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Record book number: 320379

Multiscale nonlinear Granger causality in currency exchange markets

Master's thesis
in the field of Physics

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Warsaw, September 2021

Summary

U.S. Dollar Index and crude oil price are important economic variables. According to the current knowledge, their relation to one another is complex and time-varying. Available research on this topic gives ambigous results. Examining casusality between those can provide valuable insights to economists, policy makers and other market agents.

A multiscale linear and nonlinear Granger causality analysis was performed for U.S. Dollar exchange rate and crude oil price. Time series were decomposed into multiple time scales using the discrete wavelet transform in Daubechies basis. Linear Granger causality test and nonlinear Diks-Panchenko test were applied to each pair of bands of wavelet decomposed data. Fourier phase randomisation method for surrogate data generation was employed to verify obtained results.

Some causality has been found in the analyzed time series, with no major patterns observed. However, tests for surrogate data suggest relatively high false-positive rate. Therefore, we refrain from stating definitive conclusions about causality between U.S. Dollar Index and oil price.

Keywords

Granger causality, time series analysis, wavelet decomposition, surrogate data, currency exchange market, crude oil market.

Title of the thesis in Polish language

Wieloskalowa nieliniowa przyczynowość Grangera na rynkach walutowych

I wish to show my gratitude to

dr Zbigniew R. Struzik
University of Tokyo

for invaluable input to this thesis.

Contents

1 Introduction **8**

2 Motivation **8**

3 Methodology **9**
- 3.1 Granger causality and feedback 9
  - 3.1a Linear autoregressive model 9
  - 3.1b Diks-Panchenko test 9
- 3.2 Wavelet transform 10
  - 3.2a Discrete wavelet transform 11
  - 3.2b Multiresolution analysis 13
- 3.3 Surrogate data 14
  - 3.3a Introduction to surrogate data method 14
  - 3.3b Surrogate data generation method choice 15

4 Datasets and software **16**
- 4.1 Datasets 16
- 4.2 Software 16

5 Results **17**
- 5.1 Data analysis algorithm 17
- 5.2 Wavelet decomposed data 18
- 5.3 Causality tests results 26
  - 5.3a Linear causality test results 27
  - 5.3b Nonlinear causality test results 34
- 5.4 Surrogate data results 41
  - 5.4a Linear causality in surrogate data 41
  - 5.4b Nonlinear causality in surrogate data 48
- 5.5 Results discussion 55

6 Conclusions **56**

A Surrogate data example **60**

1 Introduction

Unraveling causal relations is a fundamental object of interest in science. Causality testing is an important branch of time series analysis. Granger [1] stated a flexible theoretical problem definition for causality in his seminal paper in 1969. This framework laid ground for abundance of causality test implementations among many fields: physics [2, 3], finance [4, 5], neuroscience [6, 7, 8], environmental studies [9, 10] and other [11]. The impact of this approach was later recognized awarding him the Nobel Prize in Economics in 2003 [12].

Spectral methods of time series analysis enable investigatation of its features in different frequency domains. Wavelet approach allows to transform signals without loosing information from its original domain (usually spatial or temporal), overcoming limitations of Fourier analysis. Wavelet analysis has proven to be a prominent tools in image compression [13], pattern recognition [14], time series analysis [15] and others.

In this thesis a linear and nonlinear Granger test is employed to investigate causal relations in multiple time scales between U.S. Dollar exchange rate and crude oil price. Section 2 contains literature review and motivation for research. Section 3 decribes in details methodology used for data analysis. Data sets used in this thesis are characterized in section 4, along with software used for data analysis. Section 5 gathers important results and provides commentary to the obtained results. Lastly, closing remarks are presented in section 6.

2 Motivation

U.S. Dollar exchange rate and crude oil price are significant economic indexes. Oil, literally and figuratively, fuels the contemporary economic system. Because the U.S. is the World's biggest oil producer and a major exporter, their currency's exchange rate can have a significant impact on oil's demand. Moreover, the U.S. Dollar is the major invoicing currency on international oil markets, making the connections between currency exchange rate and oil price even more emphasized.

Quantifying causality between economic variables is crucial for better understanding of links between them. This in turn allows economists to constuct better models, improve policies by govening bodies and may even provide competitive advantage for various market agents.

Significant number of research was conducted on relationships between U.S. Dollar and crude oil price. Granger causality has been studied revealing mixed results. Relatively recent examples, containg comprehensive overview are described in the following papers: [16, 17, 18, 19].

The aim of this thesis is to deepen the understanding of causal links between U.S. Dollar exchange rate and crude oil price by testing for both linear and nonlinear Granger causality. Additional insights come from decomposing investigated time series into multiple time scales by wavelet transform approach.

3 Methodology

3.1 Granger causality and feedback

Framework for causality testing was first proposed by Granger [1] in 1969. The original formulation had a general form with an example of a linear autoregressive model provided, further described in section 3.1a.

Consider two time series X_t and Y_t of length n, which are assumed to be weakly stationary. Lets denote an optimal, least-squares predictor of some time series X_t given another time series Y_t by $P_t(X|Y)$. The residual errors of the model and its variance are defined as follows: $\varepsilon_t(X|Y) = X_t - P_t(X|Y)$ and $\sigma^2(X|Y) = var[\varepsilon_t(X|Y)]$. Lastly, all available information until time t will be denoted U_t.

Given such notation the following definitions are proposed:

Definition 1 *Causality*
If $\sigma^2(X|U) < \sigma^2(X|U - Y)$, *then* Y_t *causes* X_t, *denoted as* $Y_t \Rightarrow X_t$.

Definition 2 *Feedback*
If $\sigma^2(X|U) < \sigma^2(X|U - Y)$ *and* $\sigma^2(Y|U) < \sigma^2(Y|U - X)$, *then there is feedback between* X_t *and* Y_t, *denoted as* $Y_t \Leftrightarrow X_t$.

The completely unrealistic aspect of the formulation above is having access to the full information U_t. In practice those definitions allow to construct statistical tests and to reason for causal relations between time series.

3.1a Linear autoregressive model

A simple realization of Granger causality test can be implemented using a two-variable linear autoregressive (AR) model of order p. Two linear predictors of X_t can be constructed as follows:

$$X_t = \sum_{i=1}^{p} \alpha_i X_{t-i} + \varepsilon_{X,t} \tag{1}$$

$$X_t = \sum_{i=1}^{p} a_i X_{t-i} + \sum_{i=1}^{p} b_i Y_{t-i} + \varepsilon_{X|Y,t} \tag{2}$$

where: p is the order of AR model, α, a, b are scalar coefficients and $\varepsilon_{X,t}$, $\varepsilon_{X|Y,t}$ are residual errors of the models. Causal relationship $Y_t \Rightarrow X_t$ can be inferred by comparing variance of residual errors $\varepsilon_{X,t}$ and $\varepsilon_{X|Y,t}$, as stated in definition 1 in section 3.1.

3.1b Diks-Panchenko test

To capture effects beyond linear causality nonparametric statistical tests are used. In principle, such methods infer about some features of a system by constructing statistics on their distributions. Those methods, not relying on a linear model, in context of Granger causality, are often called nonlinear Granger causality tests. Diks and Panchenko

[20], basing on earlier approaches by Hiemstra and Jones [4] and Baek and Brock [21], developed a test for nonlinear Granger causality.

Let $X_t^l = (X_t, X_{t-1}, \dots X_{t-l+1})$ and $Y_t^l = (Y_t, Y_{t-1}, \dots Y_{t-l+1})$ be lagged vectors of time series. The null hypothesis assumes no causality from $X_t \Rightarrow Y_t$:

$$Y_{t+1}|(X_t^l, Y_t^l) \sim Y_{t+1}|(Y_t^l) \tag{3}$$

Consider the following vector $W_t = (X_t^l, Y_t^l, Z_t)$ with $Z_t = Y_{t+1}$. Equation (3) can be rewritten in terms of joint probability density function $f_{X,Y,Z}(x,y,z)$:

$$\frac{f_{X,Y,Z}(x,y,z)}{f_Y(y)} = \frac{f_{X,Y}(x,y)}{f_Y(y)} \frac{f_{Y,Z}(y,z)}{f_Y(y)} \tag{4}$$

Next, the null hypothesis can be expressed as:

$$q \equiv E[f_{X,Y,Z}(x,y,z) f_Y(y) - f_{X,Y}(x,y) f_{Y,Z}(y,z)] = 0 \tag{5}$$

An estimator of q based on indicator functions is:

$$T_n(\varepsilon) = \frac{(2\varepsilon)^{-d_X - 2d_Y - d_Z}}{n(n-1)(n-2)} \sum_i \left[\sum_{k \neq i,j} \sum_{j \neq i} \left(I_{ik}^{XYZ} I_{ij}^{Y} - I_{ik}^{XY} I_{ij}^{YZ} \right) \right] \tag{6}$$

where I_{ij}^W is the indicator function: $I_{ij}^W = I(||W_i - W_j|| < \varepsilon)$, I is equal to 1 if the condition is met and 0 otherwise, $||\cdot||$ is the supremum norm, d_W is dimension of time series vector W_t, ε is bandwith. The superscripts in the indicator function denote joint probablities, i.e. I^{XY} is the indicator function for joint probability distribution of X and Y.

Denoting local density estimator of random vector W at W_i as:

$$\hat{f}_W(W_i) = \frac{(2\varepsilon)^{-d_W}}{n-1} \sum_{j, j \neq i} I_{ij}^W \tag{7}$$

the test statistics is defined as follows:

$$T_n(\varepsilon) = \frac{(n-1)}{n(n-2)} \sum_{i, j \neq i} (\hat{f}_{X,Y,Z}(x,y,z)\ \hat{f}_Y(y) - \ \hat{f}_{X,Y}(x,y)\ \hat{f}_{Y,Z}(y,z)) \tag{8}$$

It has been shown that the test is consistent for $\varepsilon = Cn^{-\beta}$ for a positive constant C and $\beta \in (\frac{1}{4}, \frac{1}{3})$ for sample size n.

3.2 Wavelet transform

The term "wavelet" was coined by Grossmann and Morlet [22] in the 1980s. The theory for this approach was further developed by many researchers, most notably Daubechies [23].

A wavelet can be intuitively understood as a "short, localized oscillation". The so-called mother wavelet, denoted as $\psi(t)$ is a function that satisfies the following conditions:

$$\int dt\ \psi(t) = 0 \tag{9}$$

$$\int dt\, |\psi(t)|^2 = 1 \tag{10}$$

and the *admissability condition*:

$$\int d\omega\, |\omega|^{-1} \left(\mathscr{F}\psi\right)(\omega) < \infty \tag{11}$$

where $\left(\mathscr{F}\psi\right)(\omega)$ is the Fourier transform of $\psi(t)$.

This function is then dilated and translated to account for frequency and localization:

$$\psi^{(a,b)}(t) = \frac{1}{\sqrt{a}}\, \psi\left(\frac{t-b}{a}\right) \tag{12}$$

The continuous wavelet transform $W(a,b)$ is a projection of the original signal $x(t)$ on the mother wavelet:

$$W(a,b) = \int dt\, x(t)\psi^{(a,b)}(t) \tag{13}$$

There are several widely used wavelet bases, typical examples are: Haar, Morlet, "Mexican hat" and Daubechies. The mother wavelets of those examples are plotted on figure 3.1.

3.2a Discrete wavelet transform

For practical purposes, to analyze real-life discrete time series, a discrete wavelet transform (DWT) is used. It allows to decompose time series into multiple frequency scales.

The DWT is based on two discrete wavelet filters, called *mother wavelet* (which is a band-pass filter):

$$h_l = (h_0, h_1, \ldots h_{L-1}) \tag{14}$$

and *father wavelet* (a low-pass filter):

$$g_l = (g_0, g_1, \ldots g_{L-1}). \tag{15}$$

where L is length of the filter.

In the disrete case, conditions 9 and 10 for the mother wavelet become:

$$\sum_{i=0}^{L-1} h_i = 0 \tag{16}$$

$$\sum_{i=0}^{L-1} |h_i|^2 = 1. \tag{17}$$

Additionally, h_l is orthogonal to even shifts:

$$\sum_{i=0}^{L-1} h_i h_{i+2n} = 0 \tag{18}$$

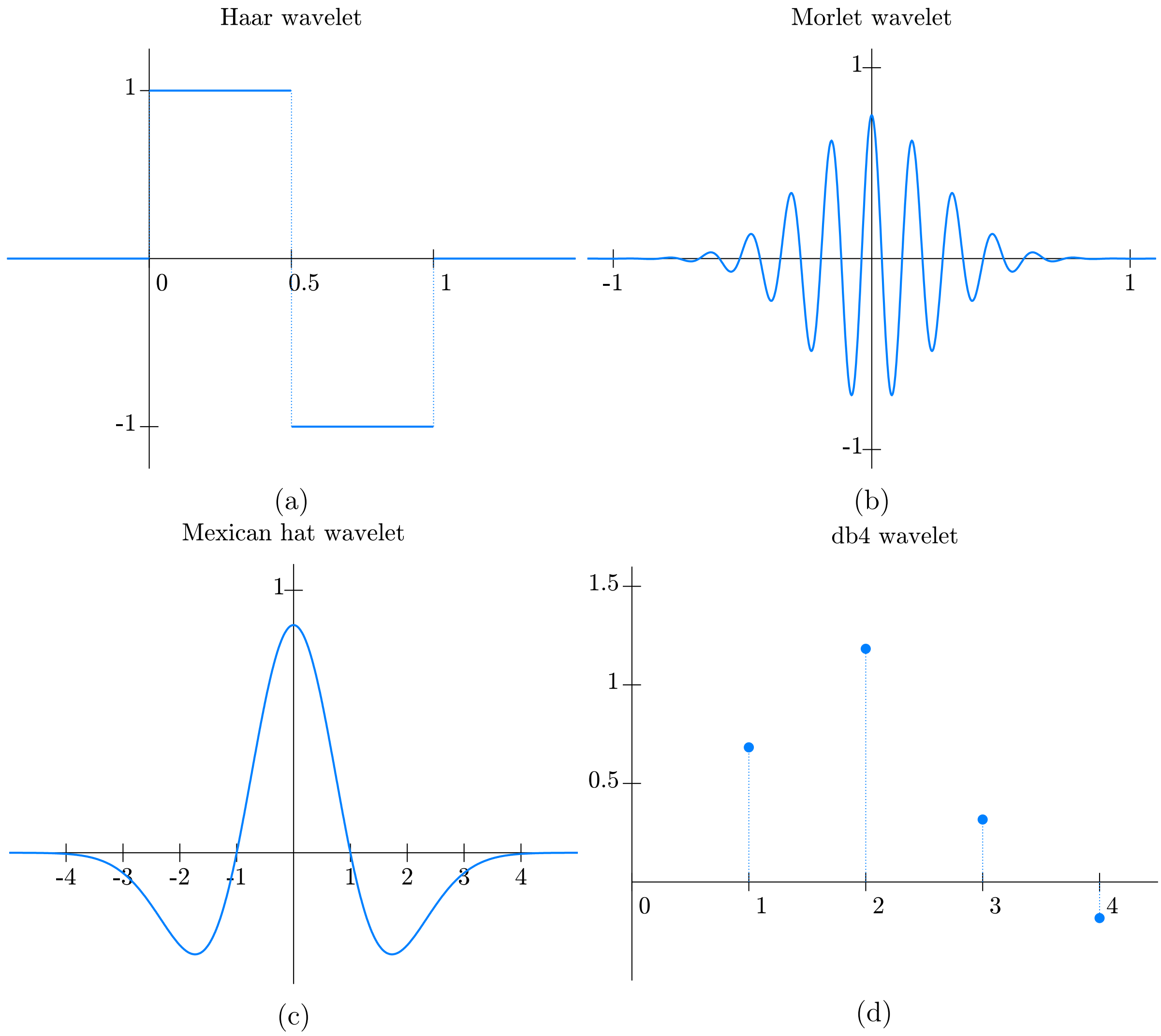

Figure 3.1: Mother wavelets of (a) Haar (b) Morlet (c) Mexican hat, (d) db4.

for all $n \neq 0$.

The father wavelet is constructed from the mother wavelet using the quadrature mirror relation:

$$g_l = (-1)^{l+1}\, h_{L-1-l}. \tag{19}$$

Moreover, the scaling filter g_l has the following properties:

$$\sum_{i=0}^{L-1} g_i = \sqrt{2}, \tag{20}$$

$$\sum_{i=0}^{L-1} |g_i|^2 = 1, \tag{21}$$

$$\sum_{i=0}^{L-1} g_i g_{i+2n} = 0, \tag{22}$$

$$\sum_{i=0}^{L-1} g_i h_{i+2n} = 0. \tag{23}$$

In the discrete case dilations and translations of mother wavelet are expressed as: $a = 2^{-j}$ and $b = k2^{-j}$, where $j, k \in \mathbb{N}$ and $j \leq J = log_2(T)$. J is the biggest number of scales and T is the length of the time series.

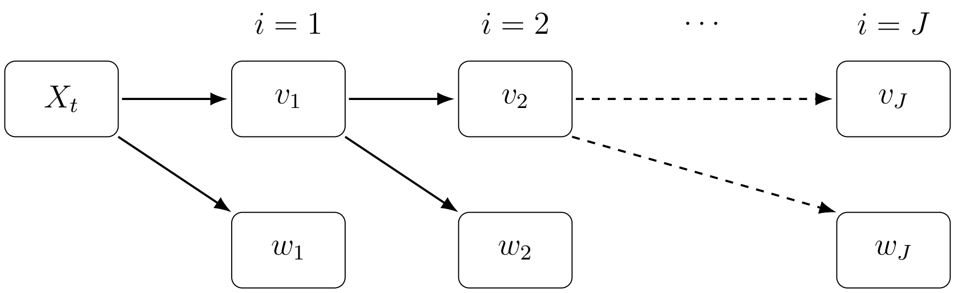

Figure 3.2: A graphical representation of discrete wavelet decomposition of time series X_t using the pyramid algorithm.

