Combinatorial topological framework for nonlinear dynamics - part 3

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Lectures

Combinatorial topological framework - Theory

Combinatorial topological framework - ODEs

Combinatorial topological framework - Maps

Combinatorial topological framework - Data

Combinatorial Methods for Dynamics of Maps

The dynamical system

$$f : X \to X \qquad \text{(continuous)}$$

The objects of interest

A set $S \subset X$ is invariant if $f(S) = S$.

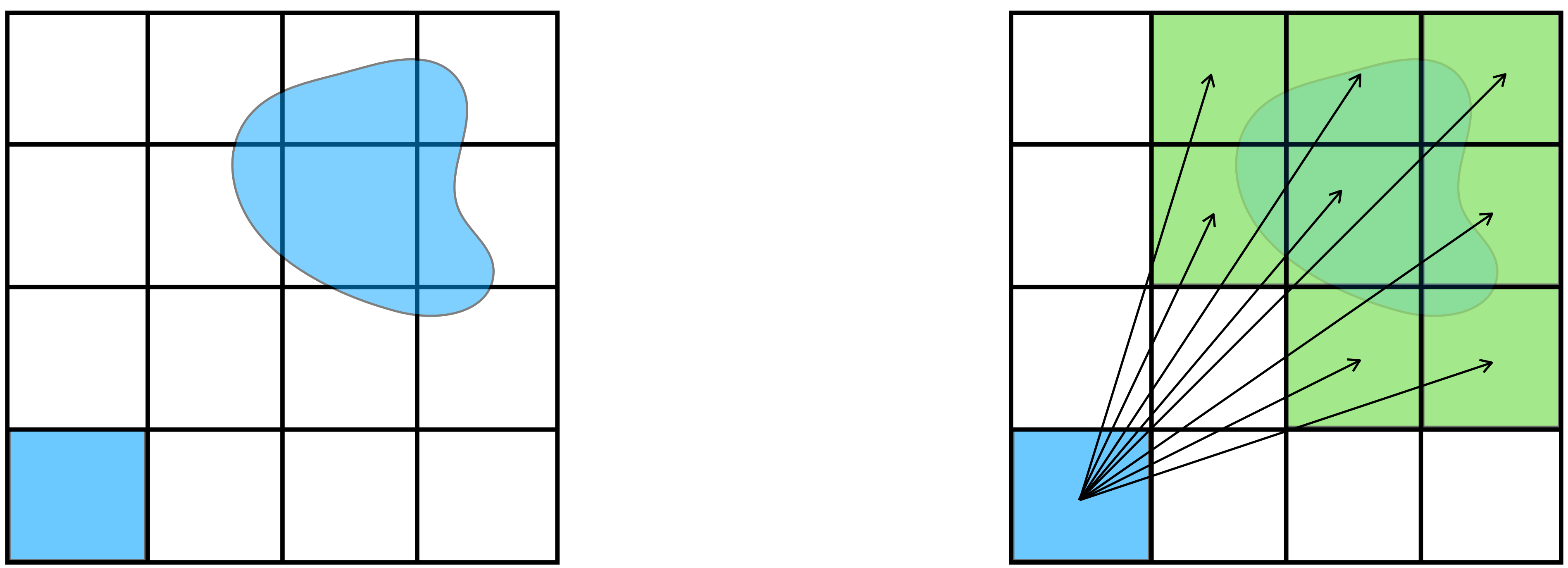
Goal: Understand the structure and connections of invariant sets

Tools: Combinatorial methods and Algebraic Topology

$\mathcal{X}$ grid decomposition of X

$\mathcal{F}\colon \mathcal{X} \rightrightarrows \mathcal{X}$ combinatorial multivalued map

