

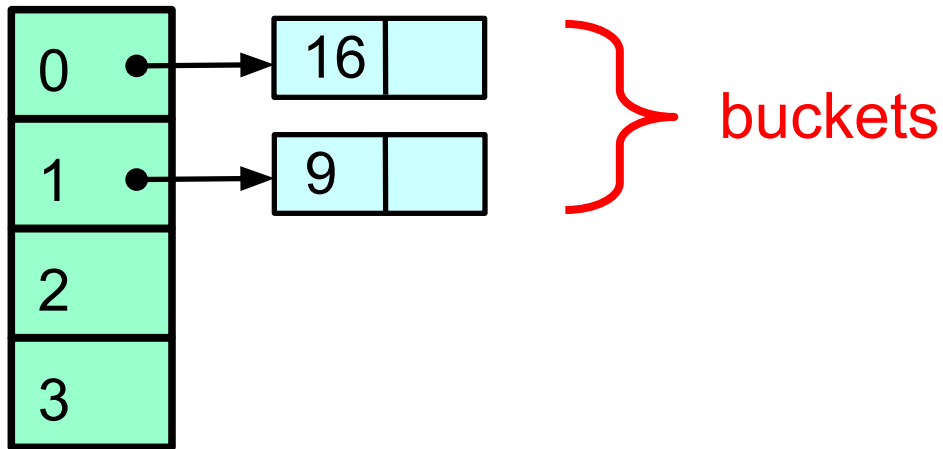
# Concurrent programming

## Hashing

Companion slides for  
The Art of Multiprocessor Programming  
by Maurice Herlihy, Nir Shavit, Victor Luchangco,  
and Michael Spear

Modified by Piotr Witkowski

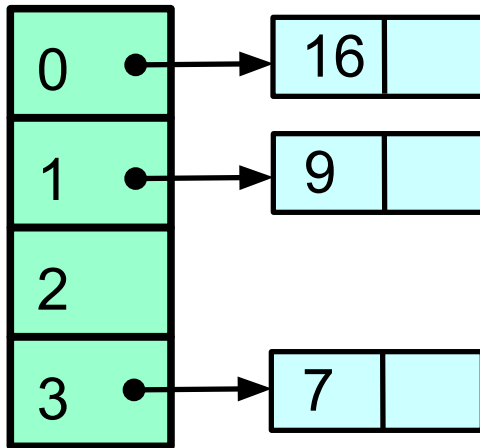
# Sequential Closed Hash Map



2 Items

$$h(k) = k \bmod 4$$

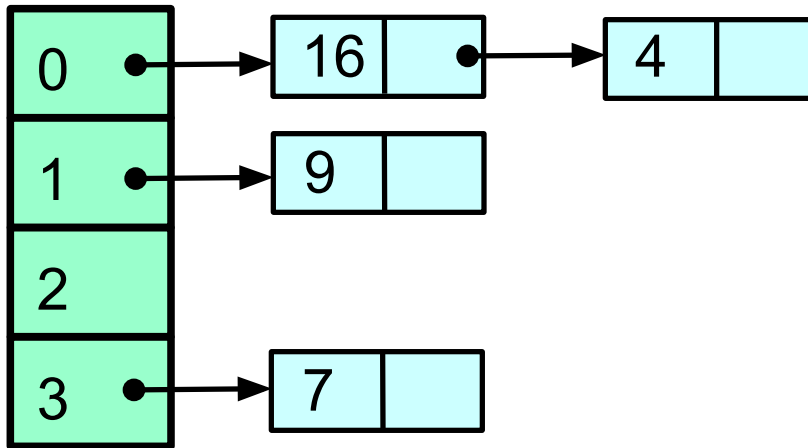
# Add an Item



3 Items

$$h(k) = k \bmod 4$$

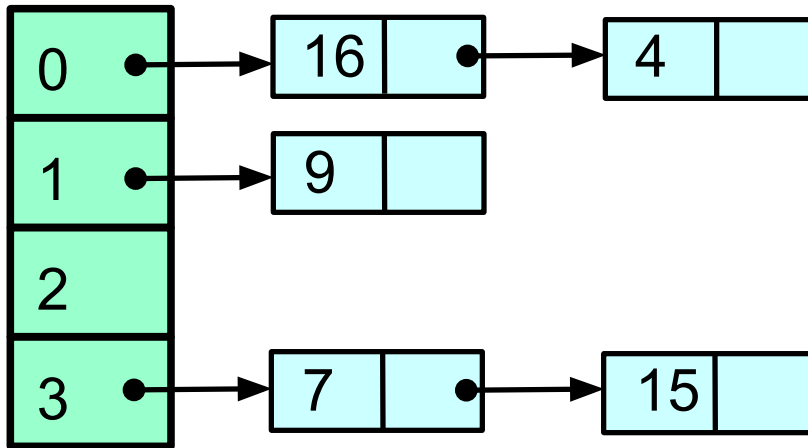
# Add Another: Collision



4 Items

$$h(k) = k \bmod 4$$

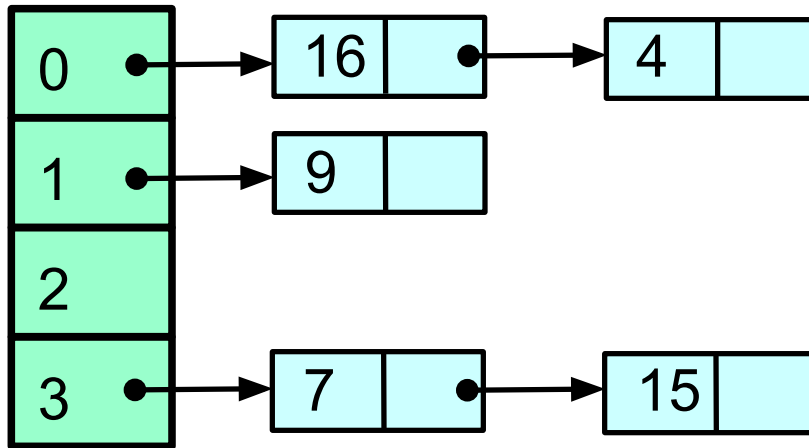
# More Collisions



5 Items

$$h(k) = k \bmod 4$$

# More Collisions

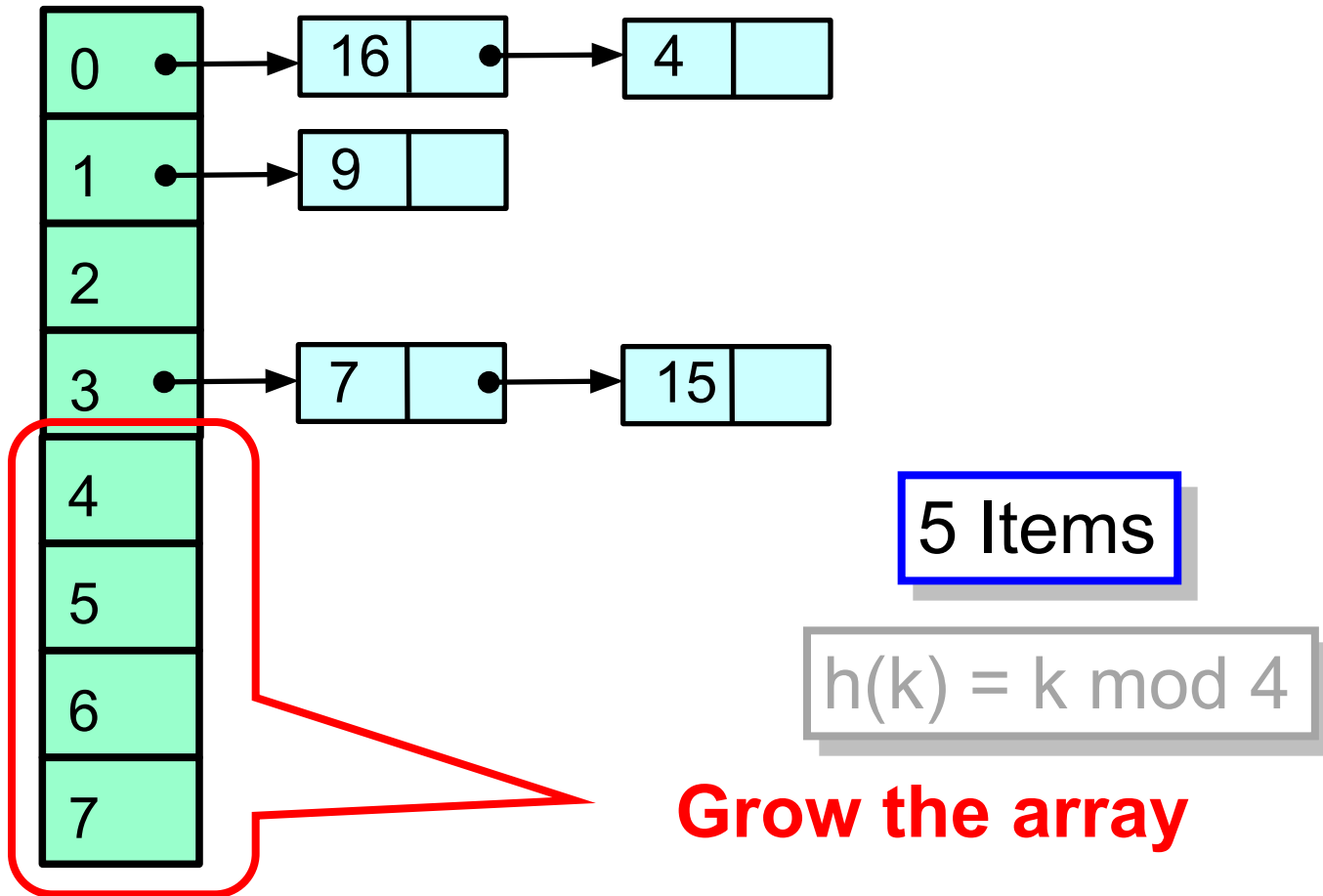


**Problem:**  
**buckets becoming too long**

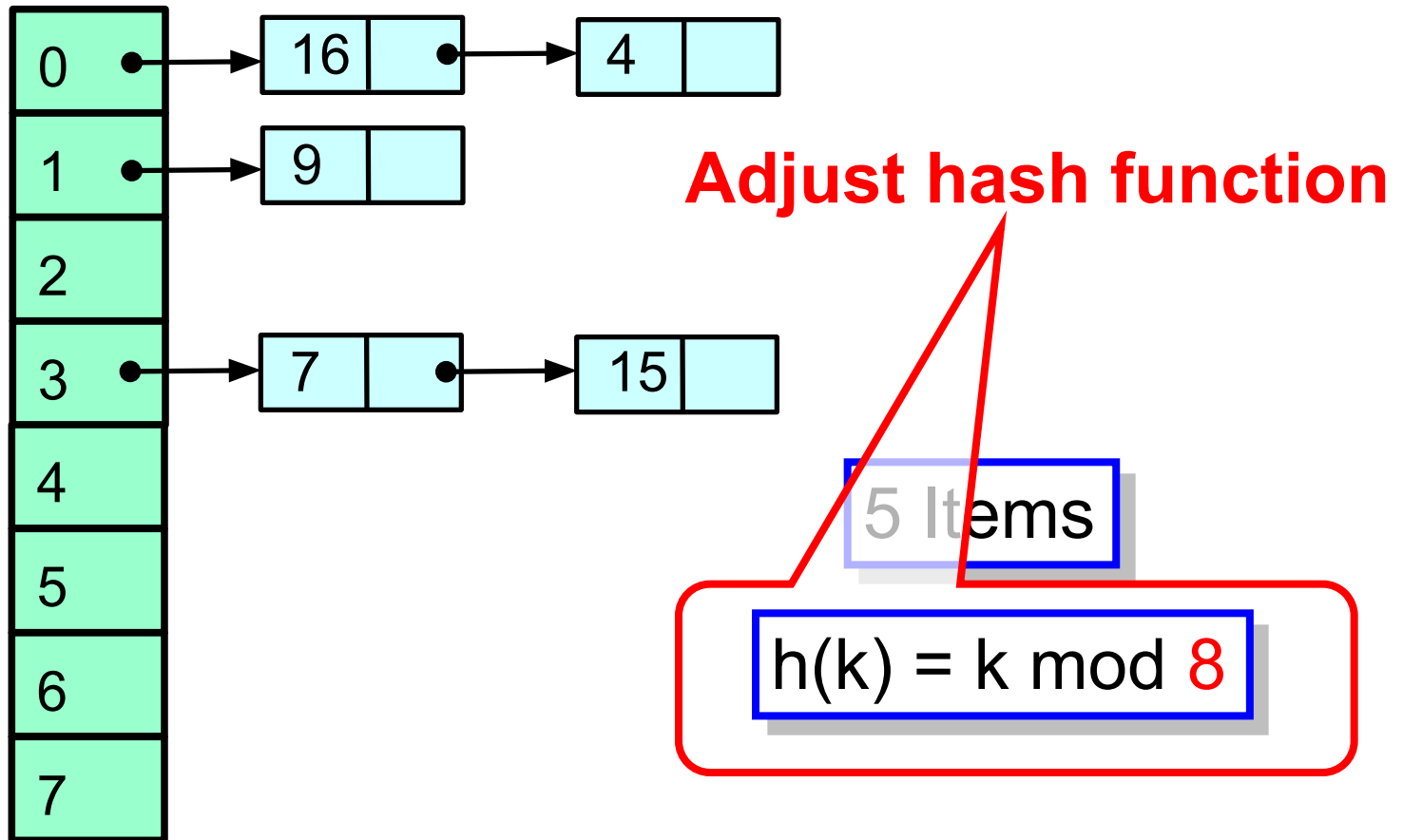
5 Items

$$h(k) = k \bmod 4$$

# Resizing

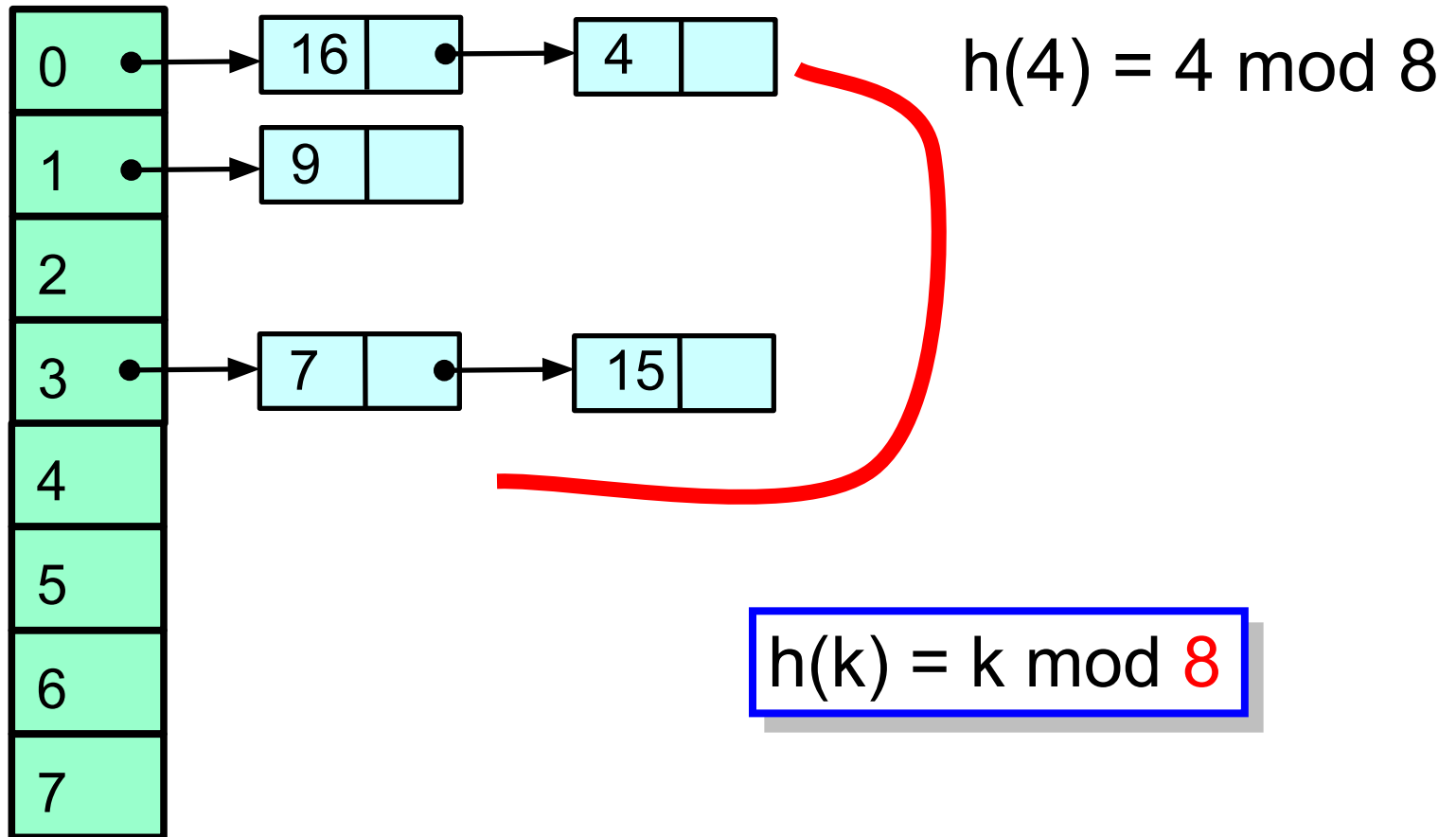


# Resizing

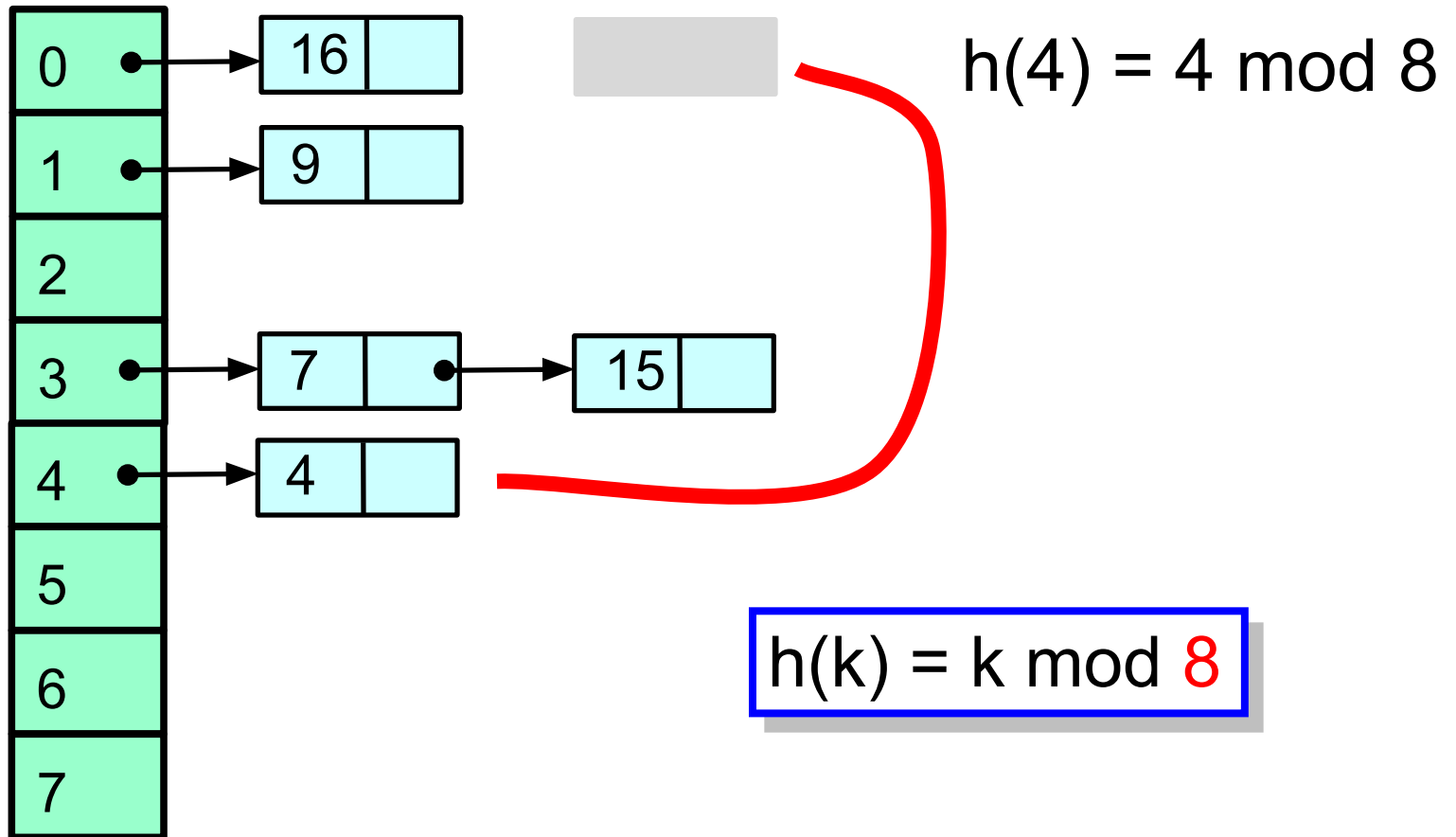




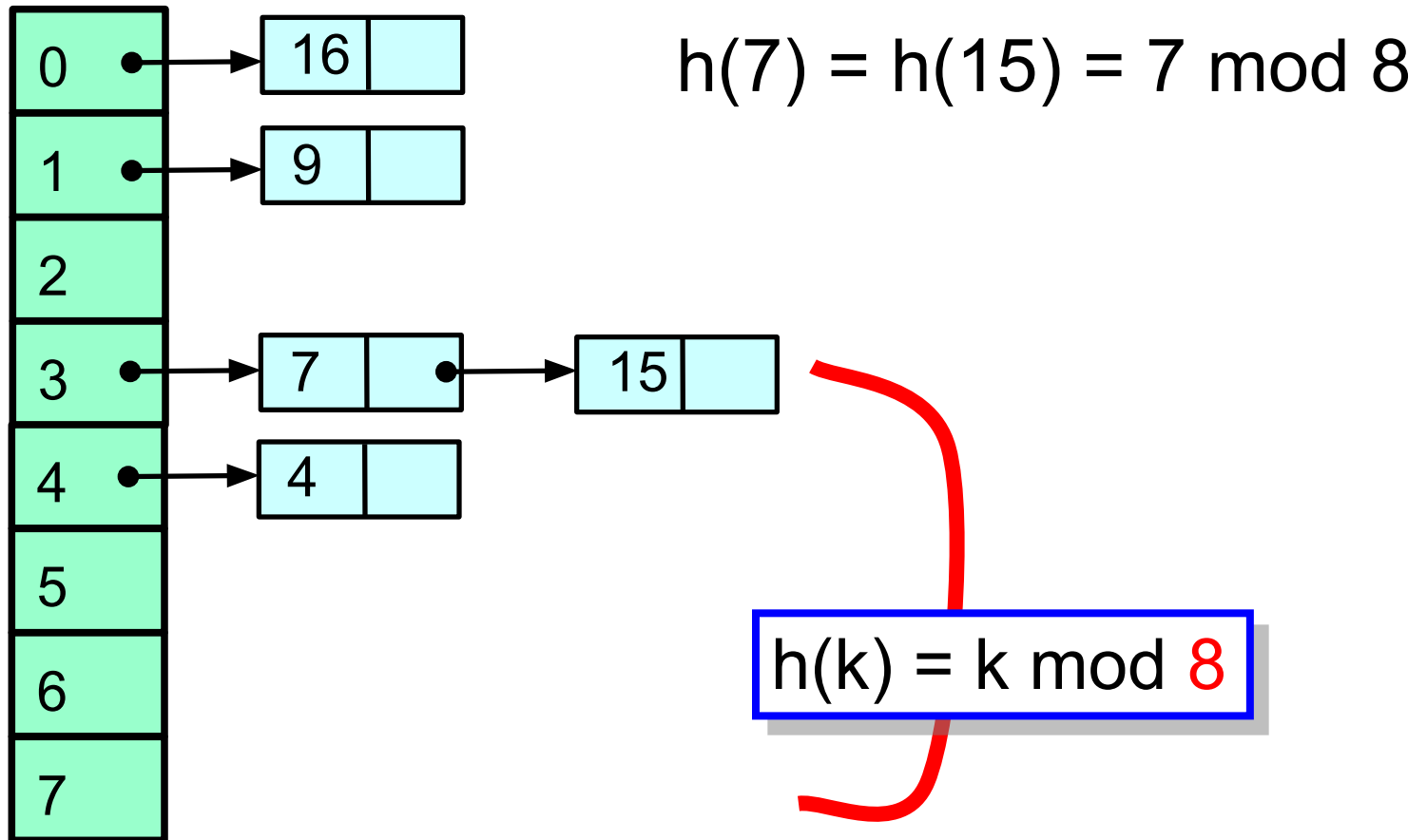
# Resizing



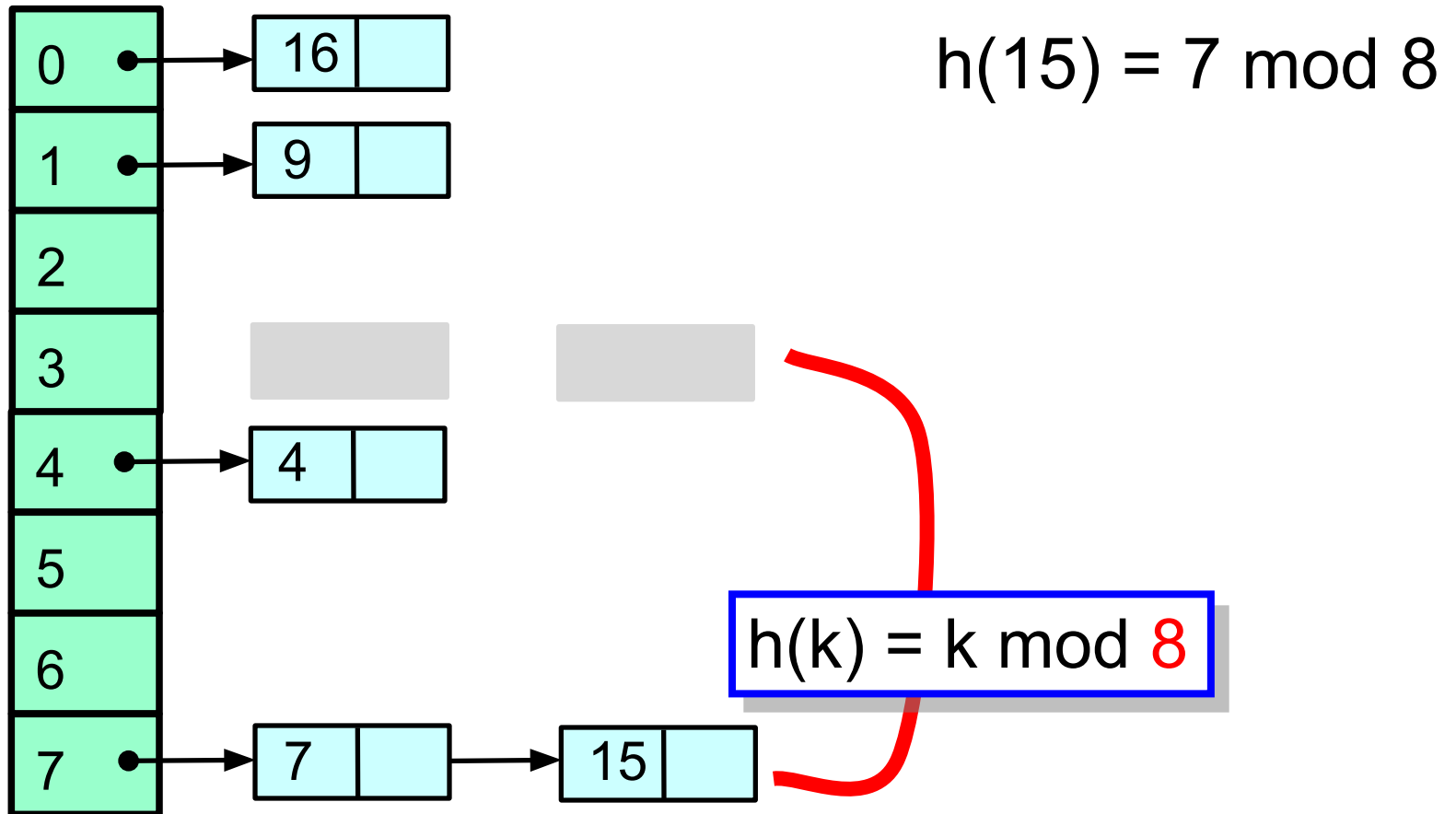
# Resizing



# Resizing



# Resizing



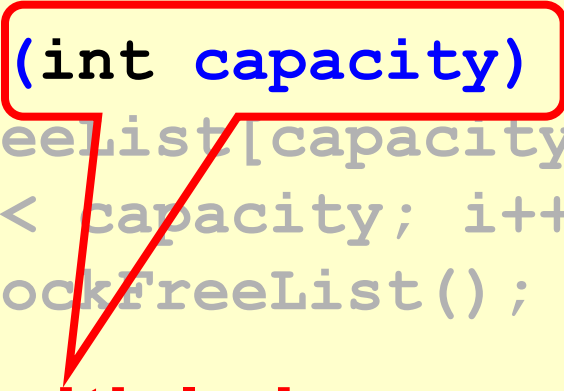
# Fields

```
public class SimpleHashSet {  
    protected LockFreeList[] table;  
  
    public SimpleHashSet(int capacity) {  
        table = new LockFreeList[capacity];  
        for (int i = 0; i < capacity; i++)  
            table[i] = new LockFreeList();  
    }  
    ...  
}
```

