

Convergence_Poisson_FV5_SQUARE_triangles

October 21, 2018

1 FV5 scheme for Poisson equation on unstructured triangular meshes

1.1 The Poisson problem on the square

We consider the following Poisson problem with Dirichlet boundary conditions

$$\begin{cases} -\Delta u = f \text{ on } \Omega \\ u = 0 \text{ on } \partial\Omega \end{cases}$$

on the square domain $\Omega = [0, 1] \times [0, 1]$ with

$$f = 2\pi^2 \sin(\pi x) \sin(\pi y).$$

The unique solution of the problem is

$$u = \sin(\pi x) \sin(\pi y).$$

The Poisson equation is a particular case of the diffusion problem

$$-\nabla \cdot (K \vec{\nabla} u) = f$$

and the associated diffusion flux is

$$F(u) = K \nabla u.$$

We are in the particular case where $K = 1$.

1.2 The FV5 scheme for the Laplace equation

The domain Ω is decomposed into cells C_i .

$|C_i|$ is the measure of the cell C_i .

f_{ij} is the interface between two cells C_i and C_j .

s_{ij} is the measure of the interface f_{ij} .

d_{ij} is the distance between the centers of mass of the two cells C_i and C_j .

The discrete Poisson problem is

$$-\frac{1}{|C_i|} \sum s_{ij} F_{ij} = f_i,$$

where u_i is the approximation of u in the cell C_i ,

f_i is the approximation of f in the cell C_i ,

F_{ij} is a numerical approximation of the outward normal diffusion flux from cell i to cell j .
In the case of the scheme FV5, we use the formula

