1. 2-D linear dynamical systems

Invariance of stability over orbits

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$

Then, holds:

- $\forall\ x' \in o(x)$:

$$\chi(x') = \chi(x)$$

Demonstration:

Follow 2 steps

Step 1 : $falta$:

$$rows$$

Step 2 : $attractiveness$:

$$\chi(x) = -1$$

$$\exists\ \varepsilon \in \mathbb{R}^{+} :$$

$$x \in B_{\varepsilon}(x) \to f^{2n}(x) \xrightarrow{n} x$$

$f \in \mathcal{C}^0(M) \to \exists\, \varepsilon_1 \in \mathbb{R}^+ :$

$$f(B_{\varepsilon_1}(x_1)) \subset B_\varepsilon(x)$$

$x \in B_{\varepsilon_1}(x_1) \to f(x) \in B_\varepsilon(x) \to f^{2n-1}(f(x)) \xrightarrow{n} x$

falta

Linear property

Let:

- $(M, \mathbb{N}, f)$ linear dynamical system

Then, holds:

- $\forall\ a, b \in \mathbb{R}$:

  $\forall\ x, y \in M$:

$$f(ax + by) = af(x) + bf(y)$$

Demonstration:

matrius

Fixed points of linear applications

Let:

· $(M, \mathbb{N}, f)$ linear dynamical system

Then, holds:

· $0 \in \mathrm{Fix}(f)$

Demonstration:

demonstration

Jordan form of 2-D real linear maps

Let:

- $A \in \mathcal{M}_{2\times 2}(\mathbb{R})$

- $\chi_A(t)$ characteristic polynomial of A

Then, holds:

- $\exists\ \beta$ base of K :
$$\begin{cases} A = \lambda, 0, 0, \mu & \#Z(\chi_A(t)) = 2 \\ A = \lambda, 1, 0, \lambda & \#Z(\chi_A(t)) = 1 \\ A = \alpha, \beta, -\beta, \alpha & \#Z(\chi_A(t)) = 0 \end{cases}$$

Demonstration:

demonstration

Topology of 2-D real linear maps

Let:

- $A \in \mathcal{M}_{2\times 2}(\mathbb{R})$
- $\lambda \neq \mu$ eigenvalues of A

Then, holds:

- $|\lambda|, |\mu| < 1 \to (0,0)$ attractive
- $|\lambda| > |\mu| \to$ tangent to y = 0
- $|\mu| > |\lambda| \to$ tangent to x = 0
- $|\mu| = |\lambda| \to$ only invariant lines
-
- $|\lambda|, |\mu| > 1 \to (0,0)$ repulsive
- equivalent to other case

Demonstration:

demonstration