**Array of lock-free lists**

# Constructor

```
public class SimpleHashSet {  
    protected LockFreeList[] table;  
  
    public SimpleHashSet(int capacity) {  
        table = new LockFreeList[capacity];  
        for (int i = 0; i < capacity; i++)  
            table[i] = new LockFreeList();  
    }  
    ...  
}
```



**Initial size**

# Constructor

```
public class SimpleHashSet {  
    protected LockFreeList[] table;  
  
    public SimpleHashSet(int capacity) {  
        table = new LockFreeList[capacity];  
        for (int i = 0; i < capacity; i++)  
            table[i] = new LockFreeList();  
    }  
    ...  
}
```

**Allocate memory**

# Constructor

```
public class SimpleHashSet {  
    protected LockFreeList[] table;  
  
    public SimpleHashSet(int capacity) {  
        table = new LockFreeList[capacity];  
        for (int i = 0; i < capacity; i++)  
            table[i] = new LockFreeList();  
    }  
    ...  
}
```

**Initialization**



# Add Method

```
public boolean add(Object key) {  
    int hash =  
        key.hashCode() % table.length;  
    return table[hash].add(key);  
}
```

# Add Method

```
public boolean add(Object key) {  
    int hash =  
        key.hashCode() % table.length;  
    return table[hash].add(key);  
}
```

**Use object hash code to  
pick a bucket**

# Add Method

```
public boolean add(Object key) {  
    int hash =  
        key.hashCode() % table.length;  
    return table[hash].add(key);  
}
```

**Call bucket's add() method**

# No Brainer?

- We just saw a
  - Simple
  - Lock-free
  - Concurrent hash-based set implementation
- What's not to like?

# No Brainer?

- We just saw a
  - Simple
  - Lock-free
  - Concurrent hash-based set implementation
- What's not to like?
- We don't know how to resize ...

# Is Resizing Necessary?

- Constant-time method calls require
  - Constant-length buckets
  - Table size proportional to set size
  - As set grows, must be able to resize

# Set Method Mix

- Typical load
  - 90% contains()
  - 9% add ()
  - 1% remove()
- Growing is important
- Shrinking not so much

# When to Resize?

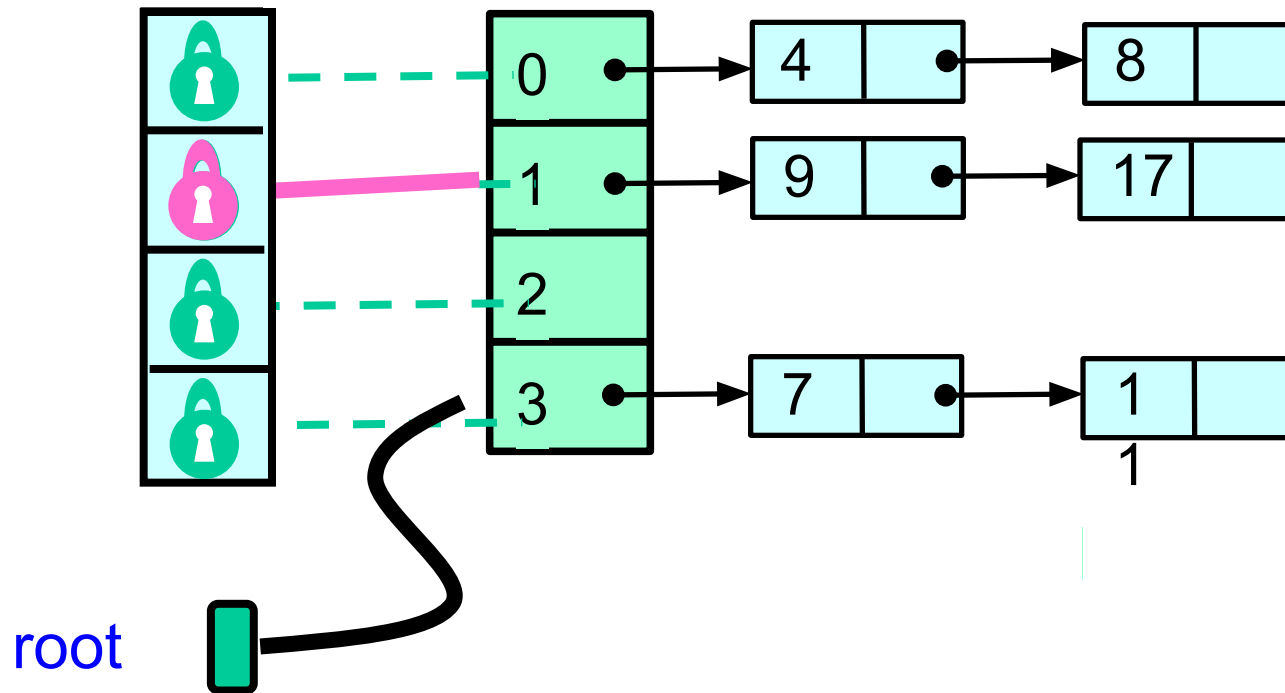
- Many reasonable policies. Here's one.
- Pick a threshold on num of items in a bucket
- Global threshold
  - When  $\geq \frac{1}{4}$  buckets exceed this value
- Bucket threshold
  - When any bucket exceeds this value



# Coarse-Grained Locking

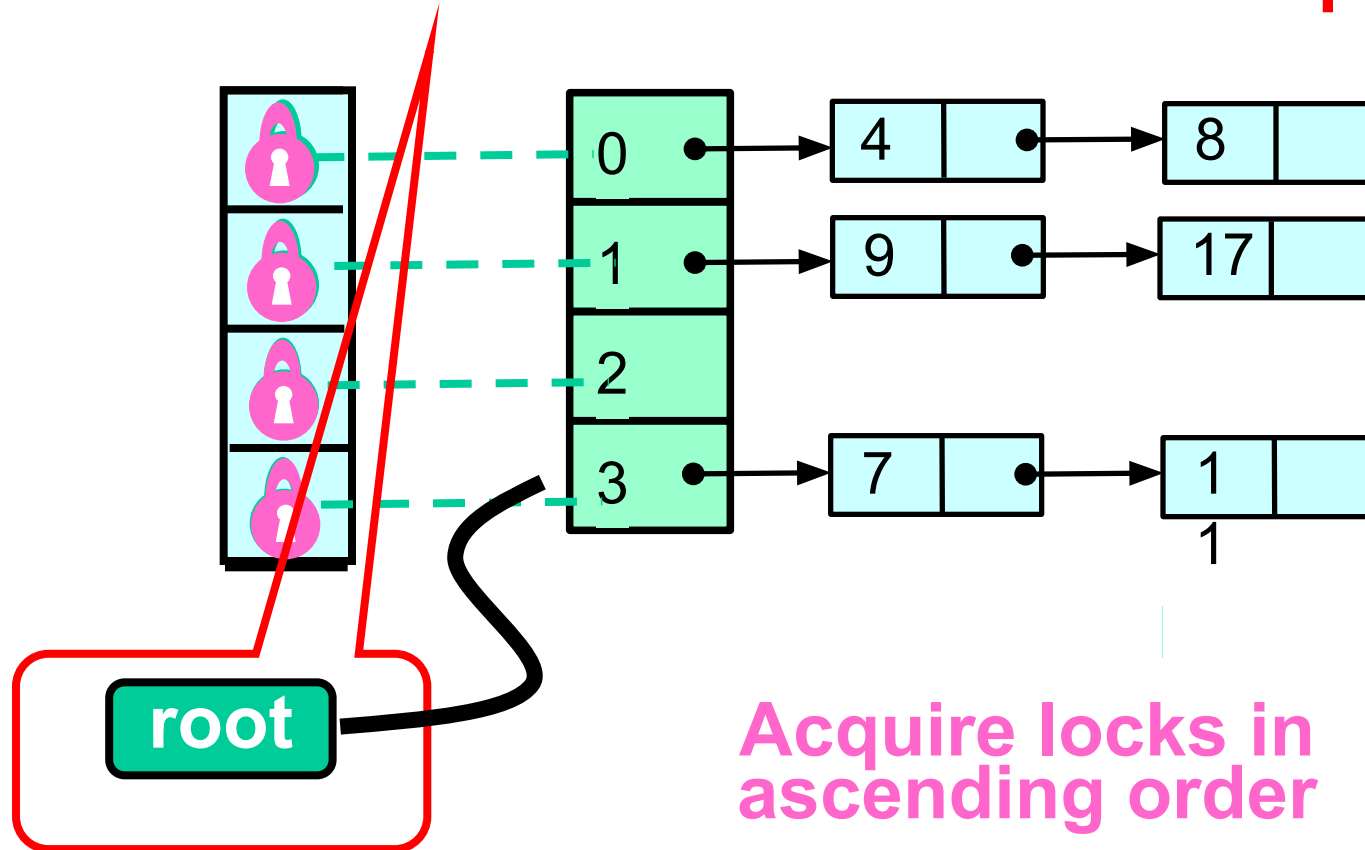
- Good parts
  - Simple
  - Hard to mess up
- Bad parts
  - Sequential bottleneck

# Fine-grained Locking

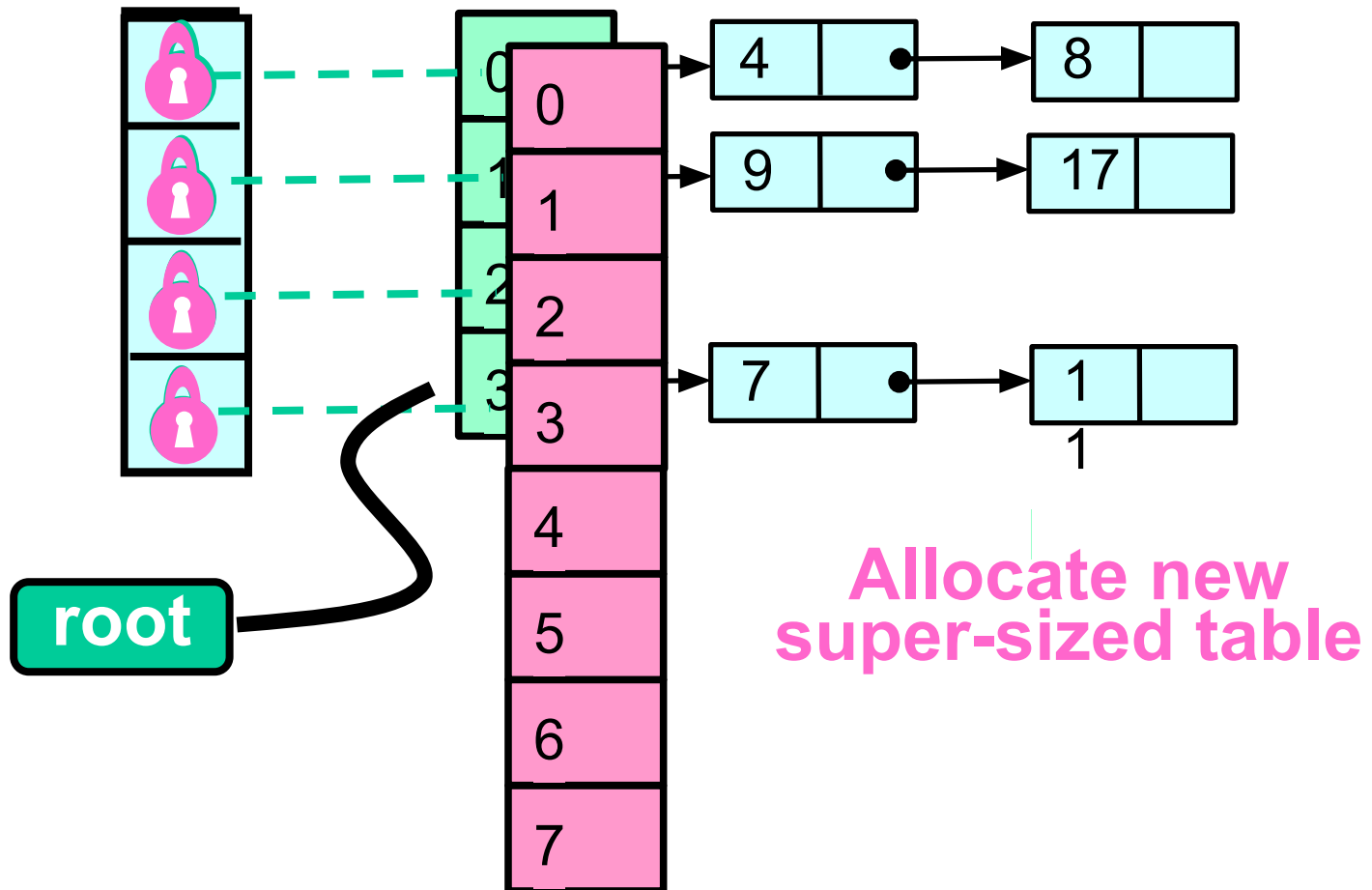


**Each lock associated with one bucket**

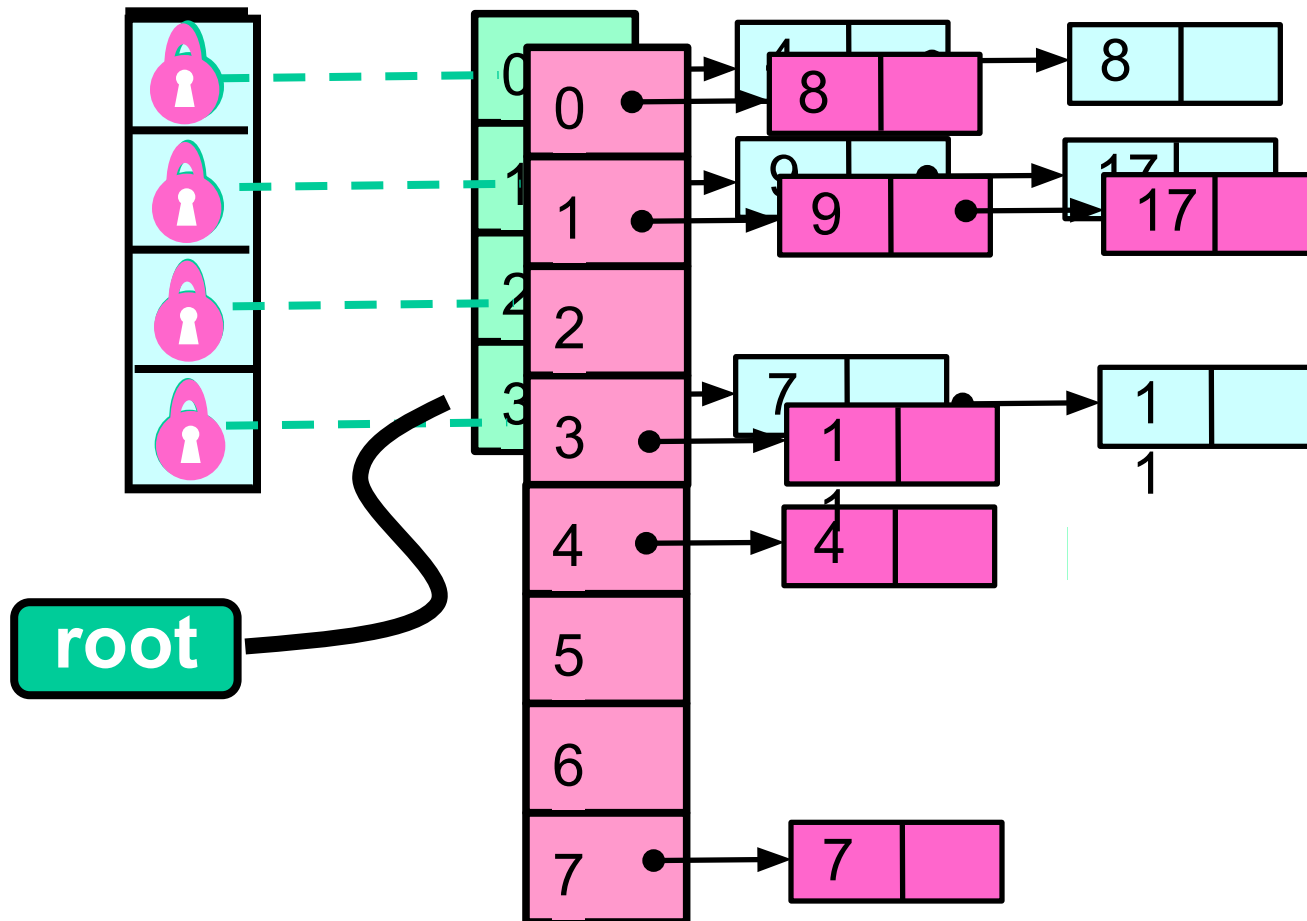
**Make sure root reference didn't change  
between resize decision and lock acquisition**



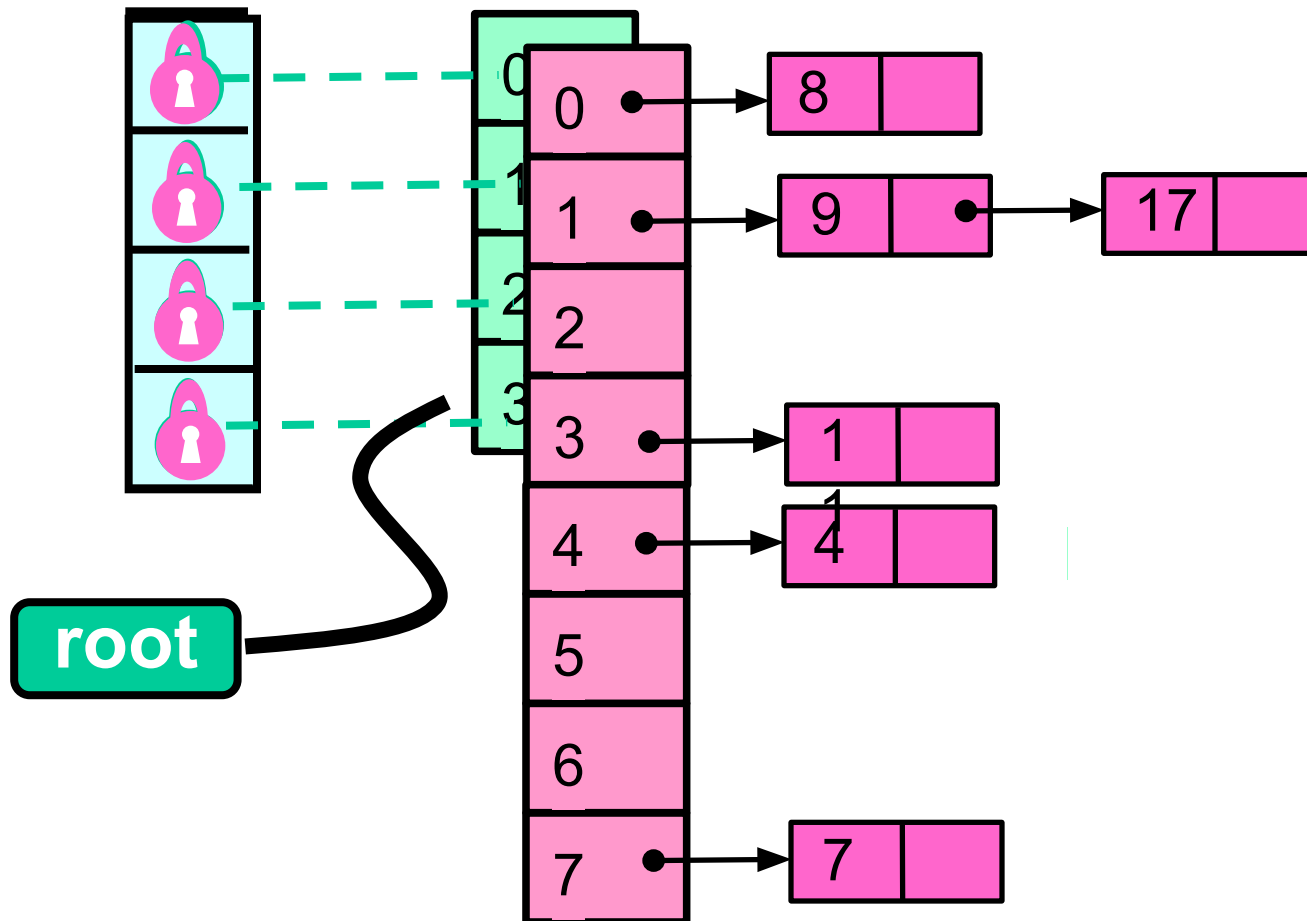
# Resize This



# Resize This

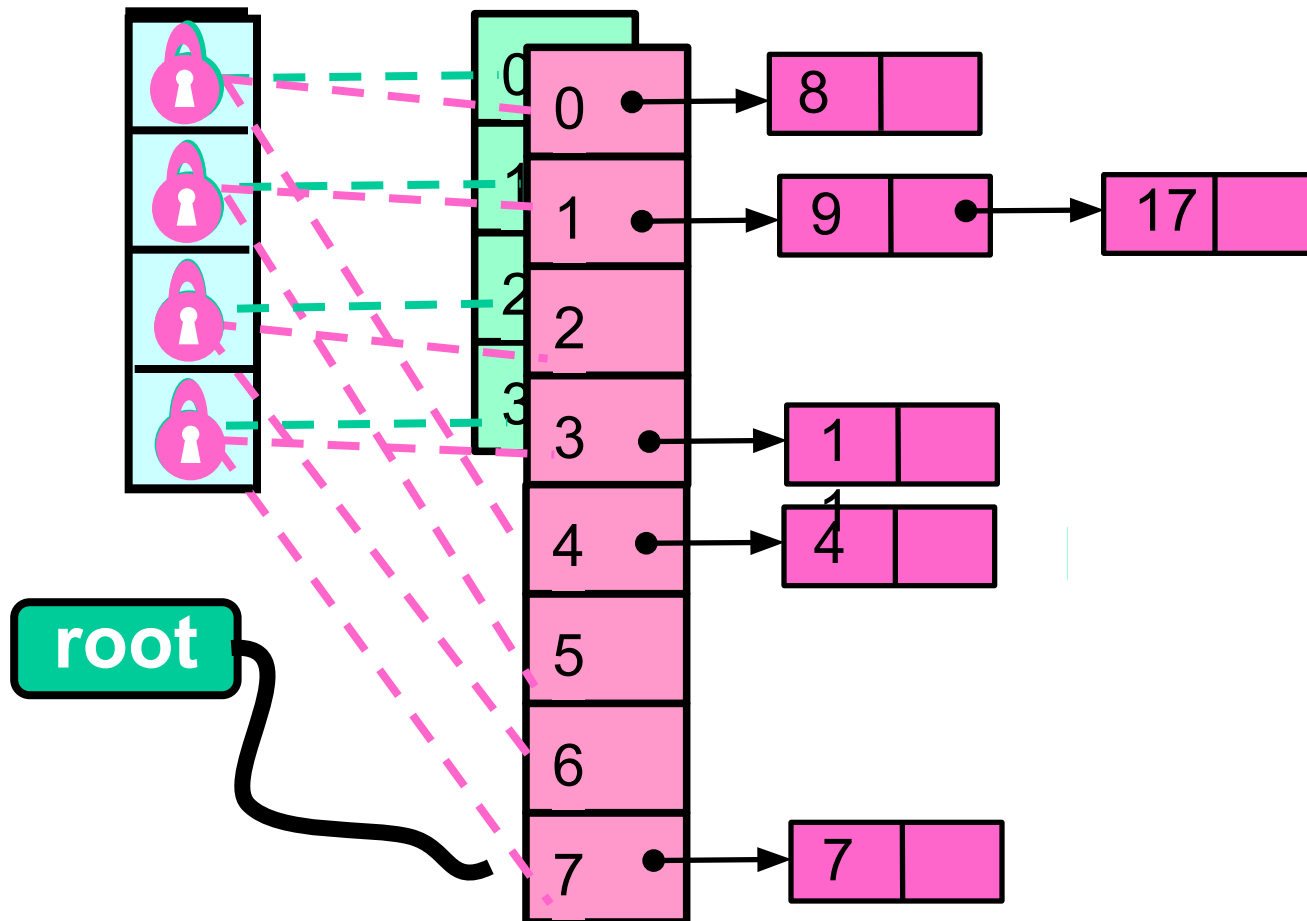


# Resize This



Striped Locks: each lock now associated with two buckets

# Resize This



# Observations

- We grow the table, but not locks
  - Resizing lock array is tricky ...
- We use sequential lists
  - Not **LockFreeList** lists
  - If we're locking anyway, why pay?



