Bargaining with Mechanisms and Two-Sided Incomplete Information

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Outline

Introduction

- Business partners want to cease partnership. Their firm cannot be divided, and if partner keeps it, the other expects a compensation.
- Two countries negotiate a peace treaty, with land swaps and reparations (or economic aid) on the table.
- Coalition parties negotiate an agreement with a support for policy traded off against number of cabinet positions.

Introduction

- Bargaining - one of the longest-studied problems in economic theory (since [Nash 50]? , earlier "bilateral monopoly")
- No satisfactory solution for incomplete information:
  - cooperative solutions: (Harsanyi 72), (Myerson 84),
  - large literature on bargaining over prices:
    - one-sided: reputational results
    - two-sided: anything goes (exception (De Clippel, Fanning, Rozen 22), but assumes verifiability of types)
- Goal here: show that a natural modification of a standard random-proposer bargaining has a "unique" outcome under
  - single good plus transfers environment,
  - private values (two types for each player).

Introduction

- Bargaining with sophisticated offers in real world
  - menus,
  - menus of menus ("I divide, you choose"),
  - mediation, arbitration (example: "trial by gods"),
  - change in bargaining protocols,
  - deadlines or delays, etc.
- Informed principal with private values (Maskin Tirole, 90)
  - informed principal types get their monopoly payoff,
- One-sided incomplete information (Peski 22),
  - informed player types get at least random monopoly payoff,
  - => uninformed player maximizes her payoff st. the above constraint.

Introduction

Results

- Theorem 0: Each (Michael) payoff correspondence is virtually and weakly implementable.
- Theorem 1: For each discount factor, each player expects at least their random monopoly payoff.
- Theorem 2: Suppose that $l_1 < l_2$. As $\delta \to 1$, *ex ante* expected payoffs of player 1 converge to a feasible maximum subject to a constraint that player 2 types get their random monopoly payoffs.

Outline

Model

Environment

- Two players $i = 1, 2$, sometimes third player ("mediator").
- Single good and transfers
- Preferences: $q_i t_i - \tau_i$,
  - t_i - type (valuation) of player i,
  - q_i - probability that pl. i gets the good,
  - τ_i - transfer from player i
  - feasibility: $q_1 + q_2 \leq 1$, $q_i \geq 0$, $\tau_1 + \tau_2 \leq 0$,

Model

Bargaining game

- Bargaining game
  - multiple rounds until offer is accepted, discounting $\delta < 1$,
  - random proposer: Player i is a proposer with probability β_i, where $\beta_1 + \beta_2 = 1$,
    - ★ includes single-proposer games $\beta_i \in \{0, 1\}$,
  - proposer proposes a mechanism ($\approx$ game, where each outcome is a good allocation and transfer)
  - once the offer is accepted, it is implemented (the mechanism game is played) and the bargaining game ends.
- Perfect Bayesian Equilibrium:
  - no updating beliefs about player i after $-i$'s action.
  - public randomization plus cheap talk.

Model

Mechanism and games

- **No revelation principle.**
- Games: actions and outcome function that maps actions to "good allocation and transfer".
- Real mechanisms: correspondences of equilibrium payoffs obtained in games.
- (Abstract) mechanisms: proper limits of such correspondences.

Model

Feasible payoffs

- Payoff vector $u(.|q,\tau) \in R^{T_1 \cup T_2}$ in allocation $q_i(.), \tau(.)$:

$$u_i(t_i|q,\tau) = \sum_{t_{-i}} p(t_{-i})(t_i q_i(t_i, t_{-i}) - \tau_i(t_i, t_{-i})) \text{ for each } t_i.$$

- Allocation $q_i(.), \tau(.)$ is IC given beliefs p iff

$$u_i(t_i|q,\tau) \geq \sum_{t_{-i}} p(t_{-i})(t_i q_i(s_i, t_{-i}) - \tau_i(s_i, t_{-i})) \text{ for each } t_i, s_i.$$

- Correspondence of feasible and IC payoffs:

$$\mathcal{U}(p) = \{u(.|q,\tau) : (q,\tau) \text{ is IC given } p\} \subseteq R^{T_1 \cup T_2}.$$

- The geometry of the correspondence $\mathcal{U}(.)$ is the true "parameter" of the model.

