

Bargaining with Mechanisms and Two-Sided Incomplete Information

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Outline

- 1 Introduction
- 2 Model
- 3 Preliminary observations
- 4 Random monopoly payoff bound
- 5 The Gap
- 6 Conclusions

Introduction

- Business partners want to cease partnership. Their firm cannot be divided, and if partner keeps it, the other expects a compensation.
- Two countries negotiate a peace treaty, with land swaps and reparations (or economic aid) on the table.
- Coalition parties negotiate an agreement with a support for policy traded off against number of cabinet positions.
- <https://bwm-payoffs.streamlit.app/>

Introduction

- Bargaining - one of the longest-studied problems in economic theory (“bilateral monopoly” before [Nash 50])
- No satisfactory solution for incomplete information:
 - ▶ cooperative solutions: (Harsanyi 72), (Myerson 84),
 - ▶ large literature on bargaining over prices:
 - ★ one-sided: uniqueness in Coasian bargaining with a gap,
 - ★ two-sided: large set of equilibria, possible refinements to eliminate some (Ausubel, Crampton, Deneckere 02 and others).
- Goal: show that a natural modification of a standard random-proposer bargaining has a “unique” outcome under
 - ▶ single good plus transfers environment,
 - ▶ private values (two types for each player).

Introduction

- Bargaining with sophisticated offers in real world
 - ▶ menus,
 - ▶ menus of menus (“I divide, you choose”),
 - ▶ mediation, arbitration (example: “trial by gods”),
 - ▶ change in bargaining protocols,
 - ▶ deadlines or delays, etc.
- Challenges:
 - ▶ how to model mechanisms as actions?
 - ▶ signaling.

Introduction

- How to model mechanisms as actions?
 - ▶ No revelation principle.
 - ▶ Games: actions and outcome function.
 - ▶ Real mechanisms: correspondences of equilibrium payoffs obtained in games.
 - ▶ (Abstract) mechanisms: proper limits of such correspondences.

Introduction

- Signaling: larger action space helps to reduce the number of equilibria.
- Informed principal with private values (Maskin Tirole, 90)
 - ▶ informed principal types get their monopoly payoff,
 - ▶ private information of the principal does not matter in private values case.
- One-sided incomplete information (Peski 22),
 - ▶ uninformed player and some of the informed player types get random monopoly payoff,

Introduction

Results

- Suppose each player has two types and, w.l.o.g., that $l_1 < l_2$.
- **Theorem 1:** For each discount factor, each player expects at least their random monopoly payoff.
- **Theorem 2:** As $\delta \rightarrow 1$, *ex ante* expected payoffs of player 1 converge to a feasible maximum subject to a constraint that player 2 types get their random monopoly payoffs.

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Outline

1 Introduction

2 Model

- Bargaining game
- Mechanisms and Implementation
- Equilibrium
- Commitment

3 Preliminary observations

4 Random monopoly payoff bound

5 The Gap

6 Conclusions

Model

Environment

- Two players $i = 1, 2$, sometimes third player (“mediator”).
- Single good and transfers
- Preferences: $q_i t_i - \tau_i$,
 - ▶ t_i - type (valuation) of player i ,
 - ▶ q_i - probability that pl. i gets the good,
 - ▶ τ_i - transfer from player i
 - ▶ feasibility: $q_1 + q_2 \leq 1$, $q_i \geq 0$, $\tau_1 + \tau_2 \leq 0$,

Model

Bargaining game

- Bargaining game
 - ▶ multiple rounds until offer is accepted, discounting $\delta < 1$,
 - ▶ random proposer: player i is chosen with prob. $\beta_i \geq 0$, where $\beta_1 + \beta_2 = 1$,
 - ▶ proposer offers a mechanism,
 - ▶ if the offer is accepted, it is implemented, and the bargaining game ends.
- Perfect Bayesian Equilibrium:
 - ▶ no updating beliefs about player i after $-i$'s action.
 - ▶ public randomization plus cheap talk.