# Fine-Grained Hash Set

```
public class FGHashSet {  
    protected RangeLock[] lock;  
    protected List[] table;  
    public FGHashSet(int capacity) {  
        table = new List[capacity];  
        lock = new RangeLock[capacity];  
        for (int i = 0; i < capacity; i++) {  
            lock[i] = new RangeLock();  
            table[i] = new LinkedList();  
        }  
    }  
    ...  
}
```

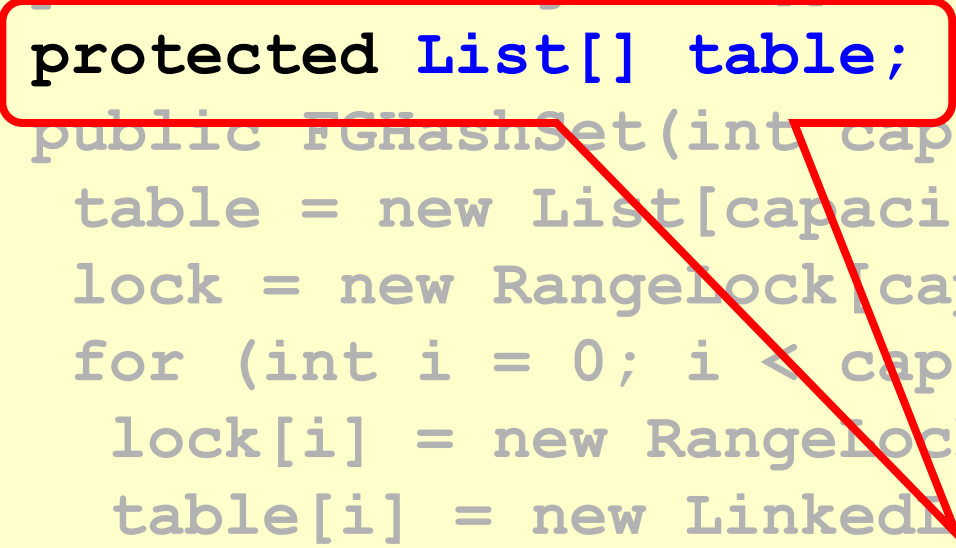
# Fine-Grained Hash Set

```
public class FGHashSet {  
    protected RangeLock[] lock;  
    protected List[] table;  
    public FGHashSet(int capacity) {  
        table = new List[capacity];  
        lock = new RangeLock[capacity];  
        for (int i = 0; i < capacity; i++) {  
            lock[i] = new RangeLock();  
            table[i] = new LinkedList();  
        }  
    }  
    ...  
}
```

**Array of locks**

# Fine-Grained Hash Set

```
public class FGHashSet {  
    protected RangeLock[] lock;  
    protected List[] table;  
    public FGHashSet(int capacity) {  
        table = new List[capacity];  
        lock = new RangeLock[capacity];  
        for (int i = 0; i < capacity; i++) {  
            lock[i] = new RangeLock();  
            table[i] = new LinkedList();  
        } ...  
    }  
}
```



**Array of buckets**

# Fine-Grained Hash Set

```
public class Initially same number of  
    protected RangeLock[] lock;  
    protected List[] table;  
    public FGHashSet(int capacity) {  
        table = new List[capacity];  
        lock = new RangeLock[capacity];  
        for (int i = 0; i < capacity; i++) {  
            lock[i] = new RangeLock();  
            table[i] = new LinkedList();  
        }} ...
```

# The add() method

```
public boolean add(Object key) {  
    int keyHash  
        = key.hashCode() % lock.length;  
    synchronized (lock[keyHash]) {  
        int tabHash = key.hashCode() %  
                        table.length;  
        return table[tabHash].add(key);  
    }  
}
```

# Fine-Grained Locking

```
public boolean add(Object key) {  
    int keyHash  
        = key.hashCode() % lock.length;  
    synchronized (lock[keyHash]) {  
        int tabHash = key.hashCode() %  
                        table.length;  
        return table[tabHash].add(key);  
    }  
}
```

**Which lock?**

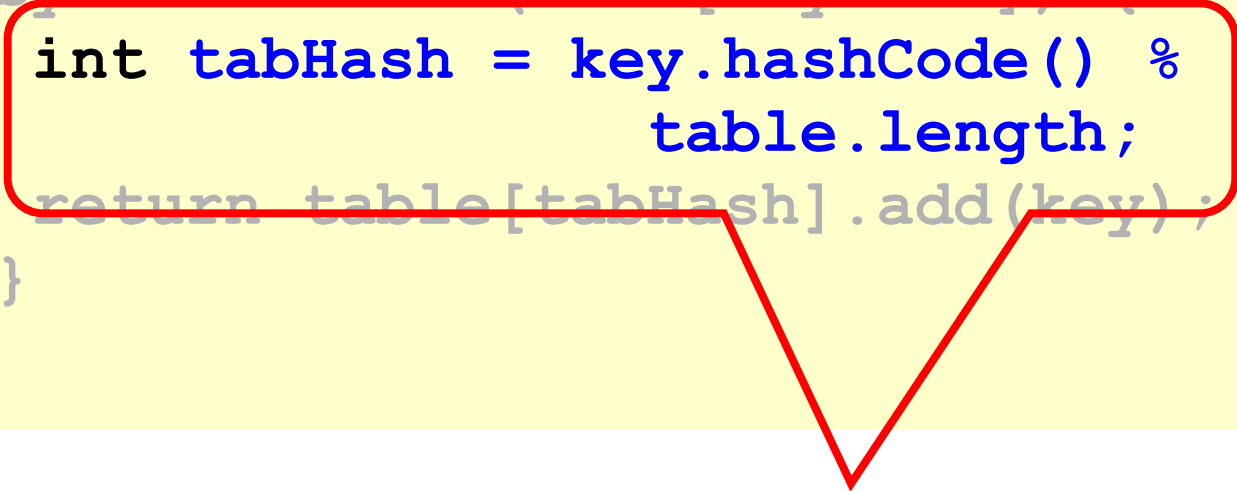
# The add() method

```
public boolean add(Object key) {  
    int keyHash  
        = key.hashCode() % lock.length;  
    synchronized (lock[keyHash]) {  
        int tabHash = key.hashCode() %  
            table.length;  
        return table[tabHash].add(key);  
    }  
}
```

**Acquire the lock**

# Fine-Grained Locking

```
public boolean add(Object key) {  
    int keyHash  
        = key.hashCode() % lock.length;  
    synchronized (lock[keyHash]) {  
        int tabHash = key.hashCode() %  
            table.length;  
        return table[tabHash].add(key);  
    }  
}
```



**Which bucket?**



# The add() method

```
public boolean add(Object key) {  
    int keyHash  
        = key.hashCode() % lock.length;  
    synchronized (lock[keyHash]) {  
        int tabHash = key.hashCode() %  
            table.length;  
        return table[tabHash].add(key) ;  
    }  
}
```

**Call that bucket's  
add() method**

# Resizing

```
private void resize(int depth,  
                    List[] oldTab) {  
    synchronized (lock[depth]) {  
        if (oldTab == table){  
            int next = depth + 1;  
            if (next < lock.length)  
                resize (next, oldTab);  
            else  
                sequentialResize();  
        }  
    }  
}
```

# Fine-Grained Locking

```
private void resize(int depth,  
                    List[] oldTab) {
```

```
    synchronized (lock[depth]) {
```

```
        if (oldTab == table){
```

```
            int next = depth + 1;
```

```
            if (next < lock.length)
```

```
                resize (next, oldTab);
```

```
            else
```

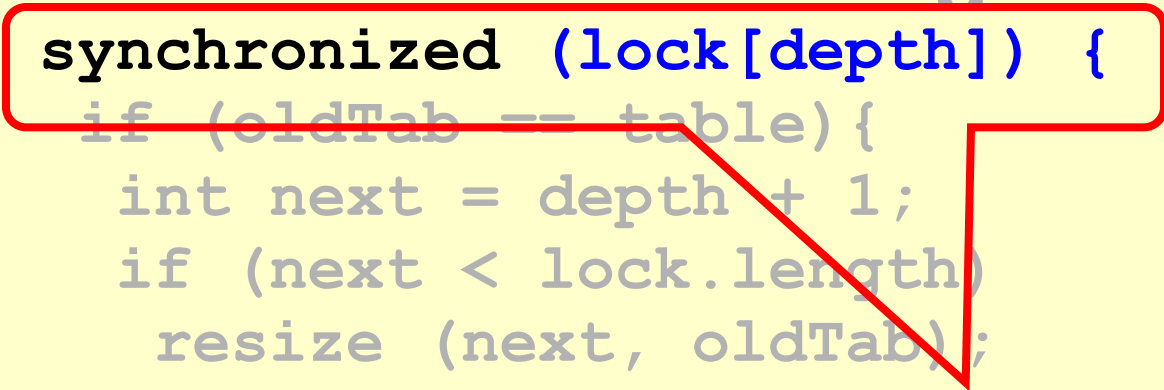
```
                sequentialResize();
```

```
        } } }
```

**resize() calls resize(0,table)**

# Resizing

```
private void resize(int depth,
                    List[] oldTab) {
    synchronized (lock[depth]) {
        if (oldTab == table) {
            int next = depth + 1;
            if (next < lock.length)
                resize (next, oldTab);
            else
                sequentialResize();
        }
    }
}
```



**Acquire next lock**

# Resizing

```
private void resize(int depth,  
                    List[] oldTab) {  
    synchronized (lock[depth]) {  
        if (oldTab == table) {  
            int next = depth + 1;  
            if (next < lock.length)  
                resize (next, oldTab);  
            else
```

**Check that no one else has resized**

# Resizing

**Recursively acquire next lock**

```
private void resize(int depth,
                    int[] oldTab,
                    int[] newTab) {
    synchronized (lock[depth]) {
        if (oldTab == table) {
            int next = depth + 1;
            if (next < lock.length)
                resize (next, oldTab);
            else
                sequentialResize();
        }
    }
}
```

# Resizing

**Locks acquired, do the work**

```
private void resize(int depth, List<List<Object>> oldTab) {  
    synchronized (lock[depth]) {  
        if (oldTab == table) {  
            int next = depth + 1;  
            if (next < lock.length)  
                resize (next, oldTab);  
            else  
                sequentialResize();  
        }  
    }  
}
```

# Another Locking Structure

- **add, remove, contains**
  - Lock table in *shared* mode
- **resize**
  - Locks table in *exclusive* mode



# Read-Write Locks

```
public interface ReadWriteLock {  
    Lock readLock() ;  
    Lock writeLock() ;  
}
```

# Read/Write Locks

**Returns associated  
read lock**

```
public interface ReadWriteLock {  
    Lock readLock();  
    Lock writeLock();  
}
```

# Read/Write Locks

```
public interface ReadWriteLock {  
    Lock readLock();  
    Lock writeLock();  
}
```

**Returns associated  
read lock**

**Returns associated  
write lock**

# Lock Safety Properties

- Read lock:
  - Locks out writers
  - Allows concurrent readers
- Write lock
  - Locks out writers
  - Locks out readers

# Lets Try to Design a Read-Write Lock

- Read lock:
  - Locks out writers
  - Allows concurrent readers
- Write lock
  - Locks out writers
  - Locks out readers

# Read/Write Lock

- Safety
  - If **readers > 0** then **writer == false**
  - If **writer == true** then **readers == 0**
- Liveness?
  - Will a continual stream of readers ...
  - Lock out writers?

# FIFO R/W Lock

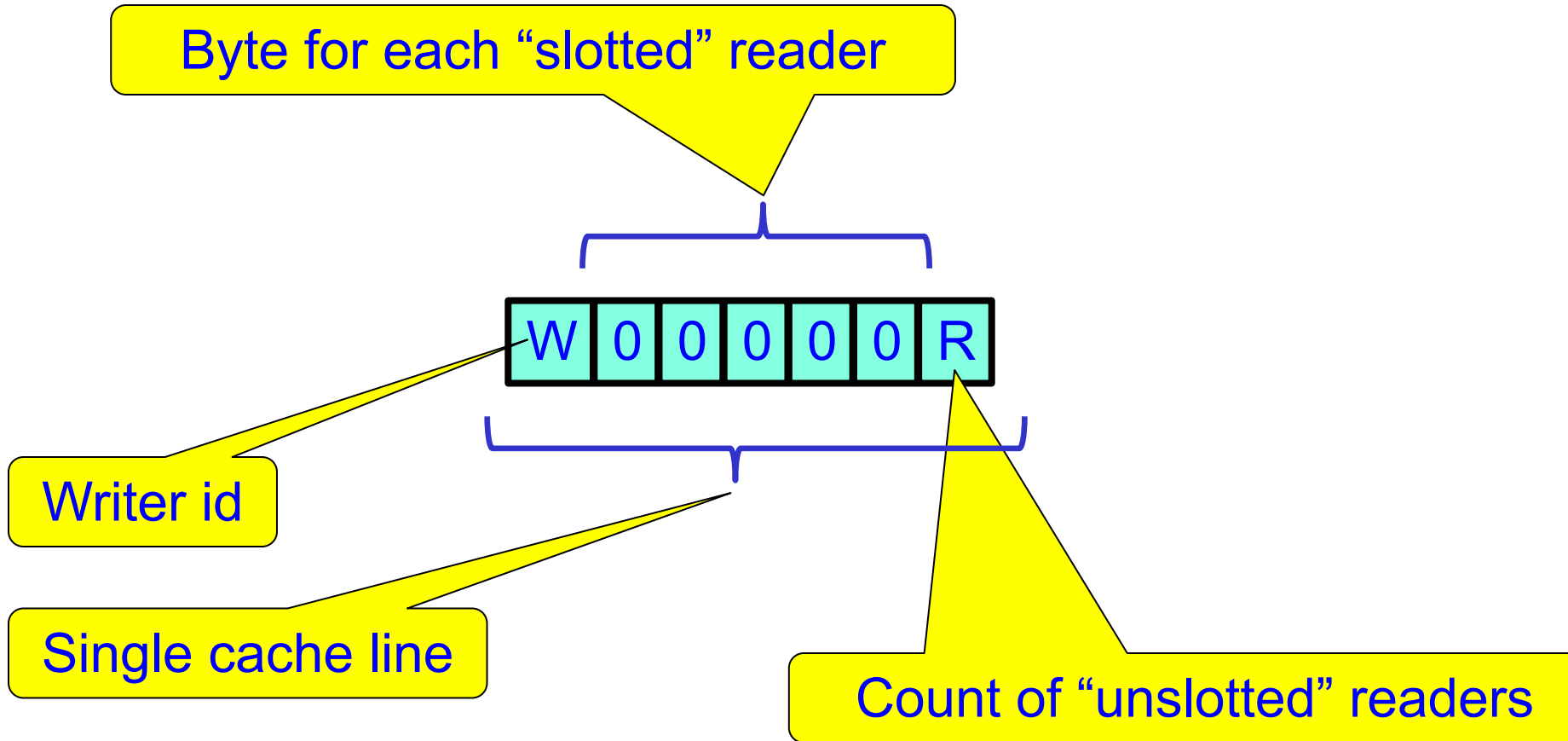
- As soon as a writer requests a lock
- No more readers accepted
- Current readers “drain” from lock
- Writer gets in

# ByteLock

- Readers-Writers lock
- Cache-aware
- Fast path for “slotted readers”
- Slower path for others

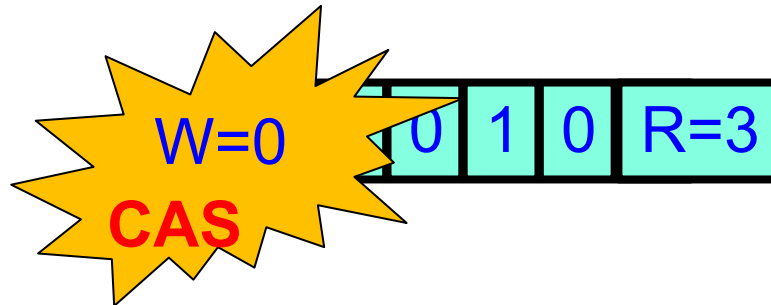
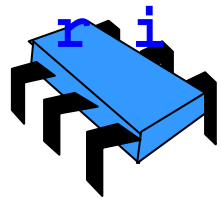


# ByteLock Lock Record



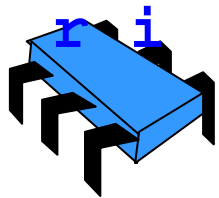
# Writer

Write



# Writer

Write

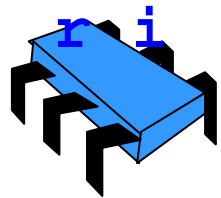


W=i	0	0	1	0	R=3
-----	---	---	---	---	-----

Wait for readers to  
drain out

# Writer

Write

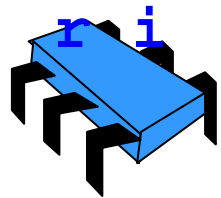


W=i	0	0	0	0	R=0
-----	---	---	---	---	-----

proceed

# Writer

Write

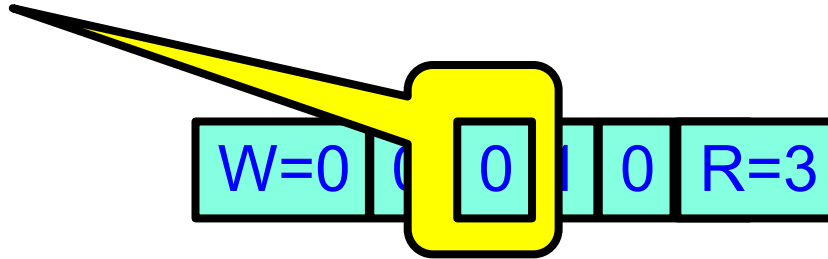
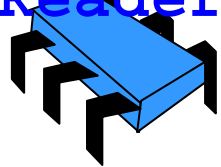


null	0	0	0	0	R=0
------	---	---	---	---	-----

release

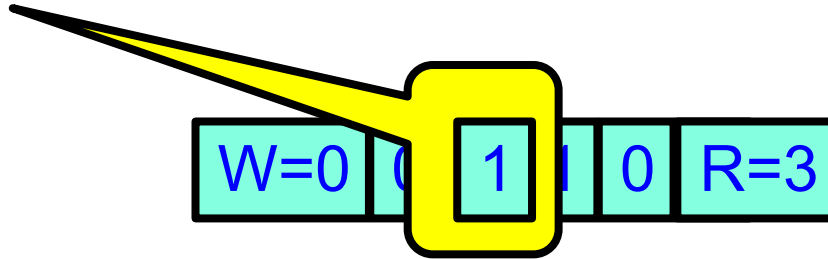
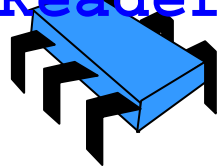
# Slotted Reader Fast Path

Slotted  
Reader



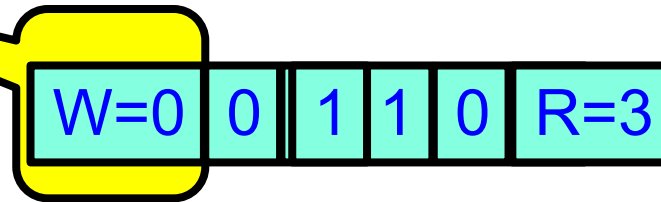
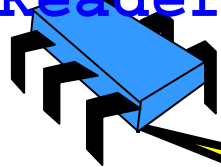
# Slotted Reader Fast Path

Slotted  
Reader



# Slotted Reader Fast Path

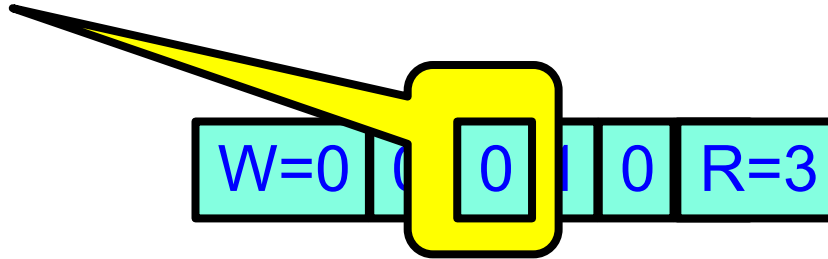
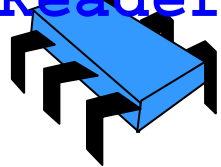
Slotted  
Reader





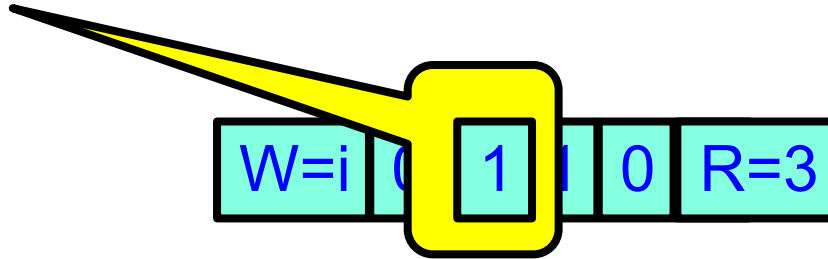
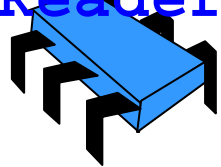
# Slotted Reader Fast Path

Slotted  
Reader



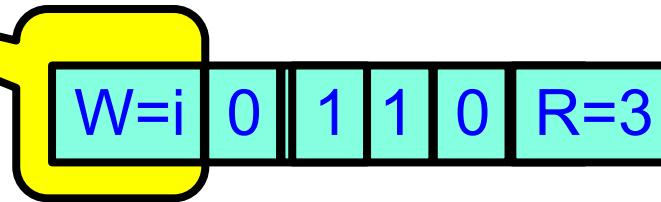
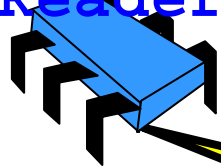
# Slotted Reader Slow Path

Slotted  
Reader



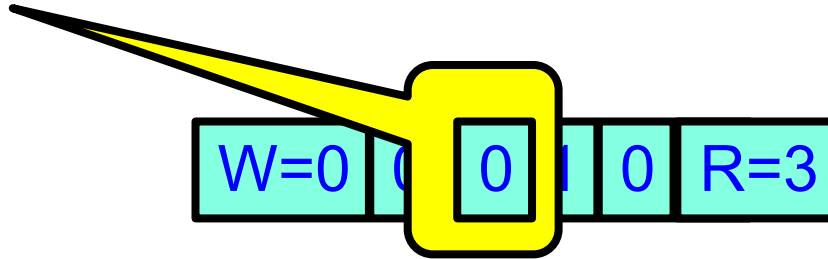
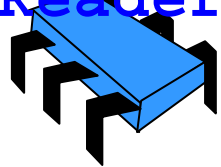
# Slotted Reader Slow Path

Slotted  
Reader



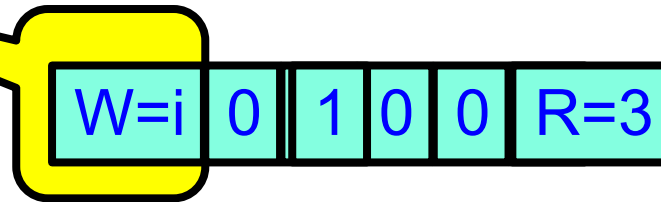
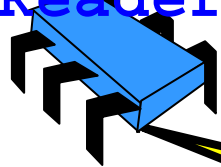
# Slotted Reader Slow Path

Slotted  
Reader



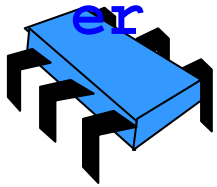
# Slotted Reader Slow Path

Slotted  
Reader



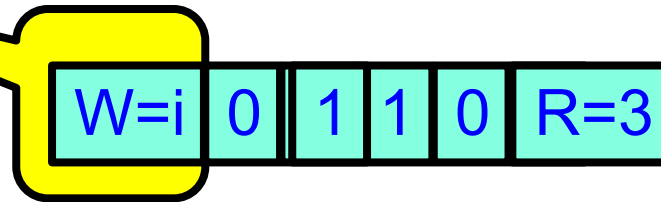
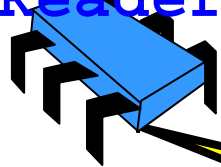
# Unslotted Reader

UnslottedRead



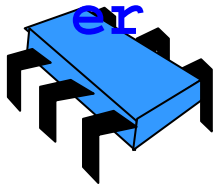
# Slotted Reader Slow Path

Slotted  
Reader



# Unslotted Reader

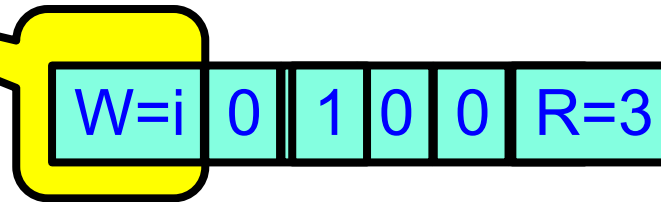
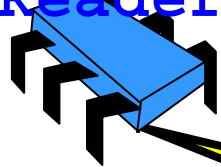
UnslottedRead





# Slotted Reader Slow Path

Slotted  
Reader



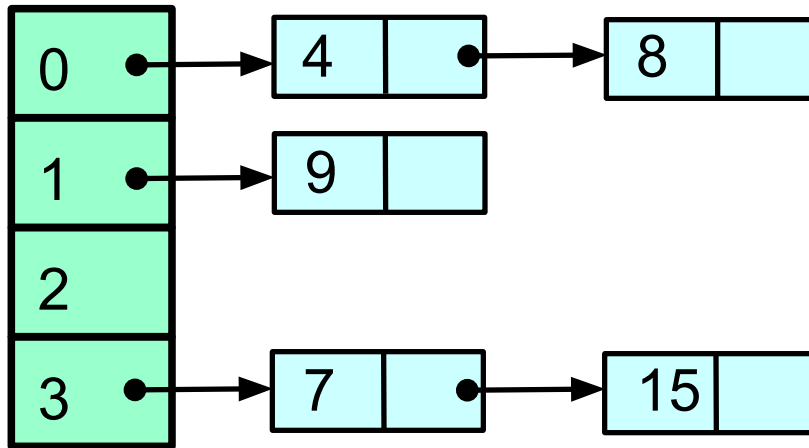
# The Story So Far

- Resizing is the hard part
- Fine-grained locks
  - Striped locks cover a range (not resized)
- Read/Write locks
  - FIFO property tricky

