CMS NOTES de la SMC

FROM THE PRESIDENT'S DESK

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IN THIS ISSUE
DANS CE NUMÉRO

Renewing the Society: This month's issue contains a copy of the President's lengthy annual report. So this column will be short, and will focus on a single critical issue, that of bringing new members to the CMS.

The CMS's ability to carry out its broad portfolio of projects and services depends crucially upon the work and financial support of volunteers across the country (and abroad). To sustain these efforts, it is vital that we engage and recruit the cohort of young faculty and recent graduates that our universities and colleges have been actively hiring over the last few years.

If you are not a CMS member, I urge you to join the Society and to volunteer for some of our committees or projects. If you are already a member, I urge you to talk to your new (and old) colleagues about also joining. Please think of inviting a member of the CMS Exeutive Committee or Board of Directors to address a meeting of your department or organization about the work of the CMS. With that in mind here are, in order of importance:

Six Reasons I should be a CMS member:

1. *I benefit from the CMS's advocacy to government, industry and the media on behalf of mathematics.* For example, the CMS attempts to both increase the total funding available for mathematics, and to ensure that funding policies (e.g., those of NSERC) deal fairly with the needs of mathematics.

2. *I benefit from the infrastructure the CMS provides*. For example: our meetings, our journals, our electronic services. If your department advertises conferences or academic jobs in Canada, chances are you use our listserv cmath to do so. If not, you should.

3. *I want to support the educational projects the CMS takes on.* For example: Mathematics competitions, Math Camps, Math education fora. Through these projects mathematicians play an important role in improving the mathematical education of school age students, and in encouraging talented students to study mathematics further.

4. *I want to build a network of friends and future collaborators* by participating in CMS meetings and activities. Professional service, or the organization of sessions at meetings are a great way to meet interesting people. And they look good in a tenure file, too.

5. *The benefits of membership*. For example: the CMS Notes or meeting and publication discounts. New members can join for two years at half-price. CMS members get a discount on reciprocal membership in the *American Mathematical Society* or the *Mathematical Association of America*.

6. *My membership dues can be reimbursed from my NSERC grant.* This is a relatively new policy. While not in itself a justification for joining, at least it makes joining easier once you are convinced by the other reasons.

Valid as all these points are, they are not the real reason you should join the CMS. In fact, rather than asking what CMS membership can do for you, many of us view joining and supporting the CMS as a responsibility - something everyone should do who genuinely seeks to advance the cause of mathematics in Canada.

In advancing that cause, the CMS is committed to working with all components of our community and with our partners, such as the research institutes, MITACS, and provincial associations. We will continue to build bridges to our sister disciplines and their professional societies. Together we will succeed in ensuring a healthy future for Canadian mathematics.

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BOOK REVIEW Another View of Trigonometry

Robert Dawson, *St. Mary's University*

Divine Proportions: Rational Trigonometry to Universal Geometry

by N. J. Wildberger

Wild Egg, Sydney 2005 xx + 300 pp \$79.95 Aus

I have on my shelves a book entitled "The Elliptic Functions As They Should Be". The author, Albert Eagle, writing in the 1950's, spent almost five hundred pages redoing the theory of elliptic functions using what he believed to be the right notation. Just about every function was redefined to standardize the periods and renamed; even where functions were left unchanged the notation was often switched to the author's preferred form; $\log_{10}(x)$ became loc(x), the factorial sign preceded its argument, and $\pi/2$ ingeniously became τ. Sadly but unsurprisingly, this never caught on.

Others have attempted to get the human race to replace base 10 with base 12 in daily use, to make English spelling phonetic, or to get everybody to speak Esperanto. In each case, the proposal had merit in the abstract; but the weight of tradition predictably kept the balance tipped firmly in its original direction. The book under review has something of the same quixotic spirit.

It attempts nothing less than a radical reinvention of trigonometry and geometry. The concepts of length and angle measure are replaced respectively by *quadrance* (length squared) and *spread* (the square of the sine of the angle). In addition to the spread, we have also the *cross* (which is 1 minus the spread) and the *twist*, the ratio of the spread to the cross. The significance of the initial letters will become evident on a moment's thought!

