

Sensitivity analysis of populations of atrial cell models uncovers different mechanisms of APD alternans behavior in normal Sinus Rhythm and Atrial Fibrillation

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Workshop on Mathematical Methods in Cardiac Electrophysiology

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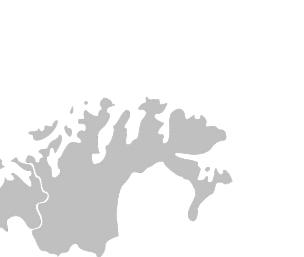
simula



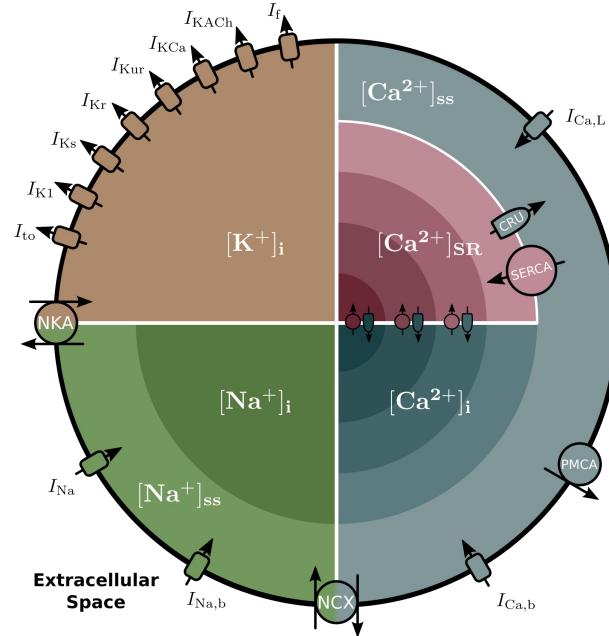
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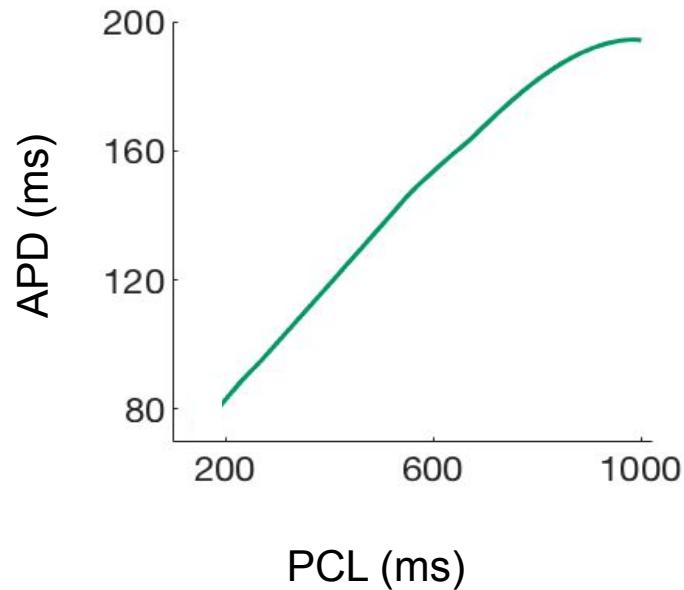


The Koivumäki model of the human atrial cell showed APD alternans under dynamic pacing



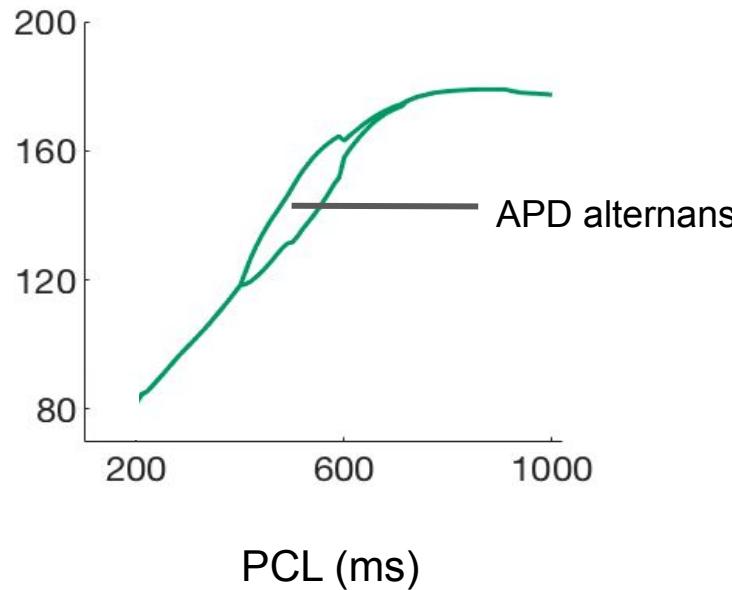
- Spatial model with detailed calcium handling system
- Human ionic currents

APD restitution can be used to characterize APD alternans



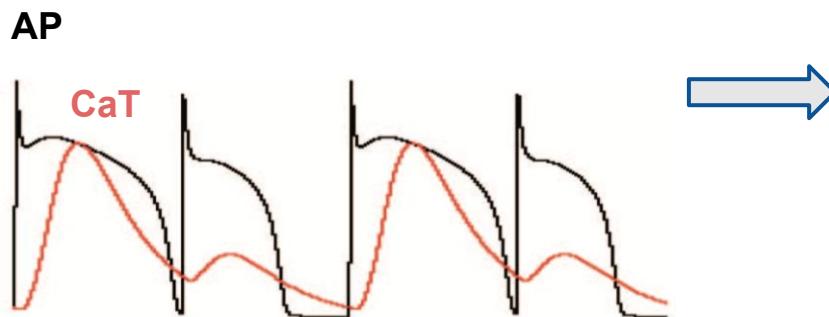
PCL: pacing cycle length

APD: Action potential duration



Cardiac alternans are correlated to AF episodes in patients and could serve as a marker for arrhythmia

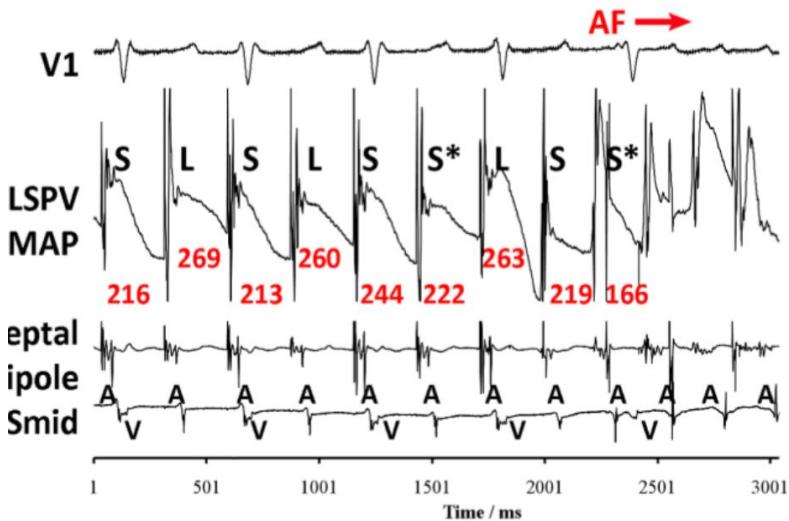
Cardiac alternans



AP: action potential → APD alternans

CaT: calcium transient → CaT alternans

Alternans precede AF

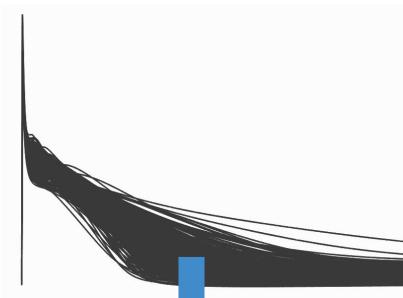


Narayan et al., Circulation, 2011

Objective

Get insight into the cellular mechanisms causing the observed APD alternans behavior in the Koivumäki model.