To decompose a signal with DWT, a *pyramid algorithm* is employed (see figure 3.2). It consists of recursively applying low-pass and band-pass filters to the time series. With each iteration of this scheme the high and low frequencies are separated. The father wavelet captures the trend of the series, while mother wavelet extracts oscillatory components around the trend.

Let $w_i(t)$ represent the high frequency and $v_i(t)$ the low frequency of time series X_t after i iterations of the pyramid algorithm. After the first pass one will get: $X_t = [w_1, v_1]$. Then, applying this scheme again: $X_t = [w_1, w_2, v_2]$, and after J iterations the time series will be decomposed into:

$$X_t = [w_1, w_2, w_3, \dots w_J, v_J] \tag{24}$$

In this thesis the Daubechies 4, or *db4* wavelet will be used. The index number refers to number N of coefficients. Each discrete wavelet has a number of vanishing moments equal to half the number of coefficients. A vanishing moment limits the wavelets ability to represent polynomial behaviour or information in a time series. For example, a D1 wavelet, with one vanishing moment encodes polynomials with one coefficient or constant signal components. D4 encodes polynomials with two coefficients, i.e. linear signal components.

3.2b Multiresolution analysis

Transformed signal can be reconstructed from the coefficients w and v using an inverse wavelet transform. Original signal can be split into different scale terms using the wavelet representation proposed by Mallat [24]:

$$X(t) = A_J(t) + \sum_{i=1}^{J} D_i(t) \tag{25}$$

where J is the number of analyzed scales, A_J and D_i are given by:

$$A_J(t) = \sum_k a_{J,k}\phi_{J,k}(t) \tag{26}$$

$$D_i(t) = \sum_k d_{i,k}\psi_{j,k}(t). \tag{27}$$

Coefficients $a_{J,K}$ and $d_{j,k}$ are projections of the original signal on father and mother wavelets respectively:

$$a_{J,k} = \int dt\, \phi_{J,k}(t) X(t) \tag{28}$$

$$d_{i,k} = \int dt\, \psi_{i,k}(t) X(t). \tag{29}$$

In the wavelet representation given by equation (25) terms D_i capture signal details around trend for frequences $2^{-i+1} \leq f \leq 2^i$. The A_J term is the remaining signal trend for scale J. Such representation of time series will allow to analyze causal links of the original signals in different time scales.

3.3 Surrogate data

3.3a Introduction to surrogate data method

Infering about statistical properties of time series can be cumbersome when the data set is limited. In some circumstances the investigated phenomena are sparse (astrophysics), happening slowly (climate studies) or impossible to reproduce (financial data). Sometimes a researcher will have access to only one realization of a physical system. Estimating confidence intervals of statistical tests in such cases can pose a real challenge.

To tackle this problem a method called *surrogate data* was adopted in various implementations. This method was first invented as a test for nonlinearity by Thelier et al. [25]. An in-depth systematic review of surrogate data for statistical hypothesis testing is presented in [26].

In principle, surrogate data method allows to obtain artificial instances of the examined data set, which preserve some of its statistics, while destroying the tested feature. This can be done either by generating a data set via some modelling (ex. Monte Carlo simulations) or by transforming the original data set in a deliberate manner. The modelling approach is often called a *typical realization*, while the data transformation approach is called *constrained realization*. The choice of the method is to be judged upon whether the underlying model of the phenomena is known and what kind of feature of data is investigated.

It is crucial to fit the method for generating surrogate data for the tested null hypothesis. Some surrogate generating methods, with examples of different types of features are:

- random permutation for temporal features and testing againt random noise,
- Fourier transform and derivative methods for nonlinear features,
- pseudo-periodic surrogates for periodic data,
- intersubject surrogates for independence testing.

Once a suitable method of data generation is chosen, a statistically significant number of surrogates is created. Then, each of this data sets is treated with exactly the same procedure as the original data. Investigating the behaviour of the discriminating statistic on the large set of surrogate data can help determine whether the results are statistically significant, or obtained simply by chance.

3.3b Surrogate data generation method choice

In this research nonlinear temporal features of time series are investigated. First, a random permutations of the original data set were tried, with no success. Then, a Fourier tranform approach was employed, called *phase randomisation*. This method has proven to be the right fit for the examined features.

The surrogate data generation scheme for phase randomisation method is as follows:

1. calculate Fourier transform of the original time series of length L,
2. generate a random vector of phase shifts of length $L/2$,
3. shift first half of the Fourier transform coefficients by the generated random vector,
4. the second half of the Fourier transform coefficients is a complex conjugate of the first half,
5. perform inverse Fourier transform of the phase shifted transform coefficients,
6. shift the resulting surrogate data by a constant to avoid negative and zero values in the resulting output, to avoid problems calculating log returns later data analysis steps.

An example of surrogate data generated using this scheme is presented in appendix A.

4 Datasets and software

4.1 Datasets

Mothly data for crude oil price and U.S. Dollar exchange rate for time period from 1974 to 2019 was analyzed.

Data for "U.S. Crude Oil First Purchase Price" comes from U.S. Energy Information Administration [27]. The price is expressed in U.S. Dollars per barrel.

Data referred to as "U.S. Dollar index" is actually "Trade Weighted U.S. Dollar Index". It's defined as "a weighted average of the foreign exchange value of the U.S. Dollar against a subset of the broad index currencies that circulate widely outside the country of issue". The index is set to 100 for March 1973. The dataset was issued by The Federal Reserve Bank of St. Louis [28].

Raw data plot is presented on figure 4.1

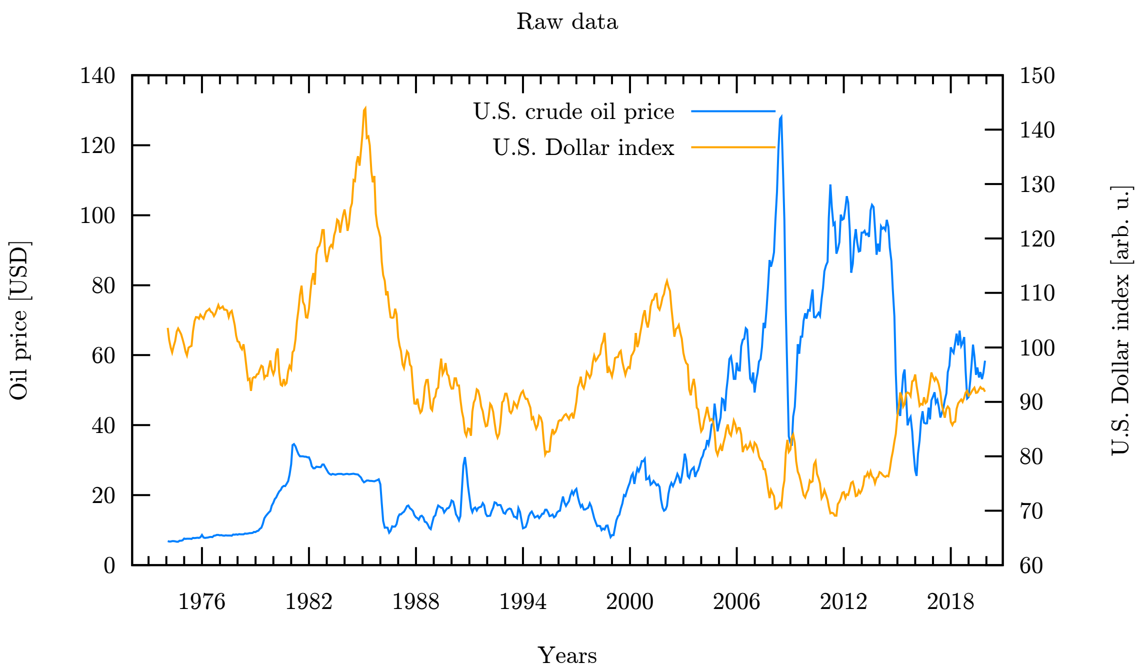

Figure 4.1: Plot of raw input data of crude oil price and U.S. Dollar Index.

4.2 Software

Data analysis was performed using Python and C programs available on open source licenses. NumPy [29] and Pandas [30] were used for array handling of input data. Fourier transform algorithms from SciPy [31] were used for surrogate data generation. PyWavelet [32] was used for wavelet decomposition of time series. StatsModels [33] has an implementation of linear Granger causality test. C code from authors of [20] was employed for nonlinear causality test. A script automating the whole process was developed using Python.

5 Results

5.1 Data analysis algorithm

Data analysis was performed in the following steps:

1. calculate log returns of the input data to make time series stationary,
2. decompose log returns in Daubechies (*db4* in PyWavelet implementation) wavelet basis,
3. reconstruct data in the original domain for each time scale,
4. test for linear Granger causality using an autoregressive model test between each wavelet scale pair, for up to 5 steps,
5. test for nonlinear Granger causality using the Diks-Panchenko test between each wavelet scale pair, for up to 5 steps.

Then, 1000 surrogate data sets were generated with phase randomisation method as described in section 3.3b and the same analysis procedure was perfomed for each realization. The number of statistically significant results among surrogates divided by number of realizations was used to asses results fidelity.

Statistical significance was assumed for p-values less than 0.05 for both linear and nonlinear tests. Those results are then filtered by false-positive rates calculated from generated surrogate data sets. A false-positive rate of $r < 0.1$ was assumed as a treshold.

Given 552 monthly observations (45 years) in the analyzed data sets, the wavelet scales correspond to the time scales as presented in table 5.1

Wavelet scale	Time scale
D1	2-4 months
D2	4-8 months
D3	8-16 months
D4	16-32 months
D5	32-64 months
D6	64-128 months
A6	>128 months

Table 5.1: Wavelet time scales

5.2 Wavelet decomposed data

Logarithmic returns of the time series are presented in 5.1 and 5.9. Wavelet decomposition of the original data is presented here in figures: 5.2-5.11. To depict how different wavelet scales relate to the input data, the logarithmic returns are displayed in light grey in the background of each plot. An example of generated surrogate data and its wavelet decomposition is presented in appendix A.

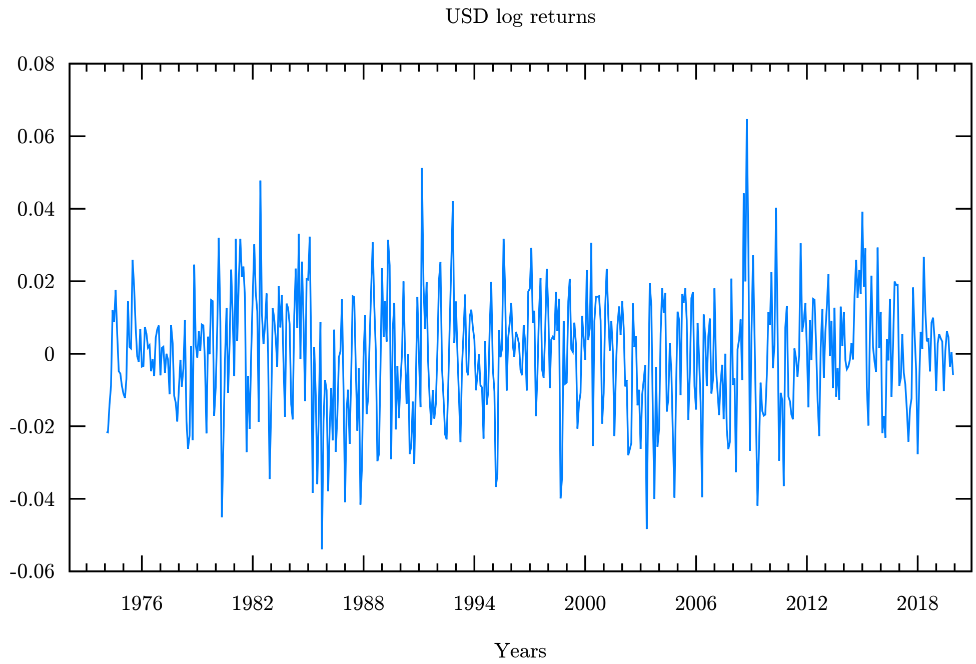

Figure 5.1: Plot of log returns of U.S. Dollar Index.

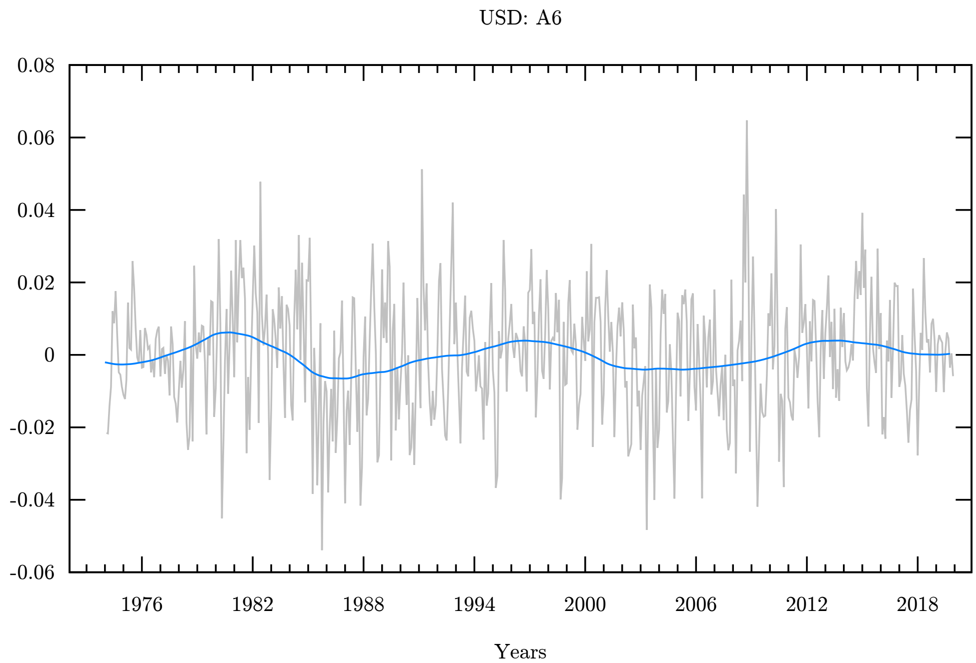

Figure 5.2: Plot of A6 component of wavelet decomposition of U.S. Dollar Index log returns. Plot of the original data in grey. A6 scale corresponds to >128 months.

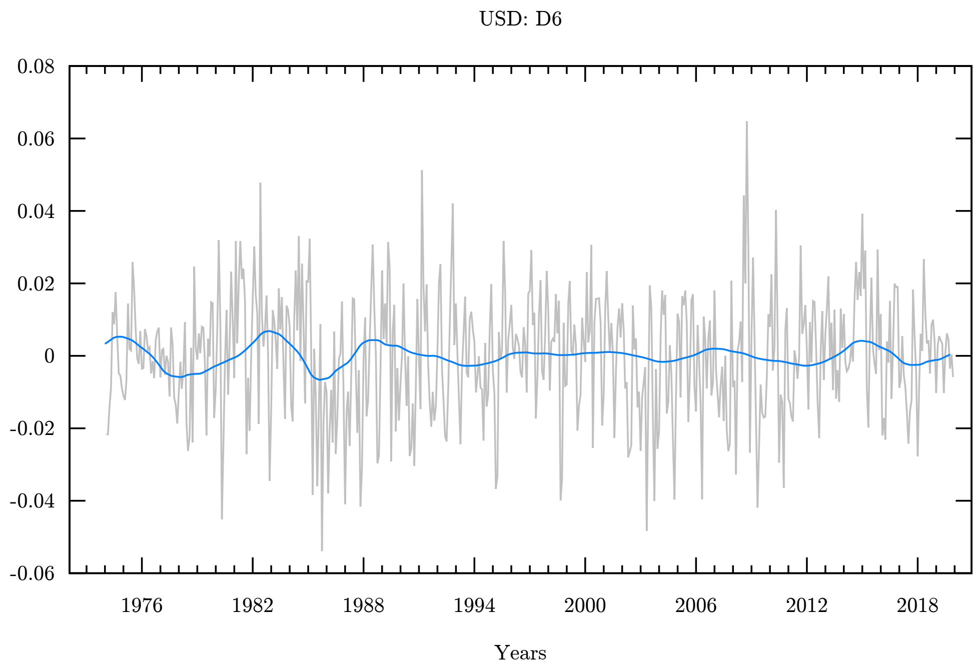

Figure 5.3: Plot of D6 component of wavelet decomposition of U.S. Dollar Index log returns. Plot of the original data in grey. D6 scale corresponds to 64-128 months.

