1. Statistic models

introduction

Statistical model

Let:

- Ω set
- $\mathcal{A}$ sigma-algebra over Ω
- $\mathcal{P} \subset \mathcal{F}(\Omega, \mathbb{R})$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is a statistical model if:

- $\forall\ f \in \mathcal{P}$:

  f probability distribution

Parametrized

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model
- $\Theta \subset \mathbb{R}^d$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is parametrized by Θif:

- $\mathcal{P} = \{P_\theta\}_{\theta\in\Theta}$

Likelihood

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ

We name likelihood function of $(\Omega, \mathcal{A}, \mathcal{P})$ to:

- $$\begin{array}{rcl} L : \Omega \times \Theta & \longrightarrow & \mathbb{R}^+ \\ (x, \theta) & \longmapsto & P_\theta(x) \end{array}$$

Exponential model

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ
- L likelihood function of $(\Omega, \mathcal{A}, \mathcal{P})$

Then, $(\Omega, \mathcal{A}, \mathcal{P})$ is exponential if:

- $\exists \; \{f_i\}_{i=1}^{r}, f \in \mathcal{F}(\Omega, \mathbb{R}) \;\; " \quad \forall \; i \in [1, r]_{\mathbb{N}}$:

  f_i, f measurable

- $\exists \; \{\phi_i\}_{i=1}^{r}, \phi \in \mathcal{F}(\theta, \mathbb{R}) \;\; " \quad \forall \; i \in [1, r]_{\mathbb{N}}$:

  ϕ_i, ϕ measurable

- $\forall \; x \in \Omega$:

  $\forall \; \theta \in \Theta$:

$$L(x, \theta) = \exp(\sum_{i=1}^{r} f_i(x)\phi_i(\theta) + f(x) + \phi(\theta))$$

description

Let:

- a

Show that:

- b

Demonstration:

c