1. The field of complex numbers

introduction

The field of complex numbers

Let:

$$
\begin{aligned}
+ &: \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}^2 \\
((a,b),(c,d)) &\longmapsto (a+c, b+d) \\
\cdot &: \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}^2 \\
((a,b),(c,d)) &\longmapsto (ac-bd, ad+bd)
\end{aligned}
$$

We name the field of complex numbers to:

$\cdot\ (\mathbb{R}^2, +, \cdot)$

We denote:

$\cdot\ (\mathbb{R}^2, +, \cdot)\ :\ \mathbb{C}$

$\cdot\ (0,1) \in \mathbb{C}\ :\ i$

$\cdot\ (a,b) \in \mathbb{C}\ :\ a + bi$

$\cdot\ \pi_1(\mathbb{C})\ :\ Re$

$\cdot\ \pi_2(\mathbb{C})\ :\ Im$

Conjugation

We name complex conjugation to:

$$
\begin{aligned}
f &: \mathbb{C} \longrightarrow \mathbb{C} \\
(a,b) &\longmapsto (a, -b)
\end{aligned}
$$

We denote:

$\cdot\ f((a,b))\ :\ \overline{(a,b)}$

Norm

We name complex norm to:

$$
\cdot \quad
\begin{aligned}
f \ :\mathbb{C} &\longrightarrow \quad \mathbb{R} \\
(a,b) &\longmapsto \quad \sqrt{a^2 + b^2}
\end{aligned}
$$

We denote:

$$
\cdot \, f((a,b)) \, : \, |(a,b)|
$$

Polar transformation

We name polar transformation to:

$$
\cdot \quad
\begin{aligned}
f \ :\mathbb{C} &\longrightarrow \quad \mathbb{R}^+ \times [0, 2\pi) \\
(a,b) &\longmapsto \quad (\sqrt{|(a,b)|}, \arctan(\tfrac{b}{a}))
\end{aligned}
$$

We denote:

$$
\cdot \, f((a,b)) \, : \, (r, \theta)
$$

Unit sphere projection

We name unit sphere projection to:

$$
\cdot \quad
\begin{aligned}
f \ :\mathbb{C} &\longrightarrow \quad S^1 \\
z &\longmapsto \quad \tfrac{z}{|z|}
\end{aligned}
$$

We denote:

$$
\cdot \, f(z) \, : \, \pi(z)
$$

Roots of unity

Let:

$\cdot\, z \in \mathbb{C}$

Then, z is a root of unity if:

$\cdot\ \exists\, n \in \mathbb{N}:$

$$z^n = 1$$

We denote:

$\cdot\, \{ z \in \mathbb{C} \mid z \text{ root of unity } \} \,:\, S^1$

Disk

Let:

$\cdot\, p \in \mathbb{C}$

$\cdot\, r \in \mathbb{R}^+ \smallsetminus \{0\}$

We name Disk centered in p and radius r to:

$\cdot\, \{ z \in \mathbb{C} \mid |z - p| < r \}$

We denote:

$\cdot\, \{ z \in \mathbb{C} \mid |z - p| < r \} \,:\, D^1$

Component decomposition

Let:

$$\cdot\, f \,:\, \mathbb{C} \to \mathbb{C}$$

We name real component of f to:

$$\cdot \quad \begin{array}{rccl} f_{Re} &:\mathbb{C} &\longrightarrow& \mathbb{R} \\ & z &\longmapsto& Re(f(z)) \end{array}$$

We name imaginary component of f to:

$$\cdot \quad \begin{array}{rccl} f_{Im} &:\mathbb{C} &\longrightarrow& \mathbb{R} \\ & z &\longmapsto& Im(f(z)) \end{array}$$

We name component decomposition of f to:

$$\cdot \quad \begin{array}{rccl} f_{\mathbb{R}^2} &:\mathbb{C} &\longrightarrow& \mathbb{R}^2 \\ & (x,y) &\longmapsto& (f_{Re}(x+yi), f_{Im}(x+yi) \end{array}$$