Bargaining with Mechanisms

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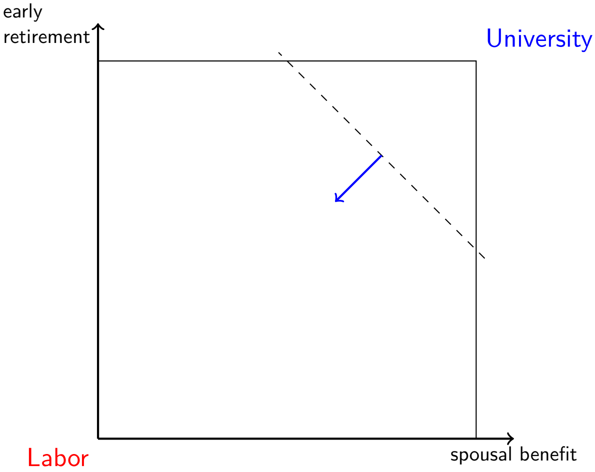
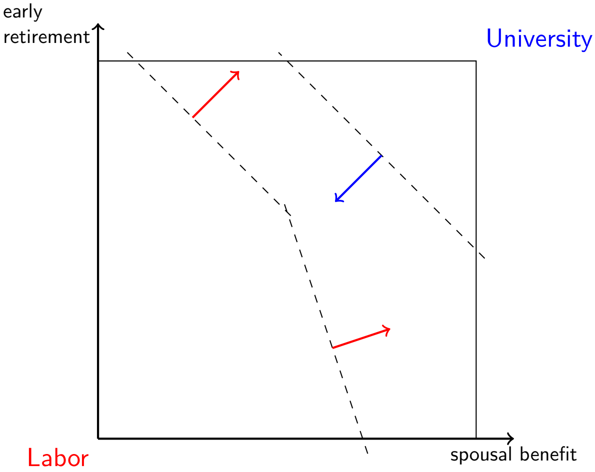
Introduction

UofT pension bargaining



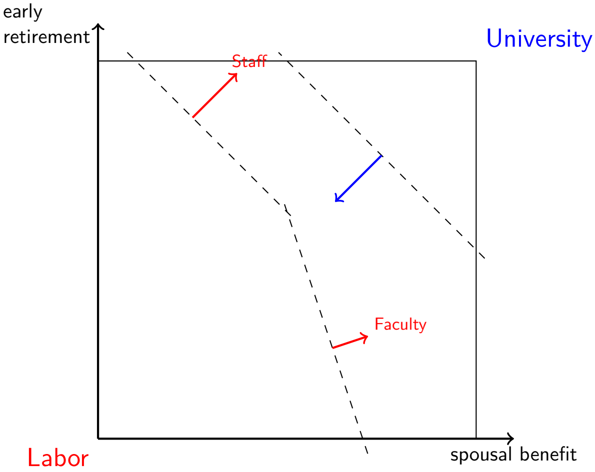
Introduction

UofT pension bargaining



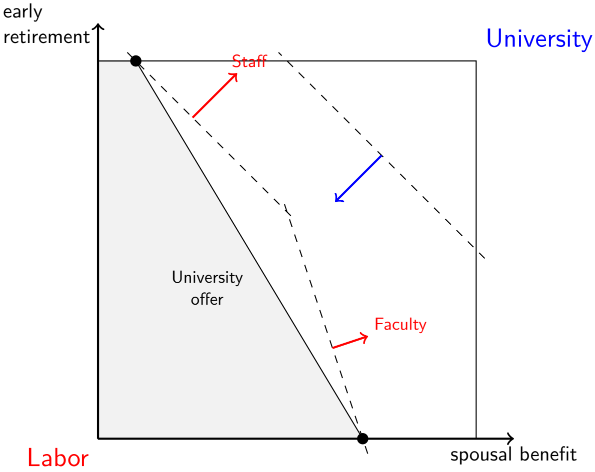
Introduction

UofT pension bargaining



Introduction

UofT pension bargaining



Introduction

Alternating-offer bargaining over heterogeneous pie,

- one-sided incomplete information about preferences,
- mechanisms as offers.

