RCD 5.0 — Causal Density Geometry and Dynamical Convergence

Refinement: core axioms, theorems, and reproducibility

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Abstract

This document presents the refined core of RCD 5.0: axioms (Gödel–Hilbert style), principal operators, the Dynamical Convergence Theorem (vacuum limit $\to$ GR), and the formal requirements for reproducibility (NASA/ESA-style pipelines + Absolute Rule). Emphasis is on concise, provable statements and operational guidance for exact replication.

1. Scope and objectives

We state the minimal axioms of RCD and derive the principal results needed for physical application and independent verification. The document emphasizes:

- formal axiomatization (dependencies and independence),
- concise statements of theorems with sketches of proof,
- computational reproducibility requirements (Absolute Rule),
- field-theory level consistency (dimensional analysis, cutoff behavior).

2. Axioms (Gödel–Hilbert style)

We list the axioms that serve as the formal basis of RCD. Every theorem below includes an explicit list of axioms on which it depends.

A1 – Causal Combinatorics: The causal density $\rho_{\text{local}}(x)$ is defined as

$$\rho_{\text{local}}(x) = \lim_{V\to 0} \frac{N(\text{causal links in } V)}{V/\ell_P^4},$$

where $N(\cdot)$ is a well-defined combinatorial count over the causal set.

A2 – Cutoff Existence: There exists a universal cutoff density $\rho_{\text{cut}} > 0$ and a finite saturation scale $s_{\text{cut}} > 0$.

A3 – Causal Coupling Constant: A dimensionless coupling C^* (empirically calibrated) relates causal density to geometric response.

A4 – Torsional Saturation: The torsional response is given by an odd, bounded operator $\Omega(s)$ satisfying $\Omega(0) = 0$, $|\Omega(s)| < 1$ and $\lim_{s\to\infty} \Omega(s) = 1$.

A5 – Arithmetic Verifiability (Absolute Rule): All numerical outputs must include stepwise arithmetic logs, units, rounding rules and a SHA-256 digest of the complete computation.

3. Fundamental operators and definitions

$$T_{\text{Causal}}(x) \;=\; \frac{\rho_{\text{local}}(x)}{\rho_{\text{cut}}}\, C^*, \tag{1}$$

$$D_T(x) \;=\; \frac{1}{1 + T_{\text{Causal}}(x)^2}, \tag{2}$$

$$\Omega(s) \;=\; \tanh\!\left(\frac{s}{s_{\text{cut}}}\right) \quad \text{(example realization).} \tag{3}$$

Corrected circular velocity:

$$V_{\text{Causal}}(r) \;=\; V_{\text{Newton}}(r)\, D_T(r).$$

4. Dynamical Convergence Theorem

Statement. (Depends on A1, A2, A3.) In any region where $\rho_{\text{local}}(x) \to \rho_{\text{cut}}$, we have $T_{\text{Causal}} \to 0$, hence $D_T \to 1$ and the RCD geometric corrections vanish; the RCD effective field equations reduce to the Einstein field equations in that limit.

Sketch of proof. From A1–A3 and (**??**) we obtain $T_{\text{Causal}}(x) \to 0$ as $\rho_{\text{local}} \to \rho_{\text{cut}}$. Substitution into (**??**) yields $D_T \to 1$. For any metric ansatz $g_{\mu\nu}^{\text{RCD}} = g_{\mu\nu}^{\text{GR}} \cdot D_T$, as $D_T \to 1$ the geometric sector matches GR term-by-term. Linearization in weak-field yields identical Poisson limit. □

5. Consistency: dimensional analysis and field-theory remarks

- **Dimensions.** ρ_{local} has dimensions $[\text{length}]^{-4}$; C^* is dimensionless; ρ_{cut} shares the same units as ρ_{local}. All equations above are dimensionally consistent.
- **Cutoff and saturation.** The presence of ρ_{cut} and s_{cut} furnishes natural regulators preventing ultraviolet blow-up. The functional form (**??**) ensures boundedness of torsional contributions to curvature invariants.
- **Renormalizability remarks.** RCD is an effective classical modification with built-in cutoff; quantization would require a prescription consistent with the causal combinatorics — this document provides the classical, renormalization-safe backbone (no uncontrolled divergences at classical level).

6. Gödel–Hilbert bookkeeping (theorem dependencies)

For each high-level result we explicitly list axioms used:

- **Dynamical Convergence Theorem:** depends on {A1, A2, A3}.
- **Singularity avoidance (bounded curvature):** depends on {A2, A4}.
- **Empirical rotation-curve reproduction (existence result):** depends on {A1, A3, A5} together with the empirical dataset (SPARC).

This explicit mapping avoids hidden circularity and allows formal auditing.

7. NASA/ESA-style reproducibility protocol (integrated)

(Depends on A5) For any reported numerical experiment the following minimal pipeline must be provided and archived:

1. **Input package:** baryonic mass model, observational errors, seed, code version.
2. **Deterministic execution:** single-run computation with fixed precision (IEEE-754 double or specified multiprecision).
3. **Complete arithmetic log:** every arithmetic operation recorded, units annotated, rounding policy declared.
4. **Output package:** $V_{\text{Causal}}(r)$, $D_T(r)$, $T_{\text{Causal}}(r)$ and residuals to data.
5. **Integrity:** SHA-256 digest of input $\|$ code $\|$ arithmetic log $\|$ output.
6. **Replication check:** reproduce result with an independent pipeline (minimal re-implementation or different language) and confirm the same SHA-256 digest on the arithmetic log.

Failure to provide the above invalidates empirical claims.

8. Minimal worked example (pipeline sketch)

Given a single galaxy:

1. load baryonic mass profile $M_b(r)$ and observational rotation curve $V_{\text{obs}}(r)$,
2. compute Newtonian circular speed $V_N(r) = \sqrt{GM_b(r)/r}$,
3. compute or estimate $\rho_{\text{local}}(r)$ from causal model (A1) or calibrated proxy,
4. evaluate $T_{\text{Causal}}(r)$ and $D_T(r)$,
5. produce $V_{\text{Causal}}(r) = V_N(r)D_T(r)$,
6. compute residuals and χ^2_{red},
7. store arithmetic log and hash as required by A5.

A complete numeric instance must accompany any submission.

9. Notes on empirical calibration

The dimensionless constant C^* and the normalization κ must be reported with uncertainties and source of calibration. Any use of SPARC or other public datasets must cite the dataset and provide a complete reproduction bundle.

10. Closing remarks

This core document is intentionally compact: it fixes the formal basis for RCD 5.0, integrates the highest-level logical bookkeeping, and elevates reproducibility to a formal requirement. All further derivations, datasets and computational artifacts must reference this core and follow A5.

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References

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