Convergence_Poisson_FV5_SQUARE_flat_triangles

October 21, 2018

1 FV5 scheme for Poisson equation on long triangles meshes

1.1 The Poisson problem on the square

We consider the following Poisson problem with Dirichlet boundary conditions

$$\left\{\begin{array}{c} -\Delta u = f \text{ on } \Omega \\ u = 0 \text{ on } \partial\Omega \end{array}\right.$$

on the square domain $\Omega = [0,1] \times [0,1]$ with

$$f = 2\pi^2 sin(\pi x) sin(\pi y).$$

The unique solution of the problem is

$$u = sin(\pi x) sin(\pi y).$$

The Poisson equation is a particular case of the diffusion problem

$$-\nabla \cdot (K \vec{\nabla} u) = f$$

and the associated diffusion flux is

$$F(u) = K \nabla u.$$

1.2 The FV5 scheme for the Laplace equation

The domain Ω is decomposed into cells C_i.

$|C_i|$ is the measure of the cell C_i.

f_{ij} is the interface between two cells C_i and C_j.

s_{ij} is the measure of the interface f_{ij}.

d_{ij} is the distance between the centers of mass of the two cells C_i and C_j.

The discrete Poisson problem is

$$-\frac{1}{|C_i|} \sum s_{ij} F_{ij} = f_i,$$

where u_i is the approximation of u in the cell C_i,

f_i is the approximation of f in the cell C_i,

F_{ij} is a numerical approximation of the outward normal diffusion flux from cell i to cell j.

In the case of the scheme FV5, we use the formula

$$F_{ij} = \frac{u_j - u_i}{d_{ij}}.$$

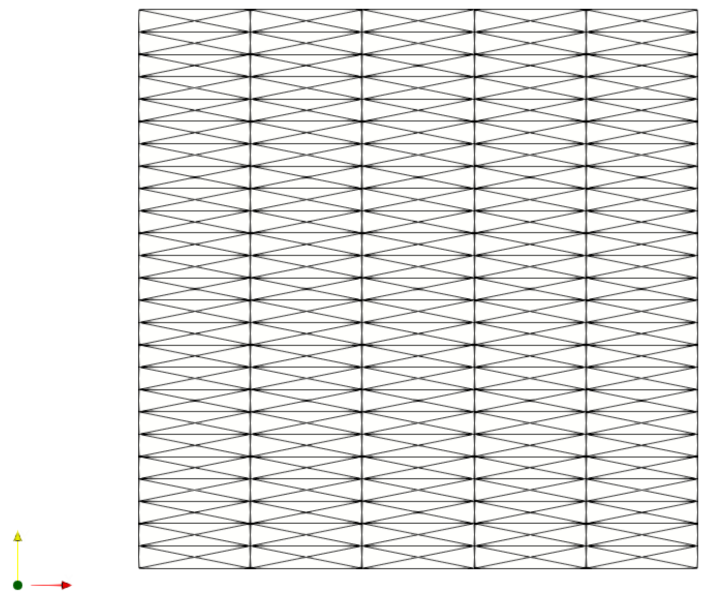
1.3 The script

```
#Discrétisation du second membre et extraction du nb max de voisins d'une cellule
#================================================================================
my_RHSfield = cdmath.Field("RHS_field", cdmath.CELLS, my_mesh, 1)
maxNbNeighbours=0#This is to determine the number of non zero coefficients in the sparse finite

for i in range(nbCells):
    Ci = my_mesh.getCell(i)
    x = Ci.x()
    y = Ci.y()

    my_RHSfield[i]=2*pi*pi*sin(pi*x)*sin(pi*y)#mettre la fonction definie au second membre de l
    # compute maximum number of neighbours
    maxNbNeighbours= max(1+Ci.getNumberOfFaces(),maxNbNeighbours)

# Construction de la matrice et du vecteur second membre du système linéaire
#===========================================================================
Rigidite=cdmath.SparseMatrixPetsc(nbCells,nbCells,maxNbNeighbours)# warning : third argument is
RHS=cdmath.Vector(nbCells)
#Parcours des cellules du domaine
for i in range(nbCells):
    RHS[i]=my_RHSfield[i] #la valeur moyenne du second membre f dans la cellule i
    Ci=my_mesh.getCell(i)
    for j in range(Ci.getNumberOfFaces()):# parcours des faces voisinnes
        Fj=my_mesh.getFace(Ci.getFaceId(j))
        if not Fj.isBorder():
            k=Fj.getCellId(0)
            if k==i :
                k=Fj.getCellId(1)
            Ck=my_mesh.getCell(k)
            distance=Ci.getBarryCenter().distance(Ck.getBarryCenter())
            coeff=Fj.getMeasure()/Ci.getMeasure()/distance
            Rigidite.setValue(i,k,-coeff) # terme extradiagonal
        else:
            coeff=Fj.getMeasure()/Ci.getMeasure()/Ci.getBarryCenter().distance(Fj.getBarryCenter
        Rigidite.addValue(i,i,coeff) # terme diagonal


# Résolution du système linéaire
#=================================
LS=cdmath.LinearSolver(Rigidite,RHS,500,1.E-6,"GMRES","ILU")
SolSyst=LS.solve()

# Automatic postprocessing :  save 2D picture and plot diagonal data
#===========================
diag_data=VTK_routines.Extract_field_data_over_line_to_numpyArray(my_ResultField,[0,1,0],[1,0,0]
plt.legend()
```