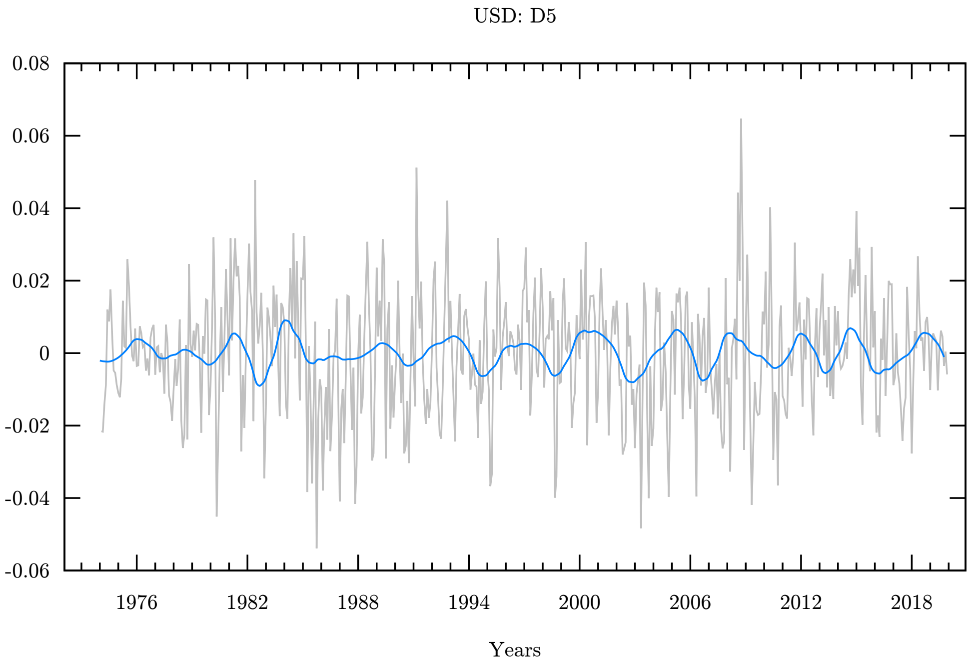

Figure 5.4: Plot of D5 component of wavelet decomposition of U.S. Dollar Index log returns. Plot of the original data in grey. D5 scale corresponds to 32-64 months.

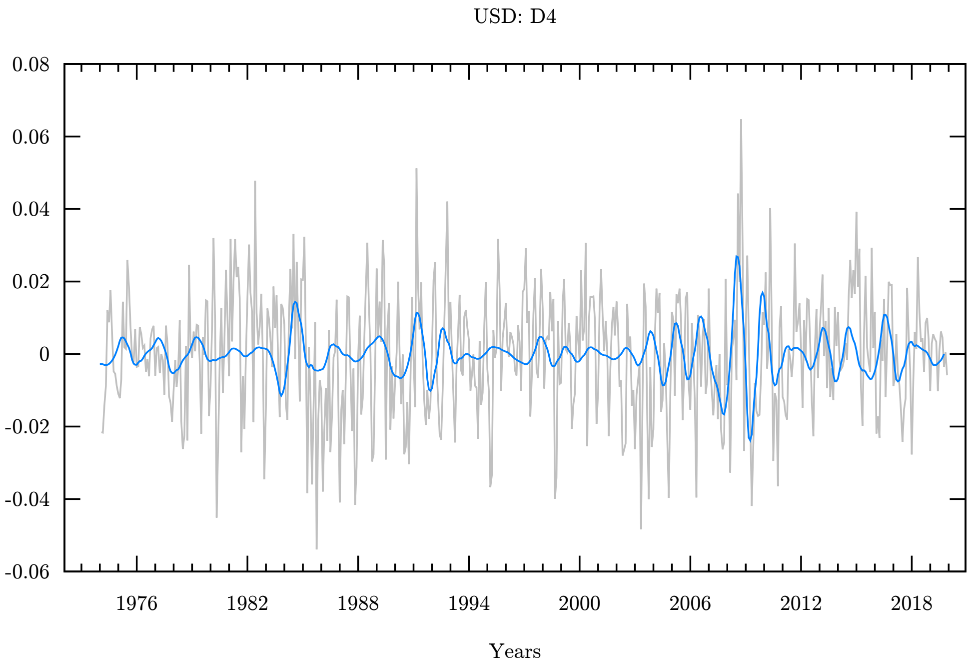

Figure 5.5: Plot of D4 component of wavelet decomposition of U.S. Dollar Index log returns. Plot of the original data in grey. D4 scale corresponds to 16-32 months.

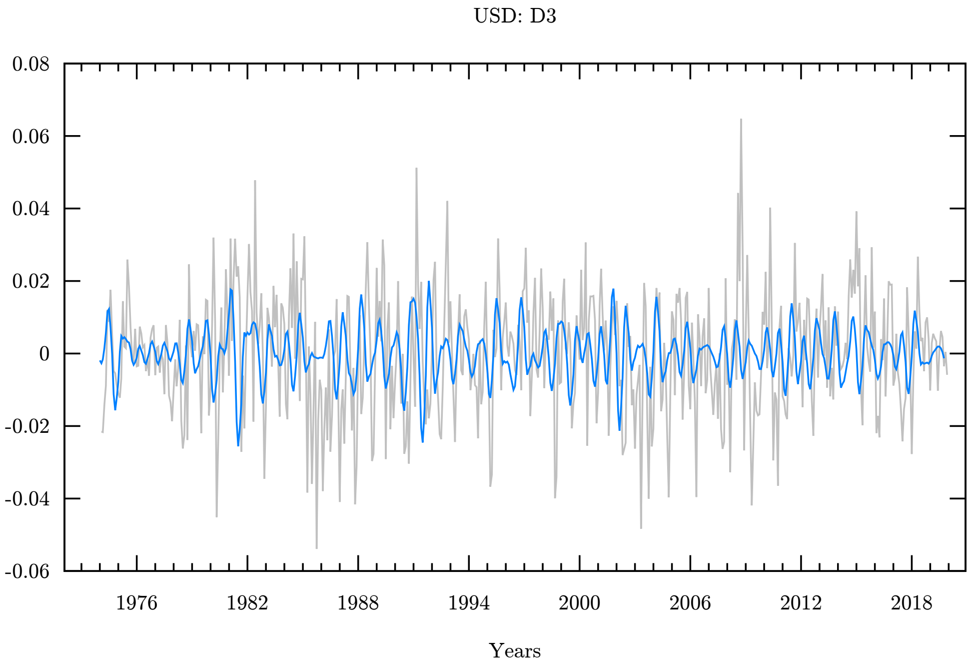

Figure 5.6: Plot of D3 component of wavelet decomposition of U.S. Dollar Index log returns. Plot of the original data in grey. D3 scale corresponds to 8-16 months.

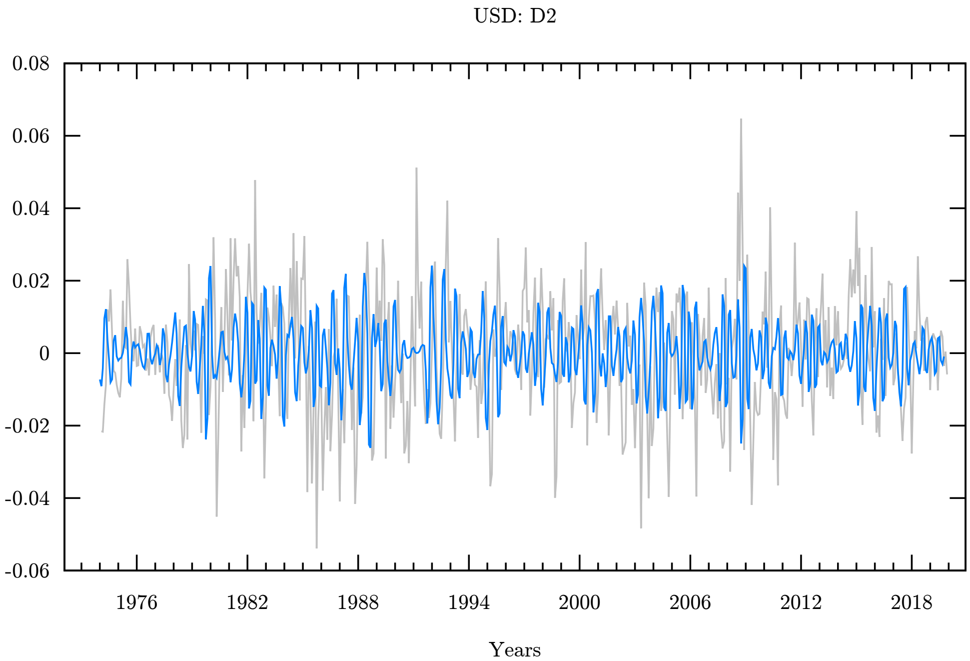

Figure 5.7: Plot of D2 component of wavelet decomposition of U.S. Dollar Index log returns. Plot of the original data in grey. D2 scale corresponds to 4-8 months.

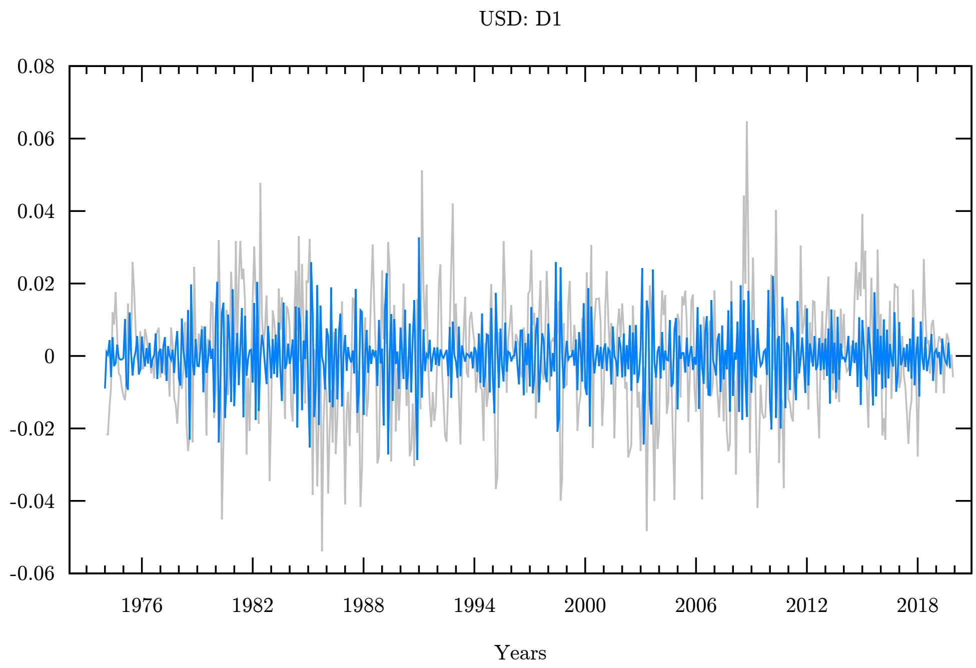

Figure 5.8: Plot of D1 component of wavelet decomposition of U.S. Dollar Index log returns. Plot of the original data in grey. D1 scale corresponds to 2-4 months.

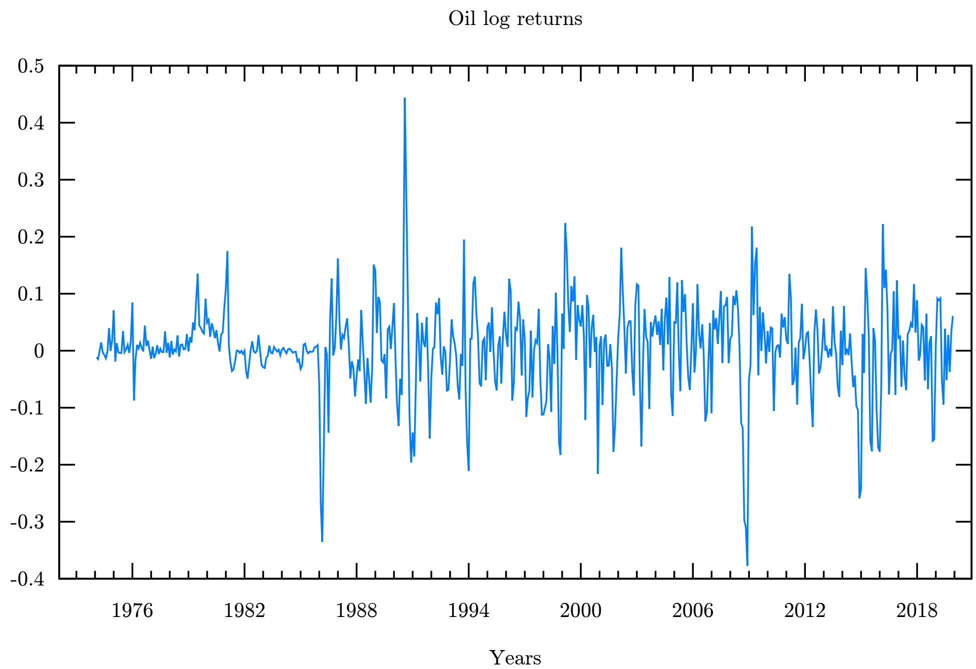

Figure 5.9: Plot of log returns of oil price.

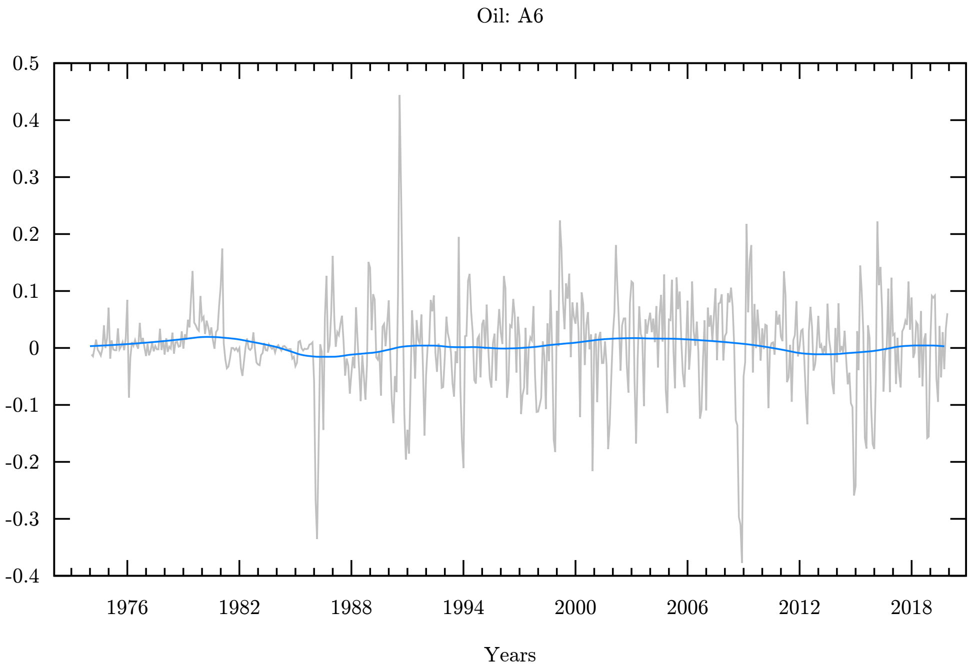

Figure 5.10: Plot of A6 component of wavelet decomposition oil price log returns. Plot of the original data in grey. A6 scale corresponds to >128 months.

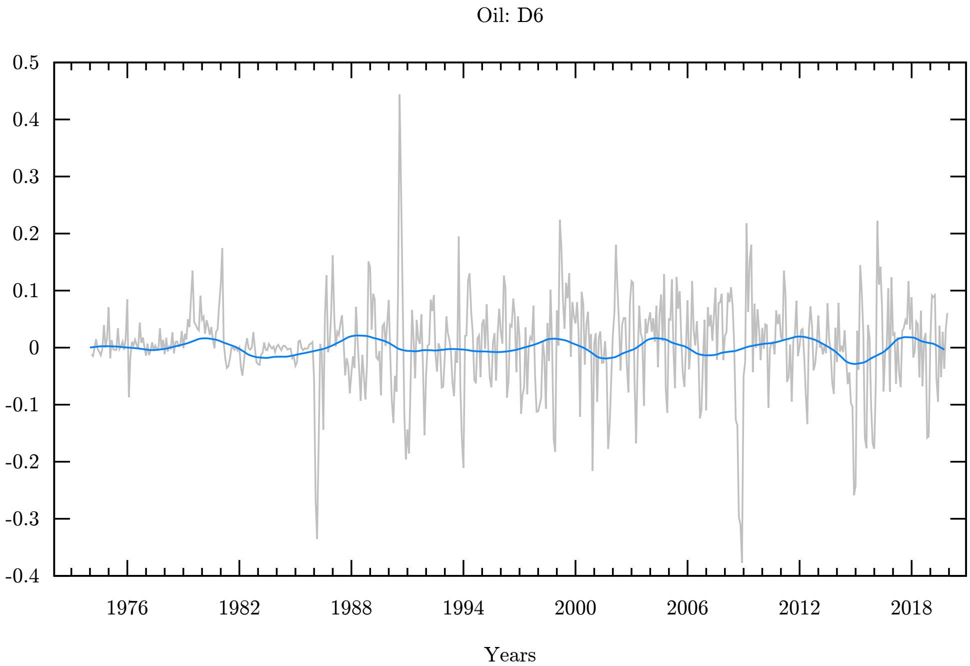

Figure 5.11: Plot of D6 component of wavelet decomposition of oil price log returns. Plot of the original data in grey. D6 scale corresponds to 64-128 months.