Populations of models

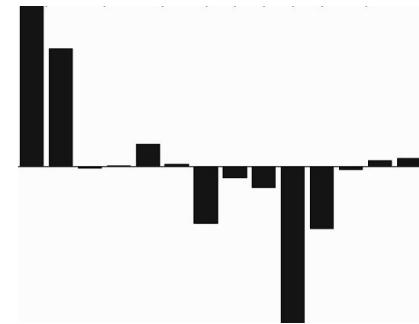


Large number of model instances
that represent responses under
different physiological conditions

Data



Sensitivity analysis



*Insight
into model*

Explore the role of individual
model parameters in the
observed model behavior

How did it start?

2009

Parameter Sensitivity Analysis in Electrophysiological Models Using Multivariable Regression

Eric A. Sobie*

Department of Pharmacology and Systems Therapeutics, Mount Sinai School of Medicine, New York, New York 10029

SA on a randomized set of models by using multivariate linear regression

2010

Regression Analysis for Constraining Free Parameters in Electrophysiological Models of Cardiac Cells

Amrita X. Sarkar, Eric A. Sobie*

Department of Pharmacology and Systems Therapeutics, Mount Sinai School of Medicine, New York, New York, United States of America

Extension by inverting the regression matrix to uniquely restrain model parameters

2013

Experimentally calibrated population of models predicts and explains intersubject variability in cardiac cellular electrophysiology

Oliver J. Britton^a, Alfonso Bueno-Orovio^b, Karel Van Ammel^c, Hua Rong Lu^c, Rob Towart^c, David J. Gallacher^c, and Blanca Rodriguez^{a,1}

Calibration of populations with experimental values of markers

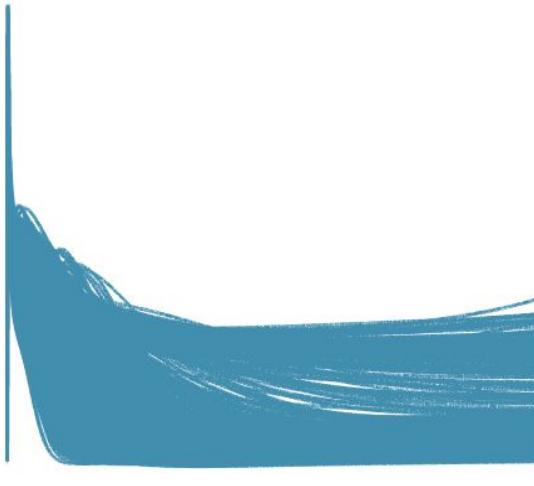
2014

Inter-Subject Variability in Human Atrial Action Potential in Sinus Rhythm versus Chronic Atrial Fibrillation

Carlos Sánchez^{1,2,3}, Alfonso Bueno-Orovio³, Erich Wettwer⁴, Simone Loose⁴, Jana Simon⁴, Ursula Ravens⁴, Esther Pueyo^{1,2}, Blanca Rodriguez^{3*}

Application of this methodology to populations of atrial cells

Populations of models simulate different model outputs



Incorporate natural **variability** observed in experimental data

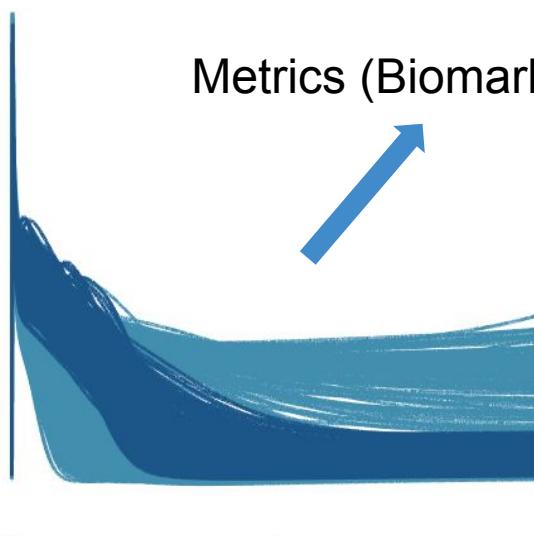
Simultaneously varying selected model parameters, related to the behavior of interest, plus “dummy” parameters.

Ion channels/pumps (maximum conductances, and gating variables)

RyR2 (time constants of close/open states)

Ionic buffering

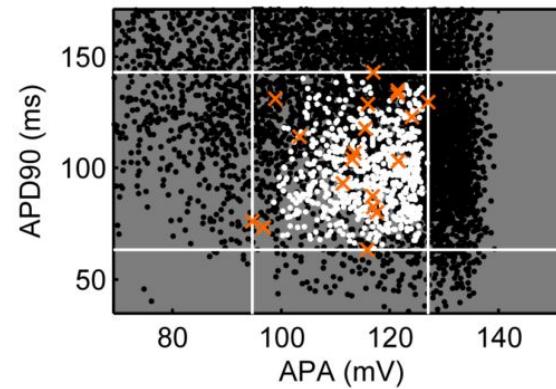
Parameters in population are **calibrated** by restricting biomarkers to within certain acceptable ranges



Data driven (eg, standard deviation)

