Bargaining with Mechanisms and Two-Sided Incomplete Information

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Introduction

- No satisfactory strategic solution for incomplete information bargaining:
  - signaling issues,
  - two-sided uncertainty: folk-theorem multiplicity, possible refinements to eliminate some equilibria.
- Here: a natural modification of a standard random-proposer bargaining has a (generically) unique outcome:
  - single good plus transfers,
  - private values,
  - two types for each player.

Introduction

- Bargaining with mechanisms (i.e., sophisticated offers) in the real world
  - menus,
  - menus of menus ("I divide, you choose"),
  - mediation, arbitration (example: "trial by gods"),
  - change in bargaining protocols,
  - deadlines or delays, etc.
- Intuition: larger space of actions help to deal with signaling issues.
- Challenge: How to model mechanisms as actions?

Model

- Two players $i = 1, 2$, sometimes third player ("mediator")
  - $T_i = \{l_i, h_i\}$,
  - belief profiles $\Delta T = \Delta T_1 \times \Delta T_2$
- Single good and transfers: preferences: $q_i t_i - \tau_i$,
  - feasibility: $q_1 + q_2 \leq 1$, $q_i \geq 0$, $\tau_1 + \tau_2 \leq 0$,
- Bargaining game
  - multiple rounds until offer is accepted, discounting $\delta < 1$,
  - player i is proposer with prob. $\beta_i \geq 0$, where $\beta_1 + \beta_2 = 1$,
  - proposer offers a mechanism,
  - if the offer is accepted, it is implemented, and the bargaining game ends (commitment!).
- Perfect Bayesian Equilibrium: no updating beliefs about player i after $-i$'s action.

Model
Mechanisms

- Game G: finite or compact actions + outcome function,
- Equilibrium payoffs correspondence: $m(p; G) \subseteq \mathcal{U}(p)$ for $p \in \Delta T$,
  - $\mathcal{U}(p) \subseteq R^{T_1 \cup T_2}$ is the set of feasible and incentive compatible payoffs.

Model
Mechanisms

- *(Abstract) mechanism* is correspondence m st. m is u.h.c., $m \subseteq \mathcal{U}$, non-empty valued, and
  - it can be *approximated* by continuous functions $m_n : \Delta T \to R^{T_1 \cup T_2}$, $m_n \subseteq \mathcal{U}$ in the sense that $\lim_n \text{Graph}(m_n) \subseteq \text{Graph}(m)$.
  - the space of mechanism is compact* under Hausdorff distance induced by d.

Theorem

(Virtual implementation) *If G is a game, then $m(.; G)$ is a mechanism. If m is a mechanism, then, there is a sequence of games G_n that "approximate" m:*

$$\lim_n Graph(m(.; G_n)) \subseteq Graph(m).$$

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Model

Derived mechanisms

- Given a mechanism or a set of mechanisms, we can construct new ones:
- $\alpha \in \Delta A$ - randomly chosen mechanism according to distribution α.
- δm - discounted mechanism m.
- $I_i(m)$ - information revelation game: public randomization plus i's cheap talk followed by m.
- $MM_i(A)$ - menu of mechanisms $a \in A$ for player i,
  - including public randomization and cheap talk by i.
- $IP_i(m)$ - informed principal problem of player i with continuation mechanism (i.e., outside option) m,

$$IP_i(m) = MM_i\{MM_{-i}\{a, m\} : a \text{ is a mechanism}\}$$

- Bargaining mechanism : the largest fixed point $\mathcal{B}^\delta$ of

$$\mathcal{B}^\delta = (IP_1(\delta\mathcal{B}))^{\beta_1} (IP_2(\delta\mathcal{B}))^{\beta_2}$$

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Random monopoly bound

Benchmarks

- Informed principal with private values $\beta_i = 1$ (or $\delta = 0$) (Maskin, Tirole 90): monopoly payoff

$$M(t_i; p_{-i}) = \max_{\tau} p_{-i}(t_{-i} \leq \tau) t_i + (1 - p_{-i}(t_{-i} \leq \tau)) \tau,$$

- Special features:
  - continuation value $= 0$ (and it does not depend on beliefs)
  - private information of the principal does not matter due to private values.
  - none of this holds in bargaining.

