



**1. One-dimensional discrete dynamical systems**
**Dynamical system**

Let:

- $M$  manifold
- $T$  monoid
- $\phi : M \times T \rightarrow M$

Then,  $(M, T, \phi)$  is a dynamical system if:

- $\forall x \in X :$

$$\phi(x, 0) = x$$

$$\forall t_1, t_2 \in T :$$

$$\phi(\phi(x, t_1), t_2) = \phi(x, t_1 + t_2)$$

**Dimension**

Let:

- $(M, T, \phi)$  dynamical system

We name dimension of  $(M, T, \phi)$  to:

- $\dim(M)$

We denote:

- $\dim(M) = n : (M, T, \phi)$  n-D dynamical system

## Discrete & Continuous

Let:

·  $(M, T, \phi)$  dynamical system

Then,  $(M, T, \phi)$  is discrete if:

·  $T \simeq \mathbb{N}$

Then,  $(M, T, \phi)$  is continuous if:

·  $T \subset \mathbb{R}$

·  $T$  open

## Defined by a function

Let:

·  $(M, T, \phi)$  dynamical system

·  $f : M \rightarrow M$

Then,  $(M, T, \phi)$  is a dynamical system defined by  $f$  if:

·  $T = \mathbb{N}$

· 
$$\begin{array}{ccc} \phi : M \times \mathbb{N} & \longrightarrow & M \\ (x, n) & \longmapsto & f^n(x) \end{array}$$

We denote:

·  $(M, T, \phi)$  dynamical system defined by  $f : (M, \mathbb{N}, f)$

·  $f \in \mathcal{C}^n(M) : (M, \mathbb{N}, f)$   $\mathcal{C}^n$  dynamical system

**Orbit**

Let:

·  $(M, \mathbb{N}, f)$  functional dynamical system

·  $x \in M$

We name orbit of  $x$  to:

·  $\{f^n(x)\}_{n \in \mathbb{N}}$

We denote:

·  $o(x)$

## n-periodic point

Let:

- $(M, \mathbb{N}, f)$  functional dynamical system
- $x \in M$
- $n \in \mathbb{N}$

Then,  $x$  is a n-periodic point if:

- $f^n(x) = x$
- $\forall n' \in \mathbb{N} \quad n' < n :$   

$$f^{n'}(x) \neq x$$

We denote:

- $\{x \in M \mid f(x) = x\} : Fix(f)$

### Attractive & Repulsive

Let:

- $(M, \mathbb{N}, f)$  metrical dynamical system
- $x \in M$  m-periodic point

Then,  $x$  is attractive if:

- $\exists \mathcal{U} \in M :$

$\mathcal{U}$  open

$\forall x' \in \mathcal{U} :$

$\exists N \in \mathbb{N} :$

$\forall n \in \mathbb{N} \quad \text{,,} \quad n \geq N :$

$f^{nm}(x') \in \mathcal{U}$

Then,  $x$  is repulsive if:

- $\forall \mathcal{U} \subset M \quad \text{,,} \quad \mathcal{U} \text{ open} \quad \wedge \quad x \in \mathcal{U} :$

$\forall x' \in \mathcal{U} \quad \text{,,} \quad x' \neq x :$

$\exists N \in \mathbb{N} :$

$\forall n \in \mathbb{N} \quad \text{,,} \quad n \geq N :$

$f^{nm}(x') \notin \mathcal{U}$

**Fixed point character**

Let:

·  $(M, \mathbb{N}, f)$  functional dynamical system

We name Fixed point character to:

$$\begin{array}{lcl} f : Fix(f) & \longrightarrow & \{+, -\} \\ \cdot & & \\ & x \longmapsto & \begin{cases} + & x \text{ repulsive} \\ - & x \text{ attractive} \end{cases} \end{array}$$

We denote:

$$\cdot f : \chi$$

**Attraction set**

Let:

·  $(M, \mathbb{N}, f)$  dynamical system

·  $x \in M$  attractive m-periodic point

·  $o(x)$  orbit of  $x$

We name attraction set of  $x$  to:

$$\cdot \{y \in M \mid \exists x' \in o(x) : f^{nm}(y) \overset{n}{\longrightarrow} x'\}$$

We denote:

$$\cdot A(x)$$

**Neutral point**

Let:

·  $(M, \mathbb{N}, f)$  differentiable dynamical system

·  $x \in M$

Then,  $x$  is a neutral point if:

·  $f'(x) \in \{-1, 1\}$

**Feeble point**

Let:

·  $(M, \mathbb{N}, f)$   $\mathcal{C}^3$  dynamical system

·  $x \in M$  neutral point

Then,  $x$  is feeble point if:

·  $f''(x) = 0$



## Saddle point

Let:

$$\cdot \mathcal{U} \subset \mathbb{R}^n$$

$$\cdot f \in \mathcal{C}^1(\mathcal{U})$$

$$\cdot x \in \mathcal{U}$$

Then,  $x$  is a saddle point if:

$$\cdot f'(x) = 0$$

## Homeomorphism

Let:

$$\cdot (X_1, \tau_1), (X_2, \tau_2) \text{ topological spaces}$$

$$\cdot f : X_1 \rightarrow X_2$$

Then,  $f$  is a homeomorphism if:

$$\cdot f \text{ bijective}$$

$$\cdot f \in \mathcal{C}(X_1)$$

$$\cdot f^{-1} \in \mathcal{C}(X_2)$$

We denote:

$$\cdot \{ f : X_1 \rightarrow X_2 \mid f \text{ homeomorphism} \} : \text{Homeo}(X_1)$$

**Multiplier**

Let:

·  $(M, \mathbb{N}, f) \mathcal{C}^1$  dynamical system

·  $x \in M$

We name multiplier of  $x$  to:

·  $f'(x)$

**Logistic**

Let:

·  $f : \mathbb{R} \rightarrow \mathbb{R}$

·  $(M, T, \phi)$  dynamical system defined by  $f$

Then,  $(M, T, \phi)$  is a logistic dynamical system if:

·  $\exists a \in \mathbb{R} :$

$$\begin{array}{rcl} f : \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & ax(1-x) \end{array}$$

## Chaos

Let:

$$\cdot f : \mathbb{R} \rightarrow \mathbb{R}$$

$$\cdot (M, T, \phi) \text{ dynamical system defined by } f$$

Then,  $(M, T, \phi)$  is chaotic if:

$$\cdot \text{Fix}(f) \text{ dense in } \mathbb{R}$$

$$\cdot \exists x \in \mathbb{R} :$$

$$o(x) \text{ dense in } \mathbb{R}$$

$$\cdot f \text{ sensibility of } x_0$$

## Sarkovskii's order

We name Sarkovskii's order to:

$$\cdot a = 2^n a', b = 2^m b'$$

$$\cdot a < b \leftrightarrow \begin{cases} m < n & a' = b' = 1 \\ * & a' = 1, b' \neq 1 \\ a' < b' & a' = b' \neq 1 \\ n < m & 1 \neq a' \neq b' \end{cases}$$

We denote:

$$\cdot a < b : a < b$$

**Topologically equivalent**

Let:

·  $(M, \mathbb{N}, f)$  functional dynamical system

·  $(M, \mathbb{N}, f')$  functional dynamical system

Then,  $(M, \mathbb{N}, f)$  is topologically equivalent to  $(M, \mathbb{N}, f')$  if:

·  $Fix(f) = Fix(f')$

·  $\forall x \in Fix(f) :$

$$character_f(x) = character_{f'}(x)$$

We denote:

·  $(M, \mathbb{N}, f) \sim (M, \mathbb{N}, f')$

## Bifurcation point

Let:

·  $(M, \mathbb{N}, f_\lambda)_{\lambda \in \Lambda}$  functional dynamical system family

·  $\lambda' \in \Lambda$

Then,  $\lambda'$  is a bifurcation value if:

·  $\forall \varepsilon \in \mathbb{R}^+ :$

$\exists \lambda'' \in (\lambda' - \varepsilon, \lambda' + \varepsilon) :$

$(M, \mathbb{N}, f_{\lambda''})$  not topologically equivalent to  $(M, \mathbb{N}, f_{\lambda'})$

**Saddle-node bifurcation**

Let:

· *statements*

·

Then, *item* is a/an entity if:

· *conditions*

·

We denote:

· *property : notation*

·

**Pitchfork bifurcation**

Let:

- *statements*
- 

Then, *item* is a/an entity if:

- *conditions*
- 

We denote:

- *property : notation*
-

**Period doubling bifurcation**

Let:

· *statements*

.

Then, *item* is a/an entity if:

· *conditions*

.

We denote:

· *property : notation*

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