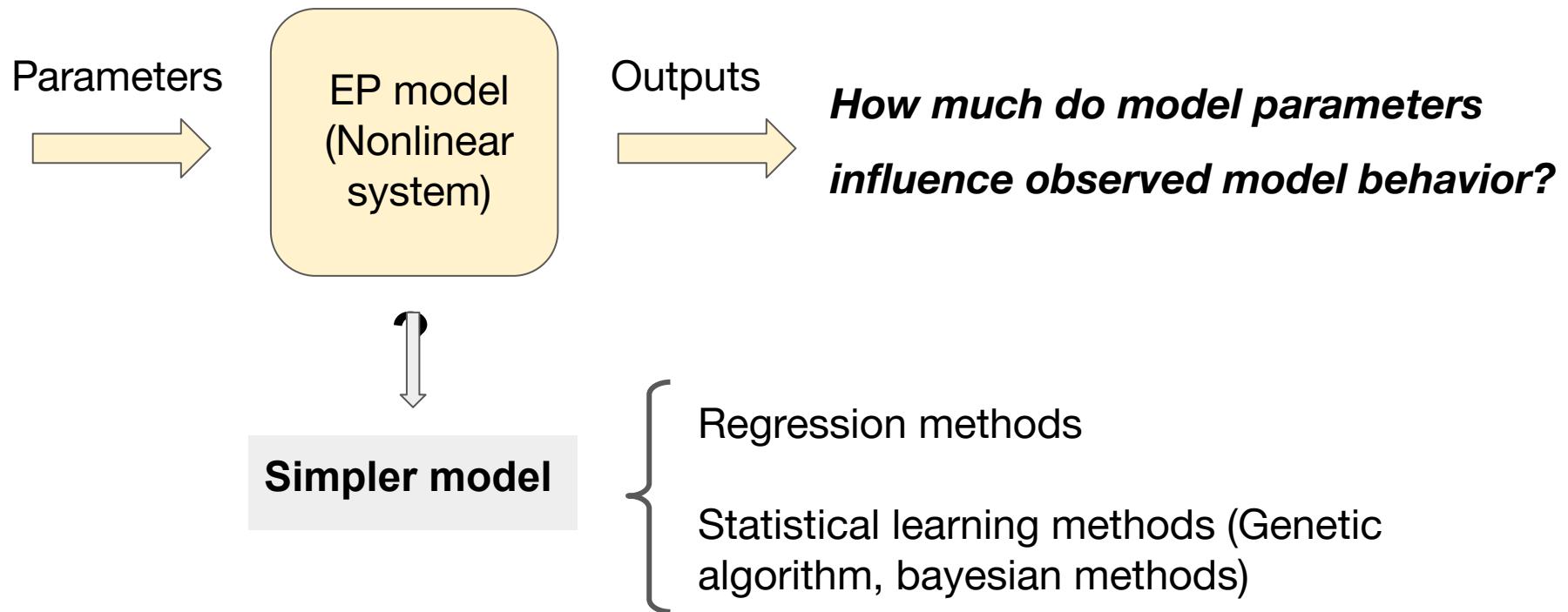
Experimental values

Restrain the parameter space



Muszkiewicz, A et al, 2014

Sensitivity analysis is useful for understanding the behaviour of a cell model



Multivariate Linear Regression method

The diagram illustrates a linear regression model equation. On the left, the word "model" is followed by an arrow pointing to a matrix X . This matrix has m rows and p columns, labeled as $(m \times p)$. The first column contains elements x_{11}, \dots, x_{1p} , and the last column contains x_{m1}, \dots, x_{mp} . Above the matrix, the label "Input parameter" is positioned with a downward-pointing arrow. To the right of the multiplication sign, the label "Regression coefficients" is placed above a second matrix. This second matrix has k rows and p columns, labeled as $(p \times k)$. Its first column contains elements $\beta_{11}, \dots, \beta_{1k}$, and its last column contains $\beta_{p1}, \dots, \beta_{pk}$. To the right of the equals sign, the label "Biomarker" is positioned with a downward-pointing arrow. To the right of the second matrix is another matrix with m rows and k columns, labeled as $(m \times k)$. Its first column contains elements y_{11}, \dots, y_{1k} , and its last column contains y_{m1}, \dots, y_{mk} .

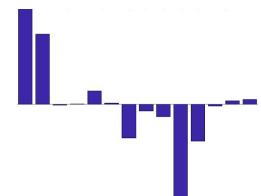
Multivariate Linear Regression method

$$Y_k = \beta_{0k} + \beta_{1k} \cdot x_{m1} + \beta_{2k} \cdot x_{m2} + \dots + \beta_{pk} \cdot x_{mp}$$

$$\begin{matrix} \mathbf{Y} = \mathbf{X} \cdot \mathbf{B} + \epsilon \\ (m \times k) \quad (m \times p) \quad (p \times k) \end{matrix} \Rightarrow \begin{matrix} \mathbf{X} \rightarrow \text{Model parameters} \\ \mathbf{Y} \equiv \mathbf{X} \cdot \mathbf{B}, \quad \mathbf{Y} \rightarrow \text{Biomarkers} \end{matrix} \quad \mathbf{B} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$$

$\mathbf{B} \rightarrow$ Regression coefficients (amount of variability in response explained by the parameter)

\mathbf{B} represents the amount of variability in Y that is explained by each of the parameters x when considered to be **independent**.



Distributions of parameters



Parameters (X)

Maximum conductance:

ICaL, INa, IK1, ICab, IKr, IKs, IKur, Ito, INaL

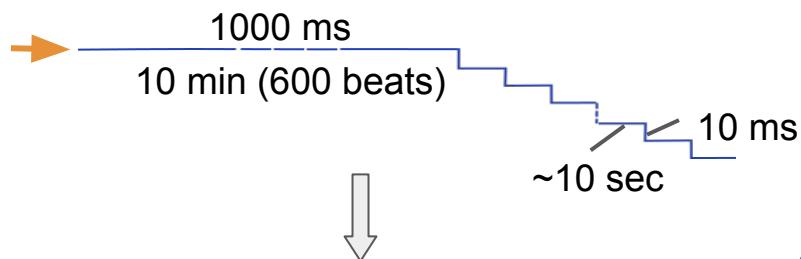
Maximum fluxes:

INaK, INCX, ICaP

Ryanodine receptors:

Maximum conductance, gating variables

Dynamic pacing protocol



Action potential (AP) traces

Calcium transient (CaT) traces

Dynamic restitution curves

Biomarkers (X)