Introduction

- Mechanisms as offers:
  - menus,
  - menus of menus,
  - "I divide and you choose" vs "you divide and I choose",
  - arbitration and general mechanisms,
  - negotiations to create or alter the bargaining protocol.

Literature

- Complete information about preferences:
  - axiomatic: Nash (50, 53)
  - alternating-offer Rubinstein (82)
  - reputational: Myerson (91), Kambe (99), Abreu and Gul (00), Compte and Jehiel (02), Fanning (16)
  - *all solutions the same -> Nash program success!*
- Incomplete information:
  - axiomatic (mechanisms): Harsanyi and Selten (72), Myerson (84)
  - Coasian-bargaining with menus (2 types only): Wang (98), Strulovici (17)
  - alternating-offer with menus (2 types only + refinements): Sen (00), Inderst (03)
  - common knowledge of surplus: Jackson et al (20).
- Dynamic mechanism design without commitment: Skreta (06), Liu et al (19), Doval, Skreta (18).
- Dynamic informed principal?

Introduction

- **Main result**: When $N = 2$, there are unique $\delta - 1$ limit PBE payoffs: Bob chooses optimal screening menu s.t. each Alice type receives complete info. payoff.
  - no refinements needed,
  - incentive- (i..e, ex ante) efficient, but not ex post efficient
  - constrained commitment solution, non-Coasian result,
    - a reputational model leads to a different result
  - equilibrium bounds when $N \geq 3$.
- Role of mechanisms:
  - menus help with screening and signaling,
  - menus of menus help with belief punishment,
  - no other mechanisms needed.

Plan

1. Introduction
2. Model
3. Complete information
4. Main result
5. Proof
6. Comments: single offers, $N \geq 3$, belief-seller environment, renegotiation.

Model

Environment

- Alice (informed) and Bob (uninformed).
- Pie $X = \{(x_{A,c}, x_{A,s}, x_{B,c}, x_{B,s}) : \sum_i x_{i,n} \leq 1 \text{ for each } n\}$.
- Linear preferences $\mathcal{U} := \{(u_c, u_s) : u_n \geq 0, \sum u_n = 1\}$
  - linear utilities $u \in \mathcal{U}$ from $x \in X$: $u(x) = \sum_n u_i x_{i,n}$,
  - Bob's preferences v,
  - Bob's beliefs $\mu \in \Delta\mathcal{U}$ about Alice's preferences u.
- Discounting $\delta < 1$.
- Alternating-offer bargaining with mechanisms as offers.

Model

Mechanisms as offers

- Each offer is a *mechanism*: a finite-horizon extensive-form game.
  - $m = \left(\left(S_A^t, S_B^t\right)_{t \leq T}, \chi \right)$
  - allocation: $\chi : \prod_{i,t} S_{i,t} \to X$,
  - $T < \infty$ and S_i^t compact.
- Examples: single-offers, menu, menu of menus
- $\mathcal{M}$ - "compact" space of all available mechanisms
  - main result hold as long as $\mathcal{M}$ contains menus and menus of menus.

Model

Equilibrium

- Perfect Bayesian Equilibrium,
  - existence is an issue.
- (Payoff) outcomes:

$$e_B \in [0,1], e_A : \mathcal{U} \to [0,1].$$

- Limit set of equilibrium outcomes $E^j(\delta, \mu)$:

