Bargaining with Mechanisms and Two-Sided Incomplete Information

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December 11, 2023

Outline

Introduction

- Business partners want to cease partnership. Their firm cannot be divided, and if partner keeps it, the other expects a compensation.
- Two countries negotiate a peace treaty, with land swaps and reparations (or economic aid) on the table.
- Coalition parties negotiate an agreement with a support for policy traded off against number of cabinet positions.

Introduction

- Bargaining - one of the longest-studied problems in economic theory ("bilateral monopoly" before [Nash 50])
- No satisfactory solution for incomplete information:
  - cooperative solutions: (Harsanyi 72), (Myerson 84),
  - large literature on bargaining over prices:
    - one-sided: uniqueness in Coasian bargaining with a gap,
    - two-sided: large set of equilibria, possible refinements to eliminate some (Ausubel, Crampton, Deneckere 02 and others).
- Goal: show that a natural modification of a standard random-proposer bargaining has a "unique" outcome under
  - single good plus transfers environment,
  - private values (two types for each player).

Introduction

- Bargaining with sophisticated offers in real world
  - menus,
  - menus of menus ("I divide, you choose"),
  - mediation, arbitration (example: "trial by gods"),
  - change in bargaining protocols,
  - deadlines or delays, etc.
- Challenges:
  - how to model mechanisms as actions?
  - signaling.

Introduction

- Challenge 1: how to model mechanisms as actions.
  - No revelation principle.
  - Games: actions and outcome function.
  - Real mechanisms: correspondences of equilibrium payoffs obtained in games.
  - (Abstract) mechanisms: proper limits of such correspondences.

Introduction

- Challenge 2: signaling.
  - larger action space helps.
- informed principal with private values (Maskin Tirole, 90)
  - informed principal types get their monopoly payoff,
  - private information of the principal does not matter in private values case.
- one-sided incomplete information (Peski 22),
  - informed player types get at least random monopoly payoff,
  - => uninformed player

Introduction

Results

- Suppose each player has two types and, w.l.o.g., that $l_1 < l_2$.
- **Theorem 1**: For each discount factor, each player expects at least their random monopoly payoff.
- **Theorem 2**: As $\delta \to 1$, *ex ante* expected payoffs of player 1 converge to a feasible maximum subject to a constraint that player 2 types get their random monopoly payoffs.

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Outline

Model

Environment

- Two players $i = 1, 2$, sometimes third player ("mediator").
- Single good and transfers
- Preferences: $q_i t_i - \tau_i$,
  - t_i - type (valuation) of player i,
  - q_i - probability that pl. i gets the good,
  - τ_i - transfer from player i
  - feasibility: $q_1 + q_2 \leq 1$, $q_i \geq 0$, $\tau_1 + \tau_2 \leq 0$,

Model

Bargaining game

- Bargaining game
  - multiple rounds until offer is accepted, discounting $\delta < 1$,
  - random proposer: player i is chosen with prob. $\beta_i \geq 0$, where $\beta_1 + \beta_2 = 1$,
  - proposer offers a mechanism,
  - if the offer is accepted, it is implemented, and the bargaining game ends.
- Perfect Bayesian Equilibrium:
  - no updating beliefs about player i after $-i$'s action.
  - public randomization plus cheap talk.

Model

Feasible payoffs

- Payoff vector $u(.|q,\tau) \in R^{T_1 \cup T_2}$ in allocation $q_i(.), \tau(.)$:

$$u_i(t_i|q,\tau) = \sum_{t_{-i}} p(t_{-i})(t_i q_i(t_i, t_{-i}) - \tau_i(t_i, t_{-i})) \text{ for each } t_i.$$

- Allocation $q_i(.), \tau(.)$ is IC given beliefs p iff

$$u_i(t_i|q,\tau) \geq \sum_{t_{-i}} p(t_{-i})(t_i q_i(s_i, t_{-i}) - \tau_i(s_i, t_{-i})) \text{ for each } t_i, s_i.$$

- Correspondence of feasible and IC payoffs:

$$\mathcal{U}(p) = \{u(.|q,\tau) : (q,\tau) \text{ is IC given } p\} \subseteq R^{T_1 \cup T_2}.$$

- The geometry of the correspondence $\mathcal{U}(.)$ is the true “parameter” of the model.