$$F_{ij} = \frac{u_j - u_i}{d_{ij}}.$$

1.3 The script

```
#Discrétisation du second membre et extraction du nb max de voisins d'une cellule
#=====
my_RHSfield = cdmath.Field("RHS_field", cdmath.CELLS, my_mesh, 1)
maxNbNeighbours=0#This is to determine the number of non zero coefficients in the sparse finite

for i in range(nbCells):
    Ci = my_mesh.getCell(i)
    x = Ci.x()
    y = Ci.y()

    my_RHSfield[i]=2*pi*pi*sin(pi*x)*sin(pi*y)#mettre la fonction definie au second membre de l
    # compute maximum number of neighbours
    maxNbNeighbours= max(1+Ci.getNumberOfFaces(),maxNbNeighbours)

# Construction de la matrice et du vecteur second membre du système linéaire
#=====
Rigidite=cdmath.SparseMatrixPetsc(nbCells,nbCells,maxNbNeighbours)# warning : third argument is
RHS=cdmath.Vector(nbCells)
#Parcours des cellules du domaine
for i in range(nbCells):
    RHS[i]=my_RHSfield[i] #la valeur moyenne du second membre f dans la cellule i
    Ci=my_mesh.getCell(i)
    for j in range(Ci.getNumberOfFaces()):# parcours des faces voisines
        Fj=my_mesh.getFace(Ci.getFaceId(j))
        if not Fj.isBorder():
            k=Fj.getCellId(0)
            if k==i :
                k=Fj.getCellId(1)
            Ck=my_mesh.getCell(k)
            distance=Ci.getBarryCenter().distance(Ck.getBarryCenter())
            coeff=Fj.getMeasure()/Ci.getMeasure()/distance
            Rigidite.setValue(i,k,-coeff) # terme extradiagonal
        else:
            coeff=Fj.getMeasure()/Ci.getMeasure()/Ci.getBarryCenter().distance(Fj.getBarryCenter())
            Rigidite.addValue(i,i,coeff) # terme diagonal

# Résolution du système linéaire
#=====
LS=cdmath.LinearSolver(Rigidite,RHS,500,1.E-6,"GMRES","ILU")
```

```

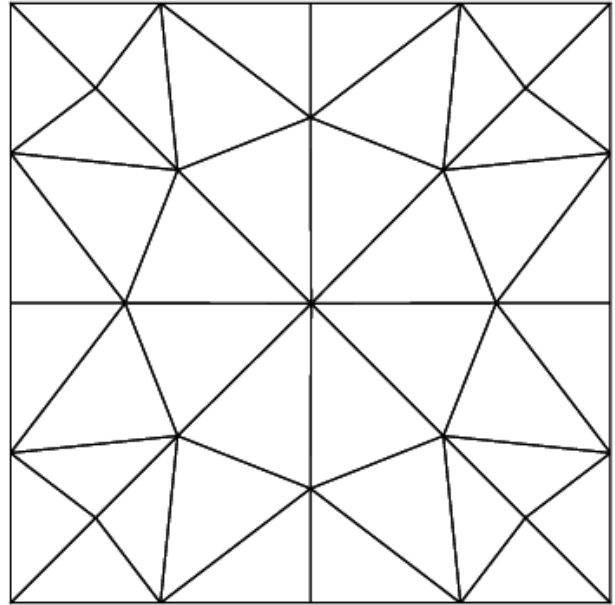
SolSyst=LS.solve()

# Automatic postprocessing : save 2D picture and plot diagonal data
#=====