Random monopoly

Theorem

For each $\delta < 1$*, each* $u \in \mathcal{B}^{\delta}(p)$*, each player* i*, each* t_i*,*

$$u_i(t_i) \geq \beta_i M_i(t_i; p_{-i}).$$

- Each player gets at least their random monopoly payoff.
- Rubinstein-style argument, but ....
- not easy to extend to more than two types.

Unique outcome

- In many cases, Theorem 2 is enough to characterize payoffs and equilibrium behavior, as there is unique interim efficient allocation that satisfies the random monopoly condition:
  - $\beta_i \in \{0, 1\}$,
  - $p_i \in \{0, 1\}$ for one of the players,
  - $l_1 = l_2$ or $l_2 = h_1$ or $h_1 = h_2$.
- In general, there is a gap between random monopoly payoffs and efficiency.
- The gap is not larger than $\text{Gap}\,(p) \leq 6.25\%$ of $\max\,(h_1, h_2)$ for all p.

Unique outcome

- Assume $l_1 < l_2$.

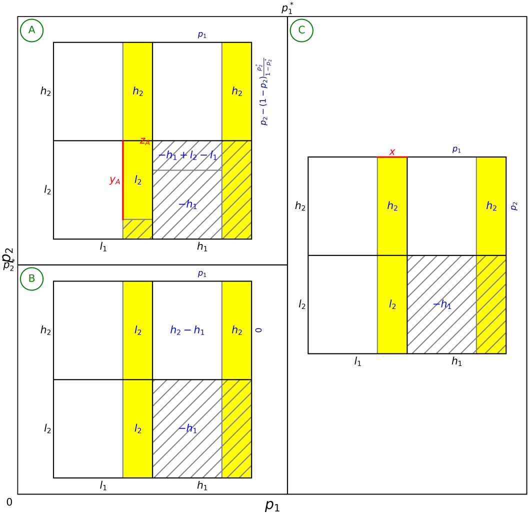
Theorem

For generic payoffs and generic p, $\mathcal{B}(p) = \lim_\delta \mathcal{B}^\delta(p)$ contains a single element $|\mathcal{B}(p)| = 1$.
The entire gap goes to player 1: If $u \in \mathcal{B}(p)$, then

$$\sum_{t_1} p_1(t_1)\, u_1(t_1) = \max_{\substack{u \text{ is incentive compatible, feasible at } p \\ u_2(t_2) \geq \beta_2 M_2(t_2;p) \text{ for } t_2 \in \{l_2, h_2\}}} \sum_{t_1} p_1(t_1)\, u_1(t_1$$

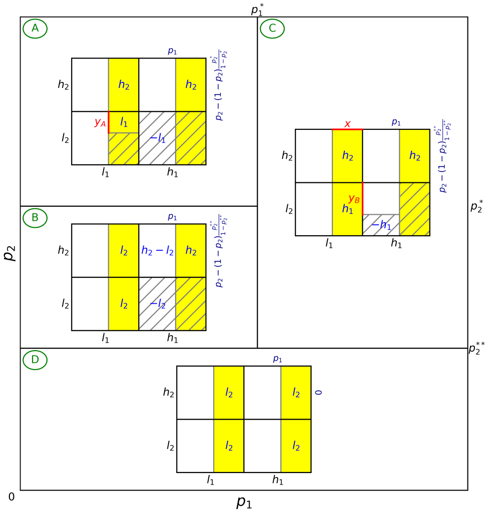
Unique outcome

$l_2 < h_1$



Unique outcome

$h_1 < l_2$



Conclusions

- A natural modification of a standard random-proposer bargaining has unique payoffs under
  - single good plus transfers, private values environment,
  - two types for each player.
- A proof of concept - better results and a general theory would be nice:
  - more types,
  - other environments,
  - better implementation results.