Model

Games

- Game G:
  - three players: 1, 2, and mediator (whose payoff is a non-negative transfer),
  - finite or compact actions,
  - continuous outcome function that maps actions to "good allocation and transfer",
  - public randomization.
- For each p, the set of equilibrium payoff vectors

$$m(p; G) \subseteq \mathcal{U}(p).$$

- Equilibrium correspondence:

$$m(.; G) : \Delta T \rightrightarrows R^{T_1 \cup T_2}, m_G \subseteq \mathcal{U},$$

u.h.c., non-empty and convex valued.

Model

Mechanisms

- *Real mechanism* is a correspondence m for which there exists a game G such that $m = m(.; G)$.
- *(Abstract) mechanism*: m is correspondence $m : \Delta T \rightrightarrows R^{T_1 \cup T_2}$ st.
  - m is u.h.c., non-empty valued, $m \subseteq \mathcal{U}$,
  - can be *approximated* by continuous functions $m_n : \Delta T \to R^{T_1 \cup T_2}$, $m_n \subseteq \mathcal{U}$such that

$$\lim_{n \to \infty} \max_{p} \min_{v, q : v \in m(q)} \| m_n(p) - v \| = 0.$$

- Using Michael's Theorem: any real mechanism is an (abstract) mechanism.
- The space of mechanism is compact* under Hausdorff distance.

Model

Implementation Theorem

Theorem

For any mechanism m, there is a sequence of real mechanism m_n converge to m:

$$\lim_{n\to\infty} \max_{u,p:u\in m_n(p)} \min_{v,q:v\in m(q)} \|u-v\| = 0.$$

- Proof:
  - mediator names the beliefs p,
  - given p, use virtual implementation of (Abreu Matsushima 92).

Model

Derived mechanisms

- Given a mechanism or a set of mechanisms, we can construct new ones:
  - each game is preceded by public randomization plus cheap-talk message.
- $MM_i(A)$ - menu of mechanisms $a \in A$ for player i
- $IP_i(m)$ - informed principal problem of player i with continuation mechanism (outside option) m,

$$IP_i(m) = MM_i(\{MM_{-i}(n, m) : n \text{ is a mechanism}\})$$

- $\alpha \in \Delta A$ - randomly chosen mechanism according to distribution α,
- δm - discounted mechanism m.

Model

Bargaining game

- Bargaining game:

$$B = (IP_1(\delta B))^{\beta_1} (IP_2(\delta B))^{\beta_1}$$

  - the largest fixed point

Model
Equilibrium

- Equilibrium in $MM_i\,(A)$:
  - modular (one-shot deviation principle),
  - extends to the existence in bargaining game.
- Existence:
  - space of mechanisms is compact,
  - each mechanism can be approximated by a payoff function,
  - the equilibrium can
- Weak solution concept:
  - relative to WPBE, a minor restriction on beliefs (no updating after the other player actions),
  - approximate equilibrium with real mechanisms.

Model

Commitment

- Players are not committed to future offers.
- Players are committed to implementing a mechanism once offered and accepted:
    - hence, less commitment than, say in the *limited commitment* literature (V. Skreta and L. Doval).
- Comments:
    - many situations
        - how to bargain about deadlines if we are not really committed to them),
        - what does it mean to allocate the good.
    - "lack of commitment" is a restriction on the space of mechanisms,
    - commitment is not necessarily helpful to the agent who can exercise it.

Outline

Preliminary observations

Complete information bargaining

- **Claim:** Assume $t_1 < t_2$ are known. Then, in each equilibrium, player i gets $\beta_i t_2$.
- **Proof**: Suppose $i = 1$ (the other argument is analogous). Let

$$x^* = \min_{u \in B} \frac{u_1}{t_2}.$$

.

- If $x^* < \beta_1$, player 1 has a profitable deviation:
  - wait until she is a proposer, and offer: player 2 gets the good and pays $(1 - \delta(1 - x^*))\, t_2$ to player 1,
  - the offer will be accepted.
- Special features:
  - linearly transferable payoffs,
  - endogenous interdependent value

Preliminary observations

One-sided incomplete information

- p_i is degenerate for one of the players $=>$ one-sided incomplete Peski (22):
- The equilibrium payoffs are unique and implemented by random monopoly mechanism is offered:
  - with probability β_j, agent j gets the good:
  - if so, she make a single take-it-or-leave-it sell offer,
  - her expected payoff is

$$M\left(t_j; p_{-;}\right) = \max_{\tau} p_{-i}\left(t_i \leq \tau\right) t_j + \left(1 - p_{-i}\left(t_i \leq \tau\right)\right) \tau.$$

  - regardless if the offer is accepted or not, the mechanism ends.
- Special features:
  - random monopoly mechanism is interim efficient.