Model

Games

- Game G:
  - players: 1, 2, and mediator (whose payoff is a non-negative transfer),
  - finite or compact actions,
  - continuous outcome function that maps actions to an allocation of a good and a transfer,
  - always assume public randomization.
- For each p, the set of equilibrium payoff vectors

$$m(p; G) \subseteq \mathcal{U}(p).$$

- Equilibrium correspondence:

$$m(.; G) : \Delta T \rightrightarrows R^{T_1 \cup T_2}, m_G \subseteq \mathcal{U}.$$

Model

Mechanisms

- *Real mechanism* is a correspondence m for which there exists a game G such that $m = m(.; G)$.
- Real mechanism m is
  - u.h.c.,
  - $m \subseteq \mathcal{U}$,
  - non-empty-valued, and
  - convex valued.

Model

Mechanisms

- *(Abstract) mechanism* is correspondence m st.
  - m is u.h.c.,
  - $m \subseteq \mathcal{U}$,
  - non-empty valued,
  - it can be *approximated* by continuous functions $m_n : \Delta T \to R^{T_1 \cup T_2}$, $m_n \subseteq \mathcal{U}$such that

    $$\lim_{n\to\infty} \max_{p} \min_{v,q:v\in m(q)} d\left(\left(m_n\left(p\right), p\right), \left(v, q\right)\right) = 0,$$

    where d is a Euclidean distance on $\Delta T \times R^{T_1 \cup T_2}$.
-
- The space of mechanism is compact* under Hausdorff distance induced by d.

Model

Implementation Theorem

Theorem

Any real mechanism is an (abstract) mechanism.
For any (abstract) mechanism m, there is a sequence of real mechanisms m_n that "approximate" m:

$$\lim_{n\to\infty} \max_{u,p:u\in m_n(p)} \min_{v,q:v\in m(q)} d\left((u,p),(v,q)\right) = 0.$$

- Proof:
  - Michael's Theorem shows that a real mechanism is an abstract mechanism.
  - For the other side;
    - mediator names the beliefs p,
    - given p, use virtual Bayesian implementation of (Abreu Matsushima 92).

Model

Derived mechanisms

- Given a mechanism or a set of mechanisms, we can construct new ones:
- $\alpha \in \Delta A$ - randomly chosen mechanism according to distribution α.
- δm - discounted mechanism m.
- $I_i(m)$ - information revelation game: public randomization plus cheap talk by i).
- $MM_i(A)$ - menu of mechanisms $a \in A$ for player i (including p.r. and cheap talk by i).
- $IP_i(m)$ - informed principal problem of player i with continuation mechanism (i.e., outside option) m,

$$IP_i(m) = MM_i\{MM_{-i}\{n, m\} : n \text{ is a mechanism}\}$$

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Model

Bargaining game

- Bargaining mechanism : the largest fixed point $\mathcal{B}$ of

$$\mathcal{B} = \left(IP_1\left(\delta\mathcal{B}\right)\right)^{\beta_1}\left(IP_2\left(\delta\mathcal{B}\right)\right)^{\beta_2}$$

Model
Equilibrium

- Equilibrium in $MM_i(A)$:
  - modular (one-shot deviation principle),
  - extends to the existence in bargaining game.
- Existence:
  - space of mechanisms is compact,
  - if A finite, approximate each mechanism by a payoff function and apply Brouwer FPT,
  - extend to compact A (cheap talk is important),
  - public randomization is important.
- Weak solution concept:
  - PBE = WPBE + "no updating after the other player actions",
  - if restricted to real mechanisms, approximate (i.e., ε-like) equilibrium.