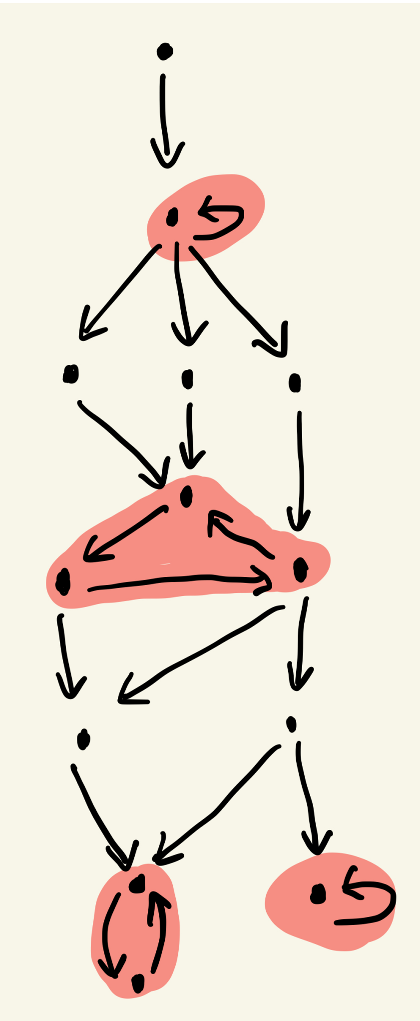
$$\xi \in \mathcal{X} \mapsto \mathcal{F}(\xi) \subset \mathcal{X}$$



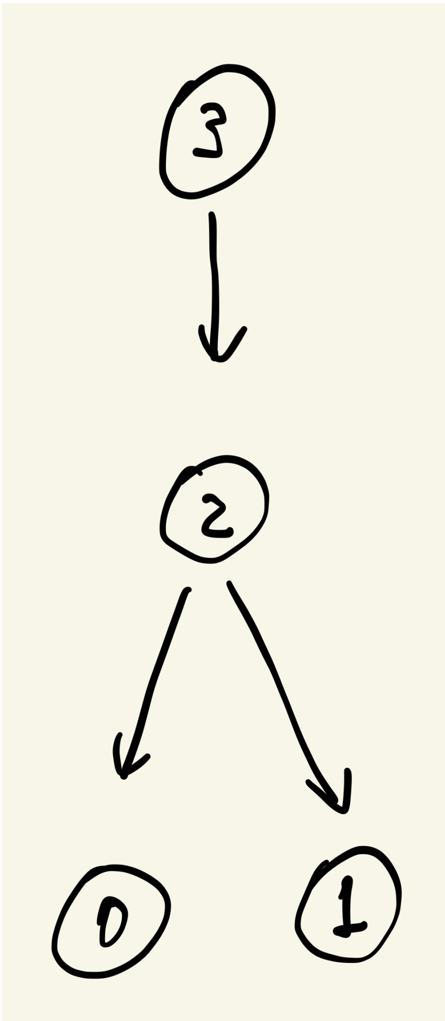
$f(\xi)$ $\qquad$ $\mathcal{F}(\xi)$

$\mathcal{F}$ can be interpreted as a directed graph (digraph)

The nontrivial strongly connected components (SCC) capture the recurrent dynamics



SCC



Morse Graph

Linear time algorithm to compute SCC

Vertices: Morse sets (Recurrent Dynamics)

Edges: Non-recurrent (gradient-like) dynamics

Use a memory efficient algorithm (by Shaun Harker)

CMGDB scheme

X rectangular region in $\mathbb{R}^n$

Adaptive grid using a (init, min, max, limit) scheme:

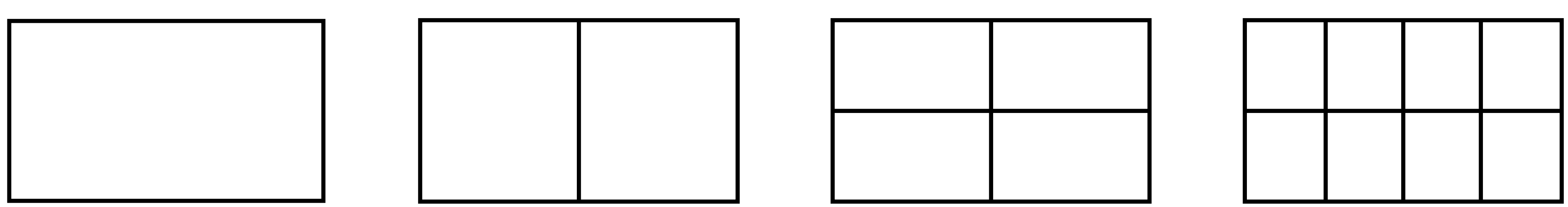
init - initial number of subdivisions

min - min number of subdivisions

max - max number of subdivisions

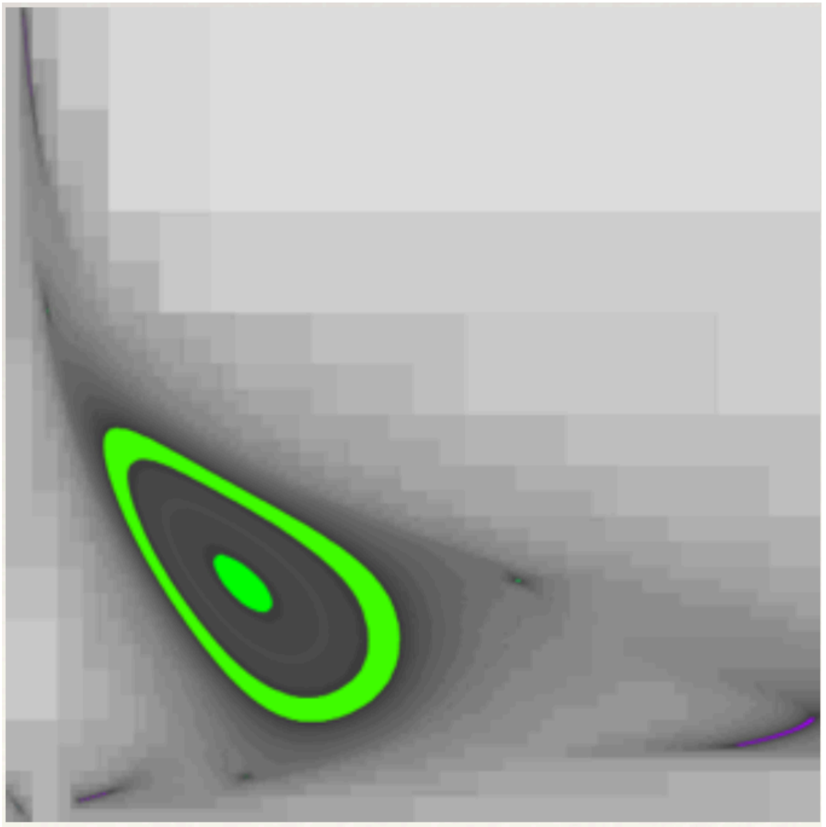
limit - max number of boxes to subdivide component
after min # of subdivisions

Initial grid (init number of subdivisions):



0 subdiv | 1 subdiv | 2 subdiv | 3 subdiv

CMGDB scheme



Compute $\mathcal{F}$ and SCC decomposition

Subdivide recurrent components

Repeat until at least the min # of subdivisions is reached

Subdivide at most the max # of subdivisions

Stop if reach max # of grid boxes after min subdivisions

Conley Morse Graph Database (CMGDB)

https://github.com/marciogameiro/CMGDB

Examples

```
import CMGDB

from interval import interval, imath

import matplotlib
import math
import time
```

```
# Define Leslie map
def f(x):
    th1 = 19.6
    th2 = 23.68
    return [(th1 * x[0] + th2 * x[1]) * math.exp (-0.1 * (x[0] + x[1])), 0.7 * x[0]]

# Define box map for f
def F(rect):
    return CMGDB.BoxMap(f, rect, padding=True)
```

```
subdiv_min = 20
subdiv_max = 30
lower_bounds = [-0.001, -0.001]
upper_bounds = [90.0, 70.0]

model = CMGDB.Model(subdiv_min, subdiv_max, lower_bounds, upper_bounds, F)
```

