BurgersEquation1D

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1 Finite volume schemes for the Burgers' equation

1.1 Conservative form of Burgers' equation

The velocity u in an incompressible 1D flow with constant pressure is given by

$$\partial_t u + \partial_x \frac{u^2}{2} = 0$$

1.1.1 Conservative discretization

The equation is discretised on a regular mesh with time step Δt and space step Δx.
The exact solution u is approximated by a piecewise constant function u_h with values u_i in each cell

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{u_{i+1/2}^2 - u_{i-1/2}^2}{2\Delta x}$$

The interfacial velocities $u_{i+1/2} \approx u_{|i+1/2}$ can be computed using for instance Godunov scheme

1.2 Non conservative form of Burgers' equation

For regular solutions, the Burgers' equation becomes

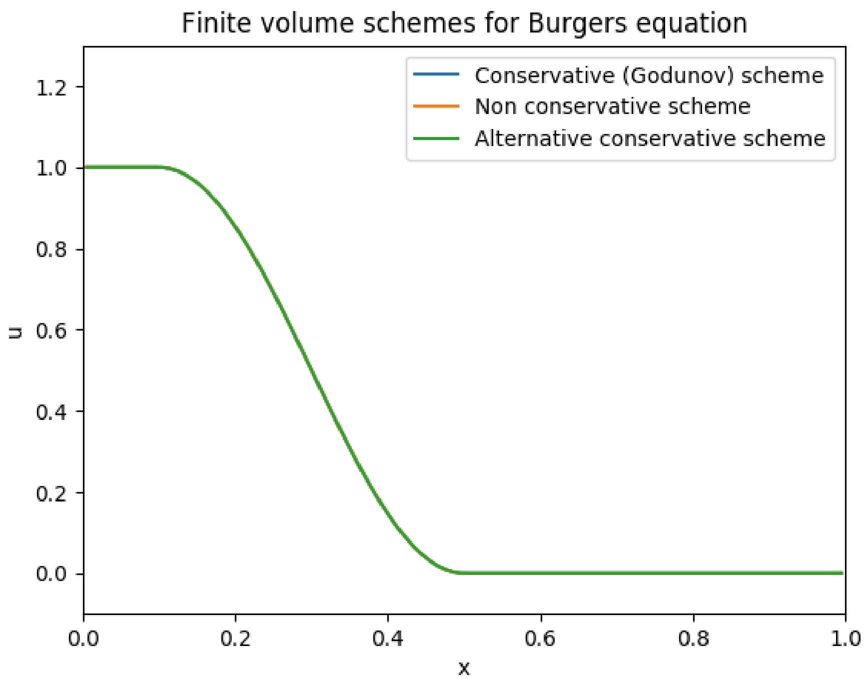
$$\partial_t u + u \partial_x u = 0$$

1.2.1 Non conservative discretization

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + u_i^n \frac{u_{i+1/2} - u_{i-1/2}}{\Delta x}$$

The interfacial velocities $u_{i+1/2} \approx u_{|i+1/2}$ can be computed using for instance the upwind principle :

$$u_{i+1/2} = \begin{cases} u_i & \text{if} \quad u_i^n < 0 \\ u_{i+1} & \text{if} \quad u_i^n \geq 0 \end{cases}$$



“Initial data”

1.3 Alternative conservative form of Burgers’ equation

Alternatively, a regular solution u of the Burgers’ equation is also a solution of the following conservative equation

$$\partial_t u^2 + \partial_x \frac{u^3}{3} = 0$$

1.3.1 Alternative conservative discretization

$$\frac{(u_i^2)^{n+1} - (u_i^2)^n}{\Delta t} + \frac{u_{i+1/2}^3 - u_{i-1/2}^3}{3\Delta x}.$$

The interfacial velocities $u_{i+1/2}$ can be computed using for instance Godunov scheme

1.4 The initial data

1.5 Numerical results

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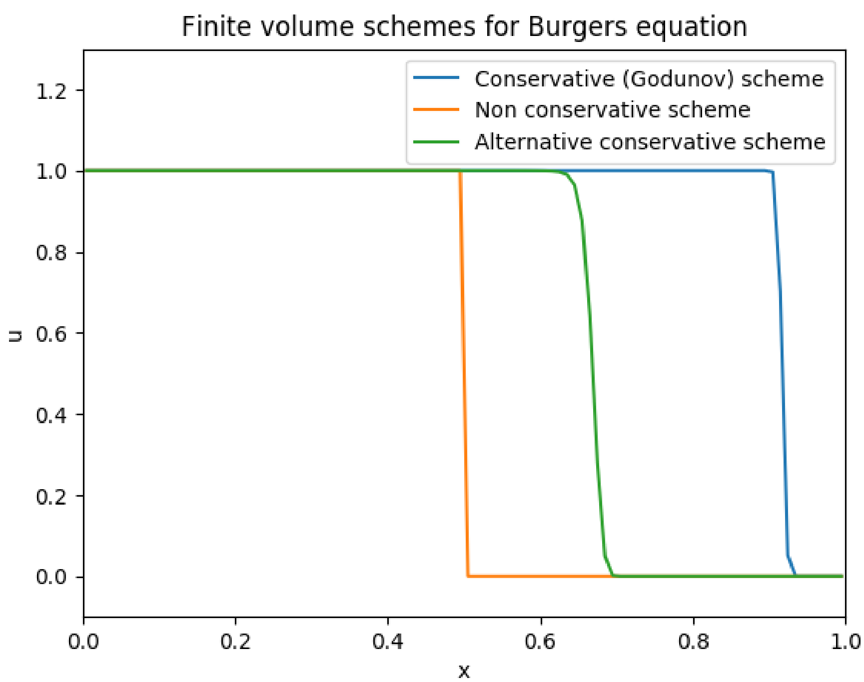
Finite volume schemes for Burgers equation
Conservative (Godunov) scheme
Non conservative scheme
Alternative conservative scheme
u
x
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

“Final data”