$$E^j(\mu) = \lim_{\delta \to 1} E^j(\delta, \mu)$$

Model

Commitment

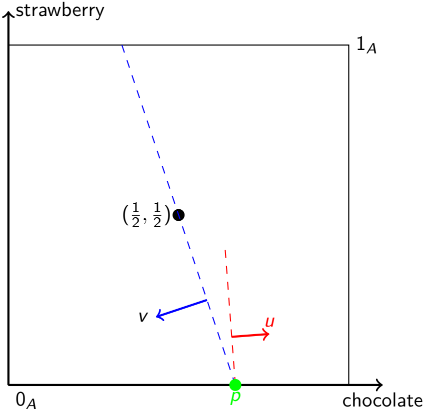
- Coasian bargaining and dynamic mechanism design without commitment: Doval, Skreta (18), Liu et al (19)
- As in that literature,
  - players cannot *unilaterally* commit to future offers,
  - players are committed to an offer for the period in which the offer is made.
- But, we allow for mechanisms, which offered and accepted *bilaterally*, may commit players to an ex post inefficient allocation.
- Would allowing renegotiation change anything? -> later.

Complete information

- Complete information bargaining: Alice u, and Bob v (fixed).
- Assume $v_c > v_s$,
  - Bob likes chocolate more than he likes strawberry.
- As $\delta \to 1$, Alice's payoffs converge to the Nash solution: $(\mathcal{N}_A(u), \mathcal{N}_B(u))$.

Complete information

Nash allocations I

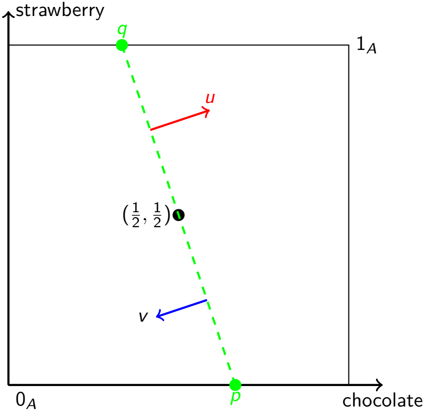


Nash allocations:

- p if $u_c > v_c$, i.e., if Alice likes chocolate more than Bob.

Complete information

Nash allocations II

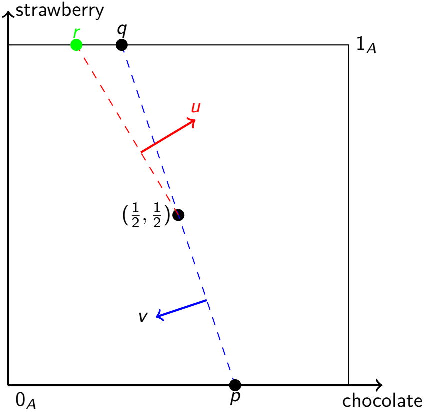


Nash allocations:

- p if $u_c > v_c$,
- $\overline{pq}$ if $u_c = v_c$

Complete information

Nash allocations III

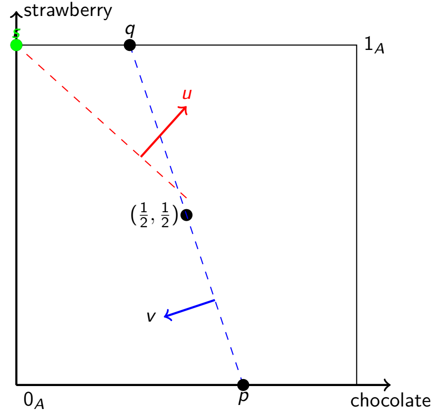


Nash allocations:

- p if $u_c > v_c$,
- $\overline{pq}$ if $u_c = v_c$,
- r if $\frac{1}{2} < u_c < v_c$,

Complete information

Nash allocations IV

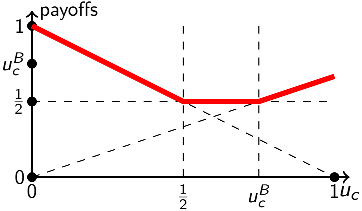


Nash allocations:

- p if $u_c > v_c$,
- $\overline{pq}$ if $u_c = v_c$,
- r if $\frac{1}{2} < u_c < v_c$,
- s if $u_c < \frac{1}{2}$ (i.e., Alice likes strawberry more)