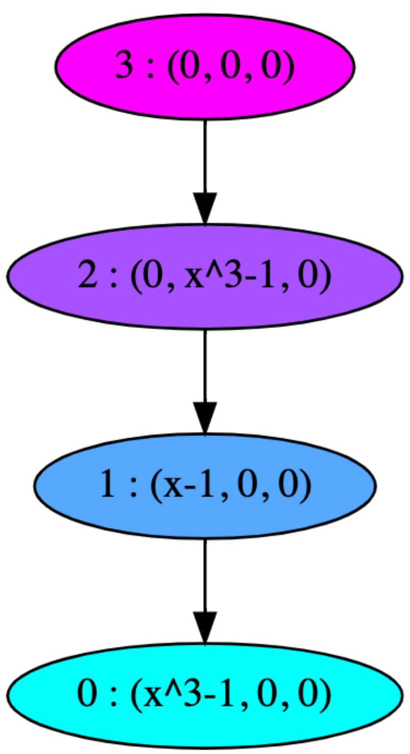
Examples

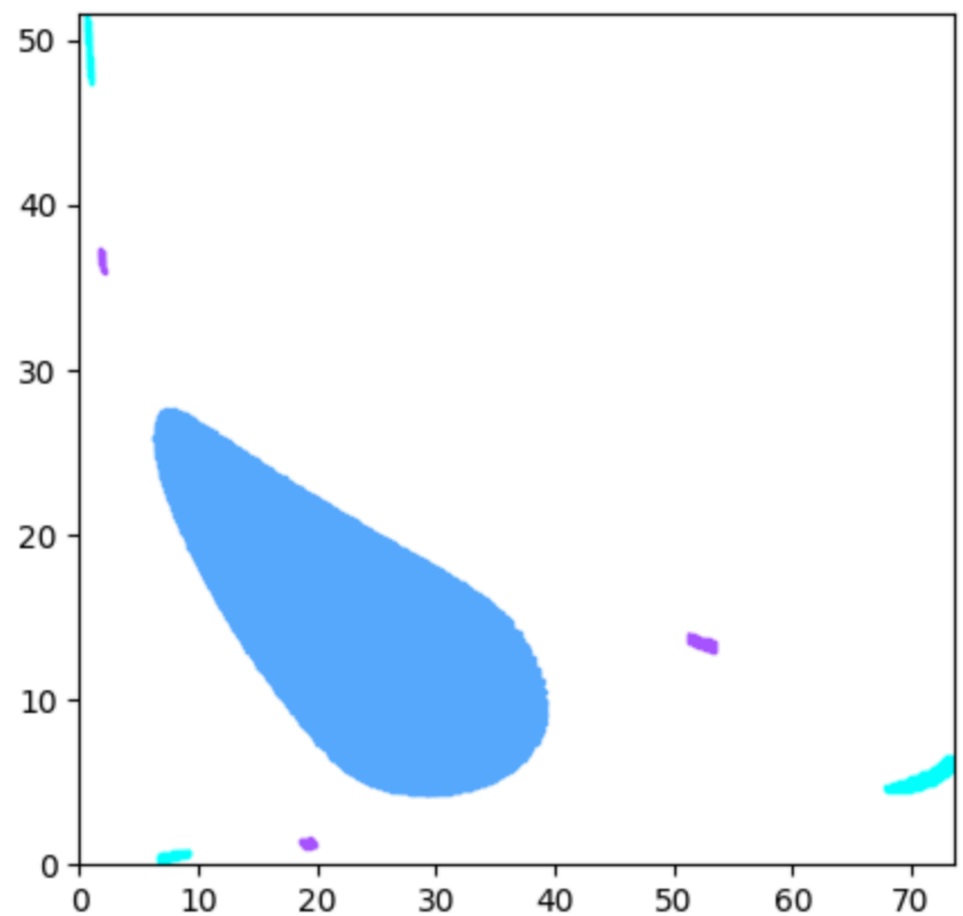
```
%%time
morse_graph, map_graph = CMGDB.ComputeConleyMorseGraph(model)
```

```
CPU times: user 17.4 s, sys: 353 ms, total: 17.8 s
Wall time: 18.6 s
```

```
CMGDB.PlotMorseGraph(morse_graph, cmap=matplotlib.cm.cool)
```

```
CMGDB.PlotMorseSets(morse_graph, cmap=matplotlib.cm.cool, fig_w=5, fig_h=5)
```