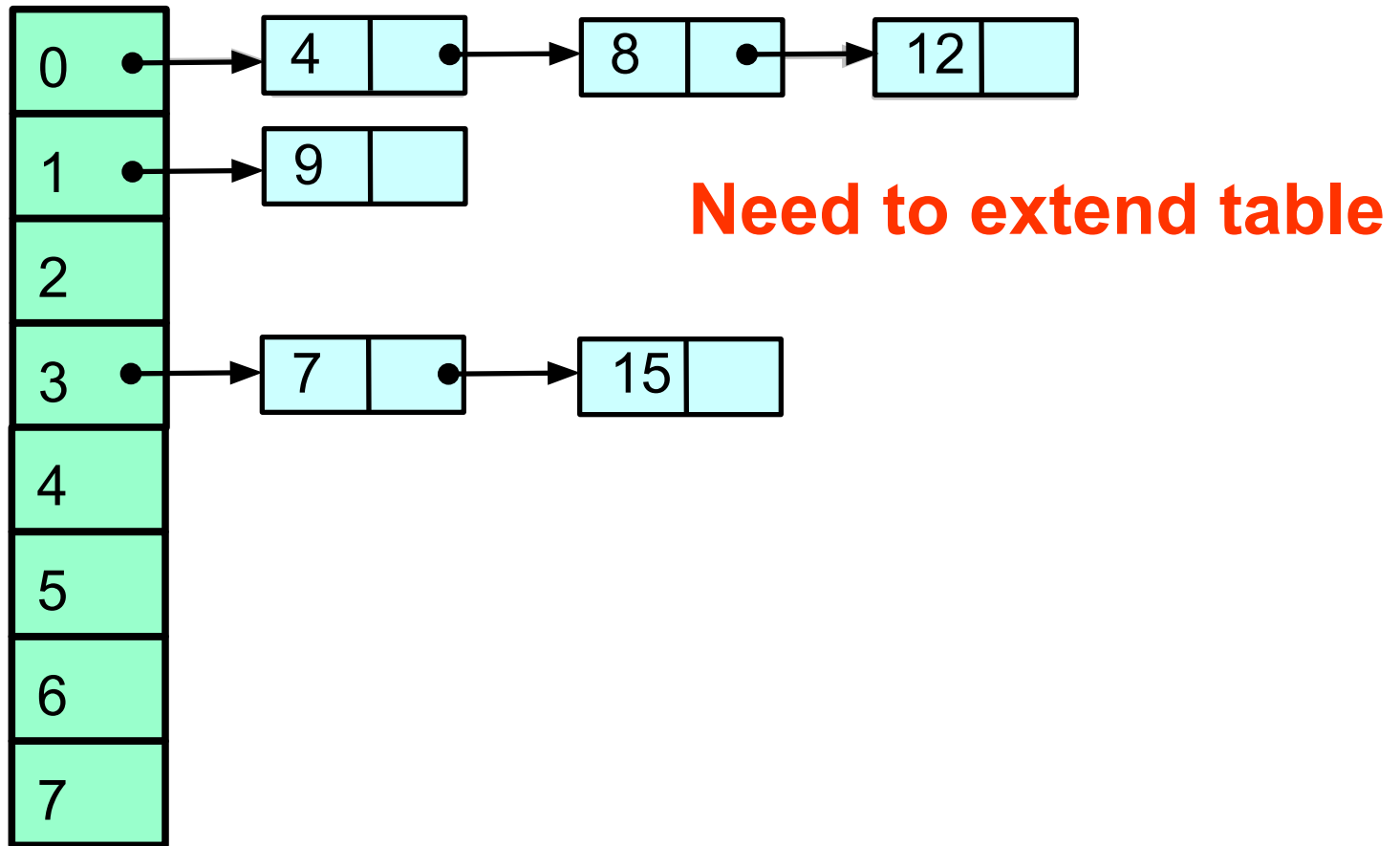
# Stop The World Resizing

- Resizing stops all concurrent operations
- What about an incremental resize?
- Must avoid locking the table
- A lock-free table + incremental resizing?

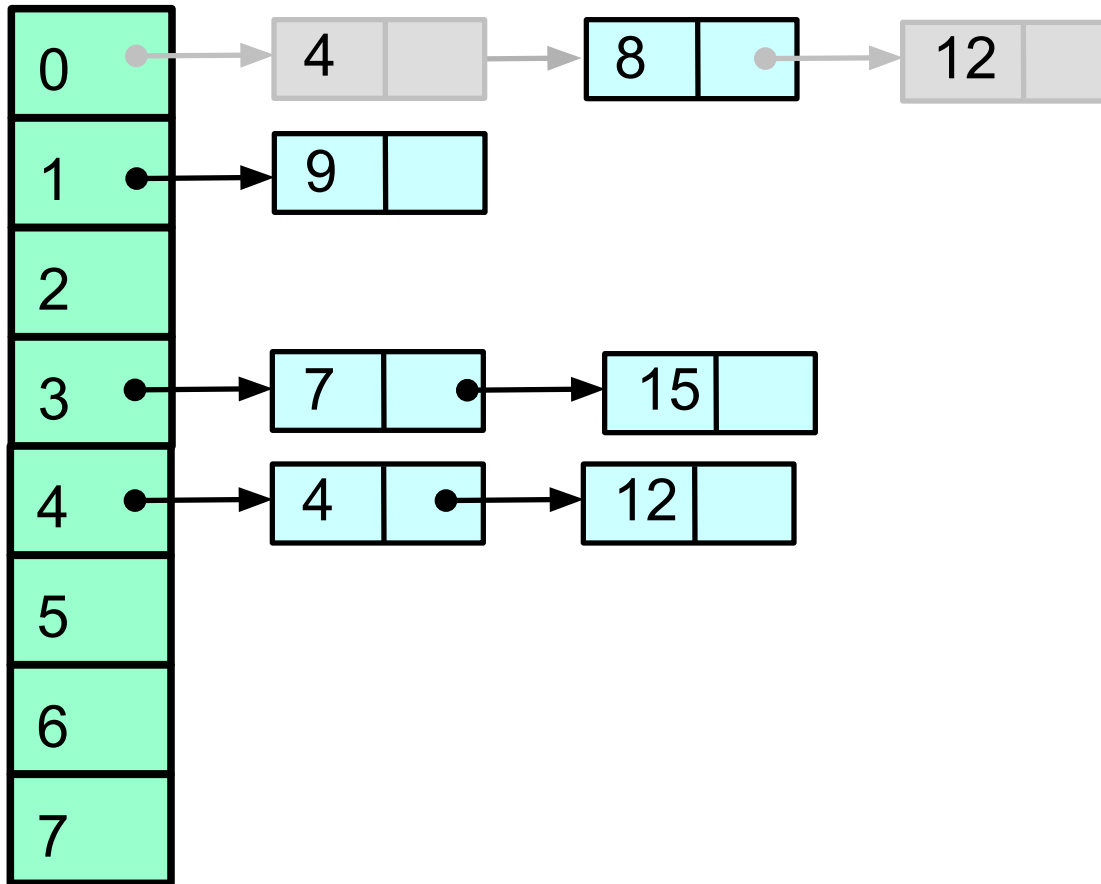
# Lock-Free Resizing Problem



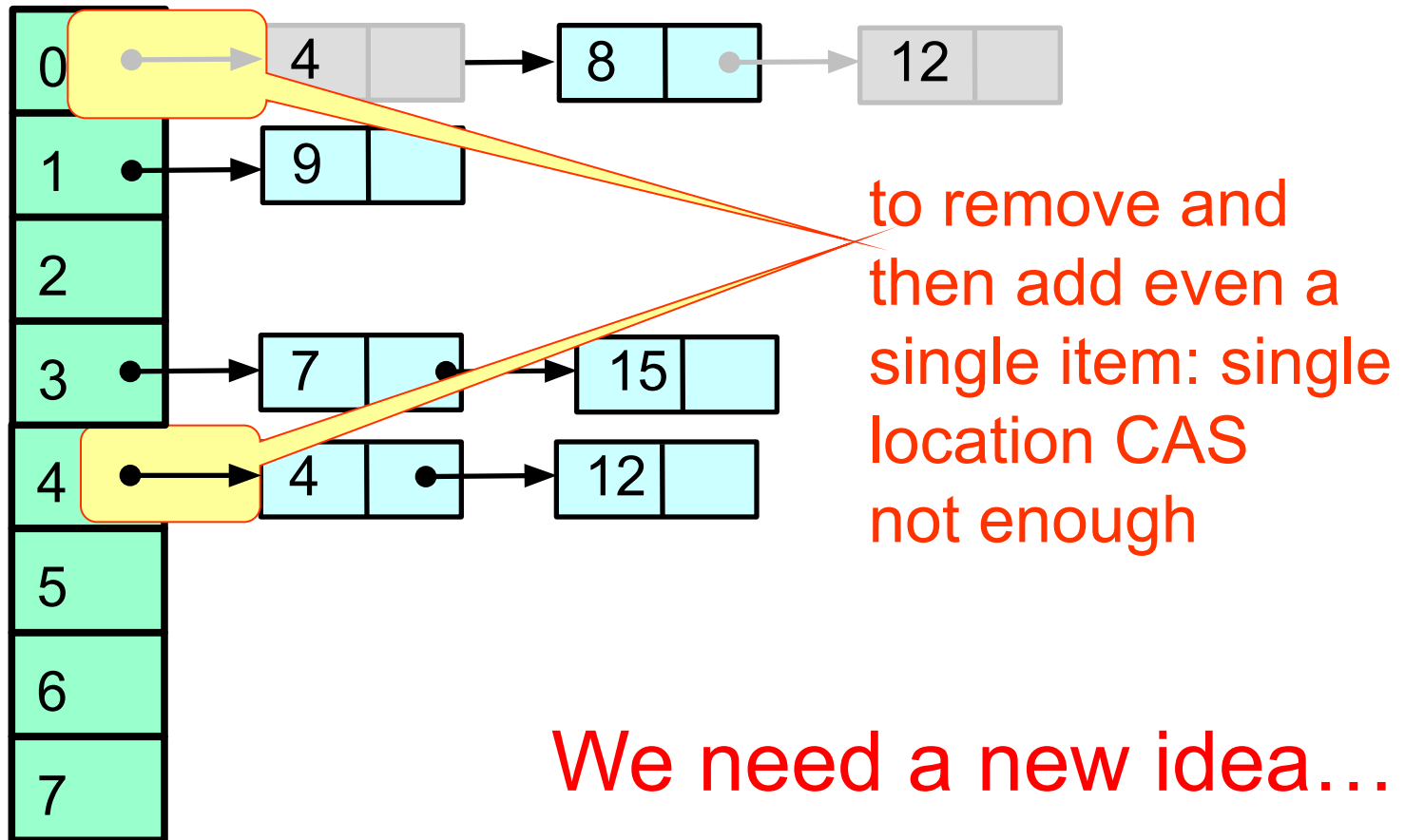
# Lock-Free Resizing Problem



# Lock-Free Resizing Problem

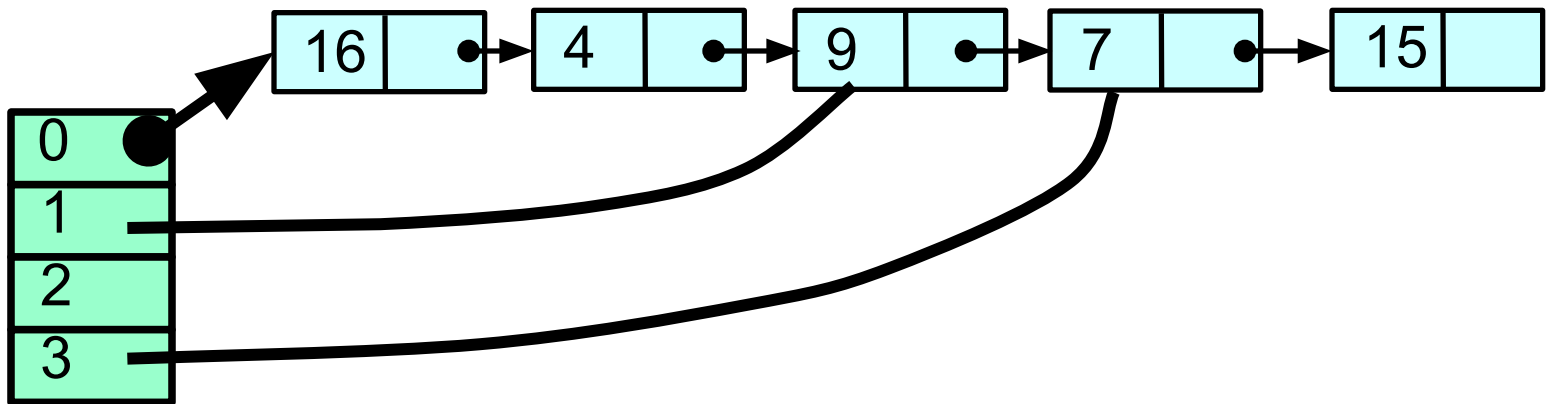


# Lock-Free Resizing Problem



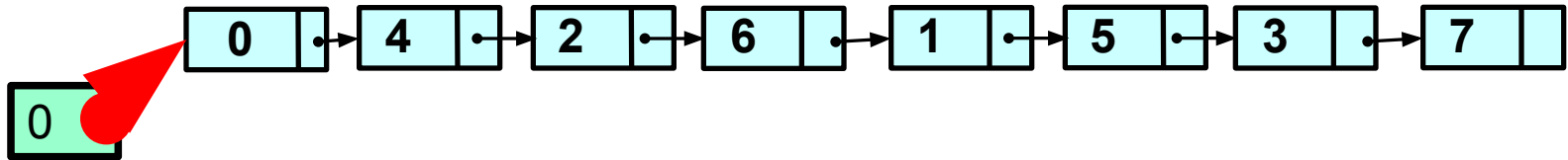
# Don't move the items

- Move the buckets instead!
- Keep all items in a single, lock-free list
- Buckets are short-cuts into the list

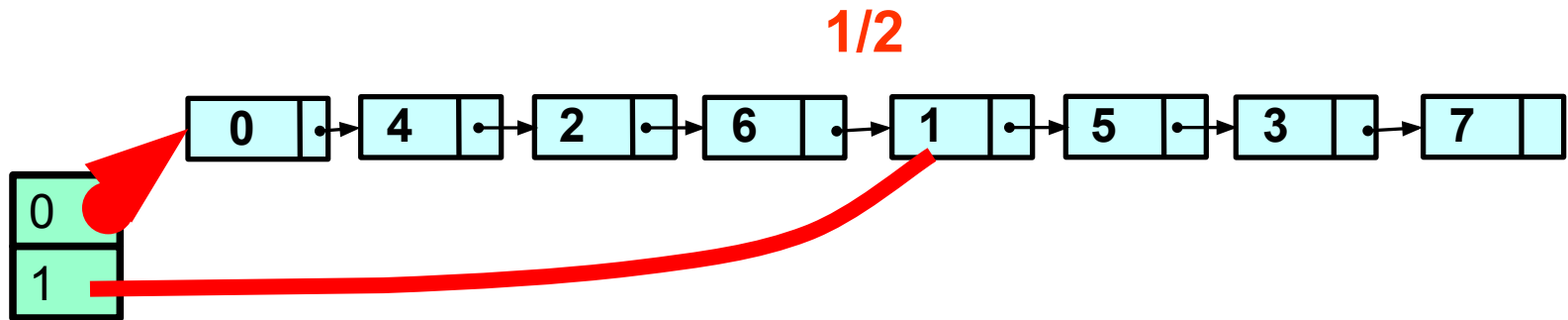




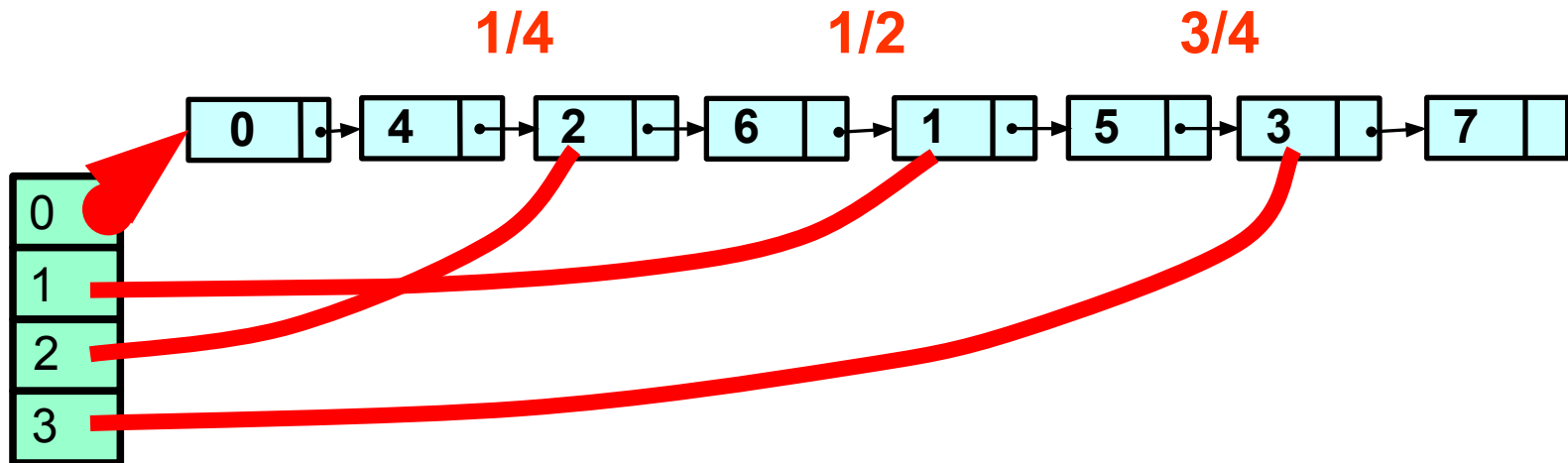
# Recursive Split Ordering



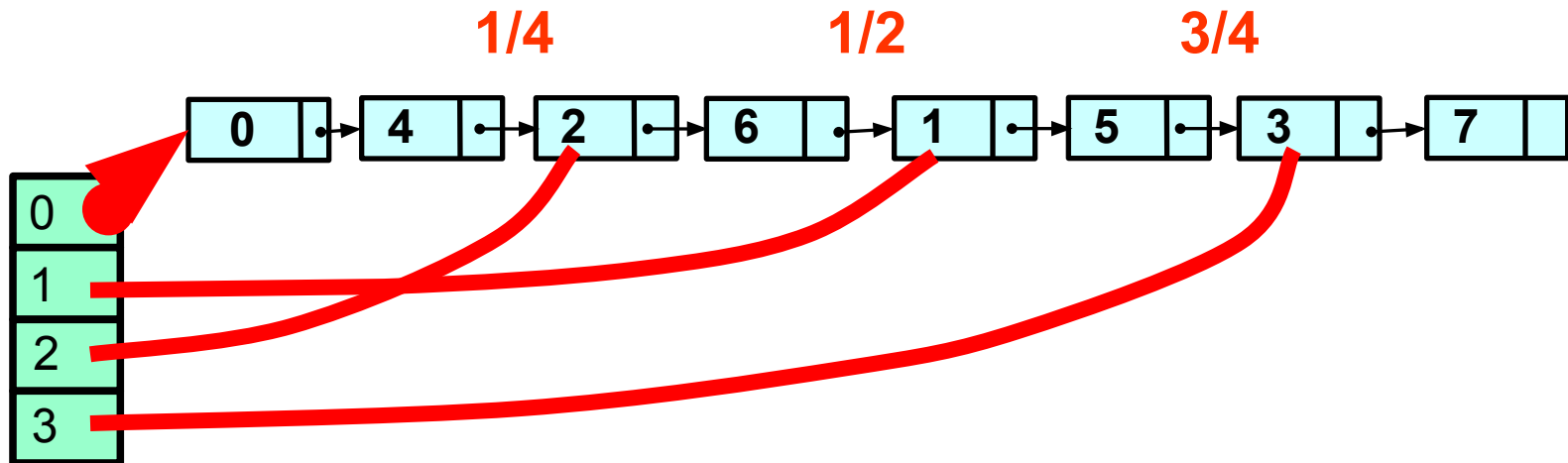
# Recursive Split Ordering



# Recursive Split Ordering

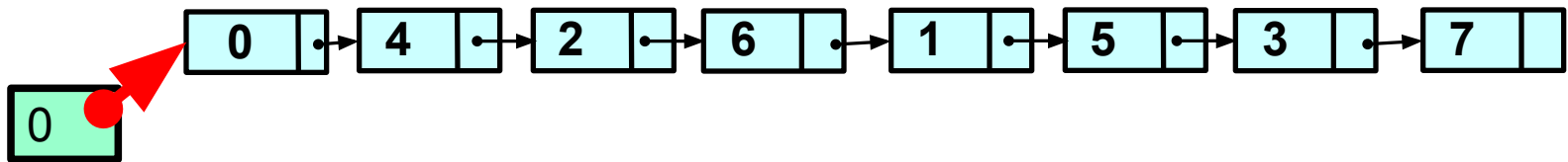


# Recursive Split Ordering

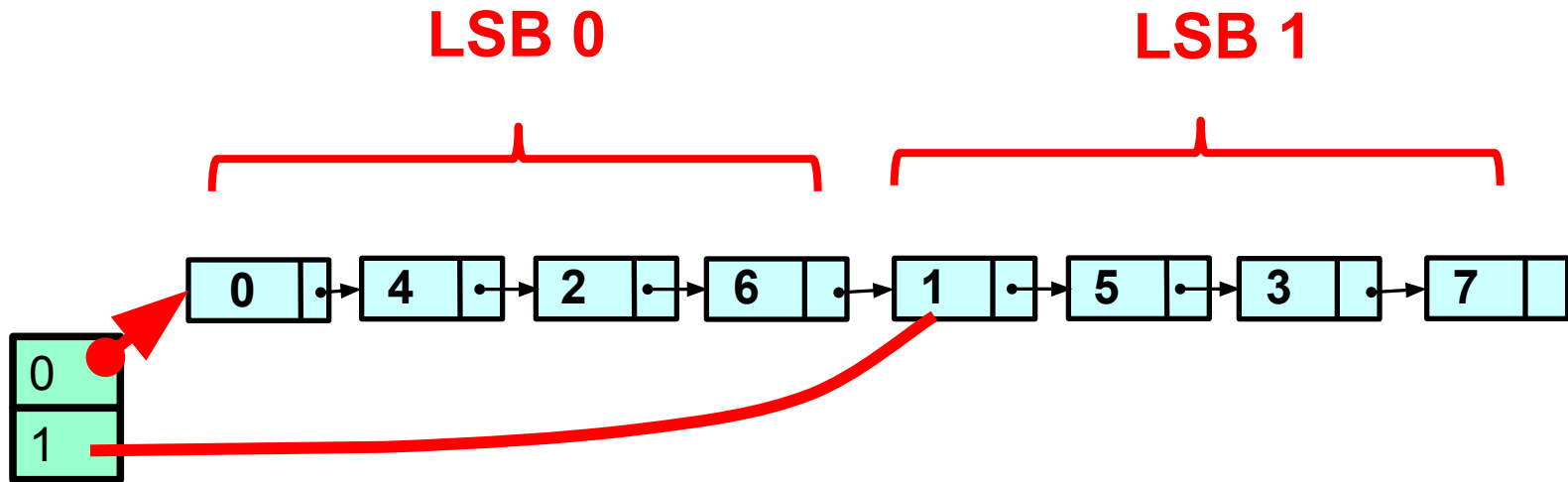


**List entries sorted in order that allows recursive splitting. How?**

# Recursive Split Ordering

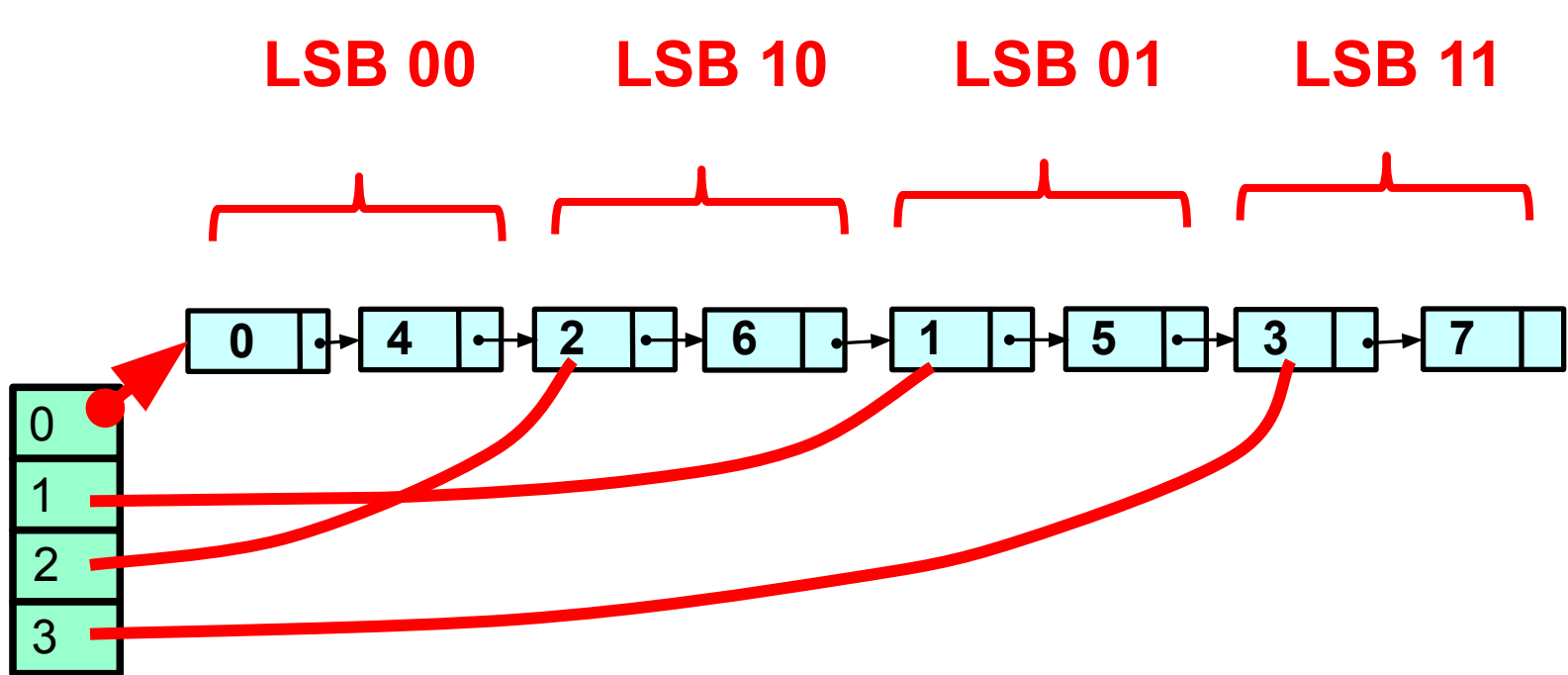


# Recursive Split Ordering



**LSB = Least significant Bit**

# Recursive Split Ordering



# Split-Order

- If the table size is  $2^i$ ,
  - Bucket  $b$  contains keys  $k$ 
    - $k = b \pmod{2^i}$
  - bucket index consists of key's  $i$  LSBs