AP

APD90-50-20

AP amplitude

RMP

dVdt max

CaT

CaT amplitude

Diastolic $[Ca^{2+}]_i$

CaT time to peak

CaT time of decay

APD restitution

Alternans threshold

Alternans range

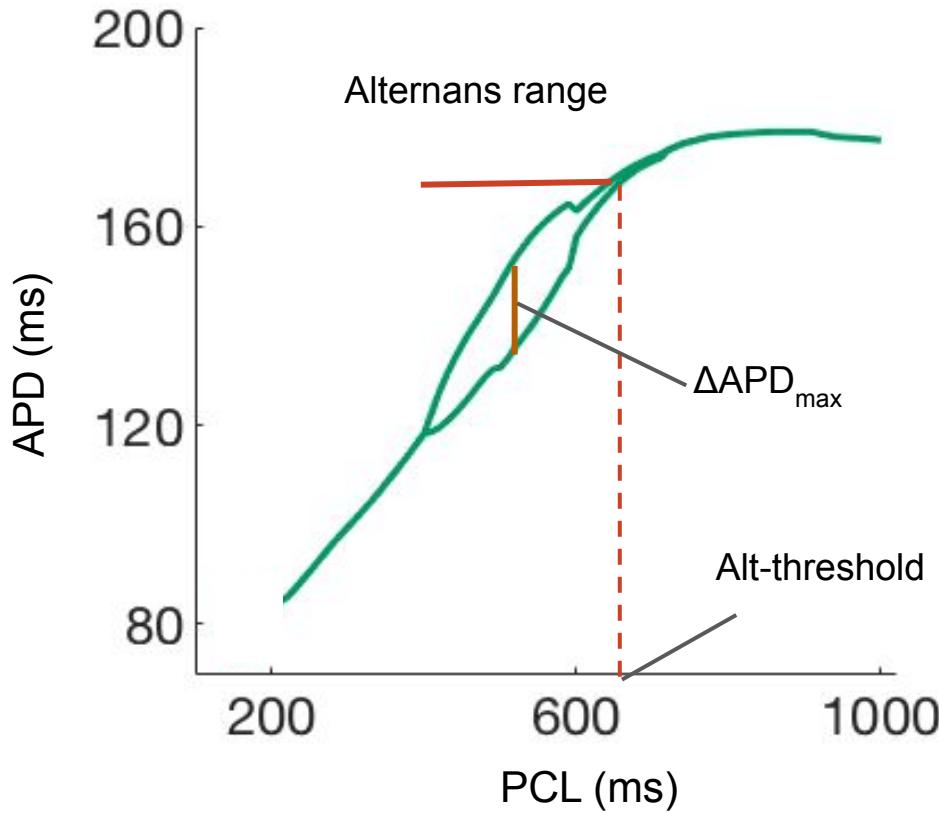
Alternans area

ΔAPD maximum

Alternans markers based on dynamic APD restitution

APD restitution

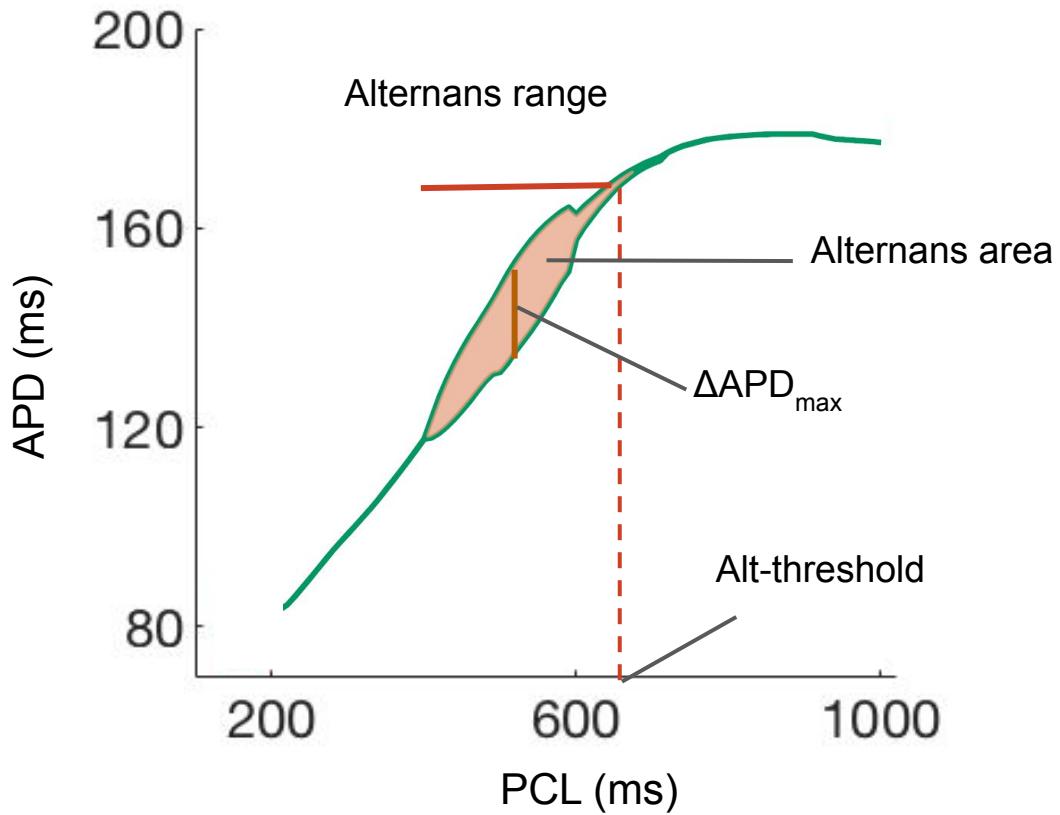
- Alternans threshold
- Alternans range
- Alternans area
- $\Delta\text{APD}_{\text{max}}$



Alternans markers based on dynamic APD restitution

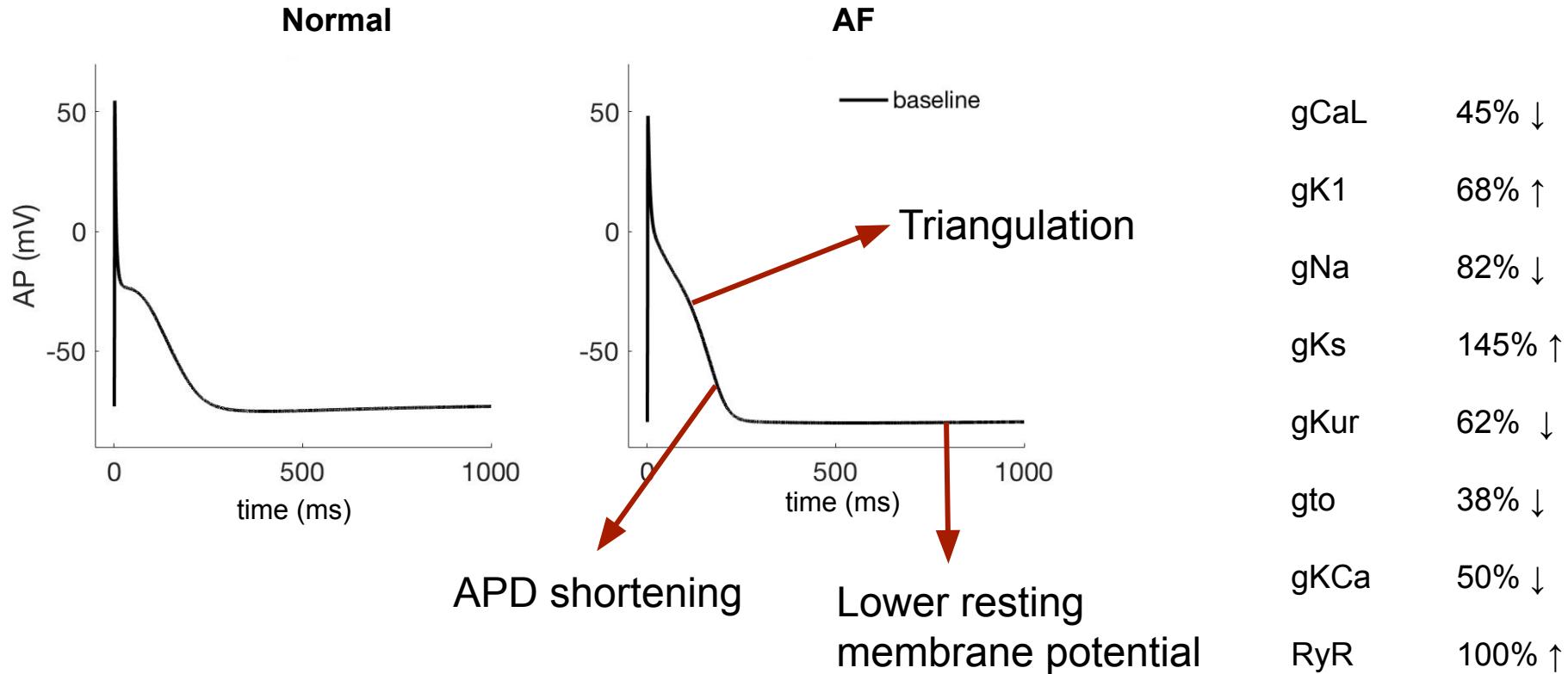
APD restitution

- Alternans threshold
- Alternans range
- Alternans area
- $\Delta\text{APD}_{\text{max}}$