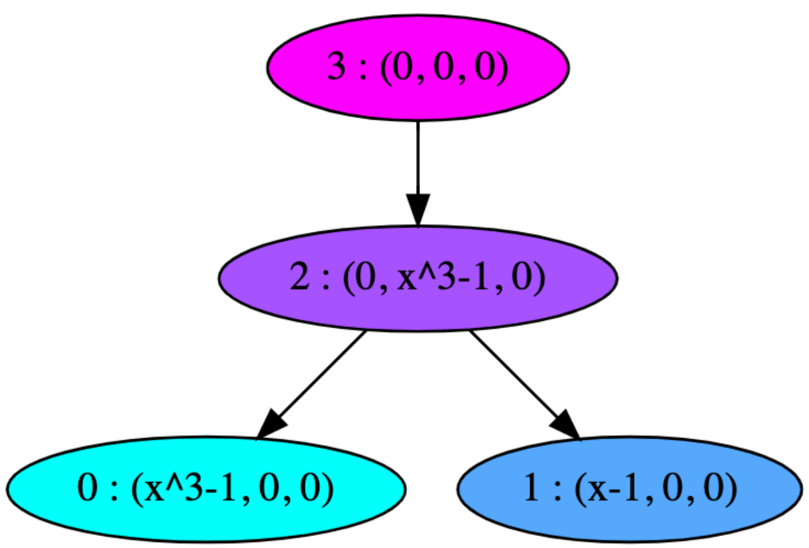


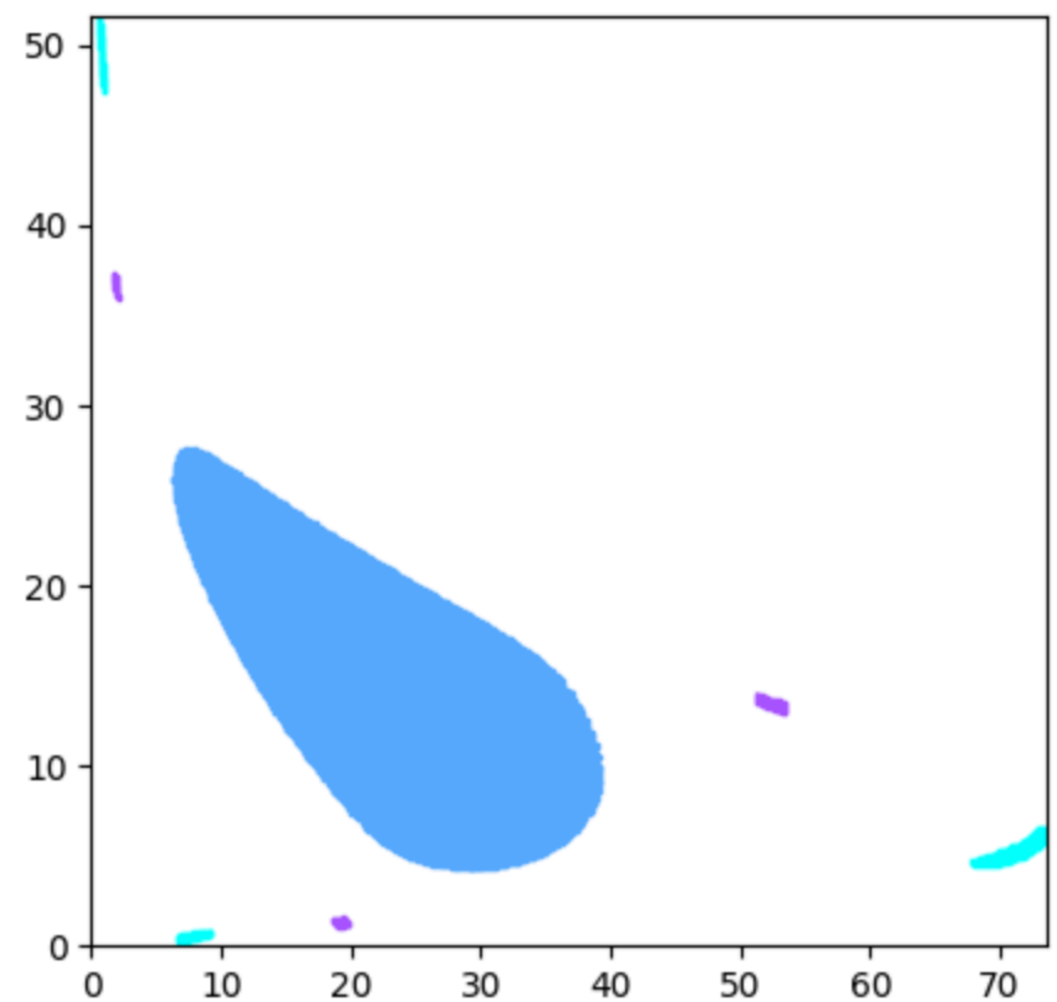


Stable 3-cycle

Examples

```
subdiv_min = 20
subdiv_max = 30
subdiv_init = 4
subdiv_limit = 10000
lower_bounds = [-0.001, -0.001]
upper_bounds = [90.0, 70.0]

model = CMGDB.Model(subdiv_min, subdiv_max, subdiv_init, subdiv_limit,
                    lower_bounds, upper_bounds, F)
```

