

# Bargaining with Mechanisms and Two-Sided Incomplete Information

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August 18, 2024

- No satisfactory strategic solution for incomplete information bargaining:
  - signaling issues,
  - two-sided uncertainty: folk-theorem multiplicity, possible refinements to eliminate some equilibria.
- This paper: a natural modification of a standard random-proposer bargaining has a (generically) unique outcome:
  - single good plus transfers,
  - private values, two types for each player.

- Bargaining with mechanisms (i.e., sophisticated offers) in the real world
  - menus,
  - menus of menus (“I divide, you choose”),
  - mediation, arbitration (example: “trial by gods”),
  - change in bargaining protocols,
  - deadlines or delays, etc.
- Intuition: larger space of actions help to deal with signaling issues.
- Challenge: How to model mechanisms as actions?

- Two players  $i = 1, 2$ , sometimes third player (“mediator”)
  - $T_i = \{l_i, h_i\}$ , assume  $l_1 \leq l_2$ ,
  - belief profiles  $\Delta T = \Delta T_1 \times \Delta T_2$
- Single good and transfers: preferences:  $q_i t_i - \tau_i$ ,
  - feasibility:  $q_1 + q_2 \leq 1$ ,  $q_i \geq 0$ ,  $\tau_1 + \tau_2 \leq 0$ ,
- Bargaining game
  - multiple rounds until offer is accepted, discounting  $\delta < 1$ ,
  - player  $i$  is proposer with prob.  $\beta_i \geq 0$ , where  $\beta_1 + \beta_2 = 1$ ,
  - proposer offers a mechanism,
  - if the offer is accepted, it is implemented, and the bargaining game ends (commitment!).
- Perfect Bayesian Equilibrium: no updating beliefs about player  $i$  after  $-i$ 's action.

- Game  $G$ : finite or compact actions + outcome function,
- Equilibrium payoffs correspondence:  $m(p; G) \subseteq \mathcal{U}(p)$  for  $p \in \Delta T$ ,
  - $\mathcal{U}(p) \subseteq R^{T_1 \cup T_2}$  is the set of feasible and incentive compatible payoffs.

- (*Abstract*) *mechanism* is correspondence  $m$  st.  $m$  is u.h.c.,  $m \subseteq \mathcal{U}$ , non-empty valued, and
  - it can be *approximated* by continuous functions  $m_n : \Delta T \rightarrow R^{T_1 \cup T_2}$ ,  $m_n \subseteq \mathcal{U}$  in the sense that  $\lim_n \text{Graph}(m_n) \subseteq \text{Graph}(m)$ .
  - the space of mechanism is compact under Hausdorff distance induced by  $d$ .

### Theorem

(**Virtual implementation**) If  $G$  is a game, then  $m(\cdot; G)$  is a mechanism. If  $m$  is a mechanism, then, there is a sequence of games  $G_n$  that “approximate”  $m$ :

$$\lim_n \text{Graph}(m(\cdot; G_n)) \subseteq \text{Graph}(m).$$

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# Model

## Derived mechanisms

- Given a mechanism  $m$  or a set of mechanisms  $A$ , we can construct new ones:
- $\alpha \in \Delta A$  - randomly chosen mechanism according to distribution  $\alpha$ .
- $\delta m$  - discounted mechanism  $m$ .
- $I_i(m)$  - information revelation game: public randomization plus  $i$ 's cheap talk followed by  $m$ .
- $MM_i(A)$  - menu of mechanisms  $a \in A$  for player  $i$ ,
  - including public randomization and cheap talk by  $i$ .
- $IP_i(m)$  - informed principal problem of player  $i$  with continuation mechanism (i.e., outside option)  $m$ ,

$$IP_i(m) = MM_i \{ MM_{-i} \{ a, m \} : a \text{ is a mechanism} \}$$

- Bargaining mechanism : the largest fixed point  $\mathcal{B}^\delta$  of

$$\mathcal{B}^\delta = (IP_1(\delta \mathcal{B}))^{\beta_1} (IP_2(\delta \mathcal{B}))^{\beta_2}$$



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# Random monopoly bound

## Benchmarks

- (Maskin, Tirole 90) Informed principal with private values ( $\beta_i = 1$  and  $\delta = 0$ ) : monopoly payoff

$$M(t_i; p_{-i}) = \max_{\tau} p_{-i}(t_{-i} \leq \tau) t_i + (1 - p_{-i}(t_{-i} \leq \tau)) \tau,$$

- Special features:
  - continuation value = 0 (and it does not depend on beliefs)
  - private information of the principal does not matter due to private values.
  - none of this holds in bargaining.