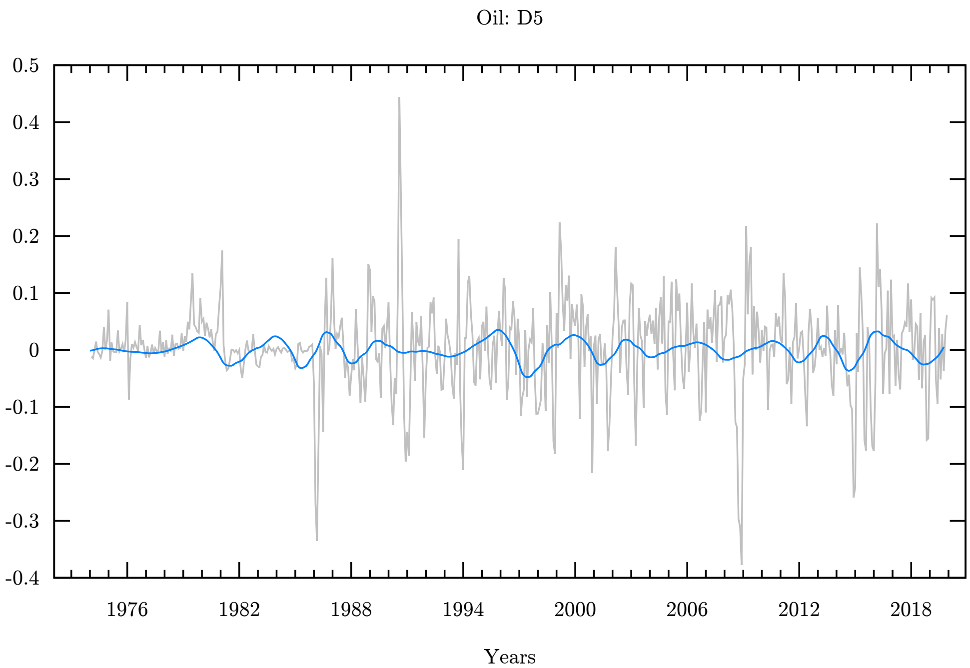

Figure 5.12: Plot of D5 component of wavelet decomposition of oil price log returns. Plot of the original data in grey. D5 scale corresponds to 32-64 months.

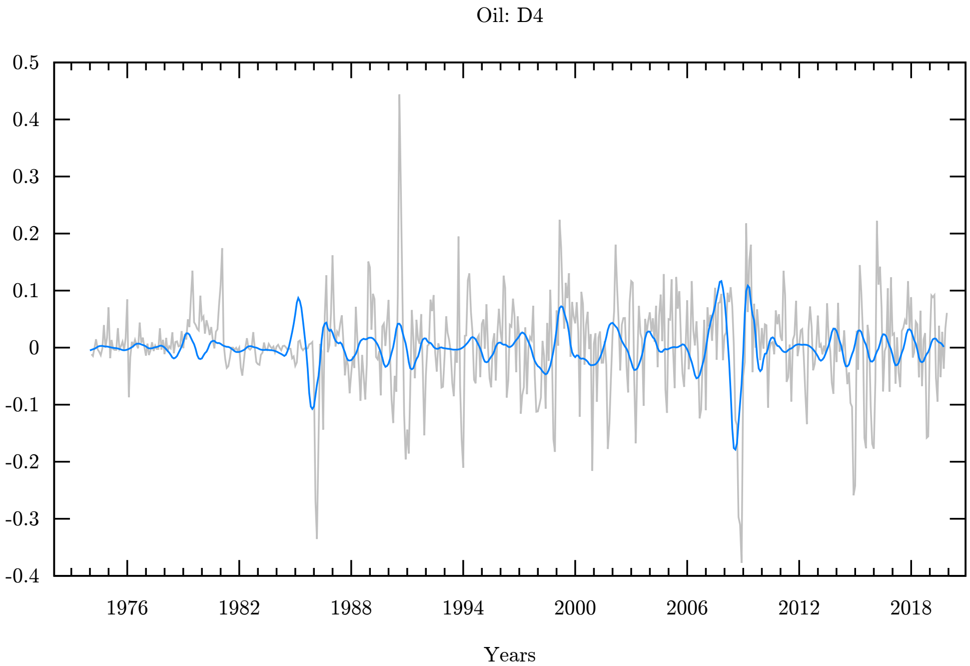

Figure 5.13: Plot of D4 component of wavelet decomposition of oil price log returns. Plot of the original data in grey. D4 scale corresponds to 16-32 months.

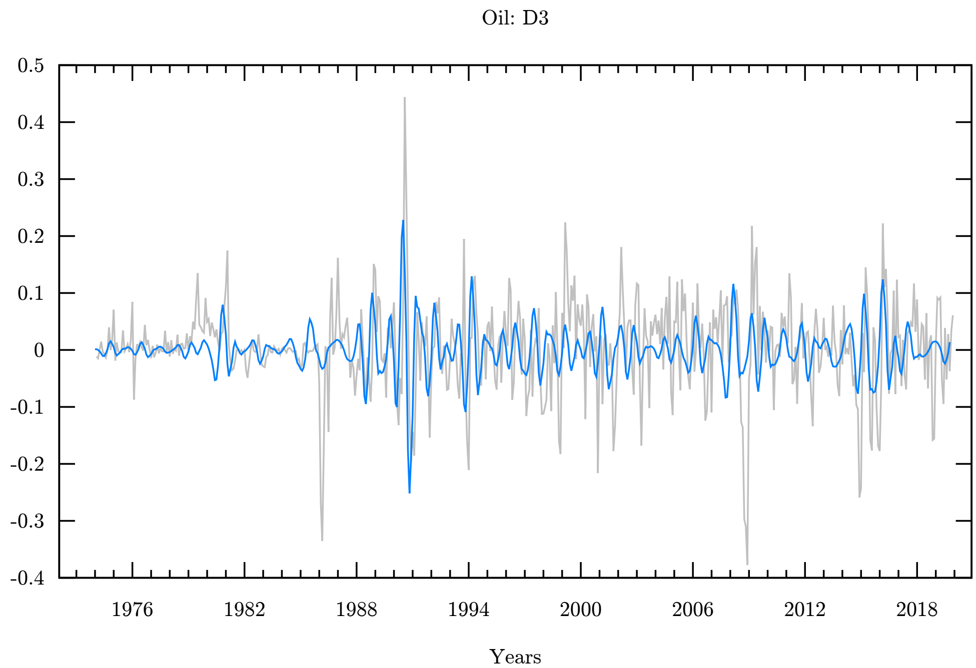

Figure 5.14: Plot of D3 component of wavelet decomposition of oil price log returns. Plot of the original data in grey. D3 scale corresponds to 8-16 months.

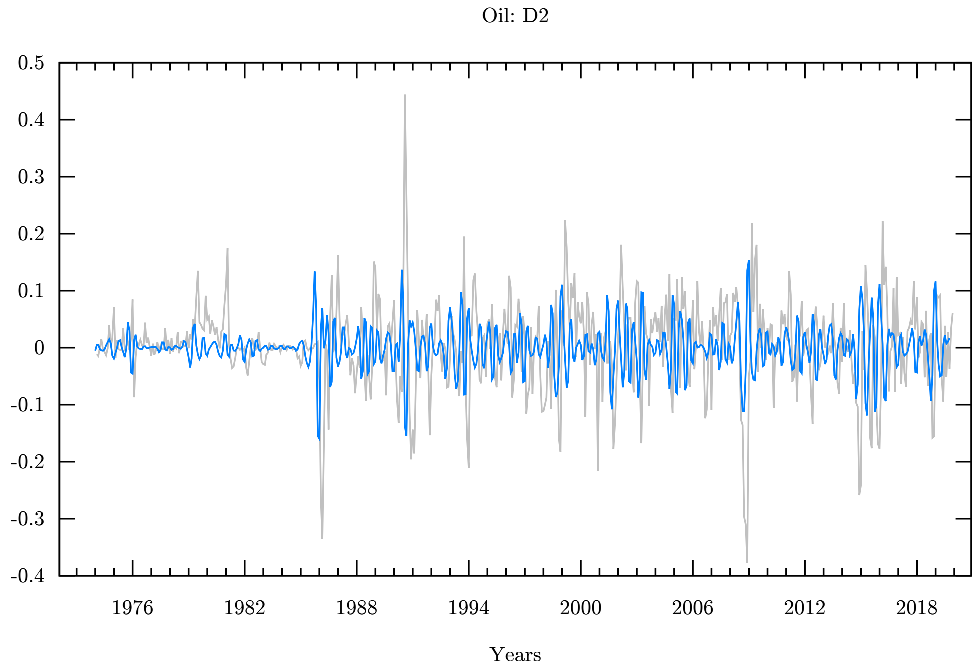

Figure 5.15: Plot of D2 component of wavelet decomposition of oil price log returns. Plot of the original data in grey. D2 scale corresponds to 4-8 months.

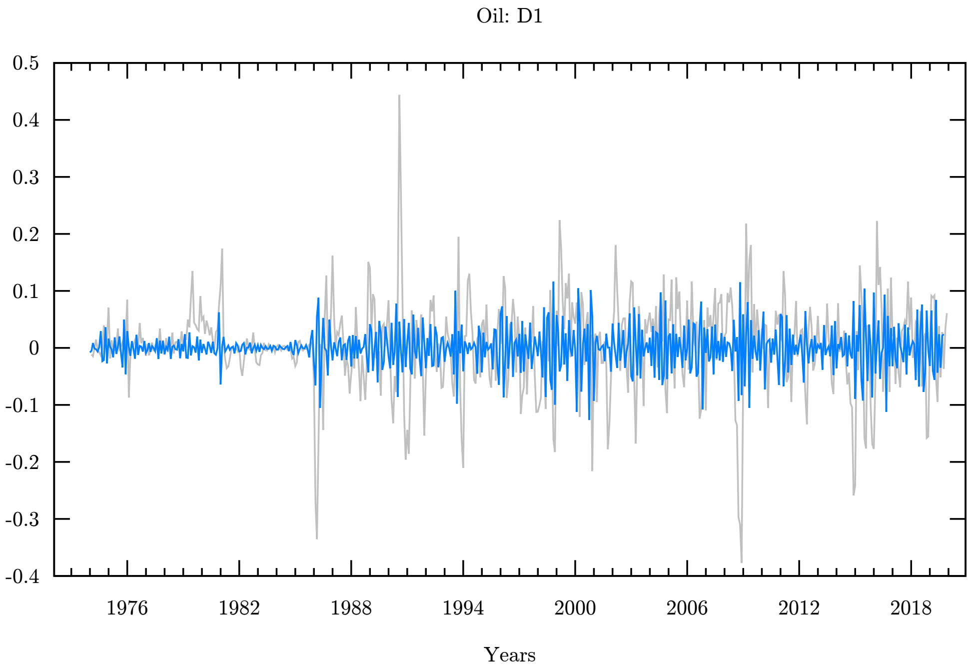

Figure 5.16: Plot of D1 component of wavelet decomposition of oil price log returns. Plot of the original data in grey. D1 scale corresponds to 2-4 months.

5.3 Causality tests results

In tables 5.2 to 5.29 results obtained from algorithm described in section 5.1 are collected. Cells containing p-values of $p < 0.05$ are marked with colored cell background: green for false-positive rates $r < 0.1$ from surrogate data, grey otherwise.

For detailed discussion regarding false-positive rates from surrogate data refer to section 5.4.

5.3a Linear causality test results

USD ⇒ Oil							
Causality from A6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.6200	0.0001	0.6100	0.9875	0.9993	0.9998	0.9935
2	0.3321	0.0077	0.6439	0.9943	0.9993	0.9998	0.9999
3	0.7979	0.2591	0.9150	0.9994	0.9968	0.9996	0.8537
4	0.8756	0.1705	0.7704	1.0000	0.9760	0.9999	0.5546
5	0.8545	0.2808	0.7016	1.0000	0.9923	0.9999	0.8883
6	0.8272	0.1676	0.7324	1.0000	0.9647	0.9966	0.9674

Table 5.2: p-values for linear Granger causality test under H_0:
U.S. Dollar Index A6 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

USD ⇒ Oil							
Causality from D6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.2686	0.0000	0.8785	0.5212	0.9635	0.9912	0.9906
2	0.0164	0.3609	0.8507	0.4577	0.9619	0.9989	0.9994
3	0.2598	0.7244	0.9835	0.7008	0.9602	1.0000	0.6645
4	0.3306	0.6557	0.9927	0.5924	0.7660	0.9979	0.4818
5	0.5460	0.6330	0.9982	0.1360	0.8186	0.9996	0.8271
6	0.3874	0.7689	0.9913	0.0524	0.8126	0.9722	0.9743

Table 5.3: p-values for linear Granger causality test under H_0:
U.S. Dollar Index D6 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

USD ⇒ Oil							
Causality from D5							
m	A6	D6	D5	D4	D3	D2	D1
1	0.0454	0.0000	0.0000	0.2192	0.8601	0.9319	0.9999
2	0.7743	0.3405	0.4076	0.3828	0.8463	0.9366	0.9906
3	0.9822	0.4960	0.6347	0.7134	0.9232	0.3698	0.7828
4	0.9105	0.6575	0.7249	0.5834	0.8687	0.7226	0.4026
5	0.9268	0.5320	0.3842	0.7342	0.8616	0.6901	0.4900
6	0.9592	0.5967	0.2039	0.6918	0.9815	0.9510	0.5241

Table 5.4: p-values for linear Granger causality test under H_0:
U.S. Dollar Index D5 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

USD ⇒ Oil							
Causality from D4							
m	A6	D6	D5	D4	D3	D2	D1
1	0.2897	0.9118	0.4934	0.9984	0.2238	0.8936	0.9500
2	0.4679	0.0000	0.9642	0.0964	0.0942	0.9349	0.9433
3	0.8458	0.0000	0.9950	0.3228	0.1420	0.8259	0.5060
4	0.8966	0.0000	0.9983	0.1069	0.1400	0.9318	0.1855
5	0.3120	0.0000	0.9997	0.4028	0.1501	0.9578	0.1915
6	0.1033	0.0000	0.9999	0.1459	0.0981	0.9892	0.4155

Table 5.5: p-values for linear Granger causality test under H_0:
U.S. Dollar Index D4 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

USD ⇒ Oil							
Causality from D3							
m	A6	D6	D5	D4	D3	D2	D1
1	0.5554	0.7389	0.2930	0.2254	0.3766	0.0293	0.9490
2	0.3045	0.5258	0.7617	0.7649	0.3534	0.0061	0.8265
3	0.2833	0.7572	0.9259	0.9097	0.4805	0.0038	0.2173
4	0.0031	0.7637	0.0946	0.8202	0.5700	0.0134	0.0468
5	0.0016	0.8090	0.1489	0.9177	0.3863	0.0166	0.1471
6	0.0028	0.9388	0.1538	0.9465	0.5367	0.0014	0.0417

Table 5.6: p-values for linear Granger causality test under H_0: U.S. Dollar Index D3 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

USD ⇒ Oil							
Causality from D2							
m	A6	D6	D5	D4	D3	D2	D1
1	0.9326	0.7834	0.5304	0.6002	0.9506	0.0569	0.0087
2	0.4000	0.4564	0.2379	0.0153	0.5042	0.4186	0.0212
3	0.3273	0.3233	0.0136	0.0274	0.5943	0.2531	0.0827
4	0.5227	0.5441	0.0821	0.2043	0.7386	0.3128	0.5504
5	0.6493	0.2822	0.0163	0.2737	0.7311	0.3002	0.0033
6	0.4422	0.0911	0.1994	0.1045	0.9044	0.5168	0.0164

Table 5.7: p-values for linear Granger causality test under H_0: U.S. Dollar Index D2 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

USD ⇒ Oil							
Causality from D1							
m	A6	D6	D5	D4	D3	D2	D1
1	0.9292	0.9711	0.9529	0.4526	0.9481	0.5671	0.6825
2	0.3159	0.7507	0.7759	0.0008	0.5673	0.7412	0.6033
3	0.1004	0.6860	0.7611	0.0001	0.7916	0.8142	0.4274
4	0.0111	0.4084	0.6206	0.0009	0.7802	0.8267	0.1251
5	0.0438	0.4081	0.6008	0.0000	0.7368	0.8271	0.2201
6	0.0416	0.0323	0.8419	0.0057	0.7761	0.9712	0.7402

Table 5.8: p-values for linear Granger causality test under H_0:
U.S. Dollar Index D1 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from A6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.7676	0.5882	0.6805	0.9183	0.9724	0.9989	0.9995
2	0.5515	0.7573	0.8023	0.7906	0.8958	0.9999	0.9980
3	0.9323	0.9680	0.9779	0.9025	0.6660	0.9999	0.5084
4	0.9239	0.9808	0.9626	0.9335	0.0183	0.7082	0.0118
5	0.9474	0.9939	0.9700	0.5962	0.0086	0.8274	0.1366
6	0.9626	0.9956	0.9727	0.3719	0.0903	0.2792	0.1779

Table 5.9: p-values for linear Granger causality test under H_0:
Oil price A6 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from D6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.0000	0.0000	0.0008	0.8216	0.9824	0.9879	0.9964
2	0.2255	0.1056	0.0405	0.7678	0.9887	0.9841	0.9969
3	0.9259	0.3966	0.3783	0.9453	0.9953	0.6416	0.9992
4	0.9175	0.3432	0.3546	0.9346	0.8935	0.2961	0.9980
5	0.7658	0.3822	0.4324	0.9864	0.9310	0.3356	0.9704
6	0.6122	0.4302	0.0664	0.9533	0.7071	0.1072	0.9829

Table 5.10: p-values for linear Granger causality test under H_0:
Oil price D6 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from D5							
m	A6	D6	D5	D4	D3	D2	D1
1	0.0461	0.5719	0.0000	0.7712	0.7659	0.8777	0.9784
2	0.0461	0.1964	0.1981	0.9494	0.7347	0.7654	0.9995
3	0.4729	0.6471	0.4561	0.9950	0.7950	0.0139	0.8992
4	0.2821	0.6982	0.5931	0.9955	0.0281	0.3864	0.8886
5	0.3916	0.8731	0.3859	0.9987	0.0251	0.0796	0.9588
6	0.4091	0.7887	0.1710	0.9985	0.0000	0.3595	0.9530