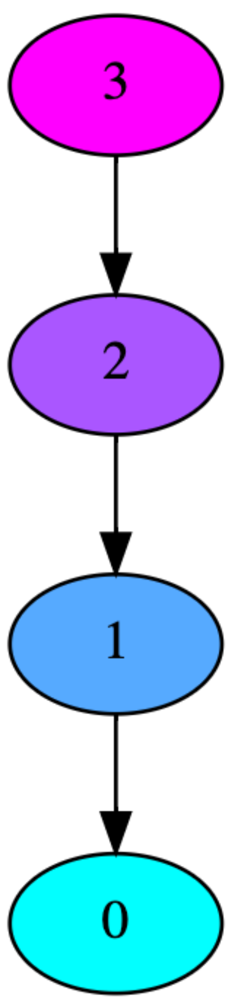


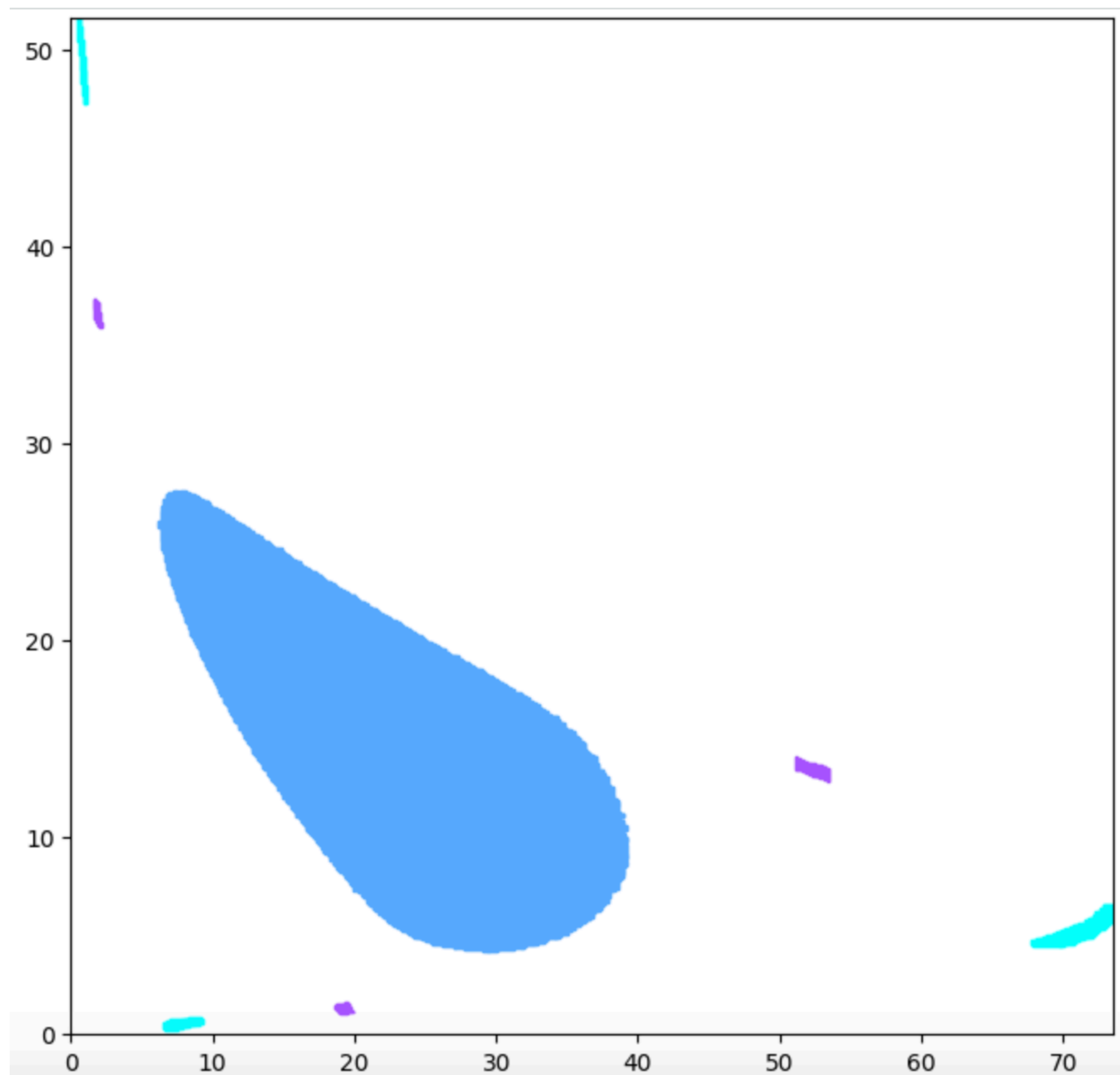


Examples

```
%%time
morse_graph = CMGDB.MorseGraphMap(subdiv_min, subdiv_max, lower_bounds,
                                  upper_bounds, morse_fname, F)
```

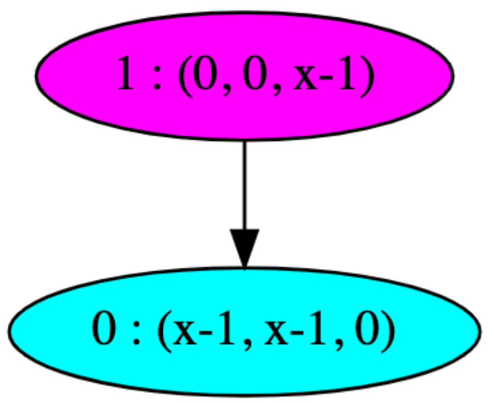
```
CPU times: user 13.1 s, sys: 373 ms, total: 13.5 s
Wall time: 14.1 s
```