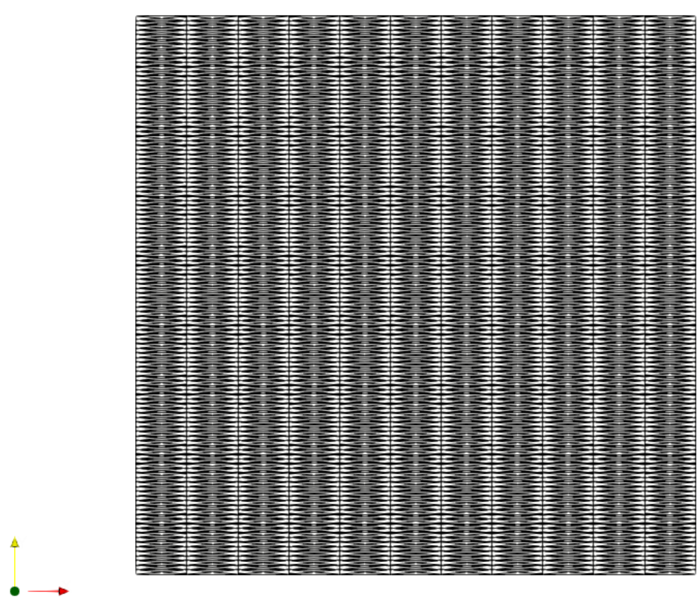
```
plt.xlabel('Position on diagonal line')
plt.ylabel('Value on diagonal line')
if len(sys.argv) >1 :
    plt.title('Plot over diagonal line for finite Volumes \n for Laplace operator on a 2D square
    plt.plot(curv_abs, diag_data, label= str(nbCells)+ ' cells mesh')
    plt.savefig("FiniteVolumes2D_square_ResultField_"+str(nbCells)+ '_cells'+"_PlotOverDiagonalL
```