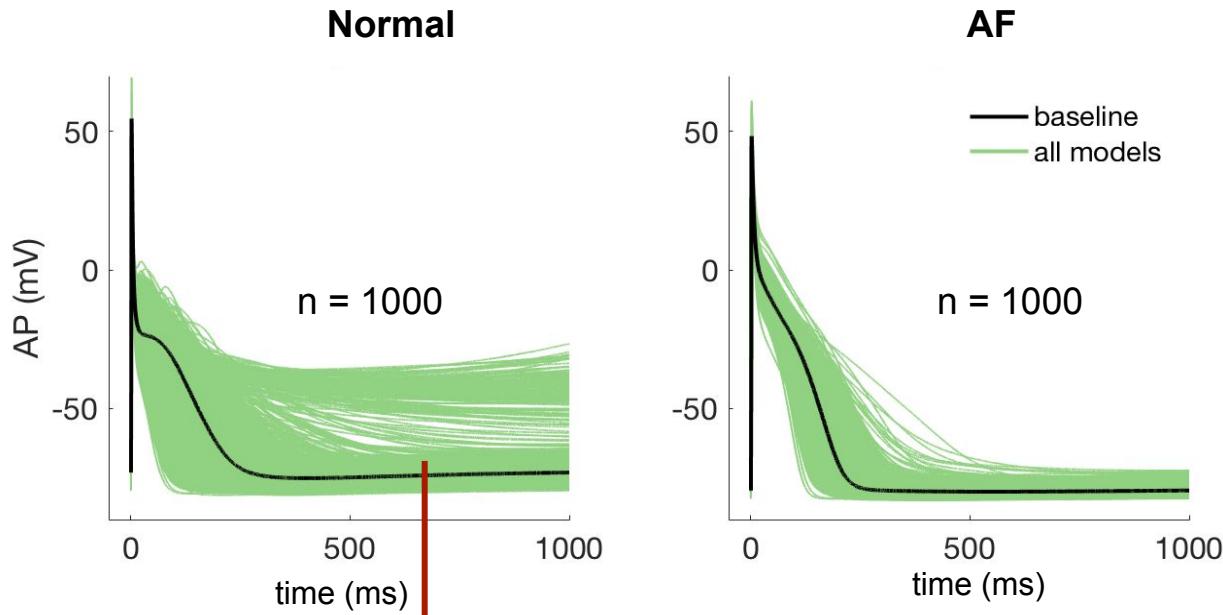


Results

Baseline of Normal and AF models

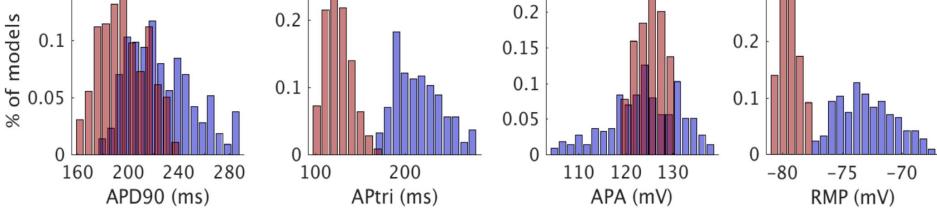
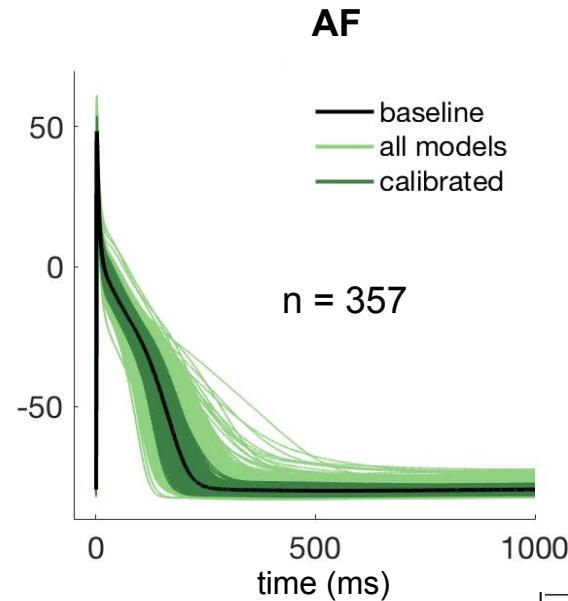
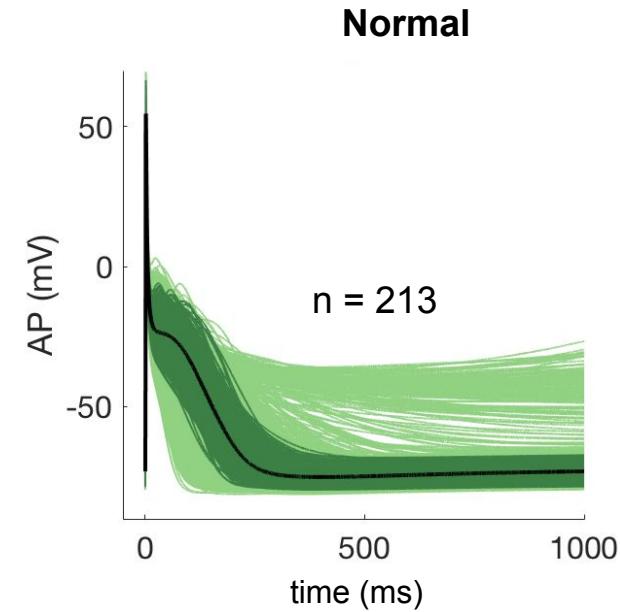


Populations of Normal and AF models



Higher intra-population
variability

Populations of Normal and AF models



Functional Calibration

Based on experimental data
on human atrial cell by
Sánchez et al (2014)

APD90
APA
RMP
Upstroke velocity

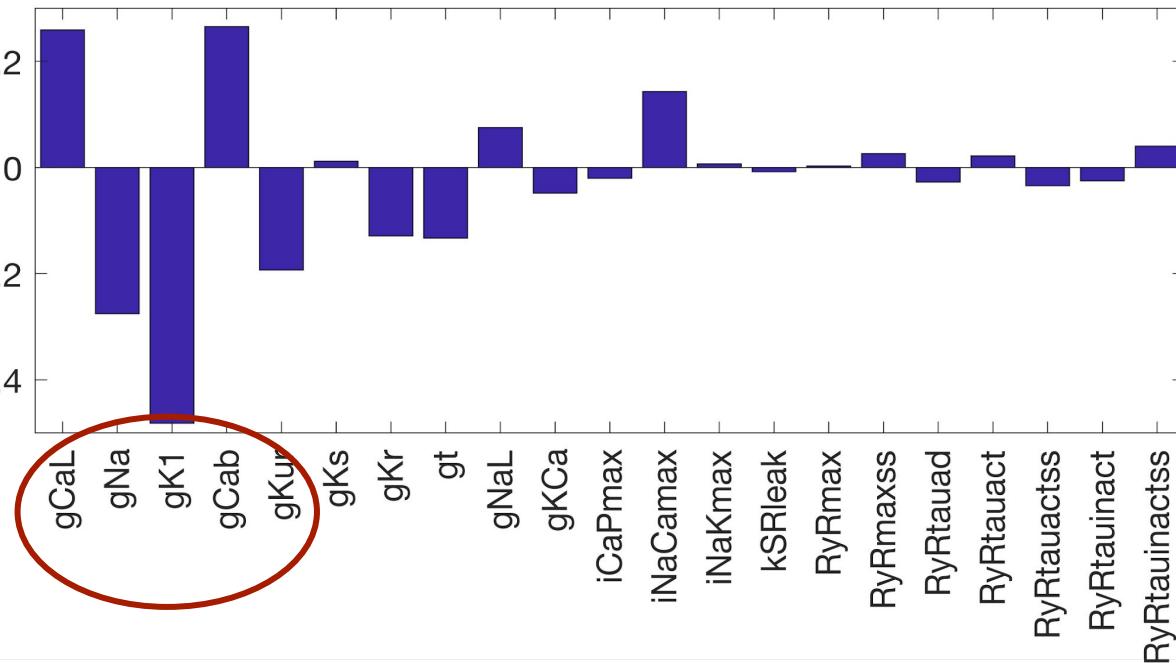
Maximum and minimum
APD in restitution curves

	SR		cAF	
	Minimum Value	Maximum Value	Minimum Value	Maximum Value
APD ₉₀ (ms)	190	440	140	330
APD ₅₀ (ms)	6	200	30	180
APD ₂₀ (ms)	1	60	1	75
APA (mV)	75	120	80	130
RMP (mV)	-85	-65	-85	-65
V ₂₀ (mV)	-35	10	-30	20
dV/dt _{max} (V/s)	40	420	40	420

Sánchez et al (2014)