Model

Commitment

- Players are not committed to future offers.
- Players are committed to implementing a mechanism once offered and accepted:
  - hence, less commitment than in the *limited commitment* literature (V. Skreta and L. Doval).
- Relevant for many situations
  - good allocation with no backsies,
  - bargaining over protocol,
- "Lack of commitment" is a restriction on the space of mechanisms,
- Commitment is not necessarily helpful to the agent who can exercise it.

Outline

Preliminary observations

Complete information bargaining

- **Claim:** Assume $t_1 < t_2$ are known. Then, in each equilibrium, player i gets $\beta_i t_2$.
- **Proof**: Suppose $i = 1$ (the other argument is analogous). Let

$$x^* = \min_{u \in \mathcal{B}} \frac{u_1}{t_2}.$$

.

- If $x^* < \beta_1$, player 1 has a profitable deviation:
  - wait until she is a proposer, and offer: player 2 gets the good and pays $(1 - \delta(1 - x^*))\, t_2$ to player 1,
  - the offer will be accepted.
- Special features:
  - linearly transferable payoffs,
  - endogenous interdependent value

Preliminary observations

Informed principal

- (Random) informed principal with private values ($\beta_i = 1$ or $\delta = 0$):
  - monopoly payoff:

$$M(t_i; p_{-i}) = \max_{\tau} p_{-i}(t_{-i} \leq \tau) t_i + (1 - p_{-i}(t_{-i} \leq \tau)) \tau,$$

  - If player i is a proposer, she offers the monopoly price to $-i$, which is accepted (the game ends),
  - i's expected payoff is $M(t_i; p_{-i})$.
- Special features:
  - continuation value $= 0$ (and it does not depend on beliefs)
  - private information of the principal does not matter due to private values.

Preliminary observations

One-sided incomplete information

- One-sided incomplete ($p_i \in \{0, 1\}$, i.e., i is uninformed):
- The equilibrium payoffs are unique and implemented by random monopoly mechanism is offered:
    - with probability β_j, agent j gets the good:
    - if so, she make a single take-it-or-leave-it sell offer,
    - player i's expected payoff of $\beta_i M(t_i; p_{-i})$,
    - some player $-i$'s types may get a bit more than $\beta_{-i} M(t_{-i}; p_i)$,
- Special features:
    - random monopoly mechanism is interim efficient.

Preliminary observations

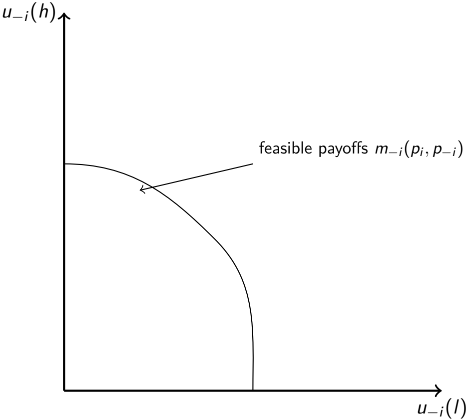
Offers that cannot be refused

- i makes an offer, $-i$ decides whether to accept or reject.
- Offer design:
  - making offers that are refused is inefficient due to surplus-burning delay,
  - control: offers should be be accepted exactly as they are.
- Two problems:
  - $\Rightarrow$ player $-i$ may have reasons to refuse the offer,
  - signaling: (possibly, off-path) offers lead to belief updating $p_i \to q_i$.

Preliminary observations

Offers that cannot be refused

- m is a continuation mechanism.
- a is an offer that is accepted exactly as it is.