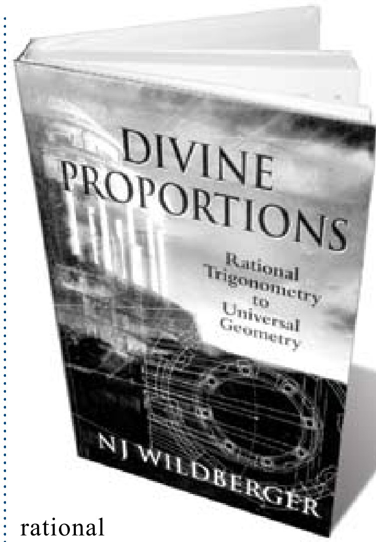
The author, a Canadian with a doctorate from Yale, now teaching in Australia, makes a reasonable case that many trigonometric calculations are simplified by this novel choice of variables. Pythagoras' Theorem is obviously simpler in this form; the sine law is replaced by the "spread law"

$$\frac{s_1}{Q_1} = \frac{s_2}{Q_2} = \frac{s_3}{Q_3};$$

and so on. At one level this is simply the strategy, often taught to calculus students and problem solving contestants, of "working with the squares" to avoid the use of square roots. A triangle with rational vertices will have edges with rational quadrance and angles with rational spread, cross, and twist. This is certainly a good thing; we are reminded of the convenience, in some circumstances, of working with the slope rather than with the angle in civil engineering, construction, plumbing, etc., although Wildberger's proposal goes much deeper.

We may also detect a similarity to the use of variance in statistics. On the one hand, standard deviation is a more concrete measure of how much a population fails to be concentrated at one point; in particular, it is dimensionally equivalent to the original data (and to any reasonable location statistic). On the other hand, for theoretical purposes, it is much easier to work with variance, which is additive when the sources are independent. The whole idea of ANOVA is based on this idea that variance can be broken up into different components, adding up to the whole, and attributable to different sources.

Another parallel situation arises in classical mechanics. The



moment of inertia of a body about an arbitrary point can be expressed as the sum of its moment of inertia about its own center of gravity, and the moment of inertia of a point mass at the center of gravity. Statisticians and physicists use some of the same algebraic tricks to simplify their calculations. Wildberger's approach would seem to allow the same bag of tricks to be applied to geometry. (It could be interesting to see rational trigonometry applied to quantum mechanics or special relativity!)

For certain very specialized sorts of numerical computation, rational trigonometry might be enormously simpler. In particular, programmers wishing to use exact arithmetic may find that being able to stay within the rational numbers rather than having to use an unwieldy tree-based data type representing constructible numbers justifies complications elsewhere. Wildberger also points out that rational trigonometry generalizes directly and transparently to geometries over finite fields; I think this is an exciting idea. In such cases, it is possible that the explicit use of quadrance and spread might actually be worthwhile.

However, in other situations it would seem that the cost is higher and the benefit lower. For instance, rational trigonometry seriously obscures the link between trigonometry and periodic functions. To work with electronics, differential equations, complex numbers, and many other things we absolutely require sinusoidal functions; spread and quadrance are not an adequate substitute. As we cannot then avoid introducing the tools of classical trigonometry, even at the high school level, the advantages of introducing a second set of foundations as well seem dubious. Finally, not every irrational number that arises in trigonometry can be avoided in this way; the trisection of the angle is a case in point.

From the point of view of classical Euclidean geometry, rational trigonometry at first seems positively obstructive. The fact that distance is additive along lines is crucially important to Euclidean geometry, as is the fact that angle is additive around an arc. The concepts of translational, reflectional, and rotational symmetry all spring from these facts; and it would seem at first glance that nothing could justify obscuring this additivity. However, I was surprised how readily the author manages to achieve a wide range of classical results, including the eyeball theorem, the theorems of Menelaus and Ceva, and results on the Euler line and nine-point circle. These proofs are similar in complexity to those in standard texts - and in some cases a little easier, though where there is a real advantage to working with the square of the length standard texts often do so too. Moreover, as mentioned above, they generalize in a transparent way to geometries over finite fields. The discussion of constructibility of regular polygons in finite fields, in chapter 14, is particularly nice.

Why was this book written? The first few chapters give a hint. The author claims a strong philosophical aversion to the fields of real numbers and what he unfortunately terms "decimal numbers". These (page 23) he apparently identifies with the computable real numbers - although (page 22) he claims the field to be complete and not countable. These views will not, as the author admits, be shared by many; nor do they force the approach that he has chosen, as exact arithmetic with algebraic numbers may be carried out formally and rigorously under more or less any axiom system. Fortunately, the rest of the book stands independently of these shaky foundations (as mathematics often does).

For the most part this book is clear and logical; one could almost imagine it as a standard undergraduate textbook from an alternate universe in which mathematical conventions had developed differently. Reading it gives an interesting illustration of how mathematics is, and is not, shaped by mathematicians (or "social construction" if you prefer). More practically, I would recommend the book as serious reading for anybody who teaches classical or (especially) finite geometry. Computational geometers may find the author's approach extremely useful for certain purposes. But, ultimately, I do not expect rational trigonometry to replace the more usual approach.