Model

Feasible payoffs

- Payoff vector $u(\cdot|q, \tau) \in R^{T_1 \cup T_2}$ in allocation $q_i(\cdot), \tau(\cdot)$:

$$u_i(t_i|q, \tau) = \sum_{t_{-i}} p(t_{-i}) (t_i q_i(t_i, t_{-i}) - \tau_i(t_i, t_{-i})) \text{ for each } t_i.$$

- Allocation $q_i(\cdot), \tau(\cdot)$ is IC given beliefs p iff

$$u_i(t_i|q, \tau) \geq \sum_{t_{-i}} p(t_{-i}) (t_i q_i(s_i, t_{-i}) - \tau_i(s_i, t_{-i})) \text{ for each } t_i, s_i.$$

- Correspondence of feasible and IC payoffs:

$$\mathcal{U}(p) = \{u(\cdot|q, \tau) : (q, \tau) \text{ is IC given } p\} \subseteq R^{T_1 \cup T_2}.$$

- The geometry of the correspondence $\mathcal{U}(\cdot)$ is the true “parameter” of the model.

Model

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Model

Games

- Game G :
 - ▶ players: 1, 2, and mediator (whose payoff is a non-negative transfer),
 - ▶ finite or compact actions,
 - ▶ continuous outcome function that maps actions to an allocation of a good and a transfer,
 - ▶ always assume public randomization.
- For each p , the set of equilibrium payoff vectors

$$m(p; G) \subseteq \mathcal{U}(p).$$

- Equilibrium correspondence:

$$m(\cdot; G) : \Delta T \rightrightarrows R^{T_1 \cup T_2}, m_G \subseteq \mathcal{U}.$$

Model

Mechanisms

- *Real mechanism* is a correspondence m for which there exists a game G such that $m = m(\cdot; G)$.
- Real mechanism m is
 - ▶ u.h.c.,
 - ▶ $m \subseteq \mathcal{U}$,
 - ▶ non-empty-valued, and
 - ▶ convex valued.

Model

Mechanisms

- (*Abstract*) mechanism is correspondence m st.
 - ▶ m is u.h.c.,
 - ▶ $m \subseteq \mathcal{U}$,
 - ▶ non-empty valued,
 - ▶ it can be *approximated* by continuous functions $m_n : \Delta T \rightarrow R^{T_1 \cup T_2}$, $m_n \subseteq \mathcal{U}$ such that

$$\lim_{n \rightarrow \infty} \max_p \min_{v, q: v \in m(q)} d((m_n(p), p), (v, q)) = 0,$$

where d is the Euclidean distance on $\Delta T \times R^{T_1 \cup T_2}$.

- The space of mechanism is compact* under Hausdorff distance induced by d .

Model

Implementation Theorem

Theorem

Any real mechanism is an (abstract) mechanism.

For any (abstract) mechanism m , there is a sequence of real mechanisms m_n that “approximate” m :

$$\lim_{n \rightarrow \infty} \max_{u, p: u \in m_n(p)} \min_{v, q: v \in m(q)} d((u, p), (v, q)) = 0.$$

- *First part: use Michael's Theorem.*
- *Second part: construct a game:*
 - ▶ *mediator names the beliefs p ,*
 - ▶ *given p , use virtual Bayesian implementation of (Abreu Matsushima 92).*

Model

Derived mechanisms

- Given a mechanism or a set of mechanisms, we can construct new ones:
- $\alpha \in \Delta A$ - randomly chosen mechanism according to distribution α .
- δm - discounted mechanism m .
- $I_i(m)$ - information revelation game: public randomization plus cheap talk by i .
- $MM_i(A)$ - menu of mechanisms $a \in A$ for player i (including p.r. and cheap talk by i).
- $IP_i(m)$ - informed principal problem of player i with continuation mechanism (i.e., outside option) m ,

$$IP_i(m) = MM_i \{ MM_{-i} \{ n, m \} : n \text{ is a mechanism} \}$$

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Model

Bargaining game

- Bargaining mechanism : the largest fixed point \mathcal{B} of

$$\mathcal{B} = (IP_1(\delta\mathcal{B}))^{\beta_1} (IP_2(\delta\mathcal{B}))^{\beta_2}$$

Model

Equilibrium

- Equilibrium: definition

- ▶ modular (one-shot deviation principle), extends to the existence in bargaining game,
- ▶ $PBE = WPBE + \text{"no updating after the other player actions"}$,
- ▶ if restricted to real mechanisms, approximate (i.e., ε -like) equilibrium.