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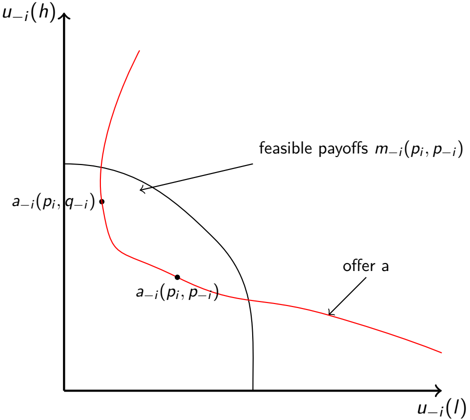

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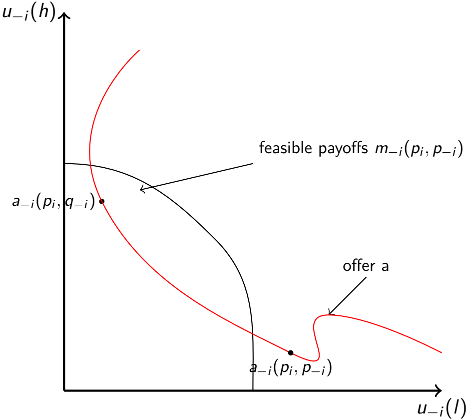

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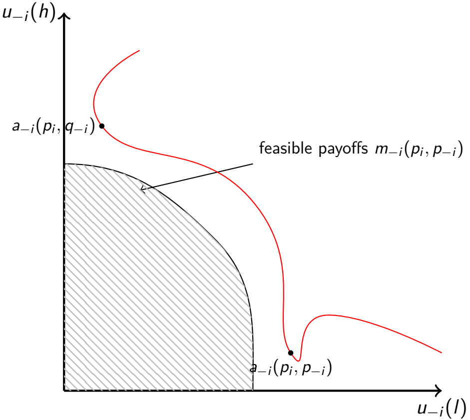

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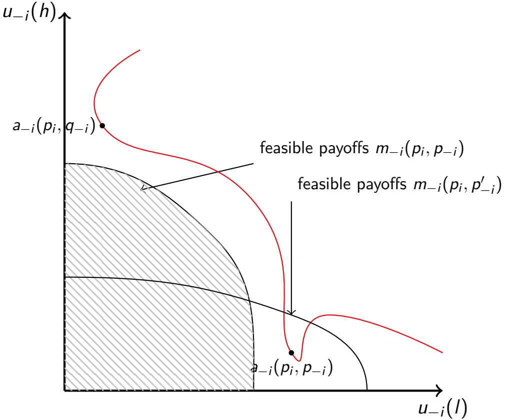

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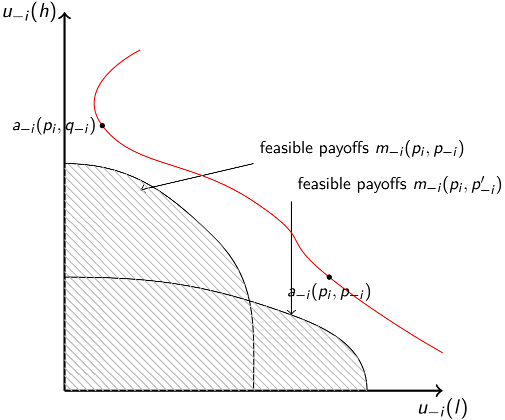

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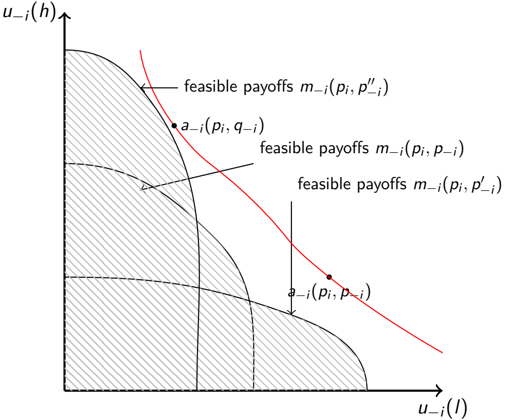

