# 1. One-dimensional discrete dynamical systems

*introduction*

## Fixed points theorem

Let:

- $I \subset \mathbb{R}$ open
- $f : I \to I$ differentiable
- $x \in I$

Then, holds:

- $|f'(x)| < 1 \to x$ attractive
- $|f'(x)| > 1 \to x$ repulsive

Demonstration:

*demonstration*

**Attractiveness of periodic points does not involve the chosen point**

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$ n-periodic point
- $\{x_i\}_{i=1}^{r}$ orbit of $x$

Then, holds:

- $x$ attractive $\leftrightarrow$ $\forall\ x' \in o(x)$:

  $x'$ attractive

Demonstration:

$\forall\ x' \in o(x)$:

$$f^{n'}(x') = \prod_{i=1}^{r} f'(x_i) = f^{n'}(x)$$

**Partition of attraction set**

Let:

$\cdot\ (M, \mathbb{N}, f)$ functional dynamical system

$\cdot\ x$ n-periodic point

$\cdot\ o(x)$ orbit of $x$

Then, holds:

$\cdot \quad \forall\, x' \in o(x)$:

$\exists\, \mathcal{U} \subset M$ open :

$\forall\, y \in \mathcal{U}$:

$$f^n(y) \xrightarrow{n} x'$$

Demonstration:

*demonstration*

### Homeomorphisms are monotonous

Let:

- $f : \mathbb{R} \to \mathbb{R}$ homeomorphism

Then, holds:

- $f$ monotonous

Demonstration:

no demonstration

### Homeomorphisms and n-periodic points

Let:

$\cdot\ f\ :\ \mathbb{R} \to \mathbb{R}$ homeomorphism $(M, T, \phi)$ dynamical system defined by $f$

Then, holds:

$\cdot\quad \forall\ n \in \mathbb{N}$:

$\nexists\ x \in M \ \ _{"}\ x$ n-periodic point

Demonstration:

graphically

### Sarkovskii's theorem

Let:

- $f : I \to I$
- $(M, T, \phi)$ dynamical system

Then, holds:

- $\exists\, x \in M$:

  $o(x)$ k-period

- $\to \ \forall\, l \in \mathbb{N}$ „ $l > k$:

  $\exists\, x' \in M$:

  $x'$ l-period