- Equilibrium: existence

- ▶ space of (abstract) mechanisms is compact,
- ▶ if A finite, approximate each mechanism by a payoff function and apply Brouwer FPT,
- ▶ extend to compact A (cheap talk is important),
- ▶ public randomization is important.

Model

Commitment

- Players are not committed to future offers.
- Players are committed to implementing a mechanism once offered and accepted:
 - ▶ hence, less commitment than in the *limited commitment* literature (V. Skreta and L. Doval).
- Relevant for many situations
 - ▶ good allocation with no backsies,
 - ▶ bargaining over protocol,
- Lack of commitment is a restriction on the space of mechanisms,
- Commitment is not necessarily helpful to the agent who can exercise it.

Outline

1 Introduction

2 Model

3 Preliminary observations

- Complete information
- Informed principal
- One-sided incomplete information
- Offer design

4 Random monopoly payoff bound

5 The Gap

6 Conclusions

Preliminary observations

Complete information bargaining

- **Claim:** Assume $t_1 < t_2$ are known. Then, in each equilibrium, player i gets $\beta_i t_2$.
- Special features:
 - ▶ linearly transferable payoffs,
 - ▶ endogenous interdependent value.

Preliminary observations

Complete information bargaining

- **Claim:** Assume $t_1 < t_2$ are known. Then, in each equilibrium, player i gets $\beta_i t_2$.
- **Proof:** Suppose $i = 1$ (the other argument is analogous). Let

$$x^* = \min_{u \in \mathcal{B}} \frac{u_1}{t_2}.$$

- If $x^* < \beta_1$, player 1 has a profitable deviation:
 - ▶ wait until she is a proposer, and offer: player 2 gets the good and pays $(1 - \delta(1 - x^*)) t_2$ to player 1,
 - ▶ the offer will be accepted.

Preliminary observations

Informed principal

- (Random) informed principal with private values ($\beta_i = 1$ or $\delta = 0$):
 - ▶ monopoly payoff:

$$M(t_i; p_{-i}) = \max_{\tau} p_{-i}(t_{-i} \leq \tau) t_i + (1 - p_{-i}(t_{-i} \leq \tau)) \tau,$$

- ▶ If player i is a proposer, she offers the monopoly price to $-i$, which is accepted (the game ends),
 - ▶ i 's expected payoff is $M(t_i; p_{-i})$.
- Special features:
 - ▶ continuation value = 0 (and it does not depend on beliefs)
 - ▶ private information of the principal does not matter due to private values.

Preliminary observations

One-sided incomplete information

- One-sided incomplete ($p_i \in \{0, 1\}$, i.e., i is uninformed):
- The equilibrium payoffs are unique and implemented by random monopoly mechanism is offered:
 - ▶ with probability β_j , agent j gets the good:
 - ▶ if so, she offers monopoly price to $-j$,
 - ▶ player i 's expected payoff of $\beta_i M(t_i; p_{-i})$,
 - ▶ some player $-i$'s types may get a bit more than $\beta_{-i} M(t_{-i}; p_i)$,
- Special features:
 - ▶ random monopoly mechanism is interim efficient.

Preliminary observations

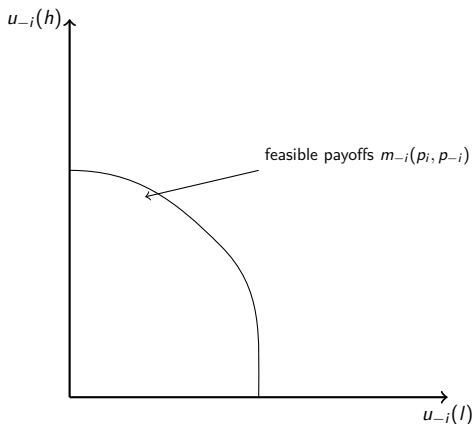
Offers that cannot be refused

- i makes an offer, $-i$ decides whether to accept or reject.
- Offer design:
 - ▶ making offers that are refused is inefficient due to surplus-burning delay,
 - ▶ control: offers should be accepted exactly as they are.
- Two problems:
 - ▶ \Rightarrow player $-i$ may have reasons to refuse the offer,
 - ▶ signaling: (possibly, off-path) offers lead to belief updating $p_i \rightarrow q_i$.