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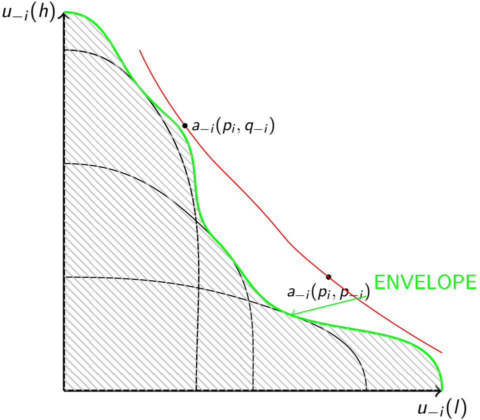


Preliminary observations

Offers that cannot be refused

Definition

Mechanism a is *an offer that player $-i$ cannot refuse given m,* if $\forall p_i, p_{-i}, q_{-i}$, $\forall u \in a(p_i, p_{-i})$, and $\forall v \in m(p_i, q_{-i})$,

$$u \text{ is } q_{-i}\text{-undominated by } v.$$

(i.e., there is a q_{-i}-positive prob. type t_{-i} such that $u_{-i}(t_{-i}) \geq v_{-i}(t_{-i})$).

- Compare with SUPO allocations in (Maskin Tirole 90) and strong neologism proof allocations in (Mylovanow Troger 14).

Preliminary observations

Offers that cannot be refused

Lemma

Suppose that a is an offer that player $-i$ strictly cannot refuse given mechanism m and

- *a is a payoff function,*
- *$I_{-i}(a) = a$. Then,*

$$MM_{-i}\{m, a\} \subseteq a.$$

- straightforward proof.

Preliminary observations

Offers that cannot be refused: Existence

- Existence of offers cannot be refused is not an issue.
- For any two mechanisms m and a, there exists continuous $w : \Delta T \to \mathbb{R}$such that

$$(a +_{-i} w)_j (p) = \begin{cases} a_i (p) + w (p) & j = -i \\ a_i (p) - w (p) & j = i \end{cases}$$

cannot be refused by $-i$ given continuation m.

Preliminary observations

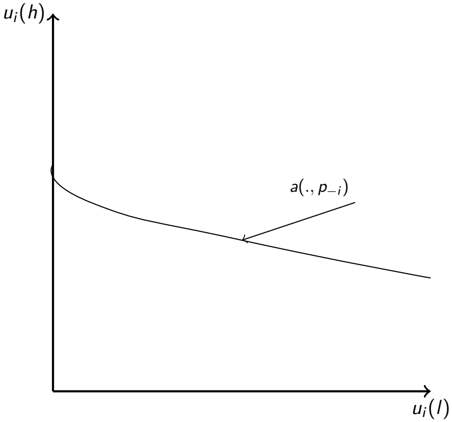
Mixing and matching offers that cannot be refused

- Two problems:
  - player $-i$ may have reasons to refuse the offer,
  - $\Rightarrow$signaling: (possibly, off-path) offers lead to belief updating $p_i \rightarrow q_i$.
- Consider informed principal problem with continuation m and suppose that $MM_{-i}\{m, a\} \subseteq a$.
  - informally, the principal should get at least a.
  - but, belief updating :(
- If $u \in IP_i(m)(p_i, p_{-i})$, then there must be q_i and $v \in a(q_i, p_{-i})$ st. $u_i \geq v_i$.