Complete information

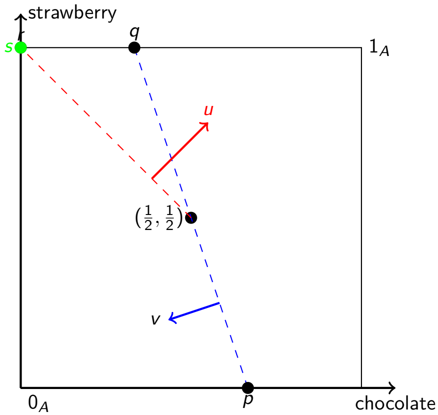
- Alice's Nash payoffs:



Alice	all strawberry	all strawberry some chocolate	some chocolate
Bob	all chocolate	some chocolate	all strawberry some chocolate
	Anti-Coase		*Coase*

Complete information

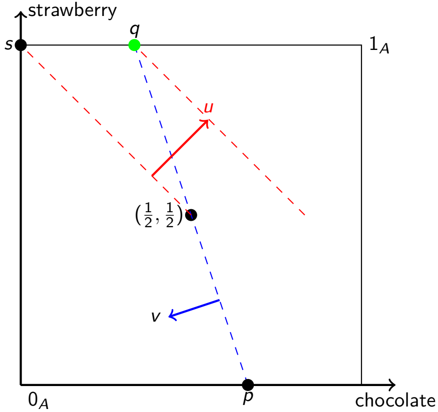
Incentive problem I



Incentive problem.

Complete information

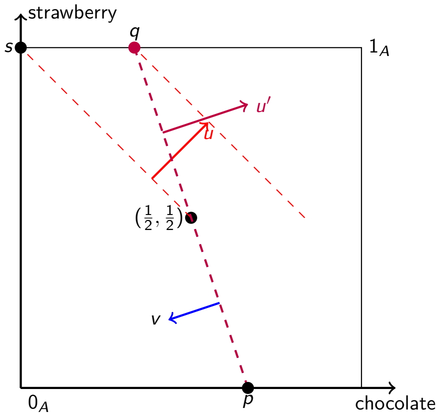
Incentive problem II



Incentive problem.

Complete information

Incentive problem III

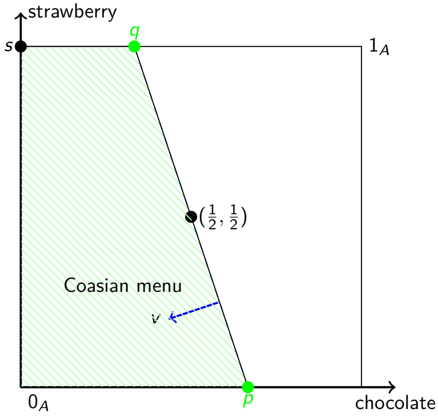


Incentive problem.

- types $u_c < v_c$ prefer to report $u'_c \approx v_c$

Complete information

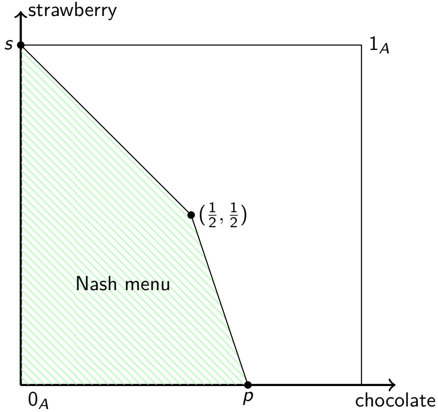
Coasian menu



- If we ignore incentive problem, Alice chooses either p or q
- Coasian menu $\{p, q\}$.
- A companion paper studies the same environment,
  - bargaining with reputational types like in Abreu-Gul (00) and Kambe (98)
  - Coasian menu is the unique equilibium outcome.

