1. Estimation

introduction

Statistic

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ m-D statistical model parametrized by Θ
- (M, Σ) measurable space
- $X : \Omega \to M$ random variable
- $f : M \to \mathbb{R}^m$

Then, f is a statistic if:

- $\forall\ B \in \mathbb{B}(\mathbb{R}^m)$:

  $f^{-1}(B) \in \Sigma$

We denote:

- $f : T$

Sufficiency

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ m-D statistical model parametrized by Θ
- (M, Σ) measurable space
- $X : \Omega \to M$ random variable
- $T : M \to \mathbb{R}^m$ statistic

Then, T is sufficient if:

- $\forall\ \theta_1, \theta_2 \in \Theta$:

  $\forall\ x \in M$:

  $\forall\ t \in \mathbb{R}^m$:

$$P_{\theta_1}(X = x \mid T \circ X = t) = P_{\theta_2}(X = x \mid T \circ X = t)$$

Estimator

Let:

· $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ

· $x_1, \ldots, x_n$ observation of $X = X_1, \ldots, X_n) : \tilde{\Omega} \to \Omega$

· $T \, : \, \Omega \to \mathbb{R}^k$ statistic

Then, T is an estimator if:

· T approaches θ

Loss function

Let:

· $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ

· $x_1, \ldots, x_n$ observation of $X = X_1, \ldots, X_n) : \tilde{\Omega} \to \Omega$

· $T \, : \, \Omega \to \mathbb{R}^k$ estimator

· $W \, : \, \mathbb{R}^k \times \Theta \to \mathbb{R}^+$

Then, W is a loss function if:

· $W(\theta, \theta) = 0$

Risk function

Let:

- $(\Omega, \mathcal{A}, \mathcal{P})$ statistical model parametrized by Θ
- $x_1, \ldots, x_n$ observation of $X = X_1, \ldots, X_n) : \tilde{\Omega} \to \Omega$
- $T \,:\, \Omega \to \mathbb{R}^k$ estimator
- $W \,:\, \mathbb{R}^k \times \Theta \to \mathbb{R}^+$ loose function

We name risk function to:

$$\cdot \begin{array}{rcl} R_T : \Theta & \longrightarrow & \mathbb{R}^+ \\ \theta & \longmapsto & E_\theta(W(T, \theta)) \end{array}$$