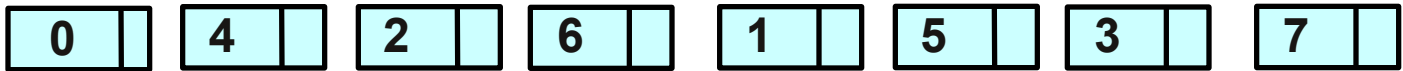


# When Table Splits

- Some keys stay
  - $b = k \bmod(2^{i+1})$
- Some move
  - $b+2^i = k \bmod(2^{i+1})$
- Determined by  $(i+1)^{\text{st}}$  bit
  - Counting backwards
- Key must be accessible from both
  - Keys that will move must come later

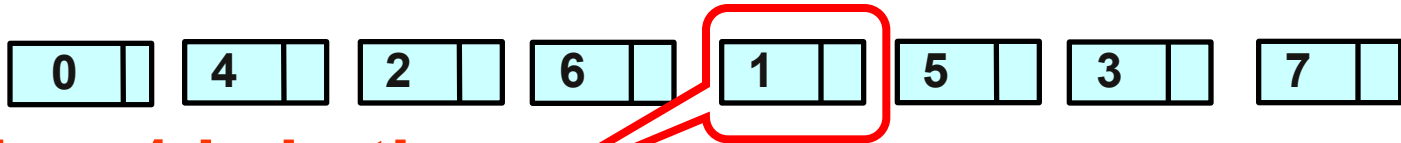
# A Bit of Magic

Real keys:



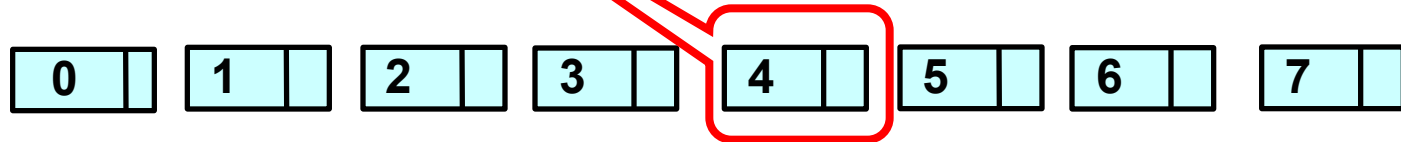
# A Bit of Magic

Real keys:



**Real key 1 is in the  
4<sup>th</sup> location**

Split-order:



# A Bit of Magic

Real keys:

0	4	2	6	1	5	3	7
000	100	010	110	001	101	011	111

**Real key 1 is in 4<sup>th</sup> location**

Split-order:

0	1	2	3	4	5	6	7
000	001	010	011	100	101	110	111

# A Bit of Magic

Real keys:

**000    100    010    110    001    101    011    111**

Split-order:

**000    001    010    011    100    101    110    111**

# A Bit of Magic

Real keys:

000 100 010 110 001 101 011 111

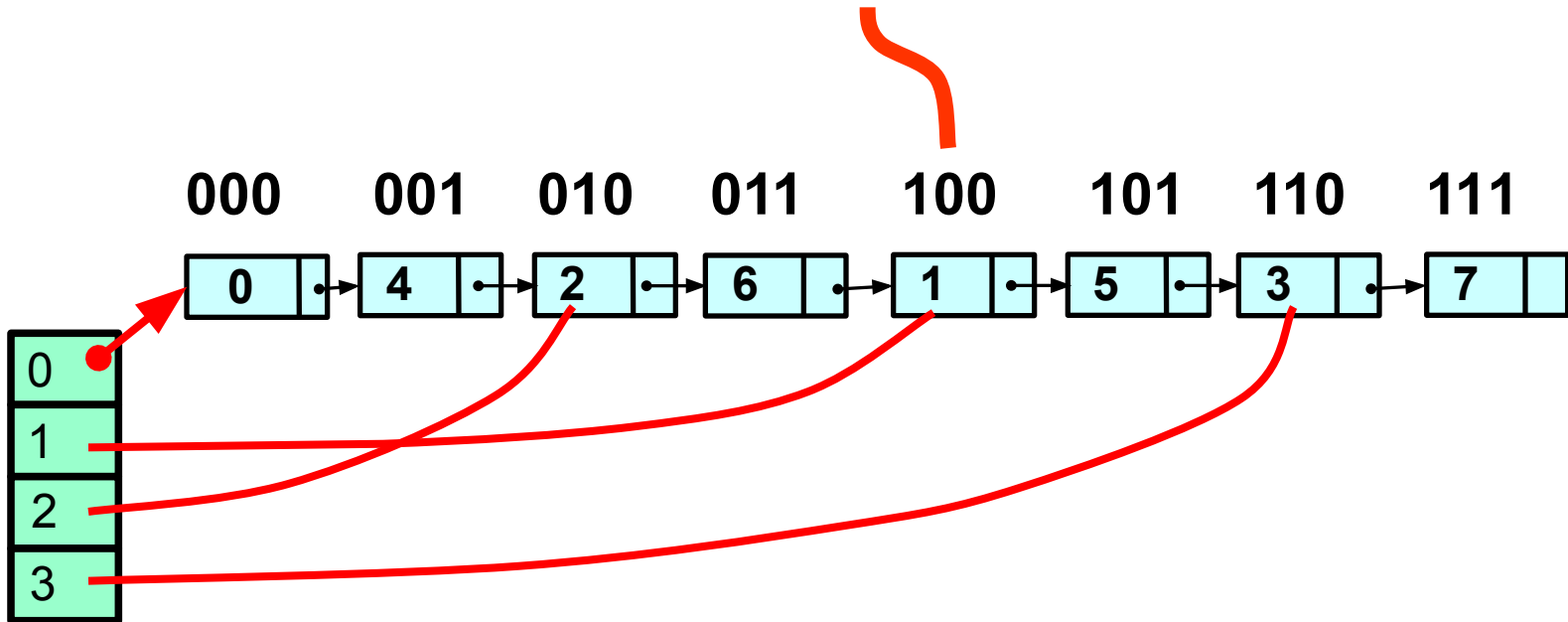
Split-order:

000 001 010 011 100 101 110 111

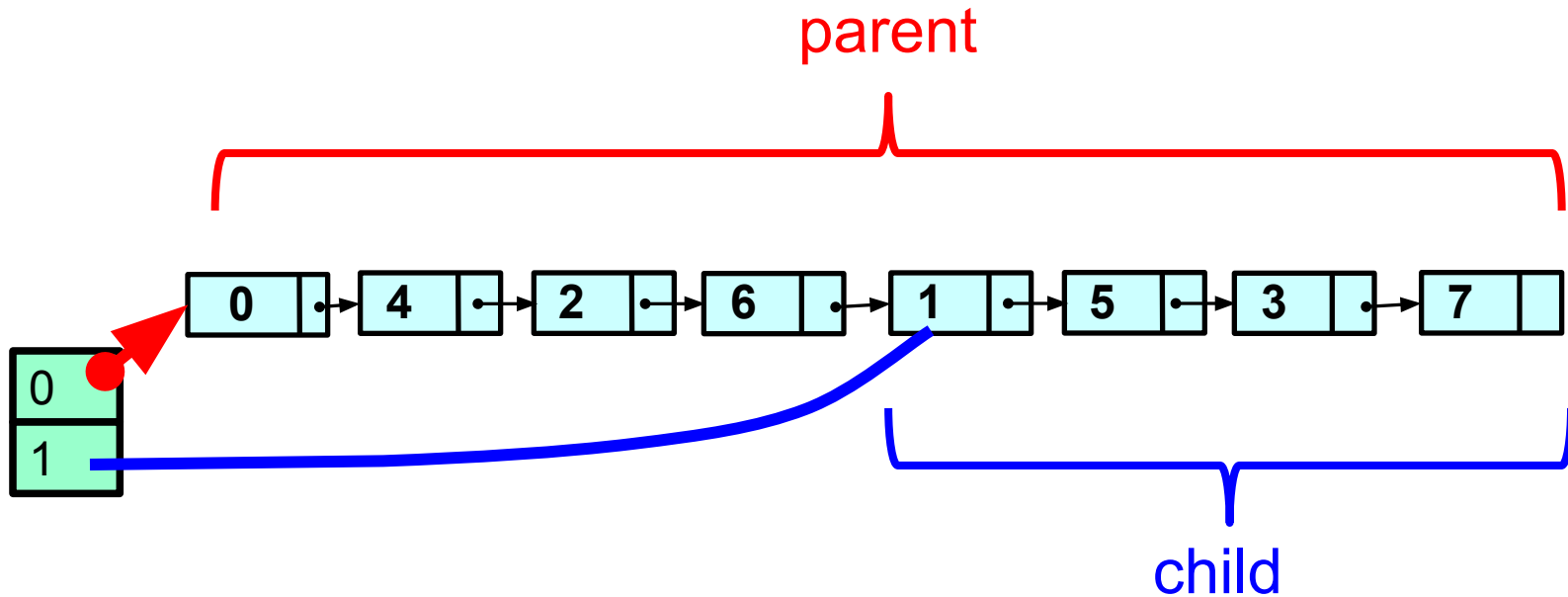
**Just reverse the order of the  
key bits**

# Split Ordered Hashing

Order according to reversed bits

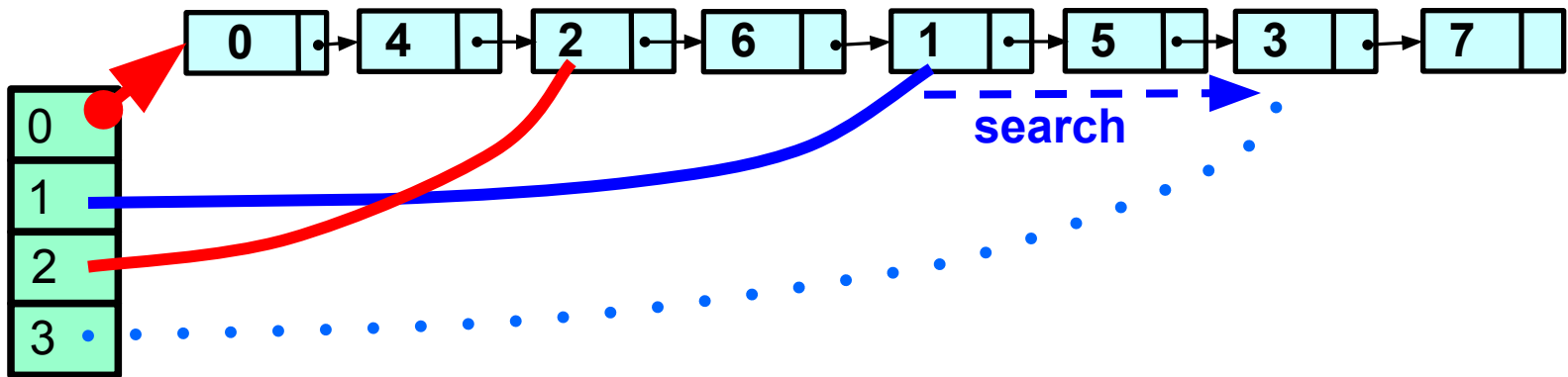


# Bucket Relations

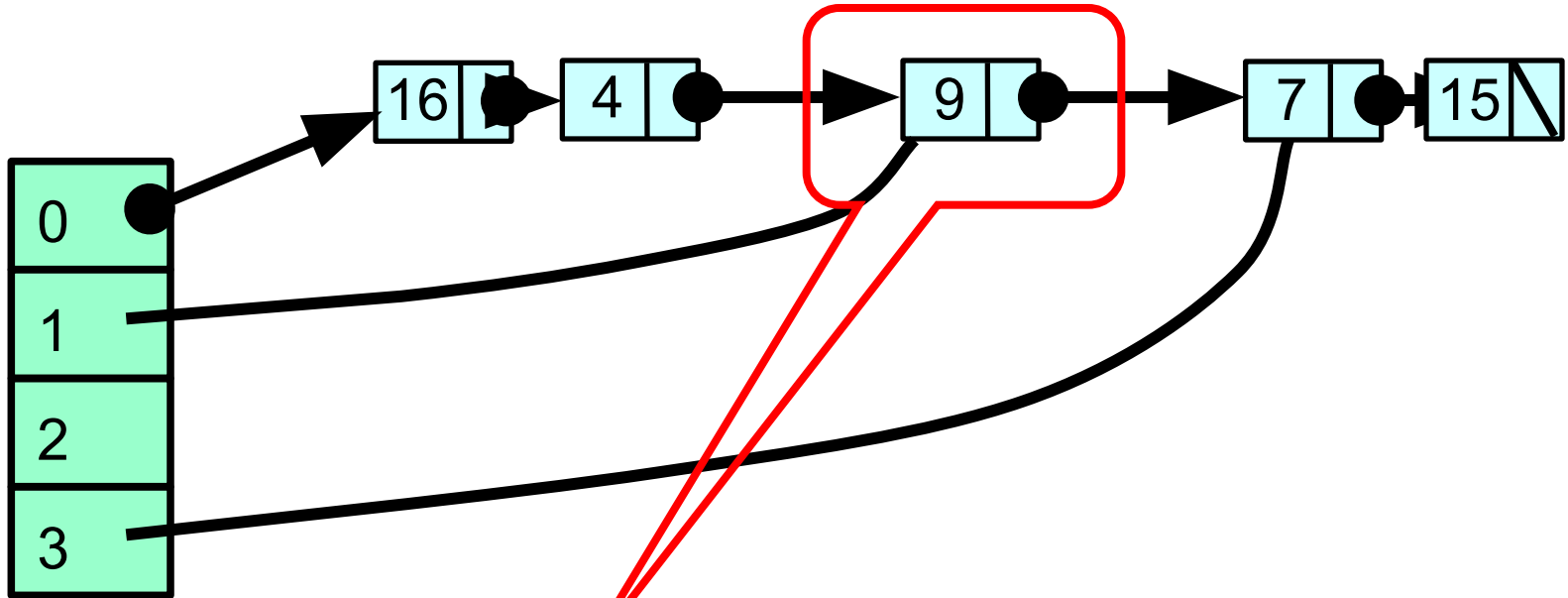




# Parent Always Provides a Short Cut

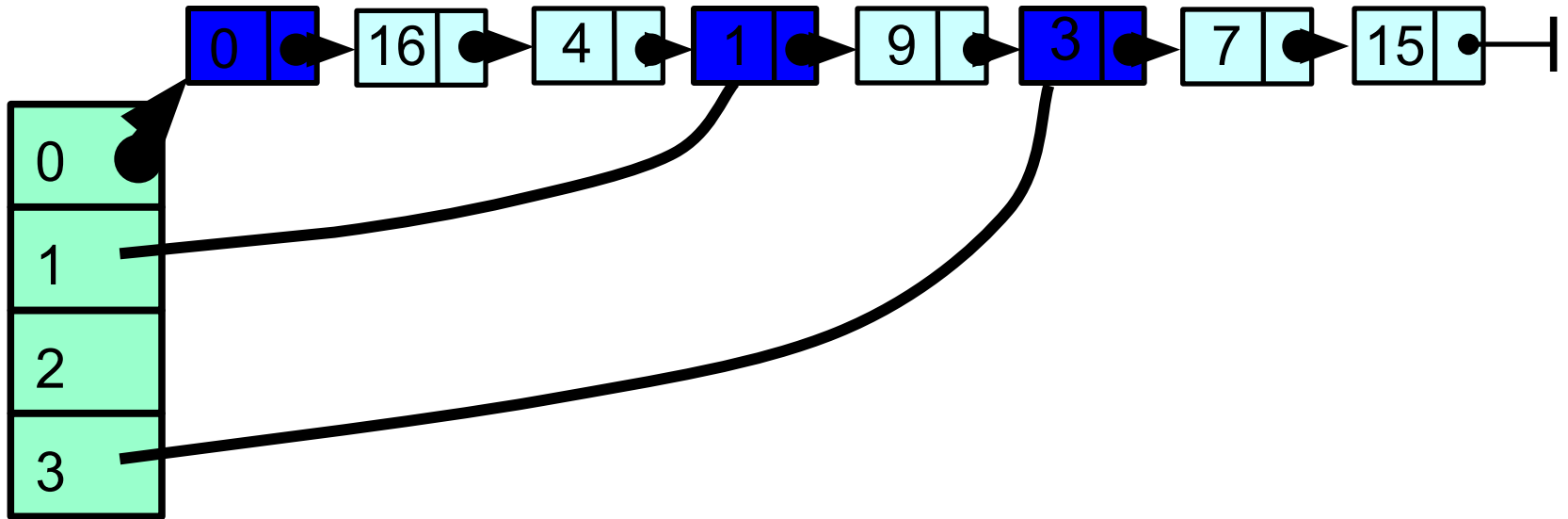


# Sentinel Nodes



**Problem: how to remove a node pointed by 2 sources using CAS**

# Sentinel Nodes



Solution: use a Sentinel node for each bucket

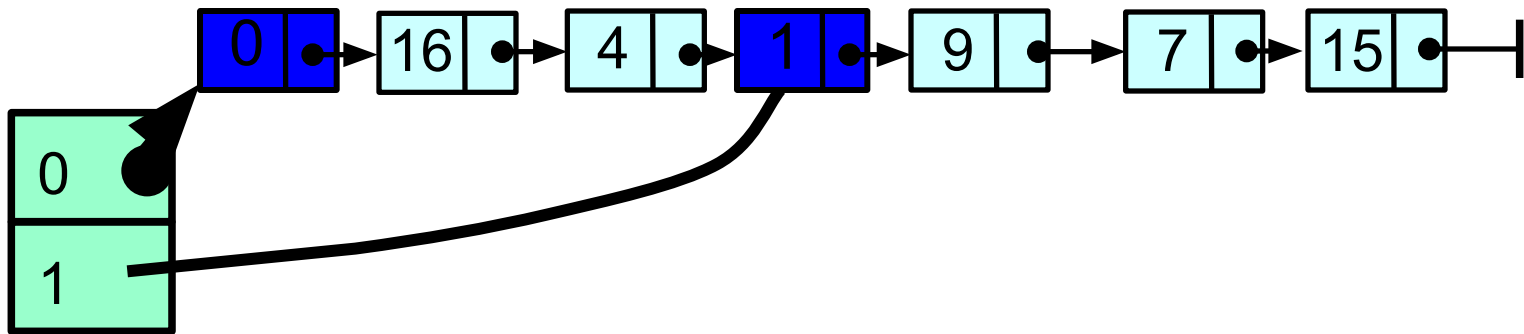
# Sentinel vs Regular Keys

- Want sentinel key for  $i$  ordered
  - before all keys that hash to bucket  $i$
  - after all keys that hash to bucket  $(i-1)$

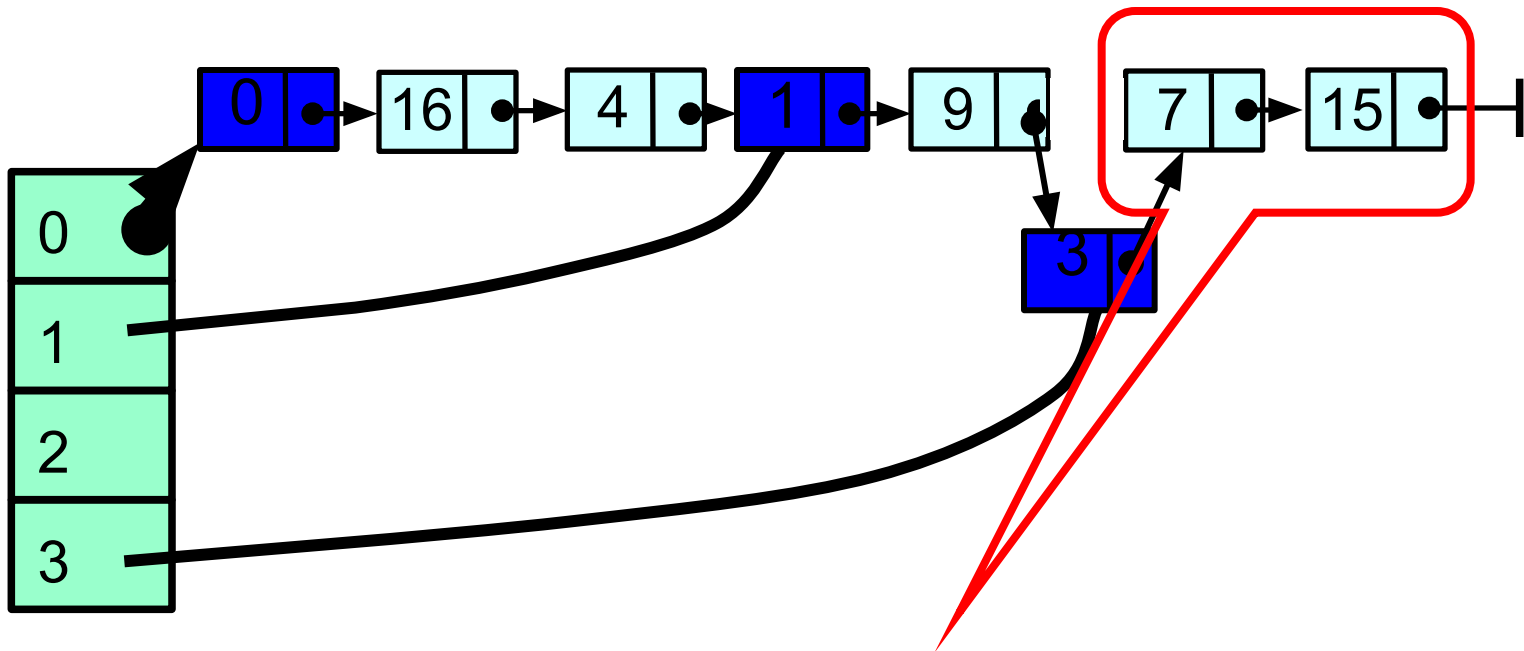
# Splitting a Bucket

- We can now split a bucket
- In a lock-free manner
- Using two `CAS()` calls ...
  - One to add the sentinel to the list
  - The other to point from the bucket to the sentinel