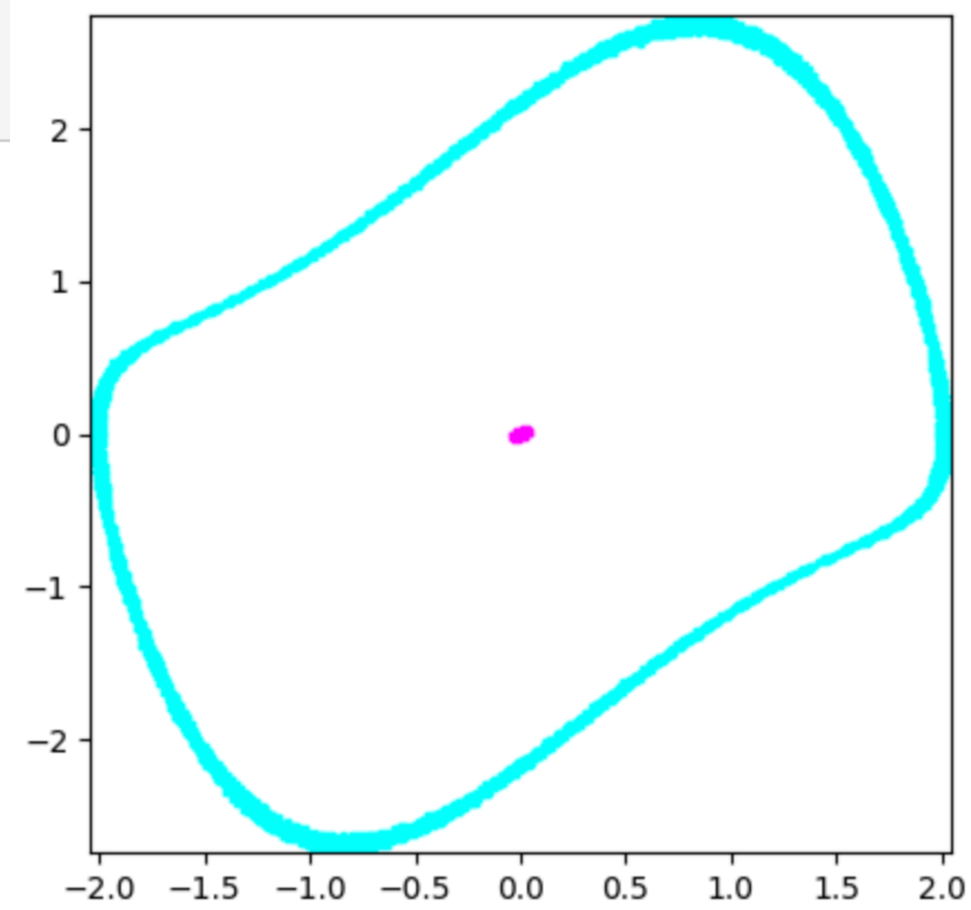




```python
# VanderPol oscillator
def vdp(t, x):
    return np.array([x[1], (1 - x[0]**2) * x[1] - x[0]])

# Time tau map
def f(x):
    tau = 0.5
    h = 0.1
    t_range = [0, tau]
    t, y = RungeKutta4(vdp, t_range, x, h)
    return y[:,-1]

# Define box map for f
def F(rect):
    return BoxMap(f, rect)
```

```python
subdiv_min = 18
subdiv_max = 25
subdiv_init = 8
subdiv_limit = 10000
lower_bounds = [-2.5, -3]
upper_bounds = [2.5, 3]

model = CMGDB.Model(subdiv_min, subdiv_max, subdiv_init, subdiv_limit,
                    lower_bounds, upper_bounds, F)
```





More Examples

Jupyter Notebooks:

https://github.com/marciogameiro/CTD_Tutorial_CRM

Thank you for your attention!

Rutgers:

- K. Mischaikow
- B. Rivas
- E. Vieira
- D. Gameiro

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- W. Kalies

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- T. Gedeon
- B. Cummins
- W. Duncan

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- H. Kokubu
- H. Oka

CMGDB software

https://github.com/marciogameiro/CMGDB

https://github.com/marciogameiro/PyCHomP2

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