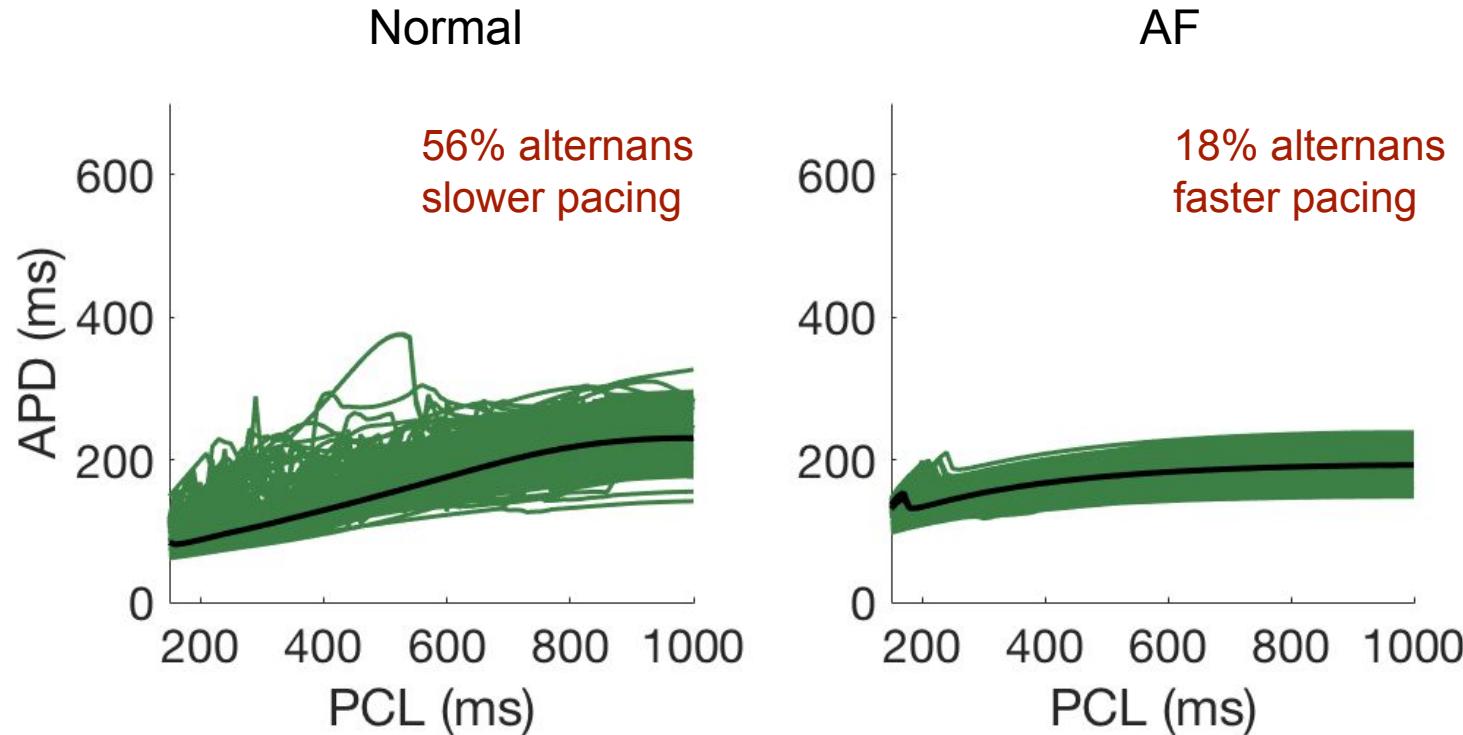
Sensitivities of AP and CaT markers of normal population were consistent with literature