1.4 Numerical results

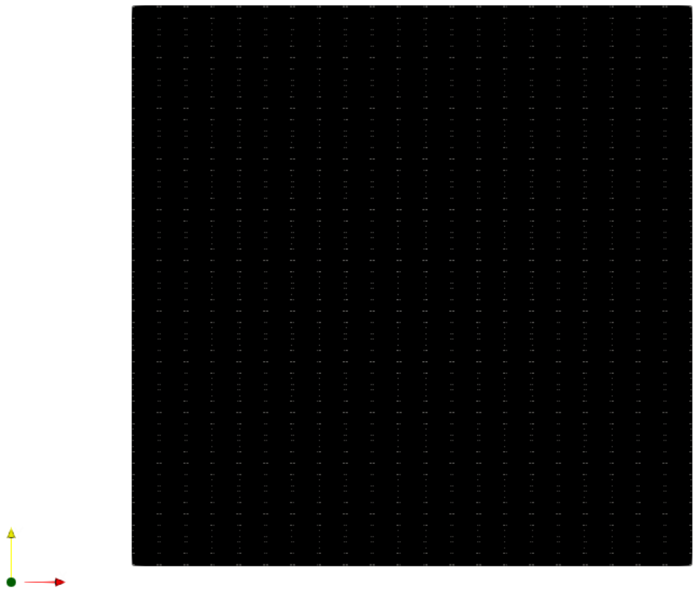
1.4.1 Cartesian meshes



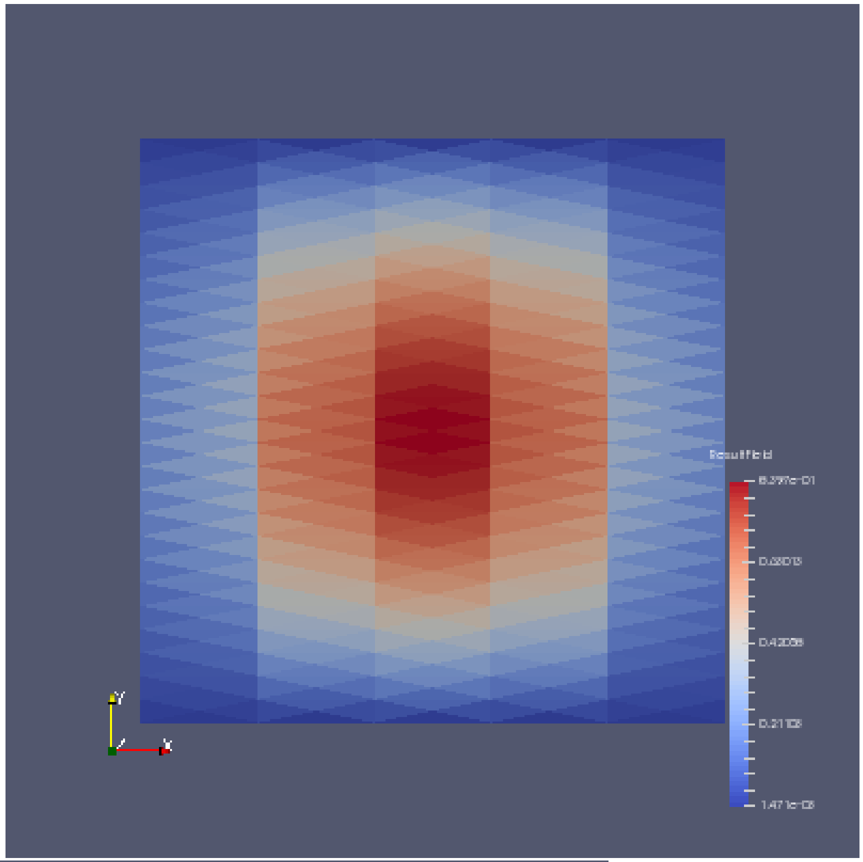
mesh 1 | mesh 2 | mesh 3 - | - - | -



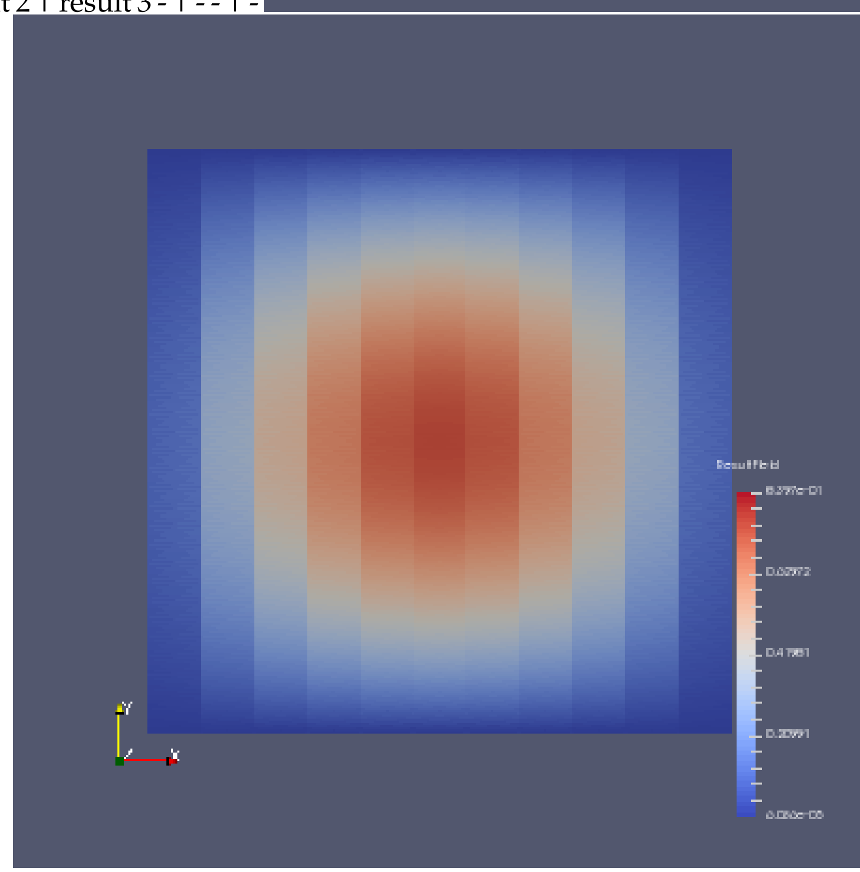
| |

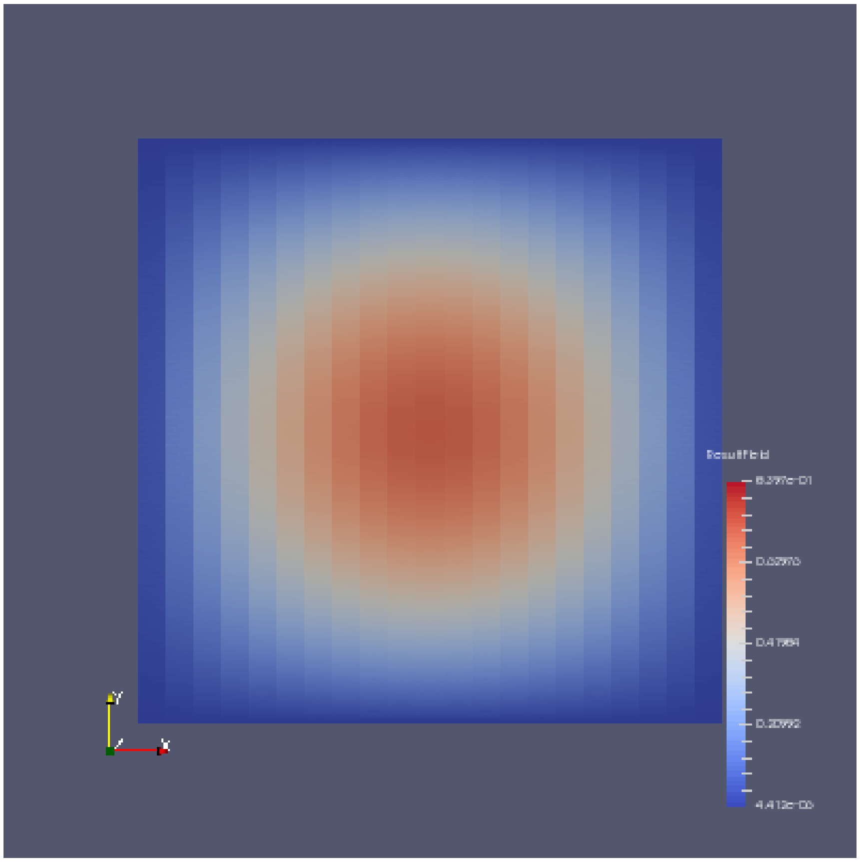


1.4.2 Numerical results

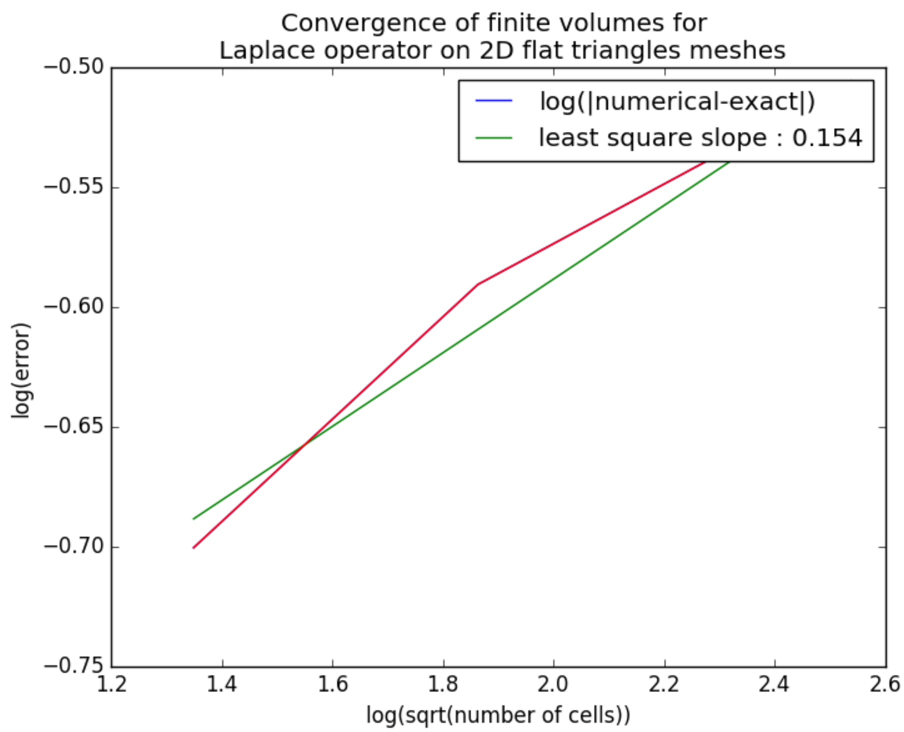