Preliminary observations

Offers that cannot be refused

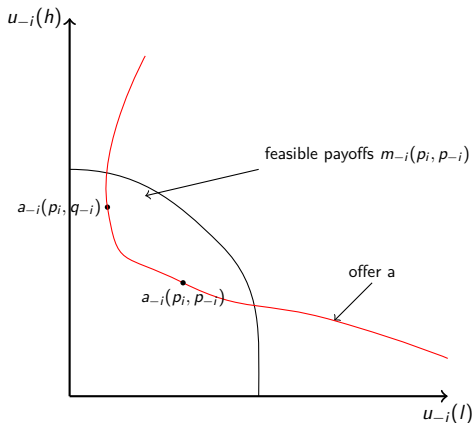
- m is a continuation mechanism.
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Preliminary observations

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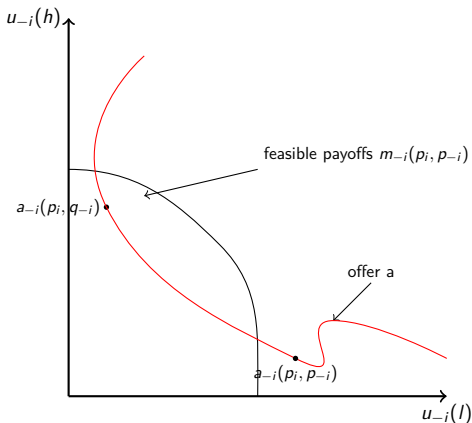
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Preliminary observations

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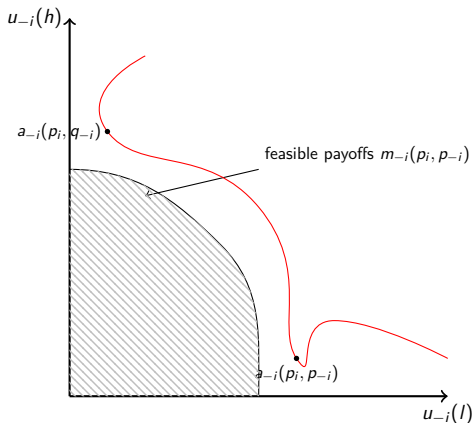
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Preliminary observations

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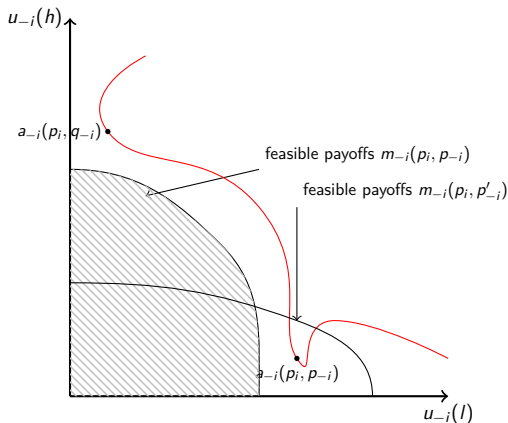
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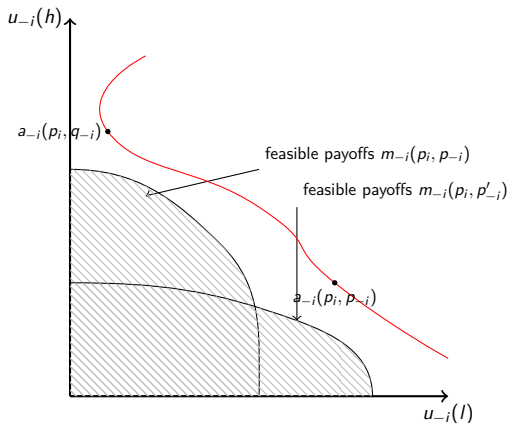
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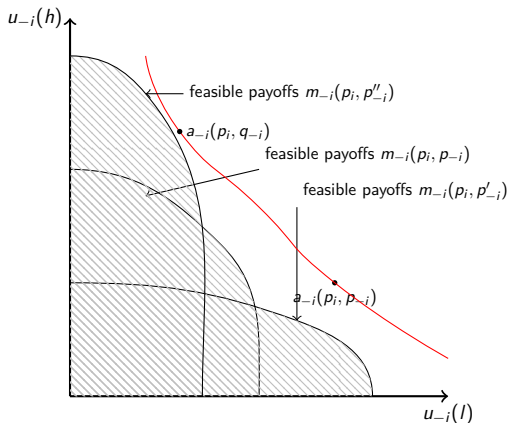
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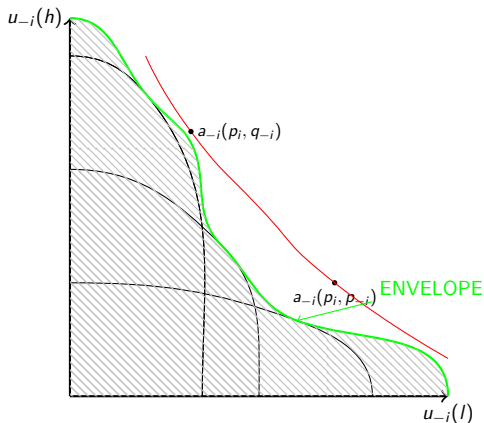
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Preliminary observations