Preliminary observations

Mixing and matching offers that cannot be refused

- Suppose that a, b are offers that cannot be refused given m



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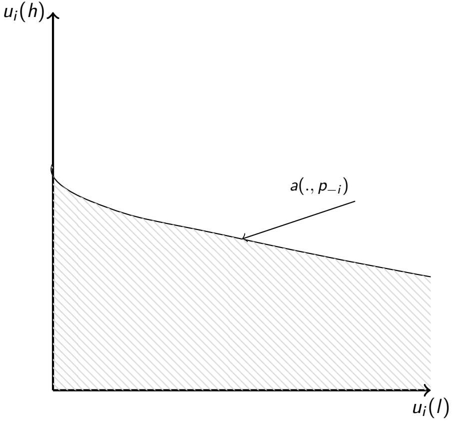

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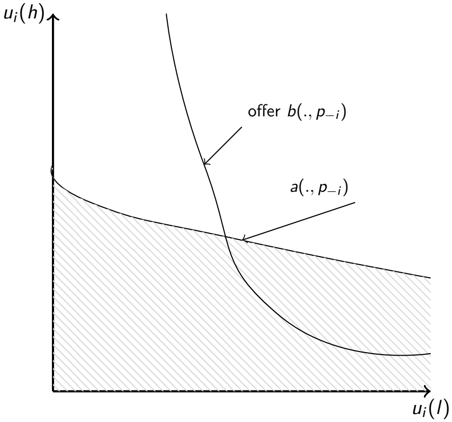

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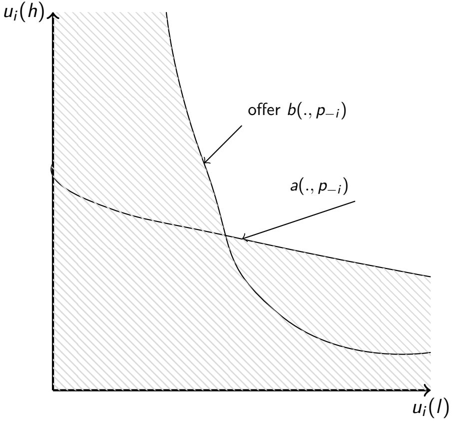

Outline

Random monopoly

- From now on, assume two types for each player $T_i = \{l_i, h_i\}$ and, w.l.o.g. $l_1 < l_2$,
  - p_i - probability of type h_i.
- I focus on

$$0 \leq l_1 \leq l_2 < h_1 \leq h_2.$$

Random monopoly

Theorem

For each $\delta < 1$, each $u \in \mathcal{B}(p)$, each player i, each t_i,
$u_i(t_i) \geq \beta_i M_i(t_i; p_{-i})$.

Random monopoly

- Each player gets at least their random monopoly payoff.
- In many cases, Theorem 4 is enough to characterize payoffs and equilibrium behavior, as there is unique interim efficient allocation that satisfies the random monopoly condition:
  - $\beta_i \in \{0, 1\}$,
  - $p_i \in \{0, 1\}$ for one of the players,
  - $l_1 = l_2$ or $l_2 = h_1$ or $h_1 = h_2$.
- In general, there is a gap between random monopoly payoffs and efficiency.

Random monopoly

Proof:

- The idea is to reproduce the complete info argument. Fix player i.
- The smallest equilibrium random monopoly share:

$$x^* = \min_{u \in \mathcal{B}} \min_{t_i} \frac{u_i}{M_i\left(t_i; p_{-i}\right)}.$$

- The set of all feasible and IC payoffs that give player i at least x share of her monopoly payoffs:

$$A_x\left(p\right) = \left\{u \in \mathcal{U}\left(p\right) : u_i \geq x M_i\left(.; p_{-i}\right)\right\}.$$

- Then,

$$\mathcal{B} \subseteq A_{x^*}.$$

Random monopoly

Proof:

- Easy to check that

$$\delta \mathcal{B} \subseteq \delta A_{x^*} \subseteq A_{1-\delta(1-x^*)}.$$

  - Instead of delaying payoffs, we can give them today with prob. δ and with prob. $1-\delta$, give player i his monopoly payoff,
  - but we can do better as well.
- Goal: find $a \subseteq A_{1-\delta(1-x^*)}$ st.
  - a cannot be refused given $A_{1-\delta(1-x^*)}$ and
  - $a \subseteq A_{1-\delta(1-x^*)}$, i.e, each type t_i receives payoff at least

$$\geq \left(1-\delta\left(1-x^*\right)\right) M_i\left(t_i; p_{-i}\right).$$

- If $x^* < \beta_i$, complete information argument shows that player i has a profitable deviation.