Preliminary observations

Informed principal

- $\beta_i = 1$ or $\delta = 0 =>$ (Random) informed principal with private values :
  - If player i is a proposer, she offers the monopoly price to $-i$,
  - the offered (single-price mechanism) is accepted and the game ends.
- Special features:
  - private information of the principal does not matter due to private values.

Preliminary observations

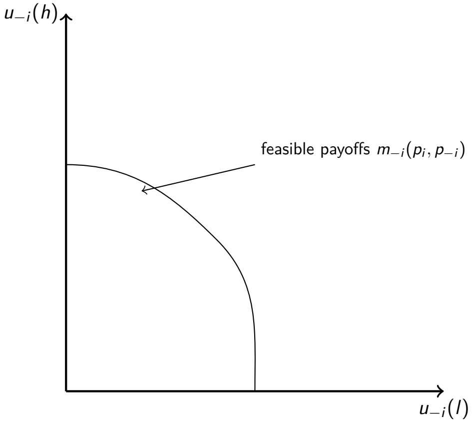
Offers that cannot be refused

- In bargaining, making offers that are refused is inefficient due to surplus-burning delay.
- Solution: make an offer that cannot be refused.
- Two problems:
  - signaling: (possibly, off-path) offers lead to belief updating $p_i \to p_i'$,
  - player $-i$ may have reasons to refuse the offer after updating.

Preliminary observations

Offers that cannot be refused

- m is a continuation mechanism.
- Design offer a that is never refused by $-i$ (and actually happens).



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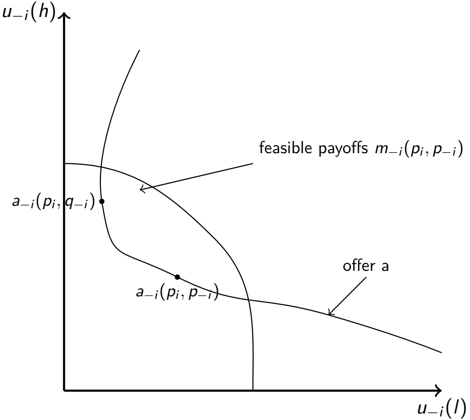

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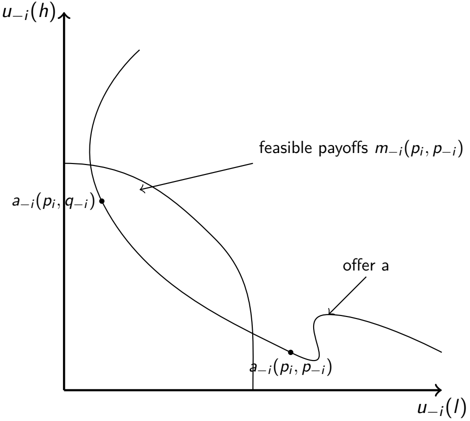

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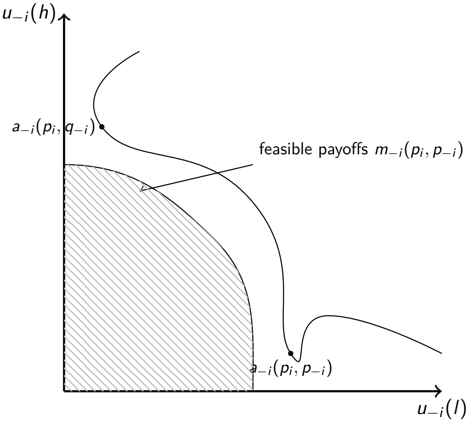

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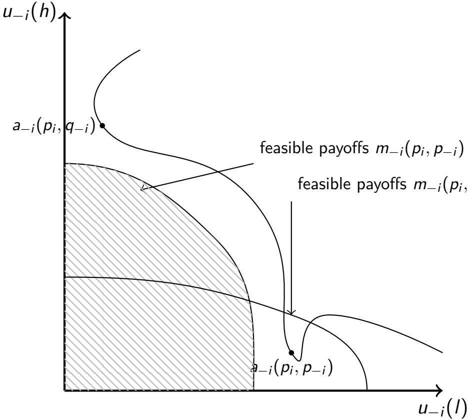