NEWS FROM THE FIELDS INSTITUTE

May 11-12, 2007, "Workshop on Global Optimization: Methods and Applications", Fields Institute
www.fields.utoronto.ca/programs/scientific/06-07/globalopt/

May 25-26, 2007, "Ottawa-Carleton Discrete Mathematics Workshop", Carleton University
www.fields.utoronto.ca/programs/scientific/06-07/discrete_math/

May 28-June 1, 2007, "Seventh Canadian Summer School on Quantum Information", University of Waterloo
http://www.iqc.ca/quantumworld/index.php?id=2&pid=15

May 28-31, 2007, "Canadian Discrete and Algorithmic Mathematics Conference (CANADAM 2007)", Banff Conference Center, Alberta
http://www.cs.ualberta.ca/%7Emreza/CANADAM/

May 31-June 2, 2007, "Lattices and Trajectories: A Symposium of Mathematical Chemistry in honour of Ray Kapral and Stu Whittington", Fields Institute
www.fields.utoronto.ca/programs/scientific/06-07/lattices/

June 1-3, 2007, "16th International Workshop on Matrices and Statistics", University of Windsor
http://www.uwindsor.ca/iwms

June 5-8, 2007, "Probability and Stochastic Processes Symposium in Honour of Donald A. Dawson's work", Carleton University
www.fields.utoronto.ca/programs/scientific/06-07/stochastic/

June 5-9, 2007, "35th Canadian Operator Symposium(COSy)", University of Guelph
www.fields.utoronto.ca/programs/scientific/06-07/COSy/

June 18-23, 2007, "Conference on Combinatorics and Optimization", University of Waterloo
www.fields.utoronto.ca/programs/scientific/06-07/CO40/

June 27-29, 2007, "Randomization of Quantum Systems Workshop Institute for Quantum Computing", University of Waterloo
www.iqc.ca/quantumworld/index.php?id=4

July 7, 2007, "Future Directions of Computational and Mathematical Neuroscience", Fields Institute
www.fields.utoronto.ca/programs/scientific/07-08/neuroscience/

July 16 - 20, 2007, "Workshop on Noncommutative Dynamics and Applications", Fields Institute
www.fields.utoronto.ca/programs/scientific/07-08/operator_algebras/

July 18-21, 2007, "CUMC 2007 Canadian Undergraduate Mathematics Conference", Simon Fraser University
http://cumc.math.ca/2007/en/

July 25-28, 2007, "Symbolic-Numeric Computation (SNC'07) and Parallel Symbolic Computation '07 (PASCO '07)", University of Western Ontario
www.orcca.on.ca/conferences/pasco2007/site/
www.orcca.on.ca/conferences/snc2007/site/

July 27, 2007, "Brain Biomechanics: Mathematical Modelling of Hydrocephalus and Syringomyelia Centre for Mathematical Medicine at the Fields Institute"
www.fields.utoronto.ca/programs/scientific/CMM/07-08/biomechanics/

July 29-August 1, 2007, "International Symposium on Symbolic and Algebraic Computation (ISSAC2007)", University of Waterloo
http://www.cs.uwaterloo.ca/%7Eissac07/

August 12-16, 2007, "2nd International Conference on Continuous Optimization ICCOPT - MOPTA07", McMaster University
http://iccopt-mopta.mcmaster.ca/

August 13-17, 2007, "6th International Conference on Unconventional Computation", Queen's University
www.cs.queensu.ca/uc07/

August 13-24, 2007, "Summer School on Operator Algebras", University of Ottawa
www.fields.utoronto.ca/programs/scientific/07-08/opalg_school/

August 27-29, 2007, "Automata 2007, 13th International Workshop on Cellular Automata", Fields Institute
www.fields.utoronto.ca/programs/scientific/07-08/automata07/

September 4-7, 2007, "Data Assimilation Workshop", Fields Institute
www.fields.utoronto.ca/programs/scientific/07-08/data_assim/

September 17 - 21, 2007, "Workshop on Free Probability, Random Matrices, and Planar Algebras", Fields Institute
www.fields.utoronto.ca/programs/scientific/07-08/operator_algebras/free/

September 22-24, 2007, "Geometrization of Probability Workshop", University of Ottawa
www.fields.utoronto.ca/programs/scientific/07-08/geometrization/

October 29 - November 2, 2007, "Workshop on von Neumann Algebras"
www.fields.utoronto.ca/programs/scientific/07-08/operator_algebras/

November 9-10, 2007, "Conference in Honour of the 60th birthday of Professor Andreas R. Blass", Fields Institute
www.fields.utoronto.ca/programs/scientific/07-08/blassconference/

November 12- 16, 2007, "Workshop on Structure of C*-Algebras"
www.fields.utoronto.ca/programs/scientific/07-08/operator_algebras/

December 11 - 15, 2007, "Workshop on Operator Spaces and Quantum Groups"
www.fields.utoronto.ca/programs/scientific/07-08/operator_algebras/