# 1. One-dimensional discrete dynamical systems

*introduction*

## Fixed points theorem

Let:

- $I \subset \mathbb{R}$ open
- $f : I \to I$ differentiable
- $x \in I$

Then, holds:

- $|f'(x)| < 1 \to x$ attractive
- $|f'(x)| > 1 \to x$ repulsive

Demonstration:

*demonstration*

**Attractiveness of periodic points does not involve the chosen point**

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$ n-periodic point
- $\{x_i\}_{i=1}^{r}$ orbit of $x$

Then, holds:

- $x$ attractive $\leftrightarrow$ $\forall\ x' \in o(x)$:

  $x'$ attractive

Demonstration:

$\forall\ x' \in o(x)$:

$$f^{n'}(x') = \prod_{i=1}^{r} f'(x_i) = f^{n'}(x)$$

## Partition of attraction set

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x$ n-periodic point
- $o(x)$ orbit of $x$

Then, holds:

- $\forall\ x' \in o(x)$:

  $\exists\, \mathcal{U} \subset M$ open :

  $\forall\ y \in \mathcal{U}$:

  $$f^n(y) \xrightarrow{n} x'$$

Demonstration:

*demonstration*