## Theorem

*For each  $\delta < 1$ , each  $u \in \mathcal{B}^\delta(p)$ , each player  $i$ , each  $t_i$ ,*

$$u_i(t_i) \geq \beta_i M_i(t_i; p_{-i}).$$

- Each player gets at least their random monopoly payoff.
- Rubinstein-style argument, but ....
- not easy to extend to more than two types.



# Unique outcome

- In many cases, Theorem 2 is enough to characterize payoffs and equilibrium behavior, as there is unique interim efficient allocation that satisfies the random monopoly condition:
  - $\beta_i \in \{0, 1\}$ ,
  - $p_i \in \{0, 1\}$  for one of the players,
  - $l_1 = l_2$  or  $l_2 = h_1$  or  $h_1 = h_2$ .
- In general, there is a gap between random monopoly payoffs and efficiency.
- The gap is not larger than  $\text{Gap}(p) \leq 6.25\%$  of  $\max(h_1, h_2)$  for all  $p$ .

## Theorem

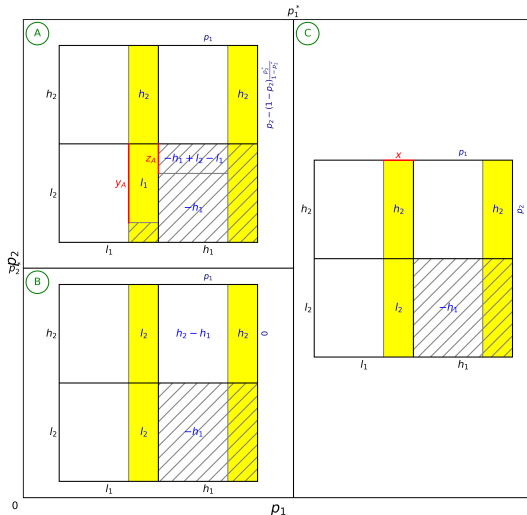
*For generic payoffs and generic  $p$ ,  $\mathcal{B}(p) = \lim_{\delta} \mathcal{B}^{\delta}(p)$  contains a single element  $|\mathcal{B}(p)| = 1$ .*

*The entire gap goes to player 1: If  $u \in \mathcal{B}(p)$ , then*

$$p_1 \cdot u_1 = \max_{\substack{u' \text{ is iIC, feasible at } p \\ u'_2(t_2) \geq \beta_2 M_2(t_2; p) \text{ for } t_2 = l_2, h_2}} p_1 \cdot u'_1$$

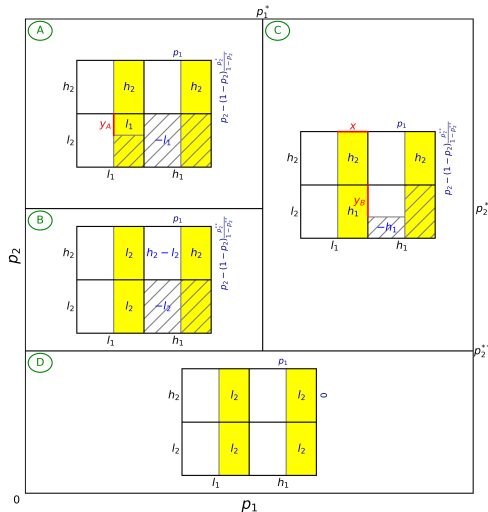
# Unique outcome

$$l_2 < h_1$$



# Unique outcome

$$h_1 < l_2$$



- A natural modification of a standard random-proposer bargaining has unique payoffs under
  - single good plus transfers, private values environment,
  - two types for each player.
- A proof of concept - better results and a general theory would be nice:
  - more types,
  - other environments,
  - better implementation results.