1. Statistic models

introduction

Statistical model

Let:

- Ω set
- $\mathcal{A}$ sigma-algebra over Ω
- $\mathcal{P} \subset \mathcal{F}(\Omega, \mathbb{R})$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is a statistical model if:

- $\forall\ f \in \mathcal{P}$:

  f probability distribution

Parametrized

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model
- $\Theta \subset \mathbb{R}^d$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is parametrized by Θif:

- $\mathcal{P} = \{P_\theta\}_{\theta \in \Theta}$

Likelihood

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ

We name likelihood function of $(\Omega, \mathcal{A}, \mathcal{P})$ to:

- $$\begin{aligned} L\ :\Omega \times \Theta &\longrightarrow \mathbb{R}^+ \\ (x, \theta) &\longmapsto P_\theta(x) \end{aligned}$$

Exponential model

Let:

$\cdot$ $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ

$\cdot$ L likelihood function of $(\Omega, \mathcal{A}, \mathcal{P})$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is exponential if:

$\cdot$ $\exists \ \{f_i\}_{i=1}^r, f \in \mathcal{F}(\Omega, \mathbb{R})$:

$\forall \ i \in [1, r]_{\mathbb{N}}$:

f_i, f measurable

$\cdot$ $\exists \ \{\phi_i\}_{i=1}^r, \phi \in \mathcal{F}(\theta, \mathbb{R})$:

$\forall \ i \in [1, r]_{\mathbb{N}}$:

ϕ_i, ϕ measurable

$\cdot$ $\forall \ x \in \Omega$:

$\forall \ \theta \in \Theta$:

$$L(x, \theta) = \exp(\sum_{i=1}^{r} f_i(x)\phi_i(\theta) + f(x) + \phi(\theta))$$