Block I

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1. 1st laboratory

Orbit analysis

Let:

$$\begin{array}{rcl} f : \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & x^3 + \frac{1}{4}x \end{array}.$$

Demonstration:

Formalization

Consider (M, T, ϕ) where:

$M = \mathbb{R}$

$T = \mathbb{N}$

$$\begin{array}{rcl} \phi : \mathbb{R} \times \mathbb{N} & \longrightarrow & \mathbb{R} \\ (x, n) & \longmapsto & f^n(x) \end{array}$$

We will denote $\phi(x, n) = f^n(x)$ as x_n

Fixed points

$\forall\ x \in \mathbb{R}$:

$$f(x) = x \leftrightarrow x^3 + \frac{1}{4}x - x = 0 \leftrightarrow x^3 - \frac{3}{4}x = 0$$

$$\leftrightarrow x = 0 \ \vee \ x^2 - \frac{3}{4} = 0 \leftrightarrow x \in \{0, \pm\frac{\sqrt{3}}{2}\}$$

Graphic analysis

Parity: f is odd

$\forall\ x \in \mathbb{R}$:

$$f(-x) = (-x)^3 + \frac{(-x)}{4} = -(x^3 + \frac{x}{4}) = -f(x)$$

Monotonicity: f is increasing

$\forall\ x \in \mathbb{R}$:

$$f'(x) = 3x^2 + \frac{1}{4} > 0$$

Convexity: f is concave over $\mathbb{R}^-$ and convex over $\mathbb{R}^+$

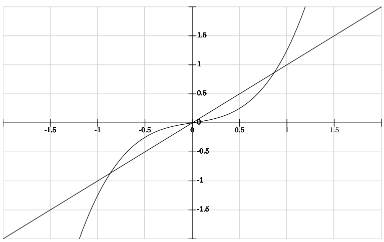
$\forall\ x \in \mathbb{R}^-$:

$$f''(x) = 6x \leq 0$$

$\forall\ x \in \mathbb{R}^+$:

$$f''(x) = 6x \geq 0$$

Graphic representation



Orbit analysis

$\forall\ x \in \mathbb{R}$ „ $x < \frac{\sqrt{3}}{2}$:

$o(x)$ is superiorly bounded by $-\frac{\sqrt{3}}{2}$

Induction over n: As f is increasing:

$$x_n < -\frac{\sqrt{3}}{2} \rightarrow f(x_n) < f(-\frac{\sqrt{3}}{2}) \rightarrow x_{n+1} < -\frac{\sqrt{3}}{2}$$

$o(x)$ decreasing

$$\begin{array}{rcl} g : \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & f(x) - x \end{array} \quad \text{increasing}$$

Induction over n: As f is increasing:

$f(x_n) - x_n = x_n^3 - \frac{3}{4}x_n$

$x_n < -\frac{\sqrt{3}}{2} \rightarrow f(x_n) < (-\frac{\sqrt{3}}{2})^3 - \frac{3}{4}(-\frac{\sqrt{3}}{2}) < 0$

$o(x)$ decreasing and no fixed point under x

$o(x) \xrightarrow{n} -\infty$

$\forall\ x \in (\frac{-\sqrt{3}}{2}, 0)$:

$o(x) \xrightarrow{n} -\frac{\sqrt{3}}{2}$

$f(x) = x(x^2 + \frac{1}{4}) < 0$

$o(x)$ superiorly bounded by 0

$f(x) - x = x(x^2 - \frac{3}{4}) > 0 \rightarrow f(x) > xo(x)$ bounded + increasing → o(x) convergent to a fixed point

The only fixed point higher than x is 0

$o(x) \xrightarrow{n} 0$