# 1. Discrete dynamical systems

*introduction*

**Fixed points theorem**

Let:

· $I \subset \mathbb{R}$ open

· $f \ : \ I \to I$ differentiable

· $x \in I$

Then, holds:

· $|f'(x)| < 1 \to x$ attractive

· $|f'(x)| > 1 \to x$ repulsive

Demonstration:

*demonstration*

**Attractiveness of periodic points does not involve the chosen point**

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$ n-periodic point
- $\{x_i\}_{i=1}^{r}$ orbit of $x$

Then, holds:

- $x$ attractive $\leftrightarrow \forall\ x' \in o(x)$ :

  $x'$ attractive

Demonstration:

$\forall\ x' \in o(x)$ :

$$f^{n'}(x') = \prod_{i=1}^{r} f'(x_i) = f^{n'}(x)$$

## Partition of attraction set

Let:

$\cdot$ $(M, \mathbb{N}, f)$ functional dynamical system

$\cdot$ $x$ n-periodic point

$\cdot$ $o(x)$ orbit of $x$

Then, holds:

$\cdot$ $\forall\ x' \in o(x)$ :

$\exists\, \mathcal{U} \subset M$ open :

$\forall\ y \in \mathcal{U}$ :

$$f^n(y) \xrightarrow{n} x'$$

Demonstration:

*demonstration*

### Homeomorphisms are monotonous

Let:

$\cdot\ f\ :\ \mathbb{R} \to \mathbb{R}$ homeomorphism

Then, holds:

$\cdot\ f$ monotonous

Demonstration:

no demonstration

**Homeomorphisms and n-periodic points**

Let:

$\cdot\ f\ :\ \mathbb{R} \to \mathbb{R}$ homeomorphism $(M, T, \phi)$ dynamical system defined

by $f$

Then, holds:

$\cdot\ \forall\ n \in \mathbb{N}$ :

$\nexists\ x \in M$ „ $x$ n-periodic point

Demonstration:

graphically

**Sarkovskii's theorem**

Let:

- $f \ : \ I \to I$

- $(M, \mathbb{N}, f)$ dynamical system

Then, holds:

- $\exists \ x \in M \ :$

  $o(x)$ k-period

- $\to \forall \ l \in \mathbb{N} \ {}_{\shortparallel} \ l > k \ :$

  $\exists \ x' \in M \ :$

  $x'$ l-period

## Invariance of stability over periods

Let:

- $(\mathbb{R}^n, \mathbb{N}, f)$ n-D dynamical system
- $p \in \mathbb{R}^n$ k-periodic point
- $\chi$ character of periodic points

Then, holds:

- $\exists\ \sigma \in \mathrm{Im}(\chi)$ :

  $\forall\ x \in o(p)$ :

  $\chi(x) = \sigma$

Demonstration:

i will