1. Punctual estimation

introduction

Neyman & Fisher's factorization theorem

Let:

$\cdot$ $(\Omega, \mathcal{A}, \mathcal{P})$ k-D statistical model parametrized by Θ

$\cdot$ $L : \Omega \times \Theta \to \mathbb{R}$ likelihood function of $(\Omega, \mathcal{A}, \mathcal{P})$

$\cdot$ (M, Σ) measurable space

$\cdot$ $X : \Omega \to M$ random variable

$\cdot$ $T : M \to \mathbb{R}^k$ statistic

Then, holds:

$\cdot$ T sufficient $\leftrightarrow \exists\, \Lambda : \mathbb{R}^m \times \Theta \to \mathbb{R}^+, h : \Omega \to \mathbb{R}^k :$

$\forall\, x \in \Omega :$

$\forall\, \theta \in \Theta :$

$$L(x, \theta) = \Lambda(T(x), \theta) h(x)$$

Cramer-Rao's inequality

Let:

$\cdot$ same conditions of above

$\cdot T \in \chi_F$

$\cdot$ regular model

$\cdot E_\theta T = g(\theta)$

Then, holds:

$\cdot Var_\theta(T) \geq \frac{g'(\theta)^2}{I(\theta)}$

Demonstration:

$|E(\partial_\theta \log(L(x,\theta))(T(x)-g(\theta)| \leq \sqrt{E_\theta(\partial_\theta \log(L(x,\theta)))^2 E_\theta(T(x)-g(\theta))^2} =$

$\sqrt{I(\theta)Var_\theta T}$

$E(\partial_\theta \log(L(x,\theta))(T(x)g(\theta) = E_\theta(\partial_\theta \log(L(x,\theta)T(x))-g(\theta)E_\theta(\partial_\theta \log(L(x,\theta))$

$E_\theta(\partial_\theta \log(L(x,\theta)T(x))$

$\int_\Omega \partial_\theta \log(L(x,\theta))T(x)L(x,\theta)dx = \int_\Omega \frac{1}{L(x,\theta)}\partial_\theta L(x,\theta)T(x)L(x,\theta)dx$

$|g'(\theta)| \leq \sqrt{I(\theta)Var_\theta(T)}$

$g'(\theta)^2 \leq I(\theta)Var_\theta(T)$

Efficient estimators are UMV

Let:

$\cdot\, statements$

$\cdot$

Then, holds:

$\cdot\, then, holds$

$\cdot$

Demonstration:

$demonstration$

Characterization of efficient estimators

Let:

- mismas condiciones

Then, holds:

- T efficient $\leftrightarrow \exists\ \lambda(\theta)$:

$$\lambda(\theta)\partial_\theta \log(L(x,\theta) = T(x) - g(\theta) P_\theta - qs$$

Demonstration:

no demonstration

Observation

Let:

- mismas condiciones

Then, holds:

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Demonstration:

$\lambda'(\theta)\log(L(x,\theta)) + \lambda(\theta)\partial_{\theta^2}\log(L(x,\theta)) = -g'(\theta)$

$E[*] = E_\theta(\lambda(\theta)\partial_\theta \log(L(x,\theta))) + \lambda(\theta)E_\theta(\partial_{\theta^2}\log(L(x,\theta))) = 0 - g'(\theta)$

$\lambda(\theta)I(\theta) = g'(\theta)$

$I(\theta) = \frac{g'(\theta)}{\lambda(\theta)}$

si existe un estimador que da la igualdad entonces es UMV