Table 5.11: p-values for linear Granger causality test under H_0:
Oil price D5 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from D4							
m	A6	D6	D5	D4	D3	D2	D1
1	0.9010	0.0443	0.0195	0.5891	0.5898	0.8336	0.8395
2	0.6622	0.0278	0.9649	0.7251	0.6616	0.8612	0.8895
3	0.8828	0.1732	0.9954	0.9078	0.8337	0.9426	0.9098
4	0.9532	0.2176	0.9851	0.8664	0.7540	0.9237	0.2216
5	0.9473	0.0392	0.9487	0.9594	0.7876	0.9820	0.0001
6	0.9352	0.0032	0.7793	0.9518	0.6759	0.7998	0.0002

Table 5.12: p-values for linear Granger causality test under H_0:
Oil price D4 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from D3							
m	A6	D6	D5	D4	D3	D2	D1
1	0.9896	0.8043	0.6714	0.0139	0.1205	0.6972	0.7198
2	0.6317	0.8826	0.6856	0.5757	0.3121	0.7095	0.8924
3	0.7328	0.9419	0.8378	0.7000	0.3889	0.8720	0.8404
4	0.8514	0.9268	0.8295	0.6842	0.0684	0.6031	0.8770
5	0.8742	0.9213	0.8948	0.8369	0.0092	0.4752	0.3290
6	0.9428	0.9862	0.9830	0.8828	0.0008	0.5216	0.3408

Table 5.13: p-values for linear Granger causality test under H_0:
Oil price D3 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from D2							
m	A6	D6	D5	D4	D3	D2	D1
1	0.9969	0.9451	0.7115	0.6052	0.0030	0.0912	0.3573
2	0.9905	0.8513	0.5932	0.7460	0.6588	0.3856	0.6155
3	0.9987	0.9245	0.3327	0.7449	0.0802	0.2836	0.6031
4	0.9722	0.9456	0.5685	0.9024	0.0379	0.4532	0.6716
5	0.9896	0.9634	0.5444	0.8651	0.0209	0.2296	0.6662
6	0.8734	0.9152	0.7554	0.9723	0.0079	0.2301	0.8294

Table 5.14: p-values for linear Granger causality test under H_0:
Oil price D2 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from D1							
m	A6	D6	D5	D4	D3	D2	D1
1	0.9916	0.9701	0.8991	0.7316	0.5127	0.0065	0.0733
2	0.9486	0.6341	0.6098	0.2369	0.1359	0.0529	0.1745
3	0.8988	0.3938	0.6711	0.2593	0.0944	0.0080	0.0579
4	0.6831	0.1661	0.7814	0.1933	0.1342	0.0978	0.0197
5	0.6617	0.0851	0.8576	0.2702	0.3237	0.0490	0.0712
6	0.7496	0.1530	0.9839	0.3712	0.3653	0.6451	0.3483

Table 5.15: p-values for linear Granger causality test under H_0:
Oil price D1 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

5.3b Nonlinear causality test results

USD ⇒ Oil							
Causality from A6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.2602	0.1801	0.0110	0.1271	0.5958	0.2226	0.9849
2	0.2475	0.1510	0.0089	0.1121	0.3139	0.3435	0.9752
3	0.2786	0.1089	0.0077	0.1259	0.1900	0.2829	0.9647
4	0.3205	0.0903	0.0059	0.1773	0.1761	0.1887	0.9437
5	0.3471	0.0815	0.0053	0.2723	0.2099	0.4024	0.8637
6	0.3779	0.0580	0.0054	0.1654	0.2445	0.4090	0.8349

Table 5.16: p-values for nonlinear Granger causality test under H_0: U.S. Dollar Index A6 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

USD ⇒ Oil							
Causality from D6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.5475	0.3714	0.7598	0.7686	0.9666	0.9451	1.0000
2	0.5699	0.4127	0.6266	0.7599	0.9394	0.9551	1.0000
3	0.5624	0.3879	0.5892	0.8440	0.9366	0.9450	1.0000
4	0.4855	0.4100	0.4303	0.9793	0.9284	0.9461	1.0000
5	0.5101	0.4298	0.3009	0.9983	0.9478	0.9782	0.9999
6	0.4764	0.4134	0.2191	0.9986	0.9645	0.9882	0.9998

Table 5.17: p-values for nonlinear Granger causality test under H_0: U.S. Dollar Index D6 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

USD ⇒ Oil							
Causality from D5							
m	A6	D6	D5	D4	D3	D2	D1
1	0.6061	0.2970	0.0204	0.3799	0.8972	0.0482	0.1244
2	0.7409	0.2068	0.0425	0.1591	0.8743	0.1381	0.2722
3	0.6984	0.0991	0.0723	0.0992	0.8222	0.1956	0.4808
4	0.5813	0.0489	0.0606	0.0706	0.8261	0.2932	0.5859
5	0.5061	0.0244	0.0521	0.0365	0.7597	0.3963	0.6216
6	0.4667	0.0113	0.0280	0.0219	0.7661	0.5343	0.7217

Table 5.18: p-values for nonlinear Granger causality test under H_0: U.S. Dollar Index D5 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

USD ⇒ Oil							
Causality from D4							
m	A6	D6	D5	D4	D3	D2	D1
1	0.2509	0.9404	0.8650	0.0451	0.0197	0.5085	0.0630
2	0.1953	0.9169	0.8571	0.0818	0.0205	0.6336	0.0561
3	0.1695	0.8912	0.9127	0.1562	0.0426	0.4878	0.0790
4	0.2226	0.8533	0.9064	0.2219	0.1136	0.5955	0.0995
5	0.1466	0.7209	0.9069	0.1638	0.1634	0.5019	0.1051
6	0.1553	0.6655	0.9161	0.1412	0.1618	0.5239	0.1448

Table 5.19: p-values for nonlinear Granger causality test under H_0: U.S. Dollar Index D4 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

USD ⇒ Oil							
Causality from D3							
m	A6	D6	D5	D4	D3	D2	D1
1	0.8321	0.1413	0.1072	0.5894	0.9903	0.2205	0.9428
2	0.8977	0.0912	0.0580	0.4987	0.8125	0.4570	0.8425
3	0.6684	0.1541	0.0120	0.5752	0.3567	0.8466	0.6201
4	0.6310	0.1402	0.0140	0.6125	0.2248	0.9323	0.5540
5	0.5897	0.1714	0.0224	0.6655	0.2655	0.8799	0.4732
6	0.6643	0.1815	0.0287	0.6823	0.2622	0.7728	0.6843

Table 5.20: p-values for nonlinear Granger causality test under H_0:
U.S. Dollar Index D3 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

USD ⇒ Oil							
Causality from D2							
m	A6	D6	D5	D4	D3	D2	D1
1	0.8236	0.2851	0.8916	0.1655	0.0635	0.3256	0.8205
2	0.8564	0.2968	0.9625	0.2509	0.0474	0.5410	0.8710
3	0.9505	0.2074	0.9691	0.4570	0.2881	0.3363	0.7351
4	0.8587	0.1915	0.8739	0.4721	0.3988	0.5711	0.9116
5	0.8576	0.2236	0.8690	0.4594	0.1780	0.1184	0.7311
6	0.7137	0.2050	0.7919	0.5035	0.1722	0.2111	0.8718

Table 5.21: p-values for nonlinear Granger causality test under H_0:
U.S. Dollar Index D2 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

USD ⇒ Oil							
Causality from D1							
m	A6	D6	D5	D4	D3	D2	D1
1	0.1175	0.7812	0.0809	0.2071	0.9418	0.2166	0.4375
2	0.0230	0.6049	0.1167	0.2027	0.8965	0.7806	0.6585
3	0.0531	0.6313	0.1682	0.0612	0.8368	0.5951	0.5796
4	0.0904	0.4633	0.1486	0.0795	0.6083	0.7955	0.7515
5	0.1078	0.4808	0.2011	0.0237	0.4726	0.7246	0.5281
6	0.1095	0.3740	0.3373	0.0129	0.5126	0.7602	0.5760

Table 5.22: p-values for nonlinear Granger causality test under H_0: U.S. Dollar Index D1 band does not cause oil price. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from A6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.0011	0.1094	0.6446	0.9838	0.0474	0.0945	0.0201
2	0.0012	0.0875	0.6406	0.9816	0.0297	0.1376	0.0168
3	0.0012	0.0747	0.6869	0.9392	0.0348	0.1419	0.0389
4	0.0013	0.0573	0.7005	0.8232	0.0462	0.1449	0.0334
5	0.0005	0.0415	0.7003	0.7989	0.0437	0.1785	0.0331
6	0.0004	0.0228	0.6946	0.7415	0.0445	0.2626	0.0377

Table 5.23: p-values for nonlinear Granger causality test under H_0: Oil price A6 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from D6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.3737	0.3527	0.1049	0.4397	0.0102	0.1951	0.3541
2	0.4282	0.3881	0.0749	0.3459	0.0054	0.3121	0.3197
3	0.4668	0.3867	0.0851	0.1555	0.0149	0.3988	0.2624
4	0.5306	0.3525	0.0890	0.0552	0.0298	0.3019	0.2146
5	0.5603	0.3219	0.0507	0.0647	0.0512	0.2429	0.2229
6	0.5967	0.2841	0.0289	0.1245	0.0260	0.2735	0.2018

Table 5.24: p-values for nonlinear Granger causality test under H_0:
Oil price D6 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from D5							
m	A6	D6	D5	D4	D3	D2	D1
1	0.2759	0.1876	0.3420	0.9583	0.5390	0.1123	0.1832
2	0.2562	0.2383	0.2151	0.9683	0.3619	0.0757	0.1138
3	0.3191	0.2216	0.1292	0.9332	0.5207	0.0642	0.1190
4	0.4248	0.1684	0.1299	0.8993	0.3067	0.0870	0.0852
5	0.4486	0.1263	0.1040	0.8386	0.1880	0.1135	0.0791
6	0.4925	0.1444	0.0723	0.7859	0.2058	0.1782	0.0797

Table 5.25: p-values for nonlinear Granger causality test under H_0:
Oil price D5 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from D4							
m	A6	D6	D5	D4	D3	D2	D1
1	0.2731	0.6277	0.3336	0.3489	0.7286	0.3420	0.0082
2	0.3138	0.6854	0.3567	0.0813	0.6757	0.4584	0.0110
3	0.3752	0.6402	0.3553	0.1819	0.8284	0.3511	0.0128
4	0.4009	0.6380	0.3047	0.3140	0.9089	0.2211	0.0147
5	0.3913	0.6700	0.2439	0.1424	0.9413	0.1619	0.0370
6	0.4511	0.7550	0.2855	0.0670	0.9260	0.1745	0.0439

Table 5.26: p-values for nonlinear Granger causality test under H_0:
Oil price D4 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from D3							
m	A6	D6	D5	D4	D3	D2	D1
1	0.9703	0.9996	0.3314	0.1016	0.5356	0.2904	0.2930
2	0.9482	0.9999	0.5522	0.0443	0.5068	0.7872	0.1803
3	0.9028	0.9998	0.4845	0.0399	0.2987	0.5428	0.1435
4	0.9083	0.9997	0.4446	0.0654	0.3645	0.4266	0.2993
5	0.8635	0.9997	0.5316	0.0831	0.4782	0.2775	0.3628
6	0.9351	0.9996	0.5272	0.0994	0.3425	0.3250	0.3012

Table 5.27: p-values for nonlinear Granger causality test under H_0:
Oil price D3 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from D2							
m	A6	D6	D5	D4	D3	D2	D1
1	0.9222	0.9999	0.4585	0.3292	0.0838	0.6833	0.4883
2	0.9756	1.0000	0.6908	0.4908	0.1561	0.7349	0.7778
3	0.9771	1.0000	0.6753	0.7212	0.4909	0.5241	0.7889
4	0.9826	1.0000	0.5577	0.7362	0.7634	0.6822	0.9084
5	0.9862	1.0000	0.3549	0.5778	0.5848	0.5475	0.7671
6	0.9824	0.9999	0.3220	0.4573	0.4876	0.7326	0.7932

Table 5.28: p-values for nonlinear Granger causality test under H_0:
Oil price D2 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

Oil ⇒ USD							
Causality from D1							
m	A6	D6	D5	D4	D3	D2	D1
1	0.9720	0.9996	0.4167	0.8559	0.4429	0.6668	0.2193
2	0.9740	1.0000	0.5017	0.7261	0.6770	0.9794	0.5355
3	0.9792	0.9999	0.4631	0.7617	0.5792	0.8982	0.5856
4	0.9518	0.9999	0.4493	0.6379	0.6806	0.9145	0.7983
5	0.9584	1.0000	0.3852	0.4724	0.5281	0.8588	0.7714
6	0.9666	0.9999	0.3033	0.4183	0.4873	0.9433	0.8510

Table 5.29: p-values for nonlinear Granger causality test under H_0:
Oil price D1 band does not cause U.S. Dollar Index. Statistical significance assumed for $p < 0.05$.

5.4 Surrogate data results

$N = 1000$ surrogate data sets were generated by phase randomisation scheme, as described in section 3.3b. Then, causality tests were run for each realization. Number of positive ($p < 0.05$) results in the generated time series were counted and divided by N. Acquired number can be iterpreted as probability of getting a false-positive result for causality tests and used to asses results fidelity. A false-positive rate of $r < 0.1$ was assumed as a treshold for causality test fidelity in the original data. Results for surrogate data tests are gathered in tables 5.30 to 5.57. Cells corresponding to a positive test in the original data are marked with colored cell background: grey for $r \geq 0.1$ and green for $r < 0.1$.

5.4a Linear causality in surrogate data

USD ⇒ Oil							
Causality from A6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.77	0.22	0.00	0.00	0.00	0.00	0.00
2	0.42	0.10	0.00	0.00	0.00	0.00	0.00
3	0.11	0.00	0.00	0.00	0.00	0.09	0.00
4	0.10	0.01	0.00	0.00	0.12	0.05	0.07
5	0.07	0.00	0.01	0.02	0.12	0.10	0.06
6	0.13	0.01	0.01	0.06	0.15	0.16	0.06

Table 5.30: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from U.S. Dollar Index band A6 to oil price.

USD ⇒ Oil							
Causality from D6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.74	0.57	0.27	0.00	0.00	0.00	0.00
2	0.40	0.13	0.15	0.00	0.00	0.00	0.00
3	0.12	0.01	0.03	0.00	0.00	0.06	0.01
4	0.12	0.02	0.04	0.01	0.12	0.06	0.04
5	0.08	0.01	0.04	0.07	0.13	0.07	0.05
6	0.13	0.01	0.15	0.11	0.12	0.10	0.05

Table 5.31: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from U.S. Dollar Index band D6 to oil price.

USD ⇒ Oil							
Causality from D5							
m	A6	D6	D5	D4	D3	D2	D1
1	0.35	0.64	0.36	0.13	0.00	0.00	0.00
2	0.41	0.10	0.13	0.12	0.01	0.00	0.00
3	0.21	0.02	0.03	0.06	0.01	0.10	0.01
4	0.21	0.02	0.03	0.12	0.13	0.07	0.05
5	0.20	0.03	0.05	0.09	0.14	0.11	0.06
6	0.24	0.05	0.17	0.16	0.18	0.13	0.06

Table 5.32: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from U.S. Dollar Index band D5 to oil price.

USD ⇒ Oil							
Causality from D4							
m	A6	D6	D5	D4	D3	D2	D1
1	0.02	0.10	0.45	0.22	0.12	0.00	0.00
2	0.20	0.27	0.06	0.10	0.15	0.01	0.00
3	0.11	0.15	0.02	0.05	0.11	0.05	0.05
4	0.10	0.15	0.03	0.15	0.15	0.09	0.09
5	0.13	0.19	0.03	0.11	0.15	0.08	0.10
6	0.18	0.25	0.10	0.16	0.20	0.09	0.05

Table 5.33: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from U.S. Dollar Index band D4 to oil price.

USD ⇒ Oil							
Causality from D3							
m	A6	D6	D5	D4	D3	D2	D1
1	0.00	0.00	0.04	0.28	0.12	0.08	0.00
2	0.09	0.07	0.23	0.06	0.09	0.14	0.01
3	0.09	0.05	0.17	0.04	0.10	0.13	0.04
4	0.19	0.11	0.27	0.09	0.14	0.13	0.07
5	0.20	0.13	0.25	0.07	0.16	0.19	0.11
6	0.17	0.07	0.19	0.07	0.17	0.18	0.13

Table 5.34: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from U.S. Dollar Index band D3 to oil price.

USD ⇒ Oil							
Causality from D2							
m	A6	D6	D5	D4	D3	D2	D1
1	0.00	0.00	0.00	0.01	0.14	0.07	0.10
2	0.03	0.07	0.07	0.12	0.06	0.09	0.09
3	0.12	0.16	0.16	0.16	0.08	0.10	0.11
4	0.10	0.12	0.11	0.12	0.08	0.10	0.11
5	0.12	0.15	0.16	0.13	0.15	0.15	0.14
6	0.14	0.15	0.15	0.09	0.12	0.13	0.13

Table 5.35: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from U.S. Dollar Index band D2 to oil price.

