1. One-dimensional discrete dynamical systems

introduction

Fixed points theorem

Let:

· $I \subset \mathbb{R}$ open

· $f : I \to I$ differentiable

· $x \in I$

Then, holds:

· $|f'(x)| < 1 \to x$ attractive

· $|f'(x)| > 1 \to x$ repulsive

Demonstration:

demonstration

Attractiveness of periodic points does not involve the chosen point

Let:

- $(M, \mathbb{N}, f)$ functional dynamical system
- $x \in M$ n-periodic point
- $\{x_i\}_{i=1}^{r}$ orbit of x

Then, holds:

- x attractive $\leftrightarrow \forall\ x' \in o(x)$:

  x' attractive

Demonstration:

$\forall\ x' \in o(x)$:

$$f^{n'}(x') = \prod_{i=1}^{r} f'(x_i) = f^{n'}(x)$$

Partition of attraction set

Let:

$\cdot\ (M, \mathbb{N}, f)$ functional dynamical system

$\cdot\ x$ n-periodic point

$\cdot\ o(x)$ orbit of x

Then, holds:

$\cdot\ \forall\ x' \in o(x)$:

$\exists\, \mathcal{U} \subset M$ open :

$\forall\ y \in \mathcal{U}$:

$$f^n(y) \xrightarrow{n} x'$$

Demonstration:

demonstration

Homeomorphisms are monotonous

Let:

· $f : \mathbb{R} \rightarrow \mathbb{R}$ homeomorphism

Then, holds:

· f monotonous

Demonstration:

no demonstration

Homeomorphisms and n-periodic points

Let:

$\cdot\ f\ :\ \mathbb{R} \to \mathbb{R}$ homeomorphism (M, T, ϕ) dynamical system defined by f

Then, holds:

$\cdot\ \forall\ n \in \mathbb{N}$:

$\nexists\ x \in M$ „ x n-periodic point

Demonstration:

graphically

Sarkovskii's theorem

Let:

- $f : I \to I$
- (M, T, ϕ) dynamical system

Then, holds:

- $\exists\, x \in M$:

  $o(x)$ k-period

- $\to \forall\, l \in \mathbb{N}$ „ $l > k$:

  $\exists\, x' \in M$:

  x' l-period

Characterization of sella-node bifurcation points

Let:

$\cdot\ I \subset \mathbb{R}\{f_\lambda\ :\ I \to I\}_{\lambda\in\Lambda}$

$\cdot\ x \in I$

Then, holds:

$\cdot\ x$ SN bifurcation point $\leftrightarrow f(x) = x, f'(x) = 1$

$\cdot\ \partial_\lambda f \neq 0$

$\cdot\ f''(x) \neq 0$

Demonstration:

demonstration

Pitchfork bifurcation

Let:

- (M, T, f_λ) dynamical system family
- $x \in M$

Then, x is pitchfork bifurcation if:

- x fixed point
- born of 2 fixed points
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Characterization of Pitchfork bifurcations

Let:

- (M, T, f_λ) dynamical system family
- $x \in M$

Then, holds:

- x Pitchfork $\leftrightarrow$
- $f(x) = x$
- $f'(x) = 1$
- $\partial_{x^2} f = 0$
- $\partial_\lambda f = 0$

Demonstration:

no demonstration