

Block I

Examples

1. Holomorphic functions

introduction

go

2. Holomorphic functions
Conjugation

Let:

$$\begin{aligned} \bar{a} : \mathbb{C} &\longrightarrow \mathbb{C} \\ z &\longmapsto \bar{z} \end{aligned}$$

Then, \bar{a} is not holomorphic :

$$u_x = 1$$

$$u_y = 0$$

$$v_x = 0$$

$$v_y = -1$$

$$\forall z \in \mathbb{C} :$$

$$-1 \neq 1 \rightarrow f \text{ not holomorphic in } z$$

Quadratic norm

Let:

$$\begin{aligned} f : \mathbb{C} &\longrightarrow \mathbb{C} \\ z &\longmapsto |z|^2 \end{aligned}$$

$\cdot f_{\mathbb{R}^2}$ component decomposition of f

Then, f is holomorphic in 0:

f differentiable in \mathbb{R}^2 polynomial

$\forall z \in \mathbb{C} :$

$$u_x(x, y) = 2x$$

$$u_y(x, y) = 2y$$

$$v_x(x, y) = 0$$

$$v_y(x, y) = 0$$

$$u_x = v_y \leftrightarrow x = 0$$

$$u_y = -v_x \leftrightarrow y = 0$$

f holomorphic function in $z \leftrightarrow z = 0$

Non preserving angles function

Let:

$$\cdot f(z) = z^2$$

Then, f is conform in $\mathbb{R} \setminus \{0\}$:

$$f(\{(x, 0) \in \mathbb{C} \mid x > 0\}) = \{(x, 0) \in \mathbb{C} \mid x > 0\}$$

$$f(\{(x, 0) \in \mathbb{C} \mid x < 0\}) = \{(x, 0) \in \mathbb{C} \mid x > 0\}$$

$$\text{ang}(A, B) = \pi \neq 0 = \text{ang}(f(A), f(B))$$

Exponential

Let:

$$\cdot a : 0$$

$$\cdot c_n : \frac{1}{n}$$

Then, $\sum_{n \geq 0} c_n (z - a)^n$ is convergent in D1 :

$$\lim_n \frac{|c_n|}{|c_{n+1}|} = \lim_n \frac{n+1}{n} = 1 \rightarrow R = 1$$

$\mathbb{CH} \rightarrow D(0, 1)$ convergent

$\mathbb{C} \setminus D(0, 1)$ divergent

$$f' = f$$

Geometric series

Let:

$$\cdot a : 0$$

$$\cdot c_n : 0$$

Then, $\sum z^n n[0]$ is convergent in \mathbb{D} :

$$R = \frac{c_n}{c_{n+1}} = 1 \quad \text{Then, holds:}$$

$$\cdot \sum z^n n[0] = \frac{1}{1-z}$$

$$\cdot \sum n z^{n-1} n[0] = \frac{1}{(1-z)^2}$$

$$\cdot \sum \frac{z^{n+1}}{n+1} n[0] = -\log(1-z)$$

Demonstration:

$$\forall z \in \mathbb{D} :$$

$$\sum z^n n[0] \text{ geometric series}$$

$$\sum z^n n[0] = \frac{1}{1-z}$$

II differentiating

III integrating

Series not centered in 0

Let:

$$\cdot a : i$$

$$\cdot c_n : \frac{n+1}{5^{n+1}}$$

Then, *item* is a/an entity :

$$\begin{aligned} & \sum \frac{n(z-i)^{n-1}}{5^n} n[1] \\ &= \frac{1}{5} \sum n \frac{z-i}{5} n^{n-1} n[1] = \frac{1}{5} \sum n u^{n-1} n[1] \end{aligned}$$

$$S(u) = \tilde{S}'(u)$$

$$\tilde{S}(u) = \frac{1}{5} \sum u^n n[1] = \frac{u}{5(1-u)}$$

$$S(u) = \frac{1}{5(1-u)^2}$$

$$S(z) = \frac{5}{(5+i-z)^2} \text{ over } D(i, 5)$$

Radius of convergence without quotient test

Let:

$$\cdot \sum \frac{(-1)^n}{n(n+1)} (z-2)^{n(n+1)} n[1]$$

Then, *R* is a/an entity :

$$\lim_{c_{n+1}} c_n \nexists$$

$$\lim_n \frac{1}{n(n+1)} \frac{1}{\frac{1}{n(n+1)}} = 1$$

ignore zeros