Offers that cannot be refused

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Preliminary observations

Offers that cannot be refused

Definition

Mechanism a is *an offer that player $-i$ cannot refuse given m* , if $\forall p_i, p_{-i}, q_{-i}, \forall u \in a(p_i, p_{-i})$, and $\forall v \in m(p_i, q_{-i})$,

u is q_{-i} -undominated by v .

(i.e., there is a q_{-i} -positive prob. type t_{-i} such that $u_{-i}(t_{-i}) \geq v_{-i}(t_{-i})$).

- Compare with SUPO allocations in (Maskin Tirole 90) and strong neologism proof allocations in (Mylovanow Troger 14).

Preliminary observations

Offers that cannot be refused

Lemma

Suppose that a is an offer that player $-i$ strictly cannot refuse given mechanism m and

- *a is a payoff function,*
- *$I_{-i}(a) = a$. Then,*

$$MM_{-i}\{m, a\} \subseteq a.$$

- straightforward proof.

Preliminary observations

Offers that cannot be refused: Existence

- Existence of offers cannot be refused is not an issue.
- For any two mechanisms m and a , there exists continuous $w : \Delta T \rightarrow \mathbb{R}$ such that

$$(a +_{-i} w)_j(p) = \begin{cases} a_i(p) + w(p) & j = -i \\ a_i(p) - w(p) & j = i \end{cases}$$

cannot be refused by $-i$ given continuation m .

Preliminary observations

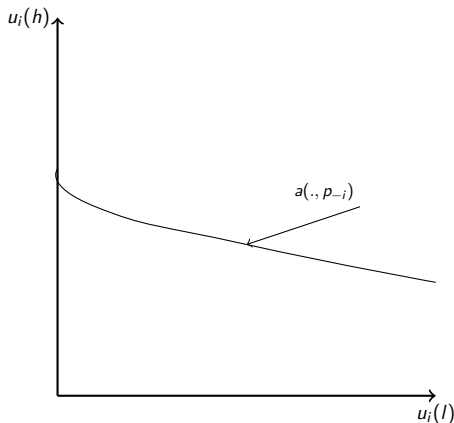
Mixing and matching offers that cannot be refused

- Two problems:
 - ▶ player $-i$ may have reasons to refuse the offer,
 - ▶ \Rightarrow signaling: (possibly, off-path) offers lead to belief updating $p_i \rightarrow q_i$.
- Consider informed principal problem with continuation m and suppose that $MM_{-i} \{m, a\} \subseteq a$.
 - ▶ informally, the principal should get at least a .
 - ▶ but, belief updating :(
- If $u \in IP_i(m)(p_i, p_{-i})$, then there must be q_i and $v \in a(q_i, p_{-i})$ st. $u_i \geq v_i$.