diag_data=VTK_routines.Extract_field_data_over_line_to_numpyArray(my_ResultField,[0,1,0],[1,0,0])
plt.legend()
plt.xlabel('Position on diagonal line')
plt.ylabel('Value on diagonal line')
if len(sys.argv) >1 :
    plt.title('Plot over diagonal line for finite Volumes \n for Laplace operator on a 2D square')
    plt.plot(curv_abs, diag_data, label= str(nbCells)+ ' cells mesh')
    plt.savefig("FiniteVolumes2D_square_ResultField_"+str(nbCells)+ '_cells'+ "_PlotOverDiagonalL

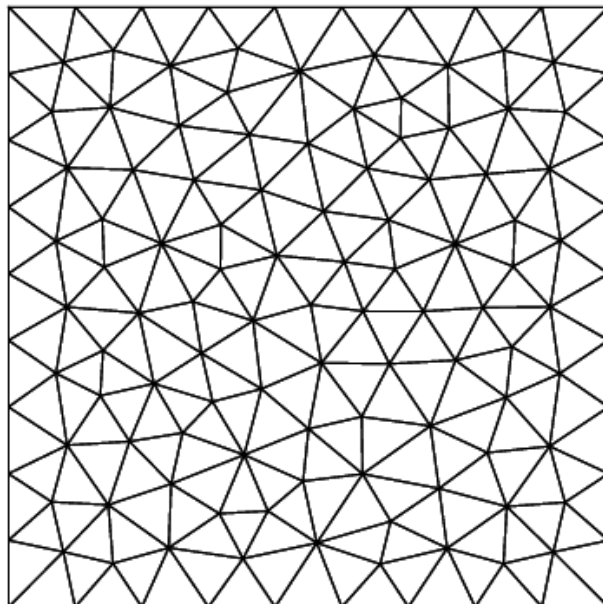
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1.4 Numerical results on triangular meshes

1.4.1 Meshes

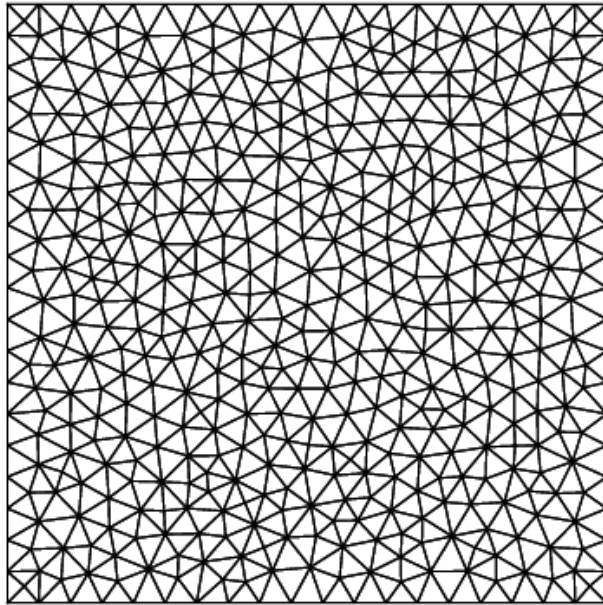


mesh 1 | mesh 2 | mesh 3 - | - - | -

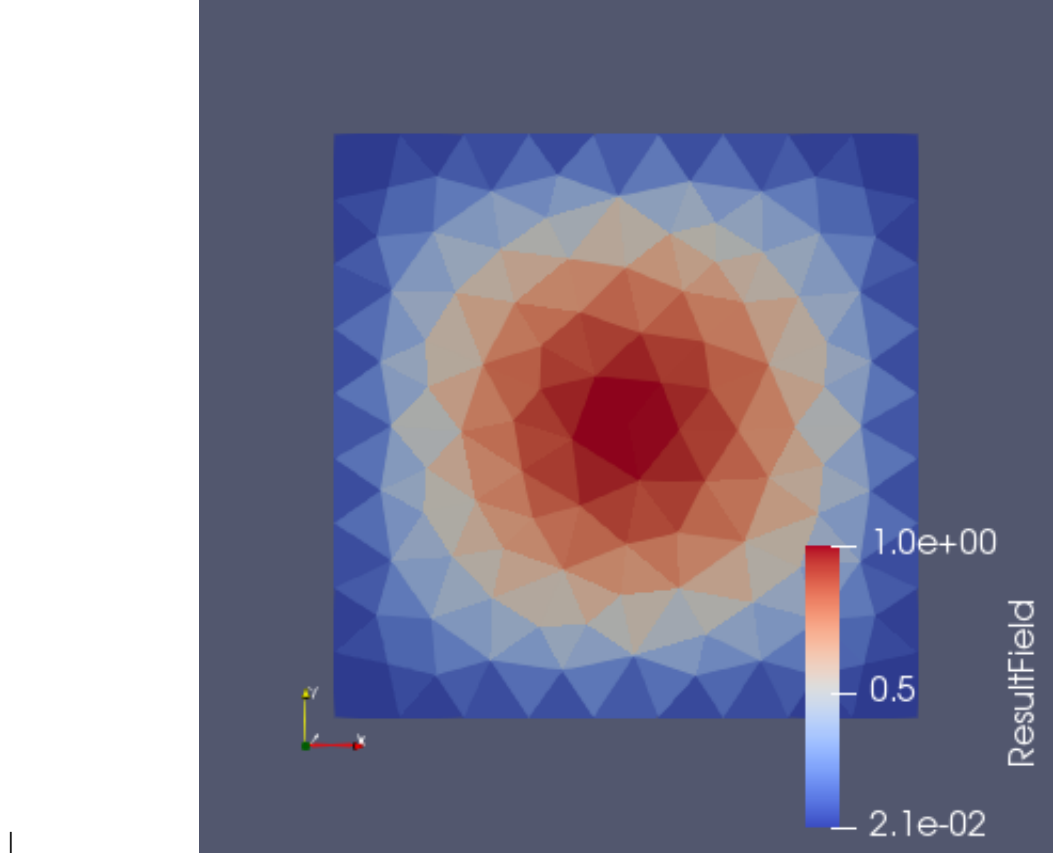
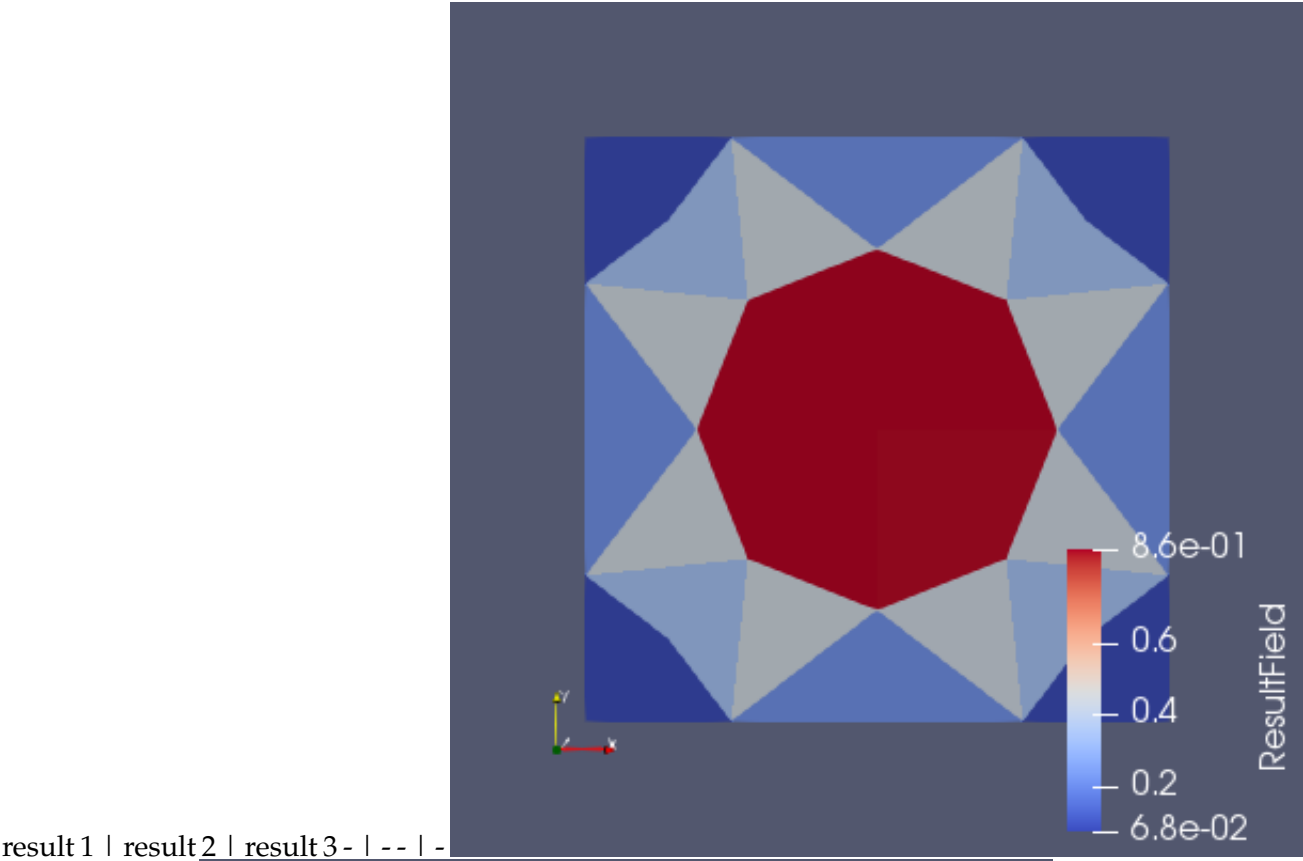


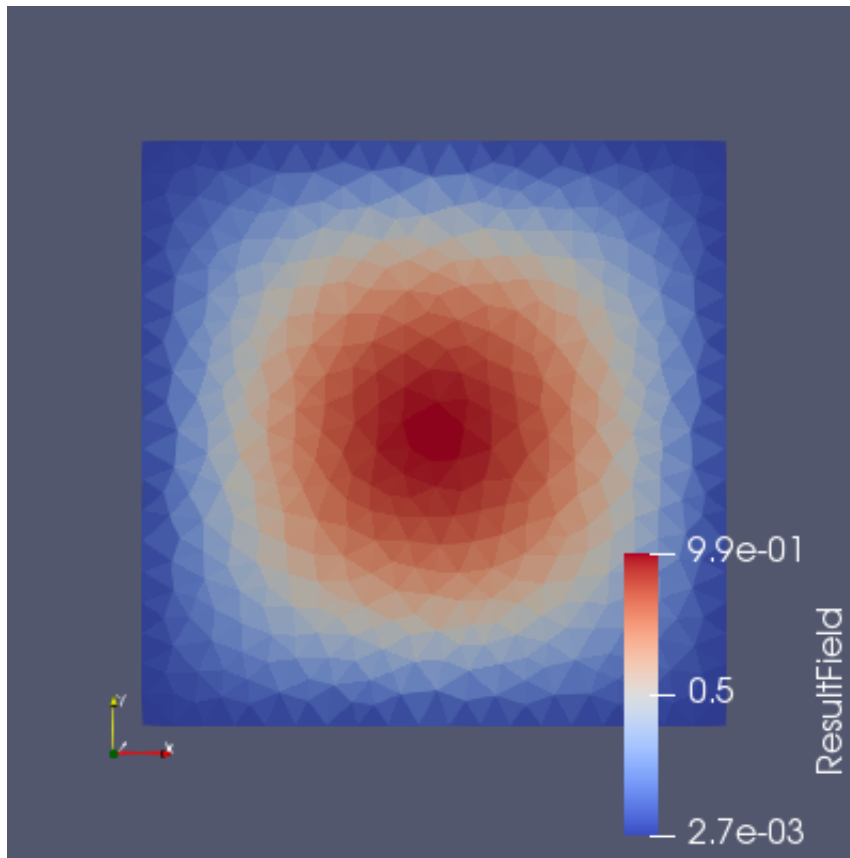
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1.5 Numerical results





1.6 Convergence