APD90

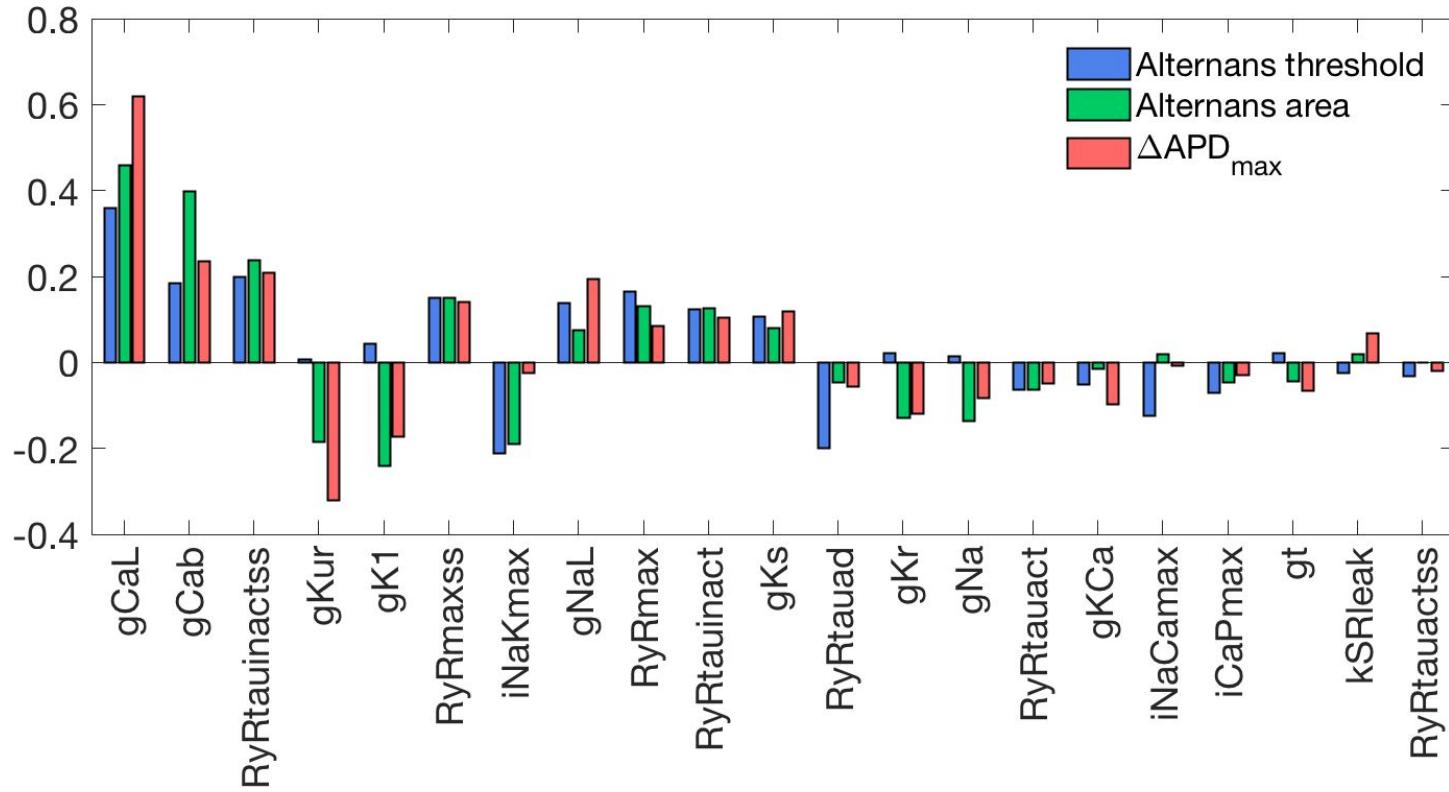


ICaL
IK1
INa
IKur

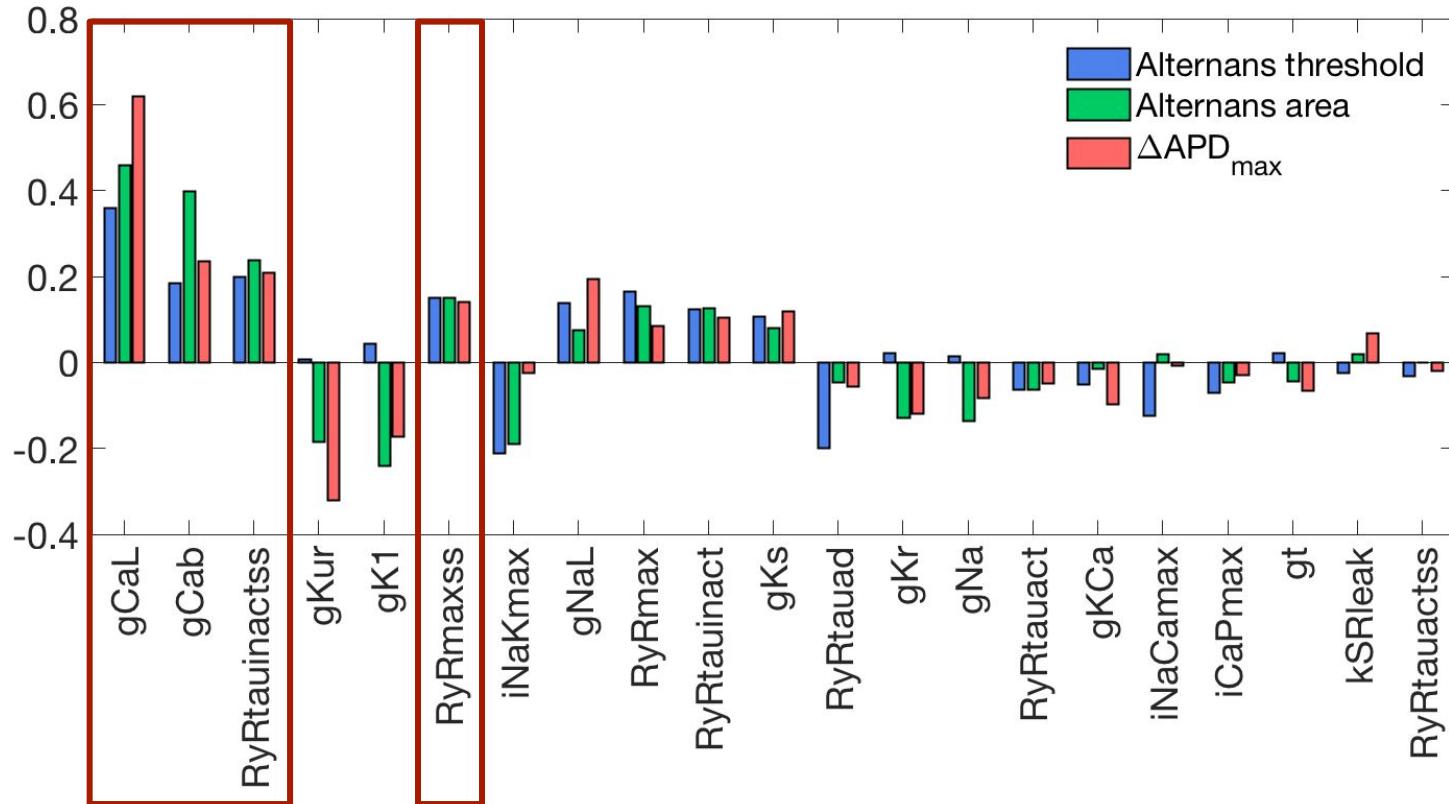
APD restitution of Normal cells revealed greater propensity to APD alternans compared to the AF cells