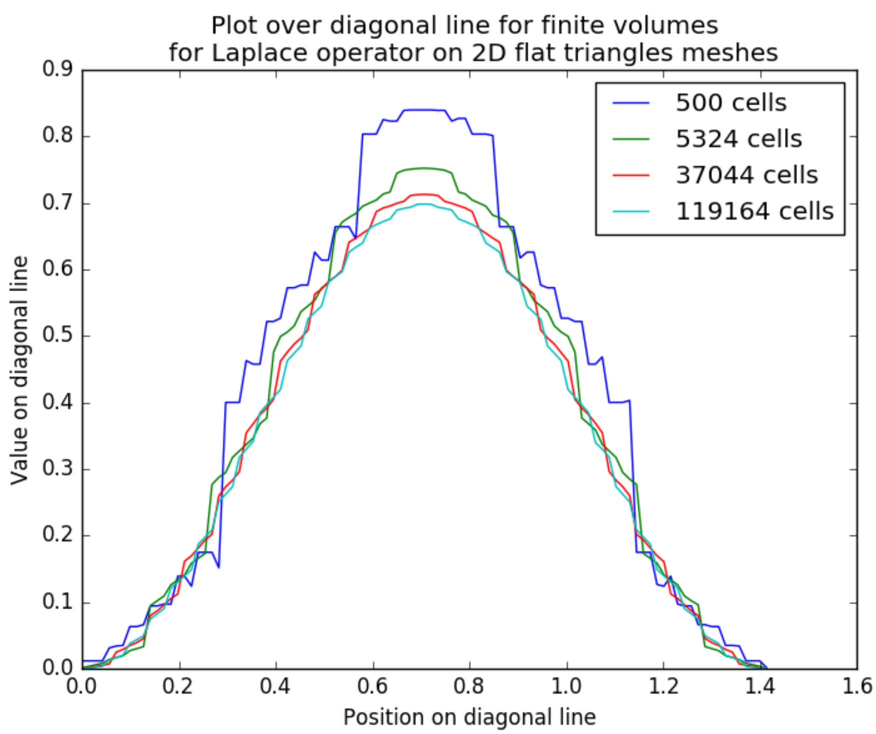
result 1 \| result 2 \| result 3 - \| - - \| -





1.5 Convergence


Convergence of finite volumes for
Laplace operator on 2D flat triangles meshes
log(|numerical-exact|)
least square slope : 0.154
log(error)
log(sqrt(number of cells))
−0.50
−0.55
−0.60
−0.65
−0.70
−0.75
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6


Plot over diagonal line for finite volumes
for Laplace operator on 2D flat triangles meshes
500 cells
5324 cells
37044 cells
119164 cells
Value on diagonal line
Position on diagonal line
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6