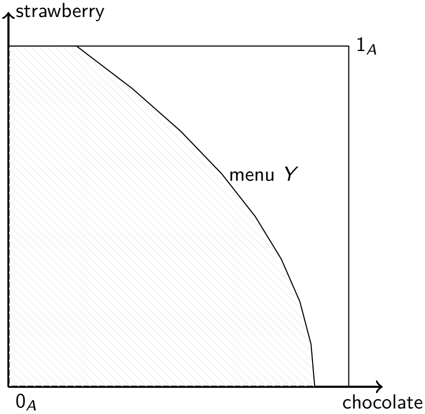
Complete information

Nash menu



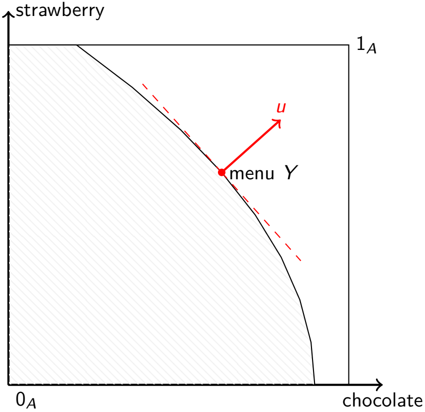
- If we want to ensure that each type of Alice receives her complete information payoff, we can offer Nash menu $\left\{s, \left(\frac{1}{2}, \frac{1}{2}\right), p\right\}$.

Main result



- Alice's menu: $Y \subseteq X$

Main result

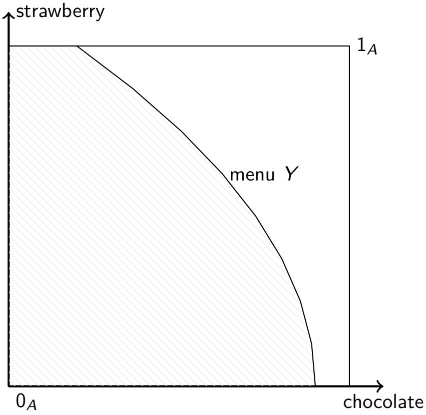


- Alice's menu: $Y \subseteq X$
- Alice's type u payoff:

$$y(u; Y) = \max_{x \in Y} u(x)$$

.

Main result



- Alice's menu: $Y \subseteq X$
- Alice's type u payoff:

$$y(u; Y) = \max_{x \in Y} u(x)$$

- Bob's expected payoff:

$$\Pi(\mu; Y)$$
$$= \int \max_{x \in \arg\max_{x' \in Y} u(x')} v(x)\, d\mu(u).$$

Main result

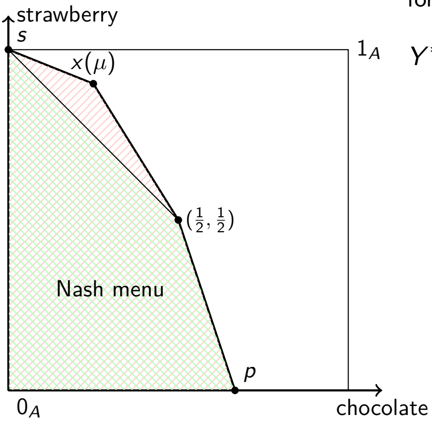
Theorem

Suppose $\mathcal{M}$ contains all menus and menus of menus. In the limit $\delta \to 1$, the payoffs are as if Bob offered

$$Y^*(\mu) \in \arg\max_{Y \supseteq \mathcal{N}} \Pi(\mu; Y)$$

i.e. optimal (for him) screening menu Y subject to Alice receiving at least her complete information payoff.

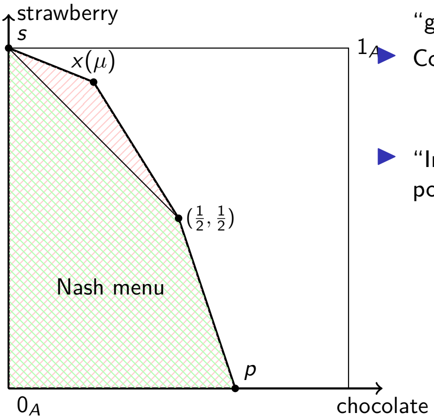
Main result



- **Theorem**: Optimal menu has a form

$$Y^*(\mu) = \mathcal{N} \cup \{x(\mu)\}$$
$$= \left\{p, \left(\frac{1}{2}, \frac{1}{2}\right), x(\mu), s\right\}$$

Main result



- Bob's payoff is unique and continuous in μ. Alice's payoff is "generically" unique.
- Constrained "commitment".
  - not a Coasian menu,
  - not a reputational result.
- "Incentive-efficient", but not ex post efficient.