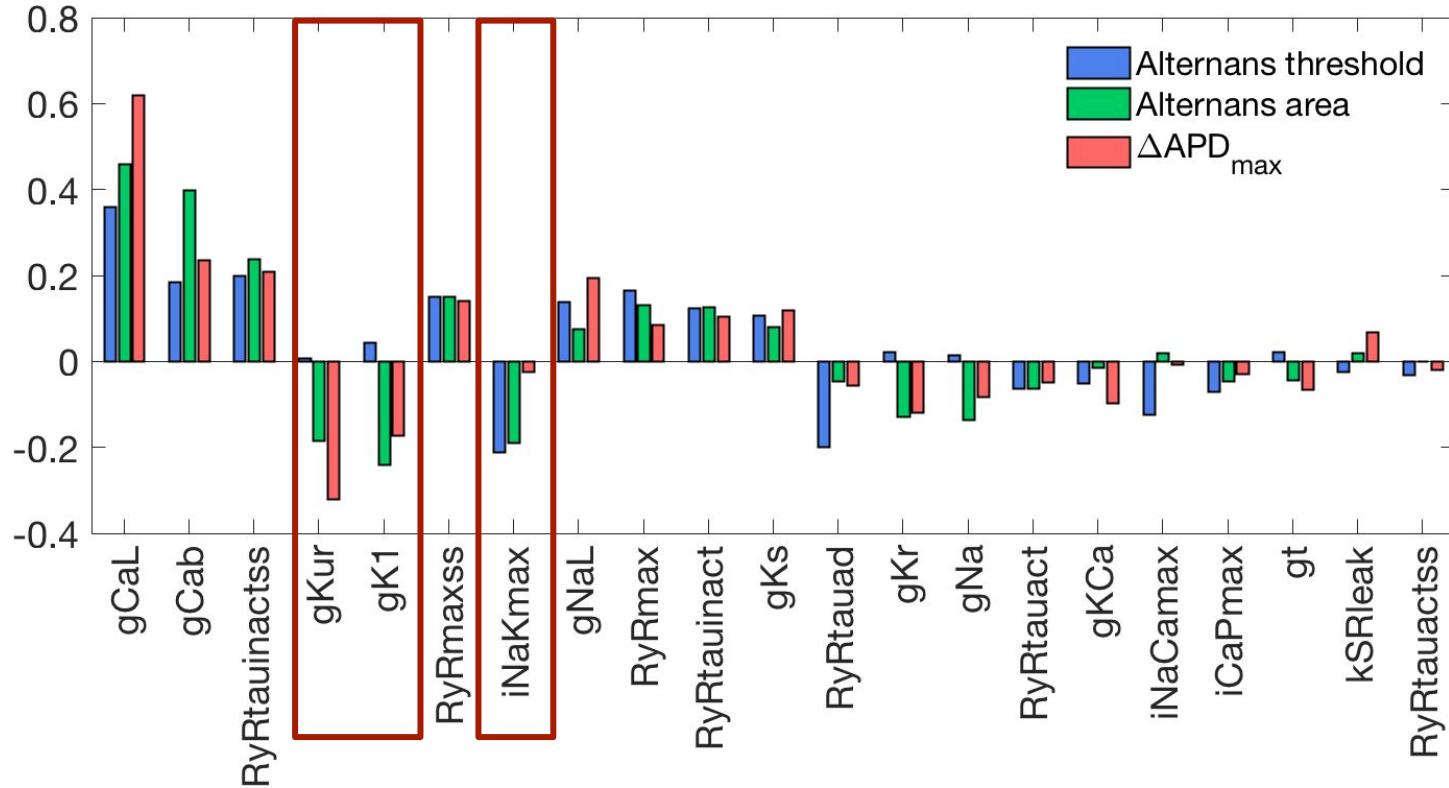
Alternans markers in Normal population



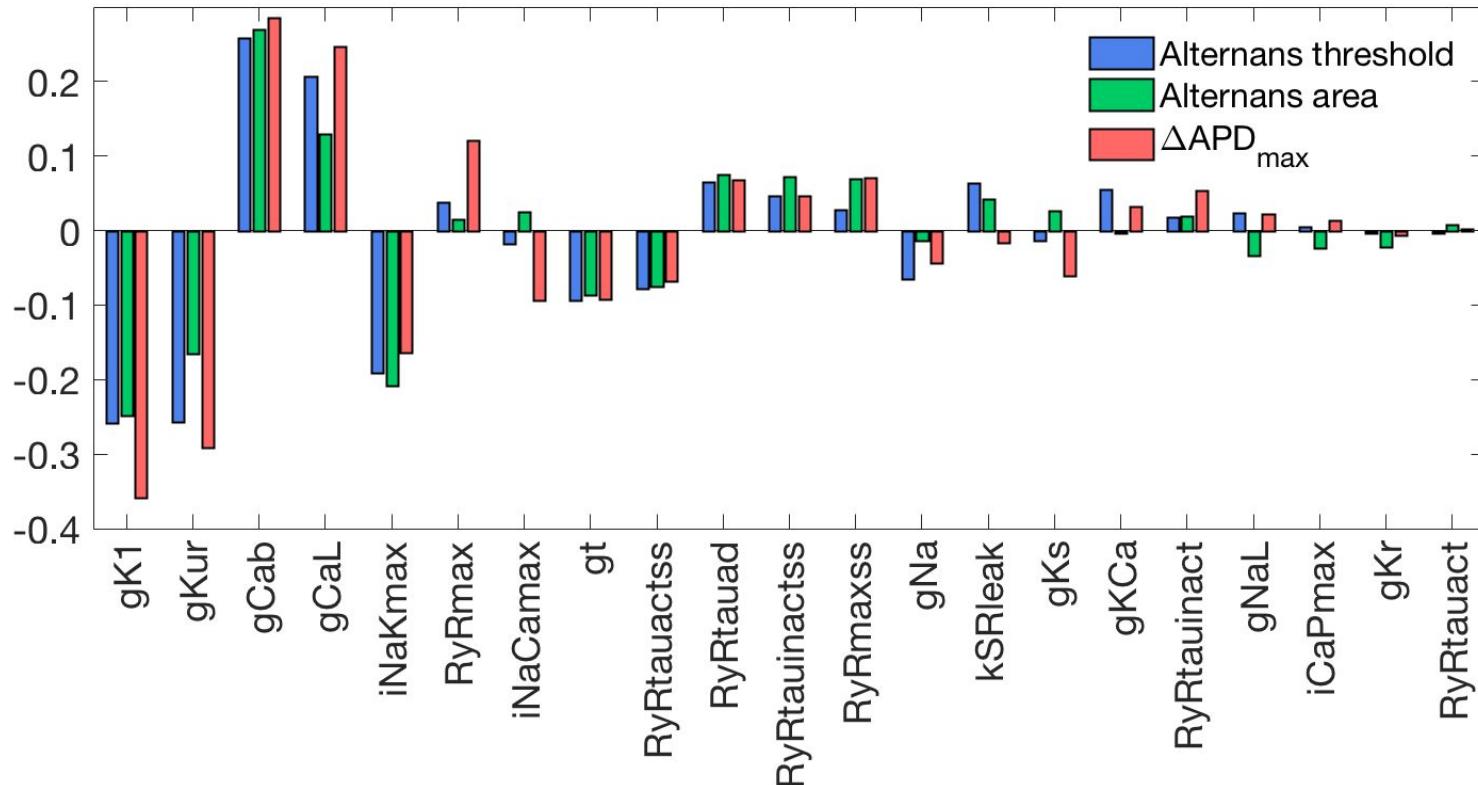
Alternans markers in Normal population showed known parameter dependencies



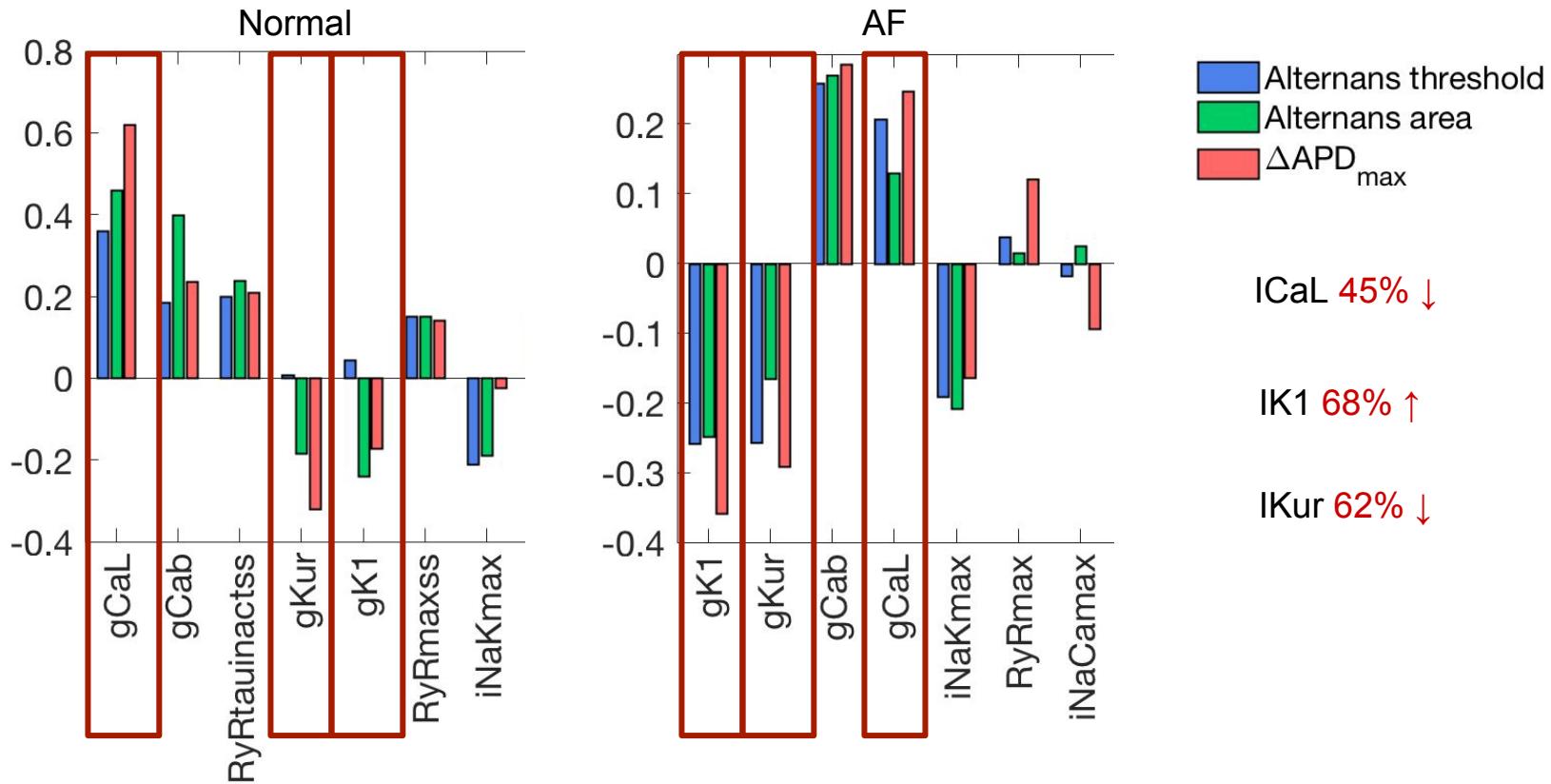
Alternans markers in Normal population provided new insights into role of IKur, IK1 and INaK



AF population showed highest sensitivities of alternans markers to IK1, IKur, ICaL and INaK



Normal and AF populations showed different sensitivities of alternans markers



Conclusions and insights

- Populations of Normal cells showed higher variability and propensity for APD alternans than the AF population
- Normal and AF populations showed differences in the sensitivities of alternans to g_{CaL} , g_{K1} , g_{Kur} , and g_{NaK} conductance
- Framework developed is a useful tool for studying mechanisms of cardiac alternans in single cells, and can be extended to tissue/organ simulations
- This methodology can be applied to study other electrophysiology mechanisms related to arrhythmia

Limitations

- Calculated B are sensitive to choice of parameters, since the coefficients represent the “relative” role of each parameter in explaining the variability in the observed response/biomarker
- The simple regression model assumes no interactions between independent variables, which sometimes cannot be neglected. Use, eg, Partial Correlation.
- Method can be extended to include nonlinear and interaction terms, but a more complicated model is also harder to interpret.
- Method is also sensitive to the calibration step, so this has to be done with a rationale keeping in mind the what are the model behaviors we want to study.

Future work

- Incorporate variable interactions in the regression model, and reduce error by defining different a priori distributions of the parameters.
- Perform mechanistic analysis of alternans, by analysis individual components of the calcium release system (eg, function of RyR and NCX).
- Move to 2D simulations (populations of tissues), and try and find relationships between observed alternans behavior at the cellular and tissue level.
- Incorporate the effect of ion channel blockade effect of drugs commonly used in the treatment of Atrial Fibrillation.
- Derive an arrhythmia score based on cellular biomarkers that works as a surrogate of pro-arrhythmic risk.

Thank you



simula



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UNIVERSITY
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