1. One-dimensional discrete dynamical system

introduction

Decreasing function orbits

Let:

· *declarations*

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Show that:

· *statements*

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Demonstration:

f corta en un punto

f decreasing $\to f^2$ increasing

$f^{2n} \xrightarrow{n}$ fixed point of f

10.

Let:

$\cdot\, f \,:\, \mathbb{R}^{+} \to \mathbb{R}^{+} \in \mathbb{C}^{\infty}$

$\cdot\, f(0) = 0$

$\cdot\, p \in \mathbb{R}^{+} \setminus \{0\} \;\text{„}\; f'(p) \geq 0$

$\cdot\, f'$ decreasing

Show that:

$\cdot \quad \forall\, p \in \mathbb{R}^{+} \setminus \{0\}$:

$$f^{n}(x) \xrightarrow{n} p$$

Demonstration:

f' decreasing $\to f'' < 0 \to f$ concave

f positive $\to f$ has no extrema $\to f' > 0 \to f$ increasing

f has only one fixed point

Suppose 2 fixed points : p, p'

$IVT \to \;\exists\, c \in (0, p')$:

$$f'(c) = 1$$

$f'(p) < 1 \to p$ attractive $IVT \to$ dont exist more fixed points $\to f'(c') = 1 \nleq 1$

$\forall\, x \in (0, p)$:

$$f(x) > x$$

$\forall\, x \in \mathbb{R} \;\text{„}\; x > p$:

$$f(x) < x$$

f increasing $\to f([0, p]) = [0, p]$