# Initialization of Buckets

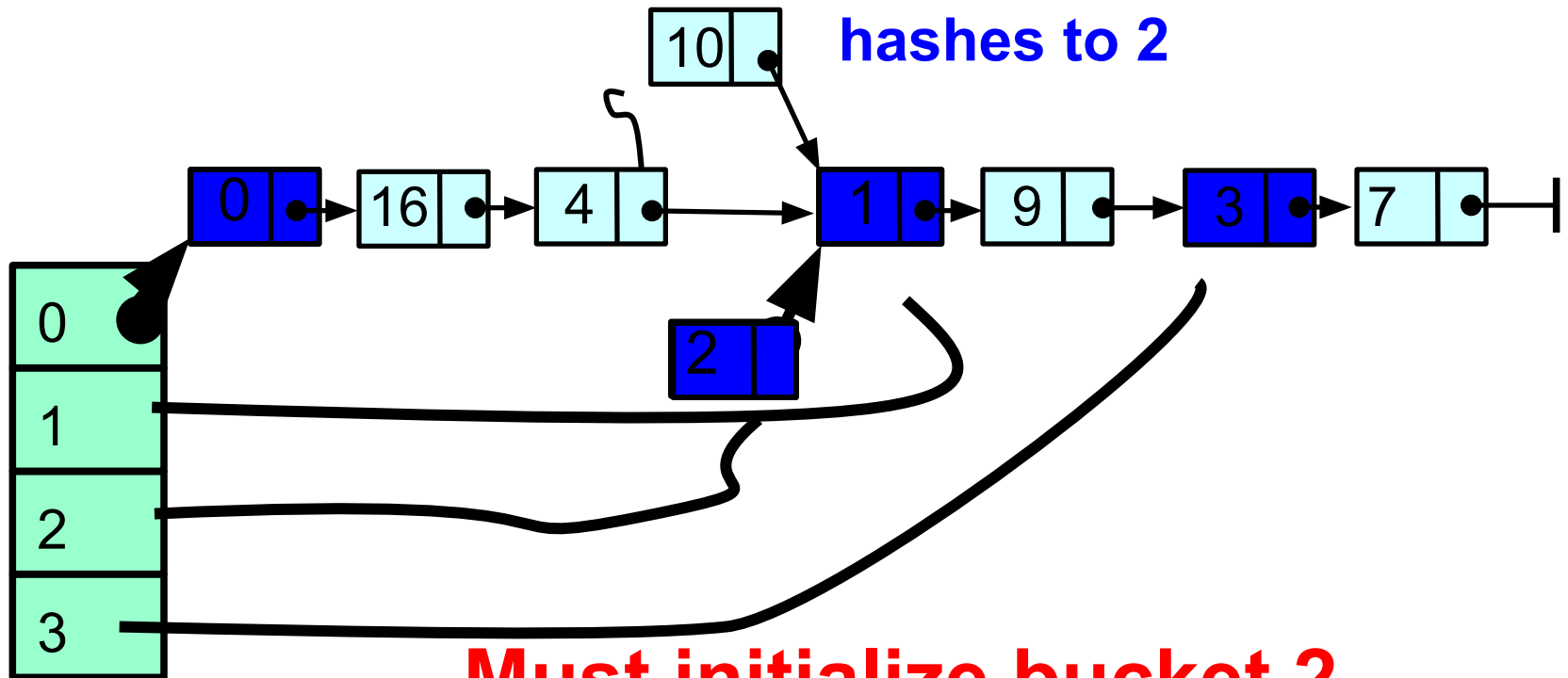


# Initialization of Buckets



**Need to initialize bucket 3 to split bucket 1**

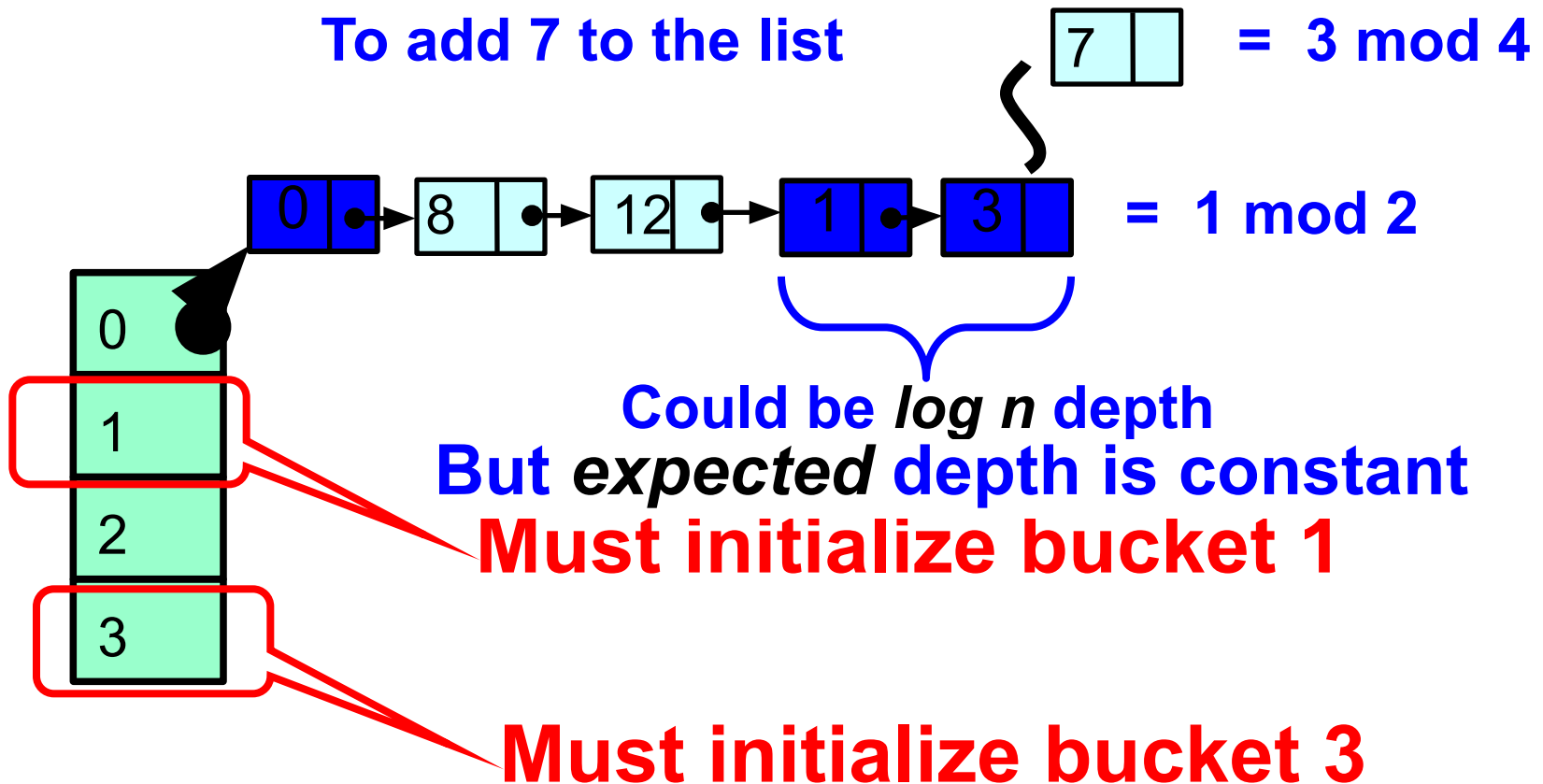
# Adding 10



**Must initialize bucket 2  
Before adding 10**



# Recursive Initialization



# Lock-Free List

```
int makeRegularKey(int key) {  
    return reverse(key | 0x80000000);  
}  
int makeSentinelKey(int key) {  
    return reverse(key);  
}
```

# Lock-Free List

```
int makeRegularKey(int key) {  
    return reverse(key | 0x80000000);  
}  
int makeSentinelKey(int key) {  
    return reverse(key);  
}
```

**Regular key: set high-order bit  
to 1 and reverse**

# Lock-Free List

```
int makeRegularKey(int key) {  
    return reverse(key | 0x80000000);  
}  
  
int makeSentinelKey(int key) {  
    return reverse(key);  
}
```

**Sentinel key: simply reverse  
(high-order bit is 0)**

# Main List

- Lock-Free List from earlier class
- With some minor variations

# Lock-Free List

```
public class LockFreeList {  
    public boolean add(Object object,  
                        int key) {...}  
    public boolean remove(int k) {...}  
    public boolean contains(int k) {...}  
    public  
        LockFreeList(LockFreeList parent,  
                      int key) {...};  
}
```

# Lock-Free List

```
public class LockFreeList {  
    public boolean add(Object object,  
                        int key) {...}  
    public boolean remove(int k) {...}  
    public boolean contains(int k) {...}  
    public  
        LockFreeList(LockFreeList parent,  
                      int key) {...};  
}
```

**Change: add takes key argument**

# Lock-Free List

**Inserts sentinel with key if not  
already present ...**

```
public class LockFreeList {  
    public boolean add(Object object,  
                        int key) {...}  
    public boolean remove(int k) {...}  
    public boolean contains(int k) {...}
```

```
    public  
        LockFreeList(LockFreeList parent,  
                      int key) {...};  
}
```



# Lock-Free List

**... returns new list starting with sentinel (shares with parent)**

```
public class LockFreeList {  
    ...  
    public boolean remove(int k) {...}  
    public boolean contains(int k) {...}
```

```
    public  
        LockFreeList(LockFreeList parent,  
                      int key) {...};  
}
```

# Split-Ordered Set: Fields

```
public class SOSet {  
    protected LockFreeList[] table;  
    protected AtomicInteger tableSize;  
    protected AtomicInteger setSize;  
  
    public SOSet(int capacity) {  
        table = new LockFreeList[capacity];  
        table[0] = new LockFreeList();  
        tableSize = new AtomicInteger(2);  
        setSize = new AtomicInteger(0);  
    }  
}
```

# Fields

```
public class S0Set {  
    protected LockFreeList[] table;  
    protected AtomicInteger tableSize;  
    protected AtomicInteger setSize;  
  
    public S0Set(int capacity) {  
        table = new LockFreeList[capacity];  
        table[0] = new LockFreeList();  
        tableSize = new AtomicInteger(0);  
        setSize = new AtomicInteger(0);  
    }  
}
```

**For simplicity treat table as big array ...**

# Fields

```
public class SOSet {  
    protected LockFreeList[] table;  
    protected AtomicInteger tableSize;  
    protected AtomicInteger setSize;  
  
    public SOSet(int capacity) {  
        table = new LockFreeList[capacity];  
        table[0] = new LockFreeList();  
        tableSize = new AtomicInteger(0);  
        setSize = new AtomicInteger(0);  
    }  
}
```

**In practice, want something that grows dynamically**

# Fields

```
public class SSet {  
    protected LockFreeList[] table;  
    protected AtomicInteger tableSize;  
    protected AtomicInteger setSize;  
  
    public SSet(int capacity) {  
        table = new LockFreeList[capacity];  
        table[0] = new LockFreeList();  
        tableSize = new AtomicInteger(0);  
        setSize = new AtomicInteger(0);  
    }  
}
```

**How much of table array are we actually using?**

# Fields

```
public class SSet {  
    protected LockFreeList[] table;  
    protected AtomicInteger tableSize;  
    protected AtomicInteger setSize;  
  
    public SSet(int capacity) {  
        table = new LockFreeList[capacity];  
        table[0] = new LockFreeList();  
        tableSize = new AtomicInteger(2);  
        setSize = new AtomicInteger(0);  
    }  
}
```

**Track set size  
so we know when to resize**

# Fields

**Initially use single bucket,  
and size is zero**

```
protected LockFreeList[] table;  
protected AtomicInteger tableSize;  
protected AtomicInteger setSize;
```

```
public S0Set(int capacity) {  
    table = new LockFreeList[capacity];  
    table[0] = new LockFreeList();  
    tableSize = new AtomicInteger(1);  
    setSize = new AtomicInteger(0);  
}
```

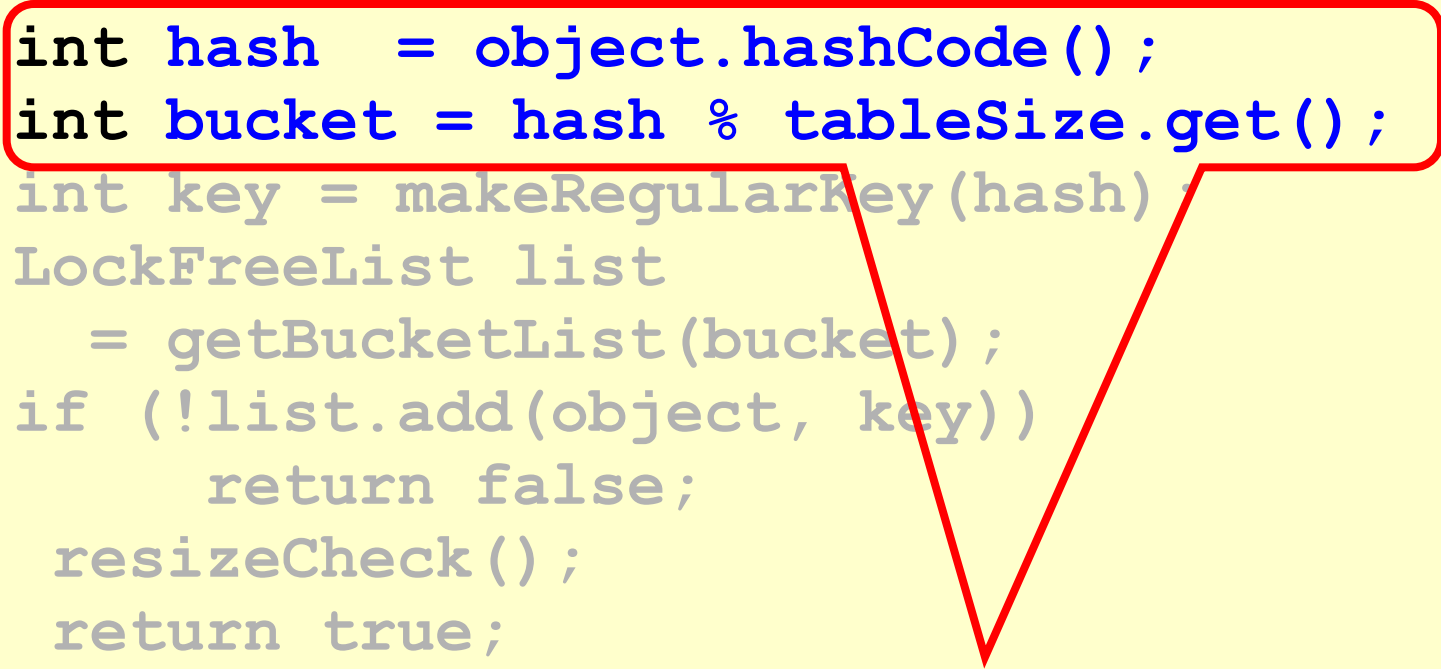
# add()

```
public boolean add(Object object) {  
    int hash = object.hashCode();  
    int bucket = hash % tableSize.get();  
    int key = makeRegularKey(hash);  
    LockFreeList list  
        = getBucketList(bucket);  
    if (!list.add(object, key))  
        return false;  
    resizeCheck();  
    return true;  
}
```



# add()

```
public boolean add(Object object) {  
    int hash = object.hashCode();  
    int bucket = hash % tableSize.get();  
    int key = makeRegularKey(hash);  
    LockFreeList list  
        = getBucketList(bucket);  
    if (!list.add(object, key))  
        return false;  
    resizeCheck();  
    return true;  
}
```



**Pick a bucket**

# add()

```
public boolean add(Object object) {  
    int hash = object.hashCode();  
    int bucket = hash % tableSize.get();  
    int key = makeRegularKey(hash);  
    LockFreeList list  
        = getBucketList(bucket);  
    if (!list.add(object, key))  
        return false;  
    resizeCheck();  
    return true;  
}
```

**Non-Sentinel  
split-ordered key**

# add()

```
public boolean add(Object object) {  
    int hash = object.hashCode();  
    int bucket = hash % tableSize.get();  
    int key = makeRegularKey(hash);  
    LockFreeList list  
        = getBucketList(bucket);  
    if (!list.add(object, key))  
        return false;  
    resizeCheck();  
    return true;  
}
```

**Get reference to bucket's  
sentinel, initializing if necessary**

# add()

**Call bucket's add() method with  
reversed key**

```
public boolean add(Object object) {  
    int hash = object.hashCode();  
    int bucket = bucketSize.get();  
    int key = makeRegularKey(hash);  
    LockFreeList list  
        = getBucketList(bucket);  
    if (!list.add(object, key))  
        return false;  
    resizeCheck();  
    return true;  
}
```

# add()

**No change? We're done.**

```
public boolean add(Object object) {  
    int hash = object.hashCode();  
    int bucket = hash % tableSize.get();  
    int key = makeRegularKey(hash);  
    LockFreeList list  
        = getBucketList(bucket);  
    if (!list.add(object, key))  
        return false;  
    resizeCheck();  
    return true;  
}
```

# add()

**Time to resize?**

```
public boolean add(Object object) {  
    int hash = object.hashCode();  
    int bucket = hash % tableSize.get();  
    int key = makeRegularKey(hash);  
    LockFreeList list  
        = getBucketList(bucket);  
    if (!list.add(object, key))  
        return false;  
    resizeCheck();  
    return true;  
}
```

# Resize

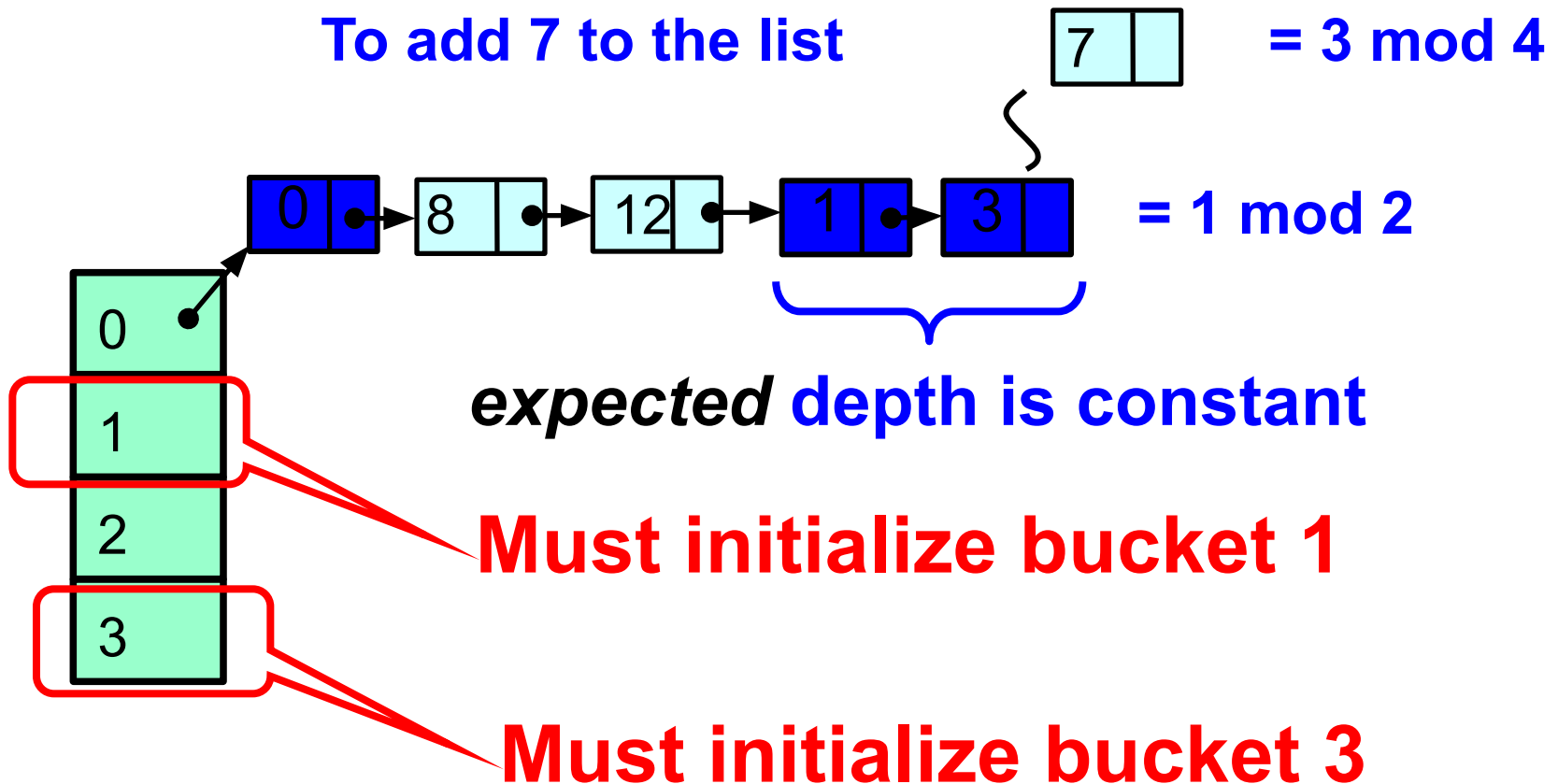
- Divide set size by total number of buckets
- If quotient exceeds threshold
  - Double **tableSize** field
  - Up to fixed limit

# Initialize Buckets

- Buckets originally null
- If you find one, initialize it
- Go to bucket's parent
  - Earlier nearby bucket
  - Recursively initialize if necessary
- Constant expected work



# Recall: Recursive Initialization



# Initialize Bucket

```
void initializeBucket(int bucket) {  
    int parent = getParent(bucket);  
    if (table[parent] == null)  
        initializeBucket(parent);  
    int key = makeSentinelKey(bucket);  
    LockFreeList list =  
        new LockFreeList(table[parent],  
                           key);  
}
```

# Initialize Bucket

```
void initializeBucket(int bucket) {  
    int parent = getParent(bucket);  
    if (table[parent] == null)  
        initializeBucket(parent);  
    int key = makeSentinelKey(bucket);  
    LockFreeList list =  
        new LockFreeList(table[parent],  
                           key);  
}
```

**Find parent, recursively  
initialize if needed**

# Initialize Bucket

```
void initializeBucket(int bucket) {  
    int parent = getParent(bucket);  
    if (table[parent] == null)  
        initializeBucket(parent);  
    int key = makeSentinelKey(bucket);  
    LockFreeList list =  
        new LockFreeList(table[parent],  
                           key);  
}
```

**Prepare key for new sentinel**

# Initialize Bucket

**Insert sentinel if not present, and get  
back reference to rest of list**

```
void initializeBucket(int bucket) {  
    int parent = getParent(bucket);  
    if (  
        initializeBucket(parent);  
        int key = makeSentinelKey(bucket);  
        LockFreeList list =  
            new LockFreeList(table[parent],  
                               key);  
    }  
}
```

# Correctness

- Linearizable concurrent set
- Theorem:  $O(1)$  expected time
  - No more than  $O(1)$  items expected between two sentinels on average
  - Lazy initialization causes at most  $O(1)$  expected recursion depth in `initializeBucket()`
- Can eliminate use of sentinels

# Closed (Chained) Hashing

- Advantages:
  - with  $N$  buckets,  $M$  items, Uniform  $h$
  - retains good performance as table density  $(M/N)$  increases  $\square$  less resizing
- Disadvantages:
  - dynamic memory allocation
  - bad cache behavior (no locality)

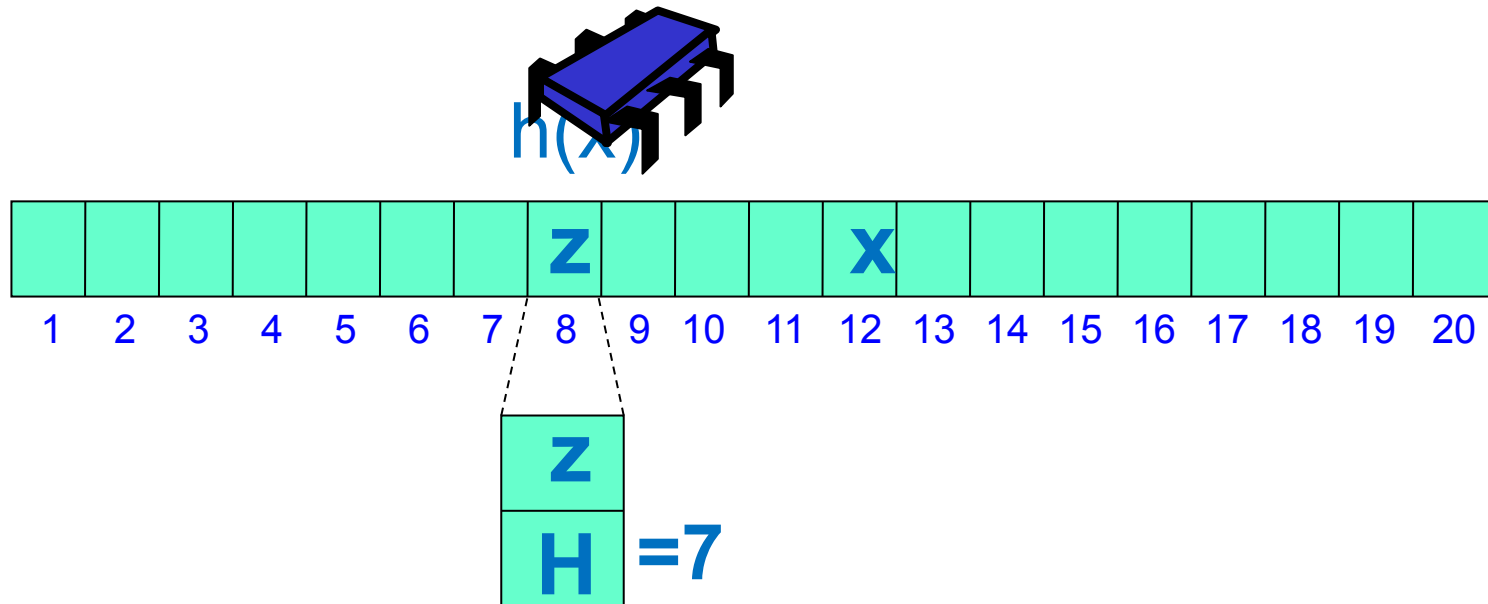
Oh, did we mention that cache behavior matters on a multicore?

# Open Addressed Hashing

- Keep all items in an array
- One per bucket
- If you have collisions, find an empty bucket and use it
- Must know how to find items if they are outside their bucket



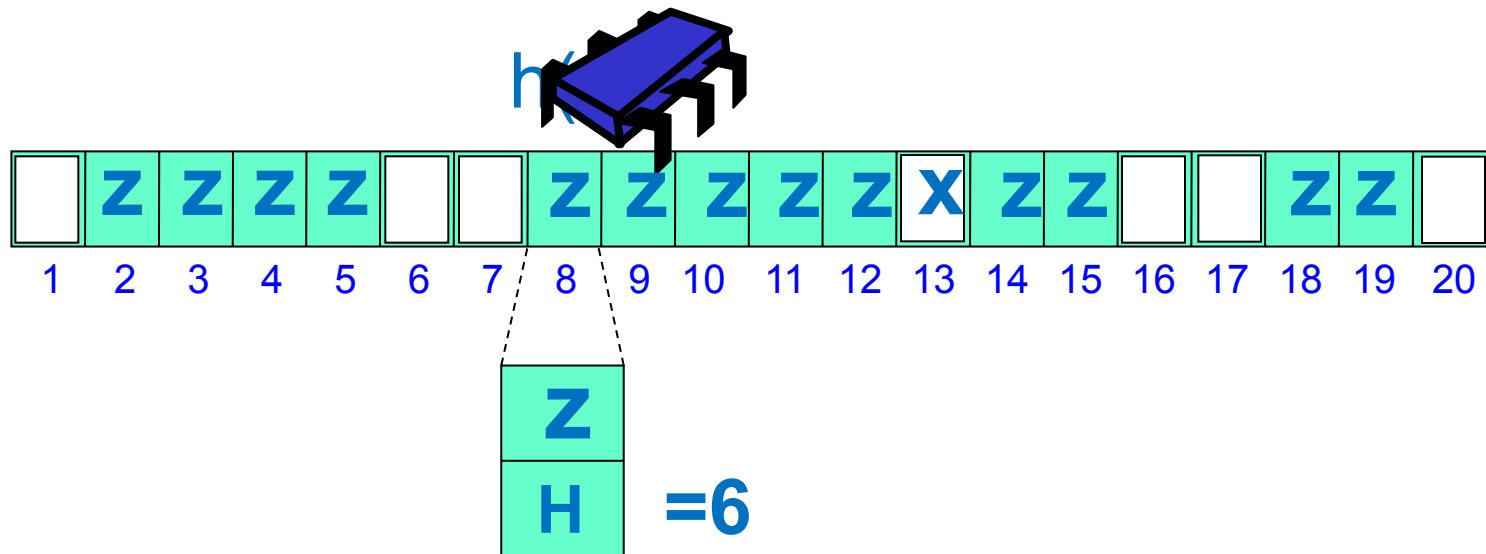
# Linear Probing\*



**contains (x)** – search linearly from  $h(x)$  to  $h(x) + H$  recorded in bucket.

\*Attributed to Amdahl...