Proof

Complete information

- Suppose that Alice's type u is known.
- Let $\Pi(y) = \max_{x:u(x)\geq y} v(x)$ be Bob's payoff.
- Let y be the highest payoff. It is *too high* if there exists $y' \geq \delta y$ such that $\delta \Pi(y') > \Pi(y)$:
  - such y' is a profitable deviation for Bob.
- The highest payoff cannot be too high. Similarly for the lowest payoff.
- Properties of Π mean that the two payoffs must be the same.

Proof

Menus

- **Revelation principle**:For each equilibrium, there is a menu-belief pair (Y, μ) such that in equilibrium
  - each Alice's type u receives $y(u; Y)$,
  - Bob receives $\leq \Pi(\mu; Y)$.
- We can assume that Y is the largest possible among all menus that satisfy the above conditions.

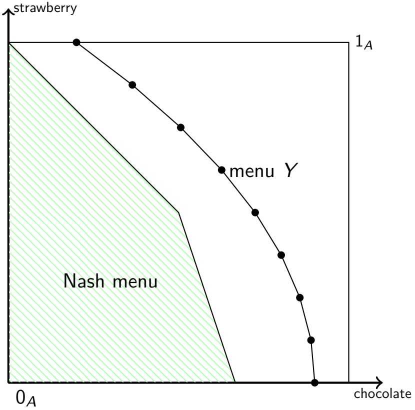
Proof

Menus

- Menu-belief pair (Y, μ) is dominated by (Y', μ') if
  - $\text{supp}\mu' \subseteq \text{supp}\mu$, and
  - $y(u, Y') \geq y(u, Y)$ for each $u \in \text{supp}\mu'$.
- There "exists" undominated equilibrium menu-belief pair.
- The undominated pair is alternative to "the highest payoff".

Proof

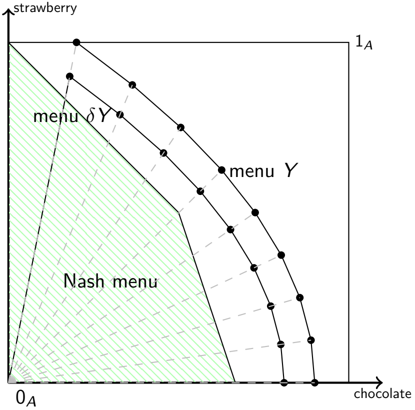
Upper bound



- Menu Y is too high for Alice if

Proof

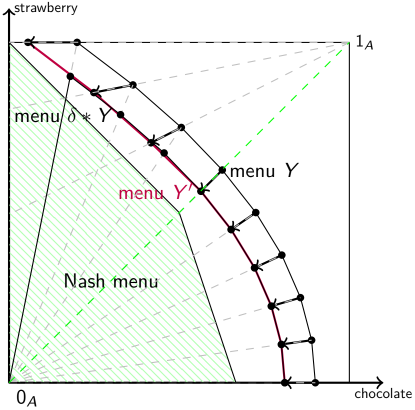
Upper bound



- Menu Y is too high for Alice if there exists $Y' \supset \delta Y$ s.t..

Proof

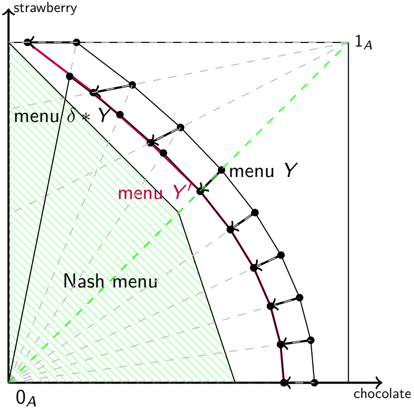
Upper bound



- Menu Y is too high for Alice if there exists $Y' \supset \delta Y$ s.t. $\delta \Pi(Y', \mu) > \Pi(Y, \mu)$.