Preliminary observations

Mixing and matching offers that cannot be refused

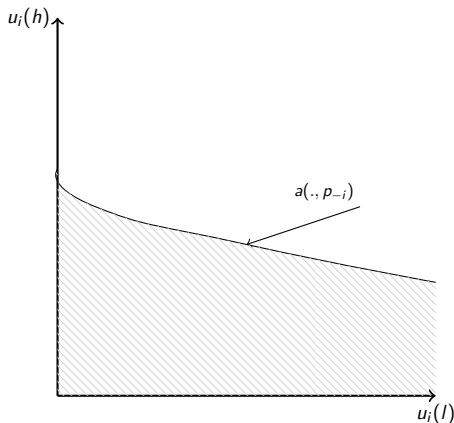
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Preliminary observations

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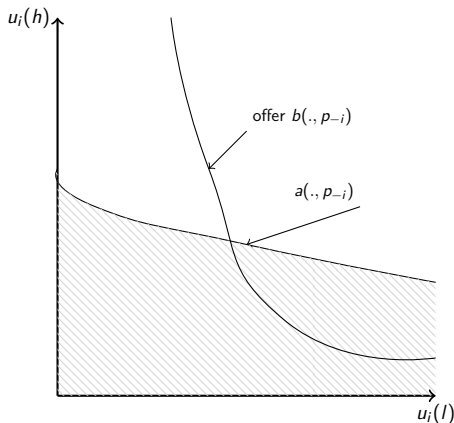
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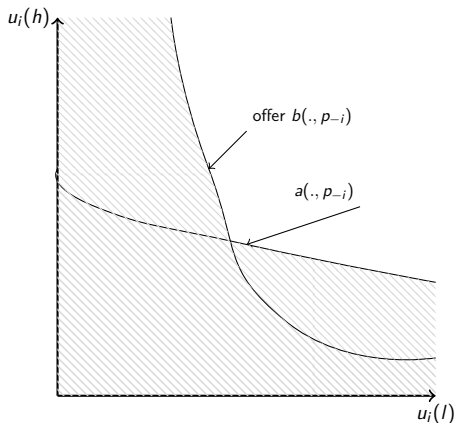
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Preliminary observations

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 - Random monopoly bound
 - Proof
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Random monopoly

- From now on, assume two types for each player $T_i = \{l_i, h_i\}$:
 - ▶ p_i - probability of type h_i .
- W.l.o.g. $l_1 < l_2$. I focus on

$$0 \leq l_1 < l_2 < h_1 < h_2.$$

Random monopoly

Theorem

For each $\delta < 1$, each $u \in \mathcal{B}(p)$, each player i , each t_i ,

$$u_i(t_i) \geq \beta_i M_i(t_i; p_{-i})$$

.

Random monopoly

- Each player gets at least their random monopoly payoff.
- In many cases, Theorem 2 is enough to characterize payoffs and equilibrium behavior, as there is unique interim efficient allocation that satisfies the random monopoly condition:
 - ▶ $\beta_i \in \{0, 1\}$,
 - ▶ $p_i \in \{0, 1\}$ for one of the players,
 - ▶ $l_1 = l_2$ or $l_2 = h_1$ or $h_1 = h_2$.
- In general, there is a gap between random monopoly payoffs and efficiency.

Random monopoly

Proof:

- The idea is to reproduce the complete info argument. Fix player i .
- The smallest equilibrium random monopoly share:

$$x^* = \min_{u \in \mathcal{B}} \min_{t_i} \frac{u_i}{M_i(t_i; p_{-i})}.$$

Random monopoly

Proof:

- The set of all feasible and IC payoffs that give player i at least x share of her monopoly payoffs:

$$A_x(p) = \{u \in \mathcal{U}(p) : u_i \geq xM_i(\cdot; p_{-i})\}.$$

- Then,

$$\mathcal{B} \subseteq A_{x^*}.$$

- We check that

$$\delta \mathcal{B} \subseteq \delta A_{x^*} \subseteq A_{1-\delta(1-x^*)}.$$

- ▶ Instead of delay, with prob. δ , deliver the payoffs now, and, with prob. $1 - \delta$, give player i his monopoly payoff.