# Linear Probing



**add( $x$ )** — put in first empty bucket, and update H.

# Linear Probing

- Open address means  $M \ll N$
- Expected items in bucket same as Chaining
- Expected distance till open slot:

$$\frac{1}{2} \left( 1 + \left( \frac{1}{1 - M/N} \right)^2 \right)$$

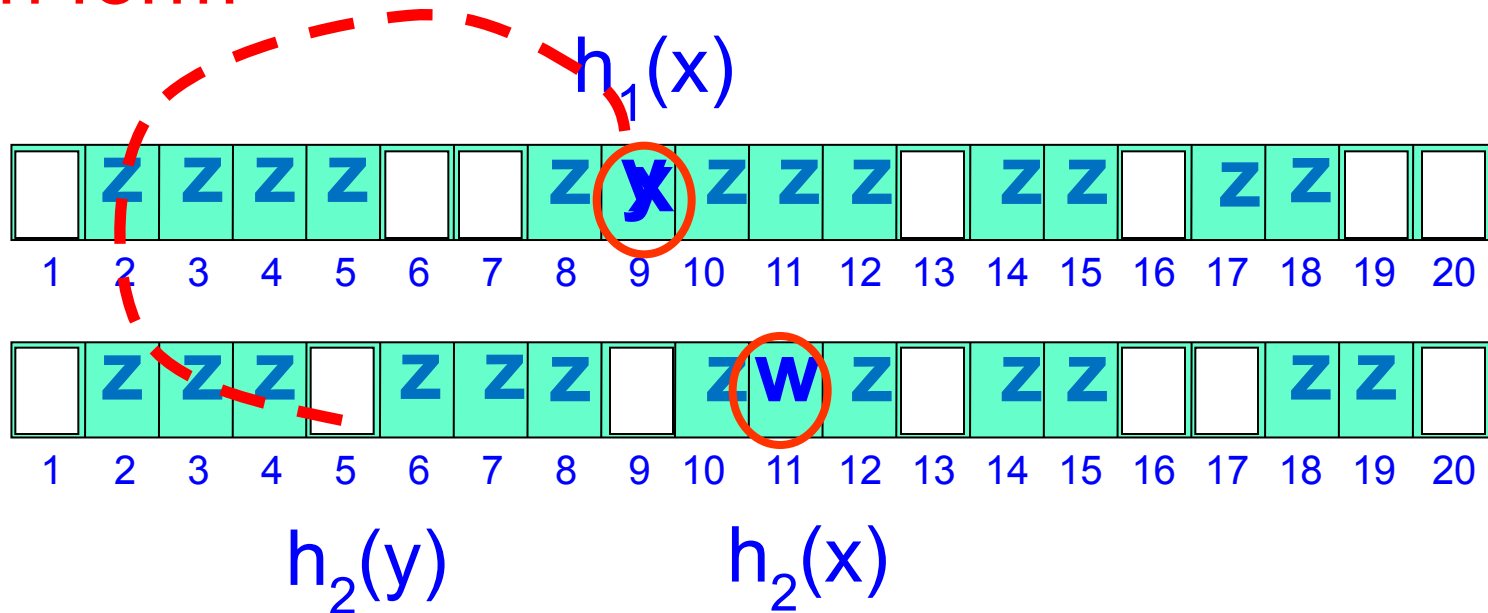
$M/N = 0.5$   $\square$  search 2.5 buckets

$M/N = 0.9$   $\square$  search 50 buckets

# Linear Probing

- Advantages:
  - Good locality  $\square$  fewer cache misses
- Disadvantages:
  - As  $M/N$  increases more cache misses
    - searching 10s of unrelated buckets
    - “Clustering” of keys into neighboring buckets
  - As computation proceeds “Contamination” by deleted items  $\square$  more cache misses

# But cycles Cuckoo Hashing can form



**add ( $\mathbf{x}$ )** — if  $h_1(x)$  and  $h_2(x)$  full evict  $y$  and move it to  $h_2(y) \neq h_2(x)$ . Then place  $x$  in its place.

# Cuckoo Hashing

- Advantages:
  - **contains (x)** : deterministic 2 buckets
  - No clustering or contamination
- Disadvantages:
  - 2 tables
  - $h_i(x)$  are complex
  - As  $M/N$  increases  $\square$  relocation cycles
  - Above  $M/N = 0.5$  Add() does not work!

# Concurrent Cuckoo Hashing

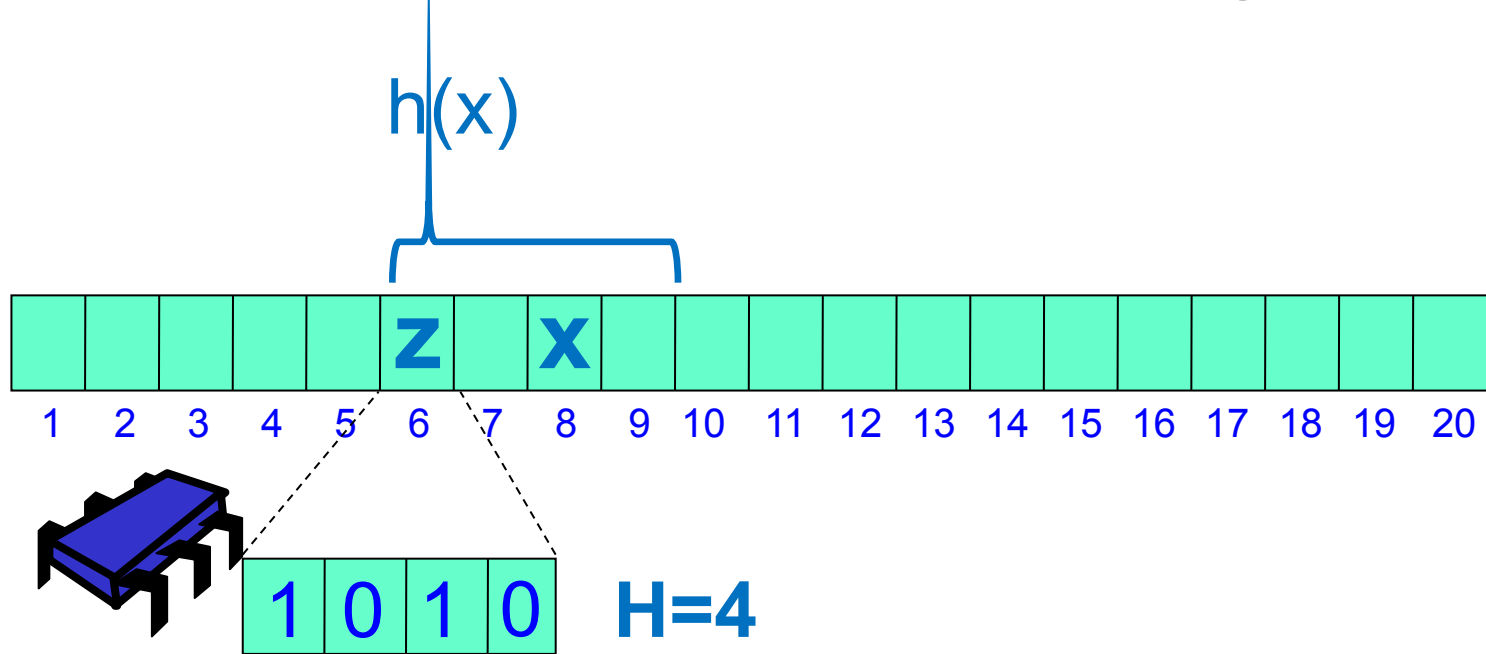
- Need to either lock whole chain of displacements (see book)
- or have extra space to keep items as they are displaced step by step.

# Hopscotch Hashing

- Single Array, Simple hash function
- Idea: define *neighborhood* of original bucket
- In neighborhood items found quickly
- Use sequences of displacements to move items into their neighborhood

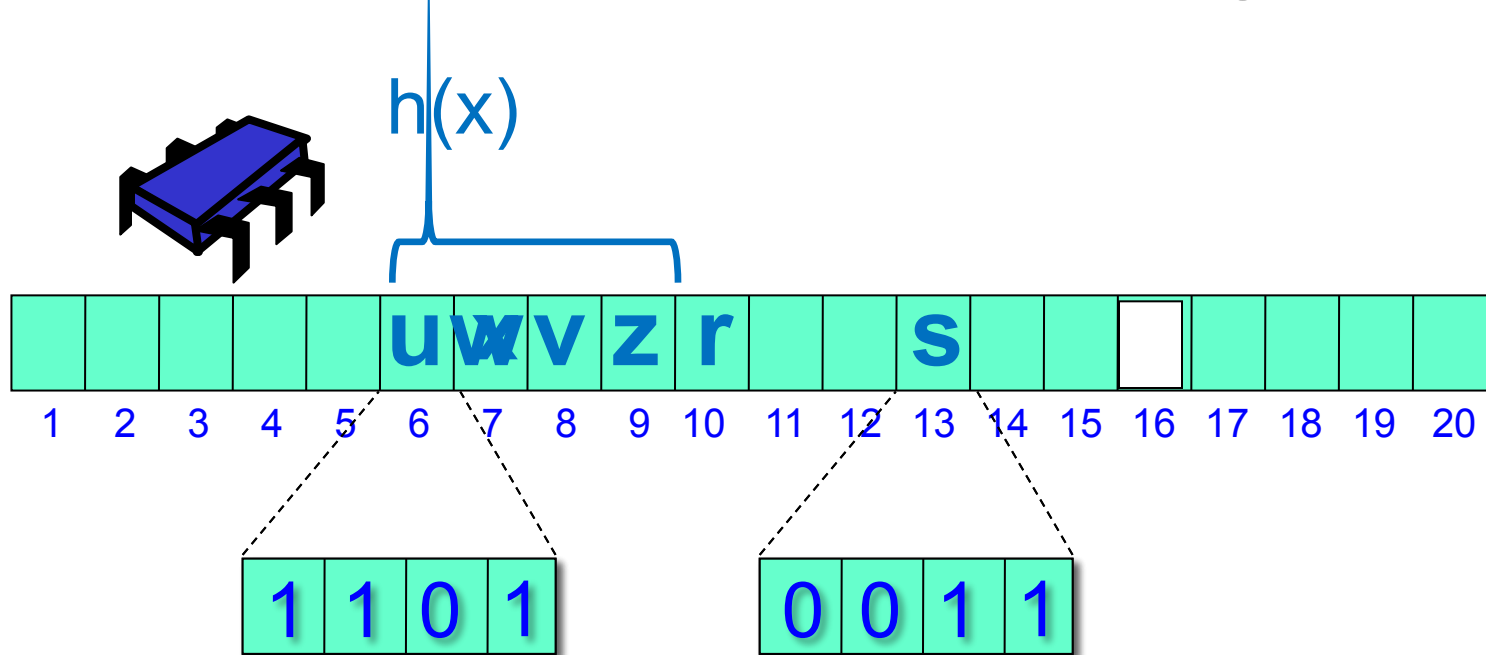


# Hopscotch Hashing



**contains (x)** – search in at most  $H$  buckets (the hop-range) based on hop-info bitmap. In practice pick  $H$  to be 32.

# Hopscotch Hashing



**add (x)** – probe linearly to find open slot.  
Move the empty slot via sequence of  
displacements into the *hop-range* of  $h(x)$ .

# Hopscotch Hashing

- *contains*
  - wait-free, just look in neighborhood

# Hopscotch Hashing

- *contains*
  - wait-free, just look in neighborhood
- *add*
  - expected distance same as in linear probing

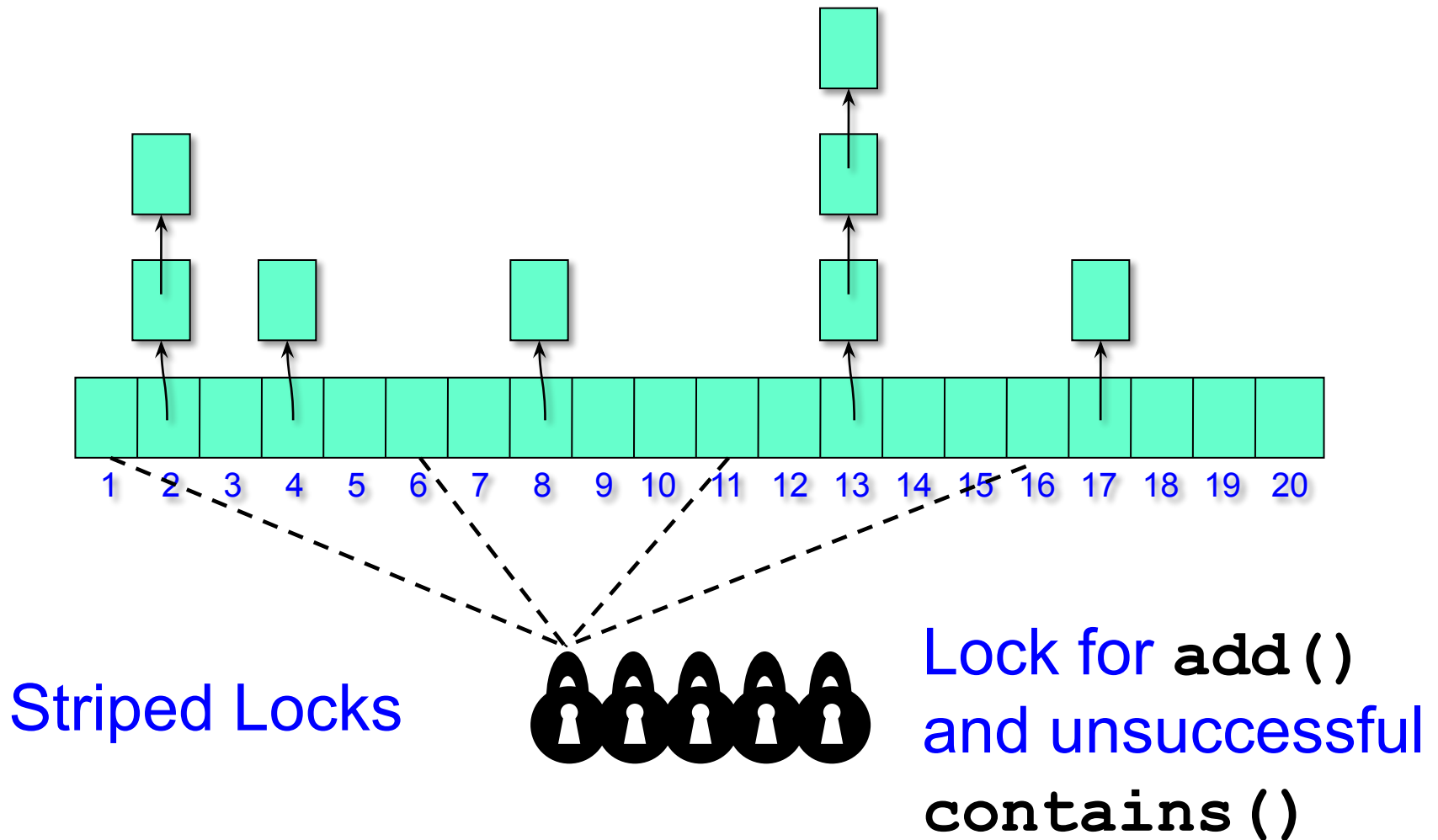
# Hopscotch Hashing

- *contains*
  - wait-free, just look in neighborhood
- *add*
  - Expected distance same as in linear probing
- *resize*
  - neighborhood full less likely as  $H \propto \log n$
  - one word **hop-info** bitmap, or use smaller  $H$  and default to linear probing of bucket

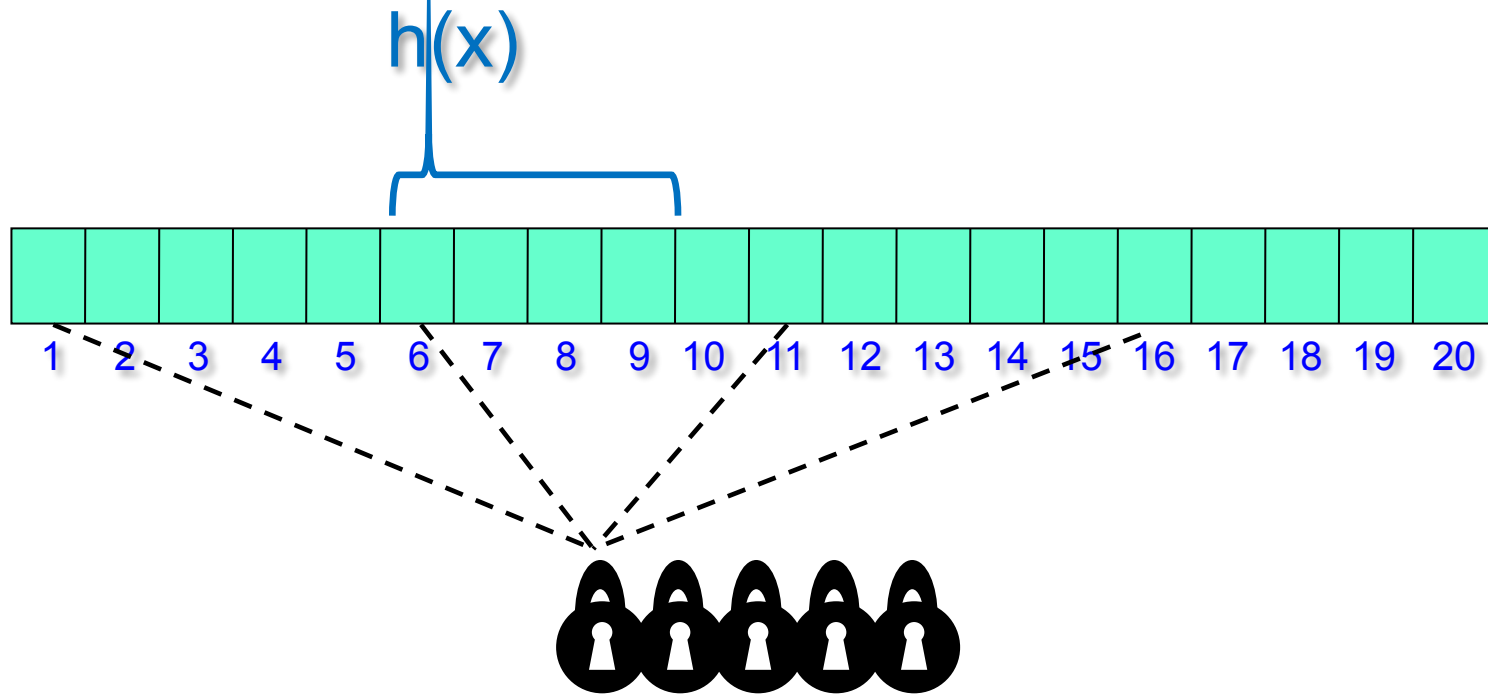
# Advantages

- Good locality and cache behavior
- As table density ( $M/N$ ) increases ☐ less resizing
- Move cost to **add ( )** from **contains (x)**
- Easy to parallelize

# Recall: Concurrent Chained Hashing



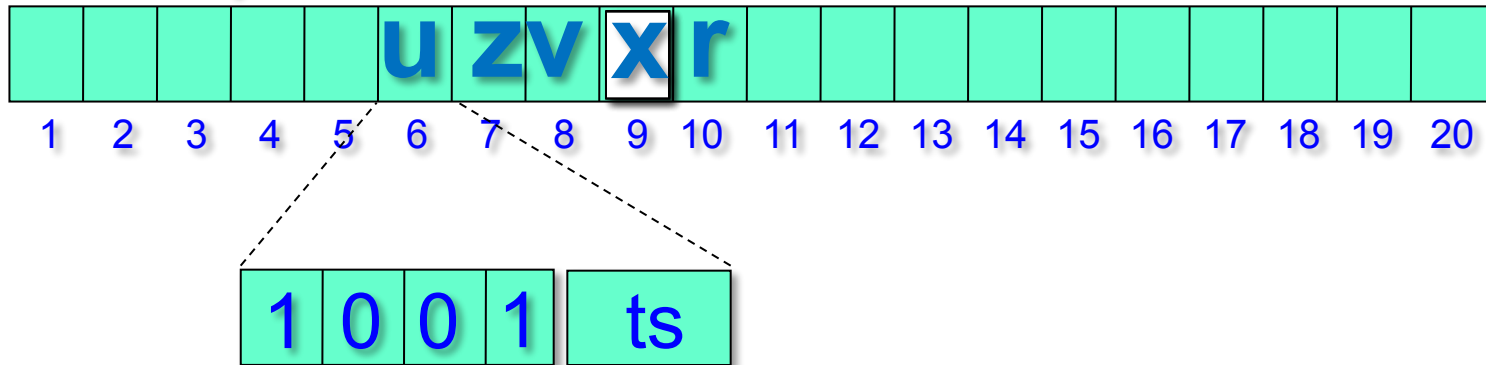
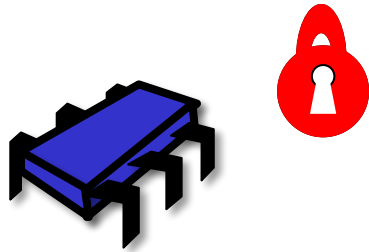
# Concurrent Simple Hopscotch



`contains()` is wait-free

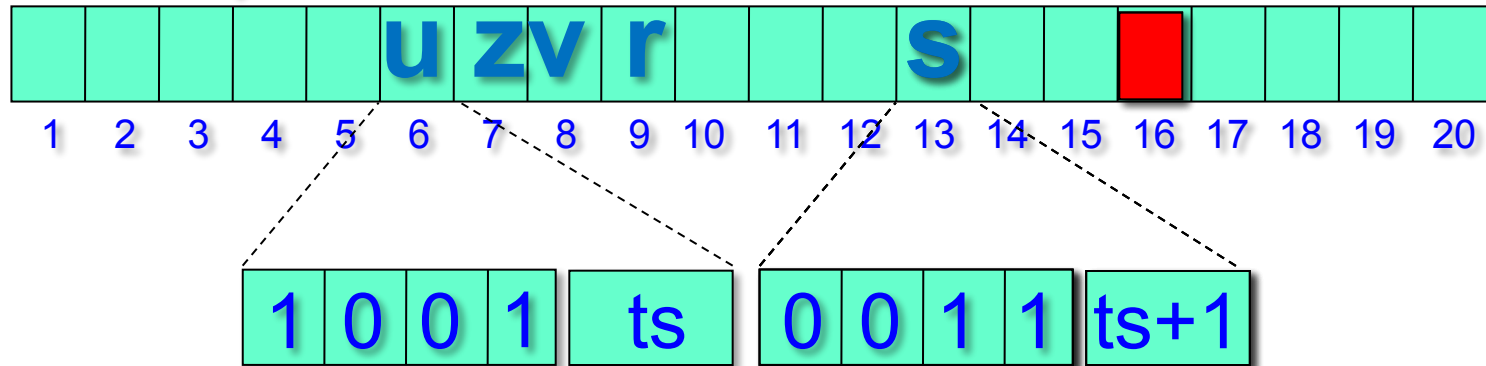
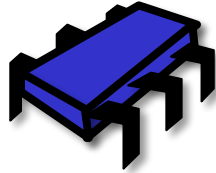


# Concurrent Simple Hopscotch



**add (x)** – lock bucket, mark empty slot using CAS, add **x** erasing mark

# Concurrent Simple Hopscotch

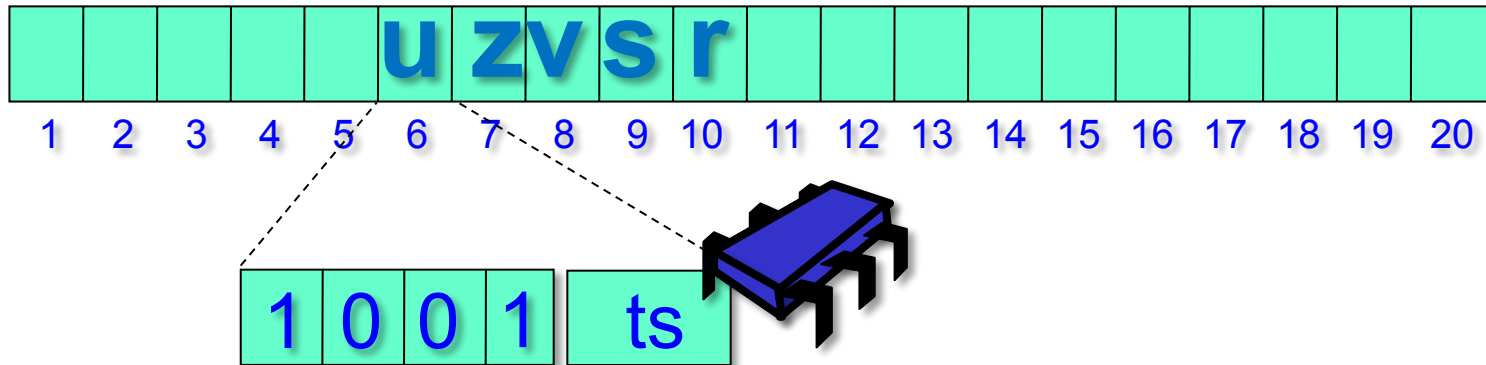


**add (x)** – lock bucket, mark empty slot using CAS, lock bucket and update timestamp of bucket being displaced before erasing old value

# Concurrent Simple Hopscotch



x not found



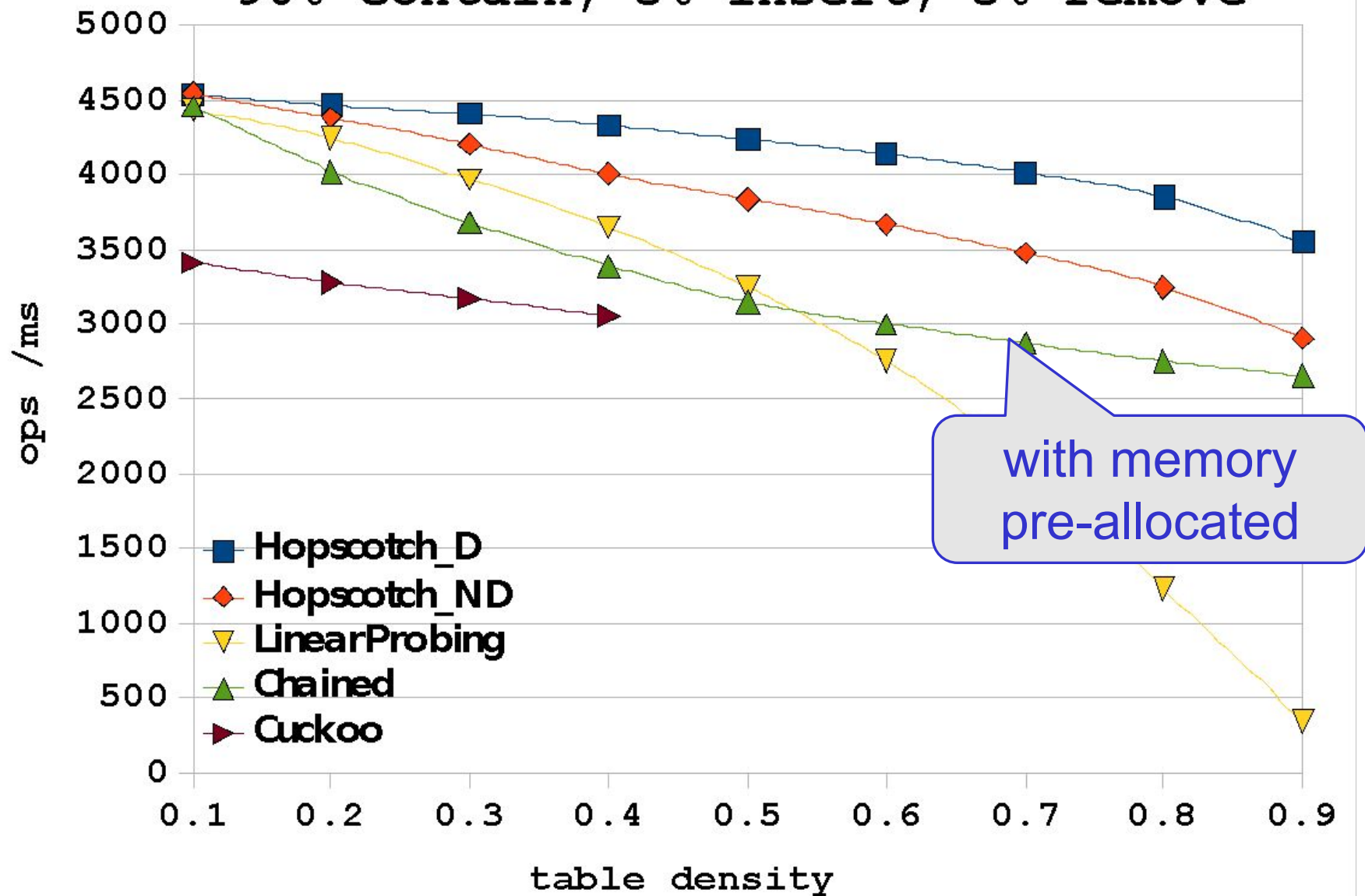
**Contains (x)** – traverse using bitmap  
and if ts has not changed after traversal  
item not found. If ts changed, after a few  
tries traverse through all items.

