Bargaining with Mechanisms

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Introduction

- Sophisticated offers in real world
  - "I split, you choose",
  - menus,
  - deadlines,
  - divide the object into "areas" and negotiate them separately,
  - delay resolution in some areas,
  - propose arbitration (example: trial by gods),
  - propose a change to bargaining protocols, etc.

Introduction

- Model of bargaining, where players offer mechanisms to find a resolution.
- Why mechanisms help?
  - screening: which type of the opponent wants what?
  - signaling: how to protect oneself from revealing information?
  - "belief threats": can opponent's adversarial beliefs be tested?

Model

Environment

- Alice (informed) and Bob (uninformed):
  - Bob's beliefs F about Alice's preferences $u \in [0, 1]$,
  - Bob's preferences $v \in [0, 1]$ are known.
- Single good + transfers,
  - Alice's utility: $qu + t$
  - Bob's utility $(1 - q)\, v - t$
- Bargaining game
  - multiple rounds until offer is accepted, discounting $\delta < 1$,
  - random proposer: Alice is a proposer with i.i.d. probability $\beta = \beta_A$ and Bob with prob. $1 - \beta = \beta_B$,
    - both sides make offers,
    - includes single-proposer games $\beta \in \{0, 1\}$.

Model

Mechanisms as offers

- Each offer is a *mechanism*: a finite-horizon extensive-form game.
  - $m = \left((S_A^t, S_B^t)_{t \leq T}, \chi \right)$
  - allocation: $\chi : \prod_{i,t} S_{i,t} \to X$,
  - $T < \infty$ and S_i^t compact.
  - examples: single-offers, menu, menu of menus
- When an offer is accepted, mechanism is implemented, and the game ends.
- Main result hold as long as $\mathcal{M}$ contains menus and menus of menus.

Model
Equilibrium

- Perfect Bayesian Equilibrium,
  - existence is an issue,
  - we show the existence if $\mathcal{M}$ is "compact".

Model
Commitment

- Coasian bargaining and dynamic mechanism design without commitment: Skreta (06), Liu et al (19), Doval, Skreta (21),
  - only uninformed party makes offers.
- As in that literature,
  - players cannot *unilaterally* commit to future offers,
  - players are committed to an offer for the period in which the offer is made,
  - once the offer is accepted, it must be implemented.
- But, mechanisms may generate ex post inefficient allocation,
  - players have also access to a large(-r) space of mechanisms,
  - applications: bargaining over protocol, bargaining without common knowledge of surplus

Main result

Complete information

- Complete information bargaining: Alice u, and Bob v (fixed).
- Surplus $\max(u, v)$.
- Both players split the surplus, and receive

$$(\beta \max(u, v), (1-\beta) \max(u, v))$$

  - the player with higher utility gets the good and pays out a fraction of its value in the form of a transfer.
- This is not incentive compatible if Alice's utility $u > v$.

Main result

Optimal mechanisms

- Alice's optimal (ICR) mechanism:
  - if $u < v$, sell at v, otherwise keep,
  - payoffs
$$(\max(u, v), 0)$$
- Bob's optimal mechanism:
  - offer for sale at price $p^* \in \arg\max vF(p) + p(1 - F(p))$
  - payoffs
$$(\max(u - p^*, 0), vF(p^*) + p^*(1 - F(p^*))).$$
  - assume for simplicity that p^* is unique.

Main result

Theorem

Suppose $\mathcal{M}$ contains all menus and menus of menus.
Then, in the unique equilibrium, the expected payoffs are as if with prob β_i, player $i = A, B$ implements their optimal mechanism.

- $\beta-$random property ("usage" + "sell") right,
- "Incentive-efficient", but not ex post efficient,
- Bob's payoffs are continuous and convex in F,
- Bob's constrained commitment.

Main result

Equilibrium

- For each α, let m^*_α be the best mechanism for Bob st. Alice receives her complete info payoffs

$$y(u) \geq \alpha \max(u, v) =: y_\alpha(u)$$

  - α-random property rights,
  - also 3-element Alice's menu $Y_{\alpha,p}$:
    - Bob gets the good and Alice receives transfer αv,
    - Alice gets the good with prob. α,
    - Alice gets the good, and pays $(1-\alpha)p$,
  - payoffs are affine in α,

$$\text{Alice payoffs: } y^*_\alpha(u) := \alpha \max(u, v) + (1-\alpha) \max(u - p^*, 0)$$
$$\text{Bob payoffs: } \Pi^*_\alpha(F) := (1-\alpha)\left[vF(p^*) + p^*(1 - F(p^*))\right],$$

Main result

Equilibrium

- In equilibrium, if player i is chosen a proposer, they offer $m^*_{\alpha_i}$, where

$$\alpha_A = 1 - \delta\,(1 - \beta) \text{ and } \alpha_B = \delta\beta.$$

  - the average payoff is as if m^*_β was implemented,
  - Bob is indifferent between accepting Alice's offer and waiting for m^*_β,
  - Alice is either indifferent or strictly prefers to accept Bob's offer than to wait for m^*_β.

Main result

Payoff bounds

- These are the only equilibrium payoffs.
- If Bob's payoff is lower, he has a profitable deviation in the form of menu Y_{α,p^*}:
  - helps with screening and signaling
- If Alice's payoff is too low, she has a profitable deviation in the form of a menu of menus:
$$\{Y_{\alpha,p} : p \in [0,1]\},$$
  - helps with "belief threats".

Comments

1. Neutral solution
2. Coasian bargaining
3. Renegotiation
4. Other bargaining environments
5. Two-sided incomplete information

Comments

Neutral solution

- Axiomatic bargainin: Harsanyi and Selten (72), Myerson (84)
  - incentive compatible mechanisms,
- (Myerson 84) - neutral solution as a minimal set of incentive compatible outcomes that satisfies three axioms
  - probability invariance
  - extension axiom,
  - random-dictatorship (with simple bargaining problems .
- In practice, equal sharing of virtual valuations.

Comments

Neutral solution

- Here: assume that $\beta = 1/2$.

Theorem

Suppose that

$$(u - v) f(u) - (1 - F(u))$$

is strictly increasing in u. Then, equal likelihood of "property rights" mechanism is the unique neutral solution.

Comments

Coasian bargaining

- When $\beta = 0$, Bob is the single proposer, the unique PBE is that Bob proposes optimal selling mechanism: sell at price $p^* > v$, which is accepted.
  - that's unlike Coasian bargaining, where Bob would sell at v:
  - in the Coasian bargaining, if offer is rejected, Bob cannot stop himself from learning that it is rejected,
  - here, rejection does not reveal any information,
- The ability of players to commit to the mechanism once accepted is important, but not crucial - see next!

Comments

Two-sided incomplete information

- Suppose that two players can have two types $u_l < u_h$.
  - beliefs $F_i \in \Delta \{u_l, u_h\}$,
- $\beta_A + \beta_B = 1$ proposer probabilities:
- β-random property right mechanism: with prob. β_i, player i gets the good and may offer to sell it at price $p = u_h$.
  - this mechanism is ex post efficient.

Comments

Two-sided incomplete information

Theorem

Suppose $\mathcal{M}$ contains all α-random property rights mechanisms for all $\alpha \in [0, 1]$. Then, in the unique equilibrium, the expected payoffs are as if β-random property rights mechanism is implemented.

Conclusion

- A model of bargaining with incomplete information and mechanisms as offers
- Main result: unique and continuous equilibrium outcome
  - role of mechanisms in bargaining,
- Proof of a concept that bargaining with mechanisms is possible and useful,
  - relation to axiomatic theory,
  - other environments,
  - two-sided incomplete information,