Proof

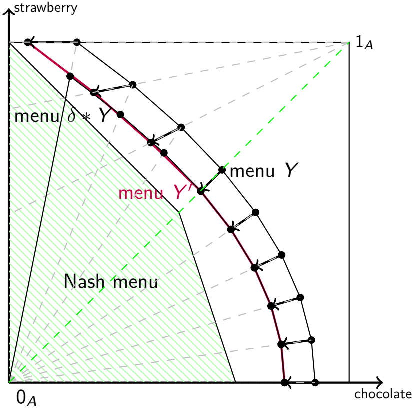
Upper bound



- Menu Y is too high for Alice if
  there exists $Y' \supset \delta Y$ s.t. $\delta \Pi(Y', \mu) > \Pi(Y, \mu)$.
- Any Y that is strictly higher than Nash menu is too high.

Proof

Upper bound



- Menu Y is too high for Alice if there exists $Y' \supset \delta Y$ s.t. $\delta\Pi(Y', \mu) > \Pi(Y, \mu)$.
- Any Y that is strictly higher than Nash menu is too high.
- But, undominated eq. menu-belief pair cannot be too high:
  - otherwise Y'' is a profitable deviation for Bob,
  - that is accepted by all Alice's types.

Proof

Upper bound

- Hence no equilibrium payoff can be uniformly higher than Nash payoffs $\mathcal{N}_A$ on the support of beliefs.
- If so, any menu with payoffs strictly above Nash must be accepted,
  - if not, some of the rejecting types must expect Nash continuation, which is not profitable for them.
- But then, Bob's payoff cannot be lower than

$$\max_{Y \supseteq \text{Nash menu}} \Pi(\mu; Y).$$

Proof

Lower bound

- If Alice's payoffs are too low, then Alice should have a profitable deviation:
  - a problem: find a deviation that is attractive for Bob *with arbitrary beliefs*,
  - solution: menu of menus

$$W(u, y_u) = \{Y \subseteq X : y(u; Y) \geq y_u\}.$$

Alice says: "I am type u and want payoff y_u, however you want to to give it to me."

Proof

Lower bound

- Payoff y_u is too low for type u if for any menu Y such that $y_u \geq y(u; Y)$, any beliefs ψ, there exists menu Y' such that

$$\delta y(u; Y') > y \text{ and } \Pi(\psi, Y') > \delta \Pi(\psi; Y).$$

- We show that
  - $y < \frac{1}{2}$ is too low for any type u,
  - $y < 1$ is too low for the type who only likes strawberries
  - $y < \frac{1}{2v_c}$ is too low for the type who only likes chocolate.
- Any equilibrium menu must contain Nash menu.

Proof

Role of menus

- Menus help with screening and signaling problems,
- menus of menus help and responding to belief threats.
- Definitions of “too high” and “too low” and the “inductive” steps likely generalize to other environments.
  The geometry of what is “too high” or “too low” does not.

Comments

1. Single offers
2. $N > 2$
3. Buyer-seller case
4. Renegotiation

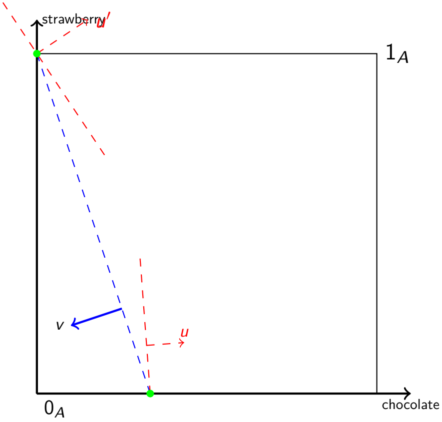
Comments

Single offer

- The ability to offer mechanisms is important for the uniqueness.
- Assume that only single offers are allowed.
- Continuum of equilibria due to signaling issues and punishment with beliefs.