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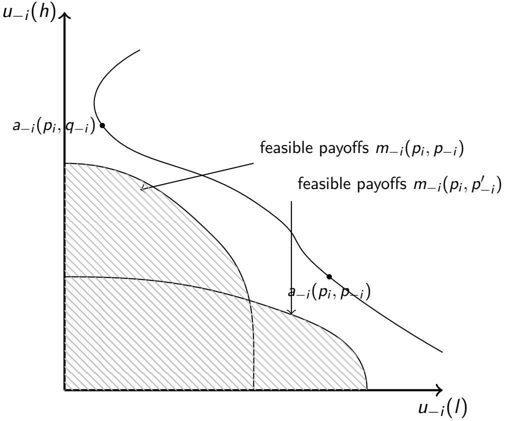

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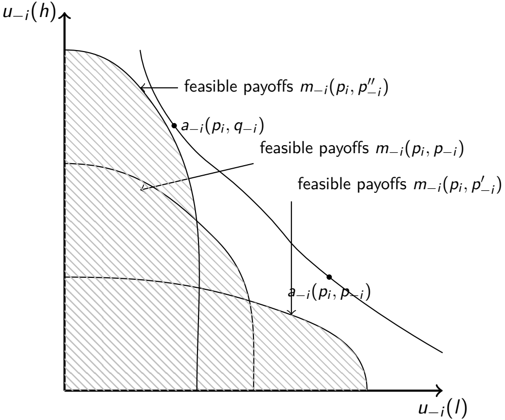

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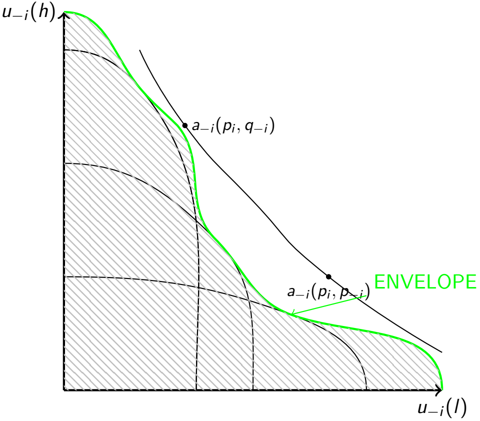

Preliminary observations

Offers that cannot be refused

Definition

Mechanism a is an offer that player $-i$ cannot refuse given mechanism m, if
for each p_i, p_{-i}, q_{-i}, each $u \in a(p_i, p_{-i})$, each $v \in m(p_i, q_{-i})$, there is a q_{-i}-positive prob. type t_{-i} such that $u_{-i}(t_{-i}) \geq v_{-i}(t_{-i})$.

- offers $u \in a(p_i, p_{-i})$ are "undominated" by offers $v \in m(p_i, q_{-i})$
- Compare with
  - SUPO allocations in (Maskin Tirole 90) and
  - strong neologism proof allocations in (Mylovanow Troger 14).

Preliminary observations

Offers that cannot be refused

Lemma

Suppose that a is an offer that player i strictly cannot refuse given mechanism m such that

- *a is a payoff function,*
- *$I_i(a) = a$. Then,*

$$MM_i\{m, a\} \subseteq a.$$

- straightforward proof.

Preliminary observations

Offers that cannot be refused: Existence

- Existence of offers cannot be refused is not an issue.
- For any two mechanisms m and a, there exists continuous $w : \Delta T \to \mathbb{R}$such that

$$(a +_i w)_j (p) = \begin{cases} a_i (p) + w (p) & j = i \\ a_i (p) - w (p) & j = -i \end{cases}$$

cannot be refused given continuation m.

Preliminary observations

Offers that cannot be refused: Informed principal

- Consider informed principal problem with continuation m.
- Suppose that $MM_{-i}\{m, a\} \subseteq a$.
- Informally, the principal should get at least a.
- But, belief updating $=>$
- If $u \in IP_i(m)(p_i, p_{-i})$, then there must be q_i and $v \in a(q_i, p_{-i})$ st. $u_i \geq v_i$.

Preliminary observations

Offers that cannot be refused: Informed principal

- Suppose that a, b are offers that cannot be refused given m
  - payoff functions st. I_i

Outline

Random monopoly

- From now on, assume two types for each player $T_i = \{l_i, h_i\}$ and $l_1 < l_2$.
- Focus on
$$0 \leq l_1 \leq l_2 < h_1 \leq h_2,$$
but the results hold for all other cases as well.
- p_i - probability of type h_i

Random monopoly

Theorem

For each $\delta < 1$, each $u \in B(p)$, each player i, each t_i,
$u_i(t_i) \geq \beta_i M_i(t_i; p_{-i})$.

- Each player gets at least their random monopoly payoff.

