1. Statistic models

Characterization of regular exponential models

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ exponential 1-D model parametrized by Θ
- $L(x,\theta) = \exp(f(x)\phi(\theta) + f'(x) + \phi'(\theta))$ exponential expression

Then, holds:

- Θ interval , $\phi, \phi' \in \mathcal{C}^2$ " $\forall\ \theta \in \Theta$:

  $\phi'(\theta) \neq 0$

  $E_\theta f^2(x) \in \mathbb{R}$

- $\rightarrow (\Omega, \mathcal{A}, \mathcal{P})$ regular

Demonstration:

no demonstration

Suficiencia

Statistic

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model m-D parametrized by Θ
- $X = (X_1, \ldots, X_n) : \tilde{\Omega} \to \Omega$
- $T : \Omega \to \mathbb{R}^m$

Then, T is statistic if:

- $\forall\ B \in \mathbb{B}(\mathbb{R})$:

  $T^{-1}(B) \in \mathcal{A}$

Sufficiency

Let:

$\cdot$ $(\Omega, \mathcal{A}, \mathcal{P})$ statistical l-D model parametrized by Θ

$\cdot$ $T : \Omega \to \mathbb{R}^m$ statistic

Then, T is sufficient if:

$\cdot$ la ley de la muestra condicionada por T no depende de θ

Noyman & Fisher's factorization test

Let:

$\cdot$ $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ

$\cdot$ $T : \Omega \to \mathbb{R}^m$

Then, holds:

$\cdot$ T sufficient $\leftrightarrow \exists \wedge \in \mathcal{F}(\mathbb{R}^m \times \Theta, \mathbb{R}^+), h \in \mathcal{F}(\Omega, \mathbb{R}^+)$ " $L(x, \theta) = \wedge (T(x), \theta) h(x)$

Punctual estimation

estimator

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ
- $x_1, \ldots, x_n$ observation of $X = X_1, \ldots, X_n) : \tilde{\Omega} \rightarrow \Omega$
- $T : \Omega \rightarrow \mathbb{R}^k$ statistic

Then, T is an estimator if:

- T approaches θ

loose function

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ
- $x_1, \ldots, x_n$ observation of $X = X_1, \ldots, X_n) : \tilde{\Omega} \rightarrow \Omega$
- $T : \Omega \rightarrow \mathbb{R}^k$ estimator
- $W : \mathbb{R}^k \times \Theta \rightarrow \mathbb{R}^+$

Then, W is a loose function if:

- $W(\theta, \theta) = 0$

Risk function

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ
- $x_1, \ldots, x_n$ observation of $X = X_1, \ldots, X_n) : \tilde{\Omega} \to \Omega$
- $T \ : \ \Omega \to \mathbb{R}^k$ estimator
- $W \ : \ \mathbb{R}^k \times \Theta \to \mathbb{R}^+$ loose function

We name risk function to:

$$\begin{array}{rcl} R_T \ : \Theta & \longrightarrow & \mathbb{R}^+ \\ \theta & \longmapsto & E_\theta(W(T, \theta)) \end{array}$$