Comments

Single offer



- Anti-Coasian equilibrium.
  - punishment of deviations with "bad" beliefs.
- This equilibrium does not survive if Alice can make menus of menus.

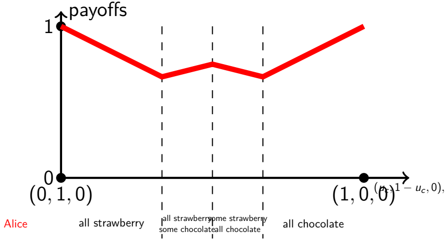
Comments

$N > 2$

- Suppose $N = 3$ (chocolate, strawberry, vanilla).
- $v = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.
- $\mathcal{N}_A$ is not a menu (it is not convex).

Comments

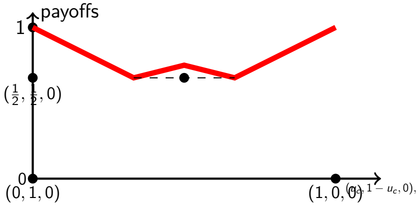
$N > 2$



- $\mathcal{N}_A$ is not a menu (it is not convex).

Comments

$N > 2$



- There is an equilibrium st.

$$e_A\left(\frac{1}{2}, \frac{1}{2}, 0\right)$$
$$\leq \left(\mathrm{Vex}\mathcal{N}^A\right)\left(\frac{1}{2}, \frac{1}{2}, 0\right)$$

.

  - punishment with beliefs

Comments

Buyer-seller environment

- Good with quality $q \in [0,1]$,
- seller's preferences $cq + t$, where $c > 0$,
- single buyer with utility $uq - t$, where u is unknown by the seller,
- "gap" case: $u \geq u_0 > c$,
- Allocation $(q,t) \in [0,1] \times \mathbb{R}$.
- Nash menu:

$$\left\{ \left(\frac{1}{2}, \frac{1}{2}c \right) \right\}.$$

Comments

Buyer-seller environment

- Identical solution.

Theorem

Suppose $\mathcal{M}$ contains all menus and menus of menus. In the limit $\delta \to 1$, the payoffs are as if Bob offered

$$Y^*(\mu) \in \arg\max_{Y \supseteq \mathcal{N}} \Pi(\mu; Y).$$

Comments

Buyer-seller environment

Theorem

Optimal menu:

$$Y^*(\mu) = \left\{ \left(\frac{1}{2}, \frac{1}{2}c \right), \left(1, \frac{1}{2}c + \frac{1}{2}p^* \right) \right\},$$

where p^* is the price chosen by the single-price discriminating monopolist.

- Optimal menu is a random dictatorship between optimal IR mechanisms for buyer and seller.

Comments

Renegotiation

- Suppose that both Alice and Bob need both to agree to come back to the negotiation table (i.e., before further offers are made):
  - after an agreement is reached, one of them may ask: can we renegotiate
  - if the other agrees, the agreement is torn apart, and the game is restarted,
  - otherwise, finish.
- Renegotiation leads to the possibility that menus are not dominant-solvable any more.

Comments

Renegotiation

- In the limit $\delta \to 1$ of equilibria of the bargaining game with renegotiation, Bob's payoffs is not smaller than

$$\max_{Y \supseteq \text{Nash menu}} \Pi(\mu; Y).$$

- The lower bound on Bob's payoffs remains the same.

Conclusion

- A model of bargaining with incomplete information about preferences and mechanisms as offers
- Main result: unique and continuous equilibrium outcome
  - role of mechanisms in bargaining
  - but not clear what to do about about Nash program,
  - also, a companion paper: reputational types lead to a different result.
- Proof of a concept that bargaining with mechanisms is possible and useful,
  - other environments, two-sided incomplete information