - \tfrac{1}{2\sigma^2}\sum_{i=1}^{n} x_i^2 + \frac{n\bar{x}\mu}{\sigma^2} - \frac{n\mu^2}{2\sigma^2})$$
$$\begin{aligned} \phi\ :\Theta &\longrightarrow \mathbb{R}^2 \\ (\mu,\sigma^2) &\longmapsto (-\tfrac{\mu}{2\sigma^2}, \tfrac{n\mu}{\sigma^2}) \\ \phi'\ :\Theta &\longrightarrow \mathbb{R} \\ (\mu,\sigma^2) &\longmapsto \tfrac{n}{2}\log(\tfrac{1}{2\pi\sigma^2}) - \tfrac{n\mu^2}{2\sigma^2} \\ f\ :\Omega &\longrightarrow \mathbb{R}^2 \\ x &\longmapsto (\sum_{i=1}^{n} x_i^2, \bar{x}) \end{aligned}$$
$$L(x,\theta) = \exp(\phi'(\theta) - \phi(\theta) f(x))$$

$\therefore)$ $(\Omega, \mathcal{A}, \mathcal{P})$ exponential model