1. Statistic models

introduction

Exponential model

Let:

- $\Omega : \mathbb{R}^n$
- $\mathcal{A} : \mathbb{B}(\mathbb{R}^n)$
- $\theta : (\mu, \sigma^2)$
- $\Theta : \mathbb{R} \times \mathbb{R}^+$
- $\mathcal{P} : \{N(\theta)\}_{\theta \in \Theta}$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is an exponential model :

$\forall\ x \in \mathbb{R}^n$:

$$f_\theta(x) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{1}{2\sigma^2}(x_i - \mu)^2)$$

$$= (\tfrac{1}{\sqrt{2\pi\sigma^2}})^n \exp(-\tfrac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2)$$

$$L(x, \theta) = \exp(\tfrac{n}{2} \log(\tfrac{1}{2\pi\sigma^2}) - \tfrac{1}{2\sigma^2} \sum_{i=1}^{n} x_i^2 + \frac{n\bar{x}\mu}{\sigma^2} - \frac{n\mu^2}{2\sigma^2})$$

$$\begin{array}{rcl} \phi : \Theta & \longrightarrow & \mathbb{R}^2 \\ (\mu, \sigma^2) & \longmapsto & (-\frac{\mu}{2\sigma^2}, \frac{n\mu}{\sigma^2}) \\ \phi' : \Theta & \longrightarrow & \mathbb{R} \\ (\mu, \sigma^2) & \longmapsto & \frac{n}{2} \log(\frac{1}{2\pi\sigma^2}) - \frac{n\mu^2}{2\sigma^2} \\ f : \Omega & \longrightarrow & \mathbb{R}^2 \\ x & \longmapsto & (\sum_{i=1}^{n} x_i^2, \bar{x}) \end{array}$$

$$L(x, \theta) = \exp(\phi'(\theta) - \phi(\theta) f(x))$$

$\therefore)$ $(\Omega, \mathcal{A}, \mathcal{P})$ exponential model