USD ⇒ Oil							
Causality from D1							
m	A6	D6	D5	D4	D3	D2	D1
1	0.00	0.00	0.00	0.00	0.00	0.05	0.08
2	0.01	0.04	0.04	0.06	0.07	0.06	0.07
3	0.09	0.09	0.07	0.09	0.07	0.09	0.09
4	0.12	0.12	0.12	0.09	0.09	0.08	0.10
5	0.13	0.13	0.13	0.13	0.10	0.12	0.10
6	0.13	0.13	0.11	0.10	0.11	0.11	0.12

Table 5.36: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from U.S. Dollar Index band D1 to oil price.

Oil ⇒ USD							
Causality from A6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.73	0.36	0.00	0.00	0.00	0.00	0.00
2	0.42	0.17	0.00	0.00	0.00	0.00	0.00
3	0.05	0.00	0.00	0.00	0.00	0.04	0.00
4	0.05	0.02	0.00	0.00	0.08	0.02	0.06
5	0.04	0.03	0.02	0.04	0.09	0.04	0.06
6	0.11	0.06	0.03	0.08	0.11	0.10	0.07

Table 5.37: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from oil price band A6 to U.S. Dollar Index.

Oil ⇒ USD							
Causality from D6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.60	0.60	0.40	0.00	0.00	0.00	0.00
2	0.12	0.13	0.19	0.00	0.00	0.00	0.00
3	0.01	0.01	0.05	0.00	0.00	0.06	0.01
4	0.01	0.02	0.06	0.00	0.06	0.05	0.07
5	0.00	0.02	0.06	0.06	0.08	0.07	0.06
6	0.01	0.02	0.16	0.10	0.07	0.10	0.06

Table 5.38: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from oil price band D6 to U.S. Dollar Index.

Oil ⇒ USD							
Causality from D5							
m	A6	D6	D5	D4	D3	D2	D1
1	0.22	0.57	0.39	0.18	0.00	0.00	0.00
2	0.31	0.13	0.13	0.15	0.01	0.00	0.00
3	0.13	0.02	0.03	0.07	0.01	0.10	0.01
4	0.12	0.02	0.04	0.15	0.12	0.05	0.07
5	0.11	0.03	0.06	0.09	0.13	0.10	0.08
6	0.15	0.04	0.17	0.15	0.19	0.15	0.09

Table 5.39: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from oil price band D5 to U.S. Dollar Index.

Oil ⇒ USD							
Causality from D4							
m	A6	D6	D5	D4	D3	D2	D1
1	0.02	0.12	0.43	0.22	0.09	0.00	0.00
2	0.14	0.28	0.07	0.11	0.14	0.01	0.00
3	0.07	0.16	0.03	0.06	0.11	0.09	0.04
4	0.06	0.16	0.04	0.15	0.13	0.10	0.09
5	0.08	0.19	0.03	0.12	0.13	0.09	0.09
6	0.13	0.24	0.10	0.16	0.20	0.11	0.05

Table 5.40: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from oil price band D4 to U.S. Dollar Index.

Oil ⇒ USD							
Causality from D3							
m	A6	D6	D5	D4	D3	D2	D1
1	0.00	0.01	0.06	0.31	0.13	0.05	0.00
2	0.16	0.12	0.22	0.07	0.10	0.13	0.00
3	0.16	0.09	0.15	0.05	0.11	0.14	0.04
4	0.25	0.21	0.27	0.13	0.16	0.16	0.07
5	0.26	0.21	0.25	0.09	0.20	0.19	0.12
6	0.22	0.15	0.19	0.09	0.21	0.20	0.14

Table 5.41: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from oil price band D3 to U.S. Dollar Index.

Oil ⇒ USD							
Causality from D2							
m	A6	D6	D5	D4	D3	D2	D1
1	0.00	0.00	0.00	0.01	0.18	0.06	0.08
2	0.07	0.06	0.08	0.12	0.06	0.10	0.07
3	0.22	0.16	0.16	0.14	0.08	0.11	0.12
4	0.19	0.13	0.12	0.11	0.07	0.10	0.11
5	0.21	0.16	0.16	0.11	0.14	0.13	0.14
6	0.24	0.18	0.15	0.09	0.12	0.11	0.13

Table 5.42: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from oil price band D2 to U.S. Dollar Index.

Oil ⇒ USD							
Causality from D1							
m	A6	D6	D5	D4	D3	D2	D1
1	0.00	0.00	0.00	0.00	0.00	0.07	0.06
2	0.02	0.03	0.03	0.06	0.07	0.06	0.07
3	0.07	0.09	0.06	0.11	0.07	0.10	0.11
4	0.11	0.10	0.10	0.09	0.08	0.09	0.11
5	0.11	0.10	0.11	0.14	0.09	0.11	0.11
6	0.13	0.12	0.10	0.11	0.09	0.11	0.12

Table 5.43: Probability of getting $p < 0.05$ for generated surrogate data for linear test for causality from oil price band D1 to U.S. Dollar Index.

5.4b Nonlinear causality in surrogate data

USD ⇒ Oil							
Causality from A6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.20	0.08	0.11	0.16	0.18	0.18	0.18
2	0.20	0.09	0.12	0.16	0.17	0.16	0.17
3	0.20	0.09	0.13	0.16	0.15	0.14	0.16
4	0.20	0.10	0.14	0.13	0.12	0.13	0.16
5	0.21	0.11	0.14	0.12	0.12	0.12	0.16
6	0.21	0.11	0.15	0.10	0.11	0.12	0.15

Table 5.44: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from U.S. Dollar A6 to oil price.

USD ⇒ Oil							
Causality from D6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.14	0.13	0.08	0.14	0.20	0.19	0.18
2	0.14	0.13	0.10	0.15	0.18	0.16	0.17
3	0.15	0.13	0.12	0.14	0.16	0.15	0.17
4	0.15	0.13	0.13	0.13	0.14	0.14	0.16
5	0.16	0.13	0.13	0.11	0.13	0.14	0.15
6	0.18	0.14	0.14	0.10	0.12	0.14	0.15

Table 5.45: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from U.S. Dollar band D6 to oil price.

USD ⇒ Oil							
Causality from D5							
m	A6	D6	D5	D4	D3	D2	D1
1	0.10	0.06	0.07	0.10	0.17	0.15	0.13
2	0.10	0.09	0.07	0.12	0.16	0.15	0.13
3	0.11	0.11	0.09	0.12	0.13	0.11	0.13
4	0.13	0.13	0.10	0.11	0.11	0.12	0.14
5	0.14	0.15	0.13	0.09	0.10	0.12	0.14
6	0.15	0.17	0.13	0.08	0.10	0.13	0.14

Table 5.46: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from U.S. Dollar Index band D5 to oil price.

USD ⇒ Oil							
Causality from D4							
m	A6	D6	D5	D4	D3	D2	D1
1	0.10	0.04	0.06	0.07	0.13	0.13	0.12
2	0.11	0.06	0.09	0.08	0.14	0.11	0.11
3	0.13	0.08	0.12	0.09	0.10	0.10	0.11
4	0.13	0.09	0.14	0.09	0.09	0.11	0.12
5	0.15	0.10	0.15	0.09	0.08	0.10	0.11
6	0.15	0.12	0.15	0.09	0.07	0.10	0.11

Table 5.47: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from U.S. Dollar Index band D4 to oil price.

USD ⇒ Oil							
Causality from D3							
m	A6	D6	D5	D4	D3	D2	D1
1	0.07	0.05	0.06	0.07	0.07	0.12	0.10
2	0.09	0.07	0.09	0.13	0.09	0.11	0.10
3	0.11	0.09	0.11	0.13	0.11	0.11	0.11
4	0.12	0.10	0.12	0.12	0.09	0.09	0.11
5	0.12	0.11	0.12	0.09	0.07	0.10	0.11
6	0.12	0.12	0.12	0.07	0.07	0.09	0.10

Table 5.48: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from U.S. Dollar Index band D3 to oil price.

USD ⇒ Oil							
Causality from D2							
m	A6	D6	D5	D4	D3	D2	D1
1	0.07	0.06	0.05	0.08	0.14	0.11	0.09
2	0.10	0.07	0.07	0.11	0.16	0.10	0.09
3	0.11	0.08	0.10	0.09	0.12	0.09	0.12
4	0.12	0.09	0.11	0.10	0.09	0.09	0.11
5	0.12	0.11	0.11	0.09	0.10	0.11	0.12
6	0.14	0.12	0.11	0.09	0.10	0.10	0.11

Table 5.49: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from U.S. Dollar Index band D2 to oil price.

USD ⇒ Oil							
Causality from D1							
m	A6	D6	D5	D4	D3	D2	D1
1	0.07	0.05	0.04	0.08	0.11	0.10	0.08
2	0.10	0.06	0.06	0.10	0.12	0.09	0.09
3	0.11	0.08	0.08	0.10	0.11	0.11	0.12
4	0.12	0.09	0.10	0.09	0.10	0.10	0.11
5	0.14	0.11	0.11	0.08	0.10	0.11	0.12
6	0.15	0.12	0.12	0.07	0.09	0.11	0.13

Table 5.50: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from U.S. Dollar Index band D1 to oil price.

Oil ⇒ USD							
Causality from A6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.20	0.01	0.14	0.18	0.22	0.22	0.22
2	0.20	0.10	0.14	0.18	0.18	0.19	0.20
3	0.21	0.12	0.15	0.16	0.16	0.16	0.19
4	0.22	0.13	0.16	0.14	0.14	0.16	0.19
5	0.22	0.14	0.16	0.12	0.13	0.16	0.18
6	0.22	0.15	0.14	0.11	0.12	0.15	0.17

Table 5.51: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from oil price band A6 to U.S. Dollar Index.

Oil ⇒ USD							
Causality from D6							
m	A6	D6	D5	D4	D3	D2	D1
1	0.09	0.14	0.08	0.12	0.17	0.18	0.18
2	0.09	0.13	0.10	0.13	0.16	0.15	0.17
3	0.09	0.12	0.12	0.11	0.12	0.13	0.15
4	0.10	0.11	0.13	0.11	0.11	0.13	0.15
5	0.11	0.11	0.14	0.10	0.11	0.12	0.16
6	0.12	0.12	0.13	0.09	0.11	0.12	0.16

Table 5.52: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from oil price band D6 to U.S. Dollar Index.

Oil ⇒ USD							
Causality from D5							
m	A6	D6	D5	D4	D3	D2	D1
1	0.07	0.08	0.07	0.11	0.16	0.15	0.16
2	0.08	0.09	0.08	0.14	0.15	0.14	0.15
3	0.10	0.11	0.09	0.13	0.12	0.12	0.15
4	0.11	0.14	0.10	0.13	0.11	0.12	0.15
5	0.13	0.15	0.11	0.12	0.10	0.12	0.15
6	0.14	0.17	0.12	0.09	0.10	0.13	0.15

Table 5.53: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from oil price band D5 to U.S. Dollar Index.

Oil ⇒ USD							
Causality from D4							
m	A6	D6	D5	D4	D3	D2	D1
1	0.08	0.06	0.06	0.06	0.13	0.13	0.12
2	0.10	0.09	0.09	0.08	0.15	0.11	0.13
3	0.12	0.10	0.11	0.08	0.12	0.10	0.13
4	0.13	0.12	0.13	0.09	0.09	0.11	0.13
5	0.13	0.13	0.14	0.10	0.10	0.11	0.13
6	0.14	0.15	0.13	0.10	0.11	0.11	0.13

Table 5.54: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from oil price band D4 to U.S. Dollar Index.

Oil ⇒ USD							
Causality from D3							
m	A6	D6	D5	D4	D3	D2	D1
1	0.08	0.06	0.07	0.10	0.08	0.15	0.14
2	0.11	0.08	0.09	0.15	0.11	0.11	0.15
3	0.12	0.10	0.11	0.15	0.12	0.11	0.14
4	0.12	0.11	0.12	0.13	0.09	0.10	0.14
5	0.13	0.12	0.11	0.11	0.08	0.09	0.13
6	0.13	0.13	0.12	0.09	0.07	0.09	0.13

Table 5.55: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from oil price band D3 to U.S. Dollar Index.

Oil ⇒ USD							
Causality from D2							
m	A6	D6	D5	D4	D3	D2	D1
1	0.08	0.06	0.05	0.08	0.16	0.11	0.11
2	0.09	0.08	0.07	0.11	0.17	0.08	0.12
3	0.11	0.09	0.08	0.09	0.11	0.09	0.14
4	0.12	0.11	0.10	0.09	0.08	0.09	0.12
5	0.13	0.12	0.11	0.09	0.09	0.11	0.13
6	0.14	0.13	0.12	0.09	0.10	0.10	0.13

Table 5.56: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from oil price band D2 to U.S. Dollar Index.

Oil ⇒ USD							
Causality from D1							
m	A6	D6	D5	D4	D3	D2	D1
1	0.07	0.04	0.03	0.05	0.08	0.10	0.10
2	0.09	0.06	0.05	0.08	0.10	0.07	0.10
3	0.10	0.08	0.07	0.09	0.08	0.08	0.11
4	0.11	0.10	0.09	0.08	0.08	0.08	0.12
5	0.13	0.11	0.10	0.09	0.09	0.09	0.12
6	0.13	0.13	0.11	0.08	0.09	0.09	0.12

Table 5.57: Probability of getting $p < 0.05$ for generated surrogate data for nonlinear test for causality from oil price band D1 to U.S. Dollar Index.

5.5 Results discussion

Examining results of conducted linear and nonlinear tests, along with tests for surrogate data, there are no regular patterns in the investigated time series. We assumed $p < 0.1$ as a treshold for surrogate data to exclude false-positive results. With such assumptions, the following causal relations were discovered:

- linear causality
  - from U.S. Dollar Index band D3 to oil price band D2 for 1 step,
  - from U.S. Dollar Index band D3 to oil price band D1 for 4 steps,
  - from U.S. Dollar Index band D2 to oil price band D1 for 2 steps,
  - from U.S. Dollar Index band D1 to oil price band D4 for 2, 3 and 4 steps,
  - from oil price band A6 to U.S. Dollar Index band D3 for 4 and 5 steps,
  - from oil price band A6 to U.S. Dollar Index band D1 for 4 steps,
  - from oil price band D4 to U.S. Dollar Index band D1 for 5 and 6 steps,
  - from oil price band D2 to U.S. Dollar Index band D3 for 4 steps,
  - from oil price band D1 to U.S. Dollar Index band D3 for 4 steps,
- nonlinear causality
  - from U.S. Dollar Index band D5 to oil price band D5 for 1 and 2 steps,
  - from U.S. Dollar Index band D5 to oil price band D4 for 5 and 6 steps,
  - from U.S. Dollar Index band D4 to oil price band D4 for 1 step,
  - from U.S. Dollar Index band D1 to oil price band D4 for 5 and 6 steps.

6 Conclusions

Causality between U.S. Dollar Index and oil price is a complex problem. There are multiple identifiable factors suggesting influence on one another in several time scales. On top of that, external economic factors are plentiful and play a big role affecting those variables. Studies show that connections between U.S. Dollar Index and oil price can be time-varying and dependent on specific characteristics of the investigated economy [19].

The methodology used in this study seems to be unable to distinguish those fine-grained contributing factors and establish high-fidelity results. Some causality has been found in the analyzed data. However, tests run for generated surrogate data suggest that many of those may be a false-positive. Therefore, in the thesis, we refrain from stating definitive conclusions about causality between U.S. Dollar Index and oil price.

This thesis is an example of using wavelet transform to analyze causality in multiple time scales, along with surrogate data to assess results fidelity.

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A Surrogate data example

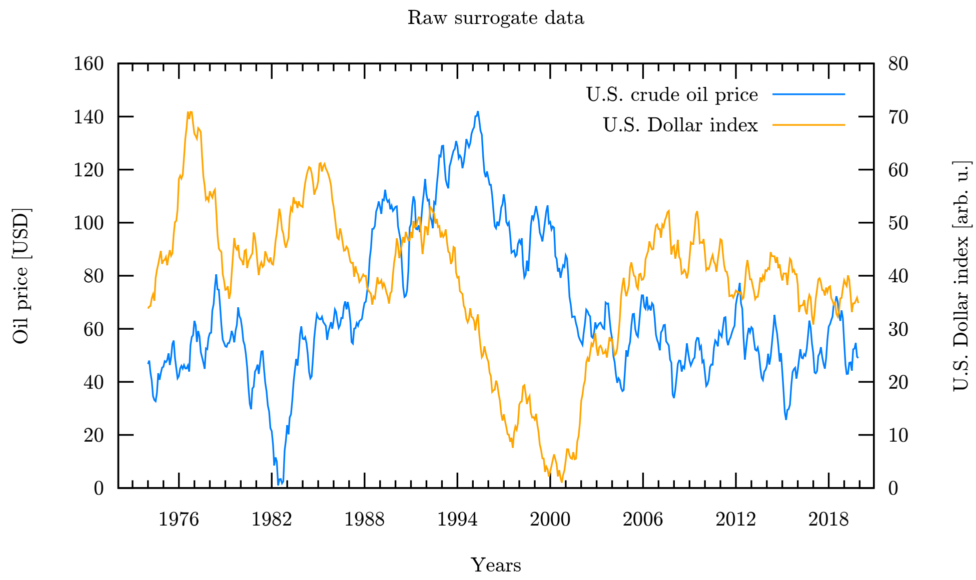

Figure A.1: Plot of raw surrogate data of crude oil price and U.S. Dollar Index.