Random monopoly

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Random monopoly

Proof:

- Goal: find mechanism a st.
 - ▶ a cannot be refused given $A_{1-\delta(1-x^*)}$ and
 - ▶ $a \subseteq A_{1-\delta(1-x^*)}$, i.e, each type t_i receives payoff at least

$$\geq (1 - \delta(1 - x^*)) M_i(t_i; p_{-i}).$$

- If $x^* < \beta_i$, complete information argument shows that player i has a profitable deviation.

Random monopoly

Offers that cannot be refused

Lemma

For each x , there exists mechanism $a^i(x)$ such that

- *$a^i(x)$ cannot be refused given $A_{i,x}$,*
 - *$a^i(x) \subseteq A_{i,x}$, and*
 - *$a^i(x)$ is (mostly) payoff function such that $I_{-i}(a^i(x)) = a^i(x)$.*
-
- <https://bwm-payoffs.streamlit.app/>

Random monopoly

Offers that cannot be refused

Definition

(Maskin Tirole 90, see also strong neologism proof of Mylovanov Troger 12, 14): u is *strongly Pareto-optimal* for $-i$ at p , if for each q_{-i} ,

$$v \in \mathcal{U}(p_i, q_{-i}) \text{ and } v_i \geq 0 \Rightarrow u \text{ is } q_{-i}\text{-undominated by } v.$$

- Mechanism a is an *offer that player $-i$ cannot refuse given A_x* , if $\forall p_i, p_{-i}, q_{-i}, \forall u \in a(p_i, p_{-i})$,

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Random monopoly

Offers that cannot be refused

- (Maskin Tirole 90) show that SUPO (closely related concept) exist with an elegant competitive equilibrium argument: types t_{-i} trade slacks in IC and IR constraints on types of player i .
- No natural way of extending this here: the IR constraint $u_i(t_i) \geq xM(t_i|p_{-i})$ is type- and belief-dependent.

Outline

- 1 Introduction
- 2 Model
- 3 Preliminary observations
- 4 Random monopoly payoff bound
- 5 The Gap**
- 6 Conclusions

The Gap

- In general, Theorem 2 does not pin down the equilibrium payoffs, as the random monopoly mechanism is not interim efficient.
- The gap between the largest *ex ante* (expected) payoffs and random monopoly payoffs:

$$\text{Gap}(p) = \max_{u \in \mathcal{U}(p) \text{ st. } \forall_{i,t_i} u_i(t) \geq \beta_i M_i(t_i|p)} p_1 \cdot (u_1 - \beta_1 M_1(\cdot|p))$$

- The gap is not larger than

$$\text{Gap}(p) \leq 6.25\% \text{ of } h_2 \text{ for all } p.$$

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The Gap

Theorem

For each p ,

$$\lim_{\delta \rightarrow 1} \sup_{u \in \mathcal{B}(p)} |p_1 \cdot u_1 - [p_1 \cdot \beta_1 M_1(\cdot | p) + \text{Gap}(p)]| = 0.$$

- As $\delta \rightarrow 1$, player 1 equilibrium *ex ante* payoffs converge to maximum possible subject to feasibility, IC, and random monopoly constraint.
 - ▶ player 1's payoffs are determined uniquely in *ex ante* sense,
 - ▶ player 2's payoffs are determined uniquely in the *interim* sense.

The Gap

- Player 1 (i.e., $I_1 < I_2$) gets the entire Gap!
 - ▶ a^2 is an example of mechanism attaining such payoffs.
- Why?
 - ▶ mix and match offers that cannot be refused:
 - ★ a^1 ,
 - ★ $a^2 - \text{Gap}(\cdot, p_2^*)$,
 - ▶ linearly transferable payoffs for $p_1 \geq p_1^*$,
 - ▶ convexity of mechanism a^2 .
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Conclusions

- A natural modification of a standard random-proposer bargaining has unique payoffs under
 - ▶ single good plus transfers, private values environment,
 - ▶ two types for each player.
- A proof of concept - better results and a general theory would be nice:
 - ▶ more types,
 - ▶ other environments,
 - ▶ better implementation results.