Random monopoly

- Each player gets at least their random monopoly payoff.
- In many cases, Theorem 4 is enough to characterize payoffs and equilibrium behavior, as there is unique interim efficient allocation that satisfies the random monopoly condition:
  - $\beta_i \in \{0, 1\}$,
  - $p_i \in \{0, 1\}$ for one of the players,
  - $l_1 = l_2$ or $l_2 = h_1$ or $h_1 = h_2$.
- In general, there is a gap between random monopoly payoffs and efficiency.

Random monopoly

Proof:

- The idea is to reproduce the complete info argument. Fix player i.
- The smallest equilibrium random monopoly share:

$$x^* = \min_{u \in B} \min_{t_i} \frac{u_i}{M_i(t_i; p_{-i})}.$$

- The set of all feasible and IC payoffs that give player i at least x share of her monopoly payoffs:

$$A_x(p) = \{u \in \mathcal{U}(p) : u_i \geq x M_i(.; p_{-i})\}.$$

- Then,

$$B \subseteq A_{x^*}.$$

Random monopoly

Proof:

- Easy to check that

$$\delta B \subseteq \delta A_{x^*} \subseteq A_{1-\delta(1-x^*)}.$$

  - Instead of delaying payoffs, we can give them today with prob. δ and with prob. $1-\delta$, give player i his monopoly payoff,
  - but we can do better as well.
- Goal: find $a \subseteq A_{1-\delta(1-x^*)}$ st.
  - a cannot be refused given $A_{1-\delta(1-x^*)}$ and
  - $a \subseteq A_{1-\delta(1-x^*)}$, i.e, each type t_i receives payoff at least

$$\geq (1-\delta(1-x^*)) M_i(t_i; p_{-i}).$$

- If $x^* < \beta_i$, complete information argument shows that player i has a profitable deviation.

Random monopoly

Offers that cannot be refused

Lemma

For each x, there exists mechanism $a^i(x)$ such that

- $a^i(x) \subseteq A_x$,
- $a^i(x)$ *is (mostly) payoff function such that* $I_{-i}(a^i(x)) = a^i(x)$.

- https://bwm-payoffs.streamlit.app/

Random monopoly

Offers that cannot be refused

- The question of existence in general seems related to the existence of
  - SUPO allocations in (Maskin Tirole 90):
  - outcome of competitive equilibrium economy, where types t_{-i} trading slacks in IC and IR constraints on types of player i
- No natural way of extending this argument.
  - the IR constraint $u_i(t_i) \geq xM(t_i|p_{-i})$ is type- and belief-dependent.

Outline

The Gap

- In general, there is a gap between the largest *ex ante* (expected) payoffs and random monopoly payoffs:

$$\mathrm{Gap}\,(p) = \max_{u \in \mathcal{U}(p) \text{ st. } \forall_{i,t_i} u_i(t) \geq \beta_i M_i(t_i|p)} p_1 \cdot (u_1 - \beta_1 M_1\,(.|p)) \tag{1}$$

- The gap is not larger than

$$\mathrm{Gap}\,(p) \leq 6.25\% \text{ of } h_2 \text{ for all } p.$$

The Gap

Theorem

For each p,

$$\lim_{\delta \leftarrow 1} \sup_{u \in B(p)} \left| p_1 \cdot u_1 - \left[p_1 \cdot \beta_1 M_1 \left(. | p \right) + Gap \left(p \right) \right] \right| = 0.$$

- As $\delta \to 1$, player 1 equilibrium *ex ante* payoffs converge to maximum possible subject to feasibility, IC, and random monopoly constraint.
  - player 1's payoffs are determined uniquely in ex ante sense,
  - player 2's payoffs are determined uniquely in the *interim* sense.

The Gap

- Player 1 (i.e., $l_1 < l_2$) gets the entire Gap!
  - a^2 is an example of mechanism attaining such payoffs,
- Why?

The Gap

- Player 1 (i.e., $l_1 < l_2$) gets the entire Gap!
  - a^2 is an example of mechanism attaining such payoffs.
- Why?
  - linearly transferable payoffs at p_1^*,
  - mixing and matching mechanisms that cannot be refused
  - convexity of mechanism a^2.
- https://bwm-payoffs.streamlit.app/

The Gap

Proof: Mixing and Matching

- Player 1 has (among many) two offers that cannot be refused by player 2 given equilibrium:
  - $a^1 = a^1(\beta)$,
  - $a^2 - \text{Gap}(., p_2^*)$

Outline

Conclusions

- A natural modification of a standard random-proposer bargaining has a "unique" outcome under
  - single good plus transfers environment,
  - private values (two types for each player).
- A proof of concept - better results and a general theory would be nice.