Random monopoly

Offers that cannot be refused

Lemma

For each x, there exists mechanism $a^i(x)$ such that

- $a^i(x) \subseteq A_x$,
- $a^i(x)$ *is (mostly) payoff function such that* $I_{-i}(a^i(x)) = a^i(x)$.

- https://bwm-payoffs.streamlit.app/

Random monopoly

Offers that cannot be refused

Definition

(Mylovanov Troger 12, 14, closely related SUPO from Maskin Tirole 90): u is *neologism-proof* for $-i$ at p if for each q_{-i}, each $u \in A_0(p_i, p_{-i})$ and each $v \in \mathcal{U}(p_i, q_{-i})$,

$$v_i \geq 0 \Rightarrow u \text{ is } q_{-i}\text{-undominated by } v.$$

- Mechanism a is *an offer that player $-i$ cannot refuse given A_x*, if $\forall p_i, p_{-i}, q_{-i}$, $\forall u \in a(p_i, p_{-i})$, each $v \in \mathcal{U}(p_i, q_{-i})$

$$v_i \geq x_i M_i(.|q_{-i}) \Rightarrow u \text{ is } q_{-i}\text{-undominated by } v.$$

- (Maskin Tirole 90) show that SUPO (closely related concept) exist with an elegant competitive equilibrium argument: types t_{-i} trade slacks in IC and IR constraints on types of player i
  - No natural way of extending this here: the IR constraint $u_i(t_i) \geq xM(t_i|p_{-i})$ is type- and belief-dependent.

Outline

The Gap

- In general, Theorem 4 does not pin down the equilibrium payoffs, as the random monopoly mechanism is not interim efficient.
- The gap between the largest *ex ante* (expected) payoffs and random monopoly payoffs:

$$\operatorname{Gap}(p) = \max_{u \in \mathcal{U}(p) \text{ st. } \forall_{i,t_i} u_i(t) \geq \beta_i M_i(t_i|p)} p_1 \cdot (u_1 - \beta_1 M_1(.|p))$$

- The gap is not larger than

$$\operatorname{Gap}(p) \leq 6.25\% \text{ of } h_2 \text{ for all } p.$$

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The Gap

Theorem

For each p,

$$\lim_{\delta \leftarrow 1} \sup_{u \in \mathcal{B}(p)} \left| p_1 \cdot u_1 - \left[p_1 \cdot \beta_1 M_1 \left(. | p \right) + Gap \left(p \right) \right] \right| = 0.$$

- As $\delta \to 1$, player 1 equilibrium *ex ante* payoffs converge to maximum possible subject to feasibility, IC, and random monopoly constraint.
  - player 1's payoffs are determined uniquely in ex ante sense,
  - player 2's payoffs are determined uniquely in the *interim* sense.

The Gap

- Player 1 (i.e., $l_1 < l_2$) gets the entire Gap!
  - a^2 is an example of mechanism attaining such payoffs.
- Why?

The Gap

- Player 1 (i.e., $l_1 < l_2$) gets the entire Gap!
  - a^2 is an example of mechanism attaining such payoffs.
- Why?
  - linearly transferable payoffs for $p_1 \geq p_1^*$,
  - mix and match offers that cannot be refused: a^1, $a^2 - \text{Gap}\,(., p_2^*)$
  - convexity of mechanism a^2.
- https://bwm-payoffs.streamlit.app/

Outline

Conclusions

- A natural modification of a standard random-proposer bargaining has unique payoffs under
  - single good plus transfers, private values environment,
  - two types for each player.
- A proof of concept - better results and a general theory would be nice:
  - more types,
  - other environments,
  - better implementation results.