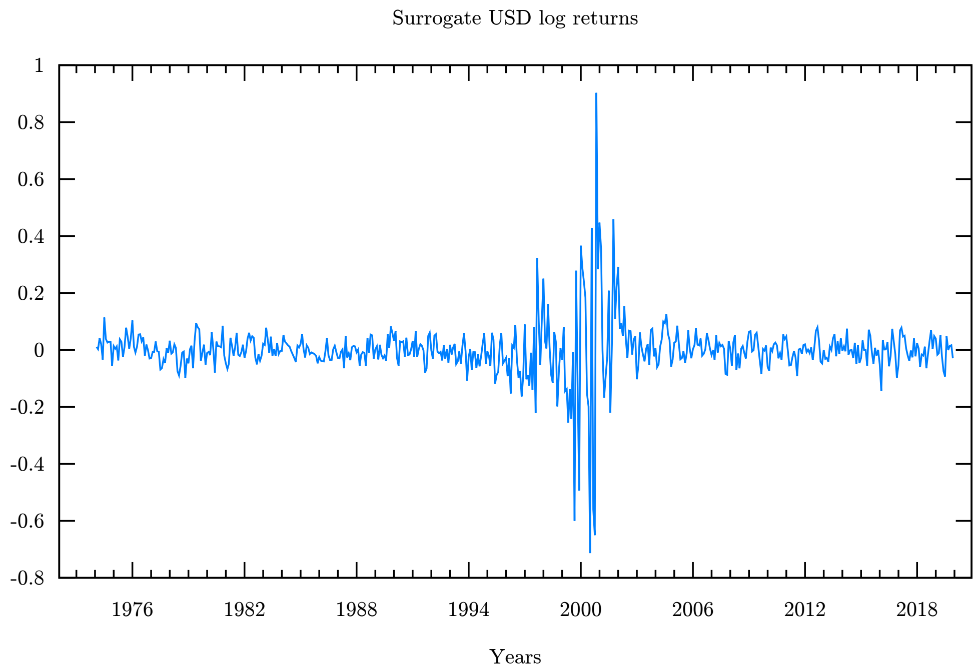

Figure A.2: Plot of a surrogate data generated from U.S. Dollar Index log returns.

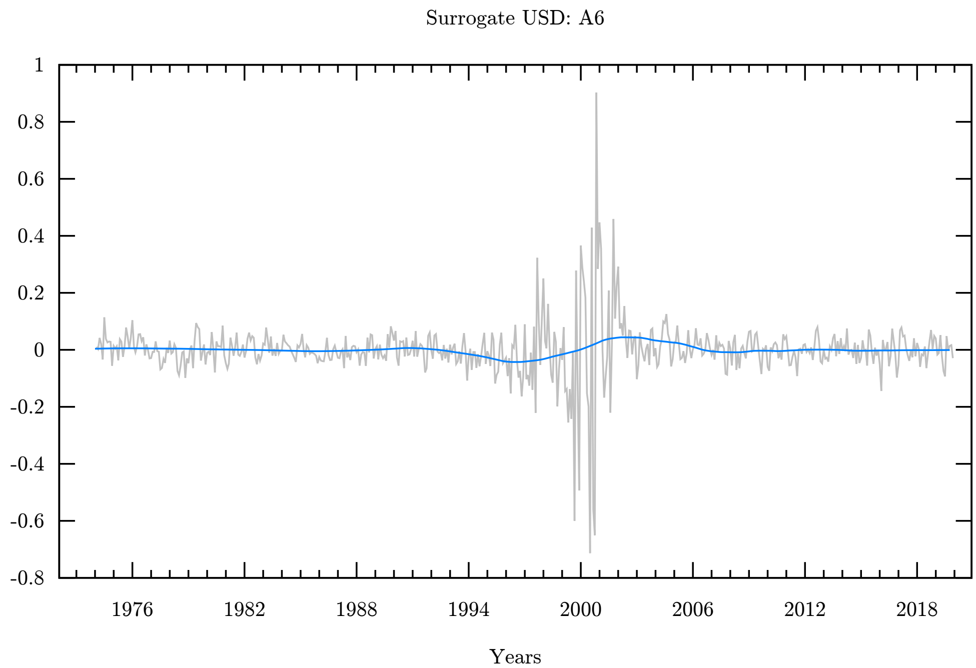

Figure A.3: Plot of A6 component of wavelet decomposition of surrogate U.S. Dollar Index log returns. Plot of the surrogate log returns in grey. A6 scale corresponds to >128 months.

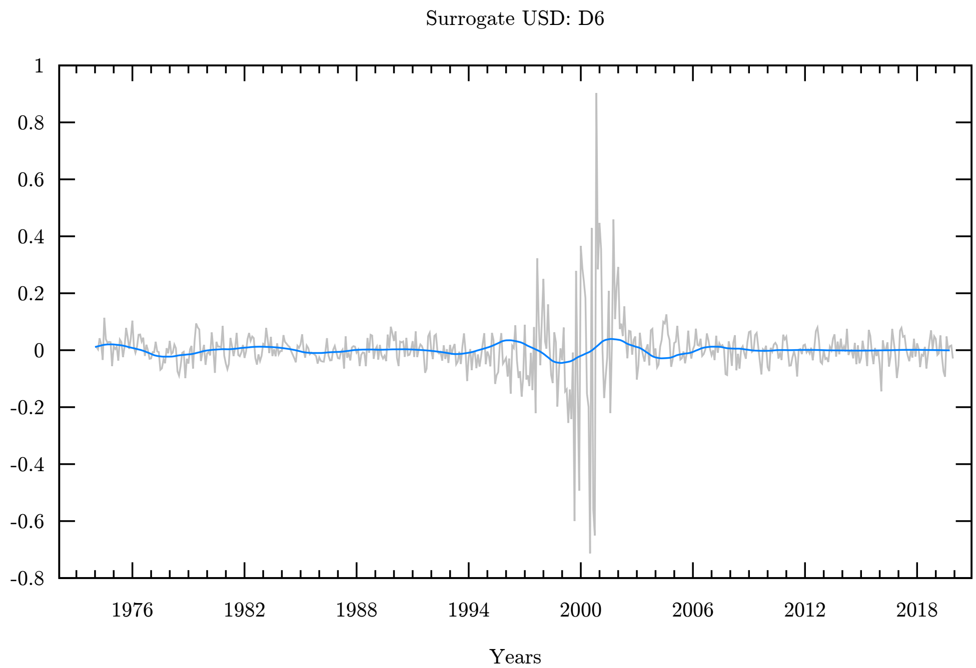

Figure A.4: Plot of D6 component of wavelet decomposition of surrogate U.S. Dollar Index log returns. Plot of the surrogate log returns in grey. D6 scale corresponds to 64-128 months.

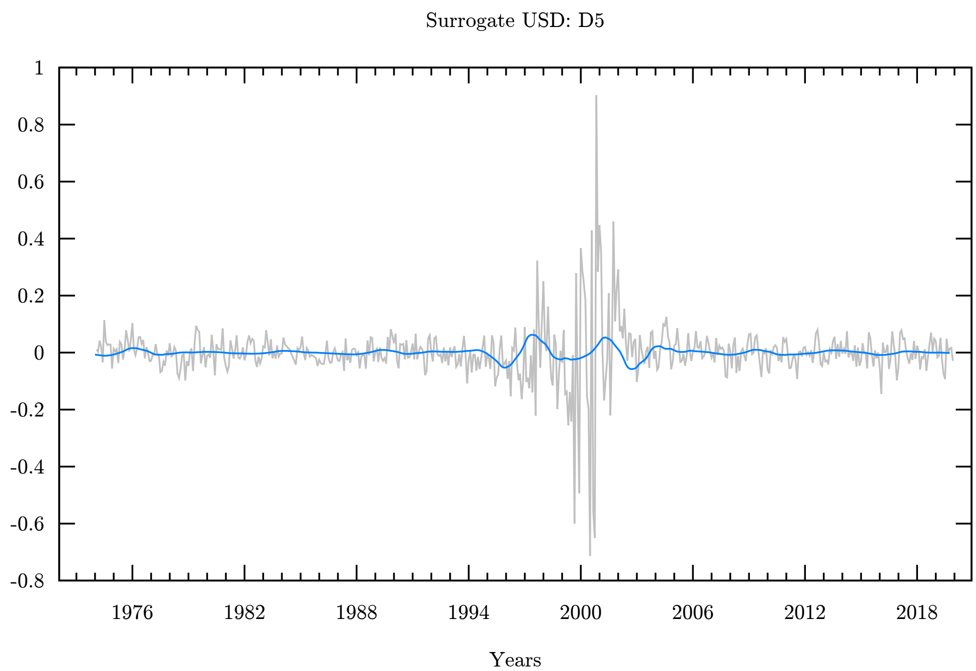

Figure A.5: Plot of D5 component of wavelet decomposition of surrogate U.S. Dollar Index log returns. Plot of the surrogate log returns in grey. D5 scale corresponds to 32-64 months.

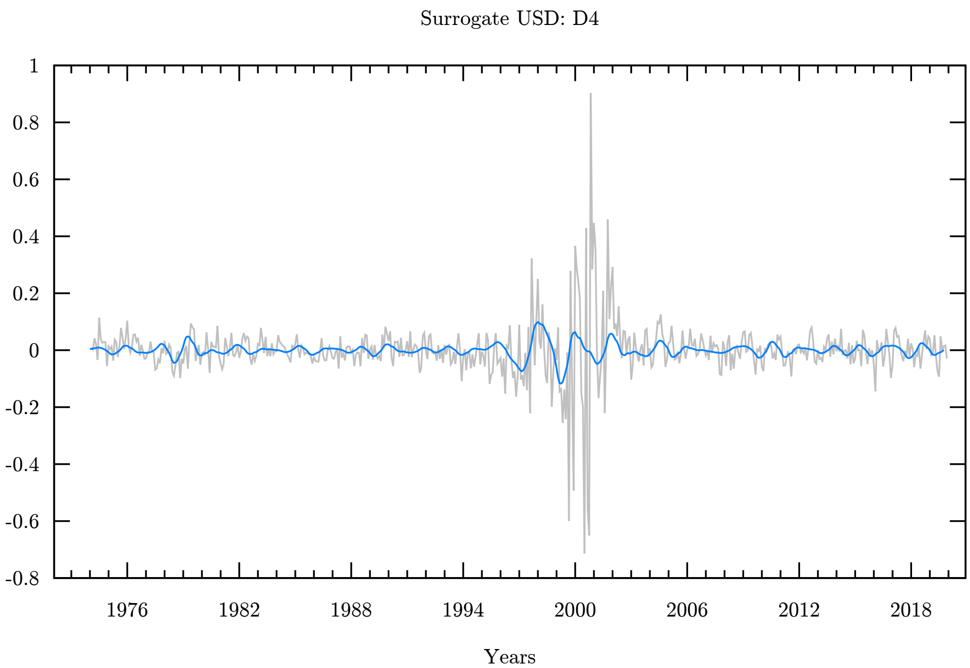

Figure A.6: Plot of D4 component of wavelet decomposition of surrogate U.S. Dollar Index log returns. Plot of the surrogate log returns in grey. D4 scale corresponds to 16-32 months.

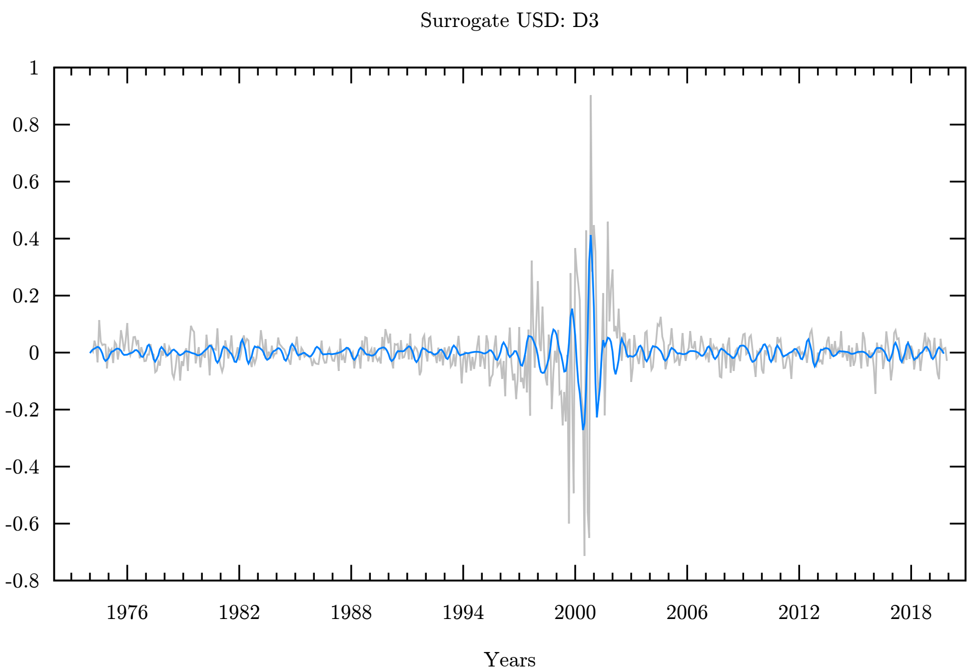

Figure A.7: Plot of D3 component of wavelet decomposition of surrogate U.S. Dollar Index log returns. Plot of the surrogate log returns in grey. D3 scale corresponds to 8-16 months.

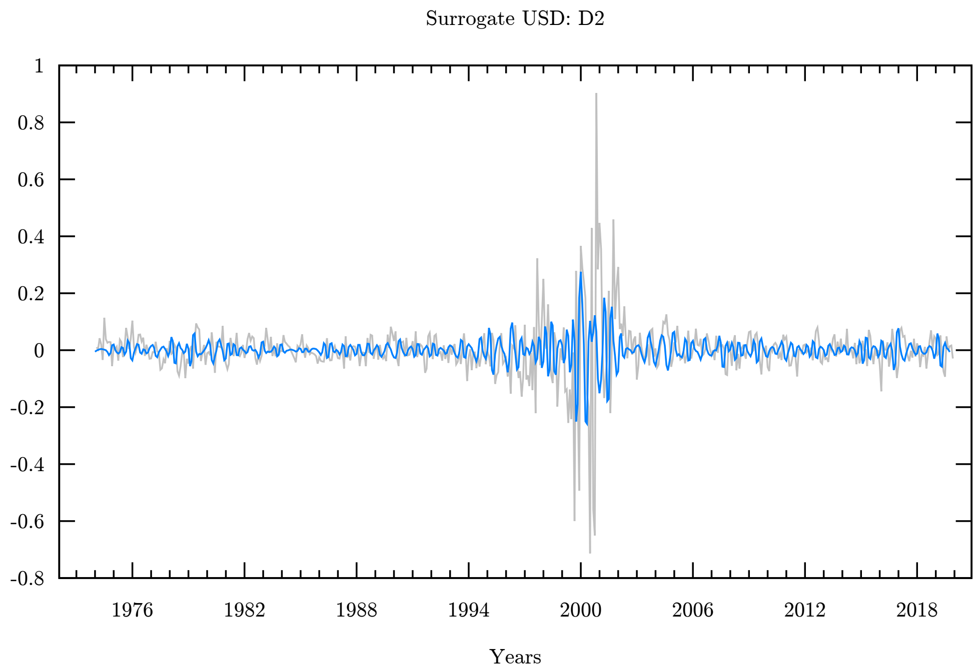

Figure A.8: Plot of D2 component of wavelet decomposition of surrogate U.S. Dollar Index log returns. Plot of the surrogate log returns in grey. D2 scale corresponds to 4-8 months.

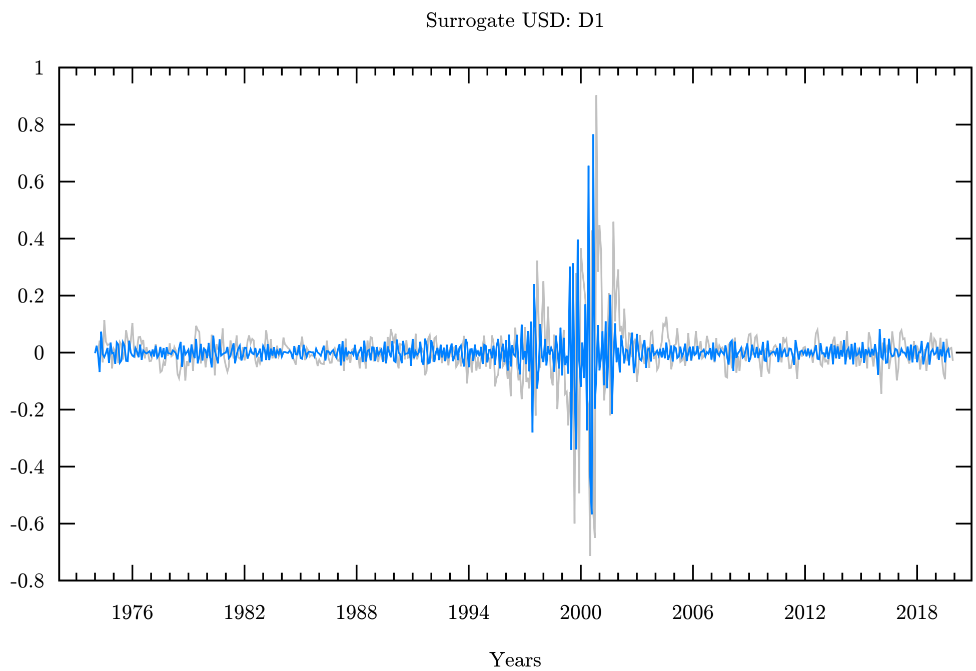

Figure A.9: Plot of D1 component of wavelet decomposition of surrogate U.S. Dollar Index log returns. Plot of the surrogate log returns in grey. D1 scale corresponds to 2-4 months.

