

1. New

UMV

Let:

- *statements*

-

Then, *item* is a/an entity if:

- *conditions*

-

We denote:

- *property : notation*

-

Efficient

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ parametric statistical model
- $X : \Omega \rightarrow \mathbb{R}$ random variable
- $\Theta \subset \mathbb{R}$ interval
- χ_F real estimator with integrable quadratic
-

Then, *item* is a/an entity if:

- $\forall \theta \in \Theta :$

$$\exists h : \mathbb{R} \rightarrow \mathbb{R} :$$

$$h \geq 0$$

$$h \text{ integrable}$$

$$\exists \mathcal{U} \subset \mathbb{R} :$$

$$\theta \in \mathcal{U}$$

$$|T(x)L(x, \theta)\theta| \leq h$$

Cramer-Rao's inequality

Let:

- same conditions of above
- $T \in \chi_F$

· regular model

$$\cdot E_{\theta}T = g(\theta)$$

Then, holds:

$$\cdot Var_{\theta}(T) \geq \frac{g'(\theta)^2}{I(\theta)}$$

Demonstration:

$$|E(\log(L(x, \theta))(T(x) - g(\theta))\theta| \leq \sqrt{E_{\theta}(\log(L(x, \theta))\theta)^2 E_{\theta}(T(x) - g(\theta))^2} = \sqrt{I(\theta)Var_{\theta}T}$$

$$E(\log(L(x, \theta))(T(x)g(\theta)\theta) = E_{\theta}(\log(L(x, \theta)T(x))\theta - g(\theta)E_{\theta}(\log(L(x, \theta))\theta) = E_{\theta}(\log(L(x, \theta)T(x))\theta$$

$$\log(L(x, \theta))\theta T(x)L(x, \theta)dx\Omega = \frac{1}{L(x, \theta)}L(x, \theta)\theta T(x)L(x, \theta)dx\Omega$$

$$|g'(\theta)| \leq \sqrt{I(\theta)Var_{\theta}(T)}$$

$$g'(\theta)^2 \leq I(\theta)Var_{\theta}(T)$$

Efficient

Let:

- mismas condiciones

Then, T is efficient if:

- $Var_{\theta}T = \frac{g'(\theta)^2}{I(\theta)}$

Efficient estimators are UMV

Let:

- *statements*
-

Then, holds:

- *then, holds*
-

Demonstration:

demonstration

Characterization of efficient estimators

Let:

- mismas condiciones

Then, holds:

- T efficient $\leftrightarrow \exists \lambda(\theta) :$

$$\lambda(\theta) \log(L(x, \theta)) = T(x) - g(\theta)P_\theta - q_s$$

Demonstration:

demonstration

si existe un estimador que da la igualdad entonces es UMV