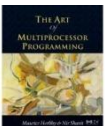
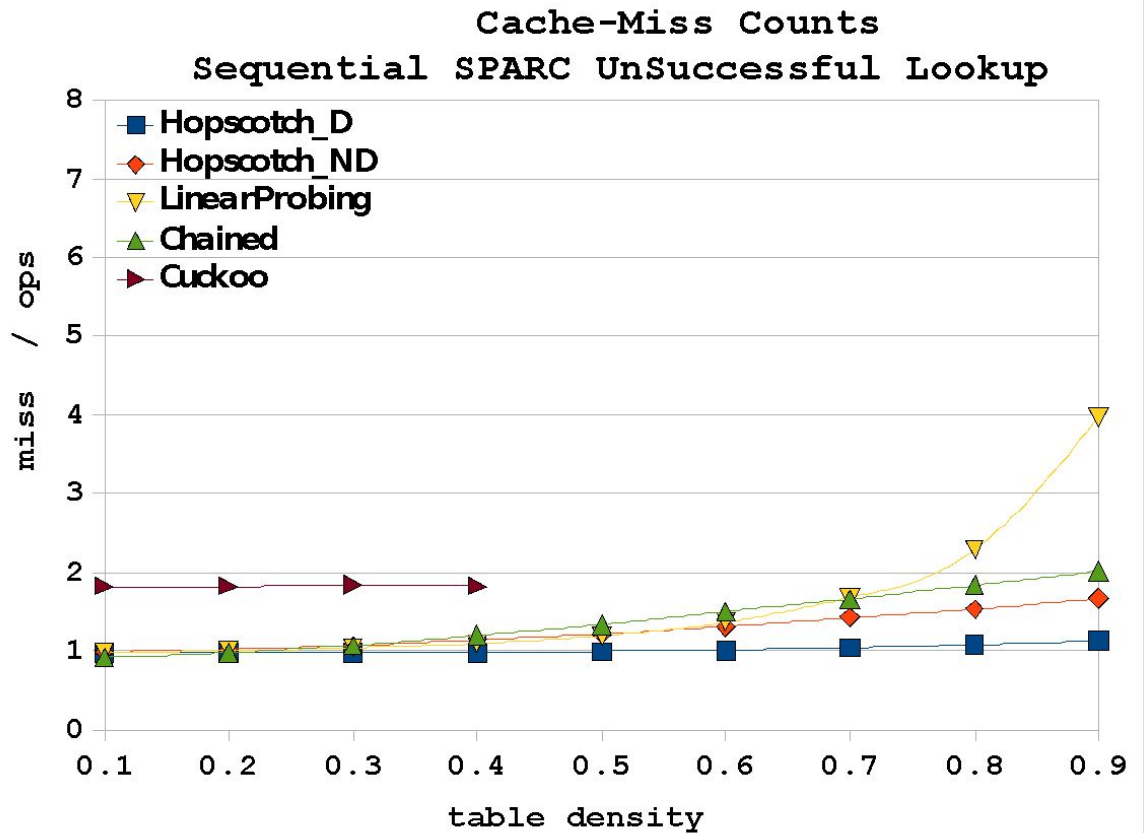
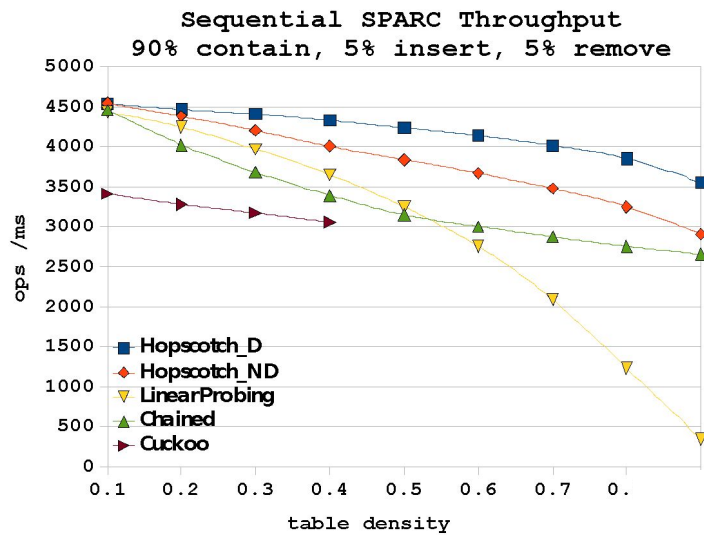
# Is performance dominated by cache behavior?

- Test on multicores and uniprocessors:
  - Sun 64 way Niagara II, and
  - Intel 3GHz Xeon
- Benchmarks pre-allocated memory to eliminate effects of memory management

# Sequential SPARC Throughput

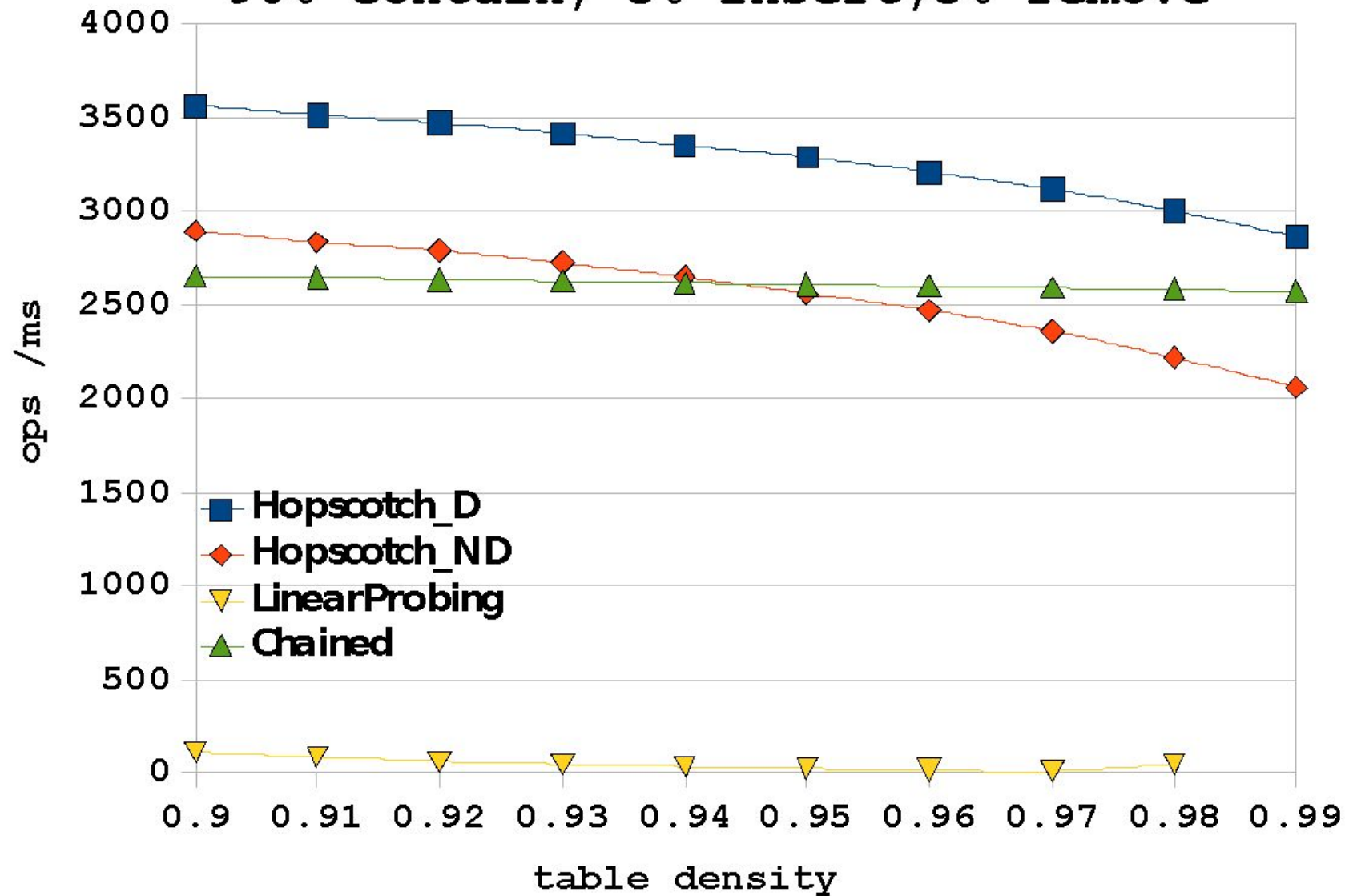
90% contain, 5% insert, 5% remove



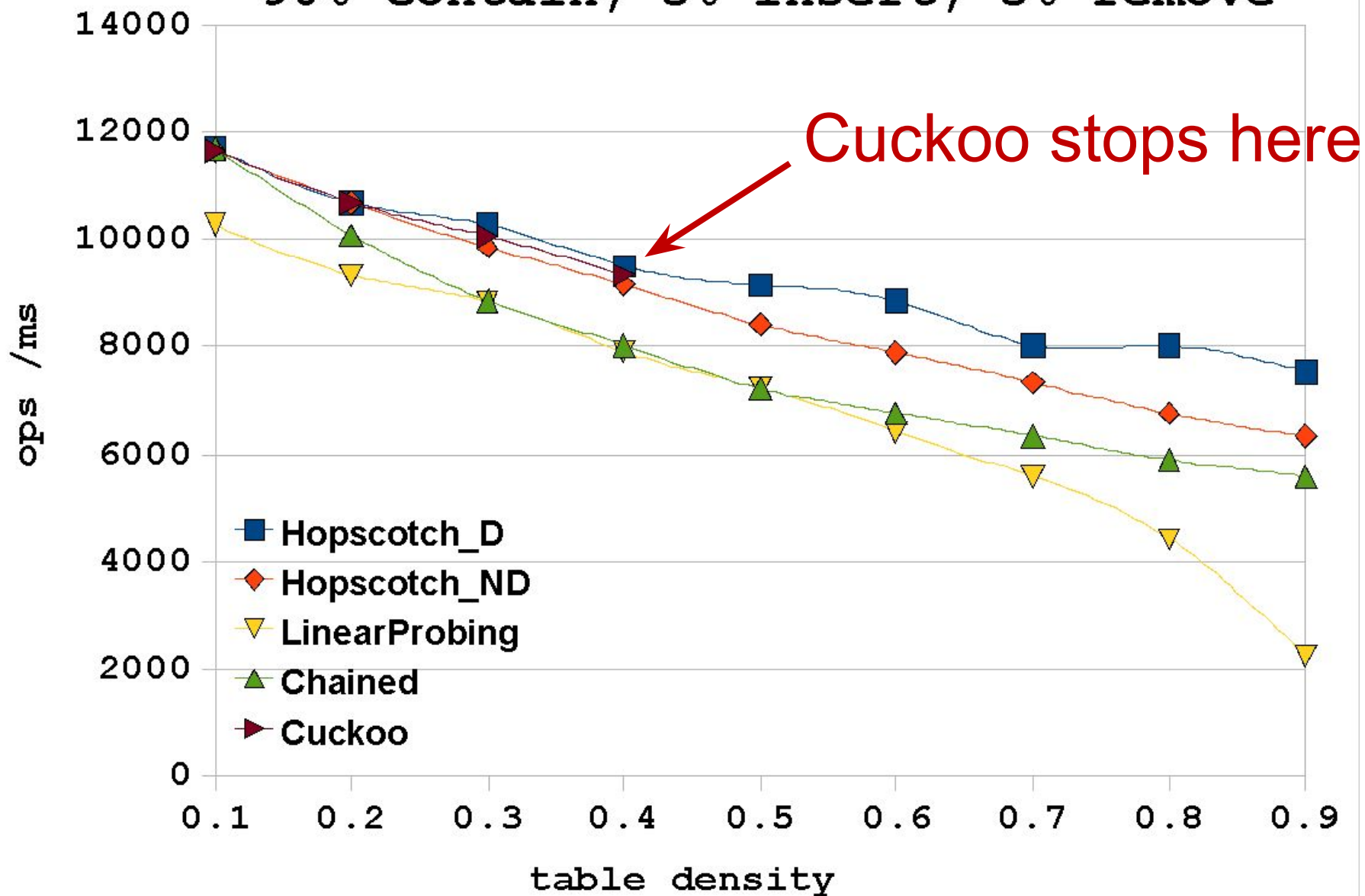


# Sequential SPARC High-Density;Throuthput

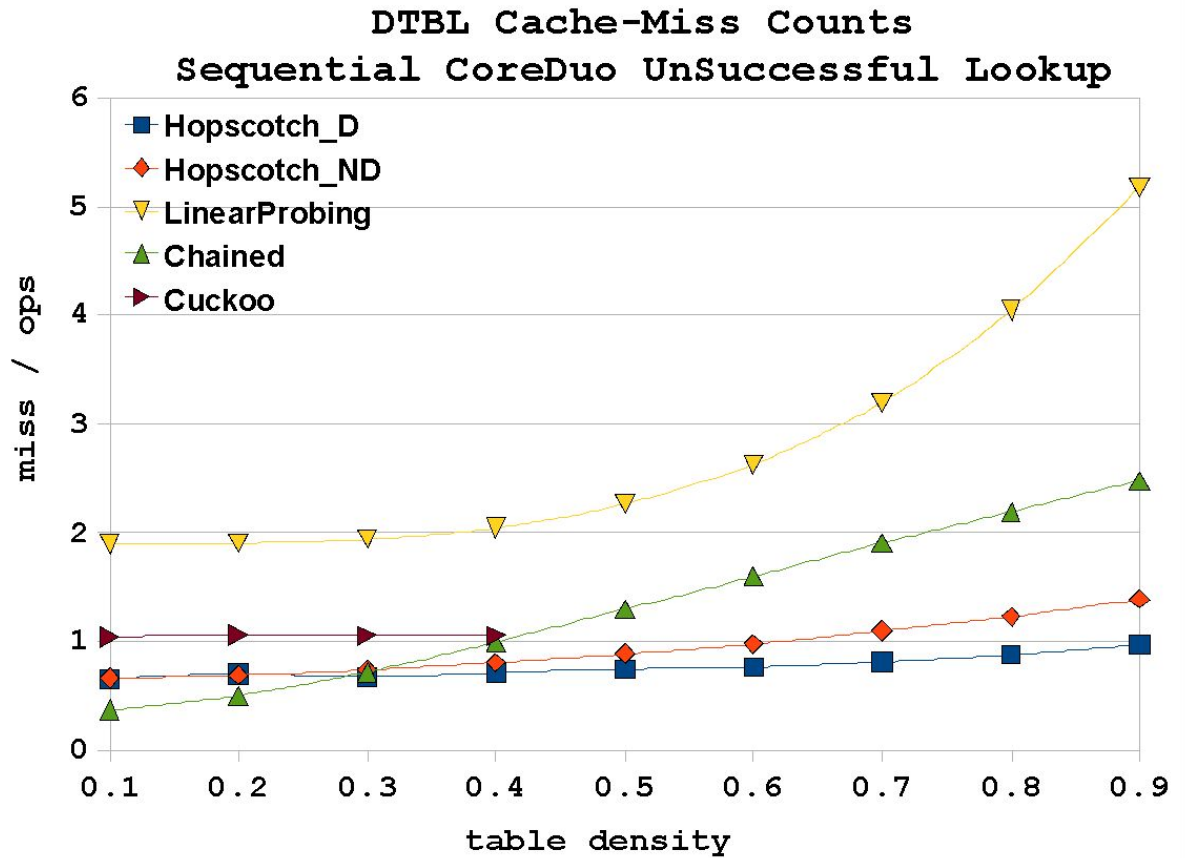
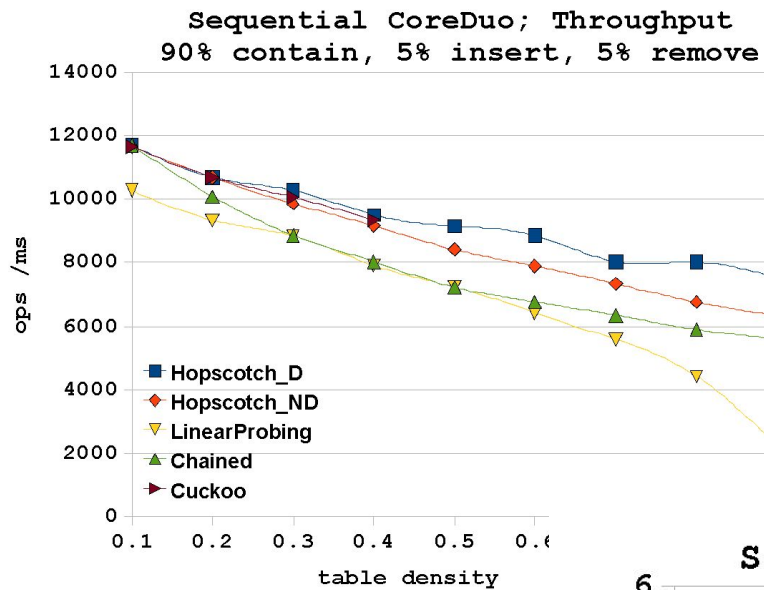
90% contain, 5% insert,5% remove



Sequential CoreDuo; Throughput  
90% contain, 5% insert, 5% remove

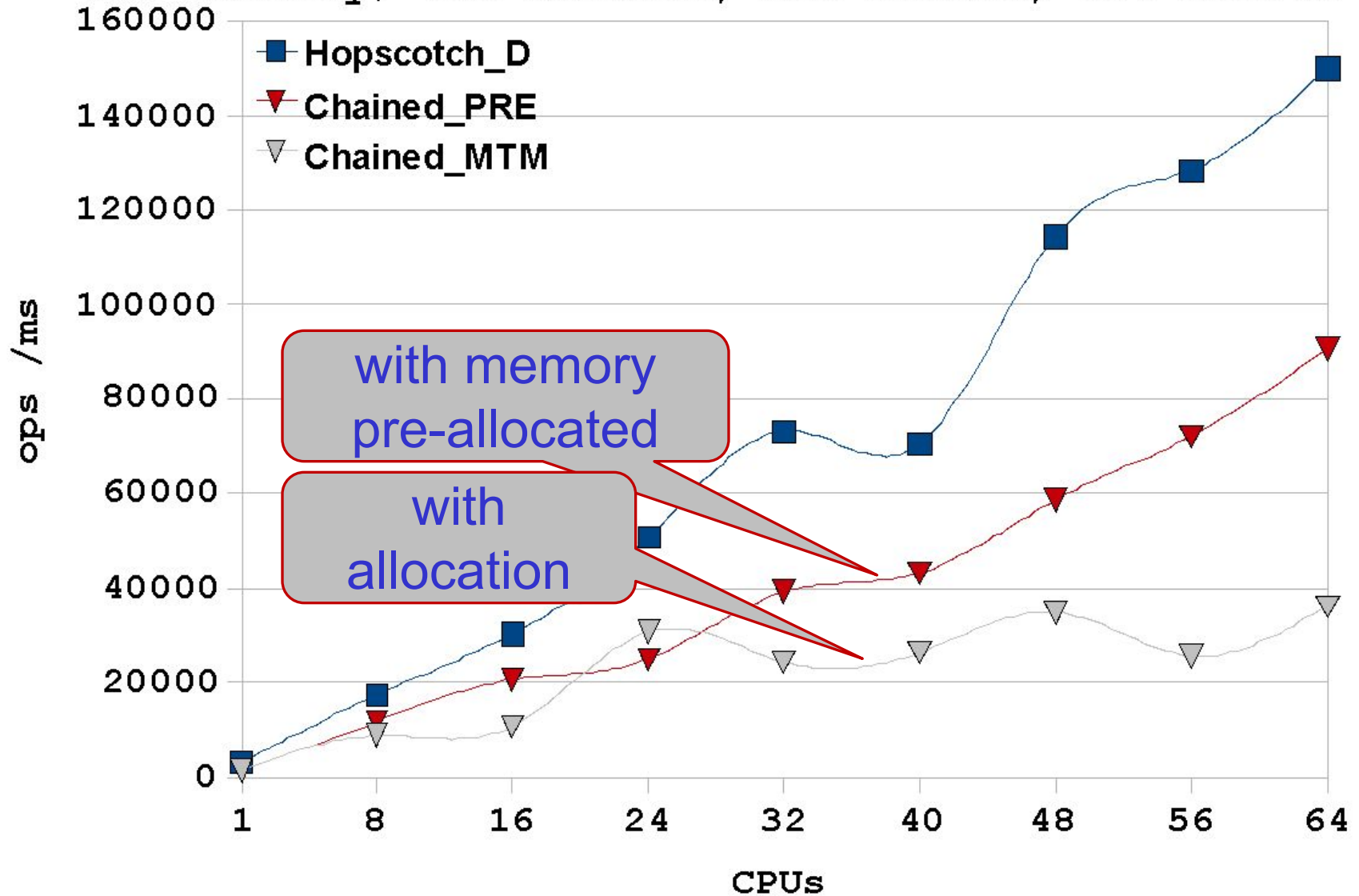






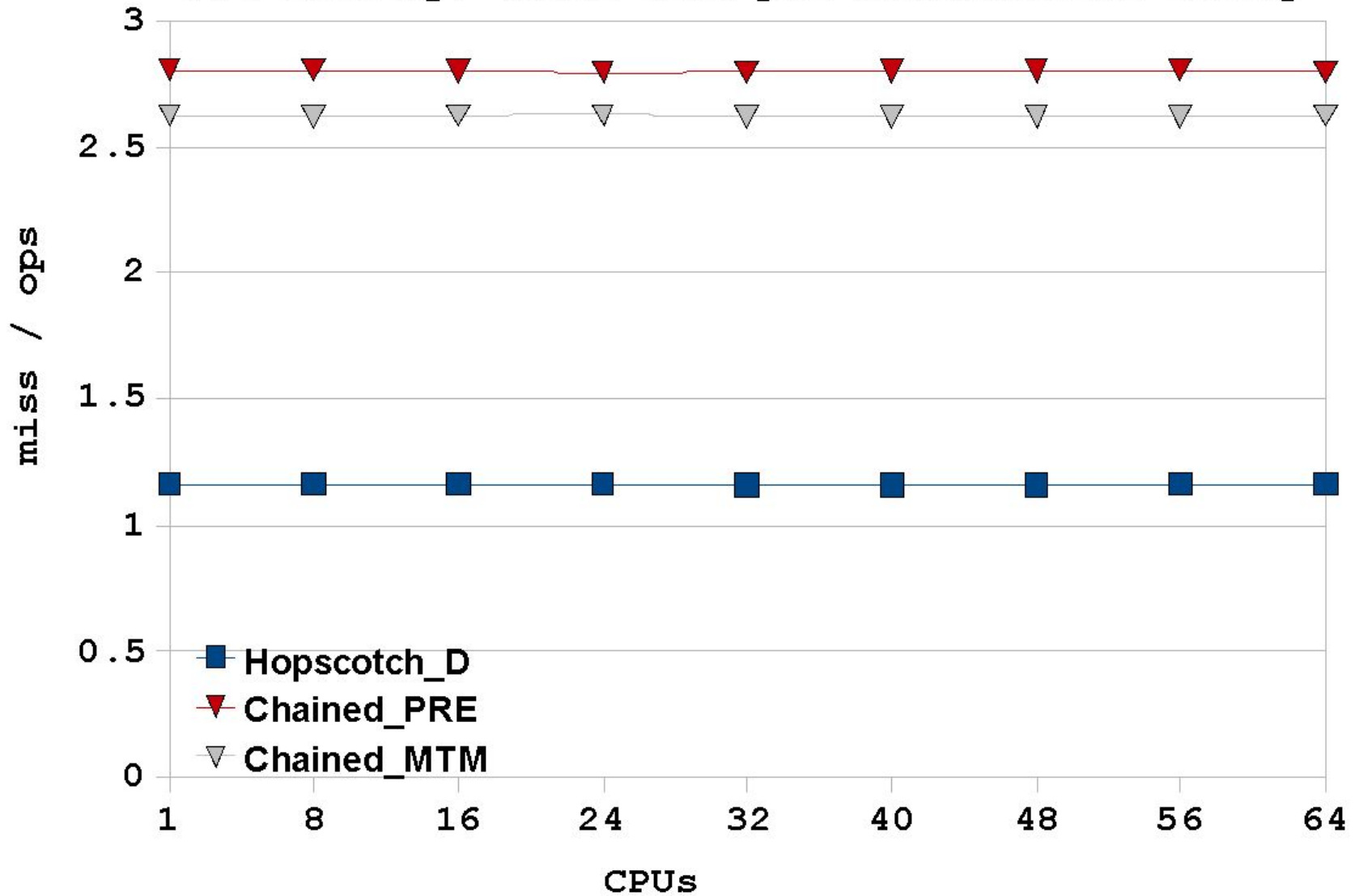
# Concurrent SPARC Throughput

90% density; 70% contain, 15% insert, 15% remove



# Concurrent SPARC Throughput

90% density; Cache-Miss per UnSuccessful-Lookup



# Summary

- *Chained hash* with striped locking is simple and effective in many cases
- *Hopscotch* with striped locking great cache behavior
- If incremental resizing needed go for *split-ordered*

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