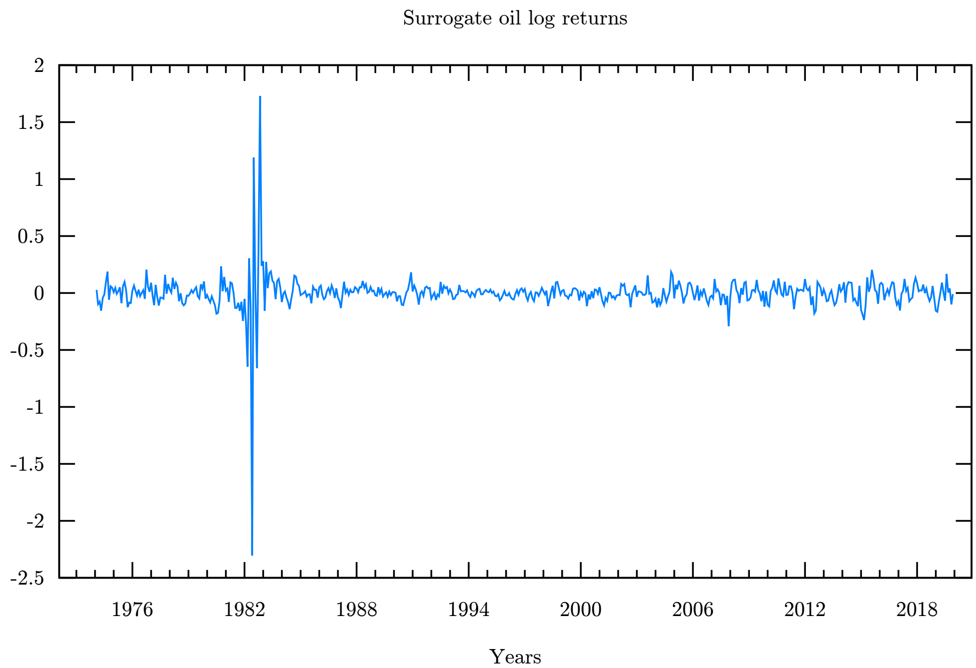

Figure A.10: Plot of a surrogate data generated from oil price log returns.

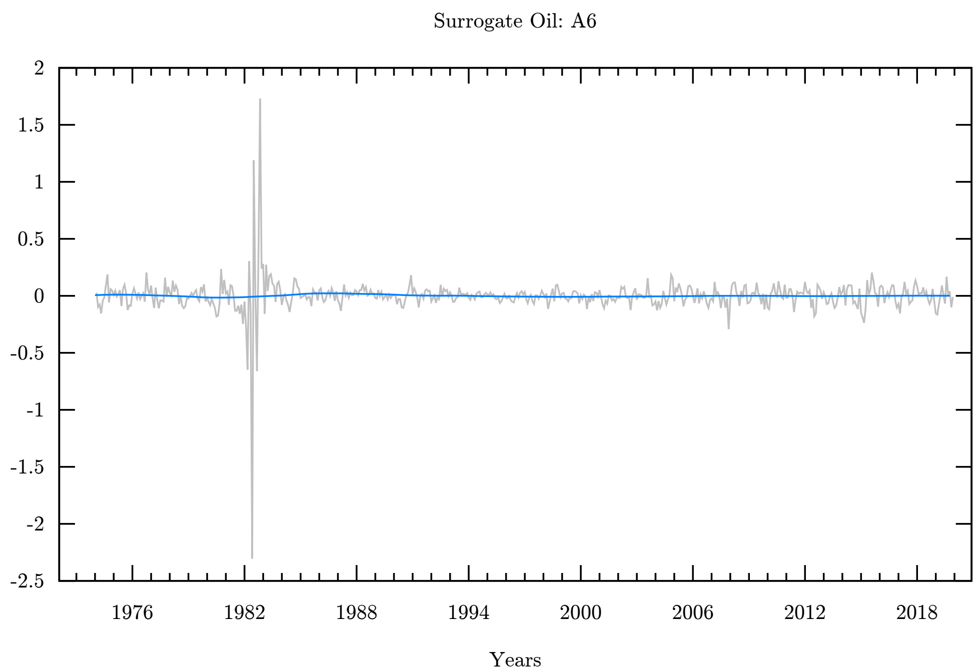

Figure A.11: Plot of A6 component of wavelet decomposition of surrogate oil price log returns. Plot of the surrogate log returns in grey. A6 scale corresponds to >128 months.

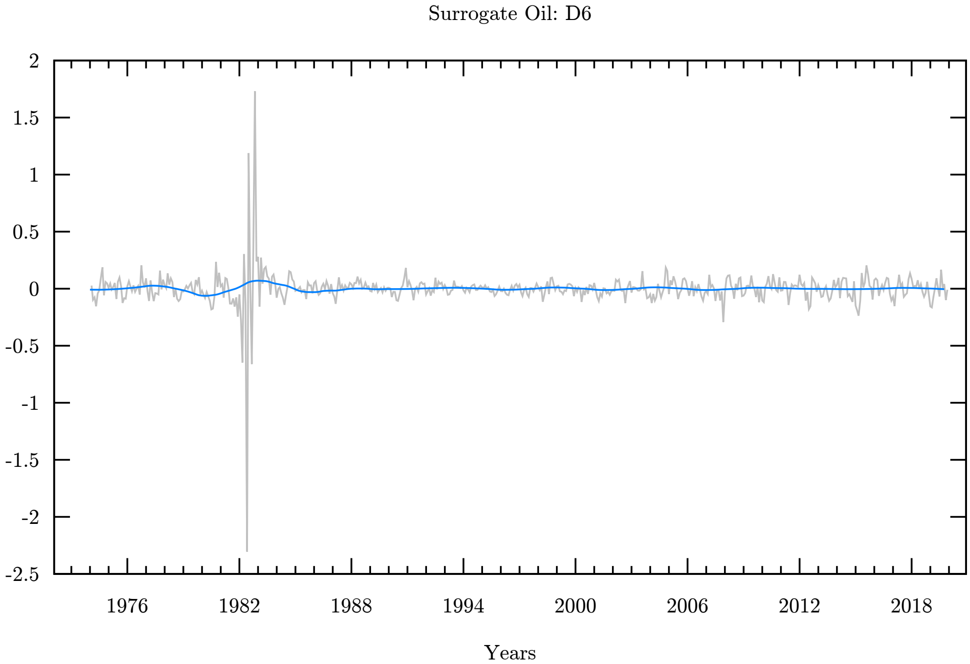

Figure A.12: Plot of D6 component of wavelet decomposition of surrogate oil price log returns. Plot of the surrogate log returns in grey. D6 scale corresponds to 64-128 months.

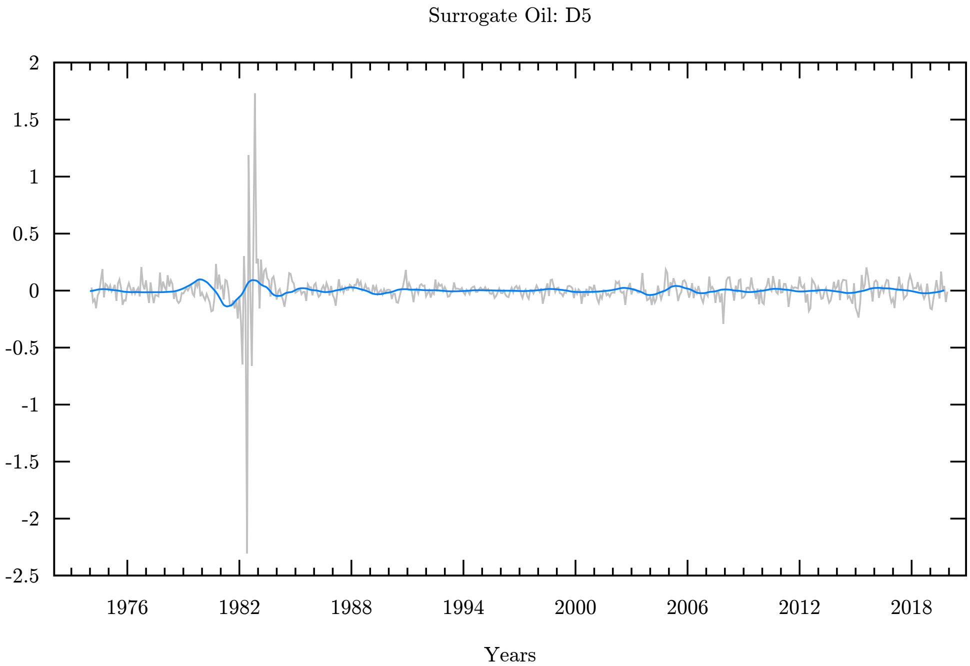

Figure A.13: Plot of D5 component of wavelet decomposition of surrogate oil price log returns. Plot of the surrogate log returns in grey. D5 scale corresponds to 32-64 months.

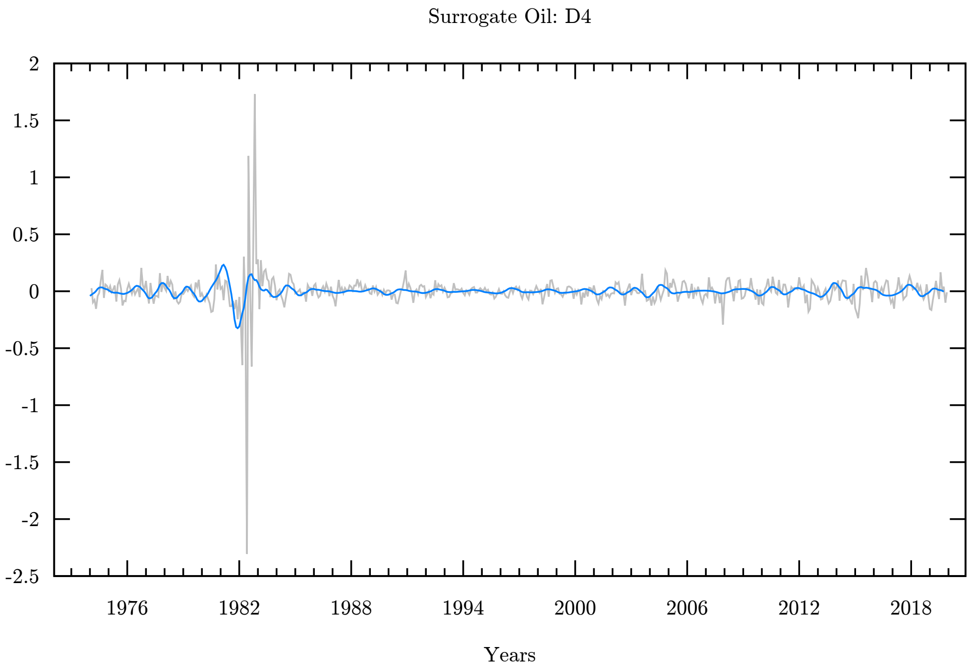

Figure A.14: Plot of D4 component of wavelet decomposition of surrogate oil price log returns. Plot of the surrogate log returns in grey. D4 scale corresponds to 16-32 months.

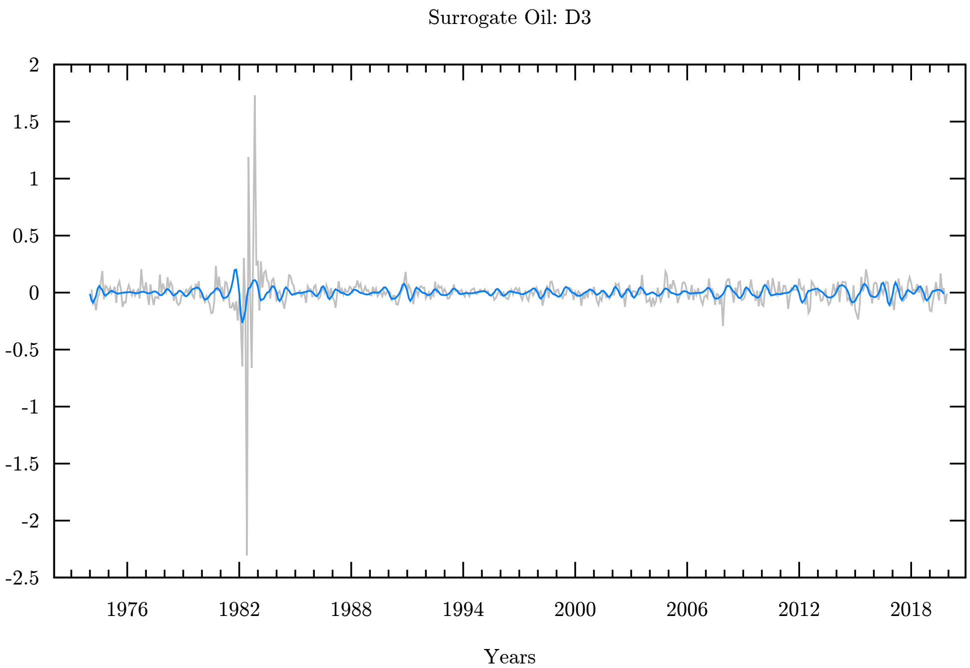

Figure A.15: Plot of D3 component of wavelet decomposition of surrogate oil price log returns. Plot of the surrogate log returns in grey. D3 scale corresponds to 8-16 months.

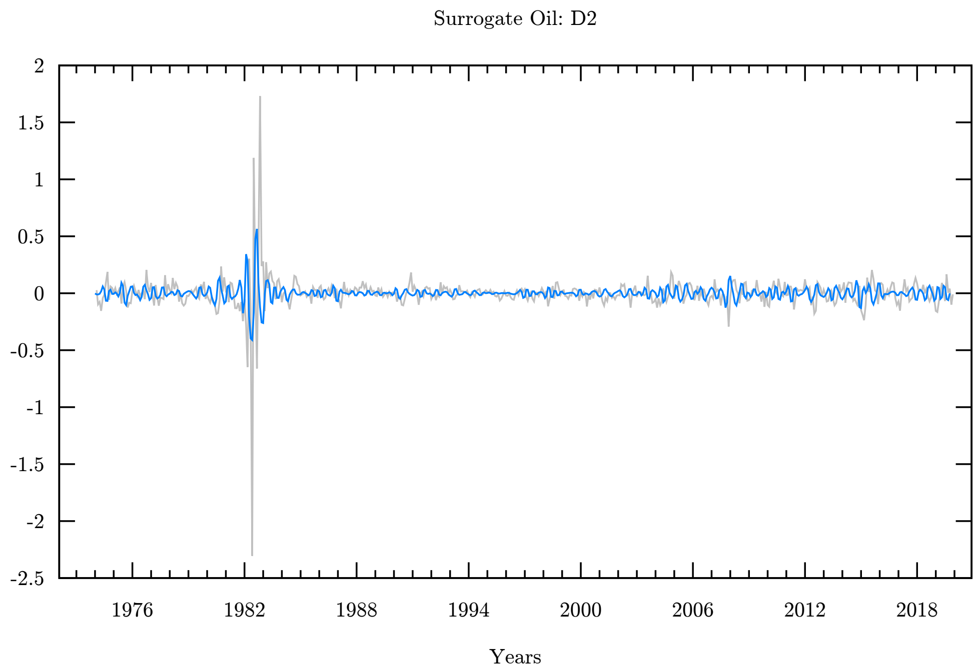

Figure A.16: Plot of D2 component of wavelet decomposition of surrogate oil price log returns. Plot of the surrogate log returns in grey. D2 scale corresponds to 4-8 months.

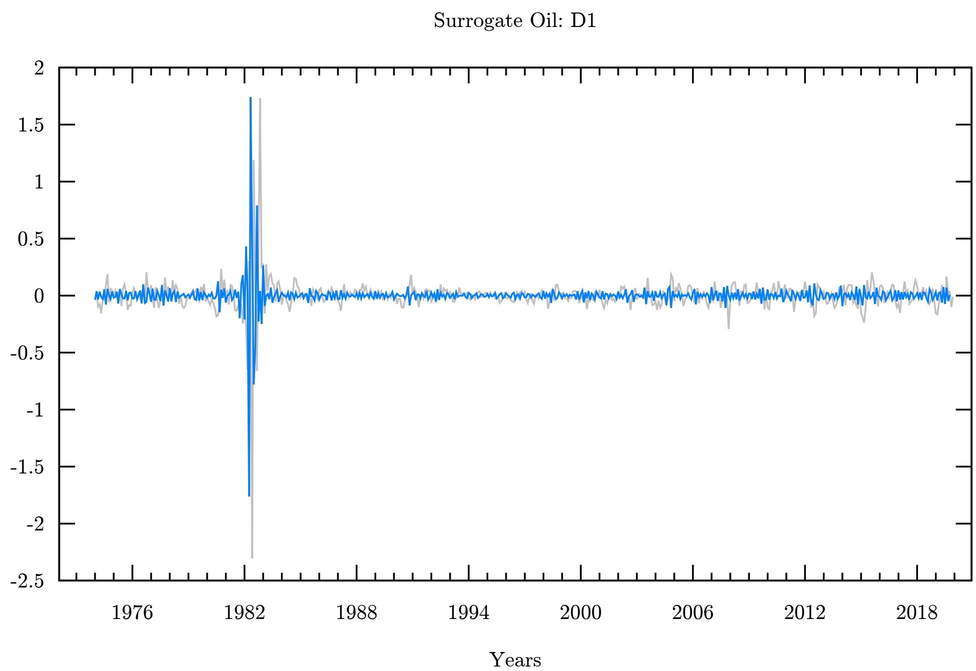

Figure A.17: Plot of D1 component of wavelet decomposition of surrogate oil price log returns. Plot of the surrogate log returns in grey. D1 scale corresponds to 2-4 months.